# GRADUATE AERONAUTICAL LABORATORIES
# CALIFORNIA INSTITUTE OF TECHNOLOGY

LITERATURE SURVEY
on the
BUCKLING OF CONICAL SHELLS

by Shigeo Kobayashi

January 1966 Grant No. NsG-18-59



Firestone Flight Sciences Laboratory

Guggenheim Aeronautical Laboratory

Karman Laboratory of Fluid Mechanics and Jet Propulsion

Pasadena

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By Shigeo Kobayashi*

California Institute of Technology

## SUMMARY

In order to prepare a comprehensive review of the state of the art of conical shell instability this literature survey was made. The bibliography is arranged in alphabetical form and is then categorized according to loading conditions. Certain key equations used by the authors are also indicated.

## INTRODUCTION

Prior to setting up a research program on the buckling load of conical shells, an extensive literature search was made. Eighty-three (83) of these reports have been reviewed and key information from each has been tabulated. The remaining thirty (30) were not available at the time of writing this report but will be reviewed when they can be obtained. This survey is presented in the hope that it will aid others contemplating research on conical shell stability to find these reports of interest to them.

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## CATEGORIES OF THE REFERENCES

1. Axial Compression

| | | |
|---|---|---|
| 1.1 | Axial Compression - Analysis | 13* |
| | 51, 20, 35, 41, 14, 45, 7, 36, 57, 33, 37, 38, 79 | |
| 1.2 | Axial Compression + Internal Pressure - Analysis | 4 |
| | 36, 46, 37, 38 | |
| 1.3 | Axial Compression - Experimental | 7 |
| | 16, 14, 17, 71, 39, 76, 81 | |
| 1.4 | Axial Compression + Internal Pressure - Experimental | 5 |
| | 15, 5, 71, 39, 83 | |

2. Bending

| | | |
|---|---|---|
| 2.1 | Pure Bending - Experimental | 1 |
| | 71 | |
| 2.2 | Transverse Shear and Bending - Experimental | 1 |
| | 16 | |
| 2.3 | Bending + Axial Compression - Experimental | 2 |
| | 5, 71 | |
| 2.4 | Bending + Internal Pressure - Experimental | 1 |
| | 71 | |
| 2.5 | Bending + Axial Compression + Internal Pressure | 1 |
| | 71 | |

3. External Pressure

| | | |
|---|---|---|
| 3.1 | External Pressure - Analysis | 23 |
| | 68, 69, 26, 25, 3, 4, 20, 9, 34, 67, 30, 29, 11, 44, 28, 54, 18, 22, 24, 53, 62, 73, 80 | |

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* Total number of references.

## AXIAL COMPRESSION - ANALYSIS - 1

| | | | | |
|---|---|---|---|---|
| Bibliography No. | 51) | 20) | 35) | 41) |
| Author | Shtayerman | Mushtari and Sachenkov | Sachenkov | Seide |
| Periodical | 1936, Non-linear Theory 52 | (PMM 1954) (NACA TM 1433, 1958) | (1955) Non-linear 56 | J. Appl. Mech. 1956 |
| Load; including internal pressure or not | no | no | no | no |
| Shell thickness; constant or not | constant | constant | constant | constant |
| Buckling stress or Post-buckling behavior | buckling stress | buckling stress | post-buckling behavior | buckling stress |
| Buckling mode; axisymmetrical or unsymmetrical or both | axisymmetrical | unsymmetrical | unsymmetrical | axisymmetrical |
| Differential equation (Strain displacement relation) | shallow shell approximation | shallow shell approximation | shallow shell approximation | Timoschenko - axi-symmetrical (They are the same as those of shallow shell approximation) |
| Boundary Condition | $w=0, M_x=0, T_x=0, \epsilon_y=0$ | $w=0, M_x=0, T_x=0, \epsilon_y=0$ | not considered | $u \sin\alpha - w \cos\alpha = 0$<br>$M_x=0$ |
| Solution; exact or approximate | | Galerkin Method ($w$ is assumed, $F$ is obtained) | Energy Method | Exact solution (but buckling stress is not obtained exactly) |
| Remarks | | $w = A_o x^{\frac{1-\nu}{2}} \sin\left(m, \log \frac{x}{x_o}\right) \cos n t$ | Approximate treatment alone which leads the equations to those of cylindrical shell | Result; $\frac{P}{P_{cyl} \cos^2\alpha} = 1$ |

AXIAL COMPRESSION - ANALYSIS - 2

| 14) Lackman and Penzien, J. Appl. Mech. 1960 | 45) Seide, J. Appl. Mech. 1961 | 7) Eringen, Gen. Tech. Corp. Rep., 1959 | 36) Schnell, ZAMM 1961 | 46) Seide, Proc. 4th U.S. Nat'l Congress. Appl. Mech. 1962 |
|---|---|---|---|---|
| | | no | no and internal pressure | internal pressure |
| | | constant | constant | constant |
| | | buckling stress | post-buckling behavior | buckling stress |
| | | axisymmetrical | unsymmetrical and axisymmetrical | axisymmetrical |
| | | | | shallow shell approximation |
| | | | not considered | (A) Clamped; $\epsilon_\theta = \frac{dw}{dx} = 0$<br>(B) Simply supported; $\epsilon_\theta = 0$ |
| | | | Energy Method | Fourier series; using special types of Galerkin Method |
| Discussion on Seide's paper Ref. 41) | Reply to Lackman and Penzien's discussion paper, Ref. 14) | Seide's result is quoted | $w = A \sin K_x \cos \eta\theta + B \cos 2K_x x + C$ | (A) Clamped; $\frac{dw}{dz} = \sum_{m=1}^{\infty} a_m \sin \frac{m\pi z}{z_0}$<br>$z = \log x/x_1 \quad (z_0 = \log x_2/x_1)$<br>(B) Simply supported;<br>$\frac{dw}{dz} = \sum_{m=1}^{\infty} a_m \cos \frac{m\pi z}{z_0}$ |

AXIAL COMPRESSION - ANALYSIS - 3

| | | | | |
|---|---|---|---|---|
| 57) | 33) | 37) | 38) | 79) |
| Singer | Ricardo, Marreco and Csokuyai | Schnell | Schnell | Singer |
| TAE Rep. 19, Sec. 4, 1962 | J. M. E. S. 1963 | Z. Flugwiss, 1962, Teil I | Z. Flugwiss, 1962, Teil II | AIAA Journal, 1965 |
| no | no | no and internal pressure | no and internal pressure | |
| constant | constant | constant | constant | |
| buckling stress | | buckling stress | post-buckling behavior | |
| unsymmetrical | unsymmetrical | unsymmetrical | unsymmetrical and axisymmetrical | |
| shallow shell approximation | | shallow shell approximation | shallow shell approximation | |
| Simply supported; $w=0, M_x=0$ ($v=0, T_x=0$ are not satisfied) $u, v$ are determined from the differential equations) | | | not considered | |
| Galerkin Method for 3rd eq. ($w$ : series expansion) | | Energy Method | Energy Method | |
| $w = \mathrm{Im} \sum_{n=1}^{N} C_n x^S \sin n t \theta$, $S = \gamma + i n \beta$ ($\gamma = \frac{1-\nu}{2}$, $\beta = \frac{\pi}{\log x_2}$ satisfy the boundary conditions $w = M_x = 0$) | Discussion like that of Yoshimura | $w = A \sin Kz \cos \eta \theta$ | $w = s_1 (\bar{A} \sin Kz \cos \eta \theta + \bar{B} \cos 2Kz + \bar{C})$ | Same as Ref. 57) |

AXIAL COMPRESSION - EXPERIMENTAL - 1

| Bibliography No. | 16) | 15) | 14) | 17) |
|---|---|---|---|---|
| Author | Lundquist and Schuette | Lofblad | Lackman and Penzien | Lutwak |
| Periodical | NACA WR, 1942 | MIT TR 1959 | J. Appl. Mech., 1960 | UCLA, MS Thesis, 1960 |
| Load; including internal pressure or not | no | internal pressure | no | no |
| Specimen material | 17 ST | 7075 ST-6 | Nickel | Mylar |
| Use of test specimen | one time | several times | one time | several times |
| Seam | Bolted with butt strap | 1/2 in. lap joint, bonded with Epon IV | Seamless, made by electroplating | 1.25 in. lap joint, fastened with double-backed adhesive scotch tape |
| Boundary support | Clamped to end bulkhead with steel band | Fixed with Cerrobend | | Clamped with tapered blocks |
| Semi-vertex angle $\alpha$ | $11.3^\circ$ | $2.9^\circ$ | $20^\circ$, $40^\circ$ | $10^\circ$. $20^\circ$, $30^\circ$, $45^\circ$, $60^\circ$ |
| Radius of Bottom Circle R - inches | 7.5 | 5 | | 5 |
| Shell Thickness h - inch | 0.011 | 0.004 | 0.005 ~ 0.0222 | 0.002 ~ 0.010 |
| Vertical Height L inches | 7.5 | 20 | | 4, 8 |
| Internal Pressure parameter $\bar{p} = \frac{p}{E}\left(\frac{R}{t}\right)^2$ | | 0 ~ 1.09 | | |
| Comparison with theory $P_{cr}/P_{cl}$ | 0.38 ~ 0.53 | | 0.25 ~ 0.5 | |

AXIAL COMPRESSION - EXPERIMENTAL - 2

| | | | | |
|---|---|---|---|---|
| 5)<br>Brown and Rea<br>Air Univ. MS Thesis, 1960 | 71)<br>Weingarten, Morgan and Seide<br>STL, TR, 1960 | 39)<br>Schnell and Schiffner<br>DVL, 1962 | 76)<br>Berkovito and Singer<br>7th Israel Am. Conf., 1965 | 81)<br>Weingarten, Morgan, and Seide<br>AIAA Jour., 1965 |
| Internal pressure | no internal pressure and internal pressure | no internal pressure and internal pressure | no | |
| 17-7 PH | Mylar (steel for comparison) | Mylar | | |
| Several times (in case of internal pressure) | Many times (changing pressure) | Many times (changing pressure) | | |
| 0.5 in. lap joint, bonded with adhesive and spot welded | 0.75 in. lap joint, bonded with eccobond | | | |
| | fixed with Cerrolow | | | |
| | $0^o$, $10^o$, $20^o$, $30^o$, $45^o$, $60^o$ | $0^o$, $10^o$, $20^o$, $30^o$ | | |
| 5 | | 200 mm | | |
| 0.005 | 0.002 ~ 0.01 | 0.125mm, 0.255 mm | | |
| 10 | | 173 mm | | |
| 0.07 ~ 0.96 | 0 ~ 5 | 0 ~ 1.5 | | |
| | no internal pressure 0.26 ~ 0.81<br>internal pressure ~ 1.55 | 0.5 ~ 1.2 | | |
| | Effect of eccentricity by seam is considered in determination of buckling load | | Buckling load is high: $P_{cr}/P_{cl}$ = 0.601 ~ 0.965 | Same as Ref. 71) |

## AXIAL COMPRESSION - EXPERIMENTAL - 3

83)

Weingarten, Morgan
and Seide

AIAA Journ. 1965

Same as Ref. 71)

EXTERNAL PRESSURE - ANALYSIS -1

| | | | | |
|---|---|---|---|---|
| Bibliography No. | 68), 69) | 26) | 25) | 3), 4) |
| Author | Tokagawa | Pflüger | Niordson | Bÿlaard |
| Periodical | JSNA (1932 1940) | Ing. Arch. 1937 | Tr. Roy Inst. Tech. Sweden, 1947 | Bell Air Co. T. R. 1953 and NASA TN D-1510 |
| Complete cone with apex or truncated cone | complete and truncated | complete | truncated | complete and truncated |
| Hydrostatic pressure alone or + axial compression, or other | hydrostatic | (1) hydrostatic<br>(2) dead weight | hydrostatic | hydrostatic |
| Pressure, uniform or not | uniform | uniform | uniform | uniform and non-uniform |
| Shell thickness; constant or not | constant | linear-variation | constant | constant |
| Small deflection theory or large deflection theory | | small deflection theory | small deflection theory | small deflection theory |
| Differential equation | | Pflüger's equation | Niordson's equation | |
| Exact solution or approximate solution | | exact solution of differential equation | Energy Method | Energy Method |
| Satisfy or not boundary condition | | satisfy B. C. at apex but do not satisfy B. C. at lower end | not considered | B. C. is simply supported. It is approximately satisfied |
| Include or not effect of prebuckling deformation | | not considered | not considered | not considered |
| Remarks | A proposal of equivalent cylinder radius and design formula, and comparison with experiment. | Solution which satisfy the differential equation;<br>$u = A\rho^n \cos\lambda\varphi$<br>$v = B\rho^n \sin\lambda\varphi$<br>$w = C\rho^n \cos\lambda\varphi;\ n=2$ | Assumed displacement;<br>$u = A\cos n\theta \cos\lambda\xi$<br>$v = B\sin n\theta \sin\lambda\xi$<br>$w = C\cos n\theta \sin\lambda\xi$<br>simple but not too good | $w = C r^2 \sin\lambda x \cos n\theta$<br>(select suitable length $\lambda$ ) |

## EXTERNAL PRESSURE - ANALYSIS - 2

| | | | | |
|---|---|---|---|---|
| 20) Mustari and Sachenkov PMM 1954 (NACA TM 1433) | 9) Grigolynk 1954, Translation in ASTIA | 34) Sachenkov 1956 (Ref. 19- 54) | 67) Taylor 3rd Midwestern, 1957 | 30), 29) Radkowski Prof. 3rd N.C. Appl. Mech. 1958 Watertown Lab. 1958 |
| truncated | complete | complete and truncated | complete | truncated (complete) |
| hydrostatic and external pressure + axial compression | hydrostatic | hydrostatic | hydrostatic | hydrostatic |
| uniform | uniform | uniform | uniform | uniform |
| constant | constant | variable thickness linear | constant | constant |
| small deflection theory | small deflection theory | small deflection theory | small deflection theory | small deflection theory |
| Donnell type | was not written | Donnell type | was not written, but corresponds to Pflüger type | |
| Galerkin Method | Energy Method | Exact solution of differential equation | Energy Method | Energy Method |
| Simply supported $w=0, M_x=0, T_x=0, \varepsilon_\theta=0$ satisfy completely | Satisfy B.C. at apex and at lower end | He considered a long shell. Did not satisfy boundary condition | Satisfy B.C. at apex and $u \sin\alpha + w\cos\alpha = 0$, $v=0$ at lower end | |
| not considered | not considered | not considered | not considered | not considered |
| Assumed deflection $w = A_0 e^{\gamma z} \sin mz \cos n\varphi$ $\left(z = \frac{x}{x_0}\right)$ | Assumed deflection $w = A_1 t^2 \sin\frac{\pi t}{b} \sin n\varphi$ $v = A_2 t^2 \sin\frac{\pi t}{b} \cos n\varphi$ $u = A_3 t^2 \cos\frac{\pi t}{b} \cos n\varphi$ | | $u=0$ $v = Bx^3(L-x)\sin n\theta$ $w = Cx^3(L-x)\cos n\theta$ Expression of strain-displacement relation in large deflection theory is shown | Assumed displacement; $u = A_1 \cos n\theta \cos \pi\xi/\ell$ $v = B_1 \sin n\theta \sin \pi\xi/\ell$ $w = C_1 \cos n\theta \sin \pi\xi/\ell$ |

## EXTERNAL PRESSURE - ANALYSIS - 3

| | | | | |
|---|---|---|---|---|
| 11) Hoff and Singer (Proc. IUTAM, 1959) | 44) Seide Proc. IUTAM, 1959 | 28) Pittner and Morton Lockheed Rep., 1961 | 54) Singer J. Mech. Eng. Sci. 1961 | 18) Mescall WAL. TR. 1961 |
| truncated | truncated and complete | truncated | truncated | truncated and complete |
| hydrostatic | hydrostatic | hydrostatic and external pressure + axial compression | (1) hydrostatic (2) external pressure (3) external pressure + axial compression | hydrostatic |
| uniform | uniform | uniform | uniform and non-uniform | uniform |
| constant | constant | constant | constant | constant |
| small deflection theory | small deflection theory | small deflection theory | small deflection theory | (large deflection theory) |
| Hoff's equation | Seide's equation (Donnell type) | was not written | Donnell type | shallow shell assumption |
| Galerkin Method | Energy Method | Energy Method | Galerkin Method | Energy Method |
| $w=0$, $M_x=0$ does not consider condition for $u$ and $v$ | satisfy $u \sin\alpha - w\cos\alpha = 0$ and $v=0$ at both ends | satisfy $v=0$, $w=0$ AND $M_x=0$ does not consider condition of | satisfy $w=0$ and $M_x=0$ does not consider conditions for $u$ and $v$ | |
| not considered | not considered | not considered | not considered | includes the effect of prebuckling deformation |
| Assumed displacement $u = J_m \sum_{n=1}^{N} A_n x^s \sin t\xi$ $v = J_m \sum B_n$ " $w = J_m \sum C_n$ " $s = \delta + i m\beta$ Correction of mistake is shown in Ref. 52 | Assumed displacement $w = \sum_{m=1}^{\infty} a_m \sin(m\pi \frac{z}{z_0}) \cos m\theta$ $e^z = s/s_1$ About 12 terms are required | Assumed displacement $u = C_1 e^{\mu z} \cos\lambda z \cos n\xi$ $v = C_2 e^{\mu z} \sin\lambda z \sin n\xi$ $w = C_3 e^{\mu z} \sin\lambda z \cos n\xi$ $(e^z = x/x_1)$ | Assumed Displacement $u = J_m \sum_{n=1}^{N} A_n x^s \sin t\phi$ $v = J_m \sum B_n x^s \cos t\phi$ $w = J_m \sum C_n x^s \sin t\phi$ | $w = w_0 + A\left[\frac{2\beta}{1+\beta^2}\left(\frac{t}{t_0}\right)^2 + (1-\beta)^2\right] \times \cos(a\tau + b) \times \cos n\theta$ etc. |

EXTERNAL PRESSURE - ANALYSIS - 4

| | | | | |
|---|---|---|---|---|
| 22) | 24) | 53) | 62) | 73) |
| Myursepp | Newman and Reiss | Singer | Singer | Williams |
| Iz Ak Ne. 1961 (FTDTT 62-342 1962) | NASA TN D 1510, 1962 | J. Appl. Mech., 1962 | AIAA Jour. 1963 | (Navy Rep. 1952) |
| truncated | complete and truncated | truncated | | |
| external pressure (= cone alone) | external pressure (= cone alone) | hydrostatic | | |
| linear variation | uniform | uniform | | |
| constant | constant | constant | | |
| small deflection theory | large deflection theory | small deflection theory | | |
| | axisymmetrical deformation in large deflection theory | Donnell type | | |
| Eigen value problem of diff. eq. Perturbation method | Finite difference iteration | Energy Method | | |
| satisfy | simply supported satisfy | $w=0$ and $v=0$ | | |
| not considered | not considered | not considered | | |
| | Axisymmetrical buckling of shallow cap. | Calculation of the effect of axial constrain $u = A \sin t\phi \cos \lambda\xi$ $v = B \cos t\phi \sin \lambda\xi$ $w = C \sin t\phi \sin \lambda\xi$ | Continuation of Ref. 54). Discussion on equivalent cylindrical shell | Numerical table of Niordson's theoretical result 25) |

EXTERNAL PRESSURE - ANALYSIS - 5

| |
|---|
| 80)<br>Thurston<br>J. Appl. Mech.<br>1965 |
| |
| |
| |
| |
| |
| |
| |
| |
| Effect of pre-buckling deformation is calculated |

EXTERNAL PRESSURE - EXPERIMENTAL - 1

| | | | | |
|---|---|---|---|---|
| Bibliography No. | 68) | 10) | 12) | 40) |
| Author | Tokugawa | Harris and Leyland | Jordan | Schroeder and Kusterer |
| Periodical | JSNA, 1932 | Tr. In. Chem. Eng. 1952 | U. Alabama, 1955 ASTIA | J. Appl. Mech. 1963 Aircraft Armaments, 1958 |
| Complete cone with apex or truncated cone | complete and truncated | complete | complete | complete |
| Load | Hydrostatic - uniform | Hydrostatic - uniform | Hydrostatic - uniform | Hydrostatic - uniform |
| Specimen material | brass | brass, copper, monel, steel | 24S-T3, 61S-T6 | yellow brass |
| Use of test specimen | one time | one time | one time | one time |
| Seam | seam | seam 1/8 in., soldered | seam welded | seam soldered |
| Boundary support | thick end plate | clamp the edge like hat | | steel ring |
| Semi-vertex angle $\alpha$ | $3^{\circ} \sim 34^{\circ}$ | $20^{\circ} \sim 75^{\circ}$ | $10^{\circ}$, $15^{\circ}$, $20^{\circ}$, $25^{\circ}$ | $25^{\circ}$, $15^{\circ}$, $10^{\circ}$ |
| Radius of bottom circle R inches | 50mm | 1.3 ~ 6 | 6 | 2 |
| Shell thickness h inches | 0.5mm | 0.013 ~ 0.0705 | 0.04, 0.05, 0.06, 0.07, 0.08 | 0.010 |
| Vertical height L | $10^{mm} \sim 180^{mm}$ | | | 2R, 4R, 6R |
| No. of test specimens or No. of test points | test specimen: 50 | | test specimen kinds = 20 numbers = 52 | test specimen: 10 (other reinforced specimen = 2) |
| Loading method | | hydraulic pressure (water) | hydraulic (oil) | hydraulic (oil) |
| | In some cases plastic buckling is observed | experimental formula is derived | | |

EXTERNAL PRESSURE - EXPERIMENTAL - 2

| | | | |
|---|---|---|---|
| 32)<br>Raetz<br>David Taylor M. B. Rep., 1960 | 71)<br>Weingarten, Morgan and Seide<br>STL Final Report, 1960 | 56)<br>Singer and Eckstein<br>Bul. Res. Con. Israel, 1962 | 60), 57)<br>Singer and Eckstein<br>Proc. 5 Israel, 1963 (TAE Rep. 19) |
| truncated | truncated | truncated | truncated |
| hydrostatic - uniform | hydrostatic - uniform *external + axial compression | hydrostatic - uniform | hydrostatic - uniform |
| steel | Mylar (steel) | Al. alloy, AG 5-X516 | stainless steel (18-8) 2024-T3 |
| | many times (changing pressure) | one time | steel; one time<br>Al. alloy; many times |
| seam welded | seam (Eccobond) | seam butt welded | St.; bull weld<br>Al.; lap bond |
| stiffening ring at both ends | fix by Cerrolow | simply supported | two types:<br>simply supported<br>clamped |
| $30^o$, $45^o$, $60^o$ | $0^o$, $10^o$, $20^o$, $30^o$, $45^o$, $60^o$(steel;$75^o$) | $10^o$, $20^o$, $30^o$ | $30^o$, $40^o$ |
| 13.5 | 4 | 317.5 mm | 317.5mm |
| 0.1 | 0.01 ~ 0.002<br>(steel;0.01 ,<br>0.02 ) | 0.5mm, 0.6mm,<br>0.8mm, 1.2mm,<br>1.6 mm | 0.4mm, 0.6mm,<br>0.8mm, 1.0mm |
| radius of upper circle = 8 in. | | | radius of upper circle = 50 mm., 127 mm. |
| test specimen = 6 | | test specimen; kinds = 30<br>numbers = 71 | kinds of test specimen = 10 |
| | evacuate inside air | hydraulic (oil) | hydraulic (oil) |
| stress distribution was measured before collapse | *to obtain interaction curve | | improvement of experiment in previous paper 56) |

EXTERNAL PRESSURE - EXPERIMENTAL - 3

| | |
|---|---|
| 31)<br>Radkowski<br>NASA TN D-1510, 1962 | 82)<br>Weingarten and Stein<br>AIAA Jour., 1965 |
| truncated | |
| external pressure + axial compression | external pressure and external pressure + axial compression |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| not original paper, review of his previous studies. Experimental formula of interaction curve for design use is derived. | same as Ref. 71) |

TORSION - ANALYSIS - 1

| Bibliography No. | 27) | 21) | 47) | 57) |
|---|---|---|---|---|
| Author | Pflüger | Mushtari | Seide | Singer and Baruch |
| Periodical | Ing. Arch. 1942 | PMM 1943, Ref. 19) 55 | J. Appl. Mech. 1962 | TAE Rep. 19, Sec. 1, 1962 |
| complete cone with apex or truncated cone | truncated | | truncated | truncated |
| Load | torsion | torsion | torsion | torsion and torsion + external pressure* |
| shell thickness; constant or not | constant | constant | constant | constant |
| small deflection theory or large deflection theory | small deflection theory | small deflection theory | small deflection theory | small deflection theory |
| differential equation | Pflüger's energy expression | Donnell's equation | Seide's equation (Donnell type) | Seide's equation (Donnell type) |
| exact solution or approximate solution | Energy Method | Energy Method | Energy Method | Galerkin Method |
| boundary condition; satisfied or not | not considered | | simply supported satisfied | modified simply supported satisfied |
| effect of prebuckling deformation included or not | not considered | not considered | not considered | not considered |
| Remarks | Assumed displacement: $u = A\rho^2 \sin[\lambda\xi - m(\rho-\rho_2)\tan\alpha]$ $v = B\rho^2 \sin[\quad " \quad]$ $w = C\rho^2 \cos[\quad " \quad]$ | Assumed displacement: $w = -2Ce^z \sin(\mu z + n\xi)\sin m_1 z$ $u = C_1 e^z \{\sin(\mu_1 z + n\xi) - \sin(\mu_2 z + n\xi)\}$ $v = C_2 e^z \{\quad " \quad\}$ | Method of analysis is the same as his paper Ref. 44). | Method of analysis is the same as his paper Ref. 54). *interaction curve is obtained |

## TORSION - ANALYSIS - 2

| | |
|---|---|
| 61) | 76) |
| Singer | Berkovito and Singer |
| TAE Rep. 30, Sec. 2, 1963 | 7th Israel Am. Conf., 1965 |
| truncated | |
| torsion + axial compression* | |
| constant | |
| small deflection theory | |
| Seide's equation (Donnell type) | |
| Galerkin Method | |
| modified simply supported satisfied | |
| not considered | |
| Method of analysis is the same as his paper Ref. 54)<br>*interaction curve is obtained | |

TORSION - EXPERIMENTAL - 1

| | | | | |
|---|---|---|---|---|
| Bibliography No. | 16) | 71) | 60), 57) | 61) |
| Author | Lundquist and Schutte | Weingarten, Morgan and Seide | Singer and Eckstein | Berkovito and Singer |
| Periodical | NACA WRL 1942 | STL Final Report, 1960 | Proc. 5 Israel Conf. 1963, TAE Rep. 19, Sec. 3 | TAE Rep. 30, Sec. 3, 1963 |
| complete cone with apex or truncated cone | truncated | truncated | truncated | truncated |
| Load | torsion | torsion | torsion and torsion + external pressure* | (1) torsion<br>(2) axial compression<br>(3) torsion + axial compression |
| specimen material | 17 S-T | steel | 2024S-T3, steel | 2024S-T3, AG 5-X 516 |
| use of test specimen | one time | - | 2024S-T3; several times<br>steel; one time | 2024S-T3; several times |
| seam | seam bolted | | seam: Al.: lap bond<br>St.: butt weld | seam: Al.: lap bond<br>AG 5-X516: butt weld |
| boundary support | | | simply supported | simply supported |
| semi-vertex angle $\alpha$ | | $30^{\circ}$, $60^{\circ}$ | $30^{\circ}$, $40^{\circ}$ | $40^{\circ}$ |
| radius of bottom circle R inches | 7.5 | 10 | 317.5 mm | 140 mm |
| shell thickness h inch | 0.011 , 0.016<br>0.022 | 0.01 , 0.02 | 0.4 mm, 0.6 mm<br>0.8 mm, 1.0 mm | 2024S-T3: 0.4 mm<br>AG5-X516: 0.53 mm |
| vertical height L | radius of upper end 6 in. | | | |
| No. of test specimens or No. of test points | No. of test specimen 5 | Test specimen 10 | | Test specimen 20 |
| | Comparison with the theory on cylinder | Comparison with the theory in Ref. 47) | Comparison with the thesis in Ref. 47) and 57)<br>*interaction curve | Interaction curve between torsion and axial compression |

TORSION - EXPERIMENTAL - 2

| | |
|---|---|
| 72) | 76) |
| Weingarten | Berkovito and Singer |
| AIAA Jour. 1964 | 7th Israel Am. Conf. 1965 |
| truncated | |
| (1) torsion<br>(2) torsion + internal pressure | (1) torsion<br>(2) compression<br>(3) torsion + compression |
| Mylar | |
| many times | |
| seam 3/4 in. lap joint | |
| fixed to Al. end plates by Cerrobend | |
| $20^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}$ | |
| 5 | |
| 0.01 and 0.005 | |
| | |
| | |
| | |
| Comparison with theory: torsion; Seide torsion + internal pressure; Singer | |

ORTHOTROPIC - ANALYSIS - 1

| Bibliography No. | 64) | 55) | 50) | 58) 59)* |
| --- | --- | --- | --- | --- |
| Author | Steyer and Zien | Singer | Serpico | Singer and Fersht-Scher |
| Periodical | U. Alabama Rep. 1961 | J. Appl. Mech. 1963 (TAE Rep. 18) | AIAA Jour., 1963 | Aero Qurt. 1964, (TAE Rep. 22) |
| complete cone with apex or truncated cone | truncated | | truncated | truncated |
| Load | external pressure (+ axial compression) | equation alone arbitrary load | external pressure (+ axial compression) | external pressure |
| orthotropic property: constant or not | constant | constant | constant | constant |
| shell thickness: constant or not | linear variation | | constant | constant |
| small deflection theory or large deflection theory | small deflection theory | small deflection theory | | small deflection theory |
| differential equation | | Singer's equation (Donnell type) | Donnell type | Singer's equation |
| exact solution or approximate solution | | | one term Rayleigh Ritz Method | Galerkin Method |
| boundary condition: satisfied or not | | | | Modified Singer type |
| effect of prebuckling deformation: included or not | not considered | not considered | not considered | not considered |
| Remarks | | Derivation of Donnell type equation for orthotropic conical shell. | | Application of Singer's method (Ref. 54) |

*NASA TN D-1510

ORTHOTROPIC - ANALYSIS - 2

| | |
|---|---|
| 2) 59)* | 61), 63) |
| Baruch and Singer | Singer and Fersht-Scher |
| TAE Rep. 28, 1963 | (TAE Rep. 30, Sec. 4, 1963) and (6th Am. Conf. 1964) |
| truncated | truncated |
| external pressure | torsion (+ external pressure) |
| not constant, stiffened shell | constant |
| constant | constant |
| small deflection theory | small deflection theory |
| Donnell type new equation | Singer's equation |
| Galerkin Method | Galerkin Method |
| Singer type, modified one | Singer type, modified one |
| not considered | not considered |
| *NASA TN D-1510 | Application of Singer's Method (Ref. 54) |

## ORTHOTROPIC - EXPERIMENTAL

| | |
|---|---|
| Bibliography No. | 61) 59) |
| Author | Singer, Eckstein, and Berkovito |
| Periodical | TAE Rep. 30, Sec. 1, 1963 |
| complete cone with apex or truncated cone | truncated |
| Load | hydrostatic pressure |
| specimen material | AG5 - X516 (3)*<br>SAE 1015 (4)*<br>2024 T3 (3)* |
| use of test specimen | one time |
| seam | AG5-X516<br>SAE 1015 butt weld<br>2024 T3 lap bond |
| stringer, ring | AG5-X516<br>SAE 1015 machining<br>2024 T3 bonding or riveting |
| boundary support | |
| semi vertex angle $\alpha$ | $22^{\circ}$, $20^{\circ}$, $40^{\circ}$<br>(3)* (4)* (3)* |
| radius of bottom circle R | 140mm, 317.5mm |
| shell thickness $h$ | 0.60mm 0.80mm |
| vertical height L | |
| loading method | |
| | Including local buckling<br>*number of test specimen |

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