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TOTAL THERMAL CONDUCTIVITY OF PARTIALLY AND FULLY IONIZED GASES

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The calculation of the total thermal conductivity within the framework of the Chapman-Enskog formulation is based on the expression for the heat flux vector[1]

$$\underset{\sim}{q} = \sum_i n_i h_i \underset{\sim}{V}_i - \lambda_t \frac{\partial T}{\partial \underset{\sim}{r}} - nkT \sum_i \frac{1}{n_i m_i T} D_i^T \underset{\sim}{d}_i \tag{1}$$

where D_i^T and D_{ij} are the thermal diffusion and multicomponent diffusion coefficients, and h_i is the total enthalpy of a particle of species i. The diffusion velocity is defined as

$$\underset{\sim}{V}_i = \frac{n^2}{n_i \rho} \sum_j m_j D_{ij} \underset{\sim}{d}_j - \frac{1}{n_i m_i T} D_i^T \frac{\partial T}{\partial \underset{\sim}{r}} \tag{2}$$

and the "forcing potential" is defined as

$$\underset{\sim}{d}_i = \frac{\partial x_i}{\partial \underset{\sim}{r}} - \frac{n_i m_i}{p \rho} \left(\frac{\rho}{m_i} \epsilon_i \underset{\sim}{E}_i - \sum_j n_j \epsilon_j \underset{\sim}{E}_j \right) \tag{3}$$

where ϵ_i is the charge, $Z_i e$, for a particle of species i, and $\underset{\sim}{E}_i$ is the electric field.

[redacted]

Recent approaches[2-5] to the derivation of the total thermal conductivity of partially and fully ionized gases have attempted to resolve the forcing potential (Eq. (3)) into two components. The first component is expressed in terms of a temperature gradient, and the second, in terms of the charge separation field. This resolution was brought about in an attempt to duplicate Spitzer and Härm's[6] expression for the heat flux vector, which is valid only for a binary plasma (i.e., equal numbers of ions and electrons). Their expression is

$$\mathbf{q} = -\lambda_s(\partial T/\partial \mathbf{r}) + \beta \mathbf{E} \tag{4}$$

In doing so, simplifications to the concentration gradient terms and/or charge separation terms were made. As a result, the charge separation correction to λ_s could then be found by use of Onsager's reciprocal relation.[7]

However, an inspection of the Chapman-Enskog expression for $\mathbf{q}$ for a binary plasma shows the following functional dependence,

$$\mathbf{q} = -\lambda_t(\partial T/\partial \mathbf{r}) + \beta' \mathbf{E} + \gamma(\partial x/\partial \mathbf{r}) \tag{5}$$

If it is assumed that both approaches are equivalent, then the question arises as to which of the two terms in Eq. (4) contains the last term, $\gamma(\partial x/\partial \mathbf{r})$, in Eq. (5). This note does not attempt to find the correspondence of these terms, but to present a method by which these problems are circumvented, and in which no simplifications of the $\partial x/\partial \mathbf{r}$ and $\mathbf{E}$ terms are necessary. The approach for a binary plasma will then be extended to obtain closed form expressions for the total thermal conductivity of a partially ionized plasma and a fully ionized ternary plasma (i.e., a mixture of singly and doubly ionized atoms and electrons).

[redacted]

The crux of this approach is not to separate the forcing potential, $\mathbf{d}$, into $\partial x/\partial \mathbf{r}$ and $\mathbf{E}$ components, but to solve for this quantity directly by making use of the conservation of the net flux for each species.[8] Physically, this means that an arbitrary value of the charge separation field will result in a readjustment of the concentration gradients so that the net flux is conserved. Therefore the combined effect in the form of $\mathbf{d}$ is a more logical variable than its components.

Binary plasma.- The conservation of fluxes of ions and electrons requires that the diffusion velocities of the ion and electron be identically zero. This is in accord with simple plasma theory where the more mobile electrons and ions travel in tandem due to the action of a charge separation field. Equation (2) then yields

$$\mathbf{d}_e = (\rho/m_I m_e n^2 D_{Ie} T) D_I^T (\partial T/\partial \mathbf{r}) \tag{6}$$

$$\mathbf{d}_I = (\rho/m_I m_e n^2 D_{eI} T) D_e^T (\partial T/\partial \mathbf{r}) \tag{7}$$

Substituting Eqs. (6) and (7) into the last term of Eq. (1) results in

$$\mathbf{q} = \left(-\lambda_t - \frac{k\rho}{n n_I m_e m_I^2} \frac{D_I^T D_e^T}{D_{eI}} - \frac{k\rho}{n n_e m_I m_e^2} \frac{D_I^T D_e^T}{D_{Ie}}\right) \frac{\partial T}{\partial \mathbf{r}} = \left(-\lambda_{total}\right) \frac{\partial T}{\partial \mathbf{r}} \tag{8}$$

It can be seen that the total thermal conductivity now can be expressed in terms of known quantities. Note that it was not necessary to employ Onsager's reciprocal relation. Numerical values of λ_{total} were obtained by combining the second-order Chapman-Enskog values of λ_t, D_i^T, and D_{ij} in Ref. 4 and the third-order corrections in Ref. 3. The resulting values were 10 to 15 percent higher than Spitzer and Härm's value for the total thermal conductivity.

<u>Partially ionized gases</u>.- The expression for the total thermal conductivity of a partially ionized gas undergoing the reaction $A \rightarrow I + e$ follows directly. The conservation of mass flux[7] requires that the diffusion velocities be related as follows

$$x_A \underset{\sim}{V}_A + x_I \underset{\sim}{V}_I = 0 \tag{9}$$

$$x_A \underset{\sim}{V}_A + x_e \underset{\sim}{V}_e = 0 \tag{10}$$

The additional equation

$$\sum_i \underset{\sim}{d}_i = 0 \tag{11}$$

which results from the choice of a mass averaged coordinate system,[1] completes the necessary system of equations. A set of linear equations in the unknowns $\underset{\sim}{d}_A$, $\underset{\sim}{d}_I$, and $\underset{\sim}{d}_e$ are obtained by combining Eqs. (2), (9), (10), and (11). The following determinantal expression for the various $\underset{\sim}{d}_i$ is then obtained by using Cramer's rule,

$$\underset{\sim}{d}_i = \frac{\partial T}{\partial \underset{\sim}{r}} \frac{\begin{vmatrix} a_{11} & a_{12} & a_{13} & -c_1 \\ a_{21} & a_{22} & a_{23} & -c_2 \\ a_{31} & a_{32} & a_{33} & -c_3 \\ \delta_{i1} & \delta_{i2} & \delta_{i3} & 0 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \equiv \delta_i \frac{\partial T}{\partial \underset{\sim}{r}} \tag{12}$$

where

$$\left.\begin{aligned}
a_{11} &= (n^2/\rho)m_A D_{IA} & a_{12} &= (n^2/\rho)m_I D_{AI} \\
a_{13} &= (n^2/\rho)m_e D_{Ae} + (n^2/\rho)m_e D_{Ie} & c_1 &= \left(D_A^T/m_A T\right) + \left(D_I^T/m_I T\right) \\
a_{21} &= (n^2/\rho)m_A D_{eA} & a_{22} &= (n^2/\rho)m_I D_{AI} + (n^2/\rho)m_I D_{eI} \\
a_{23} &= (n^2/\rho)m_e D_{Ae} & c_2 &= \left(D_A^T/m_A T\right) + \left(D_e^T/m_e T\right) \\
a_{31} &= a_{32} = a_{33} = 1 & c_3 &= 0
\end{aligned}\right\} \tag{13}$$

Combining Eqs. (1), (2), (12), and (13) gives the final expression for the total thermal conductivity.

$$\begin{aligned}
\underset{\sim}{q} = \Bigg[&-\lambda_t + \left(h_I \frac{n^2}{\rho} m_A D_{IA} + h_e \frac{n^2}{\rho} m_A D_{eA} - \frac{nkTD_A^T}{n_A m_A}\right)\delta_A \\
&+ \left(h_A \frac{n^2}{\rho} m_I D_{AI} + h_e \frac{n^2}{\rho} m_I D_{eI} - \frac{nkTD_I^T}{n_I m_I}\right)\delta_I \\
&+ \left(h_A \frac{n^2}{\rho} m_e D_{Ae} + h_I \frac{n^2}{\rho} m_e D_{Ie} - \frac{nkTD_e^T}{n_e m_e}\right)\delta_e \\
&- \frac{h_A D_A^T}{m_A T} - \frac{h_I D_I^T}{m_I T} - \frac{h_e D_e^T}{m_e T}\Bigg] \frac{\partial T}{\partial \underset{\sim}{r}} \equiv \left(-\lambda_{total}\right)\frac{\partial T}{\partial \underset{\sim}{r}}
\end{aligned} \tag{14}$$

where λ_t is the translational thermal conductivity, the sum of all terms containing the various h_i is the reactive thermal conductivity, and the sum of the remaining terms is the thermal conductivity due to thermal diffusion.

Ternary plasma.- Equation (14) is also valid for the case of a fully ionized ternary plasma undergoing the reaction

$$I^{(n)} + ne \rightarrow I^{(n+1)} + (n+1)e \tag{15}$$

where n denotes the degree of ionization. The only necessary change is to double the values of a_{21} and the last terms of a_{22} and c_2 (Eq. (13)) and replace the subscript A by $I^{(n)}$ and the subscript I by $I^{(n+1)}$.

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