THEORETICAL AND EXPERIMENTAL STUDY OF THERMAL
CONTACT RESISTANCE IN A VACUUM ENVIRONMENT

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Prepared by: A. M. Clausing

Submitted by: A. M. Clausing
Assistant Professor of
Mechanical Engineering
27 Mechanical Engineering Bldg.
University of Illinois
Urbana, Illinois
Phone: Area Code 217, 333-3556

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Nomenclature

a	radius or half width of a contact area
b	radius or half width of constriction region
d	equivalent flatness deviation (see Fig. 1)
h	interface conductance, $h = \frac{1}{A_a R}$
k	thermal conductivity
L	length of specimen
ΔL	equivalent length of contact resistance
p	contact pressure
q	rate of heat flow
q'	rate of heat flow per unit depth
R	resistance
R^*	dimensionless resistance, $\frac{R\,k\,A_a}{b}$
r	radial coordinate
T	temperature
ΔT	a temperature difference
x	constriction ratio, $x = a/b$
y	cartesian coordinate
z	axial coordinate

Subscripts

1	surface or specimen 1
2	surface or specimen 2
a	apparent contact area
ct	total contact resistance
f	interstitial substance
L	macroscopic constrictions or contact regions
o	isothermal interfacial plane
s	microscopic constrictions or contact areas
t	total

1. Current Status

1.1 Introduction

Our previous studies [1,2] have led to the development of a restrictive model which was successful at quantitatively predicting the thermal contact resistance. Its ability to qualitatively explain many of the apparent discrepancies in the literature and its agreement with the vastly varied experimental data which were obtained have demonstrated its conceptional correctness. The current endeavors are: (i) to remove the restrictions present in the original model (some of the results of such studies have already been reported [2]), and (ii) to better our understanding of the basic mechanisms of the thermal contact resistance. The latter is perhaps our most important objective. One can easily argue that the proposed model is not capable of predicting the additional resistance of a complex bolted joint; however, one must also agree that the proposed model has indeed been successful in enriching our understanding of the extremely complex problem of interest. In contrast, the numerous studies which have been made with "more realistic" surfaces have added little to our understanding and to our ability to predict the resistance of joints in other physical situations.

The new chamber which was recently constructed is in the process of being checked out. Some new instrumentation for this facility has not yet arrived. In addition, it is felt that with the small resources available, greater emphasis at the present time on analytic studies might be more fruitful. Experimental studies will then again be made to evaluate the results and conclusions drawn from these studies.

Brief descriptions of several investigations which are in progress will now be given. The thermal contact resistance problem can be divided into two parts: (i) Given the contact geometry, determine the additional thermal resistance due to the presence of the interface and (ii) Given the specimen geometry and load, determine the contact areas. The second part of the problem is being studied by Mr. McNary. Part of his work was reported in Reference [3]. Since this work is for his Ph.D. dissertation, the details will not be available until the thesis is completed. The objective of the study is to remove the restriction on the model caused by the inapplicability of the Hertz's solution when the macroscopic contact area is relatively large. A further extension of this analysis to include the influences of thermal strain is also desirable.

Section 1.2 describes preliminary results from analytical studies which show the influences of microscopic resistances on the macroscopic constriction resistance. Section 1.3 gives the formulation for the extension of the model to include an interstitial substance.

Section 1.4 describes the results of constriction resistance calculations for two-dimensional plane constrictions. These calculations are for the problem of the first type, i.e., given the contact geometry, determine the constriction resistance. The Hertz's Equation could be employed to determine the required contact area in some cases.

1.2 The Influence of Microscopic Resistances on the Macroscopic Constriction Resistance

1.2.1 Formulation of the Problem

In the study of the relative importance of the macroscopic and microscopic resistances** it is of great importance to know how the macroscopic and microscopic resistances are interrelated. It is seen that they are not independent since the presence of a microscopic resistance would, of course,

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**The macroscopic resistances always refer to the large scale or macroscopic constriction resistances. The microscopic resistances refer to the combined effects of the film resistance and the small scale or microscopic constriction resistances.

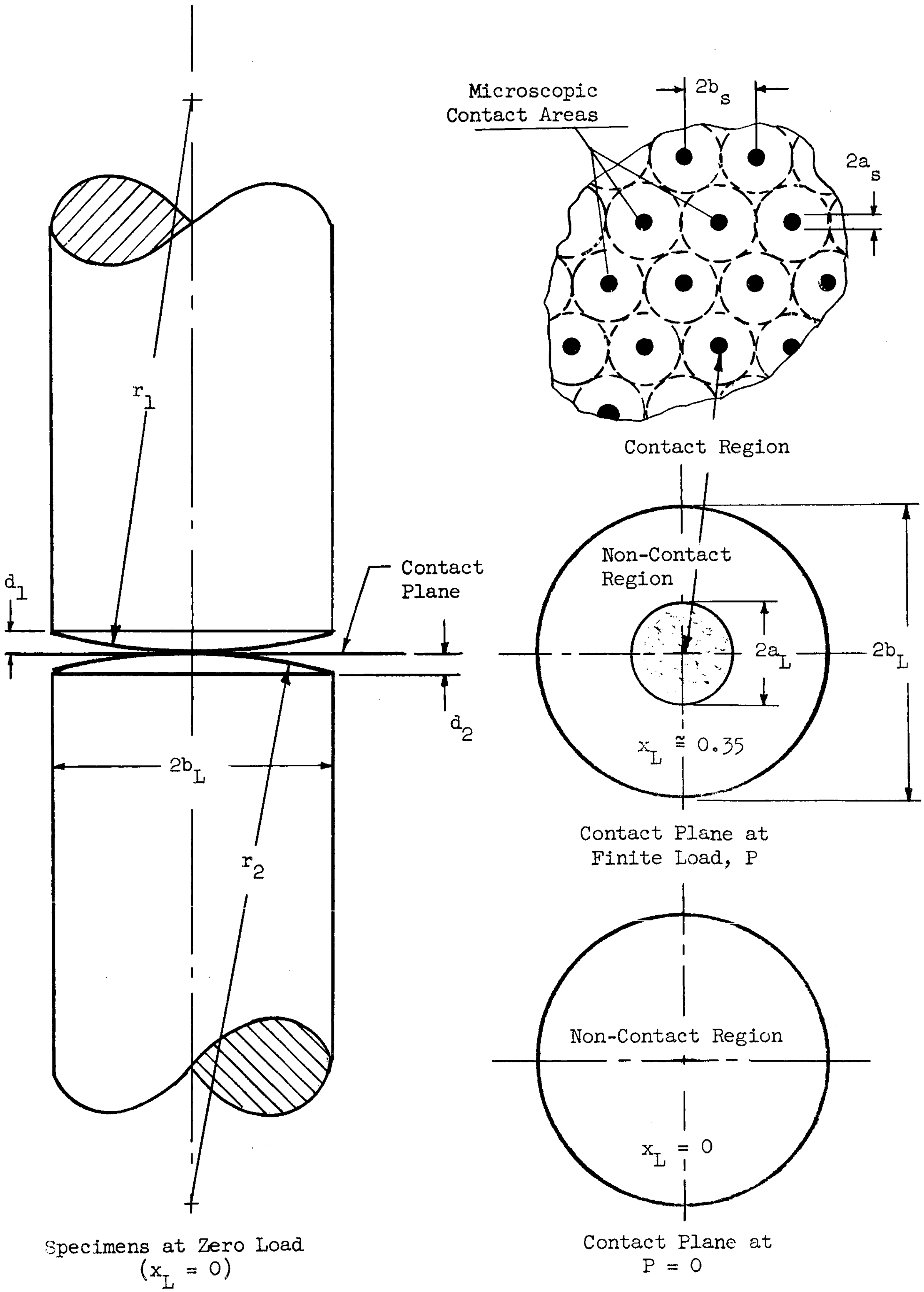

Fig. 1 Model of Contact Surface

affect the macroscopic temperature and flux distribution and consequently the macroscopic constriction resistance.

The model employed in [1] is shown in Figure 1. Only one region will be considered as shown in Figure 2. The governing differential equation and boundary conditions are as follows:

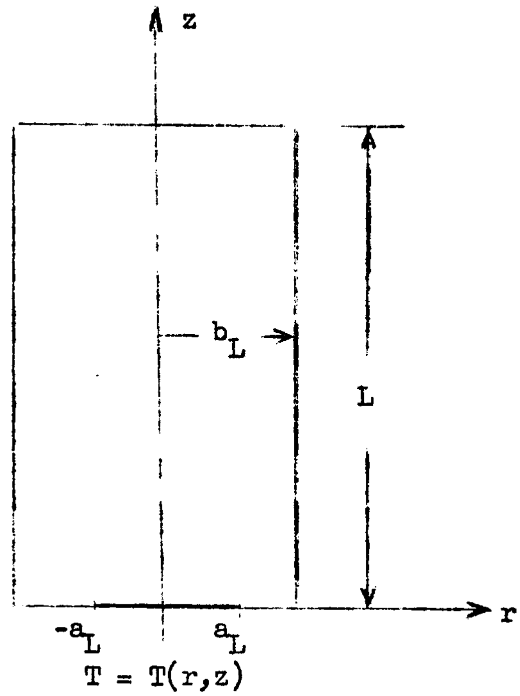

Fig. 2 Finite Cylindrical Region

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{1}$$

$$T(r,L) = T_L, \quad 0 \leq r \leq b_L \tag{2}$$

$$\frac{\partial T}{\partial r}(b_L,z) = 0, \quad 0 < z < L \tag{3}$$

$$\frac{\partial T}{\partial z}(r,o) = 0, \quad a_L < r \leq b_L \tag{4}$$

The boundary condition across the macroscopic contact remains to be specified.**

The change in the macroscopic constriction resistance, R_L, if a microscopic resistance, R_s, is uniformly distributed over the macroscopic contact was considered by Holm [4] and also was discussed in [1]. The limiting situations were considered. Holm suggested that if R_s is very large compared with R_L and is uniform over the macroscopic contact area, it followed that the heat flux through the contact region is approximately uniform. He went on to assume the flux was constant and determined the macroscopic constriction

**For a perfect macroscopic contact, i.e., neglecting R_s, this boundary is simply a constant temperature boundary.

resistance with this boundary condition. Although this boundary condition is approximately true, it cannot be employed in the calculation of R_L, a secondary resistance. A closer examination of the problem shows that if two specimens have the same radius b_L and if the distribution of the microscopic resistance is axially symmetric (it is not necessary to assume the distribution is uniform), an isothermal surface exists within the plane of contact. The presence of the microscopic resistance over the boundary is analogous to a convective type boundary condition with a finite surface conductance h.** The boundary condition to be employed in the determination of the total contact resistance is:

$$+ k \frac{\partial T}{\partial z}(r,o) = h[T(r,o) - T_o], \quad 0 \leq r \leq a_L \tag{5}$$

where h is a function of r if the microscopic resistance is not uniformly distributed; otherwise, it is a constant. T_o is the temperature of the isothermal surface in the contact plane. The contact resistance is given by:

$$R = \frac{T_L - T_o}{q} - \frac{L}{k \pi b_L^2} \tag{6}$$

where q is the rate of heat flow through the region. The dimensionless contact resistance is simply:

$$R^* = \frac{k(\pi b_L^2)\, R}{b_L} = \frac{\Delta L}{b_L} \tag{7}$$

**The region of the microscopic resistance is assumed to be of negligible thickness. This assumption is probably a valid one.

It is seen that in order to calculate the contact resistance the heat flow q is required. To determine q Equation (1) with the boundary conditions (2) through (5) must be solved; however, first h must be determined.

Consider two regions of different materials in contact as shown in Figure 1. Assume a known microscopic resistance is uniformly distributed across the macroscopic contact area. (A non-uniform axially-symmetric distribution would add little complication; however, the results would be of less general value.) If one is to consider only one region for the solution of this problem, it is required that the flux distributions across the contact area of each region are identical. From the differential equation and boundary condition it can be seen that this will be the case if the microscopic resistance is partitioned such that

$$\frac{h_1}{k_1} = \frac{h_2}{k_2} \tag{8}$$

Also $h = \dfrac{1}{\pi a_L^2 R}$ and $R_1 + R_2 = R_s$.

Therefore:

$$R_1 = R_s \frac{1}{1 + \frac{k_1}{k_2}}$$

or

$$h_1 = \frac{1 + \frac{k_1}{k_2}}{\pi a_L^2 R_s} = h_s \left[1 + \frac{k_1}{k_2}\right] \tag{9}$$

It can thus be seen that the requirement of equal flux distributions through the contact area is satisfied if region 1 has a distributed conductance given by (9). The boundary condition (5) is then valid along the interfacial plane. It is therefore possible to find the contact resistance between any two mating materials separated by a small scale resistance, R_s, by finding the contact resistance of one of the regions with the conductance at the interface given by (9). The value of the dimensionless contact resistance would be the same for both regions; hence, the total resistance could be easily calculated.

1.2.2 Solution Procedure

As previously discussed in Reference [3], little success has been achieved in solving problems with mixed boundary conditions analytically. For this reason, the numerical procedure employed in the solution of the problem for the case of negligible microscopic resistance (see Ref. 3) was employed. A difference equation for the boundary $0 \leq r \leq a_L$ is now required in addition to those given in [3]. In terms of the nomenclature of [3], a general difference equation valid for the points on this boundary is:

$$\left(1 + \frac{\Delta r}{2r_j}\right) T_{1,j+1} + 2\, T_{2,j} + \left(1 - \frac{\Delta r}{2r_j}\right) T_{1,j-1} + \frac{h_1\, \Delta r}{k_1}\, T_o - \left(4 + \frac{h_1\, \Delta r}{k_1}\right) T_{1,j} = 0 \tag{10}$$

where h_1 is given by Equation (9). The computer program was modified accordingly and the preliminary results which were obtained follow.

1.2.3 Results of Numerical Computations**

The object of this study was to determine the influence of microscopic resistances on the macroscopic constriction resistance. However, the question

**These computations were made by Mr. Bruce Spencer while taking a special problem course under the author's direction.

which one really needs to answer, from an engineering standpoint, is what error does one commit if it is assumed that the macroscopic and microscopic resistances are independent and the total contact resistance is determined by a simple addition, i.e., by assuming the two resistances are in series. Furthermore, it is seen that the macroscopic and microscopic resistances have lost their identity since one can only define a resistance between isothermal planes. Thus, the macroscopic resistance used in the calculation of the contact resistance is its value for the case when $R_s = 0$ and the microscopic resistance used in this calculation is the value of the contact resistance for the case when b_L is made equal to a_L.

A series of numerical calculations were made to determine the total dimensionless contact resistance R^*_{ct} for the problem as specified by equations (1) through (5) for several values of the constriction ratio $x_L(=a_L/b_L)$, and for several lengths. This solution was compared with the result obtained by adding $R^*_s + R^*_L$. In these calculations, values of R^*_s and x_L were assumed; R^*_L was obtained from Ref. 3.

Table 1 gives a comparison of these results. In examining the apparent trends in these values it must be remembered that no great effort was expended to remove truncation errors. The results are of an exploratory nature; minimum computer time was expended in obtaining these answers. Tables 2, 3, and 4 give the specific values employed in obtaining Table 1.

The conclusions which can be drawn from these results are:

1. The total contact resistance of an interface is always greater than the value obtained by assuming R^*_s and R^*_L are independent resistances in series.
2. The error committed by assuming that R^*_s and R^*_L are independent

Table 1 Dimensionless Ratio $\left[\dfrac{R_{ct}^*}{R_L^* + R_s^*}\right]$

	L/b_L \ x_L	.233	.500	.833
$R_s^* \simeq R_L^*$	.200	1.0405	1.0372	1.0704
	.600	1.0454	1.0621	1.0925
	1.00	1.0458	1.0641	1.0891
$R_s^* \simeq 10R_L^*$	.200	1.0076	1.0078	1.0214
	.600	1.0112	1.0163	1.0366
	1.00		1.0125	1.0366
$R_s^* \simeq .2R_L^*$	.200	1.0466	1.0377	1.0842
	.600	1.0471	1.0520	1.0924
	1.00		1.0533	1.0870

Table 2 Comparisons for $R_s^* \simeq R_L^*$

L/b_L \ x_L	.233	.500	.833
.200	$R_s^* = 2.281$ $R_L^* = \underline{1.772}$ 4.053 $R_{ct}^* = 4.217$	$R_s^* = .5380$ $R_L^* = \underline{.3830}$ $.9210$ $R_{ct}^* = .9553$	$R_s^* = .0460$ $R_L^* = \underline{.0407}$ $.0867$ $R_{ct}^* = .0928$
.600	$R_s^* = 2.281$ $R_L^* = \underline{2.258}$ 4.539 $R_{ct}^* = 4.745$	$R_s^* = .5380$ $R_L^* = \underline{.5306}$ 1.0686 $R_{ct}^* = 1.135$	$R_s^* = .0460$ $R_L^* = \underline{.0459}$ $.0919$ $R_{ct}^* = .1004$
1.00	$R_s^* = 2.281$ $R_L^* = \underline{2.281}$ 4.562 $R_{ct}^* = 4.771$	$R_s^* = .5380$ $R_L^* = \underline{.5380}$ 1.0760 $R_{ct}^* = 1.145$	$R_s^* = .0460$ $R_L^* = \underline{.0460}$ $.0920$ $R_{ct}^* = .1002$

Table 3 Comparison for $R_s^* \simeq 10\ R_L^*$

L/b_L \ x_L	.233	.500	.833
.200	$R_s^* = 22.810$ $R_L^* = \underline{1.772}$ 24.582 $R_{ct}^* = 24.77$	$R_s^* = 5.380$ $R_L^* = \underline{.3830}$ 5.7630 $R_{ct}^* = 5.808$	$R_s^* = .4590$ $R_L^* = \underline{.0407}$ .4997 $R_{ct}^* = .5104$
.600	$R_s^* = 22.81$ $R_L^* = \underline{2.258}$ 25.068 $R_{ct}^* = 25.35$	$R_s^* = 5.380$ $R_L^* = \underline{.5306}$ 5.9106 $R_{ct}^* = 6.007$	$R_s^* = .4590$ $R_L^* = \underline{.0459}$ .5049 $R_{ct}^* = .5234$
1.00		$R_s^* = 5.380$ $R_L^* = \underline{.538}$ 5.918 $R_{ct}^* = 5.992$	$R_s^* = .4590$ $R_L^* = \underline{.0460}$ .5050 $R_{ct}^* = .5235$

Table 4 Comparisons for $R_s^* \simeq 0.2\ R_L^*$

L/b_L \ x_L	.233	.500	.833
.200	$R_s^* = .4562$ $R_L^* = \underline{1.772}$ 2.2282 $R_{ct}^* = 2.332$	$R_s^* = .1076$ $R_L^* = \underline{.3830}$ .4906 $R_{ct}^* = .5091$	$R_s^* = .0092$ $R_L^* = \underline{.0407}$ .0499 $R_{ct}^* = .0541$
.600	$R_s^* = .4562$ $R_L^* = \underline{2.258}$ 2.7142 $R_{ct}^* = 2.842$	$R_s^* = .1076$ $R_L^* = \underline{.5306}$ .6382 $R_{ct}^* = .6714$	$R_s^* = .0092$ $R_L^* = \underline{.0459}$ .0552 $R_{ct}^* = .0603$
1.00		$R_s^* = .1076$ $R_L^* = \underline{.5380}$ .6456 $R_{ct}^* = .6800$	$R_s^* = .0092$ $R_L^* = \underline{.0460}$ .0552 $R_{ct}^* = .0600$

and that the total contact resistance is simply $(R_s^* + R_L^*)$ is small in all cases and, considering the nature of the problem, could easily be neglected.

3. This error vanishes if $R_s^* >> R_L^*$ or if $R_L^* >> R_s^*$.
4. The apparent macroscopic constriction resistance $(R_{ct}^* - R_s^*)$, in the presence of microscopic resistances can differ widely from the value obtained when R_s^* is equal to zero.

1.3 <u>The Generalization of the Model to Include an Interstitial Material</u>

The investigation has been centered in the past on the study of clean interfaces in vacuum environments. This approach was logical since:

1. The metal-to-metal conduction mode of heat transfer across an interface is the most fundamental. Without a thorough understanding of the metal-to-metal conduction mode incorrect conclusions could easily be drawn from experimental measurements of interfacial resistances for the combined mode case.
2. The problem is of greater importance in the absence of interstitial conduction since the contact resistance is then much larger.
3. The ability to predict the magnitude of the contact resistance was poorest for interfaces in vacuum environments and the need to know these values was most urgent in this area.

It is felt, however, that a limited parallel effort on the study of interfaces with the addition of the interstitial conduction mode would be profitable since:

1. This study could be conducted in conjunction with the present experimental and theoretical investigations with relatively little additional effort.

2. This study might reveal <u>reliable</u> methods of decreasing the magnitude of the thermal contact resistance in a vacuum environment. For example, it could lead to more successful theoretical predictions by giving a method of insuring the lack of importance of microscopic resistances which are, of course, difficult to predict.

3. The study should prove worthwhile in its own right for the following reasons. First, the problem is of interest and importance in other environments as, for example, in nuclear reactor cores, atmospheric operation and testing of space vehicles, etc. Second, satisfactory materials may be developed or may already exist for employment as interstitial substances in vacuum environments for both short and long duration flights.

The formulation of the problem and a discussion of the solution procedure being pursued will now be given.

Since the thickness of interstices is in most cases small compared with other dimensions, both natural convection and conduction parallel to the interface can be neglected in the calculation of the heat transferred through the interstitial substance.

The basic model developed in Ref. [1] will again be employed in the study of this aspect of the problem. Figure 1 gives the geometry of interest. The flatness deviation or waviness which gives rise to macroscopic constrictions is simulated by spherical caps on the end surfaces of the cylindrical contacting members. The distance, $f(r)$, from the contact plane to the surface of the specimen at zero load is, by definition:

$$f_i(r) = \left(\frac{r}{b_L}\right)^2 d_i , \quad i = 1 \text{ or } 2 \tag{11}$$

The distances d_1 and d_2 are indicated in Figure 1. The distance between the surfaces at zero load can also be obtained from (11) if d_i is replaced by the total flatness deviation $d_t(= d_1 + d_2)$. The distance between the surfaces is, in general, a function of the apparent contact pressure p, the initial geometry and the radial coordinate r. This distance will be called $f_t(p,r)$. For the geometry of the present model this distance at zero load is:

$$f_t(0,r) = \left(\frac{r}{b_L}\right)^2 d_t \tag{12}$$

This function at finite contact pressures will be known only after the finite elastic contact problem being considered by McNary [3] is solved. It is assumed that this distance is negligible within the macroscopic contact area. This is equivalent to neglecting the microscopic resistances. The presence of even a poor interstitial conductor such as air should vastly decrease the microscopic resistances which were found to be small even for interfaces without an interstitial substance. Thus, the assumption that $f_t(p,r) = 0$ if $r < a_L$ is probably valid. The results of the present macroscopic analysis should further substantiate this assumption.

The present problem is seen to be of the same nature as that presented in Section 1.2. The microscopic conductance h_s now becomes $\frac{k_f}{f_t(p,r)}$ where k_f is the thermal conductivity of the interstitial substance. Boundary condition (4) is no longer applicable since the interstitial material provides a conductive path over the complete apparent contact area. Boundary condition

(5) now can be applied over the entire contact area:

$$+k \frac{\partial T}{\partial z} = +h(T - T_o), \quad 0 \leq r \leq b_L \tag{13}$$

where

$$h_1 = \frac{k_f}{f_t(p,r)} \left[1 + \frac{k_1}{k_2}\right] \tag{14}$$

Thus, it is seen that in this case the conductance, h, of Eq. (13) is a function of the contact pressure and the radius r. For a given load, h would be a function of r only. If $r \leq a_L$, h becomes infinite since $f_t(p,r) = 0$ (a finite value could be employed if desired and the importance of microscopic resistances could then be easily seen); therefore, for $r \leq a_L$ one could either employ the boundary condition $T(0,r) = T_o$ or simply use a very large value of h in the boundary condition described by (13).

The solution procedure being pursued for the problem as formulated is similar to that employed in Section 1.2. Only minor changes in the computer program are again required. Only zero load solutions can be presently obtained since, in general, the function $f_t(p,r)$ is not known. Approximations of this function could be made to obtain the bounding values of the contact resistance as a function of load with a given initial geometry.

The fineness of the grid required for the numerical calculation of this combined mode problem should be considerably coarser than that required in the absence of the interstitial conduction mode [see Ref. 2]. The discontinuity in the heat flux at the point z = 0, $r = a_L$ is no longer present since the variation in h with respect to r is in general continuous.

1.4 The Influence of the Region Geometry on the Macroscopic Constriction Resistance--Plane Geometry

1.4.1 Introduction and Problem Formulation

It is usually assumed that the contact areas which are formed between contacting bodies are circular in nature. For this reason most of the effort expended in the calculation of constriction resistances was concentrated on circular contact areas. The most formidable of these problems is probably that of the finite cylindrical region. This problem was solved by numerical calculations and the results of these calculations were reported in Ref. 2. Extensions to this solution were reported in Sections 1.2 and 1.3 of this report.

Interest was recently expressed in two-dimensional plane constrictions [5]. The numerical solution procedure recently developed for the axially-symmetrical case [2] was easily modified to the plane problem. (Several plane cases were actually employed in the early development of the computer program.) Therefore, it seemed worthwhile to repeat these calculations for the plane case.

The region under consideration is shown in Figure 3. The governing differential equation and boundary conditions are:

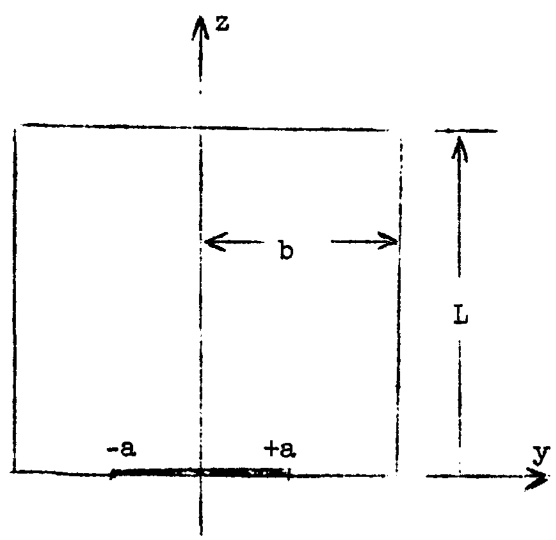

Fig. 3

$$\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \qquad (15)$$

$$T(y,L) = T_L \,, \quad -b \leq y \leq b \qquad (16)$$

$$\frac{\partial T}{\partial y}(b,z) = 0, \quad 0 < z < L \qquad (17)$$

$$\frac{\partial T}{\partial z}(y,0) = 0, \quad \begin{matrix} -b \leq y \leq -a \\ a \leq y \leq b \end{matrix} \qquad (18)$$

$$T(y,0) = T_o \,, \quad -a \leq y \leq a \qquad (19)$$

A differencing procedure similar to that used in [2] was employed in the numerical solution of these equations. The schemes used to speed convergence of the iteration procedure were also similar. The grid network employed is shown in Figure 4. Equal increments were used in the y and z directions. A coarse network was used away from the interface and a fine network was employed near the interface. These networks were joined with triangular elements. The number of columns m, and the number of fine rows k could be varied; n was dependent upon the length L and the value of m (see Fig. 4).

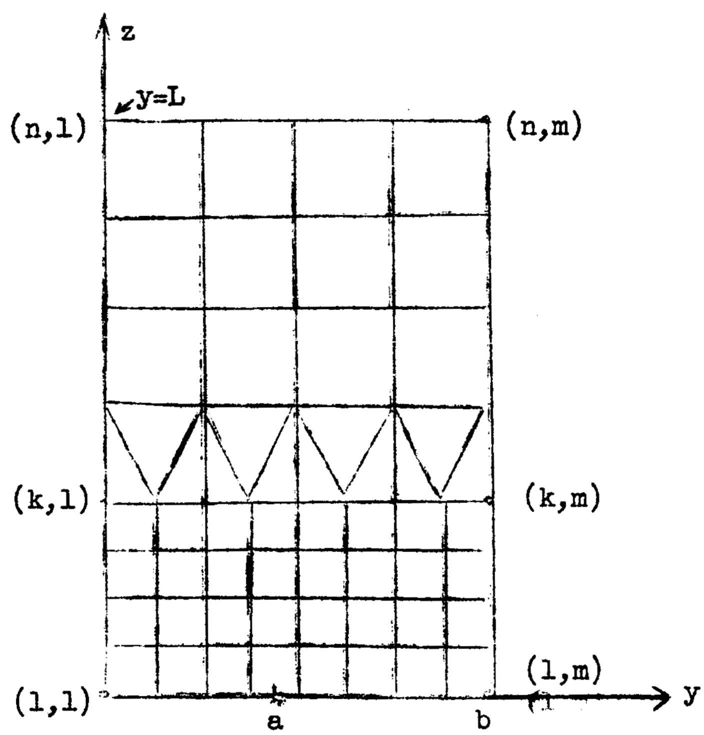

Figure 4

1.4.2 Results of Numerical Computations

Since in the particular solution procedure which was employed, solutions were obtained for two different spacial increments,** an estimate of the truncation error was possible. The error was estimated to be several percent, which is of relatively little importance for a problem of the nature of thermal contact resistance. The exact solution which was later obtained for the limiting case of $L \gg b$ showed that the average error was approximately 2%. Since this small error was of little concern, all the numerical computations could be carried out in approximately 8 minutes of production time on the 7094 digital computer. Before these results are reported, a brief discussion of the nomenclature will be given.

According to the usual definition, the constriction resistance, R, is:

$$R = R_t - R_t\,(x = 1.0) \tag{20}$$

where R_t is the total resistance and x is the constriction ratio a/b.

A dimensionless resistance as defined in Ref. 2 will be again employed. It is, by definition:

$$R^* = \frac{R\,k\,A_a}{b} \tag{21}$$

where A_a is the apparent contact area, k is the thermal conductivity of the material, and b is the half-width of the region. (b was the radius of the region in the cylindrical problem.) Since the resistance R is inversely proportional to the depth, the dimensionless resistance is independent of the depth. For convenience a unit depth will be employed. Therefore,

**17 and 49 columns were employed in the two solutions. The first solution was used as an initial approximation for the second.

$R^* = 2k\,R$ for the plane region of width 2b. The dimensionless constriction resistance is

$$R^* = \frac{2k\Delta T}{q'} \tag{22}$$

where q' is the rate of heat flow per unit depth.

The results obtained from the numerical solution are given in Table 5. The dimensionless constriction resistance as defined by Eq. (22) is listed. It is the constriction resistance of one region only. For two plane regions of the same width, length and type of material in contact, the total macroscopic constriction resistance or the so-called contact resistance is twice the value given in Table 5. Table 6 shows the variation of the ratio $R^*(L/b) \,/\, R^*(L/b = \infty)$ with x and L/b.

1.4.3 Exact Solution for the Case of $L \gg b$**

Constriction resistance problems are formidable ones due to the presence of the mixed boundary condition along the boundary forming the plane of contact. The boundary is isothermal over the contact area and is a zero flux surface over the remainder of the plane of contact. This suggests that one apply conformal mapping techniques to try to eliminate this mixed boundary condition. These techniques, of course, have only been successfully employed for plane regions; therefore, these methods of solution were not possible for the axially symmetric case studied in [2].

The boundary conditions of the types given by equations (16) through (19) are invariant with a change of variables arising from a conformal transformation. Therefore the new form of the boundaries is of primary interest.

**The nomenclature in this section was chosen to conform with standard complex variable nomenclature; consequently, there are several minor conflicts with that employed elsewhere. The complex variables used are: $z = x + iy$; $z' = x' + iy'$; $z'' = x'' + iy''$ and $w = u + iv$; therefore, a/b is now used for the constriction ratio instead of the letter x which was previously employed.

$R^* = \Delta L/b$

L/b \ x	.156	.219	.281	.344	.406	.469	.531	.594	.656	.719	.781	.844
0	0	0	0	0	0	0	0	0	0	0	0	0
.250	.6987	.5160	.3928	.3038	.2365	.1838	.1415	.1069	.07814	.05416	.03433	.01852
.375	.8096	.6107	.4710	.3667	.2858	.2213	.1689	.1257	.09010	.06087	.03742	.01953
.500	.8637	.6579	.5107	.3989	.3109	.2400	.1820	.1344	.09521	.06353	.03854	.01986
.625	.8895	.6806	.5300	.4145	.3230	.2488	.1882	.1383	.09763	.06462	.03897	.01999
.750	.9012	.6911	.5389	.4218	.3286	.2530	.1910	.1401	.09866	.06524	.03928	.02007
1.0	.9093	.6983	.5450	.4286	.3325	.2558	.1930	.1413	.09934	.06557	.03941	.02018
1.25	.9110	.6998	.5463	.4279	.3333	.2564	.1935	.1416	---	.06563	---	---
∞	(.911)	(.700)	(.547)	(.428)	(.334)	(.257)	(.1938)	(.142)	(.0997)	(.0657)	(.0395)	(.0202)

Table 5 The Dimensionless Constriction Resistance R^* as a Function of x and L/b

$R^*(L/b)/R^*(L/b = \infty)$

L/b \ x	.156	.219	.281	.344	.406	.469	.531	.594	.656	.719	.781	.844
0	0	0	0	0	0	0	0	0	0	0	0	0
.250	.767	.737	.718	.709	.708	.715	.730	.754	.784	.823	.869	.918
.375	.886	.873	.861	.857	.856	.862	.872	.885	.903	.928	.947	.967
.500	.948	.940	.934	.932	.931	.934	.939	.946	.955	.967	.975	.983
.625	.976	.972	.969	.968	.967	.967	.971	.974	.979	.983	.989	.990
.750	.989	.987	.985	.985	.984	.984	.986	.987	.989	.993	.994	.994
1.0	.998	.998	.996	.997	.995	.995	.995	.995	.997	.998	.998	.999

Table 6 The Ratio $R^*(L/b)/R^*(L/b = \infty)$ as a Function of x and L/b

According to Ref. [6] the transformation $z' = \sin z$ transforms the semi-infinite strip into the upper half of the z' plane as shown in Figure 5. Two more successive transformations are then applied. The first one $z'' = z'/\sin z$ is a simple magnification by the factor $1/\sin a$. This allows the use of the transformation $w = \sin^{-1} z''$, which is the inverse of the initial transformation. It transforms the upper half of the z'' plane into a semi-infinite strip. It is seen that the initial geometry is again obtained, however, the mixed boundary condition has been removed. The successive transformations which were employed are clearly indicated in Figure 5.

The temperature distribution in the w plane is easily seen to be:

$$T = T_o - \frac{q'}{\pi k} v \tag{23}$$

where q' is the rate of heat flow per unit depth. To obtain the solution of the original problem, v must be determined as a function of the original variables x and y.

Consider first going from T as a function of v to T as a function of x'' and y''. The transformation is:

$$z'' = \sin w = \sin u \cosh v + i \cos u \sinh v$$

Therefore:

$$\frac{x''^2}{\cosh^2 v} + \frac{y''^2}{\sinh^2 v} = 1 \tag{24}$$

Equation (24) shows that the isotherms in the z'' plane are ellipses whose centers are at the origin and the foci are at $y'' = 0$, $x'' = \pm 1$. From the definition of an ellipse it follows that:

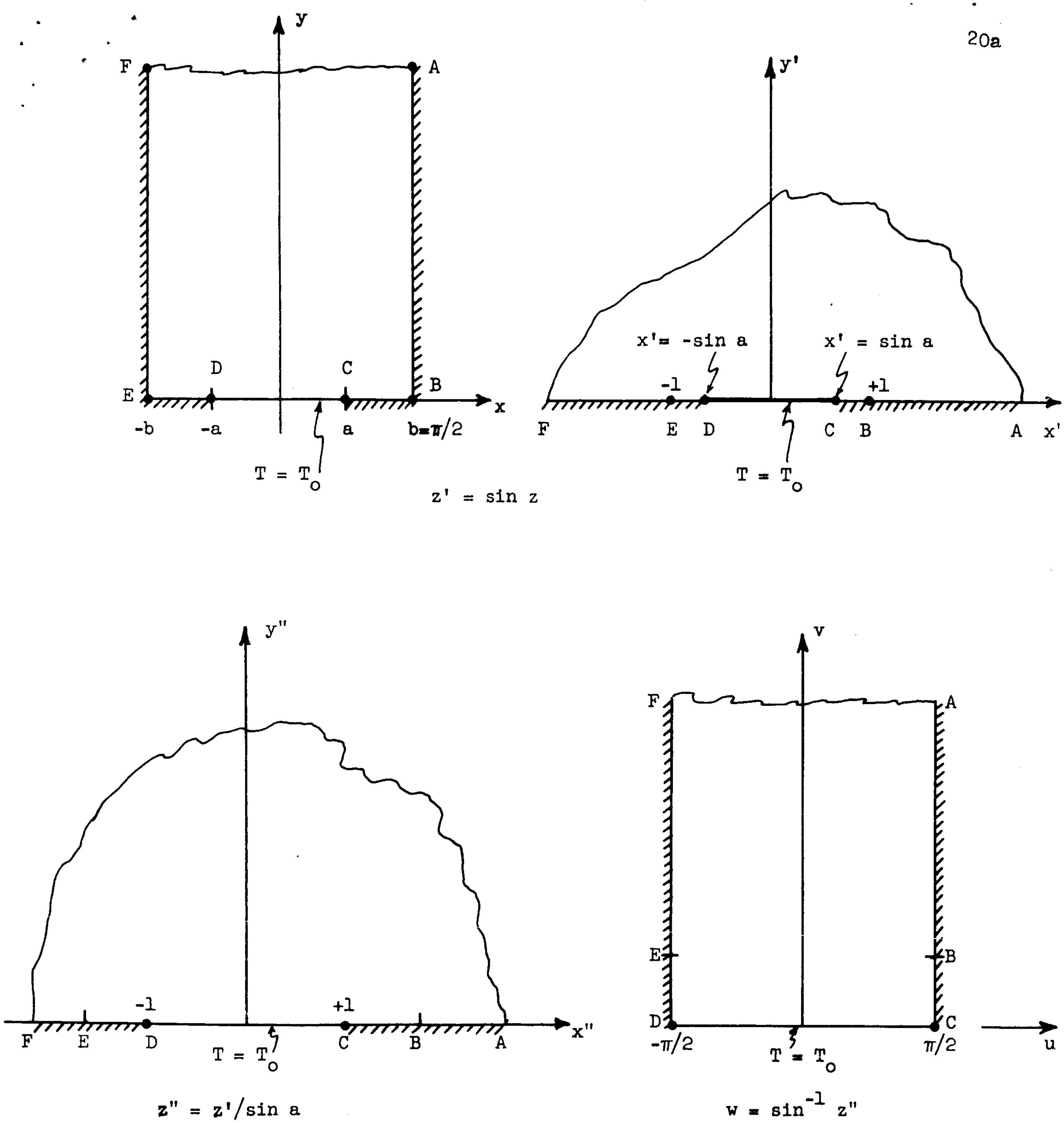

Fig. 5 Successive Transformations Employed

$$2 \cosh v = [(x'' + 1)^2 + y''^2]^{1/2} + [(x'' - 1)^2 + y''^2]^{1/2}$$

or:

$$v = \ln [Y + (Y^2 - 1)^{1/2}]$$

where:

$$Y = \frac{1}{2}\left\{[(x'' + 1)^2 + y''^2]^{1/2} + [(x'' - 1)^2 + y''^2]^{1/2}\right\}$$

Also:

$$x'' = \frac{\sin x \cosh y}{\sin \frac{\pi}{2} \frac{a}{b}} \quad \text{and} \quad y'' = \frac{\cos x \sinh y}{\sin \frac{\pi}{2} \frac{a}{b}}$$

therefore, in terms of the original variables the temperature distribution is:

$$T = T_o - \frac{q'}{\pi k} \left\{ \ln [Y + (Y^2 - 1)^{1/2}]\right\} \tag{25}$$

where

$$Y = \frac{1}{2 \sin \frac{\pi}{2} \frac{a}{b}} \left\{[(\sin x \cosh y + \sin \frac{\pi}{2}\frac{a}{b})^2 + \cos^2 x \sinh^2 y]^{1/2}\right.$$

$$\left. + [(\sin x \cosh y - \sin \frac{\pi}{2}\frac{a}{b})^2 + \cos^2 x \sinh^2 y]^{1/2}\right\} \tag{26}$$

The constriction resistance is of main interest. It can be obtained by considering a value of $y = y_\infty$ which is sufficiently large such that the temperature is independent of x. The constriction resistance is then given by:

$$R = \left| \frac{[T(y_\infty)\big|_{a/b} - T_o]}{q'\big|_{a/b}} - \frac{[T(y_\infty)\big|_{a/b=1} - T_o]}{q'\big|_{a/b=1}} \right| \tag{27}$$

If y is very large,

$$T \simeq T_o - \frac{q'}{\pi k} \ln 2Y$$

and

$$Y \simeq \frac{\cosh y}{\sin \frac{\pi}{2}\frac{a}{b}}$$

Therefore:

$$R^* \approx \frac{2}{\pi} \ln \left\{ \frac{1}{\sin \frac{\pi}{2}\frac{a}{b}} \right\} \tag{28}$$

Equation (28) is valid for all possible values of the constriction ratio, a/b; however, it fails to apply if $L/b < 1$ as can be seen from the numerical results given in Table 6. Table 7 compares the exact solution with the numerical results.

R^*

$x = a/b$	.156	.219	.281	.344	.406	.469	.531	.594	.656	.719	.781	.844
Exact Solution (Eq. 28)	.901	.693	.541	.424	.330	.253	.1909	.1395	.0977	.0643	.0383	.0194
Numerical (Table 5)	.911	.700	.547	.428	.334	.257	.1939	.142	.0997	.0657	.0395	.0202
Percentage Difference	1.2	1	1.1	.9	1.2	1.6	1.5	1.8	2.1	2.2	3.1	4

Table 7 Comparison between Exact and Numerical Solutions

2. Proposed Future Research

Future work will proceed along the lines outlined in the various subsections of Section 1, The Current Status. It is seen that many of the current studies are incomplete but hold promise of substantial reward.

Experimental investigations to substantiate the analytical studies should also be conducted; however, the funds and time available may not permit such investigations.

3. References

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