X-643-65-390

PRELIMINARY TRAJECTORY ANALYSIS OF A MARS PROBE

John McNellis

October 1965

Goddard Space Flight Center
Greenbelt, Maryland

CONTENTS

PRELIMINARY TRAJECTORY ANALYSIS OF A MARS PROBE

by

John McNellis
Goddard Space Flight Center
Greenbelt, Maryland

ABSTRACT

This report investigates the feasibility of the needle-nosed probe which is proposed as an experiment on a 1969 Voyager fly-by bus. The probe is to be separated from the bus within 20 days prior to closest approach of the bus. It is to enter the Mars atmosphere and transmit atmospheric entry information back to the bus. In order that the information be successfully transmitted to the bus there are constraints on the following parameters:

1. The entry velocity of the probe,

2. the distance between the probe and the bus,

3. the orientation of the probe relative to the bus.

Also, the separation velocity required for the probe to enter the atmosphere must be relatively small. Nine bus trajectories, considered to be representative of the actual class of potential bus trajectories, were used for generating probe trajectories. This study is of a preliminary nature and does not investigate the effects of dispersions from the nominal. All of the trajectories were analyzed by the patched conic technique.

For a given bus trajectory a set of probe trajectories (meeting the required constraints) was computed. Each of the trajectories in the set corresponded to a different separation time of the probe prior to closest approach of the bus. For each probe trajectory the separation velocity vector was varied to develop a subset of probe trajectories (with different entry times) for each separation time. Hence, the subset of probe trajectories indicates how the values of the constraint parameters vary for a given separation time.

It is found that the mission can be accomplished successfully on most of the potential bus trajectories if one has perfect a priori knowledge of the nominal bus trajectory and perfect control of the probe separation maneuver. However, even under these ideal conditions all of the constraints of the probe mission cannot be met when the bus trajectory has a hyperbolic excess velocity relative to Mars of more than 6.2 km/sec (this eliminates only 10% of the potential bus trajectories).

It is shown that by choosing the separation time sufficiently early (approximately 20 days prior to closest approach of the bus) the probe mission may be accomplished using a separation velocity under seven meters per second for all acceptable bus trajectories. Graphical presentations of the constraint parameters indicate that in most cases for a given nominal bus trajectory there exist a wide range of probe trajectories which will accomplish the probe mission.

LIST OF FIGURES

NOMENCLATURE

AA	Antenna Angle - The angle measured from the probe velocity vector to the probe-bus vector at the time of probe entry into the Mars atmosphere.
BSA	Bus-Sun Angle - The angle measured from the bus velocity vector relative to the sun to the bus-sun vector.
$d\alpha_V$	The angular displacement (in right ascension) of the velocity vector of the bus relative to the inward radial (upon entering Mars sphere of influence.)
$d\delta_V$	The angular displacement (in right ascension) of the velocity vector of the bus relative to the inward radial (upon entering Mars sphere of influence.
MSA	Mars-Sun Angle - The angle measured from the Mars velocity vector to the Mars-sun vector.
MSI	Mars Sphere of Influence - A surface (assumed spherical) around Mars such that whenever a space vehicle is within this surface its motion is considered to be due only to the attraction of Mars.
RCA	Radius of Closest Approach - The distance from the center of Mars to the point of closest approach of the bus.
RMSI	The radius of Mars sphere of influence = 574537.95 km.
SVA	Separation Velocity Angle - The angle measured from the bus velocity vector to the separation velocity vector of the probe at the time of separation.
V_{Mars}	The magnitude of the velocity vector of Mars.
V_{PL}	The hyperbolic excess velocity of the bus relative to Mars (approximately equal to the velocity of the bus upon entering MSI).
V_e	Entry Velocity - The velocity of the probe at entry into the Mars atmosphere.

ZAP The angle measured from a vector in the direction of the incoming asymptote (in relation to the two-body hyperbolic trajectory of the bus in MSI) to the Mars-sun vector.

α Right ascension.

β The angle in true anomaly that Mars goes through between the time of separation of the probe and the time of closest approach of the bus.

γ_e The entry angle of the bus into Mars atmosphere measured from the local horizontal to the entry velocity vector.

δ Declination.

ΔV Separation velocity of the probe - The velocity increment given to the probe at separation time.

ρ_{BP} Bus-Probe Distance - The distance from the probe to the bus as the probe enters the Mars atmosphere.

CONSTANTS

Gravitational constant of Mars = 4.29773×10^4 (km^3/sec^2).

Gravitational constant of sun = 1.3271544×10^{11} (km^3/sec^2).

Radius of Mars plus atmosphere = 3530.4 km.

Radius of Mars sphere of influence = 574,537.95 km.

PRELIMINARY TRAJECTORY ANALYSIS OF A MARS PROBE

I. INTRODUCTION

This report is concerned with the trajectory aspects of the needle-nosed probe which is proposed as an experiment on the 1969 Mars flyby bus.

Ths bus is expected to pass within a few thousand kms of Mars. Approximately 20 days (or less) prior to closest approach of the bus the probe is to be separated from the bus with the purpose of entering the Mars atmosphere and transmitting entry information back to the bus.

It is the object of this report to investigate the pertinent trajectory parameters which will influence the success of the probe mission. Since there are oonstraints which must be met in order that the probe enter the Mars atmosphere and transmit successfully back to the bus, it is desirable to express these constraints in terms of the trajectory parameters. Ideally, if the mission can be accomplished on a class of nominal bus trajectories then we should like to define this class and hope to reduce the proposed bus launch window to exclude those trajectories with which the probe mission cannot be accomplished. If it happens that all (or an unreasonably large portion) of the trajectories in the proposed bus launch window would have to be excluded, then the probe mission would be considered unfeasible. Fortunately, this does not appear to be the case.

The analysis performed is deterministic in nature and is concerned with the accomplishment of the probe mission under perfect knowledge of the bus trajectory and perfect control of the probe trajectory. The trajectory model is a two-body patched conic. A major part of the program is built around a generalized two-body analysis scheme developed by E. R. Lancaster and H. E. Montgomery and programmed by R. Devaney.

The results of the analysis are intended to be useful in determining when the probe mission is feasible. Cross plots of the constraint parameters should yield information helpful in tradeoff analyses necessary in selecting a nominal probe trajectory.

II. TRAJECTORY ANALYSIS

A. Probe Mission Constraints

The constraints which must be met in order to accomplish the probe mission are the following:

(1) ΔV (separation velocity) $\leq$ 15 m/sec.

(2) ρ_{BP} (bus-probe distance) $\leq$ 25,000 km.

(3) AA (antenna angle) $95° < AA < 145°$.

(4) V_e (entry velocity) $\leq$ 7.925 km/sec.

(5) γ_e (entry angle) $\cong$ 90° (vertical).

These constraints are approximate. Constraints (2) and (4) are related such that acceptable combinations of V_e and ρ_{BP} are defined by the shaded region in Figure 1. Constraint (5) which requires γ_e to be close to vertical is desirable but not absolutely necessary. In the analysis γ_e was kept within .6 degrees of vertical. ρ_{BP} and AA are shown in Figure 2.

B. Trajectory Model

All of the constraints are dependent on the bus trajectory in the vicinity of Mars. Since Mars orbit is inclined to the ecliptic plane by less than 2°, it is reasonable to consider the Mars and bus trajectories nearly coplanar relative to the sun for preliminary analysis purposes. However, upon entering Mars sphere of influence the bus may have any of a wide range of inclinations to the ecliptic in its hyperbolic trajectory relative to Mars.

We may consider the trajectory of the probe to consist of two phases: First, a sun-centered trajectory between separation from the bus and entry into Mars sphere of influence; Second, a two-body Mars centered hyperbolic trajectory which takes the probe from the Mars sphere of influence to vertical entry into the Mars atmosphere.

Since the probe is separated from the bus only in the final stage of the Earth-Mars transfer trajectory of the bus, it is desirable to define the bus trajectory in terms of its end conditions at Mars. As the bus enters the Mars sphere of influence it is convenient to define the Mars-centered two-body trajectory by the following parameters:

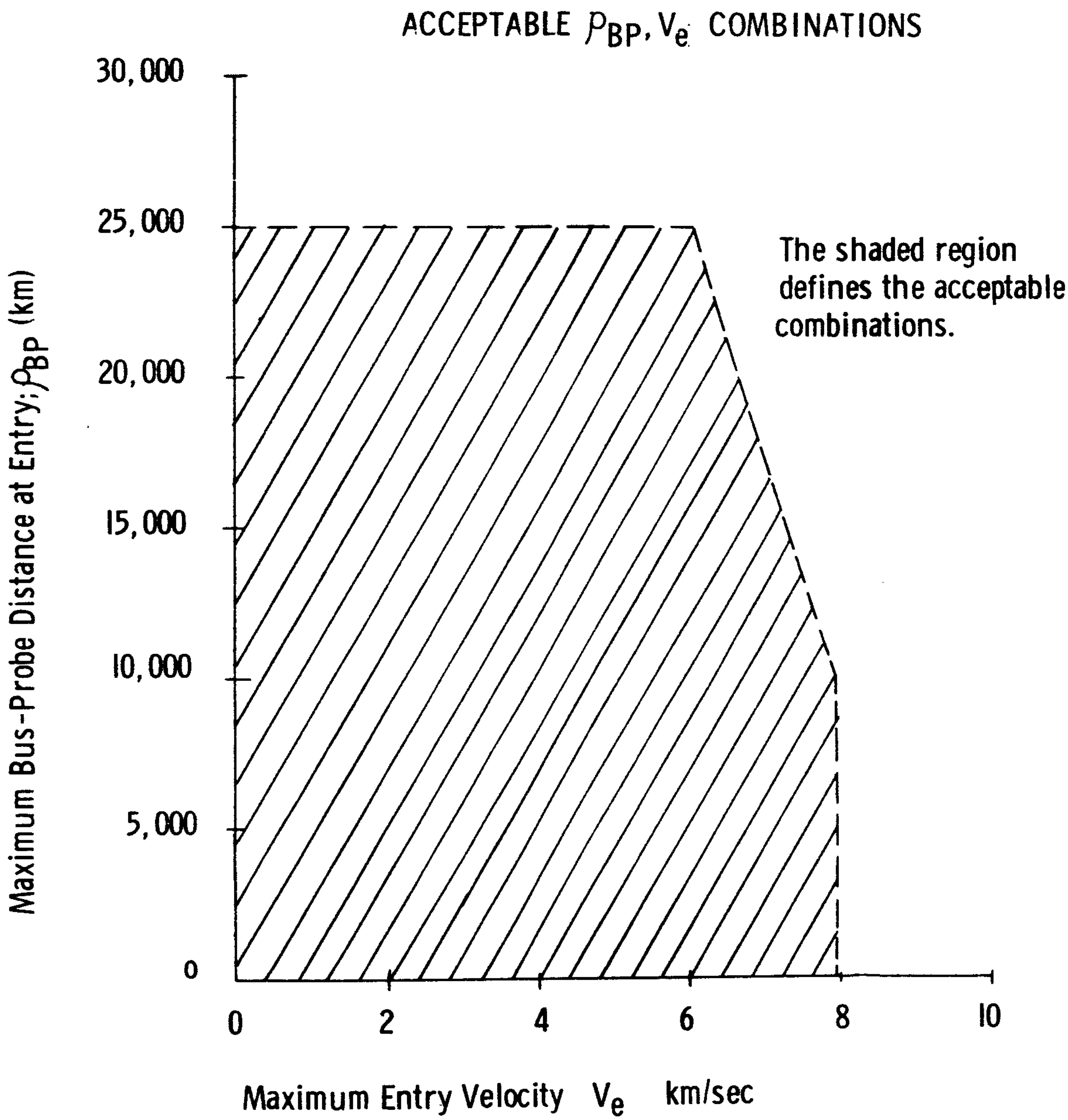
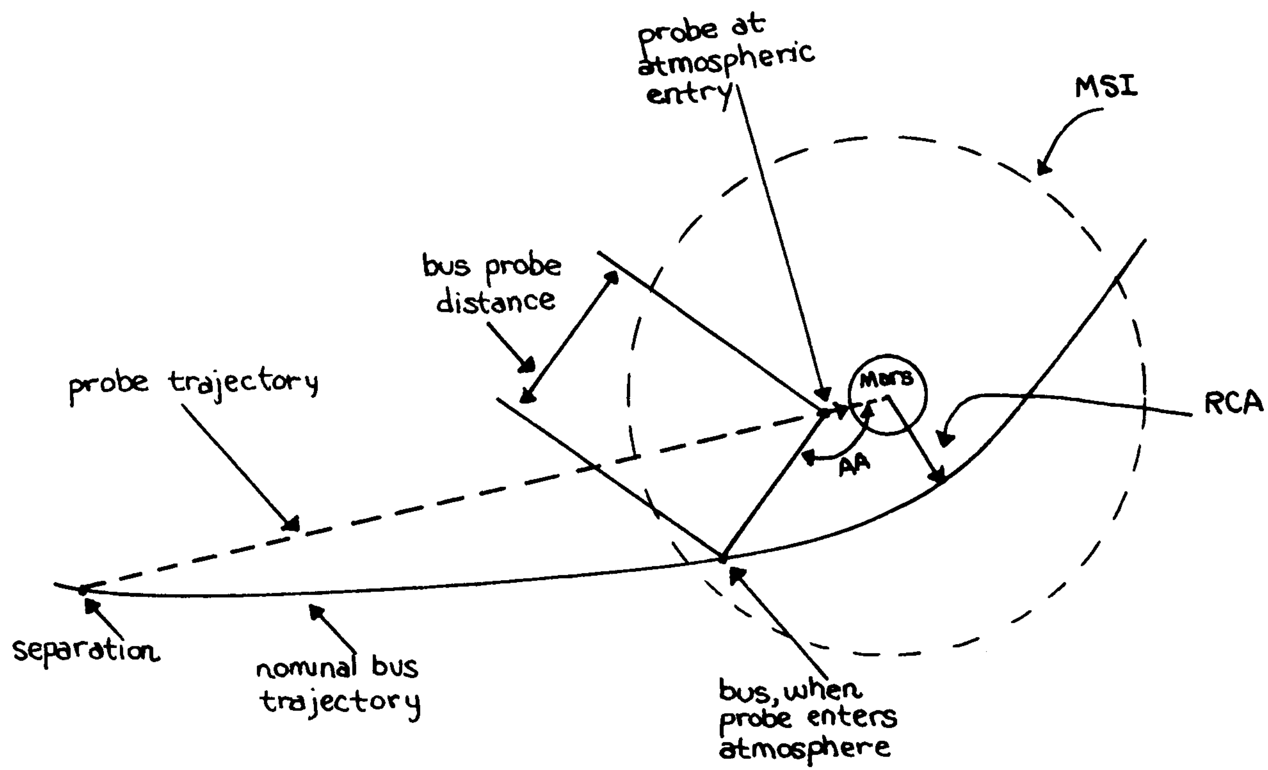


Figure 1—Acceptable ρ_{BP}, V_e combinations.



CONSTRAINT PARAMETERS

Figure 2–Constraint parameters.

(1) α (right ascension).

(2) δ (declination).

(3) $d\alpha_v$ } (the angular displacement in right ascension and declination of
(4) $d\delta_v$ } the velocity vector relative to the inward radial).

(5) V_{PL} (magnitude of velocity vector).

(6) RMSI (radius of Mars sphere of influence).

If, for the instant, we neglect the influence of the sun we can see that the important aspects of the bus trajectory in regard to the probe mission are given by the shape of the hyperbolic bus trajectory in the vicinity of Mars. The orientation in the plane and the orientation of the plane are not pertinent to the 5 constraints (on page 2) which define probe mission success. Therefore, the first two orbit parameters are not important and the third and fourth parameters may be replaced by the resultant angle that the velocity vector makes with the inward radial direction. But this resultant angle uniquely determines and may be replaced by the radius of closest approach of the bus. Hence, with the simplifying assumption of no sun, for the purpose of the probe mission the end conditions of the bus trajectory at Mars may be described by the radius of closest approach (RCA), and the magnitude of the velocity vector upon entering the Mars sphere of influence (i.e. V_{PL}, the hyperbolic excess velocity relative to Mars). That is to say that V_{PL} and RCA are the only two parameters of the bus trajectory which affect the probe mission (in the sunless force field). Now we can consider the sun and see what modifications are necessary.

By choosing the proper set of initial conditions in the sun's sphere of influence it is clear that we can enter MSI from any direction and have a given V_{PL} and RCA. We may define the direction of the sun relative to the bus by the ZAP angle which is the angle measured from the approach asymptote of the nominal bus trajectory to the Mars-Sun Vector (see Figure 3). We are interested in making a velocity correction to the probe (a certain time prior to closest approach of the bus) such that the probe will enter the Mars atmosphere radially some given time after the velocity correction. We want to know if the values of constraints (1), (2), and (3) are independent of the ZAP angle. It is reasonable to expect that for "sufficiently small" velocity corrections the constraint relationships between the nominal and perturbed trajectories will be approximately the same. However, it is not obvious that the required velocity correction for vertical entry at a given time will be "sufficiently small." Some computer runs have been made and indicate that the values of the constraint parameters are insensitive to changes in

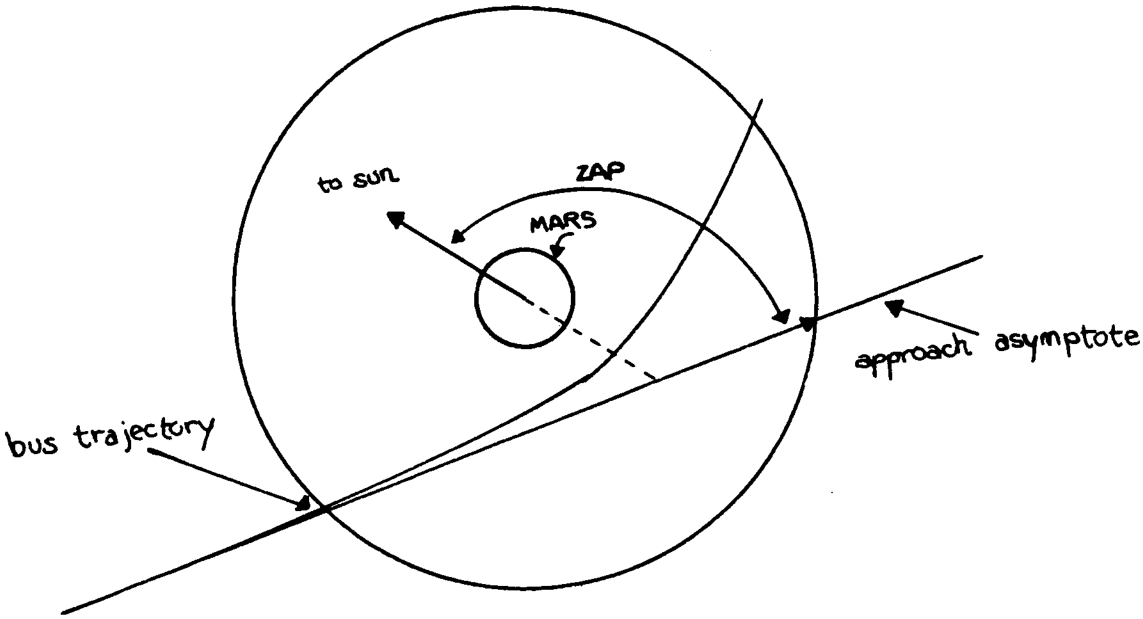


Figure 3—ZAP Angle

the ZAP angle. Hence, the ZAP angle is considered to be unimportant in regard to the constraint values. For other considerations such as the bus-sun angle and the direction of the separation velocity relative to the bus velocity vector, the ZAP angle is an important parameter and must be examined in detail.

C. Nominal Bus Trajectory

A trajectory design chart giving the characteristics of Earth-Mars transfer trajectories is given in Ref. 1. With the Atlas-Centaur as launch vehicle for the bus flyby mission it is desirable to keep the geocentric injection energy low

$$\left(c_3 \leq 10 \ \frac{\text{km}^2}{\text{sec}^2}\right)$$

But most of the trajectories with this range of injection energies result in a hyperbolic excess velocity at Mars between 4 and 6.5 km/sec. The actual trajectory of the bus is assumed to pass within 1000 to 5000 kms of Mars at closest approach. Hence, this range of V_{PL} 's and RCA's define the end conditions of the class of bus trajectories which will be considered in this report.

D. Probe Trajectory Considerations

It is worthwhile to consider the method used in generating probe trajectories for a given bus trajectory. The hyperbolic trajectory of the bus defines a plane in Mars sphere of influence. The determination of a separation velocity causing vertical atmospheric entry of the probe was considered to be equivalent to determining the velocity correction at separation time which will bring the approach asymptote of the nominal bus trajectory through the center of Mars. But the computed velocity correction corresponds to a unique time of atmospheric entry. We can compute a set of velocity corrections which result in entry times that are associated with acceptable antenna angle values. This process can be repeated for many separation times. Hence, for a given V_{PL} and RCA defining the bus trajectory one can vary entry and separation times to generate a class of probe trajectories.

III. DISCUSSION OF PROGRAM

The two-body analysis scheme mentioned in the introduction was used as a subroutine around which a program was developed to study the pertinent trajectory parameters involved in the probe mission. From a somewhat general point of view the use of the program may be thought of in terms of the following three steps:

(1) Development of a set of end segments of bus trajectories representative of the probable 1969 trajectories.

(2) Determination of probe trajectories corresponding to the set of bus trajectories.

(3) Evaluation of the parameters which may be useful in the selection of nominal probe trajectories.

The end segments of bus trajectories represent that part of the Earth-Mars transfer trajectory within 20 days of Mars. The trajectories were chosen to have a V_{PL} between 4 and 6.5 km/sec and an RCA of 4393 to 8393 km. This is the range expected on the end conditions of the actual 1969 trajectories.

The probe trajectories corresponding to a given bus trajectory are generated for a particular separation time by linear perturbation analysis. The given bus trajectory has associated with it an approach asymptote (in Mars sphere of influence). The vector from the center of Mars to the approach asymtote, and normal to it, is referred to as the impact parameter. The plane normal to the approach asymptote, and containing the impact parameter, will be called the impact plane. Let t_{sep} and t_c represent respectively the time of separation and the time of the bus crossing the impact plane. Consider the impact parameter to be a miss vector. The problem of determining the probe trajectory can now be looked upon as determining the velocity correction required at t_{sep} to null out the miss vector at t_c. The solution to the linearized equations relating deviations in the nominal trajectory at the different times can be written:

$$\begin{bmatrix} \Delta R(t_c) \\ \\ \Delta V(t_c) \end{bmatrix} = \begin{bmatrix} \Phi_{11}(t_c, t_{sep}) & \Phi_{12}(t_c, t_{sep}) \\ \\ \Phi_{21}(t_c, t_{sep}) & \Phi_{22}(t_c, t_{sep}) \end{bmatrix} \begin{bmatrix} \Delta R(t_{sep}) \\ \\ \Delta V(t_{sep}) \end{bmatrix}$$

where ΔR and ΔV are three element column vectors and the four Φ_{ij}'s are 3×3 segments of the state transition matrix. To compute the velocity required to null out the miss vector we solve the following set of three equations:

$$[\Delta V(t_{sep})] = [\Phi_{12}(t_c, t_{sep})]^{-1} [\Delta R(t_c)]$$

In the program the elements of $[\Phi_{12}(t_c, t_{sep})]$ are found numerically by varying the elements of the velocity vector at t_{sep} and finding the deviations in position at t_c. The computed ΔV is added to the state vector of the bus at t_{sep}. Using a patched conic technique theprobe state vector is computed at the time of atmospheric entry. If the atmospheric entry is not as close to vertical as desired the miss vector of this trajectory is computed and the above equation is solved to get a better estimate of the velocity correction. Two to three iterations on the linearized equations usually are sufficient to obtain the initial conditions on a probe trajectory with an atmospheric entry angle less than a half of a degree from the vertical. Often at this entry time the orientation of the bus and probe does not result in a favorable antenna angle. However, one may control the entry time by choosing the miss vector to be equal to the vector sum of the impact parameter and k times the unit vector in the direction of the approach asymptote; the constant k is suitably chosen to yield the desired entry time. By successively incrementing the previous miss vector in this manner one may obtain a set of probe trajectories corresponding to a particular separation time on a given bus trajectory. This procedure was performed for a sequence of separation times on a variety of bus trajectories covering the range of probable V_{PL} and RCA.

In addition to computing the values of the constraint parameters for each probe trajectory, the program is also used to compute BSA (bus-sun angle) and SVA (separation velocity angle) at the time of separation. Since the program uses a circular orbit for Mars the bus-sun angle could be in error by four or five degrees for some 1969 arrival times. Although the SVA and BSA are not pertinent for determining probe mission success, they are important for determining the orientation of the probe relative to the bus.

IV. RESULTS AND DISCUSSION

A. Bus-Sun Angle

The bus-sun angle (BSA) is the angle measured from the velocity vector of the bus (relative to the sun) to the bus-sun vector. In the program this angle is obtained by taking the scalar product of the two vectors. Since the direction of the velocity vector of the bus relative to Mars changes very little for the 15 to 20 days prior to entering Mars sphere of influence, it is convenient to use geometrical considerations and represents BSA by the quation:

$$BSA = 90^\circ + \tan^{-1}\left\{\frac{-V_{Mars}\cos MSA - V_{PL}\cos(ZAP - \beta)}{V_{Mars}\sin MSA + V_{PL}\sin(ZAP - \beta)}\right\}$$

where V_{Mars} = magnitude of Mars velocity vector.

β = The angle in true anomaly that Mars goes through between the time BSA is computed and the time of closest approach of the bus.

MSA = The angle measured from the Mars velocity vector to the Mars-Sun vector.

To obtain an approximate expression for BSA we can let V_{Mars} be 23.8 km/sec. and MSA be 90° (the average values of these parameters). Then we have:

$$BSA \cong 90^\circ + \tan^{-1}\left\{\frac{-V_{PL}\cos(ZAP - \beta)}{23.8 + V_{PL}\sin(ZAP - \beta)}\right\}$$

This assumption is made in the program since the model has Mars in a circular orbit. For the expected range of arrival times this should cause an error in BSA of less than 4 to 5°. Since V_{PL} is limited to slightly over 6 km/sec., BSA will be between 70 and 110 degrees.

B. Separation Velocity Angle

The separation velocity angle (SVA) is the angle measured from the velocity vector of the bus to the separation velocity vector. From the design standpoint it is useful to represent the velocity vector of the bus relative to the sun so that when SVA is combined with BSA one has information useful in determining the orientation of the probe relative to a set of body fixed bus coordinates. However, in order to generalize the results and gain insight into the guidance problem, it is useful to consider the velocity vector of the bus relative to Mars. Essentially both cases represent the same information and there is no problem as long as one knows which system is being used.

For a given time of separation one can select a set of ΔV's (and the respective SVA's) corresponding to various entry times. Figure 4 presents a typical case showing how ΔV and SVA can be traded off by varying the entry time (antenna angle). The A and B scales represent SVA when the velocity vector of the bus is taken relative to Mars and the sun respectively. It can be seen that there is a whole range of SVA which can be used to accomplish the mission even for a fixed separation time.

C. Non-Nominal Bus Trajectories

As discussed above, SVA is an important parameter from the standpoint of determining how one should align the probe when attaching it to the bus. Since, for any given bus trajectory and corresponding probe trajectory the separation velocity vector and the bus velocity vector are known, there is no problem in computing SVA. However, once SVA and ΔV are selected, this results in a fixed velocity magnitude and a fixed orientation for the probe relative to the body-fixed bus coordinates. However, if the bus trajectory is not nominal then the fixed ΔV and SVA at the predetermined t_{sep} will not give the desired probe trajectory. We can consider ΔV to have the following simplified form:

$$\Delta V = f(\text{SVA},\ t_{sep},\ \text{RCA},\ V_{PL})$$

For small deviations in these parameters we can write the linearized equation

$$\delta(\Delta V) = \frac{\partial f}{\partial \text{SVA}}\,\delta(\text{SVA}) + \frac{\partial f}{\partial t_{sep}}\,\delta t_{sep} + \frac{\partial f}{\partial \text{RCA}}\,\delta \text{RCA} + \frac{\partial f}{\partial V_{PL}}\,\delta V_{PL}$$

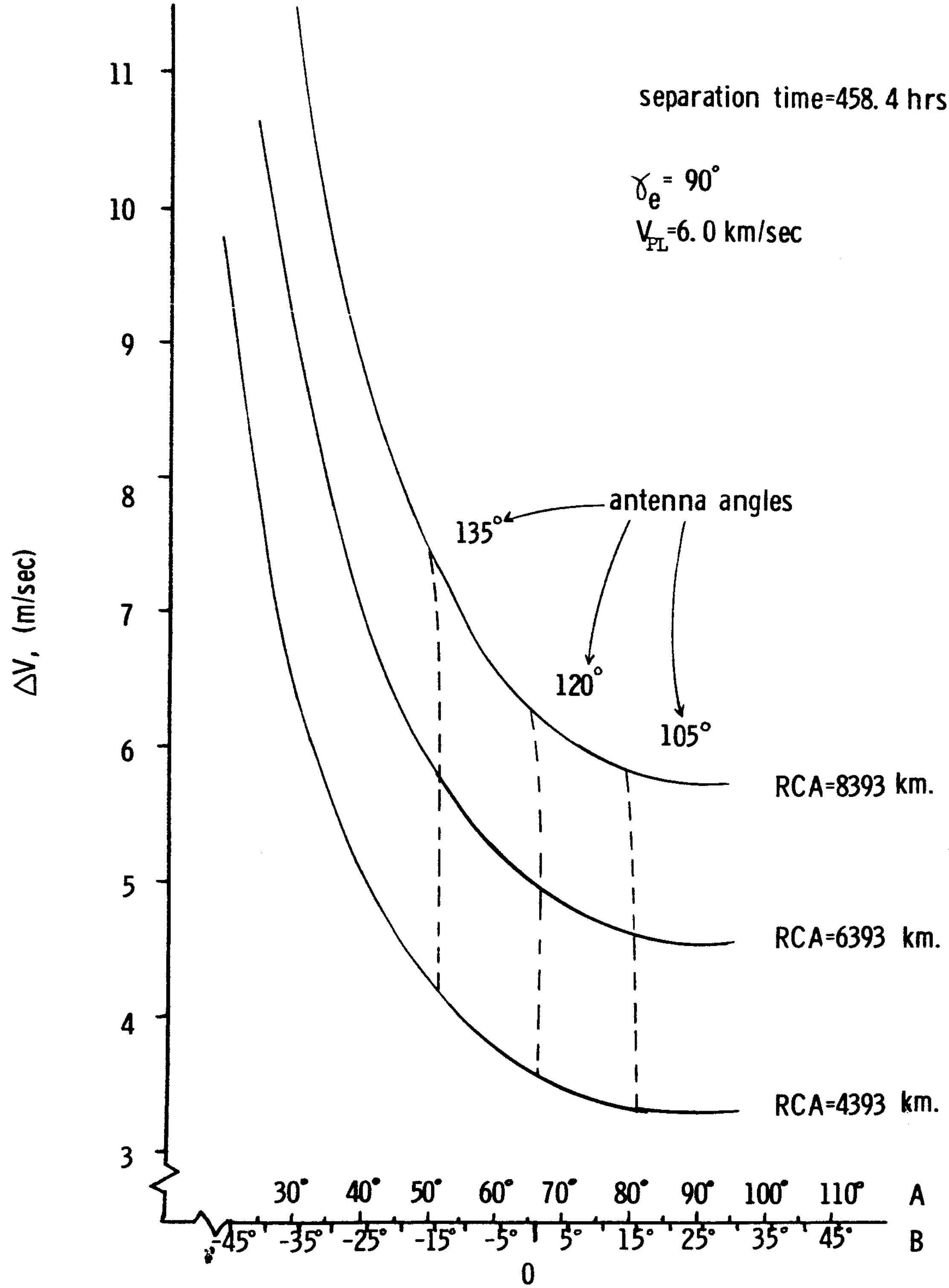


Figure 4—Separation velocity vs. separation velocity angle

But since ΔV and SVA are fixed the above equation becomes:

$$\delta t_{sep} = \frac{\dfrac{\partial f}{\partial RCA}\ \delta RCA + \dfrac{\partial f}{\partial V_{PL}}\ \delta V_{PL}}{\dfrac{\partial f}{\partial t_{sep}}}$$

That is, we want to change the separation time such that the fixed ΔV and SVA will result in a successful probe trajectory when applied to the non-nominal bus trajectory at the modified separation time. Perhaps, this is an oversimplification of the problem since we shall also be affected by non-nominal values on out-of-plane parameters of the bus trajectory. In this light, the equation for δt_{sep} being satisfied is a necessary but not sufficient condition for success. The additional condition being that the out-of-plane deviations must be small enough so that the probe mission constraints can still be met (with deviation from vertical atmospheric entry of probe being acceptable). This problem should be studied in detail once a suitable t_{sep} with the corresponding ΔV and SVA have been selected.

D. Summary

Essentially nine bus trajectories were studied. These trajectories were characterized by RCA values of 4393, 6393, and 8393 km (i.e. miss altitudes of 1000, 3000 and 5000 km). This set of RCA's was used for trajectories having V_{PL}'s of 4, 5, and 6 km/sec. Figures 5 through 7 show the values of the constraint parameters. The separation time is approximately 20 days prior to the closest approach of the bus. The antenna angle is used as the independent parameter and represents a range of entry times. Although the nominal antenna angle is 120°, deviations of up to 25° from the nominal are acceptable. From all three figures it can be seen that if one maintains the antenna angle under 130° the required separation velocity can be kept under 7 meters per second. As the antenna angle increases above 130° the separation velocity increases nonlinearly.

Figures 8 through 10 show the required separation velocity as a function of the separation time (the antenna angle is kept constant at the nominal 120°). It can be seen that as the separation time is chosen closer to the time of closest approach of the bus the required separation velocity increases rapidly. RCA becomes a critical parameter. For example, from Figure 8 the required ΔV is approximately 16 m/sec when RCA is 8393 km. As one would expect, the figures also show that it is necessary to increase ΔV for increasing values of RCA.

The only constraint which cannot be met using some of the potential bus trajectories is the requirement limiting the entry velocity of the probe. Since the separation velocity of the probe is very small the hyperbolic excess velocity of the probe relative to Mars is approximately equal to that of the bus relative to Mars. Hence, using the conservation of energy for the motion in Mars sphere of influence we can write the entry velocity as

$$V_e = \sqrt{V_{PL}^2 + \frac{2\mu}{R_e}}$$

where μ = gravitational constant of Mars.

R_e = radius at atmospheric entry.

From this relationship it is concluded that if the entry velocity is to be kept below 7.925 km/sec then the bus trajectories must not have V_{PL}'s in excess of 6.2 km/sec. This means that in order to insure probe mission success even under the most ideal circumstances (perfect control of bus and probe trajectories) the class of potential bus trajectories must be reduced to include only those with V_{PL}'s under 6.2 km/sec (less than 10% of the potential bus trajectories have V_{PL}'s greater than 6.2 km/sec.).

V. REFERENCES

1. Mariner Mars 1969 Orbiter Technical Feasibility Study. Report No. EPD-250, Jet Propulsion Laboratory, Pasadena, California, November 16, 1964.

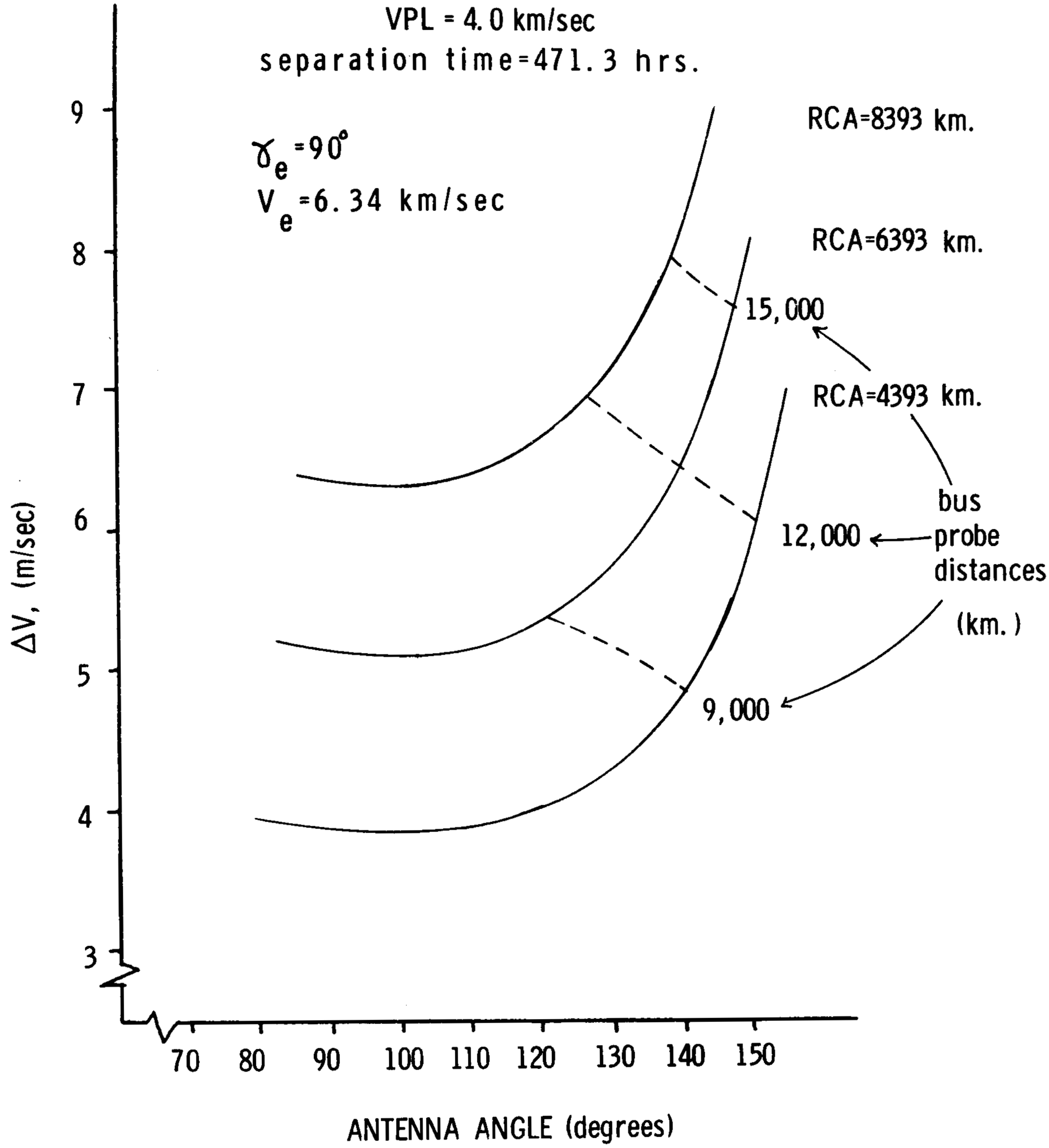


Figure 5—Separation velocity vs. antenna angle

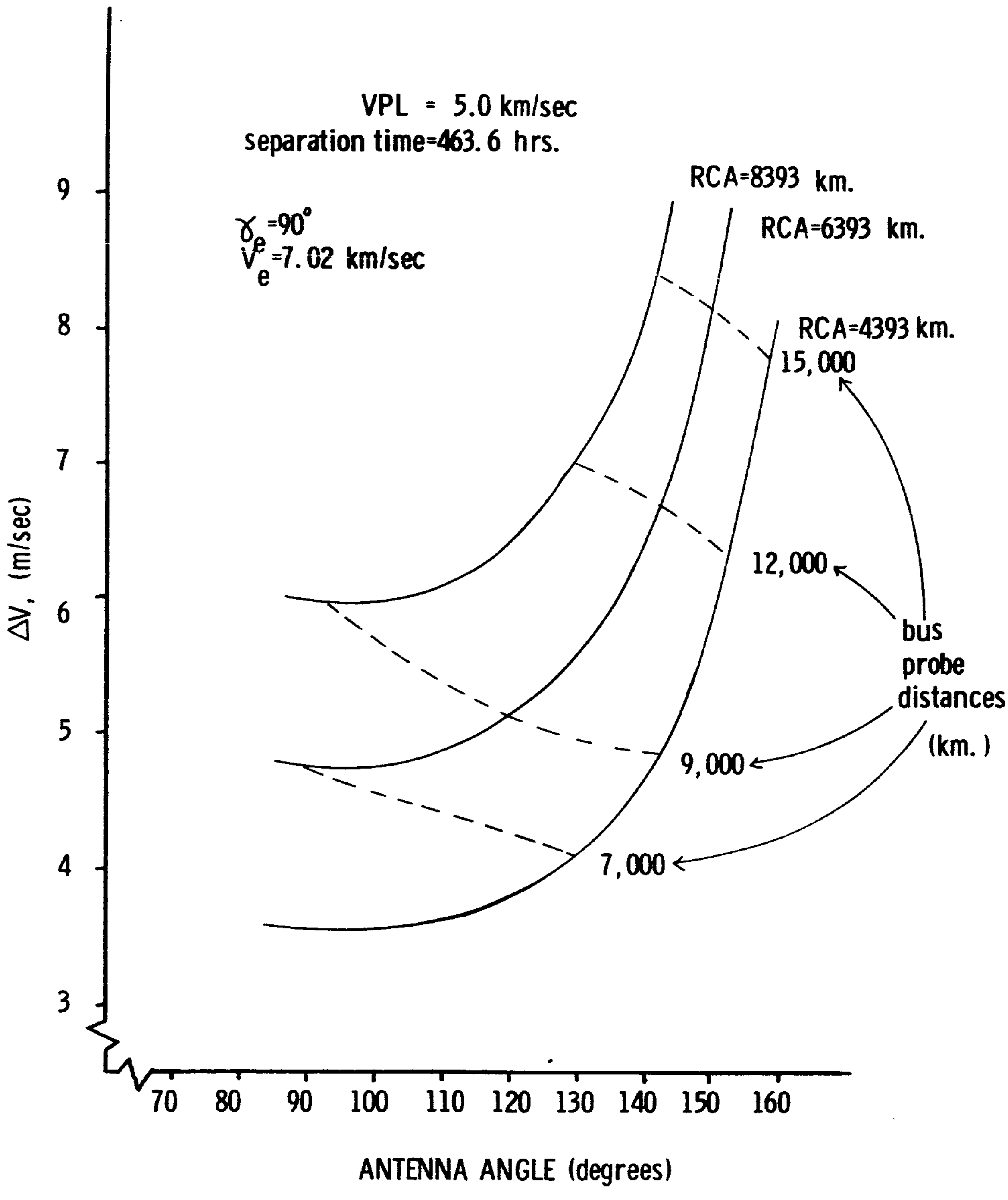


Figure 6–Separation velocity vs. antenna angle

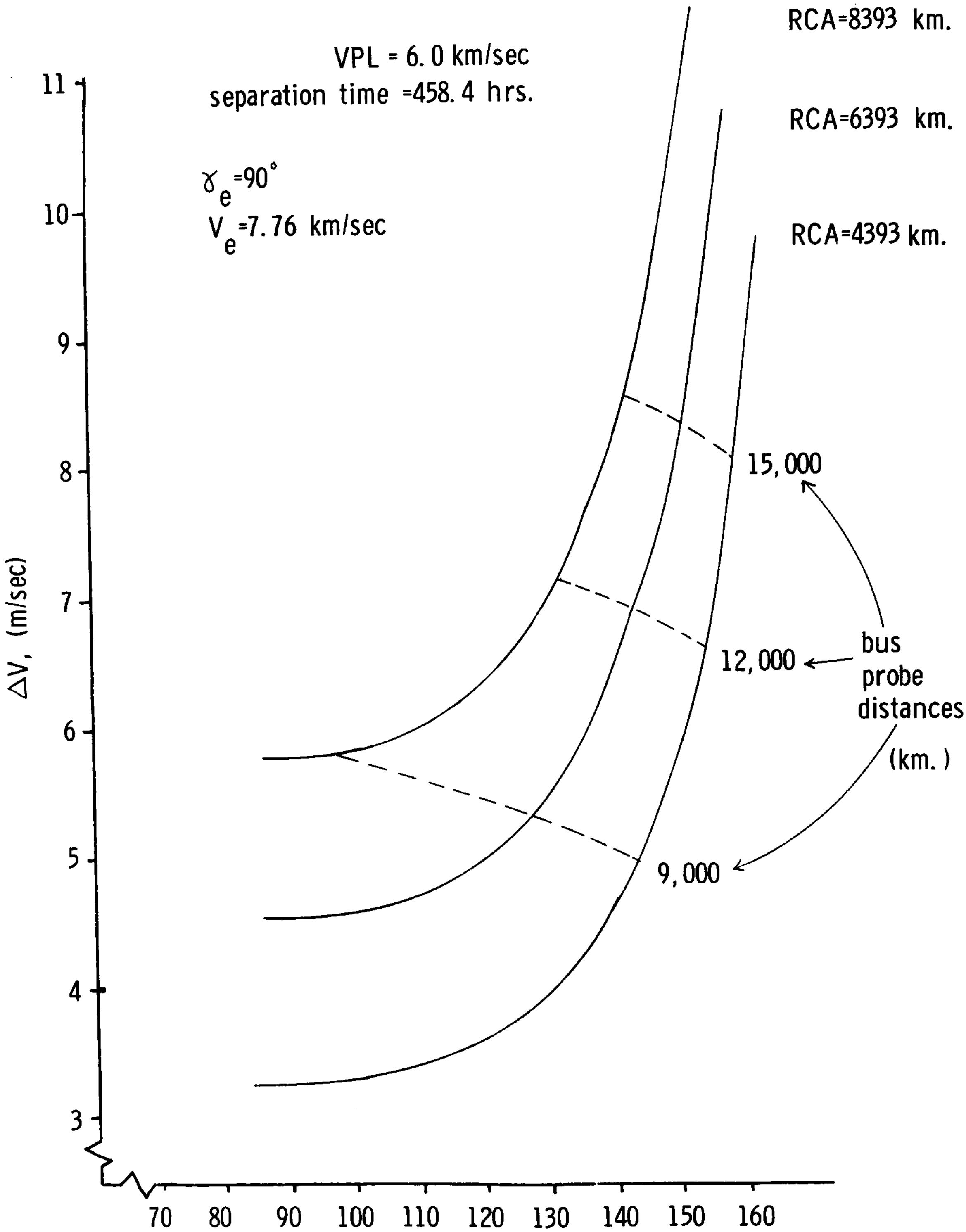


Figure 7–Separation velocity vs. antenna angle

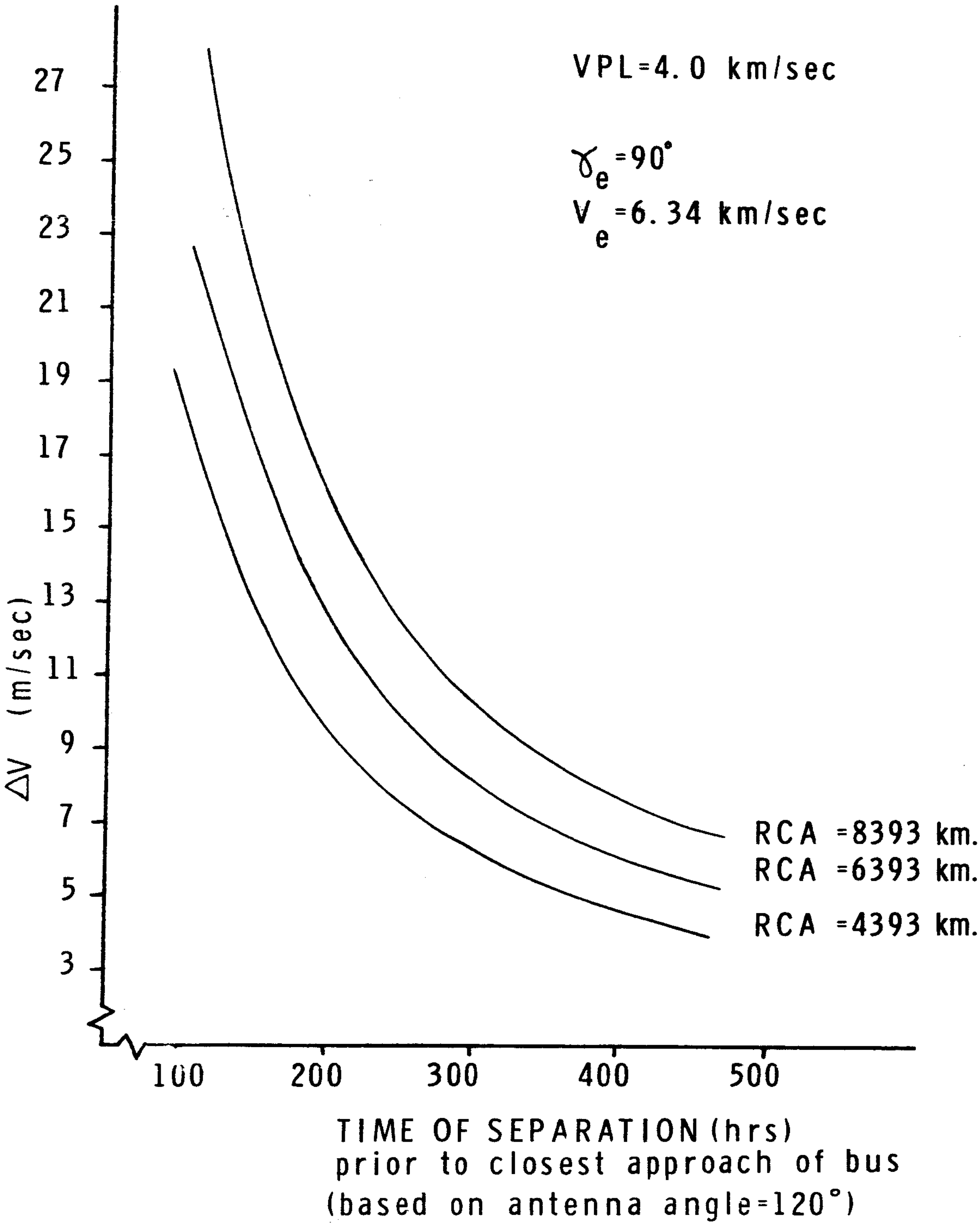


Figure 8—Separation velocity vs. separation time

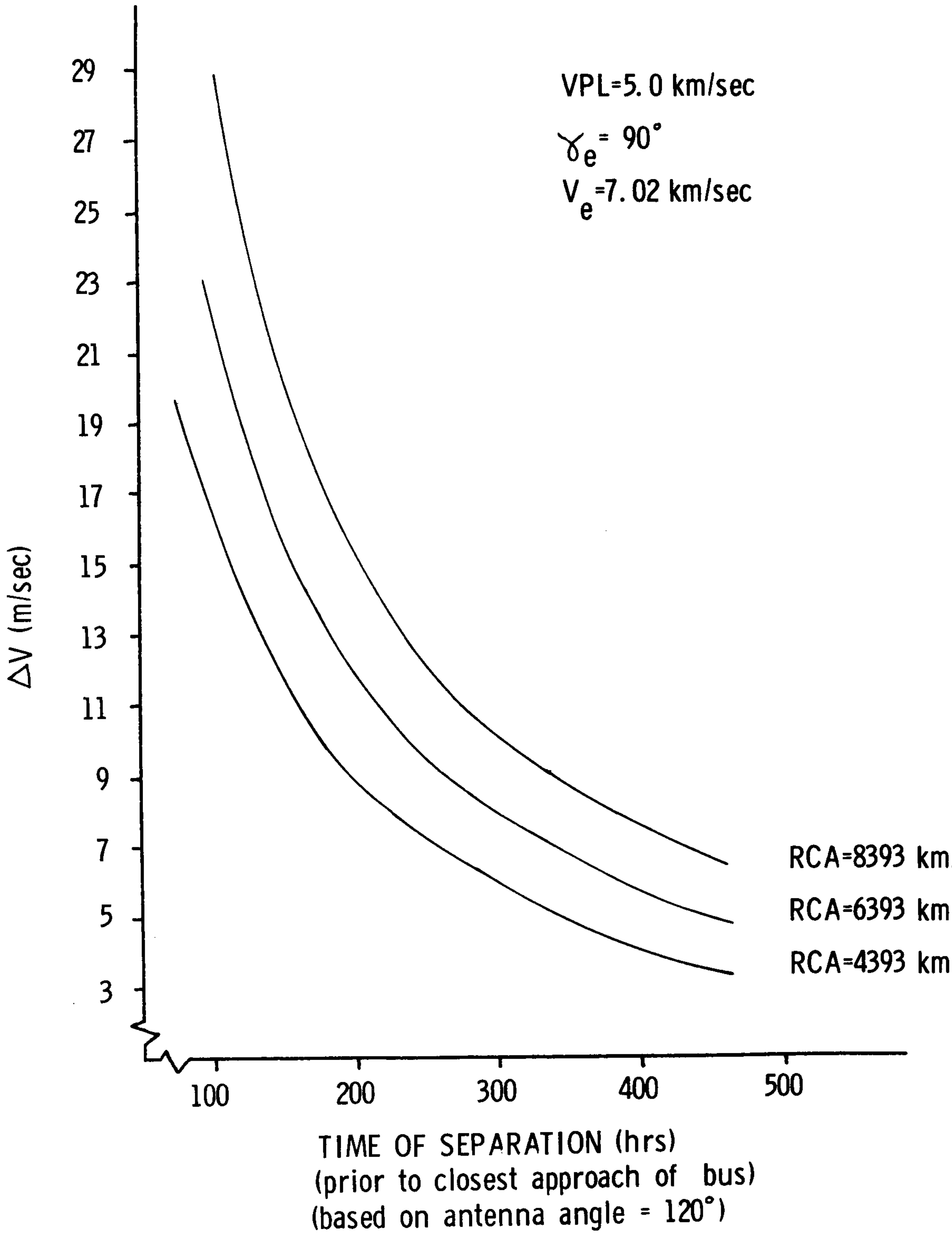


Figure 9–Separation velocity vs. separation angle

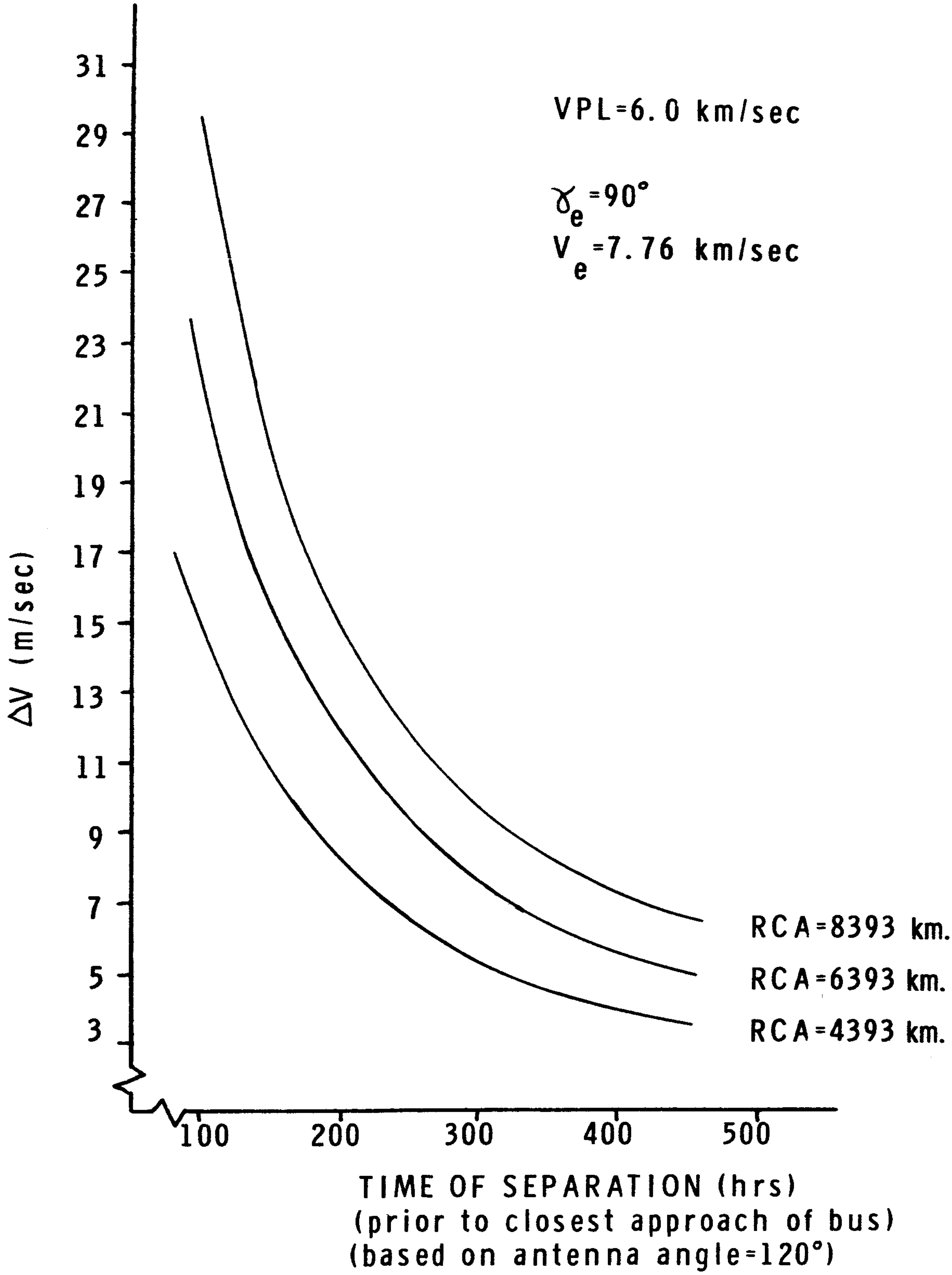


Figure 10—Separation velocity vs. separation time