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Some General Aspects of Geomagnetically Trapped Radiation*

by

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ABSTRACT

This paper provides a sketch of the physical principles of radiation belts and an abridged summary of the present state of knowledge of the geomagnetically trapped radiation. As such, it is intended as an introduction to detailed papers on specific topics which constitute the remainder of the Bergen conference.

Author

Necessary and Sufficient Conditions for the Existence of a Planetary Radiation Belt

The possibility of the durable confinement (or trapping) of a moving, electrically-charged particle in the field of a static magnetic dipole was discovered by Störmer [1907]. The rigorous theory of Störmer shows that permanently bounded motion is dynamically possible within, and only within, the bounded region $r_1 \leq r \leq r_2$ where r_1 and r_2 specify two surfaces of revolution about the axis of the magnetic dipole given by the following:

$$\frac{r_1}{b} = \frac{\cos^2 \lambda}{-(\gamma/b) + \left\{(\gamma/b)^2 + \cos^3 \lambda\right\}^{1/2}} \tag{1}$$

and

$$\frac{r_2}{b} = \frac{\cos^2 \lambda}{-(\gamma/b) + \left\{(\gamma/b)^2 - \cos^3 \lambda\right\}^{1/2}} \; . \tag{2}$$

In equations (1) and (2) the position of a point is specified by spherical coordinates about the point dipole with the magnetic dipole $\underline{M}$ directed along the negative polar axis. The radius is denoted by r, the latitude by λ, and the longitude by ω. The quantity $b \equiv (ZeM/pc)^{1/2}$ where Ze is the electrical charge on the particle, p is the scalar momentum, and c is the velocity of light. γ is an arbitrary constant of integration of the equations of motion which specifies, essentially, the conditions of injection of the particle into the field. Both b and γ have dimensions of length.

For a dipole of moment equal to that of the earth ($M = 8.06 \times 10^{25}$ gauss cm^3), Table I [cf. Van Allen, 1962] gives maximum possible values of r_2 and the corresponding values of r_1 (both at $\lambda = 0°$) for protons and electrons of the energies shown (1 earth radius = 6.371×10^8 cm). The idealized model of Störmer has a basic intellectual appeal, places firm upper limits on the trapping capability of a dipole field, and guides the understanding of actual situations.

But the real geophysical system is found to have trapping capabilities which are so greatly inferior to those illustrated by Table I as to render the idealized model of little practical utility. The principal reasons for this fact are the following:

(a) Presence of the solid earth.

(b) Presence of the earth's atmosphere, extending as it does with non-negligible density to several earth radii.

Table I

Störmer Trapping Boundaries

Maximum r_2 Earth Radii	Corresponding r_1 Earth Radii	Kinetic Energy E 10^9 eV Proton	Kinetic Energy E 10^9 eV Electron
1.00	0.41	58.6	59.5
2.00	0.83	14.0	14.9
4.00	1.66	2.90	3.72
6.00	2.49	0.96	1.65
10.0	4.14	0.173	0.60
15.0	6.21	0.036	0.264

(c) Interaction of the earth's magnetic field with hot ionized gas flowing radially outward from the sun through the solar system (the solar wind).

(d) Occurrence of electromagnetic perturbations of terrestrial origin.

Thus, the establishment of sufficient conditions for the durable trapping of particles having specified energy and injection conditions has become a dominantly observational undertaking.

The Störmer theory and other simple dynamical theories treat the motion of single, isolated particles, i.e., they neglect the interactions between particles and the magnetic perturbations which are caused by the electrical currents due to motion of the particles themselves. Thus, they yield no information whatever on the maximum intensities or number densities of particles which can be trapped, irrespective of their individual kinetic energies. Some crude guidance on this matter is provided by rudimentary theory of the magnetic confinement of plasma, which considers the diamagnetic effect of gyrating particles in a magnetic field and shows that the quantity

$$\beta = \frac{n\,\overline{E}}{(B^2/8\pi)} \tag{3}$$

must be always less than unity, and in particular cases, considerably less than unity. In (3), n is the number density of charged particles having average kinetic energy $\overline{E}$ and B is the unperturbed magnetic field strength due to fixed currents. Table II gives some sample numerical values of n and of omnidirectional intensities J_o in the equatorial plane in the geomagnetic field ($B = M/r^3$) for three classes of particles, and for $\beta = 0.1$.

The Störmer theory may be regarded as placing an absolute lower limit on the magnitude of M for a planet in order that particles of a specified energy can be durably trapped in a specified region. A considerably higher value of this lower limit is required when one considers the influence of the solar wind. The rough criterion for the value which M must have in order to permit trapping of particles of any energy out to a radial distance r is that the outward pressure of the planet's magnetic field ($B^2/8\pi$) at r must equal or exceed the inward dynamic pressure of the solar wind ($n\ m\ v^2$). In the foregoing, n is the number density of charged particles of mass m and directed

Table II

Number Densities n and Omnidirectional Intensities J_o Corresponding to $\beta = 0.1$ in the Geomagnetic Field at $\lambda = 0°$

r	$B^2/8\pi$	Protons E = 100 keV n	Protons E = 100 keV J_o	Electrons E = 40 keV n	Electrons E = 40 keV J_o	Electrons E = 1.0 MeV n	Electrons E = 1.0 MeV J_o
earth radii	ergs cm^{-3}	cm^{-3}	$(cm^2\ sec)^{-1}$	cm^{-3}	$(cm^2\ sec)^{-1}$	cm^{-3}	$(cm^2\ sec)^{-1}$
1.5	3.38×10^{-4}	211.	9.2×10^{10}	528.	5.93×10^{12}	21.1	5.96×10^{11}
2.0	6.01×10^{-5}	37.5	1.64×10^{10}	93.8	1.05×10^{12}	3.75	1.06×10^{11}
3.0	5.28×10^{-6}	3.30	1.44×10^{9}	8.25	9.26×10^{10}	0.33	9.32×10^{9}
4.0	9.40×10^{-7}	0.59	2.58×10^{8}	1.48	1.66×10^{10}	0.059	1.67×10^{9}
5.0	2.46×10^{-7}	0.154	6.75×10^{7}	0.385	4.32×10^{9}	0.0154	4.35×10^{8}
7.0	3.27×10^{-8}	0.0204	8.94×10^{6}	0.0510	5.61×10^{8}	0.00204	5.76×10^{7}

velocity v in the solar wind [Van Allen, Frank, Krimigis, and Hills, 1965]. Omitting factors of the order of unity, this criterion gives the maximum radial extent of a trapping region:

$$r_{max} \approx \left(\frac{M^2}{8\pi\ n\ m\ v^2} \right)^{1/6} . \qquad (4)$$

An alternative formula in terms of the radius of the planet r_P and the magnetic field at its surface B_P is

$$\frac{r_{max}}{r_p} \approx \left(\frac{B_P^2}{8\pi\ n\ m\ v^2} \right)^{1/6} . \qquad (5)$$

For the solar wind, $m = 1.67 \times 10^{-24}$ g and at 1 A.U. $n \approx 5$ cm^{-3} and $v \approx 5 \times 10^7$ cm sec^{-1} [Snyder, Neugebauer, and Rao, 1963]. Hence from (5), for the earth ($B_E \sim 0.31$ gauss):

$$\frac{r_{max}}{r_E} \approx 7.5,$$

in rough agreement with observation.

In equations (4) and (5), n is assumed to have an inverse square dependence on heliocentric radial distance and v, to be independent of this distance. A minimum, necessary condition for the existence of a planetary radiation belt is that M be sufficiently great that r_{max} exceeds the radial distance to the top of the appreciable atmosphere of the planet (to be defined more quantitatively in a later section).

It is generally believed that the foregoing criterion is also a sufficient condition for the presence of durably trapped, energetic particles in the magnetic field of a planet (or of a satellite of a planet). This belief has its foundation in the observational knowledge of the earth system and in the belief that all other planetary bodies of the solar system are subjected to qualitatively similar, external influences as is the earth, viz., the solar wind and solar and galactic cosmic rays (as well as the possibly irrelevant electromagnetic radiations from the sun).

There is no existing theory of a fundamental nature for predicting the detailed structure (particle composition, energy spectra, spatial distribution of intensities, etc.) in a planetary radiation belt, though equation (3) with $\beta \sim 0.1$ is believed to provide an upper limit on the kinetic energy density of charged particles at any point therein.

The theory of particle trapping in a magnetic field of higher multi-polarity than that of a dipole has not been worked out, to the author's knowledge.

General Nature of the Earth's Radiation Belts

Knowledge of the particle populations in regions around the earth, or radiation belts, has developed almost entirely as an observational undertaking, with little detailed theoretical guidance. Specific observational work has been directed toward establishing:

(a) Sufficient conditions for durable trapping.

(b) Identification of trapped particles and spatial, temporal, and spectral characteristics of each identified species.

(c) Sources of observed particles.

(d) Effective trapping lifetimes.

(e) Losses of particles.

(f) Diffusion of particles.

(g) Acceleration processes.

There are, in addition, a large number of related matters such as the configuration of the geomagnetic field, heating and ionization of the atmosphere, and the relationship of trapped particles to the aurorae.

To a first crude approximation, all durably trapped particles in the geomagnetic field lie within a toroidal region bounded by two surfaces of revolution about its magnetic axis, the inner one a spherical surface at an altitude of ~ 700 km above the solid earth [Van Allen, McIlwain, and Ludwig, 1959] [Yoshida, Ludwig, and Van Allen, 1960] and the outer one a magnetic shell of dipolar lines of force emanating from the earth at $\lambda \sim 70°$ and crossing the magnetic equator at $r/r_E \sim 8$ [Frank, Van Allen, and Macagno, 1963] [Freeman, 1964] [Frank, 1965].

The nature of the inner boundary is reasonably well understood in terms of simple energy loss in the gaseous atmosphere of the earth [Ray, 1960] [Walt, 1964]. The general plausibility of this statement can be checked by simply determining the length of time which a specified particle can travel in a gas of appropriate density, then comparing the result with a value of the order of 10^7 sec [Van Allen, 1964]. If the ratio is very much less than one, then the corresponding intensity in a quasi-stationary state will be very much less than the maximum value of that component anywhere in the radiation belt.

The general nature of the outer boundary of the durable trapping region is understood in terms of the criterion expressed by equation (5).

Particle Populations

A complete observational knowledge of the particle populations in the radiation belt in a time-stationary state at a given moment may be shown [Van Allen, 1962] to be equivalent to knowing

$$j_i \; (L, \alpha_o, E) \; . \qquad (6)$$

In expression (6), j_i is the differential unidirectional intensity of the i^{th} species of particle (i = proton, electron, alpha particle, etc.) at equatorial pitch angle α_o in the kinetic energy range dE at E, and L specifies the magnetic shell [McIlwain, 1961].

There are now reasonably comprehensive bodies of observations of j_i (or the equivalent) for electrons of 40 keV $< E_e <$ 2 MeV [Frank, Van Allen, and Hills, 1964] [Armstrong, 1965]; for protons 100 keV $< E_p <$ 4.5 MeV and electrons 10 keV $< E_e <$ 100 keV [Davis and Williamson, 1963]; for protons 10 MeV $< E_p <$ 200 MeV [Freden, Blake, and Paulikas, 1965]; and for protons 1.1 MeV $< E_p <$ 63 MeV and electrons 150 keV $< E_e <$ 1.0 MeV [McIlwain, 1963]. Recent work is described in papers by principal workers in the field at the present conference.

The identification of trapped alpha particles $E_\alpha \sim 2$ MeV has been reported [Van Allen and Krimigis, 1965], though only limited and preliminary information on their intensities and spatial distribution is available at this time.

There continues to be a glaring deficiency in the knowledge of protons $E_p < 100$ keV and electrons $E_e < 40$ keV.

General Energetic Considerations

The total magnetostatic energy in the field of a dipole of moment M external to a sphere of radius a centered on the dipole is

$$W_a = \iiint (B^2/8\pi)\ dV = \frac{M^2}{3\ a^3} \tag{7}$$

[Chapman and Bartels, 1940].

Using (7) and taking a to be the radius of the solid earth, 6.371×10^8 cm, and $M = 8.06 \times 10^{25}$ gauss cm^3 the magnetostatic energy of the geomagnetic field external to the earth is found to be

$$W_a = 8.37 \times 10^{24} \text{ ergs} . \tag{8}$$

It has been argued by Dessler and Vestine [1960] on the basis of the maximum admissible contribution of external currents to the geomagnetic field that an upper limit on the total kinetic energy of geomagnetically trapped particles is $\sim 6 \times 10^{22}$ ergs or less than one percent of the external field energy. The observed distributions of particles do not exceed this limit [Hoffman and

Bracken, 1965], which may be written as the volume average $< \beta > = 0.01$.

More detailed examination of this matter is of interest. The magnetostatic energy between magnetic shells differing in L value by dL at L (measured in units of a) external to a sphere of radius a is found to be

$$\frac{dW_a}{dL} = \left(\frac{M^2}{2a^3} \right) \frac{(L - 1)^{1/2}}{L^{5/2}} \quad . \tag{9}$$

For the earth:

$$\frac{dW_a}{dL} = 1.26 \times 10^{25} \; \frac{(L - 1)^{1/2}}{L^{5/2}} \; \frac{\text{ergs}}{\text{unit L}} \quad . \tag{10}$$

The quantity $\frac{dW_a}{dL}$ has a maximum value at $L = 1.25$.

The area of an annular ring in the equatorial plane dA_o bounded by two circles of radius a L and a (L + dL) is given by

$$\frac{dA_o}{dL} = 2\pi \, a^2 \, L \tag{11}$$

and at r = a the area of a ring bounded by the same magnetic shells and perpendicular to $\underline{B}$, dA_a, is smaller by the factor:

$$\frac{dA_a}{dA_o} = \frac{B_o}{B_a} = \frac{1}{L^3 \; (4 - 3L^{-1})^{1/2}} \quad . \tag{12}$$

Thus, combining (9), (11), and (12),

$$\frac{dW_a}{dA_a} = \frac{M^2}{8\pi \; a^5 \; L} \; (4L^2 - 7L + 3)^{1/2} \tag{13}$$

(attributed equally to both hemispheres).

or in numerical form

$$\frac{dW_a}{dA_a} = 2.47 \times 10^6 \; \frac{(4L^2 - 7L + 3)^{1/2}}{L} \; \frac{\text{ergs}}{\text{cm}^2} \quad . \tag{14}$$

The following is a numerical example of interest. At L = 5 (or $\lambda_a = 63.4°$), $\frac{dW_a}{dA_a} = 4.07 \times 10^6 \; \frac{\text{ergs}}{\text{cm}^2}$. Thus if this magnetic shell were populated with charged particles to an average $\beta = 0.01$ and the particles were subjected to a pure precipitation process involving only an appropriate change of their pitch angles, an average precipitation of 1 erg/cm^2 sec in both hemispheres could be maintained for 4×10^4 seconds or about 11 hours. This example provides a graphic illustration of the limited storage capability of the geomagnetic field.

Another formula of occasional use is the one for volume:

$$\frac{dV_a}{dL} = \frac{3}{35} \left(\frac{4\pi a^3}{3} \right) \frac{(L - 1)^{1/2}}{L^{3/2}} (5 + 6L + 8L^2 + 16L^3) . \tag{15}$$

At L = 5,

$$\frac{dV_a}{dL} = 3.71 \times 10^{28} \frac{\text{cm}^3}{\text{unit L}} .$$

The greatest known "loading" of the geomagnetic field by energetic particles, as measured by β, at low L values was due to the Starfish nuclear burst of 9 July 1962. After the initial transients of the first few hours there was an omnidirectional intensity of 1.5×10^9 (cm^2 sec)$^{-1}$ of electrons (having a fission-like spectrum) at L = 1.2 and $\lambda = 0°$. The corresponding value of β is $\sim 10^{-4}$. The natural loading of the field here is several orders of magnitude less.

By contrast, the natural loading of the field in the outer zone approaches values which suggest that the population there may be limited by plasma instabilities. The entire population of the radiation belts may be judged unstable against cooperative processes on the simple ground that the velocity distribution is everywhere anisotropic. The only question is the time rate of growth of instabilities and this question is not yet answered within a proper theoretical framework.

For example, at $L = 4$ on the equator, values of J_o for electrons $E_e > 40$ keV as great as 3×10^8 $(cm^2 \; sec)^{-1}$ have been observed [Frank, Van Allen, and Hills, 1964] and values of J_o for protons $E_p > 100$ keV of the order of 3×10^7 $(cm^2 \; sec)^{-1}$ are reported by Davis at this conference. The corresponding value of β is ~ 0.03, with the protons making the principal contribution. It is important to note that the true value, including particles below present experimental thresholds, is, of course, higher than that just estimated and may reasonably be as much as 0.10.

It is perhaps remarkable that the time variations of the population of outer zone electrons are much greater than those for protons despite the fact that the loading of the field is due principally to the latter. This observed fact may be taken to signify that the population of electrons is more closely coupled to the ambient medium, both electrostatically and electromagnetically and hence exhibits a much more rapid growth of trapping instabilities than does the proton population.

Characteristic Time Scales

In principle, one would wish to remove all energetic particles from the earth's magnetic field (as by "de-gaussing" the earth for a brief period), then to inject known populations of known particles and to follow their subsequent time histories in observational detail. The closest practical approach to this ideal thus far has been by the injection of populations of fission-product decay-electrons from atomic bursts (see pertinent papers of this conference). This approach has yielded a large body of knowledge on the gross physical dynamics of populations of energetic electrons.

It is found that for $L < 1.25$, the time histories of such particles can be well understood in terms of simple Coulomb scattering and energy loss in the residual atmosphere. Toward higher L values other, unknown loss processes dominate and observed lifetimes beyond $L \sim 1.8$ become many orders of magnitude less than those attributable to atmospheric losses.

Extended studies of the natural population of trapped particles yield results of a generally comparable nature. The following is an abridged summary of characteristic lifetimes:

[Van Allen and Lin, 1960]
[Pizzella, McIlwain, and Van Allen, 1962]
[Forbush, Pizzella, and Venkatesan, 1962]
[Frank, Van Allen, and Hills, 1964]
[McIlwain, 1963]
[Davis, This Conference]

(a) <u>For $1.2 < L < 1.3$</u>

Protons $E_p > 20$ MeV, ~ 1 year

(b) <u>For $1.5 < L < 2.5$</u>

Electrons $E_e > 1$ MeV, days to weeks

(c) <u>For $L > 2.5$</u>

Electrons $E_e > 1$ MeV, hours to weeks

(d) <u>For $L > 2.5$</u>

Electrons $E_e > 40$ keV, hours to weeks

(e) <u>For $L > 2.0$</u>

Protons $E_p > 100$ keV, markedly stable with small scale fluctuation in times of the order of days

In the outer zone, $L \gtrsim 2.0$, fluctuations of particle populations are closely associated with geomagnetic disturbances, thus strongly suggesting a common cause--specifically fluctuations in one or more of the characteristic parameters of the solar wind.

Sources of Observed Particles

The evidence cited in the preceding section leaves little doubt that the dominant source of energy of the energetic particles in the radiation belts (as well as of aurorae and magnetic storms) is the solar wind. The typical power flux of the solar wind at the earth's orbit [Snyder, Neugebauer, and Rao, 1963] is 0.1 erg/cm^2 sec. The total geomagnetic system presents a circular frontal area of radius ~ 12a and is, therefore, subjected to a typical power flow of ~ 2 x 10^{20} ergs/sec. There are fluctuations of about one order of magnitude, both positively and negatively, about this value.

The power flow in the solar wind is estimated to be adequate by a factor of the order of one hundred to power the sum of all known phenomena in the magnetospheric system.

The physical mechanisms for producing the known, natural populations of charged particles in the geomagnetic field are another matter. The current state of work on mechanisms of particle injection, diffusion, energy loss and energy gain, and precipitation is represented by other papers in this conference.

It now appears that only the protons having $E_p > 20$ MeV in the region $L \lesssim 1.8$ are not attributable fundamentally to the solar wind. These particles are probably due to the radioactive decay of neutrons which have been produced by galactic cosmic rays in the earth's atmosphere [Lenchek and Singer, 1962]. Other weak sources of particles may be (a) solar cosmic rays either by direct injection and capture or by neutron production in the atmosphere; (b) energization of particles by the earth's rotation; (c) a neutral component in the solar wind [Akasofu and McIlwain, 1963]; and (d) electromagnetic coupling with whistlers and other ionospheric disturbances of terrestrial origin.

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