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by Henry A. Cole, Jr.

Ames Research Center
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SUMMARY

Two-dimensional measurements of damping of oscillating plates in water are found to correlate with a power law of Cauchy number and geometrical parameters. These results are used to develop an equation for prediction of fuel-slosh damping by ring baffles in cylindrical tanks for situations where baffle thickness and liquid surface effects are small. The predictions are shown to be in excellent agreement with data for both small and large tanks.

INTRODUCTION

Fuel-slosh damping by ring baffles in cylindrical tanks has been investigated extensively in recent years, both theoretically and experimentally. A survey of damping measurements obtained in various experiments shows many apparent discrepancies. The purpose of the present report is to present a mathematical model for fuel-slosh damping which brings all of the results together and provides a logical basis for prediction of damping in full-scale tanks.

The most widely used damping equation at present is the one obtained by Miles (ref. 1) which is based on experiments of Keulegan and Carpenter (ref. 2). This equation has been used in predicting damping in tanks from 1 to 8 feet in diameter (refs. 3, 4, and 5). However, the coefficients needed to correlate with experimental data have varied from 2.25 to 4.5 for various tank sizes.

Another method for predicting damping proposed (ref. 6) using data obtained by oscillating two-dimensional plates in the fluid. The data obtained were an extension of the Keulegan and Carpenter data in that the experiments were conducted on larger plates at higher velocities. Although this technique showed good correlation with ring damping in large tanks, it showed errors of the order of a factor of 2 when extrapolated to small tanks in the range of the data of Keulegan and Carpenter.

During the recent review of fuel-sloshing work, the basic data of references 2 and 6 were found to be consistent under the damping law developed in the following sections.

NOTATION

A double amplitude of motion at baffle edge

a cylindrical tank radius

C_D drag coefficient

d depth of baffle measured from quiescent liquid surface

F measured damping force on a baffle per foot of length

g acceleration of gravity

K bulk modulus, lb/ft^2

n acceleration of vehicle, g

t baffle thickness

V maximum velocity, ft/sec

V_s velocity of sound

w chord of baffle measured perpendicular to the tank wall

y_s amplitude of motion of the liquid surface at the tank wall

ζ damping ratio

ζ_o damping ratio of tank without baffle

ρ density

ω frequency of oscillation, radians/sec

MEASURED TWO-DIMENSIONAL DAMPING FORCES

The damping data of references 2 and 6 were reduced to baffle-damping force per unit chord width of the baffle and were plotted against amplitude-to-width ratio. The resulting curves were then interpolated to obtain values of unit damping force at constant values of amplitude-to-width ratio. Constant amplitude-to-width ratio was chosen as a parameter for reasons of dimensional similarity. This procedure is consistent with the findings of Keulegan and Carpenter that a period parameter correlated their data (period parameter is equal to the amplitude-to-width ratio multiplied by $\pi/2$). The unit damping force values are shown plotted on logarithmic scales on figure 1. It may be seen that the data very nearly fall on straight lines with a slope of 1.75. This is quite remarkable since the data were obtained by different methods, with plates which differed greatly in size (1/2 to 12 in.), and over a velocity range that varied by an order of magnitude.

Curves of constant velocity are shown on figure 2. It may be seen that straight lines with a slope of -0.4 fit the data quite well. Consequently, the data from the two references which fall within the amplitude-to-width ratio of 1/4 to 3 can be closely represented by the power law

$$F/w = 0.003(A\omega)^{1.75}(A/w)^{-0.4} \tag{1}$$

DEVELOPMENT OF THE DAMPING EQUATION

The damping force equation of the previous section is obviously not complete because the dimensions of the constant must satisfy dimensional similarity. Equation (6) of reference 6 may be used to reduce equation (1) to coefficient form:

$$C_D = 11.4V^{-0.25}(A/w)^{-0.4} \tag{2}$$

From the conditions of similarity (ref. 7), it is known that the constant must be some function of the Reynolds number, Cauchy number (Mach number squared), and the cavitation parameter which satisfies dimensional similarity. The answer to this question may be obtained from figure 7 of reference 6 which shows measured drag coefficients in water at temperatures of 70° and 212° F. The particular curve which provides the key is for a 6-inch chord at an amplitude-to-width ratio of 1/2. This curve drops off rapidly with an increase of the product of a length times a velocity. However, when viscosity is varied from 1 to 0.284 centipoise by heating the water (density variation 0.998 to 0.958), the curve does not change appreciably. Hence, the process is independent of Reynolds number. The temperature change also causes the vapor pressure to vary from 0.34 to 1.47 lb/sq in.; so, since the curve does not change appreciably, cavitation effects can be ruled out. The velocity of sound, on the other hand, remains relatively constant (4700 to 5120 ft/sec) with the temperature increase, and thus can be introduced into the equation without disturbing the correlation. We assume, therefore, that the process must depend solely on the Cauchy number. When the speed of sound is introduced into equation (2),

$$C_D = 1.38(V/V_s)^{-0.25}(A/w)^{-0.4} \tag{3}$$

This equation applies only to oscillatory flow and to a limited range of amplitude-to-width ratios since it does not approach the steady-state drag coefficient of 2 and since for large amplitudes it is unlikely that the effects would be the same (because the vortices shed would have dissipated by the time the baffle reversed its path). Although it might seem unlikely that compressibility would be an important factor in plates which have such small velocities, it should be remembered that the local velocities can be exceedingly high at the edge of the plate, and it is this region which has the largest influence on the shedding of the vortices.

By substituting equation (3) into equation (13) of reference 6, one may obtain the equation for the damping ratio of a cylindrical tank:

$$\zeta = \zeta_o + 0.9\left(\frac{V_s^2}{ang}\right)^{1/8} \left(\frac{w}{a}\right)^{1.4} \left(1 - \frac{w}{2a}\right) \left(\frac{y_s}{a}\right)^{0.35} e^{-4.2(d/a)} \tag{4a}$$

or

$$\zeta = \zeta_o + 0.9\left(\frac{K}{\rho ang}\right)^{1/8} \left(\frac{w}{a}\right)^{1.4} \left(1 - \frac{w}{2a}\right) \left(\frac{y_s}{a}\right)^{0.35} e^{-4.2(d/a)} \tag{4b}$$

in which $\rho ang/K$ corresponds to Cauchy number. The fact that the coefficient is nearly 1 and is dimensionless indicates that the equation very nearly represents the fundamental law of damping for the conditions being considered.

On the basis of equation (4), it may be seen that Cauchy number as well as geometric parameters should be maintained for proper scaling of fuel-sloshing experiments. Several interesting predictions may be made from this equation. If the size of the tank were increased by a factor of 10, the damping ratio would decrease by 25 percent. If a rocket were to accelerate to 5 times gravity, the damping ratio would decrease by 18 percent. If the liquid were changed from water to liquid oxygen, the damping would decrease by 12 percent. Unfortunately, there are not enough data to verify the equations under all of these various conditions. However, information on scaling does exist and will be used for comparison in the following section. Predictions by the Miles' equation will also be shown for comparison. This equation is:

$$\zeta = \zeta_o + 3\left[1 - \left(\frac{a-w}{a}\right)^2\right]^{3/2} e^{-4.60(d/a)} \left(\frac{y_s}{a}\right)^{1/2} \tag{5}$$

Note that the coefficient of 3 is the one in Miles' original paper.

COMPARISON WITH MEASURED DAMPING IN CYLINDRICAL TANKS

The data from which the damping equation was derived were obtained from thin flat plates oscillating in a direction normal to the plates. In reference 6, plates of other shapes were investigated and their effectiveness relative to thin plates was shown. For example, increasing the thickness of the baffles could reduce their effectiveness as much as 50 percent. It was also shown in reference 6 that the proximity of the plates to the fluid surface could influence the effectiveness of the baffles. These effects in various experiments may account for the wide range of coefficients obtained for the Miles' equation (2.83 in ref. 5, 2.25 to 4.5 in ref. 3). Since surface and baffle thickness effects are not included in equations (4) and (5), they should not be applied without appropriate corrections to situations where these effects are large.

To test the damping equations, damping measurements in both small and large tanks were selected for comparison.

3-Foot-Diameter Tank

The tank and technique used for measuring damping are described in reference 6. The baffle was made of steel, 0.0625 inch thick, and the outer edge was ground sharp to represent zero thickness. The baffle was therefore quite rigid with no apparent flexibility.

The results of the damping measurements are shown in figure 3. It may be seen that equation (4) fits the experimental points very well until surface effects become important. Equation (5) also fits quite well and only falls slightly below equation (4). Hence, it may be seen that under the new damping law of equation (4), the measurements of reference 6 very accurately predict the damping of a ring baffle in a small tank.

95-Inch-Diameter Tank

Data for a relatively large tank were obtained from reference 3 and are presented in figure 4. These data were selected because the baffle was at a depth where surface effects should be small, and the lip baffle was nearly equivalent to a flat plate as shown in reference 6. It may be seen that equation (4) shows excellent agreement with the data at low amplitudes and even predicts the sharp rise in damping. At amplitudes above 2 inches, there is considerable scatter in the data and it is difficult to judge the agreement. Equation (5) is also shown on the figure for comparison, and it may be seen that it falls considerably below the experimental data at small amplitudes. It should be noted that the depth of the fluid in this experiment was only 1.1 times the tank radius and that equations (4) and (5) are only valid for cases where the depth of fluid is greater than the tank radius. Hence, the comparison shown here is near the limit of application of the equations where the error is estimated to be of the order of 20 percent. If the theoretical curves are revised downward by this amount, the comparison with the data favors equation (4).

On the basis of the comparisons in a small and large tank in which surface and baffle thickness effects are small, it is concluded that equation (4) accurately predicts not only the trends but also the magnitude of fuel-slosh damping. The comparisons with Miles' equation are made to demonstrate the effect which the addition of the Cauchy number has on the predicted damping trends. The difference between these damping laws would be much larger in predicting the damping of various fluids in large tanks in accelerated flight. The comparisons shown here do not cover such conditions. A large-scale test in accelerated flight is needed to establish the full scope of equation (4).

CONCLUSION

A fuel-slosh damping law has been found which correlates data for a two-dimensional plate oscillating in water in the amplitude-to-width range from 1/4 to 3. When applied to damping of ring baffles in cylindrical tanks where thickness and surface effects are small, the law shows excellent correlation with damping measured in a 36-inch and a 95-inch-diameter tank. On the basis of these comparisons, it is concluded that fuel-slosh damping depends on the Cauchy number and geometric parameters and is relatively independent of Reynolds number and cavitation under the conditions for which the data were obtained. The damping equation provides predictions for fluids other than water, for accelerated motion, and for large tanks, but sufficient data are not available for establishing the limits of application.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Aug. 26, 1965

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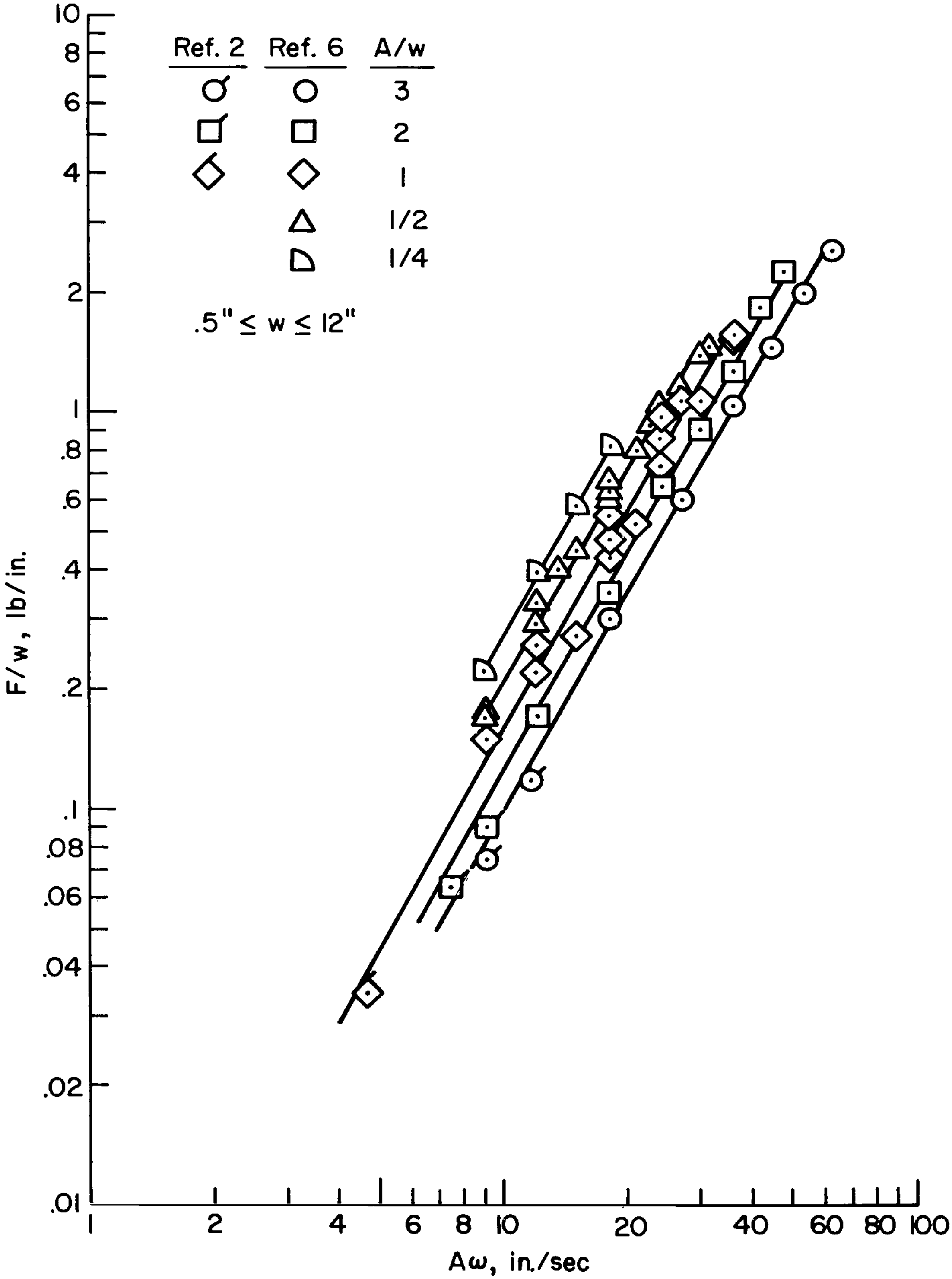

Figure 1.- Variation of unit damping force with velocity at constant amplitude-to-width ratio (average slope = 1.75).

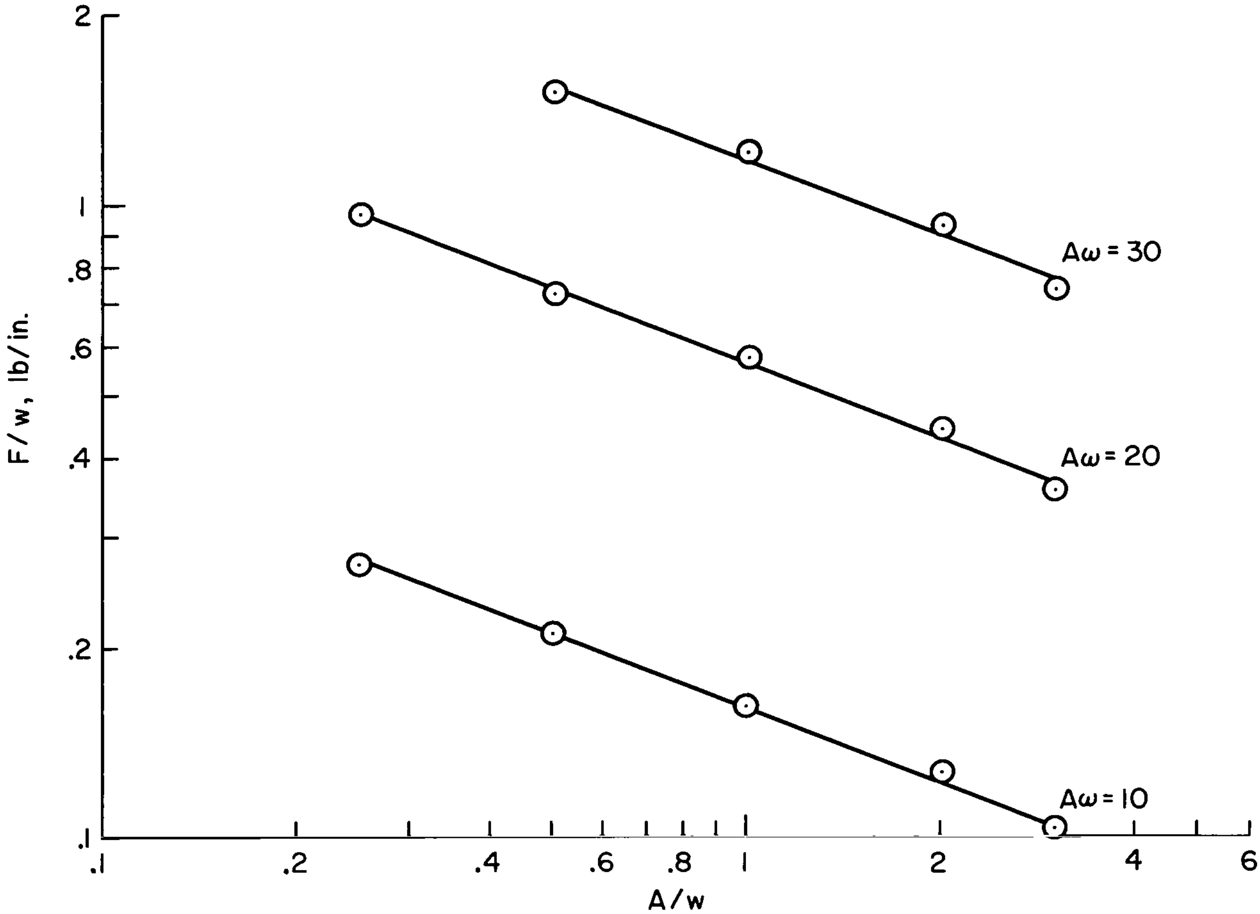

Figure 2.- Variation of unit damping force with amplitude to width at constant velocity (average slope = -0.4).

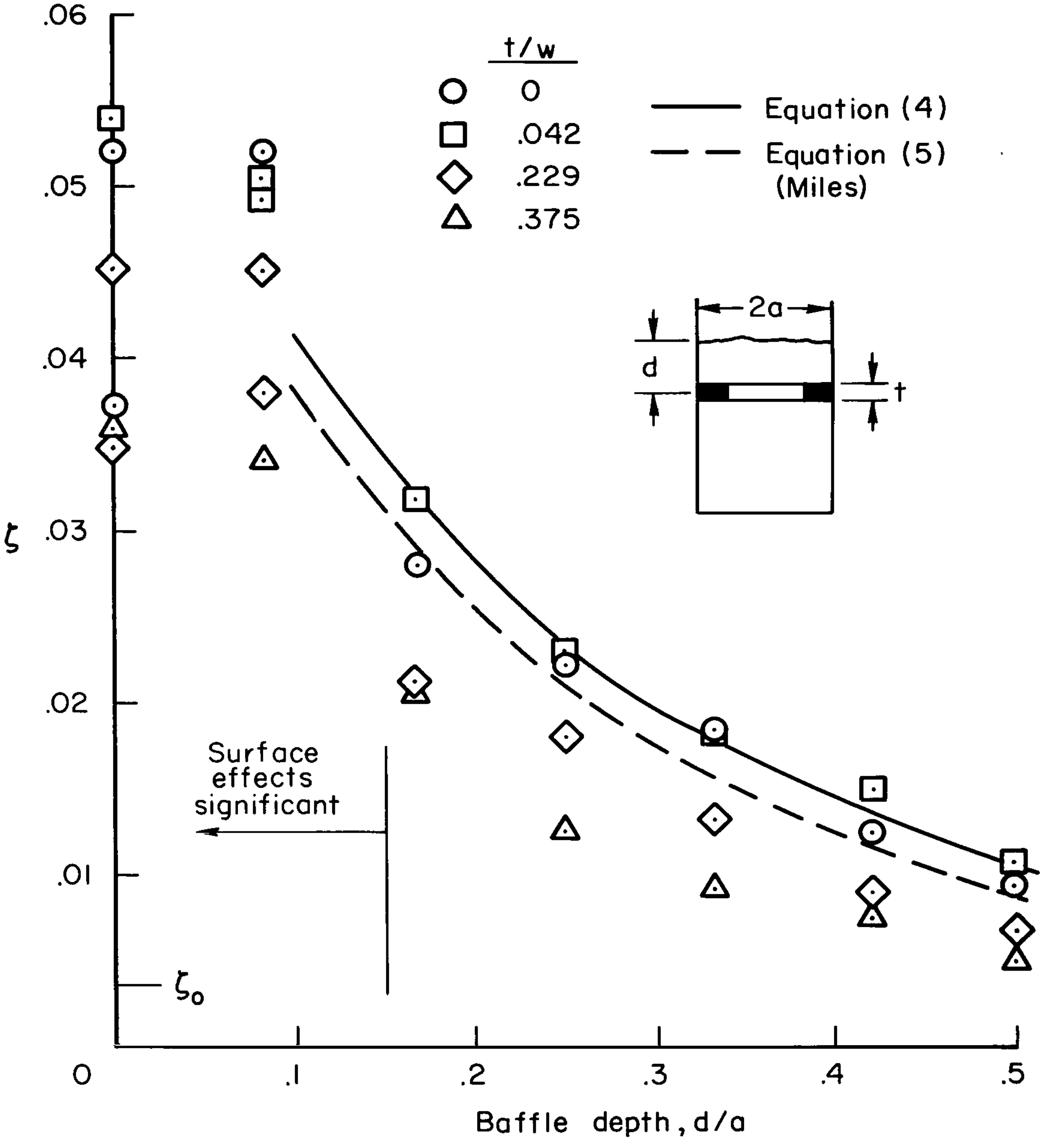

Figure 3.- Measured and predicted damping in a 3-foot-diameter tank with 1-1/2-inch-chord baffles with various thickness ratios (wave amplitude = 1-1/2 in.).

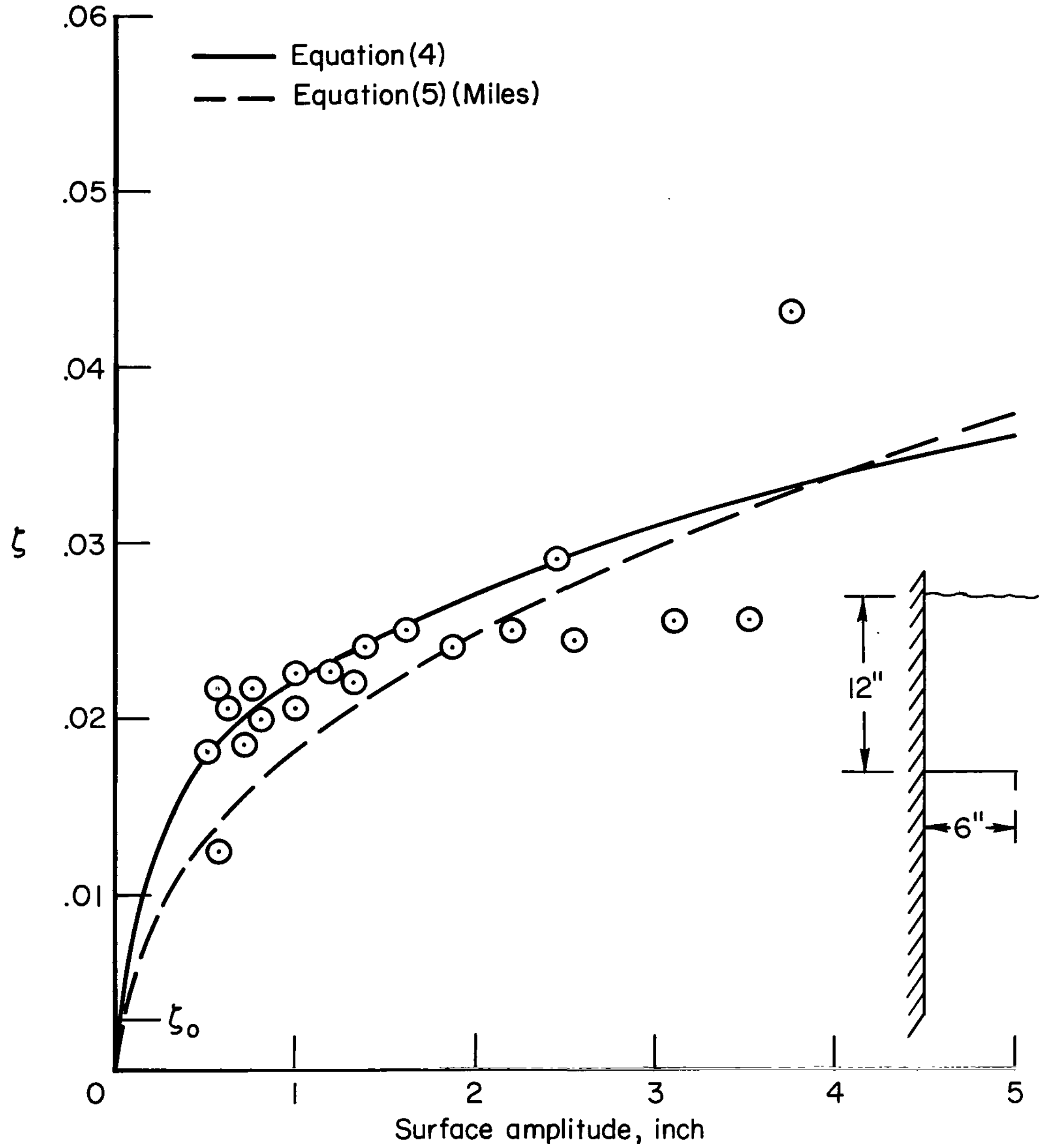

Figure 4.- Measured (ref. 3) and predicted damping in 95-inch-diameter tank.

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