7

PLASMA PHYSICS LABORATORY

THE COLLEGE OF WILLIAM AND MARY IN VIRGINIA

Williamsburg, Virginia

NONLINEAR RESPONSE OF AN OSCILLATING DISCHARGE TO VERY HIGH FREQUENCY SIGNALS

by D. B. Raiford

Technical Report No. 3, Aug. 1965

This work was submitted to the College of William and Mary in Virginia as a thesis in partial fulfillment of the requirements for the degree of Master of Arts in Physics, ~~June 1963~~ [redacted] August 1965.

The work was supported by the National Aeronautics and Space Administration under Research Grant NsG 106-61.

Approved: Frederic R Crownfield Jr.

Frederic R. Crownfield, Jr.
Principal Investigator

ACKNOWLEDGMENTS

The author wishes to express his appreciation to Professor Frederic R. Crownfield, Jr., under whom this work was done, for his help and guidance throughout this investigation. The author is especially indebted to Professor James D. Lawrence, Jr. and to Professor John L. McKnight for their careful reading and criticism of the manuscript. Thanks also go to Mr. Stan Hummel and the shop staff for their contributions of time and effort in the construction of some of the apparatus used in this investigation, and especially to David Holt for his help and discussion of experimental techniques. This investigation was supported by the National Aeronautics and Space Administration under research grant number NsG 106-61.

TABLE OF CONTENTS

LIST OF FIGURES

ABSTRACT

A general discussion of the appearance of nonlinearities in the governing equations for the behavior of a plasma is presented. An historical survey of nonlinearities in plasmas and their observation and description by earlier investigators are discussed in an effort to achieve a basic foundation for the subsequent presentation. Experimental apparatus, used to produce and investigate nonlinearities in an oscillating discharge, was assembled. A very high frequency signal applied to the anode of the discharge affects the direct current drawn by the discharge. This effect appears as a resonant change in the DC level near frequencies characteristic of the oscillating discharge. A quadratic dependence of the response on the applied signal level indicates a second-order nonlinearity. These nonlinear effects are observed only when the frequencies involved correspond to the characteristic modes of the system.

17095

Author

NONLINEAR RESPONSE OF AN OSCILLATING DISCHARGE TO AN APPLIED VERY HIGH FREQUENCY SIGNAL

I. INTRODUCTION AND HISTORICAL SURVEY

Various nonlinear effects may play an important role in plasmas. The aim of this thesis is to discuss some nonlinear effects which we have experimentally observed in a laboratory plasma and to attempt to relate them to the plasma nonlinearities observed by other investigators.

In our experiments, a Penning Ionization Gauge[1] was used to produce the plasma and we investigated the nonlinear interaction between a very high frequency signal applied to the anode of the discharge and the plasma oscillations which have been described previously.[2]

A plasma is a partially ionized gas with the average number density of the electrons and ions such that charge neutrality is preserved. Due to their relatively large mass, the ions can be assumed to be stationary under the influence of high-frequency electro-magnetic fields. They serve only to balance the DC space charge due to the electrons. When charged particles move under the influence of an electromagnetic field, the relationship between the current density and the electric field is not linear.

In order to see more explicitly where the nonlinearity enters we consider the equations which control the behavior of a plasma. First,

---

[1]F. M. Penning, Physica 4, 71 (1937).

[2]F. R. Crownfield, Jr., VI International Conference on Ionization Phenomena in Gases, Paris 1963 (S.E.R.M.A. Publishing Co., Paris, 1964) 2, 451; R. N. Dennis, Jr., Masters Thesis, College of William and Mary, Williamsburg, Va., (1962).

of course, are the four Maxwell field equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1}$$

$$\nabla \times \vec{B} = \mu_o \left(\vec{J}_{total} + \epsilon_o \frac{\partial \vec{E}}{\partial t} \right) \tag{2}$$

$$\nabla \cdot \vec{E} = \frac{\rho_{total}}{\epsilon_c} \tag{3}$$

$$\nabla \cdot \vec{B} = 0. \tag{4}$$

The electric current density $\vec{J}$ due to the motion of charge is defined as

$$\vec{J} = \rho\vec{v}. \tag{5}$$

The equation of continuity can be written by taking the divergence of Eq. (2) and with the aid of Eq. (3), we get

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0. \tag{6}$$

Finally we have the equation of motion for an electron, written in the form

$$\frac{D\vec{v}}{Dt} + \nu(\vec{v})v = \frac{-e}{m} (\vec{E} + \vec{v} \times \vec{B}),$$

where (7)

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla$$

is the convective derivative which gives the time rate of change of a quantity in a frame of reference moving with the instantaneous velocity $\vec{v}$. $\nu(\vec{v})$ represents the collision frequency for momentum transfer, generally expressed as a function of electron energy. This is introduced in a non-rigorous fashion to account for collisions in an inherently collisionless equation.

The term involving the collision frequency is non-linear because ν depends on $\vec{v}$. It is included because under certain conditions it can give rise to the well-known "Luxembourg effect", first observed when modulation from Radio Luxembourg was heard on other signals traversing the same region of the ionosphere (hence its name). This was described by Tellegen in 1933.[3] Bailey and Martyn[4] proposed a theory for this phenomenon and the effect was observed experimentally in the laboratory by Goldstein, Anderson, and Clark.[5] These authors showed that an electromagnetic wave traveling in an ionized region could, under certain circumstances, interact with a second wave in such a way that a modulation imposed on one of the waves was transferred to the other. This cross-modulation is a nonlinear interaction which is provided by the absorption of energy from radio waves in an ionized region, causing an increase in the thermal velocity of the electrons. This will change the frequency of collisions of electrons in that region. Since the absorption of an electromagnetic wave in an ionized medium depends on the collision frequency of the electrons in that medium the absorption of another ("wanted") wave is altered by the presence of the absorbed interacting wave. Thus, the "wanted" wave will become weaker or stronger in the presence of an interacting wave. If the amplitude of the interacting wave varies, the velocity of the electrons varies too, and the absorption of the "wanted" wave will also vary. In this manner, there is a superposition of the modulation of the interacting wave on the "wanted" wave. Basically, the collision frequency changes because of the absorption of energy by the plasma, and this changes the resistivity of the medium.

In our plasma, the collision frequency is vanishingly small in the

---

[3] B. D. Tellegen, Nature 131, 840 (1933).

[4] V. A. Bailey and D. F. Martyn, Phil. Mag. 18, 369 (1934).

[5] L. Goldstein, J. M. Anderson, and G. L. Clark, Phys. Rev. 90, 151 (1953).

pressure regions at which we are working; the term was included in Eq. (7) only to show its nonlinear nature.

Another nonlinear term appearing in our set of equations is the product $(\vec{v}\cdot\nabla)\vec{v}$ in Eq. (7) which contributes a nonlinear relationship between $\vec{v}$ and the fields $\vec{E}$ and $\vec{B}$. Another nonlinearity appears in Eq. (5) because $\vec{J}$ depends on $\vec{E}$. $\vec{E}$, in turn, depends on ρ which is related to $\vec{J}$. The dependence of $\vec{J}$ and $\vec{E}$ on each other results in the nonlinearity.

In short, nonlinearities occur in a plasma because the fields which influence the particle motions depend, themselves, on these motions. If the fields are small it is customary to use a linearized version of Eqs. (1) through (7).

A second-order nonlinear response produced by an alternating driving force can be expected to contain both the second harmonic of the driving frequency and a steady component. Consider a driving force

$$F = A \cos wt \tag{8}$$

The response can in general be expanded as a Taylor's series in F, viz.

$$R = BF + CF^2 + \cdots, \tag{9}$$

where the constant term is omitted because no force implies no response. Using the identity

$$\cos^2 wt = \frac{1 + \cos 2\, wt}{2} \tag{10}$$

the response can be written as

$$R = AB \cos wt + \frac{A^2C}{2}(1 + \cos 2\, wt) + \cdots \tag{11}$$

Thus, Eq. (11) contains the fundamental and the second harmonic of the driving frequency as well as a DC or zero frequency term. It is to be noted that these "second-order" effects are proportional to A^2.

Nonlinear effects in plasma have been described by many authors. In

1960, Takayama, et al.,[6] presented results of experimental investigations of the behavior of the DC component of the electron current drawn by a metal probe inserted in a plasma as a function of frequency of a signal applied to the probe. The frequency of a low voltage rf signal superimposed on a probe is swept over an adequate frequency range and the DC component of the electron current is measured as a function of frequency. Preliminary results showed a resonant increase in the current which appeared near the electron plasma frequency. The authors assumed that it occurred at the plasma frequency, and suggested that this resonance probe method would enable one to determine density, temperature, and effective collision frequency of the electrons in the plasma.

The fact that a change in the DC level is observed when an alternating signal is applied to a probe has been known for many years.[7] The fact that the DC characteristic is nonlinear implies that if one applies an rf modulating signal to a probe at a fixed DC potential, rectification will occur. This results in a change in the direct current drawn. This effect is independent of frequency provided that the frequency is low enough to allow the plasma to follow the applied signal as it does under DC operating conditions. Takayama, et al., attributed the resonance in the DC component to a flucuation in the electron density in the region of the plasma frequency.

A theoretical description of the resonance probe was proposed in 1962

---

[6] K. Takayama, H. Ikegami, and S. Miyazaki, Phys. Rev. Letters 5, 238 (1960).

[7] L. D. Loeb, Basic Processes of Gaseous Electronics (University of California press, Berkeley, 1955), p. 345.

by Ichikawa and Ikegami.[8] Wimmel,[9] however, criticized their description on the basis that the theory neglected the plasma sheath, which causes the resonant increase of the direct current to occur at frequencies below the electron plasma frequency. (The sheath consists of variations of number density, electric field, and potential in the region of a plasma boundary.) Wimmel also pointed out an improper normalization of the rf field in the theory of Ichikawa and Ikegami which provided at least part of the increase of the direct current near the electron plasma frequency.

Harp and Crawford[10] have given a description of the resonance probe which takes into account the above discrepancies. They compared their results with experiments, and found good agreement. Their description determines that the resonance in the rectified current occurs below the electron plasma frequency and predicts the observed dependence on probe size and voltage.

In some recent work, O'Brien, Gould, and Parker[11] studied longitudinal wave-propagation modes along the positive column of a discharge. Their experimental configuration consisted of a discharge tube inserted through two holes in the small sides of a waveguide in the manner often used for the measurement of Tonks-Dattner[12] resonances. (These resonances appear as

---

[8] Y. H. Ichikawa and H. Ikegami, Progr. Theoret. Phys. 28, 315 (1962).

[9] H. K. Wimmel, Institute of Plasma Physics, Report No. 6/11, Garching bei München, Germany (1963).

[10] R. S. Harp and F. W. Crawford, Aerospace Research Lab., Report No. ARL 64-139 (1964) (DDC No. AD 608613).

[11] B. O'Brien, R. W. Gould, and J. Parker, Phys. Rev. Letters 14, 630 (1965).

[12] L. Tonks, Phys. Rev. 37, 1458 (1931); A. Dattner, Phys. Rev. Letters 10, 205 (1963).

a series of minima in the absorption of an electromagnetic wave incident normally on the plasma column as a function of the plasma electron density.) Transmission along the column, from one such coupler to another, was observed as a function of electron density. The authors observed a series of small minima superimposed on the more well-defined minima at the Tonks-Dattner resonances. This superimposed spectrum was attributed to nonlinear coupling between the wave-propagation modes.

They have further suggested that experiments using high frequency probes may also exhibit nonlinearities in the region of resonances. When a probe was used for a transmitter coupler they observed a maximum in the rectified current at electron densities corresponding to resonances in transmitted power.

In other work, Stern[13] has recently observed nonlinear phenomena related to the interaction of resonant plasma oscillations. His experimental apparatus contained a hot-cathode glow discharge tube inserted across a rectangular waveguide with the axis of the tube normal to both the electric field and the direction of propagation. Low-power microwave signals of different frequencies in the 2 to 4 Gc/sec. band were fed into the wave guide in the TE_{01} mode.

If one of the signals, which he called the "wanted" wave, was turned on and held at a fixed level for a period of time while the second signal, called the "disturbing" wave, was turned on for a shorter period of time, the two signals could be represented by a narrow pulse superimposed on the broader "wanted" wave signal. If the two signal frequencies are chosen

---

[13] R. A. Stern, Applied Phys. Letters 4, 80 (1964).

such that they excite the main and the first Tonks-Dattner resonances for a plasma, the nonlinear interaction between the corresponding oscillation modes appears as cross modulation between the two signals. The interaction between the oscillations is characterized by a sharp rise time and appears as an increase or a decrease in reflected power at the "wanted" frequency during the time interval in which the "disturbing" wave is turned on. The reflected power is enhanced or attenuated depending on the ratio of the Tonks-Dattner frequency to the microwave signal frequencies.

Stern points out that this phenomenon cannot be the Luxembourg effect because of the low power levels and high electron temperatures at which he was working (nor can the Luxembourg effect explain the sharp rise time of the interaction or the reversal in sign in cross modulation with varying frequency).

In some more recent work, Stern[14] has observed other nonlinear effects related to electron-plasma oscillations of a plasma column. With the same experimental configuration used in the earlier studies, he observed the generation of the second harmonic of the incident signal as well as beats between the fundamental frequencies of two simultaneously excited oscillations. These nonlinear effects were observed only when the frequencies involved coincided closely with the Tonks-Dattner resonances. Between resonances, no harmonic or beat frequency signals were detected. He also observed that the amplitude of the second harmonic signal depended quadratically on the power of the incident signal. He attributed the observed phenomena to the inherent nonlinear properties of plasma-electron oscillations.

---

[14] R. A. Stern, Phys. Rev. Letters 14, 538 (1965).

All of the experiments indicate that nonlinear effects are frequently observed when the signals involved coincide with some characteristic frequency or frequencies of the plasma system. These frequencies are characteristic of the plasma together with its boundary conditions. The only exception which we have discussed is the Luxembourg effect, which was included for historical reasons.

II. EXPERIMENTAL APPARATUS

In order to produce the plasma for our investigation the experimental apparatus must include a discharge electrode assembly, a means of adjusting the composition and pressure of the gas, a high voltage source to operate the discharge, a magnetic field to contain the particles by forcing them to travel in a helical path along the field lines, and means of measuring the discharge parameters.

The discharge electrode assembly and a vacuum system were constructed from available parts and materials. The composition and pressure of the gas in the discharge region were adjusted by pumping the system against a controlled leak. The high voltage was produced by a regulated power supply while the magnetic field was produced by two large coils connected to a magnet power supply. The high voltage was set by the control knobs on the high voltage supply. The DC component of the discharge current was measured by using a resistor in the high voltage line and measuring the voltage drop across the resistor with an electrometer. A Hall-effect Gaussmeter was used to measure the magnetic field and an alphatron vacuum gauge was used to measure the pressure.

A variable signal generator and modulator were used to apply a very high frequency (VHF) signal to the anode of the discharge, resulting in the observed nonlinear effects in the discharge. The experimental apparatus must provide a means of detecting a small change in the DC level due to the applied rf signal; this was done by means of an amplifier and a synchronous detector which detected the modulation of the direct current. The frequency of the VHF signal was varied by driving the tuning dial of the signal generator with a synchronous motor and was read from a dial on the signal generator. The amplitude of this signal was read from the signal generator controls.

A block diagram of the experimental arrangement is shown in Figure 1. The discharge assembly was placed coaxially within the air-core magnet. A spectrum analyzer was used to detect whatever oscillations might occur in the discharge. An internal signal from the lock-in "amplifier" (to be described in more detail later) was used to modulate the VHF signal applied to the anode of the discharge assembly. The synchronous motor used to sweep the VHF signal operated at 1 RPM. The motor, as well as a power plug, starting capacitor, an on-off switch, and a forward-reverse switch were all mounted in an aluminum box which in turn was mounted on the front panel over the frequency control knob shaft of the signal generator.

The current measuring resistor had a resistance of 1000 ohms and was placed between the negative terminal of the high voltage power supply and ground. The potential difference across this resistor, in volts, was thus equal to the discharge current in milliamperes. The electrometer which measured the voltage drop across this resistor was also used as an amplifier for small changes in current. The output for the latter use was fed to a synchronous detector. The output of this detector was applied to the Y axis of an X-Y Recorder. The X-input was operated by an internal, calibrated time base.

A junction box was used as a mechanical support for the radio frequency and direct current connections to the anode. A 51 ohm resistor between

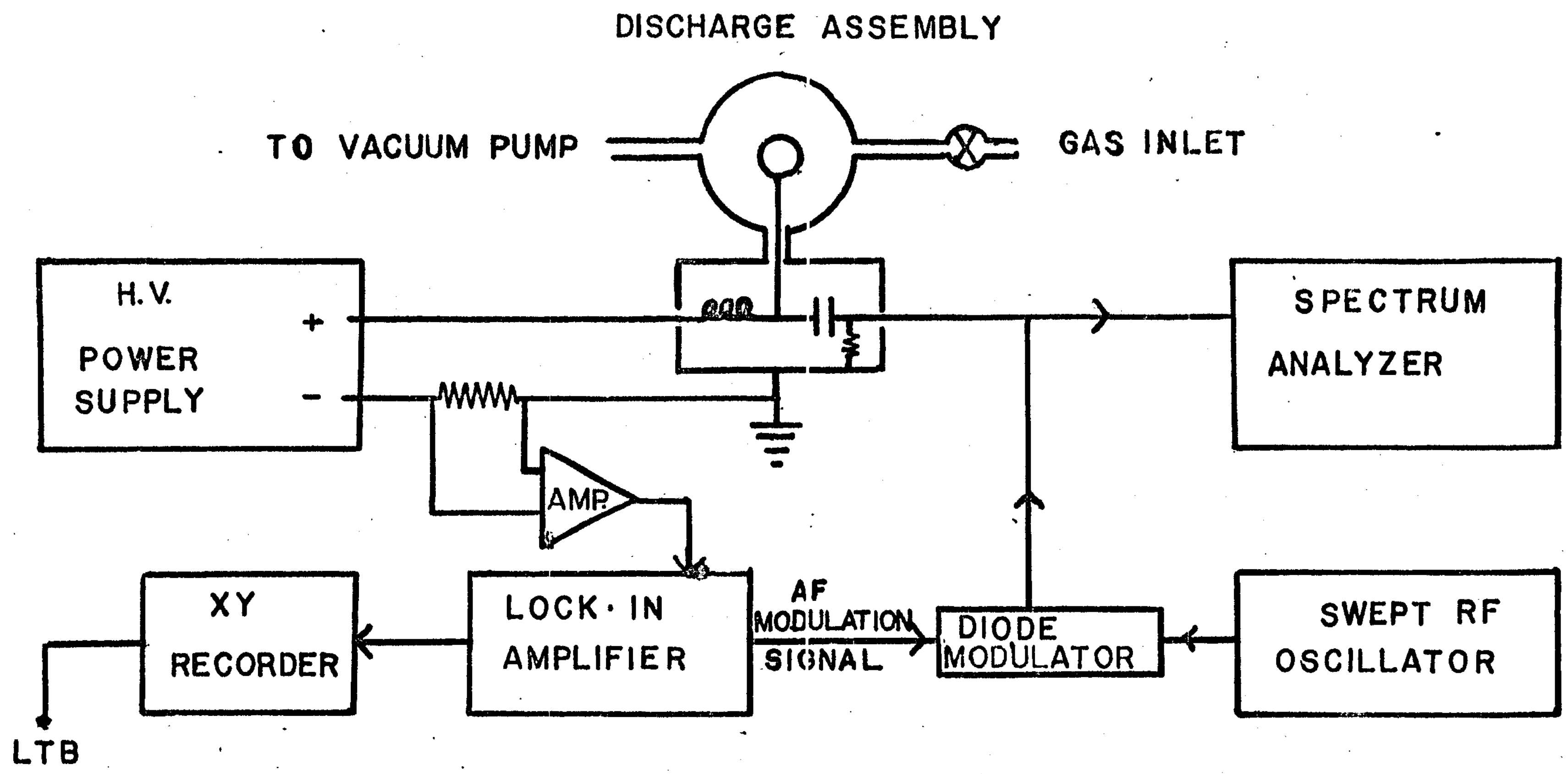

FIGURE I. BLOCK DIAGRAM OF EXPERIMENTAL ARRANGEMENT the capacitor and ground was provided for impedence matching purposes. The junction box also served as a radio frequency shield over the anode connections. The box was of aluminum with a removable cover which allowed access to the coupling connection to the anode lead.

The high voltage power supply had an output voltage adjustable from 500-6000 volts and was capable of supplying up to 20 ma. A 56,000 ohm, 50 watt resistor in series with the high voltage output was used to limit the current, should breakdown occur across the discharge.

The magnetic field was provided by a pair of coils mounted in a Helmholtz configuration, shown in Figure 2. Calculations of the field along the axis of the coils show that there should be only to the order of .1% variation over a distance of 2 inches on either side of the midplane.[15] Magnetic field measurements, using a Hall-effect Gaussmeter, substantiate these calculations. The magnetic field strength in this configuration was variable from 0 to 1500 gauss.

The vacuum system was of standard design. The high vacuum was provided by a mechanical roughing pump which backs an 85 liter/sec. air cooled oil diffusion pump. The remainder of the system consists of a ballast tank, a liquid nitrogen cold trap, and the 4.75 inch I.D. vacuum chamber. The vacuum chamber was equipped with a viewing port to allow observation of the discharge. A system of valves and the ballast tank were incorporated to permit the vacuum chamber to be exposed to atmospheric pressure without cooling the diffusion pump. Removal of the end plate of the vacuum chamber then allowed access to

---

[15] F. R. Crownfield, Jr., Rev. Sci. Inst. 35, 240 (1964).

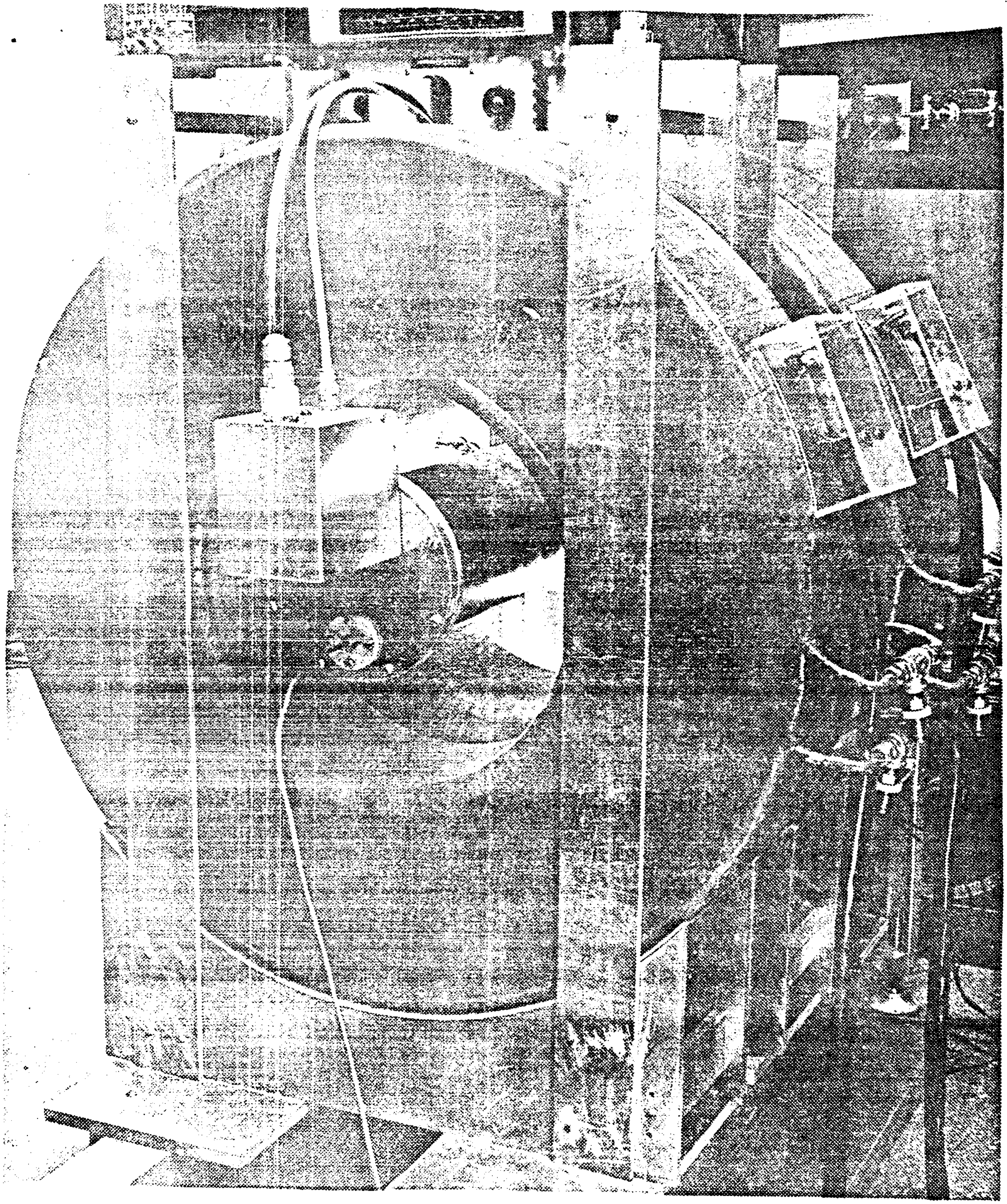

FIGURE 2. Magnet Coil and Vacuum Chamber Arrangement

the electrodes for mechanical adjustments. Provision was also made in the end plate to connect the controlled leak. The leak was adjustable to allow operation at a chosen value of pressure, and provided a means of admitting gases other than air into the system.

The forepressure was monitored by a thermocouple gauge mounted in the top of the ballast tank. An alphatron vacuum gauge was used to measure the pressure in the high vacuum chamber. The arrangement of the alphatron vacuum gauge with respect to the controlled leak and the vacuum chamber is shown in Figure 3. The alphatron used had a range from 10^{-5} mm. Hg. to atmospheric pressure.

A photograph of the electrode configuration is shown in Figure 4. The basic structure consists of an anode placed midway between two end plates in a plane parallel geometry. A cylindrical copper screen encloses the end plates and connects them electrically. The end plates plus the screen thus form a cylindrical cathode. The anode is a wire ring, allowing the electrons to pass back and forth. The mechanical support for the discharge components consists of two brass rods mounted to the removable end plate of the vacuum chamber. The cathode end plates are 6 cm. diameter aluminum discs mounted on metal stand-offs, which are attached to brass mounting brackets. These brackets slide along the brass rods allowing any desired plate spacing. They may be fixed in position by nylon set screws. An aluminum arch, also mounted on the brass rods, supports the anode. The anode was constructed of #18 Chromel-A wire which allowed operation at higher discharge currents than was possible with copper wire. The anode was designed to be interchangeable

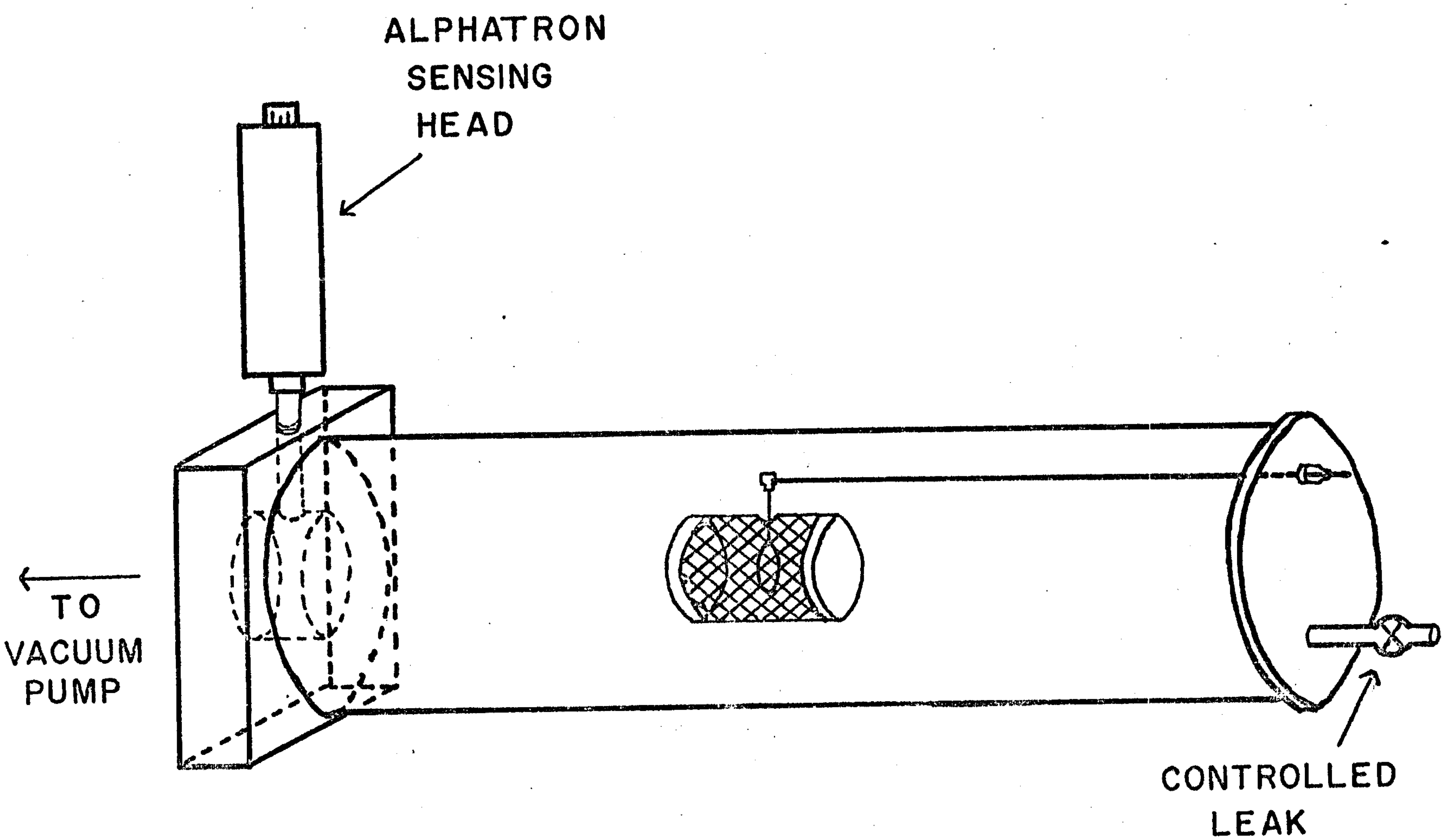

FIGURE 3. PLACEMENT OF PRESSURE GAUGE WITH RESPECT TO VACUUM CHAMBER AND CONTROLLED LEAK.

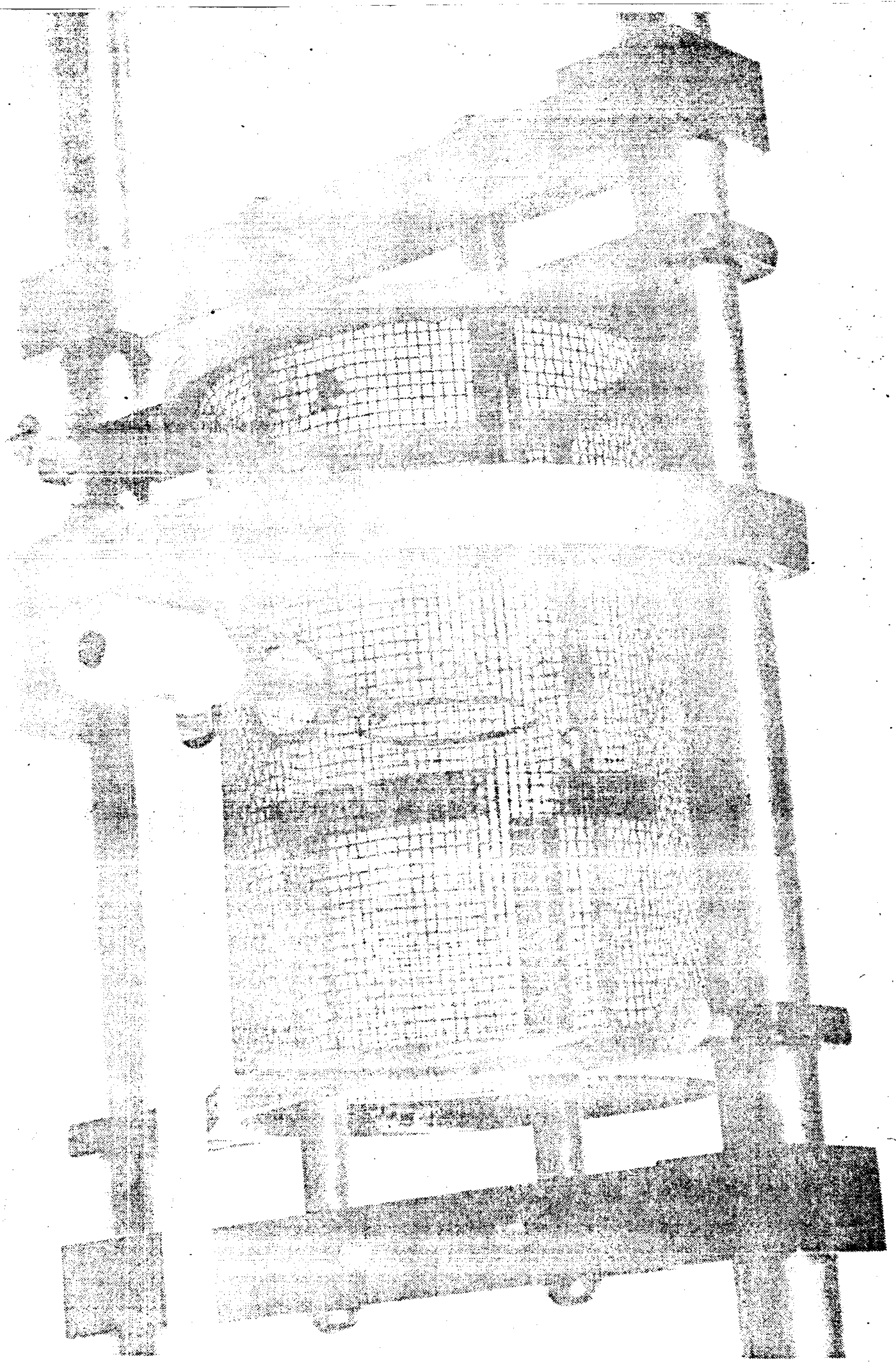

FIGURE 4. Discharge Electrode Assembly

allowing various sizes to be used. The diameter used for the present experiments was 2.5 cm. The cylindrical configuration of the electrode assembly was used to obtain the slow wave modes observed and reported earlier.[2] The support rod for the anode is glass-insulated and is mounted in a section of half-inch Teflon rod. This is used to insulate the anode from the arch and from the brass rods, which are at cathode (ground) potential. The anode is connected to a metal-to-glass "Housekeeper" seal in the end plate of the vacuum chamber by means of an 8 inch length of small brass rod insulated with Teflon tubing. This insulation consists of two overlapping sections which were slotted to allow trapped air to escape. This increased the stability of the discharge, eliminating a problem which plagued our earlier experiments. The glass seal insulates the anode lead from the vacuum chamber, which is at ground potential.

The signal in which we are interested is a small change in the relatively large direct current drawn by the discharge. The detection system must be able to selectively amplify the change in the DC level due to the VHF signal without responding to changes due to drift in discharge conditions, etc. The desired signal is therefore modulated at audio frequency and the change in current detected synchronously with a lock-in "amplifier" at the modulation frequency.

Basically, the lock-in "amplifier" is a synchronous detector which operates with an exremely narrow equivalent noise bandwidth. A block diagram of the instrument is shown in Figure 5. From an applied signal spectrum at the input, a band of frequencies is selected and the information in that band is converted to an equivalent bandwidth

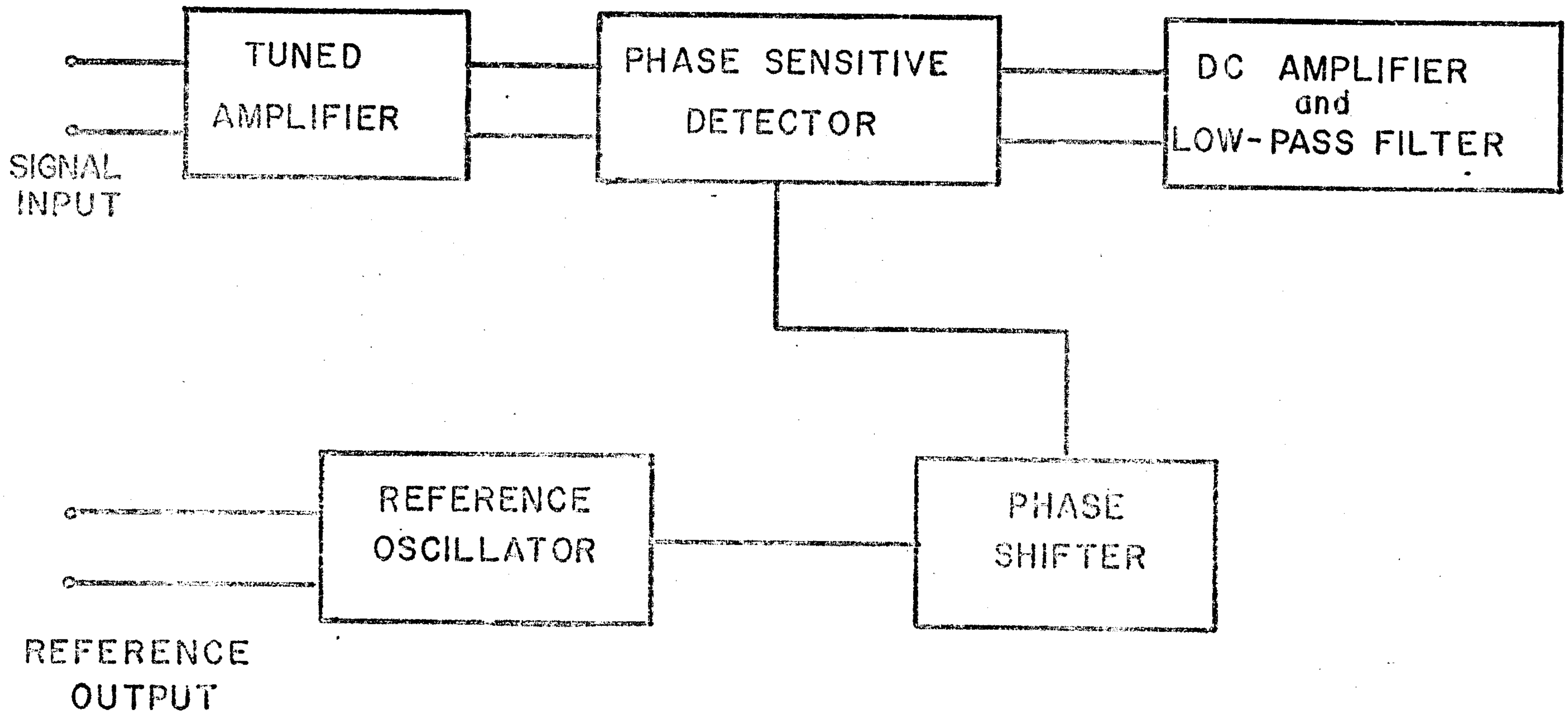

FIGURE 5. BLOCK DIAGRAM OF LOCK-IN AMPLIFIER

at DC. The center frequency of the band is chosen to be equal to the frequency being used as a reference (the audio modulating frequency, in our case).

With the signal to be detected applied at the input, a desired fraction of the signal voltage can be selected by a "signal level" control and sent to the signal channel tuned amplifier. The tuned amplifier considerably restricts the bandwidth over which signals are accepted. The output of the tuned amplifier is then fed to the phase sensitive detector where the signal and reference frequencies are mixed, producing sum and difference frequencies at the detector output. The signal is next applied to a low-pass filter and a DC amplifier which passes only the difference frequency. The difference frequency is direct current proportional to the signal level at the reference frequency. Difference frequencies due to components of the signal differing from the reference frequency by less than the bandwidth of the tuned amplifier are attenuated by the low-pass filter. This results in an output due only to the narrow band of frequencies in the signal spectrum that lie near the reference frequency, the bandwidth being determined by the pass band of the filter. The variable phase shifter may be used to adjust the relative phase of the measured signal and the reference signal.

The detector is phase sensitive because, for a fixed amplitude, the DC output from it due to a signal at the reference frequency is proportional to the cosine of the phase difference between the detected signal and the reference signal. The detector acts as a double-pole double-throw switch being driven at the reference frequency. Detector output wave forms for several phase differences are shown in Figure 6. For a phase difference of 90° between the signal and reference frequencies, the time-average output of the detector is zero. For in-phase operation, the time-average output is positive while for a phase difference of 180°, the time-average is negative. It can be seen that the detector is sensitive to odd harmonics of the reference frequency, as well as the fundamental. However, the signal channel tuned amplifier serves to attenuate the harmonic components of the signal spectrum.

The reference signal is obtained from a 5 volt peak to peak sine wave produced by an internal oscillator. It is fed to the variable phase shifter and then converted to a square wave for application to the phase-sensitive detector. The sine wave, prior to the phase shifter, is also available at the reference output terminals. It is this signal which is used to modulate the VHF signal applied to the anode of the discharge using a crystal diode modulator. The crystal diode resistance varies very markedly with the voltage across it and thus the diode can be used as a switch by applying an alternating voltage. By choosing the operating conditions, it is also possible to operate so that the envelope of the output signal nearly reproduces the modulating signal. Inserted between the signal generator and the load, the modulator produces variations in amplitude corresponding to the variations in voltage output from the reference oscillator.

This system, using the electrometer as an amplifier, is capable of detecting a signal of approximately 10^{-10} amperes superimposed on the steady discharge current of the order of 10^{-3} amperes to several milliamperes.

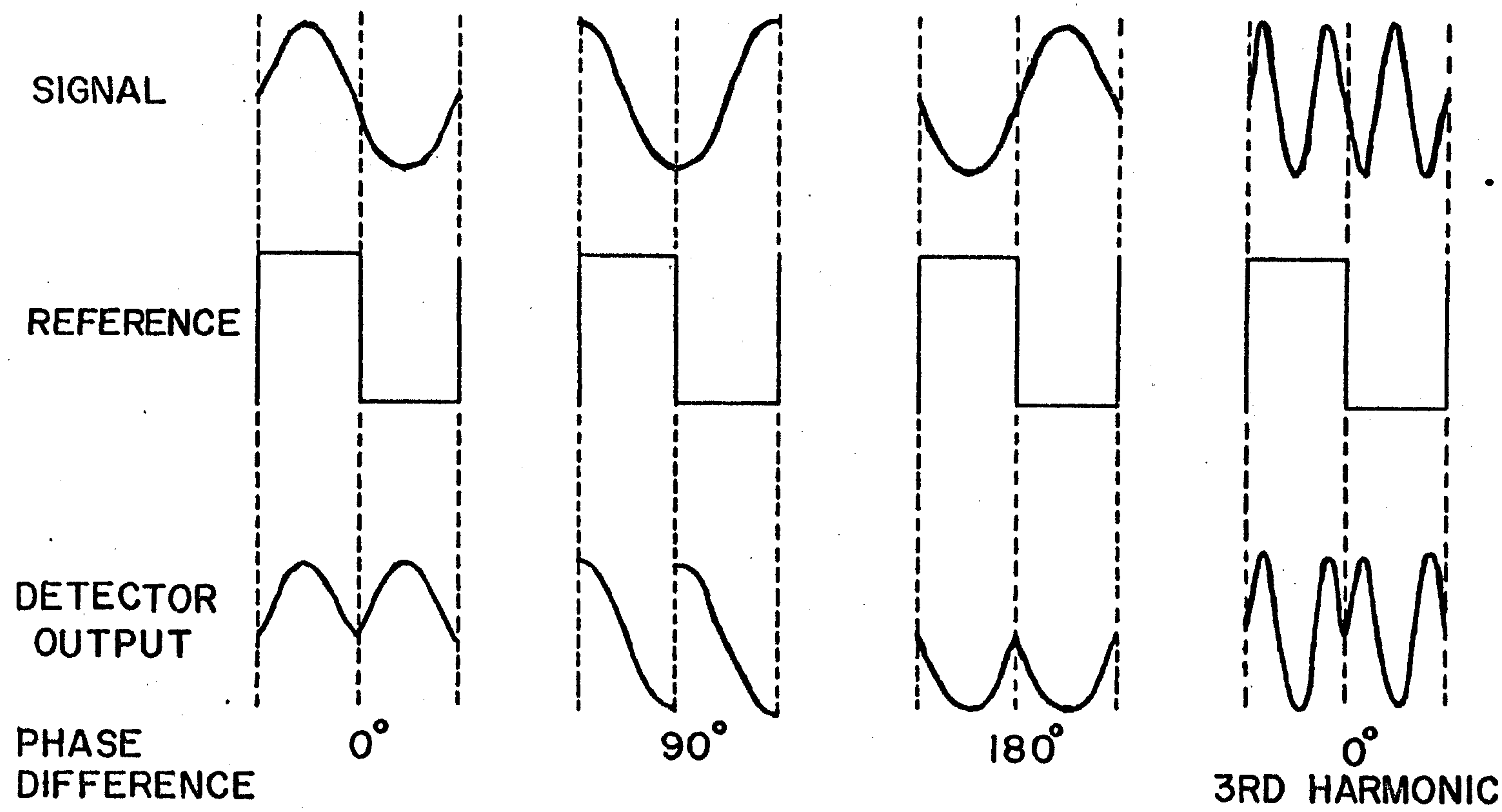

FIGURE 6. DETECTOR OUTPUT WAVEFORMS

III. PROCEDURE AND RESULTS

The procedure was to set the magnetic field at a value for which oscillations were detectable in the earlier experiments,[2] and adjust the variable leak so that the pressure was of the order of 5×10^{-4} mm. Hg. Except as noted later, dry air was used in the discharge. The anode voltage was varied until oscillations occurred, as observed by the spectrum analyzer. The voltage was then changed so that the oscillations just disappered. Following this the modulating rf signal was applied to the anode and the frequency swept through a range on either side of the previously observed oscillation frequency. The sweep was started by manually energizing the sweep motor. The Y-range of the recorder was set so that the maximum response corresponded to a convenient deflection. The X-range was set on the internal time base such that a convenient frequency range could be covered in one sweep. The variation of the frequency with time was not linear, so that a calibration of the frequency axis was necessary. This calibration was accomplished by manually switching an input signal to the Y axis on or off at frequency values for which markers were desired, as the frequency was swept. Since this procedure was quite reproducible, one calibration sufficed for each combination of motor speed and sweep speed used.

Typical DC response as a function of frequency is shown in Figure 7 and Figure 8. For this data the magnetic field was 185 gauss, and air was used at a pressure of 5×10^{-4} mm. Hg. The anode voltage

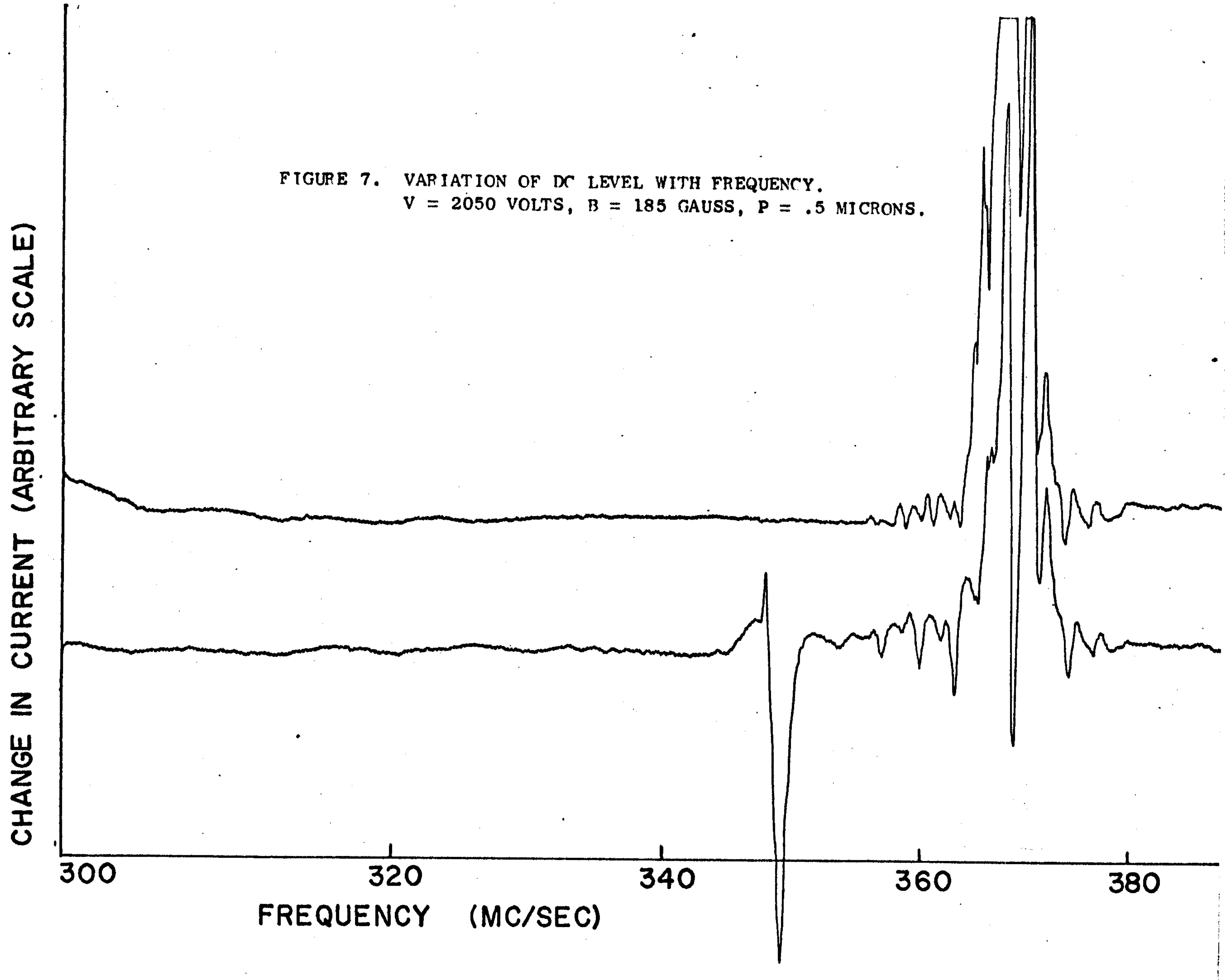

FIGURE 7. VARIATION OF DC LEVEL WITH FREQUENCY.
V = 2050 VOLTS, B = 185 GAUSS, P = .5 MICRONS.

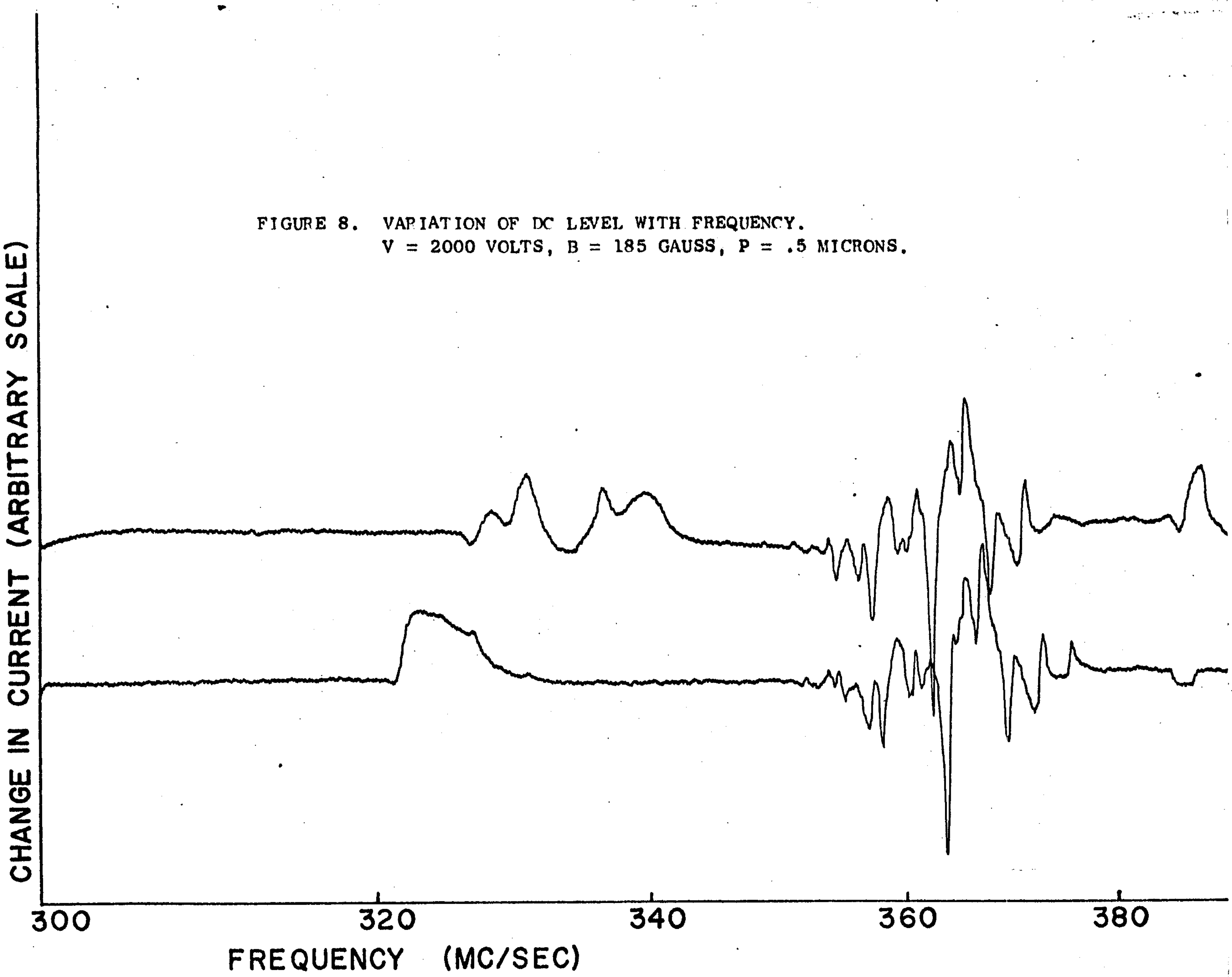

FIGURE 8. VARIATION OF DC LEVEL WITH FREQUENCY.
V = 2000 VOLTS, B = 185 GAUSS, P = .5 MICRONS.

was 2050 volts for the data in Figure 7, and 2000 volts in Figure 8. The lower trace in each case was made under similar conditions in an attempt to reproduce the data. The zero of the Y-axis was shifted to avoid overlap. For these runs a 3.2 RPM synchronous motor was used to sweep the frequency. The internal time base of the recorder was set at 5 seconds per inch.

Pressure transients in the discharge often cause the electron oscillation frequency to jump discontinuously by as much as 10 Mc/sec. and may somtimes appear on the recordings as two closely separated but distinct oscillations. This, as well as noisy or broadband oscillations, often gives spurious responses.

The changes in current occurred at frequencies near the oscillation mode frequencies observed previously[2] for similar conditions of the discharge. The structure within the resonance were highly repeatable. The spike to the left of the resonance in Figure 7 was due to a transient. The traces in Figure 8 are for different discharge conditions, which might be expected to shift the frequency slightly.

In order to find the shift of the resonance frequency due to changing the anode voltage of the discharge, we made several traces of change in current vs. frequency, changing the anode voltage about one percent from one trace to the next. The frequency of the maximum and minimum DC response of each resonance was found the is plotted in Figure 9. No maximum or minimum was observed above 2230 volts or below 1820 volts. For these data, the magnetic field was set at 240 gauss and the pressure was 4×10^{-4} mm. Hg. The 1 RPM drive motor was used on these and all subsequent runs. The internal time base was

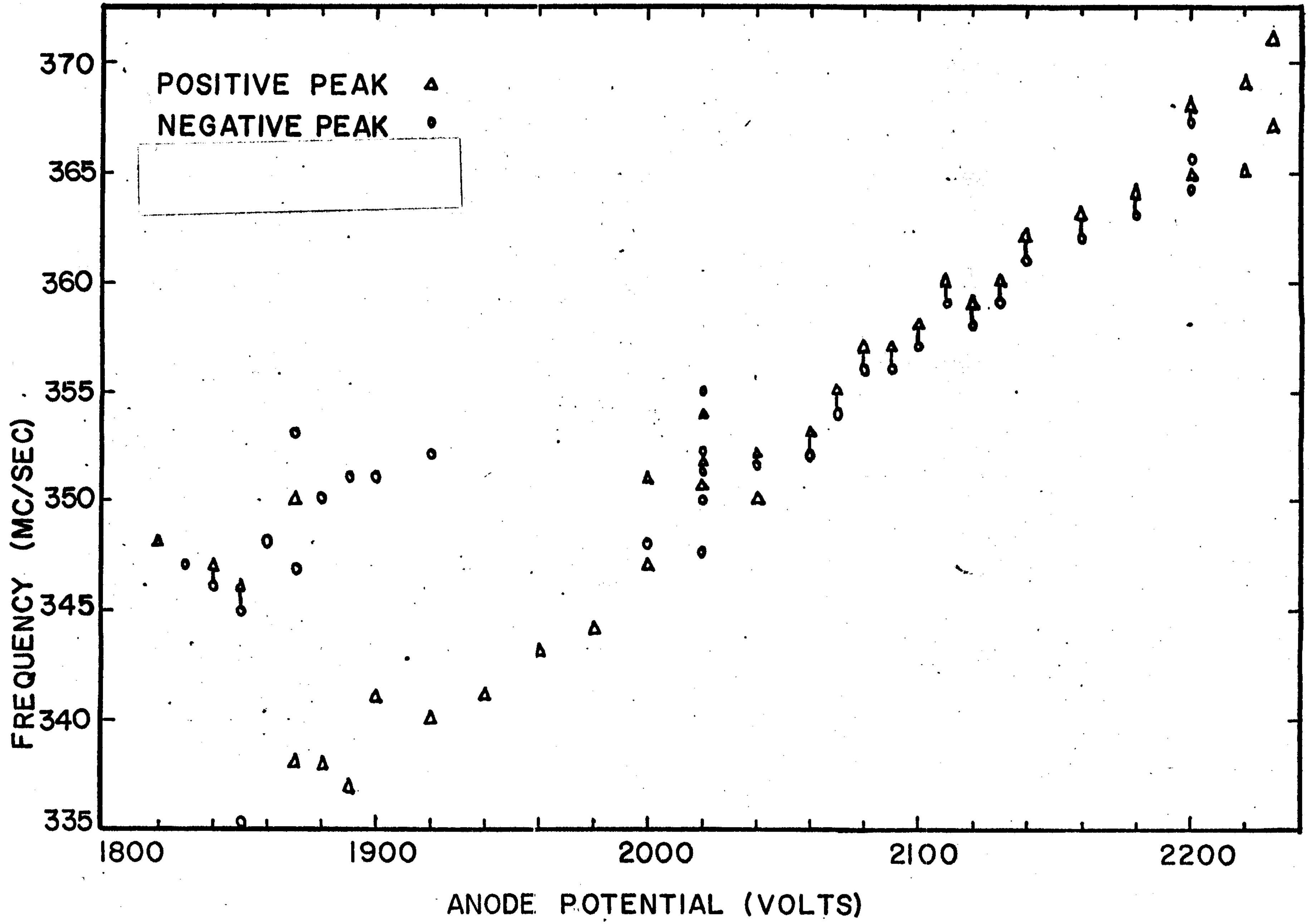

FIGURE 9. OSCILLATION FREQUENCY VS. ANODE VOLTAGE.
B = 240 GAUSS, P = .4 MICRONS.

set at 10 seconds per inch.

A change in the shape of the resonances for slight changes in voltage was seen to occur. This is shown in Figure 10, where several recorder tracings are shown with anode voltage as a parameter. The Y-axis zero has again been displaced for each trace. The pressure, magnetic field and internal time base were the same as in the previous graph. The voltage for the top trace was 2140 volts, increasing in 20 volt steps to 2220 volts for the lowest trace.

The changes in the DC maxima and minima as a function of voltage are shown clearly in Figures 9 and 10. As the voltage was increased, the peak shifted toward higher frequencies. A straight line drawn through the points in Figure 9 shows a change in frequency of approximately 7 Mc/sec. per 100 volts. Measurement of the change in frequency of the oscillations described in reference 2. also showed change in frequency of the same order for modes in the frequency range we are considering. The change in shape of the resonance at voltages corresponding to higher oscillation frequencies leads one to suspect the possibility that the modulating signal couples simultaneously to several of the oscillation modes of the discharge. The change in shape is often accompanied by a change in strength of the signal.

We have also made runs in which we varied the level of the applied rf signal. A plot of the peak-to-peak amplitude of the DC response at resonance, as a function of applied voltage, is shown in Figure 11. The voltage was varied by means of the attenuation control knob on the signal generator. For these runs, the magnetic field was set at 195 gauss, the high voltage on the anode of the discharge at 1750 volts,

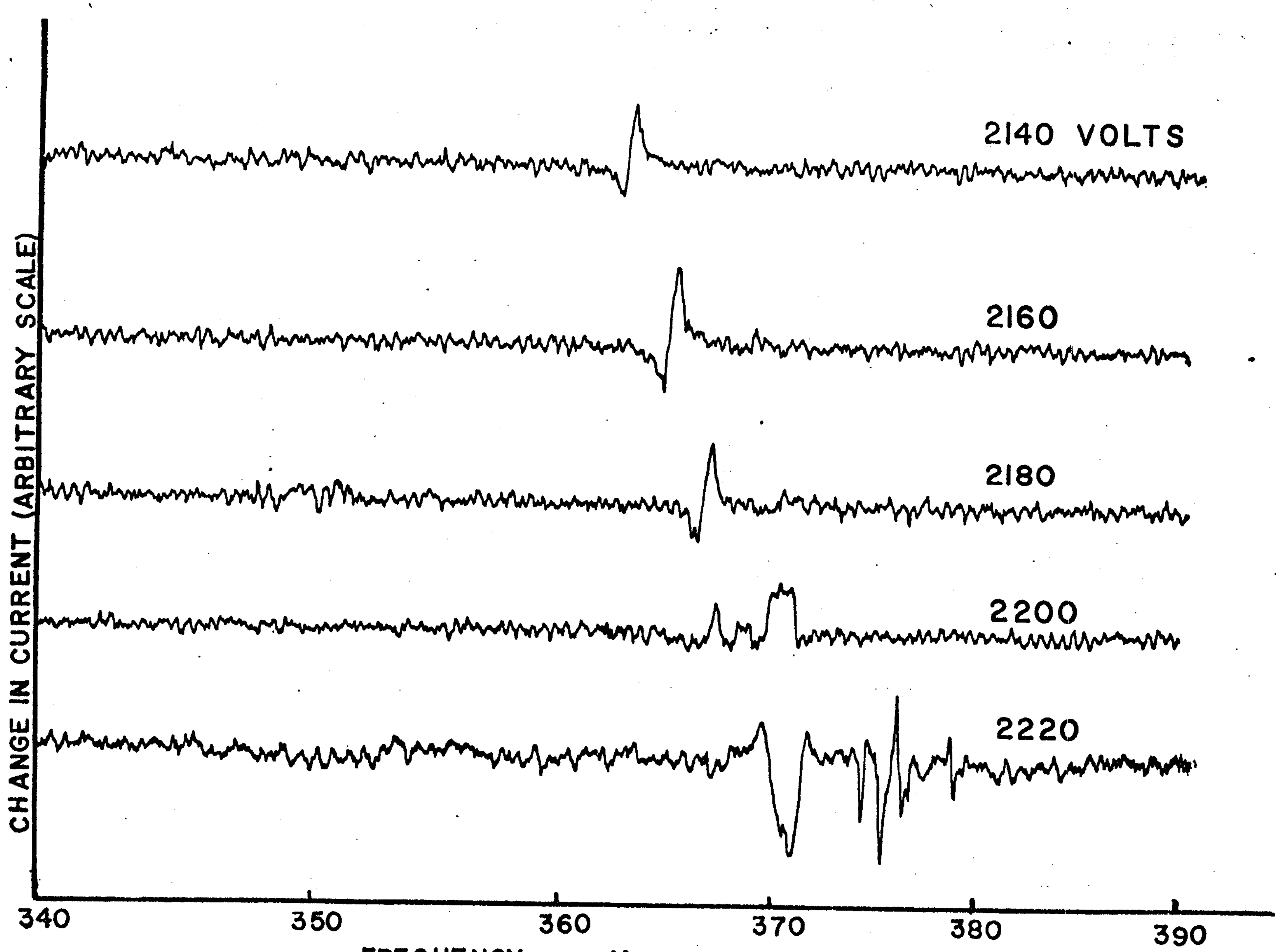

FIGURE 10. CHANGE IN RESONANCE SHAPE. VARIATION OF DC LEVEL WITH FREQUENCY. B = 240 GAUSS, P = .4 MICRONS.

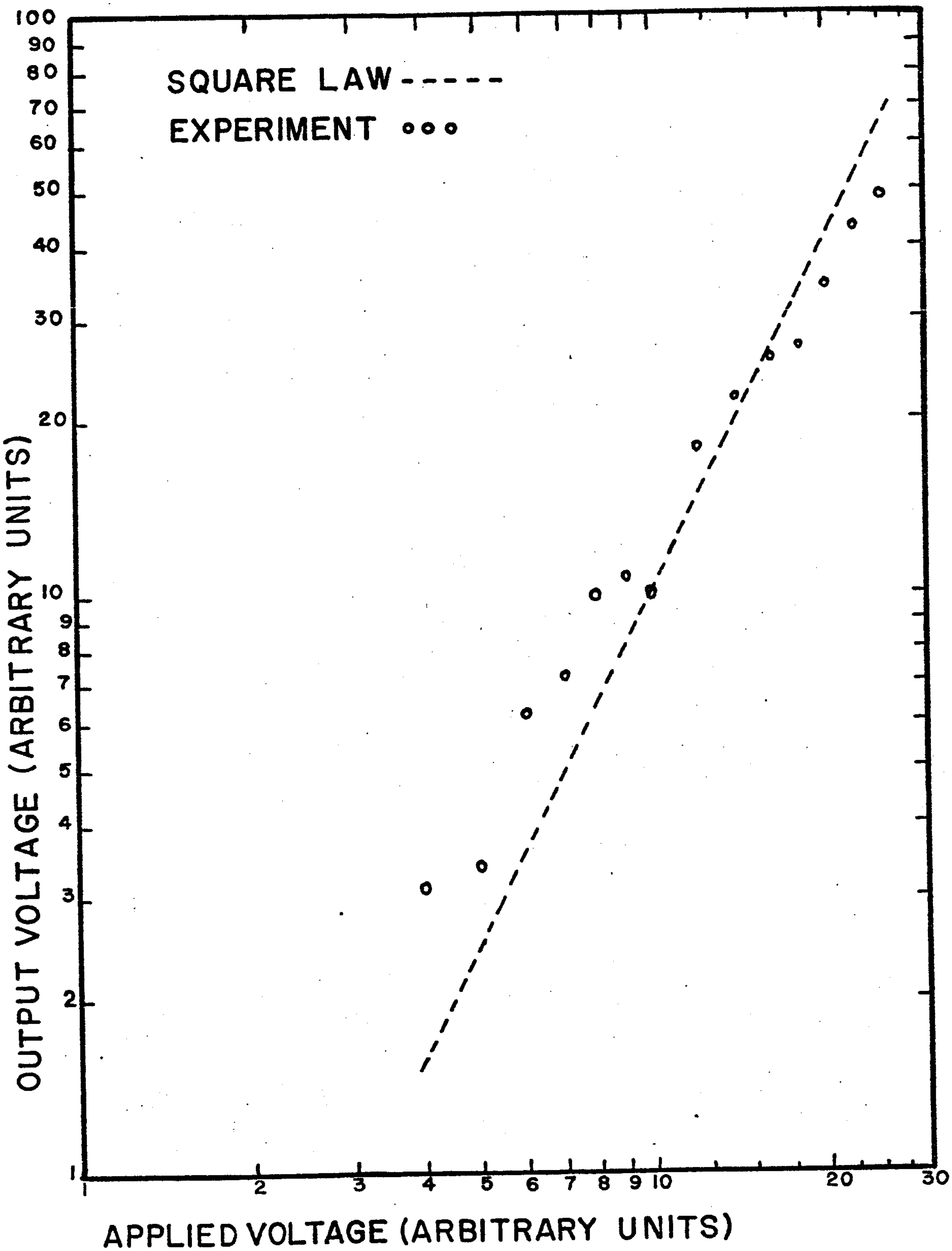

FIGURE 11. PEAK TO PEAK AMPLITUDE AT RESONANCE OF DC RESPONSE VS. APPLIED VOLTAGE. V = 1750 VOLTS, B = 195 GAUSS, P = .35 MICRONS.

and the pressure to 3.5×10^{-4} mm. Hg.

The results shown in Figure 11 indicate an approximately quadratic dependence as would be expected of a second-order nonlinear phenomenon. The deviation from quadratic dependence we attribute to an inherent shortcoming in the method used to modulate the VHF signal. As the applied signal is increased by the attenuation control knob on the signal generator, the depth of modulation decreases; the detected response thus does not increase as rapidly as it should. This effect becomes more pronounced at the upper end of the power scale as evidenced by the falling off of the DC response from the expected value.

Some experimental runs were made with helium rather than air in the discharge with no detectable differences in the results. It thus appears that, in accord with our earlier remarks, the phenomena observed are essentially associated with the electrons rather than the ions.

More recently, an electronically swept oscillator was used for performing the same experiments. This allows the frequency to be swept more rapidly and through a wider range of frequencies. The faster sweep reduces spurious responses due to pressure transients and discharge drift effects, and thus improves reproducibility; the wider range will allow us to examine other parts of the spectrum in the same experiment. Preliminary results show good agreement with results obtained by the earlier method, and reproducibility is considerably improved. In addition, the signal generator used has built-in provisions for modulation which may obviate the changes in depth with signal level.

IV. CONCLUSIONS

The direct current drawn by the discharge shows a definite "resonance" for applied frequencies near those characteristics of the oscillating discharge. As the anode potential of the discharge varies, the frequency at which resonance occurs varies with essentially the same proportionality of change in frequency to change in voltage as observed for the oscillations described in reference 2 with no applied VHF signal. This strongly substantiates that the observed resonances are closely related to the characteristic frequencies of the discharge. The observed resonances often show much structure, apparently corresponding to the fact that there are many oscillation modes in the frequency range of these experiments. The nonlinear effect appears to be a second-order one since the amplitude of the response is quadratic in the amplitude of the applied VHF signal.

The effect cannot be explained by the nonlinear volt-ampere characteristic of the discharge, described in reference 2, since this would be frequency independent.

The observed behavior is somewhat analogous to the behavior of the resonance probe.[10] One should note, however, that the frequencies are characteristic of the gross geometry of the discharge, rather than the plasma frequency alone. This indicates that an infinite plasma, slightly modified by a sheath, cannot be assumed, and confirms the comments of O'Brien, et al.[11]

Further, it should be noted that our results agree with the observations of O'Brien, et al.,[11] and of Stern[13] in that nonlinear effects are noted only when the frequencies involved correspond to characteristic modes of the system.

Results of experiments to date are consistent with a second-order effect. This would be expected to predominate since the next order which gives a DC term is of fourth order. This part of the experiment, as well as attempts to see frequencies corresponding to other modes for which oscillations are observed, should be repeated with certain refinements. These refinements should include the use of the electronically swept signal generator, with a wider sweep width. The signal generator used in the present experiments had a low frequency limit of 250 Mc/sec. which is above many of the oscillation modes of reference 2. A more careful investigation of the quadratic dependence should also be undertaken. The gain controls in the detection system should be calibrated to insure against change of calibration when changing level control settings. Also, the placement of a calibrated attenuator after the modulator would permit changing signal level without altering the depth of modulation.

Occasionally, the experiments of reference 2 found two modes at approximately frequencies of f and 2f. It may be possible to detect the second harmonic if the fundamental is applied to the anode, although this would be difficult because mixers, modulators, and detectors tend to generate harmonics.

Theoretically, it might be expected that a system having discrete modes might be easier to treat with "mode coupling theory" than an infinite system (which allows all spatial wave numbers instead of the discrete set prescribed by the boundary conditions.) One should also investigate exact methods of solution (Bernstein[16]) to see if they may be applicable. In addition, a detailed theory will probably require a detailed discussion of (DC) conduction in a discharge in a magnetic field; this is not a trivial problem.

---

[16] I. R. Bernstein in Radiation and Waves in Plasmas, Morton Mitchner, ed. (Stanford University Press, Stanford, (1961)).

BIBLIOGRAPHY

Bailey, V. A. and Martyn, D. F., Phil. Mag. 18, 369 (1934).

Bernstein, I. B. Radiation and Waves in Plasmas, Morton Mitchner, ed. (Stanford University Press, Stanford, 1961).

Crownfield, F. R., Jr., Proc. VIth International Conference on Ionization Phenomena in Gases, Paris 1963 (S.E.R.M.A. Publishing Company, Paris, 1964) 2, 451.

Crownfield, F. R., Jr., Rev. Sci. Inst. 35, 240 (1964).

Dattner, A., Phys. Rev. Letters 10, 205 (1963).

Dennis, R. N., Jr., Maters Thesis, College of William and Mary, Williamsburg, Va., (1962).

Goldstein, L., Anderson, J. M., and Clark, G. L., Phys. Rev. 90, 151 (1953).

Harp, R. S. and Crawford, F. W., Aerospace Research Lab., Report No. ARL 64-139 (1964) (DDC No. AD 608613).

Ichikawa, Y. H. and Ikegami, H., Progr. Theoret. Phys. 28, 315 (1962).

Jackson, John David, Classical Electrodynamics (John Wiley & Sons, Inc., New York, 1962).

Loeb, L. D., Basic Processes of Gaseous Electronics (University of California Press, Berkeley, 1955).

O'Brien, B., Gould, R. W., and Parker, J., Phys. Rev. Letters 14, 630 (1965).

Penning, F. M. Physica 4, 71 (1937).

Simpson, K. M., High Vacuum Equipment and Technique, A. Guthrie and R. K. Wakerling, eds., (McGraw-Hill Book Company, Inc., New York, 1949).

Stern, R. A., Applied Phys. Letters 4, 80 (1964).

Stern, R. A., Phys. Rev. Letters 14, 538 (1965).

Takayama, K., Ikegami, H., and Miyazaki, S., Phys. Rev. Letters 5, 238 (1960).

Tellegen, B. D., Nautre 131, 840 (1933).

Tonks, L., Phys. Rev. 37, 1458 (1931).

Wimmel, H. K., Institute of Plasma Physics, Report No. 6/11, Garching bei München, Germany (1963).

VITA

Daniel Burnley Raiford

Born [redacted] Graduated from Henry Clay High School, Ashland, Virginia in 1955. Served four years in the U. S. Air Force (1956-1960). Attended the Daytona Beach Junior College, Daytona Beach, Florida. Received the B. S. degree in mathematics from Guilford College, North Carolina in June, 1963.

In September 1963, the author entered the College of William and Mary as a graduate student and research assistant in the Department of Physics.