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Shock Waves in the Solar Wind and Geomagnetic Storms

By P. A. Sturrock* and J. R. Spreiter

National Aeronautics and Space Administration
Space Sciences Division
Ames Research Center, Moffett Field, California

[redacted]

*Permanent Address: Institute for Plasma Research, Stanford University, Stanford, California

ABSTRACT

Certain geomagnetic storms exhibit, in addition to the usual initial sudden positive impulse, a subsequent sudden negative impulse. The former is normally ascribed to a shock wave in the interplanetary medium, and it has recently been suggested that the latter may be ascribed to a reverse shock convected away from the sun by the solar wind.

If the velocity of efflux of gas from a source is supersonic (with respect to the source), if the velocity is instantaneously increased, and if certain subsidiary conditions are met, a pair of shock waves will be produced which propagate away from the source. The "fast" shock propagates away from the contact surface in the ambient gas (which was ejected from the source before the change in efflux velocity), while the "slow" shock propagates away from the contact surface in the driver gas, but has an outward velocity when this velocity is measured relative to the source.

This problem (which may be identified with a classical problem considered by Riemann) is discussed in its relation to the production of pairs of shock waves by the enhanced solar wind produced by a solar flare. The equations giving the relationship between the velocities of the shock waves and of the ambient and driver gases become very simple in the strong-shock approximation.

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It is shown that the propagation times of the positive and negative impulses of the July 11, 1959 magnetic storm may be explained satisfactorily on the basis of this theory.

1. INTRODUCTION

Following the original suggestion of Gold [1955], it is now generally accepted that the positive sudden impulse which normally marks the beginning of a geomagnetic storm is caused by a shock wave in the interplanetary medium (now referred to as the "solar wind" [Parker, 1963]) which, upon arrival at the earth, compresses the magnetosphere. Sonett and Colburn [1965] have recently drawn attention to the less frequent phenomenon of negative sudden impulse and suggested that this might be attributed to a "reverse" shock wave traveling away from the sun, relative to the sun itself, but toward the sun, relative to the solar wind.

One of the possible mechanisms for the generation of such a pair of shock waves, in addition to other mechanisms described by Sonett and Colburn [1965], is that both shock waves originate near the sun as a result of a sudden increase in the velocity (and perhaps density) of the solar wind. This increase in flux is of course to be ascribed to the solar flare responsible for the geomagnetic storm. Since the quiescent solar wind is supersonic (with respect to the sun), one expects that the enhanced solar wind produced by a flare will also be supersonic. The situation with which we are concerned is therefore, in its simplest representation, that of a driver gas being ejected from a source with supersonic speed (with respect to the source) into an ambient gas which has been ejected with supersonic speed from the same source. This situation already possesses one of the essential requirements for the production of a shock pair (consisting of a normal shock in the ambient gas and a propagating reverse shock in the driver gas), namely, that the driver gas should have supersonic velocity with respect to the source from which the gas originates. The purpose of this article is to set up the simple basic equations for the phenomenon in order to determine what further conditions should be met, and to inquire into the likelihood of their being met.

This problem is closely related to a problem considered by Riemann and quoted by Courant and Friedrichs [1948 (p. 181)]: A straight tube of uniform cross section is divided into two parts by a plane diaphragm; to the left of the diaphragm is the gas we refer to as the "driver" gas and to the right of the diaphragm is the gas we refer to as the "ambient" gas. We now suppose that means exists for setting each gas instantaneously into uniform motion. Then the initial conditions on the problem are that the driver gas and ambient gas have pressure, density, and velocity p_d, ρ_d, v_d and p_a, ρ_a, v_a, respectively, and that, at the time these conditions are established, the diaphragm is removed. (See figure 1.) Fig. 1

Riemann showed that the system will behave in one of four possible ways involving either a shock wave or a rarefaction wave in each gas. However, for a rarefaction wave to be set up in each gas, it is clearly necessary that the gases recede from each other, that is, that $v_d < v_a$. Hence, if we stipulate that $v_d > v_a$, there must be either two shock waves (one in each gas) or a shock wave in one gas and a rarefaction wave in the other gas.

Since we are concerned only with the possibility of two shock waves being established, this is the only case we shall discuss in detail. By considering that the strength of one shock wave is zero, we can find the condition which must be met in order that two shock waves should be set up.

Figure 2 shows the situation in which a shock wave is excited in each gas. For later purposes, the shock wave in the ambient gas will be termed the "fast shock" and that in the driver gas the "slow shock." If we consider that $v_d > v_a > 0$, it is clear that the velocity of the contact surface, v_c, must be positive. Hence v_{fs}, the velocity of the fast shock wave, must be positive. However, the velocity v_{ss} of the slow shock may have either sign. Fig. 2

The case of particular interest to us is that v_{ss} is positive. In this case, we may imagine that both the ambient gas and the driver gas are ejected by a source, with respect to which velocities are measured. Up to time $t = 0$, the pressure, density, and velocity of the gas ejected by the source are p_a, ρ_a, v_a; for times subsequent to $t = 0$, the pressure, density, and velocity of the ejected gas are p_d, ρ_d, v_d.

One would expect (and this is verified in the next section) that v_{ss} is necessarily negative if the driver gas is in subsonic flow (and the conditions of the two gases are such that two shock waves are generated). Hence a subsonic driver gas can never produce a shock pair, of the form shown in Figure 2, propagating away from the source.

We now proceed to a discussion of the mathematical relations governing the production of a pair of shock waves by a supersonic driver gas.

2. THE SHOCK RELATIONS

We now consider in more detail the configuration depicted in Figure 2 in which p'_d is the pressure of the shocked driver gas, etc. The velocities of the shocked gases and of the contact surface are all equal,

$$v'_a = v'_d = v_c \tag{2.1}$$

and the pressure is the same on both sides of the contact surface:

$$p'_a = p'_d \tag{2.2}$$

It is convenient to introduce Mach numbers in discussing the shock relations. We therefore introduce the speed of sound, c, where

$$c = (\gamma p/\rho)^{1/2} \tag{2.3}$$

γ being the ratio of specific heats. We now characterize the velocities of the ambient gas and shocked ambient gas, with respect to the fast shock wave, by Mach numbers M_a and M'_a, writing

$$v_{fs} - v_a = M_a c_a \tag{2.4}$$

$$v_{fs} - v_a' = M_a' c_a' \tag{2.5}$$

Similarly, we write

$$v_d - v_{ss} = M_d c_d \tag{2.6}$$

$$v_d' - v_{ss} = M_d' c_d' \tag{2.7}$$

Since gas flows into a shock wave at supersonic velocity (measured relative to the shock wave) and emerges at subsonic velocity, M_a and M_d are greater than unity, whereas M_a' and M_d' are less than unity.

We may note immediately from equation (2.6) that $v_{ss} < 0$ if $v_d < c_d$, that is, that the slow shock has negative velocity if the driver gas is in subsonic flow, a result referred to in Section 1.

The following relations (the Rankine-Hugoniot equations [Courant and Friedrichs, 1948 (p. 129)]) hold at the fast shock,

$$\rho_a(v_{fs} - v_a) = \rho_a'(v_{fs} - v_a') \tag{2.8}$$

$$\rho_a(v_{fs} - v_a)^2 + p_a = \rho_a'(v_{fs} - v_a')^2 + p_a' \tag{2.9}$$

$$\frac{1}{2}(v_{fs} - v_a)^2 + \frac{\gamma}{\gamma - 1}\frac{p_a}{\rho_a} = \frac{1}{2}(v_{fs} - v_a')^2 + \frac{\gamma}{\gamma - 1}\frac{p_a'}{\rho_a'} \tag{2.10}$$

and similar relations at the slow shock.

The values of ρ, p, and M (or, alternatively, the values of ρ, c, and M) on one side of a shock wave determine the values on the other side. For instance, one may determine M' from M (or vice versa) from the relation

$$M'^2 = \frac{(\gamma - 1)M^2 + 2}{2\gamma M^2 - (\gamma - 1)} \qquad (2.11)$$

Then the densities, pressures, and sound speeds are related by

$$\frac{\rho'}{\rho} = \frac{(\gamma + 1)M^2}{(\gamma - 1)M^2 + 2} \qquad (2.12)$$

$$\frac{p'}{p} = \frac{2\gamma M^2 - (\gamma - 1)}{\gamma + 1} \qquad (2.13)$$

$$\frac{c'}{c} = \frac{\{[2\gamma M^2 - (\gamma - 1)][(\gamma - 1)M^2 + 2]\}^{1/2}}{(\gamma + 1)M} \qquad (2.14)$$

If ρ, c, and v are given for the driver gas and for the ambient gas, the above relations for the contact surface and for the two shock waves provide nine equations for determining the nine unknowns ρ'_a, ρ'_d, c'_a, c'_d, v'_a, v'_d, v_{fs}, v_{ss}, and v_c.

We may now inquire about the response of the magnetosphere to the arrival of a fast or slow shock wave. Quite detailed calculations would be necessary for determining the precise change in the magnetosphere on the arrival of a shock wave. At this time, however, we wish simply to verify that there will be a compression of the magnetosphere, with a corresponding increase in the horizontal component of the magnetic field at the surface of the earth, at the time of arrival of the fast shock, and opposite changes at the time of arrival of the slow shock. The calculations of Spreiter and Hyett [1963] show that the "side" and "tail" of the magnetosphere are sensitive to the value of p and insensitive to the value of ρv^2. Since $p' > p$, the side and rear of the magnetosphere should be compressed when the fast shock arrives and expanded when the slow shock arrives.

One may expect further that the response of the front of the magnetosphere will be related to the total flux of momentum, which is given by

$$P = p + \rho v^2 \tag{2.15}$$

One may verify that

$$P' - P = (\rho' - \rho)v_s^{\,2} \tag{2.16}$$

where v_s is the velocity of the shock wave (v_{fs} or v_{ss}). Since $\rho' > \rho$, $P' > P$. However, we may instead consider the stagnation pressure (the pressure at the "nose" of the magnetosphere) as an indicator of the influence of the solar wind on the front of the magnetosphere. For flow which is substantially supersonic with respect to the earth, we may use the formula [Landau and Lifschitz, 1959 (p. 459)]

$$p_s = \left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma+1}{\gamma-1}} \gamma^{-\frac{\gamma}{\gamma-1}} \rho v^2 \tag{2.17}$$

In this case, ρv^2 is much larger than p, so that the change in ρv^2 at the shock wave is given approximately by (2.16). The stagnation pressure will therefore increase with the arrival of the fast shock and decrease with the arrival of the slow shock. These changes in pressure will lead to a sudden compression and expansion, respectively, of the magnetosphere, leading to a sudden increase and a sudden decrease, respectively, in the horizontal component of the geomagnetic field at the earth's surface.

We now consider the conditions which must be satisfied in order to obtain two shock waves rather than one shock wave and one rarefaction wave. We may arrive at one of the boundaries separating the two regimes by considering that the strength of one of the shock waves, say the fast shock wave, has zero strength. Then the conditions at the fast shock are as follows:

$$\left.\begin{aligned} M_a = 1,\ M_a' = 1 \\ \rho_a' = \rho_a,\ p_a' = p_a \\ c_a' = c_a,\ v_a' = v_a \end{aligned}\right\} \tag{2.18}$$

By using equation (2.1) and (2.2) and inspecting the relations which hold at the slow shock, we arrive at

$$v_d - v_a = 2\,\frac{M_d^{\,2} - 1}{(\gamma + 1)M_d}\,c_d \tag{2.19}$$

We also see from (2.2), (2.13), and (2.18) that $p_a > p_d$ if the fast shock wave is to be of zero strength and the slow shock wave of nonzero strength.

Equation (2.19) may be expressed as a relation between $c_d/(v_d - v_a)$ and p_a/p_d. If we derive this relation and then note that higher relative gas velocities (or lower temperatures) clearly favor the process of the formation of two shocks, we find that one of the conditions for the formation of two shocks is [Landau and Lifschitz, 1959 (p. 363)]

$$\frac{c_d}{v_d - v_a} < \frac{\left\{\frac{1}{2}\gamma\left[(\gamma + 1)\frac{p_a}{p_d} + (\gamma - 1)\right]\right\}^{1/2}}{\frac{p_a}{p_d} - 1} \quad \text{if} \quad \frac{p_a}{p_d} > 1 \qquad (2.20)$$

We see immediately that, if $p_a/p_d < 1$, the condition should be replaced by

$$\frac{c_a}{v_d - v_a} < \frac{\left\{\frac{1}{2}\gamma\left[(\gamma + 1)\frac{p_d}{p_a} + (\gamma - 1)\right]\right\}^{1/2}}{\frac{p_d}{p_a} - 1} \quad \text{if} \quad \frac{p_d}{p_a} > 1 \qquad (2.21)$$

The condition represented by (2.20) and (2.21) is shown schematically in Figure 3. It appears from this diagram that any significant sudden increase in velocity of the solar wind, if it is not associated with a remarkable change in pressure, is likely to produce the double-shock configuration. Fig. 3

3. STRONG-SHOCK APPROXIMATION

The relations derived in the preceding section simplify if the Mach numbers M_a and M_d are large compared to unity. It appears that this requirement is met by the shock waves produced by the enhanced outflow of solar wind which occurs at the time of a major solar flare, such as produces a significant geomagnetic storm. In the following formulas, we adopt the value $\gamma = 5/3$ appropriate to a fully ionized gas such as the solar wind.

If $M \gg 1$, we obtain from (2.11)

$$M' \approx \left(\frac{\gamma - 1}{2\gamma}\right)^{1/2} = 0.45 \qquad (3.1)$$

Hence (2.12) leads to the familiar result

$$\frac{\rho'}{\rho} \approx \frac{\gamma + 1}{\gamma - 1} = 4 \qquad (3.2)$$

(2.13) leads to

$$\frac{p'}{p} \approx \frac{2\gamma}{\gamma + 1} M^2 = 1.25 \; M^2 \qquad (3.3)$$

and (2.14) leads to

$$\frac{M'c'}{Mc} \approx \frac{\gamma - 1}{\gamma + 1} = 0.25 \qquad (3.4)$$

We now find from (2.4) and (2.5) that v_{fs} and v'_a may be related to v_a, c_a, and M_a by

$$v_{fs} = v_a + M_a c_a \qquad (3.5)$$

$$v'_a = v_a + 0.75 \; M_a c_a \qquad (3.6)$$

Similar relations follow from (2.6) and (2.7). If, in addition, we use (2.1), these relations lead to

$$v_{ss} = v_a + 0.75 \; M_a c_a - 0.25 \; M_d c_d \qquad (3.7)$$

$$v_d = v_a + 0.75 \; M_a c_a + 0.75 \; M_d c_d \qquad (3.8)$$

In order to find M_a and M_d, we need another relation in addition to (3.8). This is provided by the pressure condition (2.2). We write

$$\frac{\rho_d}{\rho_a} = \lambda^2 \qquad (3.9)$$

so that λ^2 is the ratio of the density of the driver gas to that of the ambient gas. Equations (3.2) and (3.3) show that

$$M_a{}^2 p_a = M_d{}^2 p_d \tag{3.10}$$

from which, with the help of (2.3) and (3.9), we find that

$$M_a c_a = \lambda M_d c_d \tag{3.11}$$

We now find from (3.8) and (3.11) that

$$M_a c_a = \frac{4\lambda}{3(1 + \lambda)} (v_d - v_a) \tag{3.12}$$

It is now possible to express the velocities of the fast and slow shock waves in terms of v_a and v_d as follows:

$$v_{fs} = \frac{3 - \lambda}{3(1 + \lambda)} v_a + \frac{4\lambda}{3(1 + \lambda)} v_d \tag{3.13}$$

$$v_{ss} = \frac{4}{3(1 + \lambda)} v_a + \frac{3\lambda - 1}{3(1 + \lambda)} v_d \tag{3.14}$$

4. DISCUSSION

It is now necessary to inquire whether the above formulas may be fitted to observations of geomagnetic storms displaying both positive and negative sudden impulses. It is necessary to find a geomagnetic storm for which the associated flare is well identified. For this reason, we select the flare of July 10, 1959 and the resulting geomagnetic storm of July 11, 1959. The positive sudden impulse occurred 38.3 hours after the flash phase of the flare, and the negative sudden impulse occurred 8.5 hours thereafter.

When we come to relate our simple theory to observations of geomagnetic storms, some of the limitations of our treatment become evident. According to theory [Parker, 1963], the speed of the solar wind does not vary greatly beyond a few solar radii from the sun. Although one would not expect that the enhanced flux of solar wind produced by an explosive phenomenon, such as a flare, would be constant over a long interval of time, the time scale of the main phase of a large solar flare has a duration of several hours, and this can be sufficient for discussion of the double-shock phenomenon. Since the Mach numbers of the two shock waves depend only on the relative velocity of the driver and ambient gases, on the density ratio of these gases, and on their temperatures, it is strictly necessary that all these quantities remain constant if the shock Mach numbers are to remain constant. One would expect the ratio of densities to remain constant, since each gas stream is expanding in the same way as the gas moves away from the sun. However, the temperatures of both gases will drop as the gases travel from the sun to the earth. For these reasons, and because we are neglecting the role of the magnetic field, one should not look for too close a correspondence between observations and the simple theories of Sections 2 and 3. It appears that correspondence will be better for strong shocks than for weak shocks, since the actual values of the temperatures of the ambient and driver gases do not appear in formulas (3.13) and (3.14).

We therefore begin by applying equations (3.13) and (3.14) to the geomagnetic storm phenomenon. The above data concerning the time delay between the occurrence of a flare and the occurrence of the sudden impulses lead to the estimates $v_{fs} = 1085$ km/sec and $v_{ss} = 888$ km/sec. We readily find from (3.13) and (3.14) that

$$v_d - v_a = 3(v_{fs} - v_{ss}) \qquad (4.1)$$

so that, in our case, $v_d - v_a = 591$ km/sec, and that

$$\lambda = \frac{v_{fs} - v_a}{3v_{fs} - 4v_{ss} + v_a} \qquad (4.2)$$

If v_a is known, v_d and λ are now determined. Since no measurements of the solar wind were made in 1959, we must consider various possible values of v_a. In this way, we find that the observed time delays may be explained by any of the sets of parameters listed in Table 1. The Mach numbers given in Table 1 have been estimated on the assumption that $c_a = c_d = 50$ km/sec, corresponding to a temperature of 10^5 deg of fully ionized hydrogen gas. It is clear that, with this choice of temperature, the strong-shock assumption is unacceptable for $v_a = 400$, 500 km/sec, but is a fair approximation for $v_a = 600$ km/sec. A smaller value of the gas temperature would, of course, make the strong-shock assumption more acceptable. Table 1

It is possible to draw up a similar table on the basis of the formulas of Section 2, provided one can relate Mc to $M'c'$ for each shock. We find from (2.11) and (2.14) that

$$\frac{M'c'}{c} = \frac{M^2 + 3}{4M} \qquad (4.3)$$

for $\gamma = 5/3$. The procedure is as follows:

For known values of v_{fs}, v_{ss} and assumed values of v_a, c_a, c_d, one may determine M_a from (2.4) and then, using (4.3), one may determine v_a' (which is the same as v_c and v_d') from (2.5). One may then determine $M_d'c_d'$ from (2.7) and, determining M_d from (4.3), one may finally find v_d from (2.6). If this procedure is applied to the event considered above in the strong-shock approximation, we arrive at the sets of parameters given in Table 2. The density ratio is evaluated from

Table 2

$$\frac{\rho_d}{\rho_a} = \frac{2\gamma M_a^2 - (\gamma - 1)}{2\gamma M_d^2 - (\gamma - 1)} \frac{c_a^2}{c_d^2} = \frac{5M_a^2 - 1}{5M_d^2 - 1} \frac{c_a^2}{c_d^2} \qquad (4.4)$$

which is derivable from (2.2), (2.3), and (2.13). We note that for $v_a = 600$ km/sec, the estimate of the driver gas velocity is not very different from that obtained in Table 1, although there is a substantial difference in the estimate of ρ_d/ρ_a. For $v_a = 500$ km/sec, there is an ambiguity in determining M_d. For $v_a = 400$ km/sec, no solution is possible.

One may verify from Figure 3, that, for all sets of parameters listed in Tables 1 and 2, the criterion for the generation of two shocks is satisfied.

Although many approximations have been made in the course of this analysis, it appears from this discussion that positive and negative sudden impulses may, in fact, be attributable to a pair of shock waves generated by an enhanced flux of the solar wind at the time of a solar flare.

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TABLE 1. Possible Parameters for July 11, 1959 Event

Based on Strong-Shock Approximation of Section 3

v_a(km/sec)	M_a	v_d(km/sec)	M_d	ρ_d/ρ_a
400	13.7	991	2.1	44
500	11.7	1091	4.1	8.3
600	9.7	1191	6.1	2.6

TABLE 2. Possible Parameters for July 11, 1959 Event

Using Equations of Section 2

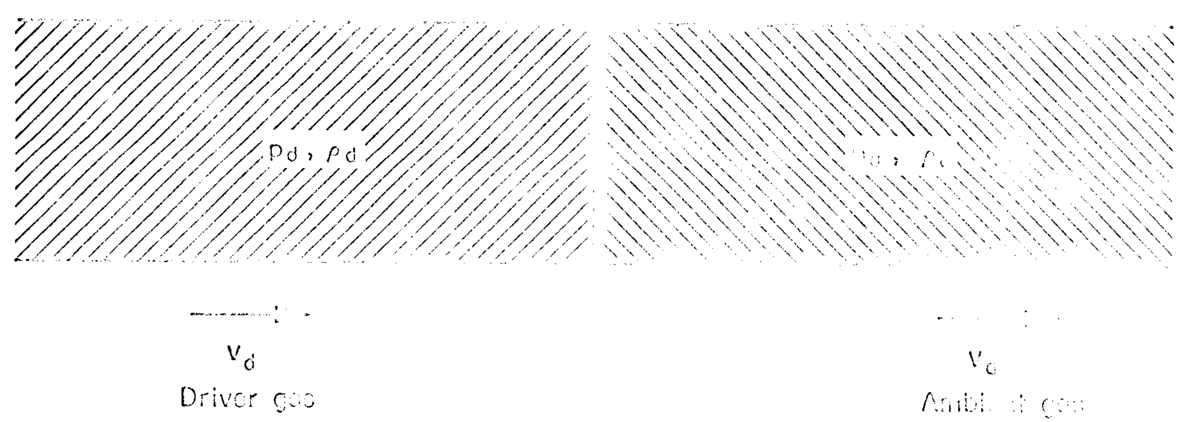
v_a(km/sec)	M_a	v_d(km/sec)	M_d	ρ_d/ρ_a
400		NO SOLUTION		
500	11.7	948 or 983	1.2 or 1.9	111 or 41
600	9.7	1128	4.8	4.1

FIGURE TITLES

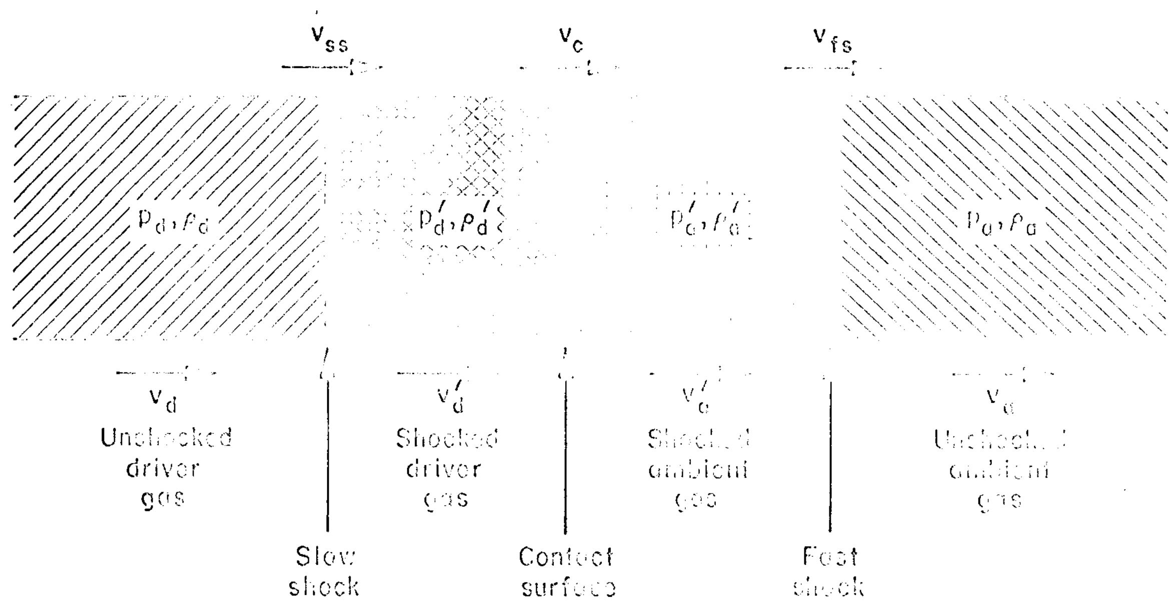
Fig. 1. Initial configuration in Riemann's problem.

Fig. 2. Configuration after collision of gas streams.

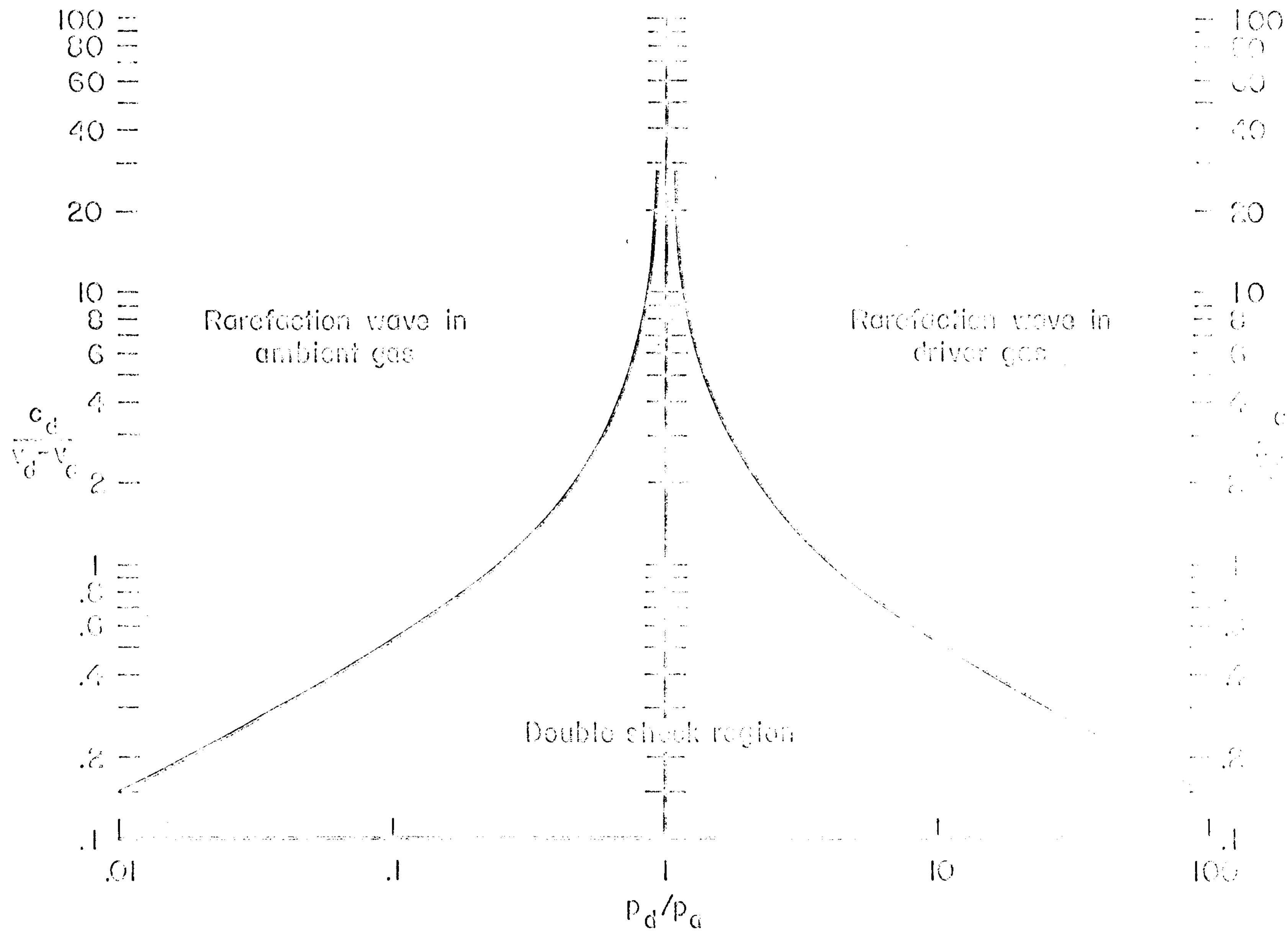
Fig. 3. Condition to be satisfied to produce two shock waves ($\gamma = 5/3$).



pd , ρd
vd
Driver gas


v_{ss}
v_c
v_{fs}
p_d, ρ_d
p_d', ρ_d'
p_a', ρ_a'
p_a, ρ_a
v_d
Unshocked driver gas
v_d'
Shocked driver gas
v_a'
Shocked ambient gas
v_a
Unshocked ambient gas
Slow shock
Contact surface
Fast shock

2



100
80
60
40
20
10
8
6
4
2
1
.8
.6
.4
.2
.1
$\frac{c_d}{V_d - V_a}$
Rarefaction wave in ambient gas
Rarefaction wave in driver gas
Double shock region
.01
.1
1
10
100
p_d/p_a