V

[redacted]

X-615-65-409

NASA TM X-55384

MAGNETOPLASMA DIFFUSION AT F2 REGION ALTITUDES

FACILITY FORM 602

N66-17257	
(ACCESSION NUMBER)	(THRU)
21	1
(PAGES)	(CODE)
	13
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

GPO PRICE $ ______

CFSTI PRICE(S) $ ______

Hard copy (HC) 1.00

Microfiche (MF) .50

ff 653 July 65

OCTOBER 1965

NASA

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

Ionospheric Physics Preprint Series

MAGNETOPLASMA DIFFUSION AT F2 REGION ALTITUDES

P. C. Kendall*

October 1965

Goddard Space Flight Center
Greenbelt, Maryland

*Permanent address: The University, Sheffield 10, England

MAGNETOPLASMA DIFFUSION AT F2 REGION ALTITUDES

P. C. Kendall

ABSTRACT

The magnetoplasma diffusion equation of the F2 region of the ionosphere is derived in its primitive form, allowing for electric currents flowing between conjugate points of the dynamo region, and for temperature variations.

It appears that the standard view of the ambipolar diffusion problem is correct. The ion-electron pairs may be thought of as sliding down the magnetic field lines with diffusive motion, while the whole field line drifts at right angles to itself.

The electric current along a field line would affect the ionization only if its magnitude were 100 times greater than Dougherty's estimate[1] of 10^{-7} amps/m^2.

CONTENTS

MAGNETOPLASMA DIFFUSION AT F2 REGION ALTITUDES

P. C. Kendall

INTRODUCTION

In this note the magnetoplasma diffusion equations of the F2 layer are derived in an attempt to lay down a solid foundation for future work. This is necessary for two reasons. First, it is clear that many future computations will be based upon current theory, which should therefore be refined as far as possible. Second, there has been much recent discussion, as evidenced by the work of Chandra[2], Kendall[3], Ferraro[4], Varnasavang[5] and Gliddon[6].

The possible effects of a hypothetical electric current nearly along the magnetic field lines will be included because Dougherty[1] has suggested that such an electric current may arise from asymmetry in the dynamo region.

The electron and ion temperatures, T_e and T_i, will be retained as different functions of position throughout, so that even in the case when there is no electric current along the field lines, the equations developed will be as useful as possible. The acceleration of the plasma will be ignored.

EQUATIONS OF THE PROBLEM

Suffixes i, e and n will be used to distinguish quantities associated with the ions, electrons and neutral atoms. For convenience, it will be assumed that at F2 region levels (300 km) the neutral atmosphere is 0 and the ions are 0^+. The plasma will be assumed to be electrically neutral, so that if N denotes the electron density,

$$N = N_i = N_e \quad . \tag{1}$$

Denote by m the mass, by $\underset{\sim}{v}$ the velocity and by p the partial pressure of a particular constituent; by $\underset{\sim}{g}$ the acceleration due to gravity with

$$|\underset{\sim}{g}| = 890 \text{ cm/sec}^2 \quad , \tag{2}$$

by e the electronic charge

$$e = 1.602 \times 10^{-20} \text{ e.m.u.} \quad , \tag{3}$$

by $\underset{\sim}{B}$ the magnetic induction vector

$$|\underset{\sim}{B}| \sim 0.3 \text{ gauss} , \tag{4}$$

by $\underset{\sim}{J}$ the density of electric current

$$\underset{\sim}{J} = eN(\underset{\sim}{v}_i - \underset{\sim}{v}_e) \tag{5}$$

and by ν_{in} , ν_{ie} , ν_{in} the collision frequencies. At F2 region levels

$$\nu_{in} \sim 1 \text{ sec}^{-1}, \; \nu_{en} \sim 35 \text{ sec}^{-1}, \; \nu_{ei} \sim 1500 \text{ sec}^{-1}. \tag{6}$$

Taking $m_n = m_i$ and assuming that $m_e << m_i$, the equation of motion of the ion-electron gas is

$$Nm_e \nu_{en} (\underset{\sim}{v}_e - \underset{\sim}{v}_n) + \frac{1}{2} Nm_i \nu_{in} (\underset{\sim}{v}_i - \underset{\sim}{v}_n)$$

$$= - \underset{\sim}{\nabla}(p_e + p_i) + Nm_i \underset{\sim}{g} + \underset{\sim}{J} \times \underset{\sim}{B} \tag{7}$$

Using Equation (5), Equation (7) may be transformed to either of the two identical forms

$$\frac{1}{2} N(m_i \nu_{in} + 2m_e \nu_{en}) (\underset{\sim}{v}_i - \underset{\sim}{v}_n)$$

$$= - \underset{\sim}{\nabla}(p_e + p_i) + Nm_i \underset{\sim}{g} + \frac{m_e \nu_{en}}{e} \underset{\sim}{J} + \underset{\sim}{J} \times \underset{\sim}{B} \tag{8}$$

and

$$\frac{1}{2} N(m_i \nu_{in} + 2m_e \nu_{en}) (\underset{\sim}{v}_e - \underset{\sim}{v}_n)$$

$$= - \underset{\sim}{\nabla}(p_e + p_i) + Nm_i \underset{\sim}{g} + \frac{m_i \nu_{in}}{2e} \underset{\sim}{J} + \underset{\sim}{J} \times \underset{\sim}{B} . \tag{9}$$

Denote by $\underset{\sim}{a}^{\parallel}$ the component of an arbitrary vector $\underset{\sim}{a}$ along a field line in the same direction as the magnetic field and by $\underset{\sim}{a}^{\perp}$ the component of $\underset{\sim}{a}$ at right angles to a field line such that $\underset{\sim}{a}^{\perp} = \underset{\sim}{a} - \underset{\sim}{a}^{\parallel}$. For future use the component of Equation (8) along a field line will be required along with the component of Equation (9) at right angles to a field line. Thus

$$\frac{1}{2} N\left(m_i \nu_{in} + 2m_e \nu_{en}\right) \left(\underset{\sim}{v}_i^{\parallel} - \underset{\sim}{v}_n^{\parallel}\right)$$

$$= - \underset{\sim}{\nabla}^{\parallel} \left(p_e + p_i\right) + Nm_i \underset{\sim}{g}^{\parallel} + \frac{m_e \nu_{en}}{e} \underset{\sim}{J}^{\parallel} \tag{10}$$

and

$$\frac{1}{2} N\left(m_i \nu_{in} + 2m_e \nu_{en}\right) \left(\underset{\sim}{v}_e^{\perp} - \underset{\sim}{v}_n^{\perp}\right)$$

$$= - \underset{\sim}{\nabla}^{\perp} \left(p_e + p_i\right) + Nm_i \underset{\sim}{g}^{\perp} - \frac{m_i \nu_{in}}{e} \underset{\sim}{J}^{\perp} + \underset{\sim}{J}^{\perp} \times \underset{\sim}{B} . \tag{11}$$

Equations (10) and (11) together constitute the equation of motion of the ion-electron gas. The equation of motion of the electrons is

$$Nm_e \nu_{en} \left(\underset{\sim}{v}_e - \underset{\sim}{v}_n\right) + Nm_e \nu_{ei} \left(\underset{\sim}{v}_e - \underset{\sim}{v}_i\right)$$

$$= - \underset{\sim}{\nabla} p_e - Ne\left(\underset{\sim}{E} + \underset{\sim}{v}_e \times \underset{\sim}{B}\right) , \tag{12}$$

where $\underset{\sim}{E}$ denotes the electrostatic field.

Denote by $\underset{\sim}{E}_1$ the electrostatic field related to motion of the electrons, by $\underset{\sim}{E}_2$ the electrostatic field due to pressure gradients and by $\underset{\sim}{E}_3$ the external electrostatic field. As the geomagnetic field is supposed to be unchanging, these electric fields are supposed without loss of generality to arise from potentials Ω_1, Ω_2 and Ω_3, where

$$\underset{\sim}{E}_1 = - \underset{\sim}{\nabla} \Omega_1, \quad \underset{\sim}{E}_2 = - \underset{\sim}{\nabla} \Omega_2, \quad \underset{\sim}{E}_3 = - \underset{\sim}{\nabla} \Omega_3, \tag{13}$$

and if s denotes the arclength and $\hat{\underset{\sim}{T}}$ denotes the unit tangent along a field line

$$\underset{\sim}{E}_1 = -\left[\frac{m_e \nu_{en}}{e}\left(\underset{\sim}{v}_e^{\parallel} - \underset{\sim}{v}_n^{\parallel}\right) + \frac{m_e \nu_{ei}}{e}\left(\underset{\sim}{v}_e^{\parallel} - \underset{\sim}{v}_i^{\parallel}\right)\right] \tag{14}$$

and

$$\underset{\sim}{E}_2 = -\frac{1}{eN}\underset{\sim}{\nabla}^{\parallel} p_e = -\frac{\hat{\underset{\sim}{T}}}{eN}\frac{\partial p_e}{\partial s}\ , \tag{15}$$

giving

$$\Omega_1 = \frac{1}{e}\int_0^s \left[m_e\, \nu_{en}\left(v_e^{\parallel} - v_n^{\parallel}\right) + m_i\, \nu_{ei}\left(v_e^{\parallel} - v_i^{\parallel}\right)\right] ds \tag{16}$$

and

$$\Omega_2 = \int_0^s \frac{1}{eN}\frac{\partial p_e}{\partial s}\, ds\ , \tag{17}$$

where $\underset{\sim}{v}^{\parallel} = v^{\parallel}\, \hat{\underset{\sim}{T}}$ for all constituents, and the level $s = 0$ is conveniently chosen. Then Ω_1 and Ω_2 are known in terms of the other variables, and Equation (12) shows that

$$\frac{\partial \Omega_3}{\partial s} = 0\ . \tag{18}$$

This shows that as defined here, the external potential Ω_3 is constant along a field line. Note that functions F and G, arbitrary apart from satisfying the equations

$$\frac{\partial F}{\partial s} = 0,\ \frac{\partial G}{\partial s} = 0, \tag{19}$$

may be added to Ω_1 and Ω_2. However, it is clear that these could be absorbed in Ω_3. It follows that the definitions of internal and external electrostatic fields are a matter of convenience only. From Equation (12), all three potentials together cause a velocity $\underset{\sim}{v}_e^{\perp}$ of the electrons given by

$$\underset{\sim}{v}_e^{\perp} \times B + \frac{m_e \nu_{en}}{e} \underset{\sim}{v}_e^{\perp}$$

$$= \underset{\sim}{\nabla}^{\perp} \Omega - \frac{1}{eN} \underset{\sim}{\nabla}^{\perp} p_e - \frac{m_e \nu_{ei}}{e^2 N} \underset{\sim}{J}^{\perp} + \frac{m_e \nu_{en}}{e} \underset{\sim}{v}_n^{\perp} \tag{20}$$

The component of Equation (12) along a field line (i.e., Ohm's Law) has already been used in Equation (14). Taking $m_e = 9.108 \times 10^{-28}$ gm, Equations (3) and (6) show that

$$m_e \nu_{en}/eB << 1 . \tag{21}$$

Thus, to a high degree of approximation,

$$\underset{\sim}{v}_e^{\perp} = \frac{\underset{\sim}{B}}{B^2} \times \left(\underset{\sim}{\nabla}^{\perp} \Omega - \frac{1}{Ne} \underset{\sim}{\nabla}^{\perp} p_e\right)$$

$$+ \frac{m_e \nu_{ei}}{e^2 NB^2} \underset{\sim}{J}^{\perp} \times \underset{\sim}{B} - \frac{m_e \nu_{en}}{eB^2} \underset{\sim}{v}_n^{\perp} - \underset{\sim}{B} . \tag{22}$$

In fact, the last two terms in this equation are also likely to be small. However, it is preferable at this stage not to jeopardize the argument by using approximations which are not immediately justifiable.

The system of equations has now been reduced to two Equations (11) and (22) in the vector quantities $\underset{\sim}{J}^{\perp}$ and $\underset{\sim}{v}_e^{\perp}$.

ANALYSIS OF THE EQUATIONS

Substituting back into Equation (11) from Equation (22) gives an equation for $\underset{\sim}{J}^{\perp}$ in the form

$$- \frac{m_i \nu_{in}}{e} \underset{\sim}{J} + \left\{1 - \frac{m_e \nu_{ei}}{2e^2 B^2} \left(m_i \nu_{in} + 2m_e \nu_{en}\right)\right\} \underset{\sim}{J}^{\perp} \times \underset{\sim}{B}$$

$$= \frac{N}{2} \left(m_i \nu_{in} + 2m_e \nu_{en}\right) \left[\frac{\underset{\sim}{B}}{B^2} \times \left(\underset{\sim}{\nabla}^{\perp} \Omega - \frac{1}{eN} \underset{\sim}{\nabla}^{\perp} p_e\right) - \frac{m_e \nu_{en}}{eB^2} \underset{\sim}{B} \times \underset{\sim}{v}_n^{\perp}\right]$$

$$+ \underset{\sim}{\nabla}^{\perp} \left(p_e + p_i\right) - Nm_i \underset{\sim}{g}^{\perp} \tag{23}$$

Taking $m_i = 13.38 \times 10^{-24}$ gm, Equations (3) and (6) show that

$$\frac{m_e \nu_{en}}{eB} << \frac{m_i \nu_{in}}{eB} << 1 . \tag{24}$$

Using this result in Equation (23) it is seen that all terms on the right hand side are negligible in comparison with $\underset{\sim}{J}^{\perp} \times \underset{\sim}{B}$. Thus, to a high degree of approximation, vector multiplication by $\underset{\sim}{B}$ using the expansion formula for triple vector products, gives

$$\underset{\sim}{J}^{\perp} = \frac{Nm_i \nu_{in}}{2B^2} \left(\frac{m_e \nu_{en}}{e} \underset{\sim}{v}_n^{\perp} - \underset{\sim}{\nabla}^{\perp} \Omega + \frac{1}{Ne} \underset{\sim}{\nabla}^{\perp} p_e - \underset{\sim}{B} \times \underset{\sim}{v}_n^{\perp} \right)$$

$$+ \frac{\underset{\sim}{B}}{B^2} \times \left[\underset{\sim}{\nabla}^{\perp} (p_e + p_i) - Nm_i \underset{\sim}{g}^{\perp} \right] . \tag{25}$$

The corresponding relative velocity of ions and electrons at right angles to a field line is obtained from Equation (5) and is

$$\left| \underset{\sim}{v}_e^{\perp} - \underset{\sim}{v}_i^{\perp} \right| = \left| \frac{m_i \nu_{in}}{2eB^2} \left(\frac{m_e \nu_{en}}{e} \underset{\sim}{v}_n^{\perp} - \underset{\sim}{\nabla}^{\perp} \Omega + \frac{1}{Ne} \underset{\sim}{\nabla}^{\perp} p_e \right) \right.$$

$$\left. + \frac{\underset{\sim}{B}}{eNB^2} \times \left[\underset{\sim}{\nabla}^{\perp} (p_e + p_i) - Nm_i g^{\perp} \right] \right| . \tag{26}$$

The term $\underset{\sim}{\nabla}^{\perp} \Omega$ leads to electrodynamic drifts which are believed at most to be of the order of 20m/sec (Maeda[7]). The neutral air speed $|\underset{\sim}{v}_n|$ is unlikely to exceed this value. Moreover, the pressure gradients will each be at most of the same order of magnitude as the gravitational force. It follows that as far as orders of magnitude are concerned, the relative velocity at right angles to a field caused by the electric field is

$$\left| \underset{\sim}{v}_e^{\perp} - \underset{\sim}{v}_i^{\perp} \right| \sim \frac{1000 m_i \nu_{in}}{eB} \sim 2.8\text{cm/sec} , \tag{27}$$

while the relative velocity caused by the mechanical forces is

$$\left| \underset{\sim}{v}_e^{\perp} - \underset{\sim}{v}_i^{\perp} \right| \sim \frac{m_i g}{eB} \sim 2.5\text{cm/sec}. \tag{28}$$

It is clear that these relative velocities are both small compared with the estimated drift speed of 20m/sec and the estimated gravitational terminal speed $g\nu_{in}^{-1} \sim 10$m/sec of a falling ion-electron pair. Therefore, if the electron drift velocity is even as high as 1 m/s the ion velocity at right angles to a field line may be taken to be approximately the same as $\underset{\sim}{v}_e$. Neglecting terms which again prove to be small, Equation (22) gives

$$\underset{\sim}{v}_i^{\perp} \approx \underset{\sim}{v}_e^{\perp} \approx \frac{\underset{\sim}{B}}{B^2} \times \left(\underset{\sim}{\nabla}^{\perp} \Omega - \frac{1}{eN} \underset{\sim}{\nabla}^{\perp} p_e \right) \quad . \tag{29}$$

This well known result shows that plasma in the F2 layer can move at right angles to the magnetic field lines only when an electric field is present at right angles to the field lines. Thus, so far, Equations (14), (18) and (29) have shown that the following electric fields and simultaneous drift motions may be present in the F2 region.

<u>An Electric Field Arising From or Causing Electric Currents Along the Field Lines</u>

This electric field $\underset{\sim}{E}_1 = - \underset{\sim}{\nabla}\Omega_1$ is given by Equation (14). If there is an imposed electrostatic field of this nature from the dynamo region it will be caused by a difference in potential between the Northern and Southern conjugate points.

It will appear later that the effects of $\underset{\sim}{J}^{\parallel}$ caused by $\underset{\sim}{E}^{\parallel}$ are only apparent in Equation (10). As $m_e \nu_{en} << m_i \nu_{in}$ this implies that the only currents of this nature which might affect the electron density are such that

$$|\underset{\sim}{v}_e^{\parallel}| >> |\underset{\sim}{v}_i^{\parallel}| \text{ or } |\underset{\sim}{v}_n^{\parallel}| \quad , \tag{30}$$

so that to be consistent with $\operatorname{div} \underset{\sim}{J} = 0$,

$$\operatorname{div}\left(N\underset{\sim}{v}_e^{\parallel}\right) \approx 0 \quad . \tag{31}$$

Denote the equation of a magnetic field line by

$$a(r, \theta) = \text{constant} , \tag{32}$$

where r, θ, ϕ are spherical polar coordinates whose axis is the axis of symmetry of the magnetic field. Then if $B(r, \theta)$ denotes the magnetic field strength, since $\operatorname{div} \underset{\sim}{B} = 0$, Equation (31) gives

$$v_e^{\parallel} = \Psi(a, \phi)\, B(r, \theta)/N , \tag{33}$$

where Ψ is an arbitrary function of a and ϕ which is constant along the magnetic field lines. At this stage, it is convenient to use the direct conductivity σ along the field lines, giving

$$\underset{\sim}{E}_1^{\parallel} = - e\sigma^{-1}\, \Psi(a, \phi)\, B(r, \theta) . \tag{34}$$

Then

$$\underset{\sim}{E}_1^{\perp} = - \underset{\sim}{\nabla}^{\perp} \int_0^s \left[\sigma^{-1}\, \Psi(a, \phi)\, B(r, \theta)\right] ds . \tag{35}$$

The corresponding drift is then

$$\frac{\underset{\sim}{E}_1^{\perp} \times \underset{\sim}{B}}{B^2} = \frac{\underset{\sim}{B}}{B^2} \times \underset{\sim}{\nabla}^{\perp} \int_0^s \left[e\sigma^{-1}\, \Psi(a, \phi)\, B(r, \theta)\right] ds . \tag{36}$$

As the primary cause of the currents lies in the dynamo region, the magnitude of the current depends crucially on the processes occurring there. Dougherty[1] has estimated that the current density along a magnetic field line may be of the order of 10^{-7} amp/m^2, which is 10^{-12} e.m.u. Thus, so far as orders of magnitude are concerned

$$e\Psi B \sim 10^{-12} \text{ e.m.u.} \tag{37}$$

It is also clear that the function ΨB will vary only slowly with position, whereas the conductivity σ is likely to vary more rapidly. It follows that

$$\frac{\underset{\sim}{E}_1^{\perp} \times \underset{\sim}{B}}{B^2} \approx \frac{e\Psi}{B}\, \underset{\sim}{B} \times \underset{\sim}{\nabla}^{\perp} \int_0^s \frac{ds}{\sigma} .$$

The order of magnitude of this can be immediately found in the case when $\sigma = \sigma(z)$ is a function of only the height z. If n denotes a local coordinate at right angles to a field line, such that, locally, $z = s \sin I + n \cos I$ we have

$$\frac{\underset{\sim}{E}_1^{\perp} \times \underset{\sim}{B}}{B^2} \approx \frac{e\Psi}{B}\, \underset{\sim}{B} \times \underset{\sim}{\nabla}^{\perp} \int_{z_e}^{z} \frac{dz}{\sigma \sin I} ,$$

where z_e denotes the height of a field line at the dip equator. Thus

$$\left| \frac{\underset{\sim}{E}_1^{\perp} \times \underset{\sim}{B}}{B^2} \right| \approx e\Psi \left(\frac{\cot I}{\sigma(z)} - \frac{\operatorname{cosec} I}{\sigma(z_e)} \right)$$

For functions $\sigma(z)$ which increase with height fairly rapidly, this shows that the above drift velocity for a particular field line is zero at the dip equator and increases away from the dip equator. At moderate latitudes

$$\left| \frac{\underset{\sim}{E}_1^{\perp} \times \underset{\sim}{B}}{B^2} \right| \sim \frac{e\Psi}{\sigma} \sim \frac{10^{-12}}{\sigma B} \ \text{cm/sec} \,. \tag{38}$$

It seems likely that this estimate of the drift due to electric currents along the field lines will apply to most functional forms of σ. Also, in Equation (38) it should be noted that σ is the local value of the direct conductivity, giving the local drift of this kind.

Taking $B \sim 0.3$ gauss and $\sigma \sim 10^{-8}$ or 10^{-9} e.m.u. Equation (38) shows that the electrodynamic drift arising from $\underset{\sim}{E}_1$ is negligible, even for currents 100 times larger than those predicted by Dougherty[1].

However, there is still a component of electric current $\underset{\sim}{J}^{\parallel}$ in Equation (10), and this will be discussed later.

A Pressure Gradient Electric Field $\underset{\sim}{E}_2 = -\underset{\sim}{\nabla}\Omega_2$

This electric field is given by Equation (15), but the combined drift caused by both electric field and pressure gradients is given by Equation (29) to be

$$\left(\underset{\sim}{E}_2^{\perp} + \frac{1}{eN} \underset{\sim}{\nabla}^{\perp} p_e \right) \times \frac{\underset{\sim}{B}}{B^2} \,. \tag{39}$$

Using Equation (17) and assuming that the operators $\underset{\sim}{\nabla}^{\perp}$ and $\int_0^s ds$ commute gives

$$\underset{\sim}{E}_2^{\perp} + \frac{1}{eN} \underset{\sim}{\nabla}^{\perp} p_e = \int_0^s \frac{k}{eN} \left(\frac{\partial N}{\partial s} \underset{\sim}{\nabla}^{\perp} T_e - \frac{\partial T_e}{\partial s} \underset{\sim}{\nabla}^{\perp} N \right) ds \,, \tag{40}$$

where k is the Boltzmann constant,

$$k = 1.38 \times 10^{-16} \ \text{erg/deg} \,. \tag{41}$$

It will be sufficient to ignore the curvature of the field lines in estimating the magnitude of this expression. Thus, if $\underset{\sim}{n}$ denotes the meridional unit normal to a field line, $\underset{\sim}{\hat{e}}_\phi$ denotes the azimuthal unit vector and I denotes the dip angle

$$\frac{\partial}{\partial s} \equiv \sin I \frac{\partial}{\partial r} + \frac{\cos I}{r} \frac{\partial}{\partial \theta} \tag{42}$$

and

$$\underset{\sim}{\nabla}^{\perp} \equiv \underset{\sim}{\hat{n}} \cos I \frac{\partial}{\partial r} - \frac{\sin I}{r} \frac{\partial}{\partial \theta} + \frac{\underset{\sim}{\hat{e}}_\phi}{r \sin \theta} \frac{\partial}{\partial \phi} \tag{43}$$

giving

$$\underset{\sim}{E}_2^{\perp} + \frac{1}{eN} \underset{\sim}{\nabla}^{\perp} p_e = \int_0^s \frac{k\underset{\sim}{\hat{n}}}{eNr} \frac{\partial(T_e, N)}{\partial(r, \theta)} ds + \int_0^s \frac{k\underset{\sim}{\hat{e}}_\phi}{eNr \sin\phi} \frac{\partial(N, T_e)}{\partial(s, \phi)} ds . \tag{44}$$

Here we denote the Jacobian of any two functions f, g of variables ζ, η by

$$\frac{\partial(f, g)}{\partial(\zeta, \eta)} = \frac{\partial f}{\partial \zeta} \frac{\partial g}{\partial \eta} - \frac{\partial f}{\partial \eta} \frac{\partial g}{\partial \zeta} .$$

The drift arising from the first integral on the right-hand side will be in the EW direction, while the drift arising from the second term will be in a meridian plane. It follows that as both N and T_e are likely to vary most rapidly in the vertical direction, the major contribution to the continuity equation is likely to arise from the second term on the right-hand side of Equation (44). The order of magnitude of the important drift component (judged by this criterion) is therefore,

$$\frac{kT_e}{erB} \sim 3 \times 10^{-2} \text{ cm/sec} , \tag{45}$$

where $T_e \sim 1000°K$, using Equations (3) and (41) with $r \sim 7 \times 10^8$ cm. This also is negligibly small.

A Magnetohydrodynamic (or $\underset{\sim}{E} \times \underset{\sim}{B}$) Drift

The electric field which gives rise to this drift is such that according to Equation (18), Ω_3 is constant along a field line. This particular form of electrodynamic drift is well known. Martyn[8] first suggested the importance of this type of transport in the F2 region, and according to calculations by Moffett and Hanson[9] and Bramley and Peart[10] it plays a great part in the geomagnetic control of the F2 layer.

THE CONTINUITY EQUATION

Denote by D_a the coefficient of ambipolar diffusion defined by

$$2kT_i \, D_a^{-1} = \frac{1}{2}\left(m_i \, \nu_{in} + 2m_e \, \nu_{en}\right) \tag{46}$$

Equation (10) then gives

$$\underset{\sim}{v_i}^{\parallel} = \underset{\sim}{v_n}^{\parallel} - \frac{D_a}{2NkT_i}\left[\underset{\sim}{\nabla}^{\parallel}\left(p_e + p_i\right) - Nm_i \, \underset{\sim}{g}^{\parallel}\right] + \frac{2m_e \nu_{en}}{eN\left(m_i \nu_{in} + 2m_e \nu_{en}\right)} \underset{\sim}{J}^{\parallel} \tag{47}$$

At this stage it is possible to show finally that $\underset{\sim}{J}^{\parallel}$ may be omitted. Equations (6) and the values $m_e \sim 10^{-27}$ gm, $m_i \sim 10^{-23}$ gm give

$$\frac{m_e \, \nu_{en}}{m_i \, \nu_{in}} \sim 4 \times 10^{-3} \, . \tag{48}$$

Also Dougherty's estimate of 10^{-12} e.m.u. for $\underset{\sim}{J}^{\parallel}$, together with $N \sim 10^6$ cm^{-3} and Equation (3) give

$$\left|\frac{\underset{\sim}{J}^{\parallel}}{eN}\right| \sim 10^2 \text{ cm/sec} \, . \tag{49}$$

As $|\underset{\sim}{v_i}^{\parallel}|$ is likely to be of order of magnitude 1m/sec it appears that the last term in Equation (47) is likely to be inappreciable; in comparison so that for most purposes

$$N\underset{\sim}{v_i}^{\parallel} = N\underset{\sim}{v_n}^{\parallel} - \frac{D_a}{2kT_i}\left[\underset{\sim}{\nabla_n}^{\parallel}\left(p_e + p_i\right) - Nm_i \, \underset{\sim}{g}^{\parallel}\right] \, . \tag{50}$$

It follows that if Q denotes the rate of production, and L the loss rate, a good approximation to the F2 region continuity equation is

$$\frac{\partial N}{\partial t} = Q - L - \operatorname{div}\left(N\underset{\sim}{v}_n^{\parallel}\right) - \operatorname{div}\left(N\underset{\sim}{v}_i^{\perp}\right) + \operatorname{div}\left(\frac{D_a}{2kT_i}\left[\underset{\sim}{\nabla}^{\parallel}\left(p_e + p_i\right) - Nm_i\, \underset{\sim}{g}^{\parallel}\right]\right), \tag{51}$$

where $\underset{\sim}{v}_n^{\parallel}$ denotes the known component of the neutral air velocity along a field line, and $\underset{\sim}{v}_i^{\perp}$ denotes the electrodynamic drift.

$$\underset{\sim}{v}_i^{\perp} = \frac{\underset{\sim}{B}}{B^2} \times \operatorname{grad} \Omega_3 , \tag{52}$$

at right angles to a field line.

CONCLUSION

Under normal F2 region conditions the standard view of the ambipolar diffusion problem appears to be correct. The ion-electron pairs may be thought of as sliding down the magnetic field lines with a diffusive motion, while at the same time the whole field line drifts at right angles to itself. The continuity equation is then Equation (51).

The current $\underset{\sim}{J}^{\parallel}$ between conjugate points of the dynamo layer (Dougherty[1]) appears to have little effect if its magnitude is as small as 10^{-7} amps/m^2. However, if $\underset{\sim}{J}^{\parallel}$ were to become 100 times larger, its effects on $\underset{\sim}{v}_i^{\parallel}$ might become appreciable. Even then, its effects upon $\underset{\sim}{\nabla} \cdot N\underset{\sim}{v}_i^{\parallel}$ would be noticeable only when ν_{en}/ν_{in} in Equation (47) varies appreciably in a vertical distance of one scale height. (This would occur, for example, in a thin layer where the electron temperature were greater than the ion temperature.)

ACKNOWLEDGMENT

I am very grateful to Dr. Jay R. Herman for his useful criticism, and to the National Academy of Sciences and National Aeronautics and Space Administration for the award of a Senior Postdoctoral Associateship held during the summer of 1965.

REFERENCES

1. Dougherty, J. P., 1963, Some Comments on Dynamo Theory, J. Geophys. Res., 68, 2383-2384.

2. Chandra, S., 1964, Plasma Diffusion in the Ionosphere, J. Atmos. Terr. Phys., 26, 113-122.

3. Kendall, P. C., 1964, On the Equations Governing Diffusion in the F2 Layer, J. Atmos. Terr. Phys., 26, 624-626.

4. Ferraro, V. C. A., 1964, Some Remarks on Ambipolar Diffusion in the Ionosphere, J. Atmos. Terr. Phys., 26, 913-917.

5. Varnasavang, V., 1965, Diffusion in the F2 Region, J. Atmos. Terr. Phys., 27, 657-661.

6. Gliddon, J. E. C., 1965, Ambipolar Diffusion and Drift Motions in the F2 Region, Plan. Space Sci., In Press.

7. Maeda, H., 1963, World-Wide Pattern of Ionization Drifts in the Ionospheric F Region as deduced from Geomagnetic Variations, The Ionosphere, Inst. Phys. and Phys. Soc., Lond., 187-190.

8. Martyn, D. F., 1947, Atmospheric Tides in the Ionosphere I Solar Tides in the F2 Region, Proc. Roy. Soc. Lond. A 189, 241-260.

9. Moffett, R. J. and Hanson, W. B., 1965, Effect of Ionization Transport on the Equatorial F2 Region, Nature, Lond., 206, 705-706.

10. Bramley, E. N. and Peart, M., 1965, Effect of Ionization Transport on the Equatorial F2 Region, Nature, Lond., 206, 1245.