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BY
GILBERT D. MEAD

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THE MOTION OF TRAPPED PARTICLES IN A DISTORTED FIELD

by

Gilbert D. Mead
NASA-Goddard Space Flight Center
Greenbelt, Maryland

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Abstract

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The motion of charged particles in a geomagnetic field distorted by the solar wind is examined, using adiabatic theory. It is shown that the drift motion of particles trapped on high L-shells depends strongly on their equatorial pitch angle. Those mirroring near the equator move in on the night side, while those mirroring at low altitudes move out, thus producing a significant splitting of the L-shell with pitch angle. One consequence is that the very high latitude lines on the day side may contain trapped particles near the equator, but none near the ends of the lines. The opening up of field lines in the geomagnetic tail may be the controlling factor governing the high-latitude limit of trapping on both the day side and the night side.

author

I. Introduction

The theory of adiabatic invariants enables one to determine the motion of trapped particles, so long as a complete description of the magnetic field is available. For the inner radiation belt, i.e., for $L \lesssim 2$, the internal earth's field dominates, and very accurate ordering of the data has been possible by using a spherical harmonic expansion of the earth's field with internal source terms only. In fact, the accuracy in this region seems to be limited only by the uncertainty in the values of the coefficients in the expansion. As more accurate magnetic field data are obtained, the resulting improvement in the field model is expected to increase even further our ability to order the trapped particle data in the inner belt.

However, at higher L values, the effect of external current sources becomes more and more important. Experimenters have found that beyond $L \approx 5$, a two-dimensional coordinate system based on a field model with internal field sources only is no longer able to order the data. With polar-orbiting satellites, particle fluxes are strongly dependent upon local time, in addition to the usual B and L coordinates. Satellites in highly eccentric orbits find large time variations in particle fluxes which depend upon magnetic activity, in addition to variations with local time which occur even during magnetically quiet times.

In this paper we shall examine the consequences of adiabatic theory in regions of the magnetosphere where external current sources produce significant distortions of the geomagnetic field. The variation of particle fluxes with local time will be investigated. We shall also demonstrate the substantial splitting of L shells as a function of equatorial pitch angle. That is, it will be shown that particles at large L which mirror near the equator (i.e., with large equatorial pitch angles) follow substantially different drift paths from those particles which mirror near the earth (i.e., with small equatorial pitch angles). One consequence of this is that field lines at large L on the day side of the earth may be only partially loaded with trapped particles. The absence of trapped particles at the ends of these lines, near the earth, is related to the inability of these particles to drift completely around the earth without entering the geomagnetic tail, where they no longer remain trapped.

II. Model of the Distorted Field

The distortions of the geomagnetic field at large altitudes are primarily due to two sources: currents at the magnetosphere boundary and currents in the tail of the magnetosphere. We shall ignore here currents due to the drift motions of the trapped particles themselves ("ring currents"), which are generally small during magnetically quiet times, and ionosphere currents, which in this region of space represent internal sources whose field falls off rapidly along with the earth's main field.

The effect of magnetopause currents has been calculated in some detail (Mead, 1964). Basically, the coefficients of a spherical harmonic expansion with external source terms have been determined, using the solution to the Chapman-Ferraro problem as given by Mead and Beard (1964). This expansion is then used to calculate field topologies, field line distortions, and the effect of changes in the boundary position. The topology of the distorted field in the noon-midnight meridian plane is shown in Fig. 1, and Fig. 2 shows a comparison of this distorted field with the experimental measurements made by Cahill on a sample pass of Explorer 12. It is seen that the predicted distortions are in reasonable agreement with the measurements.

The expression for the distorted field in the equatorial plane is particularly simple if we restrict ourselves to the first two terms in the external source expansion, which turn out to be the most important ones:

$$B_{eq} = \frac{M}{r^3} + \frac{a_1}{r_b^{\;3}} + \frac{a_2}{r_b^{\;4}} \; r \cos \varphi \tag{1}$$

Here M = 0.31 gauss (the equatorial value of the dipole field at the earth's surface), r_b is the distance to the magnetopause in the solar direction, φ is the azimuthal angle as measured from the noon meridian, and a_1 and a_2 are the first two non-vanishing external coefficients (note that φ and a_2 are defined somewhat differently in Mead [1964]). Similar, but more complicated expressions give the three components of the field off the equator. The values of the coefficients are shown in Table 1 for Mead's model and for an image dipole model (Parker, 1960).

Table 1. Values of the coefficients a_1 and a_2 in the external source expansion of the distorted field.

	a_1	a_2
Image Dipole (Parker, 1960)	$\frac{M}{8}$ = .04 Gauss	$\frac{3M}{16}$ = .06 Gauss
Mead (1964)	0.816 M = 0.25 Gauss	0.673 M = 0.21 Gauss

The expansion of the field near the earth is the same in both models, except for the values of the coefficients. In the image dipole model, r_b is the distance to the infinite plane surface. Table 1 indicates that for a given value of r_b, the image dipole model greatly underestimates the magnitude of the distortion near the earth.

From Eq. (1) the ratio of external field to internal field is given by

$$\frac{\langle B_{ext} \rangle}{B_{int}} = \frac{a_1}{M} \frac{r^3}{r_b{}^3} \approx 0.8 \frac{r^3}{r_b{}^3} \qquad (2)$$

where $\langle B_{ext} \rangle$ is the external field averaged over local time. This ratio is about 0.1% at the earth's surface, 2.5% at 3 R_E, 10% at 5 R_E, and 40% at 8 R_E, assuming a boundary at 10 R_E. Thus the external terms rapidly become important as the outer regions of the magnetosphere are approached.

Recent experimental measurements (Ness, 1965) have shown that this model is incapable of accurately describing the topology of the field in the tail region. In the region beyond about 10 R_E near the midnight meridian, a "neutral sheet" has been found which separates antisolar directed fields in the southern hemisphere from solar directed fields in the northern hemisphere. The presence of this neutral sheet implies the existence of strong plasma currents inside the magnetosphere. Since the Chapman-Ferraro problem assumes that no plasma exists inside the boundary, the original model must be modified to include effects in the magnetosphere tail. This can be done in a somewhat ad hoc manner by calculating the field due to a current sheet in the tail and vectorially adding this to the field obtained from the Chapman-Ferraro problem, as was done by Williams and Mead (1965). The general effect is to stretch out the lines of force on the night side. Lines of force at high latitudes are no longer closed, but stretch out into the tail. The configuration in the noon-midnight meridian is shown in Fig. 3. With the strength of the tail field used in this calculation, all lines above approximately 67° at the earth's surface in the midnight meridian are open, and thus cannot hold stably trapped particles. The critical latitude separating open field lines from closed field lines will decrease if the tail field increases. Such increases have been observed by Ness during magnetically disturbed periods. Thus one would expect the upper trapping boundary on the night side to move to lower latitudes during magnetic disturbances.

III. Motion of Trapped Particles in a Distorted Field

In order to study the motion of particles trapped in a distorted field, we shall assume that adiabatic theory is valid and that the energy of the particles remains constant. This is equivalent to assuming a static magnetic field and the absence of electric fields. This situation is expected to be valid for relatively energetic particles during magnetically quiet times. Any electric fields present will affect primarily the lower-energy particles, i.e., those with energies up to 50 or 100 kev. Above this energy, electrical fields do not seem to be an important factor.

We shall consider two kinds of particles: those whose mirror points are near the magnetic equator and those which mirror near the ends of the field lines at low altitudes. The drift motion of these two classes of particles will be substantially different.

A consequence of the conservation of the first invariant, μ, is that a particle will always mirror at the same value of magnetic field strength, B_m. For particles that mirror near the equator, therefore, the second invariant $J \approx 0$, and the drift paths will follow contours of constant field strength along the equator. Since the field is compressed by the solar wind on the day side, at a given radial distance the field is stronger here than on the night side. Thus in order to remain on contours of constant field strength, particles must drift closer to the earth on the night side. An estimate of the magnitude of this effect is obtained by equating the field magnitude (in gammas) given by Eq. (1) in the subsolar and antisolar direction:

$$\frac{31,000}{r_d^3} + 2.1\ r_d = \frac{31,000}{r_n^3} - 2.1\ r_n \tag{3}$$

where r_d and r_n are the dayside and nightside distances, r_b is assumed to be 10 R_E, and Mead's value for a_2 is used. Some typical values of r_n vs. r_d are given in Table 2, showing that the effect is strongly dependent upon radial distance.

Table 2. Regions of constant magnetic field intensity in the solar direction (r_d) and antisolar direction (r_n)

r_d	r_n	Δr
10	7.7	2.3
8	7.0	1.0
6	5.7	0.3
4	3.96	0.04

Particles trapped near the magnetopause on the solar side will move several earth radii closer to the earth as they drift around to the night side. Frank et al. (1963) have found that contours of constant omnidirectional fluxes of electrons >40 kev are displaced towards the sun by distances corresponding roughly to those given in Table 2. That is, fluxes fall off more rapidly on the night side than on the day side, in accordance with what one might expect from considerations of particle drifts.

Particles on high L-shells with small equatorial pitch angle, i.e., those that mirror near the earth, will follow substantially different drift paths. They will mirror at approximately constant altitude as they drift around the earth, since the external source terms produce only negligible change in the field magnitude at low altitudes. Assuming constant energy, the second invariant requires that

$$I = \int_{M}^{M^*} \left(1 - \frac{B}{B_m}\right)^{\frac{1}{2}} ds \qquad (4)$$

remain constant. B_m is the field magnitude at the mirror point, and the line integral is taken along the field line between mirror points. For high-latitude lines, the value of the integrand is essentially unity over almost the entire length of the line (for

Thus the drift paths of particles trapped on high-latitude field lines depend strongly on their equatorial pitch angle. Those with large pitch angle, i.e., those that mirror near the equator, drift to lower altitudes on the night side, the difference in altitude amounting to as much as several earth radii. Those with small equatorial pitch angles mirroring near the earth move out on the night side. This is equivalent to a splitting of L shells with pitch angle. The splitting appears to be much more than the maximum of about 1% that McIlwain (1961) found using internal source terms only for the earth's field.

We turn now to another question: what controls the high-latitude limit of trapping on the day side? For the most part, trapped particles are not observed with polar-orbiting satellites beyond an invariant latitude of about 70-75° on the day side, the exact upper limit being somewhat variable and dependent upon the threshold energy of the detector. It has often been assumed that this latitude corresponds to that separating the field lines crossing the equator on the day side from those which pass over the pole and are drawn back into the tail. However, Table 3 shows that this cannot be the case.

Table 3. Latitude where field line enters earth vs. equatorial crossing distance on the noon meridian for a pure dipole and a distorted dipole with a magnetosphere boundary at 10 R_E.

Equatorial Crossing Distance	Latitude at Earth's Surface	
	Pure Dipole	Distorted Dipole
4	60.0	60.4
5	63.4	64.4
6	65.9	67.6
7	67.8	70.3
8	69.3	73.0
9	70.5	76.5
10	71.6	82.0

An undistorted dipole field line crossing the equator at 10 R_E would enter the earth at a latitude of 71.5°. On the day side the lines are strongly compressed, and the line crossing at 10 R_E actually enters the earth at a much higher latitude, most likely above 80° (Mead, 1964). Thus there exists a bundle of field lines between approximately 70° and 80° on the day side which contain trapped particles in the equatorial regions, but which seem to be devoid of particles near the earth.

The most likely explanation of this is found in the phenomenon of L-shell splitting. Those particles trapped on this bundle of field lines whose mirror points are near the equator will drift closer to the earth on the night side and stay in the region of the magnetosphere where the field lines remain closed. Those which mirror near the earth, however, move out on the dark side and find themselves in the region of the tail, where the field lines are no longer closed. Since complete drift paths around the earth are not available to these particles, they become lost and this region is relatively free of trapped radiation.

It thus appears that the geomagnetic tail may be the controlling factor governing the high-latitude limit of trapping. On the night side particles are trapped on all field lines up to about 65-70°.

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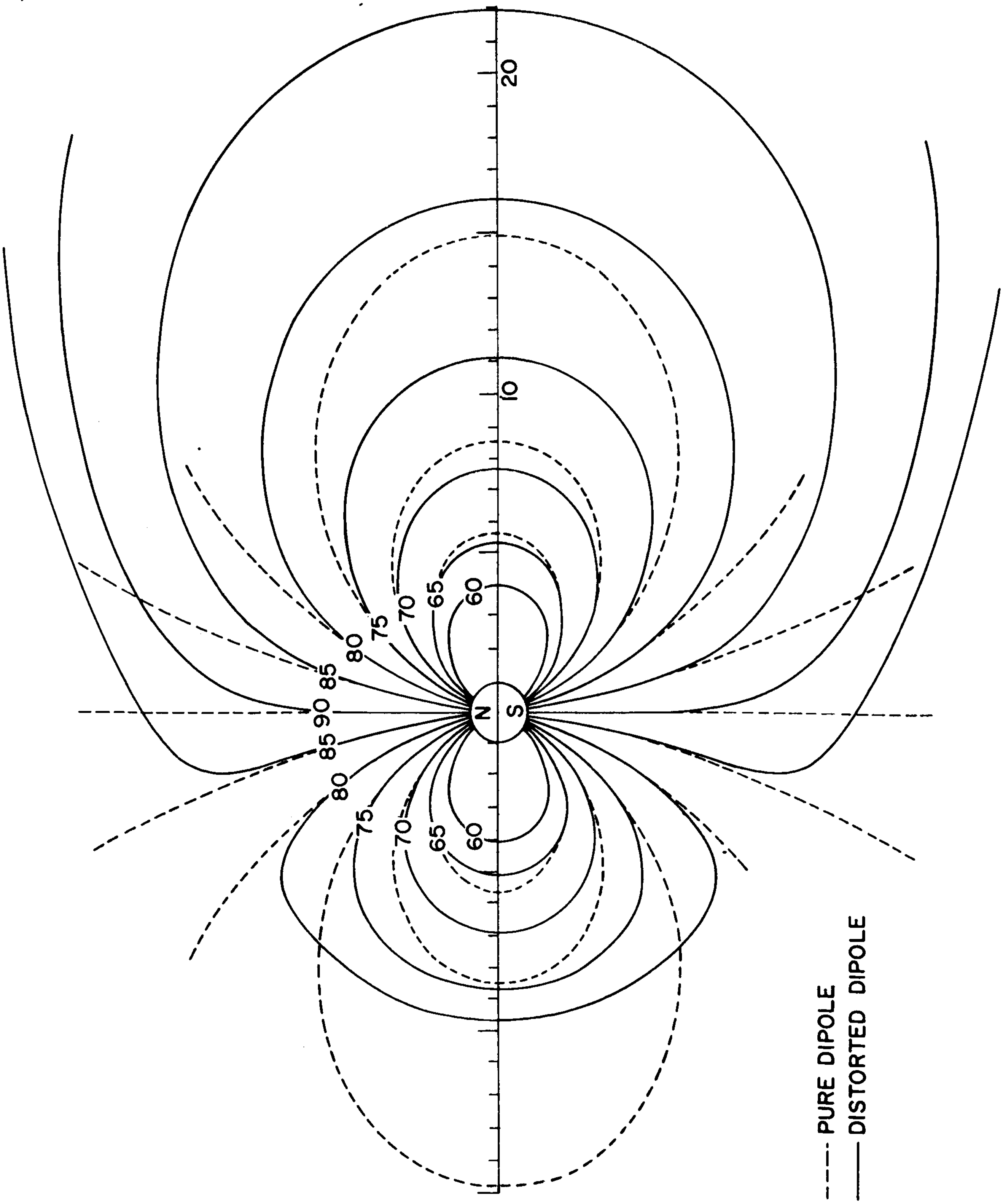
FIGURE CAPTIONS

Figure 1. Shape of the field lines in the noon-midnight meridian, considering only magnetopause currents.

Figure 2. Comparison of model of distorted field with experimental data from Explorer 12. $|F|$ is the field magnitude, and α and ψ determine its direction in spacecraft coordinates (Cahill and Amazeen, 1963). This pass is near the noon meridian.

Figure 3. Model of distorted field including a current sheet in the tail. The shaded region represents the region of stable trapping.

Figure 4. Shapes of pure dipole and compressed dipole lines having approximately the same length between low-altitude mirror points and same value of I. The compressed line crosses the equator closer to the earth, but enters the earth at a higher latitude.



20
10
N
S
90
85
80
75
70
65
60
85
80
75
70
65
60
PURE DIPOLE
DISTORTED DIPOLE

Figure 1

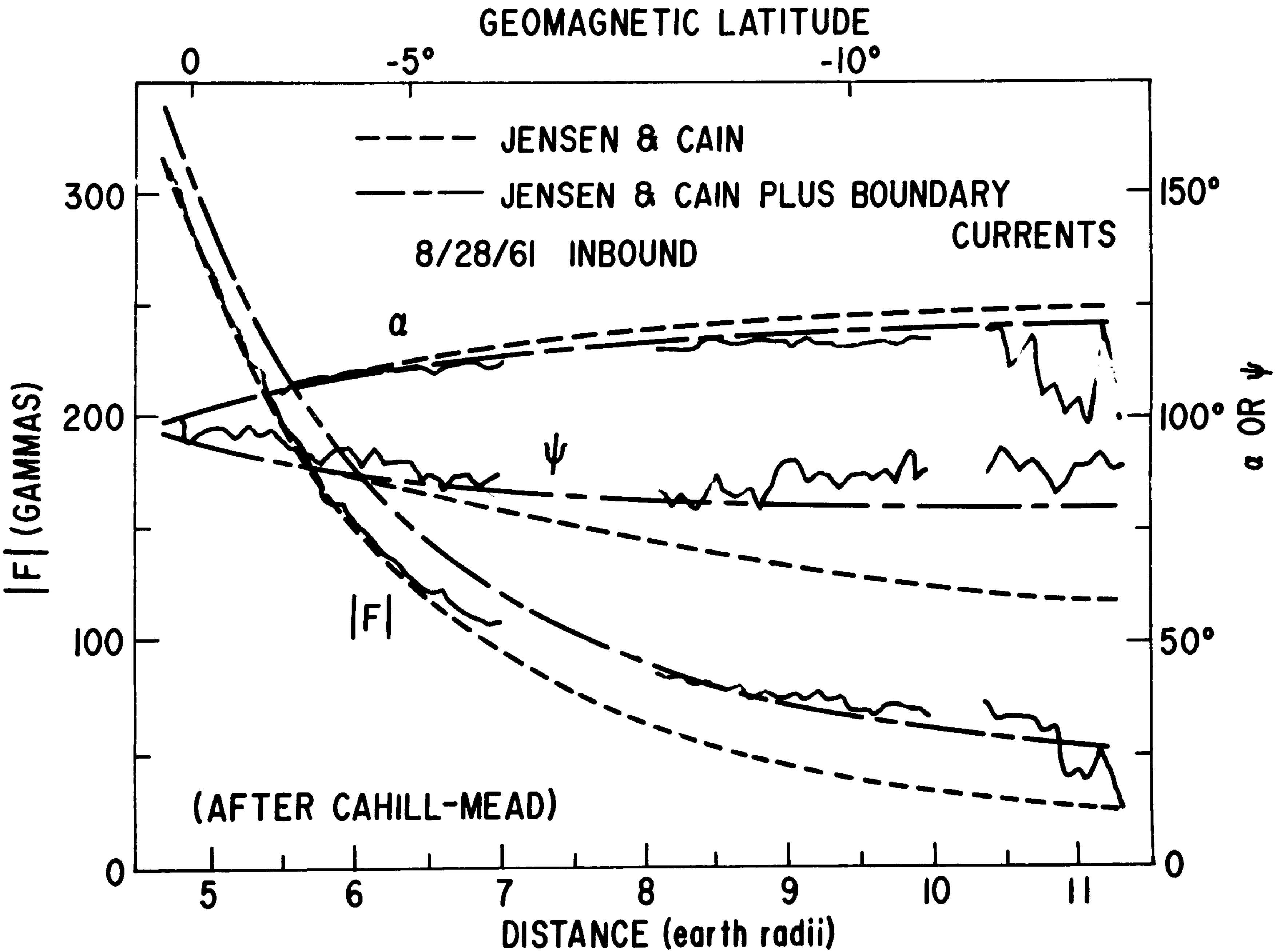

GEOMAGNETIC LATITUDE
0
-5°
-10°
JENSEN & CAIN
JENSEN & CAIN PLUS BOUNDARY CURRENTS
8/28/61 INBOUND
α
ψ
|F|
(AFTER CAHILL-MEAD)
|F| (GAMMAS)
300
200
100
0
150°
100°
50°
0
α OR ψ
5
6
7
8
9
10
11
DISTANCE (earth radii)

Figure 2

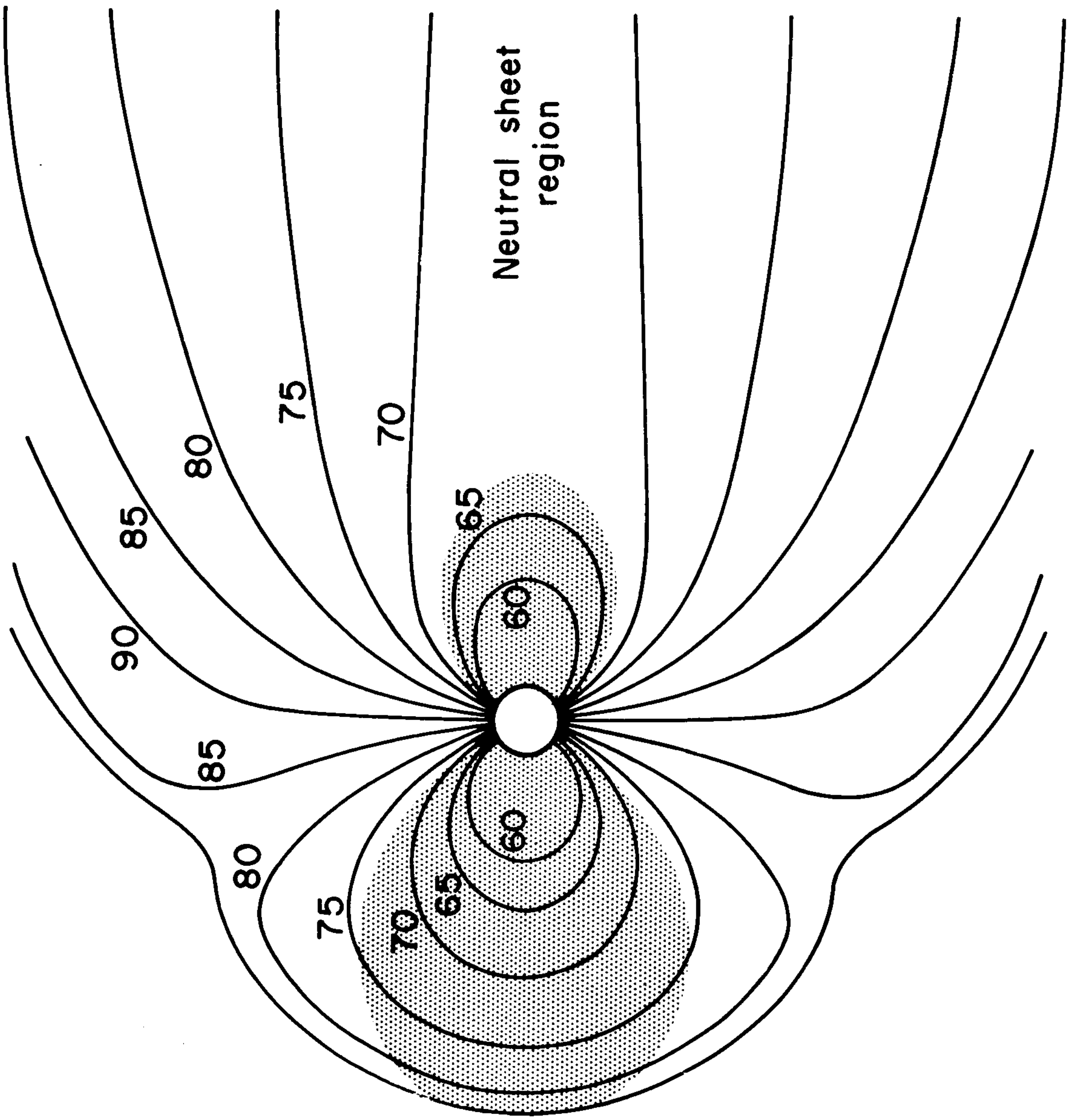


Figure 3

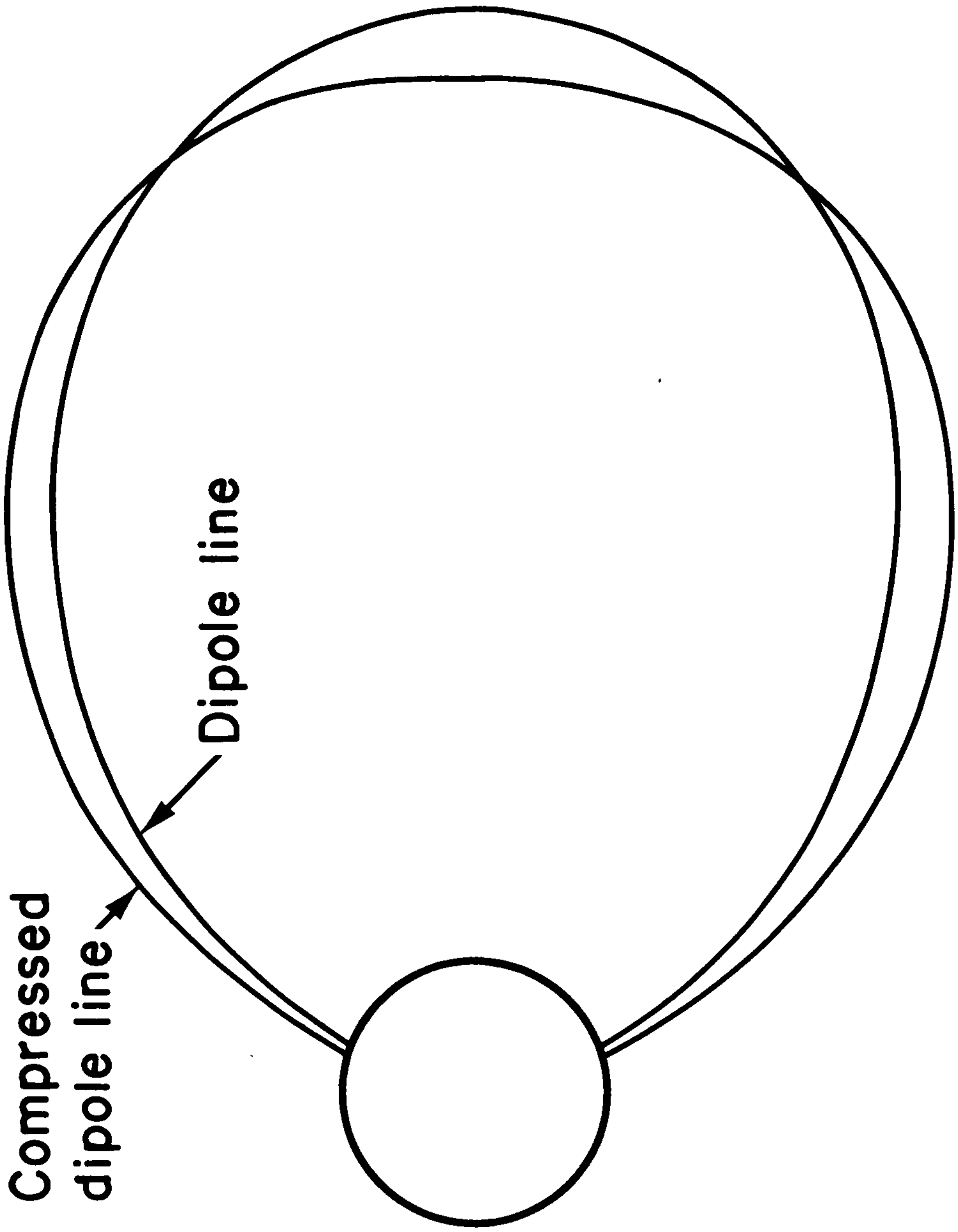


Figure 4