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Theoretic electron emission at high electric fields reacts strongly to the value at which the free-space potential terminates on the emitter surface. To indicate this effect, it is traditional to tack the simple image potential to the bottom of the bulk-metal conduction band, to talk of the implications of the refined model, and to trade its complications for the ease of the ordinary image at the mathematic outset.

This paper uses a terminated image potential throughout. The particular potential resulted from consideration of surface conditions. At the metal boundary, several mechanisms collect excess electrons and drive the potential up abruptly. However, the surface potential is limited by the near-equilibrium assumption and the large number of electrons around the Fermi level.

So for this work, the free-space potential connects to the Fermi level at the emitter surface,[1] and there the potential drops directly to the bottom of the conduction band in a silent tribute to interfacial ignorance. This wall and corner combination is a mere mathematic approximation for a rapidly but smoothly changing potential. The actual potential path is probably similar in severity to the near verticality of the ordinary image near the emitter face. Thus, in line with previous field emission theory, reflections at abrupt changes in potential are neglected.

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[1]F. M. Propst, Phys. Rev. 129, 7 (1963). [redacted]

~~The structure of~~ the barrier is sketched in Fig. 1. In this model, the conventional image merely shifts to intercept the emitter surface at the Fermi level; it is an 0.8 Å move for a 4.5-V work function.

This translation seems negligible, but as it was stated previously, image potentials of the nonterminated (NIP = $-e^2/4x$) and terminated (TIP = $-e^2/[4x + e/\varphi]$) types differ significantly in their effects at high fields. For example, the Schottky equation,

$$j_{NIP} = 120\ T^2\ \exp\left\{-[e\varphi - (e^3E)^{1/2}]/(KT)\right\},$$

predicts current densitites over 0.1 percent higher than those of the zero-order approximation for the TIP expression when $E(V/cm) > 2.4\ \varphi T\ (V - {}^{o}K)$. The complete TIP equation is

$$j_{TIP} = 120\ T^2 \left[e^{-\frac{e\varphi - (e^3E)^{1/2} + \frac{e^2E}{4\varphi}}{KT}} - \frac{e^{-2\frac{e\varphi - (e^3E)^{1/2} + \frac{e^2E}{4\varphi}}{KT}}}{4} + \frac{e^{-3\frac{e\varphi - (e^3E)^{1/2} + \frac{e^2E}{4\varphi}}{KT}}}{9} - \cdots \right].$$

The zero- and first-order approximations for TIP supra-barrier emission differ in current densities by more than 0.1 percent only when

$$E > 0.28\ \varphi^2 \left[1 + \left(\frac{T}{2.1\ \varphi} \times 10^{-3}\right)^{1/2}\right]^2 \times 10^8 (E \text{ in } V/cm,\ \varphi \text{ in } V,\ T \text{ in } {}^{o}K).$$

So the zero-order approximation for emission over the TIP barrier holds as well as the potential model itself up to 10^8 V/cm. However, even before these fields are reached tunneling makes the dike pretty leaky. Thus, emission through as well as over the barrier must be considered.

NIP field-emission functions were published by Burgess, Kroemer, and Houston.[2] Now this paper presents TIP penetration probabilities, and, as usual, it all begins with the WKB approximations and restrictions.[3]

$$P \approx f(V,\epsilon)\exp\left\{-\frac{2}{\hbar}\int_{x_1}^{x_2}\left[2m(eV-\epsilon)\right]^{1/2}dx\right\} \approx \exp\left\{-\left(\frac{8m}{\hbar^2}\right)^{1/2}\int_{x_1}^{x_2}\left[\mu + e\varphi - eEx - \frac{e^2}{4\left(x+\frac{e}{4\varphi}\right)} - \epsilon\right]^{1/2}dx\right\} = \exp\left\{-\left(\frac{\alpha}{2}\right)^{-3/2}\left(\frac{\xi}{eE}\right)^{1/4}\int_{\eta_1}^{\eta_2}\left[1 + \left(\frac{\alpha}{2}\right)^2\delta - \eta - \frac{(\alpha/2)^2}{\eta}\right]^{1/2}d\eta\right\} = \exp\left[-C(\alpha,E)I(\alpha,\delta)\right],$$

where f varies slowly and is near unity, V is electron potential, ϵ is kinetic energy of the positive-x-directed component of velocity for an electron within the emitter, $\hbar$ is Planck's constant divided by 2π, x is distance from the emitter surface, x_1 and x_2 are electron turning points (at $eV - \epsilon = 0$), e and m are electron charge and mass, μ is Fermi level, φ is work function, E is electrostatic field,

$$\alpha^2 = e^3E/\beta^2,\ \beta = \mu + e\varphi - \epsilon,\ \delta = \beta/(e\varphi),\ \xi = e^6m^2/\hbar^4,$$

$$\eta = eE\left(x + \frac{e}{4\varphi}\right)/\beta,\ \eta_{2,1} = \frac{1+(\alpha/2)^2\delta}{2}\left\{1 \pm \sqrt{1 - \frac{4(\alpha/2)^2}{\left[1+\left(\frac{\alpha}{2}\right)^2\delta\right]^2}}\right\},$$

[2] R. E. Burgess, H. Kroemer, and J. M. Houston, Phys. Rev. 90, 515 (1953).

[3] D. Bohm, "Quantum Theory," Prentice-Hall, 1961.

$$C(\alpha,E) = \frac{2}{3}\left(\frac{\alpha}{2}\right)^{-3/2}\left(\frac{\xi}{eE}\right)^{1/4},$$

and

$$I(\alpha,\delta) = \frac{3}{2}\int_{\eta_1}^{\eta_2}\left[1 + \left(\frac{\alpha}{2}\right)^2\delta - \eta - \frac{(\alpha/2)^2}{\eta}\right]^{1/2} d\eta .$$

Of course, at $\delta = 0$ for nonzero β,

$$I = \frac{3}{2}\int_{\eta_1}^{\eta_2}\left[1 - \eta - \frac{(\alpha/2)^2}{\eta}\right]^{1/2} d\eta \quad \text{and} \quad \eta_{2,1} = \frac{1}{2}\left[1 \pm (1 - \alpha^2)^{1/2}\right],$$

which are identical with the NIP expressions.

Because distance, field, and potential are real and positive, the allowed range of α depends on δ. This δ effect limits α to values from zero (where $I = 1$) to those indicated in table I.
However, for $\epsilon \leq \mu$ or $\delta \geq 1$, $x_1 = 0$ and $\eta_1 = \delta(\alpha/2)^2$ for the TIP case.

So the definite integral in the TIP penetration probability is, for $\delta < 1$,

$$I(\alpha,\delta) = 3\int_{\eta_1^{1/2}}^{\eta_2^{1/2}}\left[(\eta_2 - \eta)(\eta - \eta_1)\right]^{1/2} d(\eta^{1/2})$$

$$= \left\{\left[1 + \left(\frac{\alpha}{2}\right)^2\delta\right]^3\left(\frac{1+\gamma}{2}\right)\right\}^{1/2}\left\{E\left[\frac{\pi}{2}, \left(\frac{2\gamma}{1+\gamma}\right)^{1/2}\right] - \left(1 - \gamma\right)F\left[\frac{\pi}{2}, \left(\frac{2\gamma}{1+\gamma}\right)^{1/2}\right]\right\},$$

and for $\delta \geq 1$,

$$I(\alpha,\delta) = \int_{\left[\left(\frac{\alpha}{2}\right)^2 \delta\right]^{1/2}}^{\eta_2^{1/2}} \left[(\eta_2 - \eta)(\eta - \eta_1)\right]^{1/2} d(\eta^{1/2})$$

$$= \left\{\left[1 + \left(\frac{\alpha}{2}\right)^2 \delta\right]^3 \left(\frac{1+\gamma}{2}\right)\right\}^{1/2} \left(E(\varphi,k) - (1-\gamma)F(\varphi,k) - \left\{\frac{\delta\left(\frac{\alpha}{2}\right)^4 (\delta - 1)}{\left[1 + \left(\frac{\alpha}{2}\right)^2 \delta\right]^3 \left(\frac{1+\gamma}{2}\right)}\right\}^{1/2}\right).$$

Here, $\gamma = \left\{1 - 4(\alpha/2)^2/[1 + (\alpha/2)^2\delta]^2\right\}^{1/2}$, k = modulus of the elliptic integral = $[2\gamma/(1+\gamma)]^{1/2}$, $\varphi = \sin^{-1}\left(\left\{(1+\gamma)/2 - (\alpha/2)^2\delta/[1 + (\alpha/2)^2\delta]\right\}/\gamma\right)^{1/2}$, and $F(\varphi,k)$ and $E(\varphi,k)$ are incomplete (complete when $\varphi = \pi/2$ as for $\delta < 1$) elliptic integrals of the first and second kinds, respectively.

Values of $I(\alpha,\delta)$ and $C(\alpha,E)$ are listed in tables II and III. These can be used to compute TIP penetration probabilities;[4] they can also be compared with those for the NIP case, which correspond to $I(\alpha,\delta)$ at $\delta = 0$.

¶ Where the NIP and TIP theories apply, they probably bracket real thermal field-emission.

In the use of these transmission coefficients, the usual precautions must be taken. For example, the distance from the emitter to the outside of the potential barrier must never decrease to lengths near the size of surface imperfections, and the emission density cannot be a significant fraction of the internal electron density. These and other extremes destroy

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[4]J. F. Morris, "Thermal Emission in Electric Fields," proposed NASA Technical Note.

the simple emission models; so moderation is the rule for TIP's as well as NIP's.

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TABLE I. - MAXIMUM α VALUES

δ	0	0.1	0.2	0.4	0.6	0.8	0.9	1.0	>1
α	1	1.03	1.06	1.13	1.23	1.38	1.52	2.0	∞

TABLE II. - $C(\alpha,E)$

α	E								
	10^5	$10^{5.5}$	10^6	$10^{6.5}$	10^7	$10^{7.5}$	10^8	$10^{8.5}$	10^9
0.0000	∞	∞	∞	∞	∞	∞	∞	∞	∞
.1000	898.05	673.44	505.01	378.71	283.99	212.96	159.70	119.76	98.805
.2000	317.51	238.10	178.55	133.89	100.41	75.293	56.462	42.341	31.751
.4000	112.26	84.180	63.126	47.338	35.499	26.620	19.962	14.970	11.226
.6000	61.105	45.822	34.362	25.768	19.323	14.490	10.866	8.1484	6.1105
0.8000	39.689	29.762	22.319	16.737	12.551	9.4117	7.0577	5.2926	3.9689
.9000	33.261	24.942	18.704	14.026	10.518	7.8875	5.9148	4.4354	3.3261
1.0000	28.399	21.296	15.970	11.976	8.9805	6.7344	5.0501	3.7871	2.8399
1.0263	27.313	20.482	15.359	11.518	8.6371	6.4769	4.8570	3.6422	2.7313
1.0557	26.180	19.632	14.722	11.040	8.2789	6.2083	4.6556	3.4912	2.6180
1.1270	23.736	17.799	13.348	10.009	7.5059	5.6287	4.2209	3.1652	2.3736
1.2000	21.604	16.201	12.149	9.1102	6.8317	5.1231	3.8418	2.8809	2.1604
1.2251	20.942	15.704	11.777	8.8312	6.6224	4.9661	3.7241	2.7927	2.0942
1.3820	17.481	13.109	9.8300	7.3715	5.5278	4.1453	3.1085	2.3311	1.7481
1.4000	17.144	12.856	9.6407	7.2295	5.4214	4.0655	3.0487	2.2862	1.7144
1.5195	15.162	11.370	8.5261	6.3937	4.7946	3.5954	2.6962	2.0219	1.5162
2.0000	10.041	7.5293	5.6462	4.2341	3.1751	2.3810	1.7855	1.3389	1.0041
4.0000	3.5499	2.6620	1.9962	1.4970	1.1226	.84180	.63126	.47338	.35499
10.0000	.89805	.67344	.50501	.37871	.28399	.21296	.15970	.11976	.089805
20.0000	.31751	.23810	.17855	.13389	.10041	.075293	.056462	.042341	.031751
50.0000	0.080324	0.060235	0.045170	0.033872	0.025401	0.019048	0.014284	0.010711	0.0080324

TABLE III. - I (α,δ)

(a) 0 ≤ δ ≥ 1.

α	δ							
	0	0.1	0.2	0.4	0.6	0.8	0.9	1.0
0.0000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
.1000	.98168	.98206	.98243	.98319	.98394	.98469	.98507	.98545
.2000	.93704	.93855	.94007	.94311	.94615	.94919	.95072	.95224
.4000	.78876	.79496	.80117	.81362	.82611	.83865	.84494	.85126
.6000	.57681	.59110	.60545	.63431	.66338	.69266	.70739	.72216
0.8000	0.31166	0.33774	0.36399	0.41697	0.47060	0.52488	0.55226	--------
.9000	.16132	.19476	.22846	--------	--------	--------	--------	--------
1.0000	.00000	.041847	.084081	.16970	.25682	.34546	.39033	.43561
1.0263	--------	.00000	--------	--------	--------	--------	--------	--------
1.0557	--------	--------	.00000	--------	--------	--------	--------	--------
1.1270	--------	--------	--------	0.00000	--------	--------	--------	--------
1.2000	--------	--------	--------	--------	0.029275	0.16280	0.23065	--------
1.2251	--------	--------	--------	--------	.00000	--------	--------	--------
1.3820	--------	--------	--------	--------	--------	0.00000	--------	--------
1.4000	--------	--------	--------	--------	--------	--------	.081262	.17954
1.5195	--------	--------	--------	--------	--------	--------	0.00000	--------
2.0000	--------	--------	--------	--------	--------	--------	--------	0.00000

(b) 1 ≤ δ ≥ 5

α	δ					
	1.0	1.2	1.4	2.0	3.0	5.0
0.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
.10000	.98545	.98599	.98638	.98721	.98809	.98914
.40000	.85126	.86033	.86699	.88105	.89594	.91379
1.00000	.43561	.49868	.54289	.63051	.71342	.79787
2.00000	.00000	.15455	.25607	.44241	.60023	.74176
4.00000	------	.081782	.17392	.37586	.55919	.72254
10.00000	------	.069981	.15588	.35710	.54672	.71668
20.00000	------	.068509	.15348	.35423	.54491	.71515
50.00000	------	.065027	.15285	.35630	.54466	.67671

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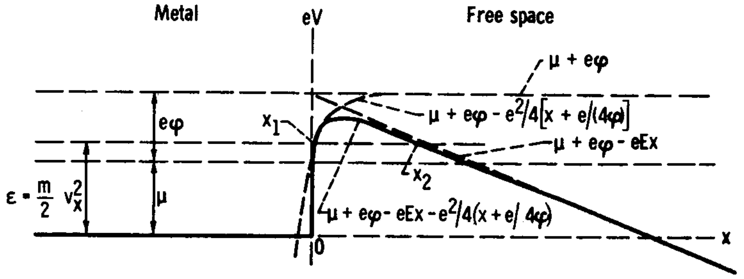

Fig. 1. Emission barrier for terminated image potential.