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WING-SLOPE TECHNIQUES FOR THE ANALYSIS OF LANGMUIR PROBE CHARACTERISTICS

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ABSTRACT

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The volt-ampere characteristics of Langmuir probes employing ideal geometries (spherical or cylindrical) contain a considerable amount of information, much of which is not utilized in the usual analysis programs. The slopes of the V-i curve for large accelerating potentials contain a substantial amount of information on mass density and temperature. We have developed equations for the current and its slope in this region for both of the above geometries and have applied them to a few typical situations. We conclude that the usefulness of Langmuir probe techniques may be appreciably improved by performing a "total analysis" on the available information. In some cases, measuring the slopes directly will supply the same information with a substantial saving in experimental complexity.

Author

INTRODUCTION:

Langmuir probes have been used extensively for ionospheric investigation for some time. They have been applied primarily to the measurement of electron temperatures by employing a retarding potential analysis and to the measurement of electron densities, usually by observing the current in the "electron saturation" region. Bettinger[1] has suggested an expansion of the analysis to include properties of the positive ions by using the slope of the volt-ampere curve at large accelerating potentials. This "wing-slope" technique was suggested as a means of measuring positive ion temperature with a small diameter spherical probe. The equation governing the behavior of the slope of the volt-ampere characteristic in the region where the retarded charge component is negligible when compared to the accelerated component, is[1]

$$\frac{di}{dV_p} = \left(\frac{8\pi}{mkT}\right)^{\frac{1}{2}} ne^2r^2 , \tag{1}$$

where i is the current to the probe; V_p is the potential of the probe with repect to the plasma; m is the mass of the accelerated constituent; k is the Boltzmann constant; T is the temperature of the accelerated constituent; n is the ambient charge density; e is the electronic charge; and r is the probe radius.

This relation is notable for the absence of a potential dependence, eliminating the need for a knowledge of the vehicle potential and for its dependence on the ambient concentration, mass, and temperature of the species under consideration. If we consider the electron acceleration region and use the electron temperature derived from a retarding potential analysis, we may obtain the charge density with excellent accuracy since the results are independent of the origin of both the current and voltage scales. Assuming charge neutrality, the ion saturation region can then be examined to obtain the product of the ion mass and temperature. If either is known from an independent source, then the other is readily obtainable. In general, the mean ion mass is assumed to be known and this technique yields ion temperatures.

Brace and Reddy[2] have suggested the use of cylindrical geometry which is inherently less sensitive to plasma temperature. The current to a small diameter cylindrical probe has been given by Mott-Smith and Langmuir[3] as

$$i = 2\pi r L n_e e \left(\frac{kT}{2\pi m}\right)^{\frac{1}{2}} \cdot \frac{2}{\sqrt{\pi}} \left(\frac{eV_p}{kT} + 1\right)^{\frac{1}{2}} \qquad (2)$$

where i is the contribution of any given component of the plasma and L is the probe length. The slope of this curve is given by the relation

$$\frac{di}{dV_p} = rLn_e e^2 \left(\frac{2}{eV_p m}\right)^{\frac{1}{2}} \left\{\frac{1}{1 + \frac{kT}{eV_p}}\right\}^{\frac{1}{2}} , \qquad (3)$$

where the term in braces approaches unity as the potential of the probe becomes large with respect to the equivalent thermal potential $\left(\frac{kT}{e}\right)$. This complete absence of a temperature dependence for large probe potentials is the most notable feature of this relation. Functionally, Eq. (1) and Eq. (3) have merely interchanged the temperature and the probe potentials. Eq. (3) may be used to determine any one of the parameters (n_e, m, V_p) in terms of the other two. Since the ambient density and the probe potential are more readily obtained by other means, this relation generally supplies information on the ion mass. If we consider the electron saturation region, where the mass is a known constant, any two points may be used to solve Eq. (3) for both remaining parameters n_e and V_p. In a manner analogous to that suggested for use with the spherical probe, this information may be used to evaluate these constants in the positive saturation region, leading to a value for the mass of the ions.

CYLINDRICAL PROBE:

The foregoing analysis assumes probes at rest with respect to the plasma, and while this approach is valid when considering the ionospheric electrons, it is often not even approximately correct for the heavier ions and seldom completely valid. Let us consider a small diameter cylindrical probe in the extreme case of a very large vehicle velocity, in which

$$U = \tfrac{1}{2}mu^2 \gg kT \quad , \tag{4}$$

where u is the translational velocity of the probe with respect to the plasma at rest. If we neglect end effects, i.e., a probe of infinite length, and orient the probe parallel to the velocity vector u, Eq. (3) will continue to apply. If on the other hand, we orient the probe normal to the velocity vector, then the current to the probe is, according to Mott-Smith and Langmuir[3], of the form

$$i = 2rLn_e eu \left(1 + \frac{eV_p}{U}\right)^{\frac{1}{2}} . \tag{5}$$

The slope of this relation is

$$\frac{di}{dV_p} = rLn_e e^2 \left(\frac{2}{eV_p m}\right)^{\frac{1}{2}} \left(\frac{1}{1 + \frac{U}{eV_p}}\right)^{\frac{1}{2}} . \tag{6}$$

It will be noted that Eqs. (3) and (6) are identical except for the interchange of the energy terms (kT [Eq. (3)] for U [Eq. (6)]). In cases where the probe potential is large compared with these energies, the expressions are identical. This condition may be roughly satisfied at altitudes below 500km with sounding rockets, but at satellite velocities and at the higher altitudes where the ion mass is less, the more general expression must by employed. Under these conditions, the dependence of the slope on the ion mass becomes somewhat stronger, falling between the inverse half power and the inverse power.

This above expression ignores the thermal velocity of the ions and this assumption generally is not valid. A more general expression for the current to the small diameter cylindrical probe has been derived by Kanal[4]

$$i = \frac{2}{\pi^{\frac{1}{2}}} e^{-c^2} \sum_{n=o}^{\infty} \frac{c^n}{n!} \frac{e^{V_o}}{V_o^{n/2}} \gamma(n + \tfrac{3}{2}, V_o) \, J_n(2cV_o^{\frac{1}{2}}) , \tag{7}$$

where

$$c = \frac{u}{v_o} \quad \sin\Theta \tag{8}$$

$$V_o = \frac{eV_p}{kT}$$

where Θ is the angle between the probe axis and the velocity vector $\bar{u}$ and where

$$v_o = \left(\frac{2kT}{m}\right)^{\frac{1}{2}} \quad . \tag{9}$$

The inverse of the incomplete gamma function is defined as

$$\gamma(n + \frac{3}{2}, V_o) = \int_{V_o}^{\infty} e^{-t}\ t^{n+\frac{1}{2}}\,dt \quad , \tag{10}$$

and $J_n(x)$ is the n^{th} order Bessel function of argument x. The slope of this volt-ampere characteristic is given by

$$\frac{di}{dV_p} = \frac{2}{\pi^{\frac{1}{2}}}\left(\frac{e}{kT}\right)e^{-c^2}\sum_{n=o}^{\infty}\frac{c^n}{n!}\frac{e^{Vo}}{V_o^{\,n/2}}\Big\{\gamma(n + \frac{3}{2}, V_o)\ Jn(2cV_o^{\frac{1}{2}}) +$$

$$V_o^{\,n+\frac{1}{2}}\ e^{-V_o}\ \ Jn(2cV_o^{\frac{1}{2}}) - \gamma(n + \frac{3}{2}, V_o)\ \ Jn + 1(2cV_o^{\frac{1}{2}})\Big\} \tag{11}$$

Unfortunately, this expression converges very slowly for large values for the argument (V_o large and $c \geq 1$). As an alternative we seek an approximate expression which will make use of the experimentally controllable condition

$$kT << eV_p \tag{12}$$

The simplest assumption we can make is that the distribution of thermal velocities is of the form

$$\frac{dn}{n} = \frac{1}{2v_o}\,dv \qquad -v_o \le v \le v_o$$
$$= o \qquad v^2 > v_o^2 \tag{13}$$

This replaces the Maxwellian distribution with the first term of its general power expansion, a constant. The limits are chosen to satisfy the normalization condition. The differential of the current to a small diameter cylindrical probe due to the particles of velocity v is given by rewriting Eq. (5)

$$di = 2rLe\left(\frac{2}{m}\right)^{\frac{1}{2}}\left(\tfrac{1}{2}mv^2 + eV_p\right)^{\frac{1}{2}} d^2n \quad , \tag{14}$$

where v is the particle velocity in the plane normal to the probe axis. We chose a coordinate system ($\dot{x}'$, $\dot{y}'$, $\dot{z}'$, x', y', z') at rest with respect to the plasma in which ions possess a two dimensional velocity distribution given by

$$d^2n = \frac{n_e}{4v_o^2}\,d\dot{x}'d\dot{y}' \qquad -v_o \le \dot{x}' \le v_o$$
$$-v_o \le \dot{y}' \le v_o \tag{15}$$

We transform to an unprimed set of coordinates moving with the probe and with the z axis coinciding with the probe axis. The new velocity components in terms of the old are

$$\begin{aligned} \dot{x} &= \dot{x}' + u \sin \Theta \\ \dot{y} &= \dot{y}' \\ \dot{z} &= \dot{z}' + u \cos \Theta \end{aligned} \tag{16}$$

Transforming Eq. (14) to the new coordinate system and integrating over the appropriate limits we obtain

$$i = \tfrac{1}{2} r L n_e e v_o \int_{-1}^{1} \int_{c-1}^{c+1} (\alpha^2 + \beta^2 + \gamma^2)^{\frac{1}{2}} \, d\alpha d\beta \quad ; \tag{17}$$

where $\alpha = \dfrac{\dot{x}}{v_o}$, $\beta = \dfrac{\dot{y}}{v_o}$ and $\gamma^2 = \dfrac{eV_p}{\frac{1}{2} m v_o}$

Integration over β yields

$$i = \tfrac{1}{2} r L n_e e v_o \int_{c-1}^{c+1} \left\{ (1 + \gamma^2 + \alpha^2)^{\frac{1}{2}} + \tfrac{1}{2}(\gamma^2 + \alpha^2) \ln \left[\frac{(\gamma^2 + \alpha^2 + 1)^{\frac{1}{2}} + 1}{(\gamma^2 + \alpha^2 + 1)^{\frac{1}{2}} - 1} \right] \right\} d\alpha \tag{18}$$

Utilizing the assumption of Eq. (12), we may expand the ln term. This reduces Eq. (18) to the form

$$i = r L n_e e v_o \int_{c-1}^{c+1} \left\{ (1 + \gamma^2 + \alpha^2)^{\frac{1}{2}} - \tfrac{1}{2}(1 + \gamma^2 + \alpha^2)^{-\frac{1}{2}} \right\} d\alpha \tag{19}$$

$$\simeq 2 r L n_e e \left(\frac{2}{m} \right)^{\frac{1}{2}} \left\{ eV_p + U \sin^2 \Theta \right\}^{\frac{1}{2}} \tag{20}$$

This result is identical to Eq. (5) although it was derived for arbitrary values of U. The first order terms in the temperature $\tau = kT/(eV_p + U \sin^2 \Theta)$ have vanished and the neglected terms are of the order of τ^2 or greater.

We might improve the accuracy of our result by using a better approximation for the thermal velocity distribution function. To this end we modify (13) for the one dimensional function by adding the next term of the general expansion:

$$\frac{dn}{n} = c_1(c_2 - \nu^2)\, d\nu \qquad -\ell \le \nu \le \ell$$
$$= 0 \qquad \nu^2 > \ell^2 \tag{21}$$

where $\nu = \frac{v}{v_o}$ and c_1, c_2, and ℓ are constants to be determined from auxiliary conditions.

The current to the cylindrical probe becomes:

$$i = 2rLn_e ev_o c_1^2 \int_{-\ell}^{\ell} \int_{-\ell}^{\ell} (c_2 - \alpha^2)\,(c_2 - \beta^2)\,([\alpha + c]^2 + \beta^2 + \gamma^2)^{\frac{1}{2}}\, d\alpha d\beta \tag{22}$$

Integration over β yields:

$$i = 2rLn_e ev_o c_1 \int_{-\ell}^{\ell} (c_2 - \alpha^2)\; 2\ell\zeta \sum_{n=o}^{\infty} \left(\frac{\ell^2}{2n+3} - c_2\right) \frac{1}{(2n+1)\;(2n-1)} \left(\frac{\ell}{\zeta}\right)^{2n} d\alpha \tag{23}$$

where $\zeta = ([a + c]^2 + \gamma^2 + \ell^2)^{\frac{1}{2}}$ (24)

This series is obtained by expanding the ln function resulting from the integration and then combining terms. Invoking Eq. (12), we neglect terms of (ℓ/ζ) of order less than -1. Eq. (23) then reduces to:

$$i = 2rLn_e ev_o c_1^2 \int_{-\ell}^{\ell} (c_2 - \alpha^2) \left\{ \left(c_2 - \frac{\ell^2}{3}\right) 2\ell\zeta - \left(c_2 - \frac{\ell^2}{5}\right) \frac{2\ell^3}{3\zeta} \right\} d\alpha \tag{25}$$

This expression may be integrated to obtain

$$i = 2rLn_e ev_o c_1^2 (2\ell)(c_2 - \frac{\ell^2}{3}) \left\{ \left[\frac{c^2}{12} - \frac{\gamma^2}{8} - \frac{3\ell^2}{8} + \frac{c_2}{2}\right] \ell(X^+ + X^-) \right.$$
$$+ \left[\frac{11\ell^2}{24} - \frac{c^2}{12} + \frac{13\gamma^2}{24} + \frac{c_2}{2}\right] c(X^+ - X^-)$$
$$\left. + (\frac{\gamma^2}{2} + \frac{\ell^2}{2})(c_2 + \frac{\gamma^2}{4} + \frac{\ell^2}{4} - c^2) \ \ln \left[\frac{X^+ + c + \ell}{X^- + c - \ell}\right] \right\} \tag{26}$$

where: $X^{\pm} = (\gamma^2 + c^2 + 2\ell^2 \pm 2c\ell)^{1/2} = (\S^2 + 2\ell^2 \pm 2c\ell)^{1/2}$ (27)

Expansion of Eq. (27) yields:

$$X = \S\left(1 + \frac{\ell(\ell \pm c)}{\S^2} - \frac{1}{2}\frac{\ell^2(\ell \pm c)^2}{\S^4} + \frac{1}{2}\frac{\ell^3(\ell \pm c)^3}{\S^6} - \frac{5}{8}\frac{\ell^4(\ell \pm c)^4}{\S^8} + \ldots\right) \tag{28}$$

The sum and difference terms are:

$$(X^+ + X^-) = 2\S + \frac{2\ell^2}{\S} - \frac{\ell^2(\ell^2 + c^2)}{\S^3} + \frac{\ell^3(\ell^3 + 3\ell c^2)}{\S^5} - \ldots \tag{29}$$
$$\simeq 2\S + \frac{2\ell^2}{\S}$$

$$(X^+ - X^- = \frac{2c\ell}{\S} - \frac{\ell^2(2c\ell)}{\S^3} + \frac{\ell^3(3\ell^2 c + c^3)}{\S^5} - \ldots$$
$$\simeq \frac{2c\ell}{\S} \tag{30}$$

The neglected terms are of the order of $\frac{\ell^2 c}{\S^3}$ or less. Since ℓ^2 is of the order of unity, this term is negligibly small as compared to the previous term ($\ell^2/\S$) unless $c^2 \gg \gamma^2$. In this case, these terms are comparable and both negligible.

The ln term may be handled in a similar fashion

$$\ln\left[\frac{X^+ + c + \ell}{X^- + c - \ell}\right] = \ln\left[\frac{1 + \frac{\ell^2}{\S^2} + \frac{c\ell}{\S^2} + \frac{c}{\S} + \frac{\ell}{\S}}{1 + \frac{\ell^2}{\S^2} - \frac{c\ell}{\S^2} + \frac{c}{\S} - \frac{\ell}{\S}}\right]$$

$$\approx \ln\left[\frac{1 + \frac{\ell}{\S} + \frac{\ell^2}{\S^2}}{1 - \frac{\ell}{\S} + \frac{\ell^2}{\S^2}}\right]$$

$$\simeq \frac{2\ell}{\S} \qquad (31)$$

Applying Eq. (29), (30), and (31) to Eq. (26), we obtain relatively simple expression for the current.

$$i = 2rLn_e ev_o(4\ell^2 c_1^2)(c_2 - \frac{\ell^2}{3})(c_2 - \frac{\ell^2}{4})(\gamma^2 + c^2) \qquad (32)$$

This expression is notable for its lack of an explicit temperature dependence as was the case previously with the less general assumptions. These results are largely independent of our assumptions concerning the form of the thermal velocity distribution function, requiring only that:

$$4\ell^2 c_1^2 (c_1 - \frac{\ell^2}{3})(c_2 - \frac{\ell^2}{4}) = 1 \qquad (33)$$

The normalization requirement

$$\int_{-\ell}^{\ell} c_1 (c_2 - \nu^2)\, d\nu = 1 \tag{34}$$

yields the result:

$$2\,\ell c_1 (c_2 - \frac{\ell^2}{3}) = 1 \tag{35}$$

Applying Eq. (35) to Eq. (33) yields:

$$c_2 = \frac{5}{12}\,\ell^2 \tag{36}$$

which is a second restriction on the form of the assumed velocity distribution.

One suspects that a more vigorous treatment of the expansions of Eq. (26) might yield the result:

$$i = 2rLn_e ev_o (4\ell^2\, c_1^2)(c_2 - \frac{\ell^2}{3})^2\, \xi \tag{37}$$

Where the discrepancy between the terms $(\frac{\ell^2}{4})$ and $(\frac{\ell^2}{3})$ result from truncation error. This would then remove the condition of Eq. (2). In any case, it follows that Eqs. (5) and (6) are applicable whenever the condition of Eq. (12) is satisfied.

SPHERICAL GEOMETRY

Let us now turn our attention to the case of a spherically symmetric probe moving with an arbitrary velocity u in a neutral Maxwellian plasma. We assume the diameter of the probe to be small so that sheath distortions and wake effects can be ignored. We follow our previous approach and consider the current to the probe due to a monoenergetic beam of charges with velocity v given by Mott-Smith and Langmuir[3]:

$$i = \Pi r^2 e n_e v \quad \left(1 + \frac{eV_p}{\frac{1}{2}mv^2}\right) \tag{38}$$

We consider a set of spherical coordinates (v, Θ, ϕ), fixed with respect to the probe, with the prime axis $(\Theta = o)$ oriented parallel with the drift velocity u. The Maxwellian velocity distribution function in this coordinate system is:

$$dn = \frac{n_e}{(\Pi^{\frac{1}{2}} v_o)} \quad \exp\left\{-\frac{1}{v_o^2}(v^2 + u^2 - 2uv\cos\Theta)\right\} v^2 \sin\Theta \, dv d\Theta d\phi \tag{39}$$

We convert Eq. (38) to differential form, combine it with Eq. (39) to obtain the differential of the current, and integrate over the appropriate limits:

$$i = \frac{\Pi r^2 e n_e}{(\Pi^{\frac{1}{2}} v_o)^3} \int_0^{2\Pi} \int_0^{\Pi} \int_0^{\infty} \left[1 + \frac{eV_p}{\frac{1}{2}mv^2}\right] v^3 \exp\left\{-\frac{1}{v_o^2}(v^2 + u^2 - 2uv\cos\Theta)\right\} \sin\Theta \, dv d\Theta d\phi \tag{40}$$

This expression may be readily integrated:

$$i = 2\Pi^{\frac{1}{2}} r^2 n_e e v_o \left\{ e^{-c} + \frac{1}{c} \int_0^c (\gamma^2 + c + \beta^2)\, e^{-\beta^2} d\beta \right\} \tag{41}$$

The integral is the sum of three incomplete Γ functions. This reduces to the expression:

$$i = 2\Pi^{\frac{1}{2}} r^2 n_e e v_o \, (1 + \gamma^2)^{\frac{1}{2}} \tag{42}$$

for zero drift velocity. For values of u such that $u > 2v_o$

$$i \simeq \Pi r^2 n_e e u \left(1 + \frac{eV_p}{\frac{1}{2} m u^2} \right)^{\frac{1}{2}} \tag{43}$$

The slope of Eq. (41) is:

$$\frac{di}{dV_p} = \frac{2\Pi r^2 n_e e^2}{mu} \, \mathrm{erf}\,(c) \tag{44}$$

where erf(c) is the error function defined as:

$$\mathrm{erf}(c) = \frac{2}{\Pi^{\frac{1}{2}}} \int_0^c e^{-\beta^2} d\beta \tag{45}$$

This has the limiting values:

$$\mathrm{erf}\,(c) \simeq \frac{2}{\Pi^{\frac{1}{2}}} c \qquad 0 \le c \le \tfrac{1}{2}$$

$$\mathrm{erf}\,(c) \simeq 1 \qquad c \ge 2 \tag{46}$$

For small values of u Eq. (44) reduces to Eq. (1). It is interesting to note that in every case, for both probe geometries, the slope is inversely proportional to a momentum.

APPLICATIONS

If the drift velocity is small ($u < \frac{1}{2}v_o$), then the slope of the cylindrical probe is characterized by the momentum associated with the accelerating potential. The sphere on the other hand, is sensitive to the ion thermal velocity. The slope with both geometries is dominated by the drift momentum when u is sufficiently large. This criteria usually will be met by the sphere at satellite velocities in the ionosphere. (For H^+ at 2000^0K, v_o = 5.75 km/sec and $u \approx 8$ km/sec.) Hence, the sphere would appear to be an excellent device for measuring ion mass in this situation. Its dependence is strong ($\alpha \frac{1}{m}$) and depends only on n_e, u and constants. The same probe could be used to measure the electron temperature since the electron thermal velocity is much larger than the vehicle velocity.

The cylindrical probe has inherent aspect sensitivity so that a low drift velocity may always be approximated by orienting the probe axis parallel to the velocity vector. In practice, the sphere, because of the mounting problem, also has aspect sensitivity which requires correction. (This problem has been treated elsewhere[5].)

Although the sphere displays a greater sensitivity to ion mass, it requires an independent measurement of charge concentration (or alternatively electron temperature). The cylinder can measure n_e by electron acceleration, only requiring a knowledge of the vehicle potential. Error in this last parameter may be minimized by making the accelerating potential large.

If we measure the ratio (R) of the slopes of the $(V_p - i)$ curves at large positive and negative potentials we obtain:

$$R = \frac{\left.\frac{di}{dV_p}\right|_{V_p^-}}{\left.\frac{di}{dV_p}\right|_{V_p^+}} = \left[\left(\frac{m_p}{m_e}\right)\left(\frac{V_p^+}{V_p^-}\right)\left(M + M^2 \frac{m_p u^2 \sin^2\Theta}{2eV_p^+}\right)\right]^{\frac{1}{2}} \tag{47}$$

where $V_p^{\pm}$ are the absolute values of the probe to plasma potentials; M is the mass of the ions in units of m_p, the proton mass. This may be solved for M to obtain:

$$M = \left(\frac{eV_p^+}{m_p u^2 \sin^2\Theta}\right)\left\{\left[1 + R^2\left(\frac{m_e}{m_p}\right)\left(\frac{V_p^-}{V_p^+}\right)\frac{2m_p u^2 \sin^2\Theta}{eV_p^+}\right]^{\frac{1}{2}} - 1\right\} \tag{48}$$

The foregoing analysis assumes a single ionic species, an assumption which is often not valid in the ionosphere. The possible ion types range from H^+ to N_2^+ and it is important to note that the effective mass to be employed in the previous equations is not in general the mean mass $(\bar{m})$ usually defined as:

$$\bar{m} = \frac{\sum_{i=1}^{j} n_i m_i}{\sum_{i=1}^{j} n_i} \tag{49}$$

since this requires a linear mass dependence. The mean ionic mass has often (and incorrectly) been employed by investigators in evaluating the ion current to various types of Langmuir probes. For instance, for a sphere at rest, the appropriate effective mass (m_{eff}) is given by

$$m_{eff} = \left[\frac{\sum_{i=1}^{j} n_j}{\sum_{i=1}^{j} \frac{n_i}{m_j^{1/2}}} \right]^2 \tag{50}$$

This phenonmena is illustrated by Figure 1 where we have plotted the slope for the (V_p-I) characteristic of a cylinder assuming "typical" ionospheric conditions. The slope is the sum of the individual contributors since:

$$i = (8r^2L^2e^3V_p)^{1/2} \sum_{i=1}^{j} \frac{n_i}{m_i^{1/2}}$$

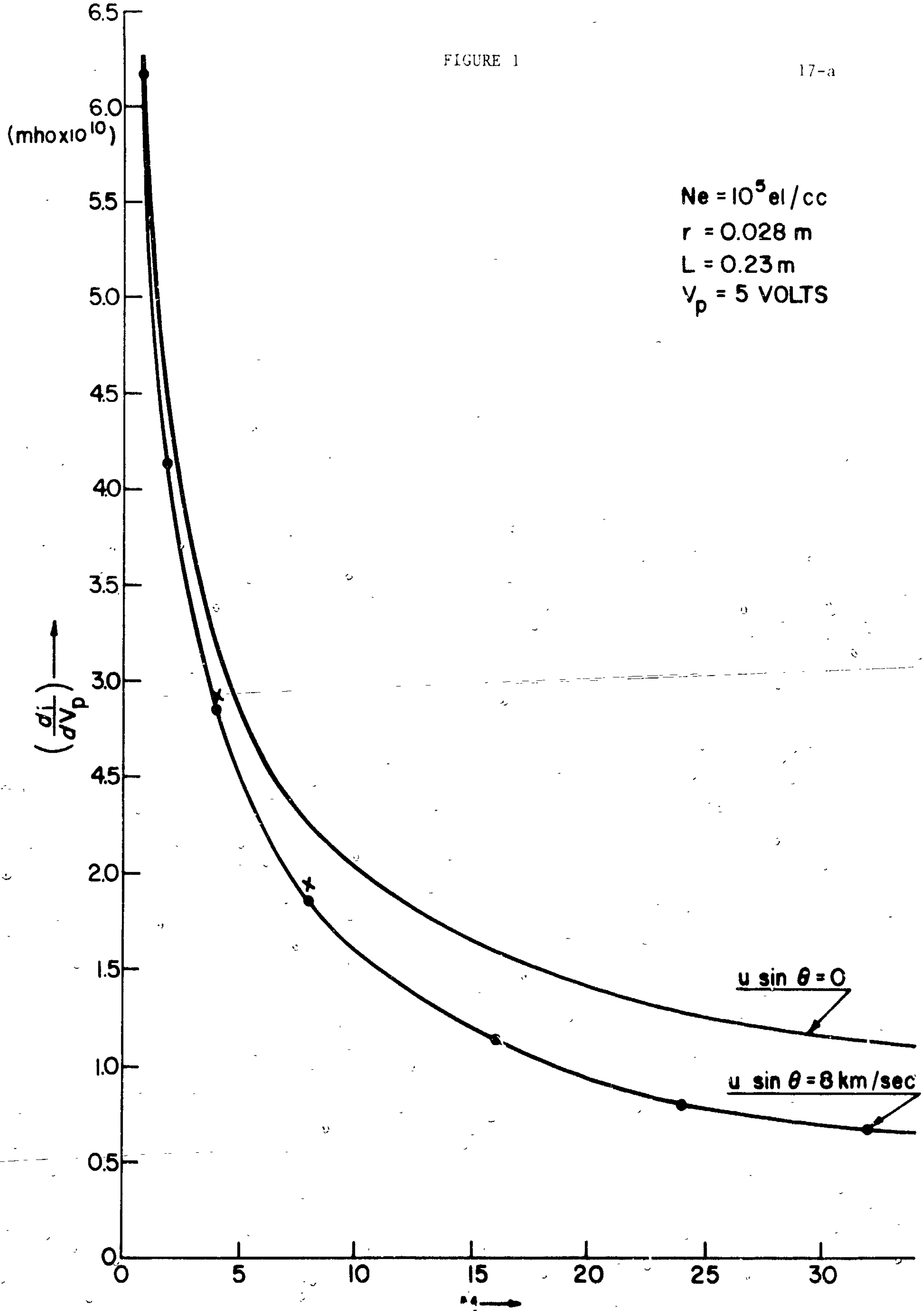
$$= \left(\frac{8r^2L^2e^3V_p n^2_e}{m_{eff}} \right)^{1/2} \tag{51}$$

where we have assumed Eq. (12) to be applicable. It is apparent that the lower masses are much more heavily weighted. For instance, if we assume u sinΘ= 8km/sec; a slope of 2.0 x 10^{-10} mho's and ion species of O^+ and H^+, we find that the fraction of H^+ is only 13%.

If we measure the dependence of the slope ($S = \frac{di}{dV_p}$) or the current (i) as a function of aspect angle we can, theoretically at least, determine the mass and density distributions of ions. We may write the slope in the form:

$$S(\Theta) = K_1 \sum_{i=1}^{j} \frac{n_i}{m_i^{1/2}} (1 + K_2 m_i \sin^2\Theta)^{-1/2} \tag{52}$$

FIGURE 1


6.5
6.0
(mhox10^10)
5.5
5.0
4.5
4.0
3.5
3.0
4.5
2.0
1.5
1.0
0.5
0
(di/dVp)
Ne = 10^5 el/cc
r = 0.028 m
L = 0.23 m
Vp = 5 VOLTS
u sin θ = 0
u sin θ = 8 km/sec
0
5
10
15
20
25
30

This equation involves 2j unknowns and, by selecting 2j values of Θ, we can, in principle, solve for these unknowns. In practice, this approach suffers from two difficulties. First, since our measurement techniques will introduce finite errors, the number of independent equations in Θ is sharply limited. Three ion types is probably the upper limit of resolution for the best experimental accuracy and two types would be more typical. The second problem is the non-linear nature of the relation which makes a closed form solution impossible. The equations can, however, by solved by iteration.

This situation is somewhat better than it might appear since we have several auxillary conditions we can apply. First we have the summation of the concentrations:

$$n_e = \sum_{i=1}^{j} n_i \tag{53}$$

More importantly, we have prior knowledge of the probable ion masses so that this approach should yield reasonable information on the ion species and their concentrations.

This general approach finds a number of applications. For instance, when considering the electron dominated regions we may write;

$$V_T = \frac{i}{\left(\frac{di}{dV_p}\right)} = \frac{kT}{e} + f\ (i_i,\ i_p,\ i_o,\ V_p) \qquad \frac{eV_p}{kT} < 0 \tag{54}$$

where the second term takes into account the ion and photoelectric currents and a possible zero shift of the current axis. One could eliminate the second term by taking the ratio of the first to the second derivative; however, experimental accuracies will not usually permit this procedure without appreciable smoothing of the data. While the correction term is usually quite small, it can lead to substantial errors in evaluating $\left(\frac{kT}{e}\right)$ with both Equation (54) and the more conventional ln i versus V_p curves. If we plot V_T versus V_p, we may eliminate f from Equation (54). (This approach is treated in some detail elsewhere.)

The foregoing are only two of many possible applications of wing-slope techniques. We believe that "ideal geometries" should be employed with Langmuir probes for ionospheric investigations so that a "total analysis" of the data will be possible. These internal cross checks with the same detector can appreciably improve accuracy and support a substantial increase in our confidence in the results obtained from such devices.

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