A METHOD FOR ESTIMATING ACCELERATIONS OF SHIPPING CONTAINERS MOUNTED ON AN IMPACTING RAILROAD CAR

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ABSTRACT

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A method is presented herein for estimating the response of a container and its contents when a railroad car on which they are mounted impacts a group of braked cars standing in a classification railroad yard.

Equations are derived for the container car impact equivalent velocity, container car acceleration, car body transmissibility, and container transmissibility. By applying characteristic data of the car and its coupler, the natural frequency of the car is calculated and the equivalent impact velocity to the container car is obtained from the actual impact speed, the car weights and a calculated braking weight. Then, input accelerations of the car and shipping container are calculated over a 1-14 mph range of input velocities, for bottoming and non-bottoming of the coupler during impact.

From these results, the container transmissibility is obtained knowing the inner container design natural frequency, its damping characteristics, and the forcing frequencies resulting from the impact at the car coupler. The resultant accelerations at the inner container are obtained for 0 and 10% coupler viscous damping ratios.

The calculated values are compared with the upper bound values of the accelerations obtained from impact tests. The results indicate that if a 20% value for equivalent viscous damping of the car coupler is used, for car impact speeds above 7.[illegible] mph, accelerations at the inner container and at the car bed adjacent the outer container will exceed 1.8 g and 20 g respectively.

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I. INTRODUCTION

Railroad impact tests were conducted to confirm the suitability of a shipping container rigidly attached to the floor of a railroad car. The container consisted of an outer steel shell supporting internally a spring and damper mounted inner container to which was firmly attached a dummy weight. These tests, consisted of impacting the container transport car traveling at a speed of 7 mph against 3 standing braked freight cars. The tests intended to simulate the majority of impacts experienced during normal railroad switching operations in railroad classification yards.

The tests indicated that for the 8 mph impact the shipping container attenuated the lateral accelerations to the inner container to 1.8 g. The results of these tests at impact speeds between 5 and 7.5 mph are shown superimposed on Figure 3 of this report. Because these impact speeds may be exceeded, Reference 3, the analysis which follows was performed to:

1. Correlate the tests results and determine the responses of the components of the system during impact.

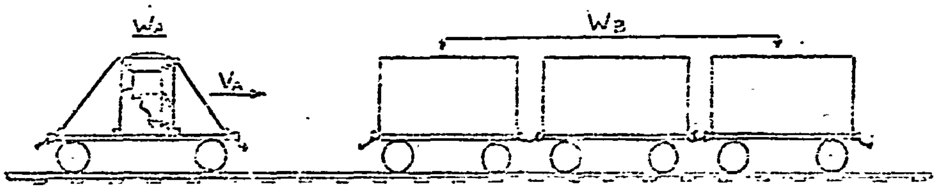
2. Obtain the accelerations at the outer container and inner container for impact speeds other than those of the tests.

3. Establish the limiting impact speed for the acceptable specified inner container acceleration.

II. METHOD OF ANALYSIS

To calculate the effect on the inner container of impact loads experienced by the container railroad car for a given environment, the equations to be used in the analysis will be indicated first. Equations will be derived for the container car equivalent input velocity, car and coupler acceleration, outer container acceleration, container transmissibility, and acceleration at the inner container. Since the car coupler may bottom upon impact and may have different damping characteristics depending on its conditions, these parameters will be considered in calculations.

1. Container Car Input Velocity Start



The kinetic energy of the impacting car will be used to impart a kinetic energy to the standing cars and compress the impacting couplers. The following balance of energy equation for the strain energy, the initial kinetic energy and the kinetic energy of the entire system after impact can be written:

$$\frac{Fd}{2} = \frac{W_A V_A^2}{2g} - \frac{\left(W_A + W_B + W_F\right) V_{AB}^2}{2g} \qquad (1)$$

where W_F represents a weight, simulating the braking forces applied to the standing cars. This weight can be estimated by assuming that the movement of the cars (sliding of wheels) occurs only after the adhesion between wheels and rails is exceeded (wheel sliding is presumed to occur rather than wheel turning).

The adhesion is provided by the friction force between the rails and wheels. Its value is given by

$$W_F = \psi\, W_B$$

The adhesion coefficient ψ in the above is generally given as 0.35 for general dry rail conditions (Reference 2). W_B is the weight of the standing cars. In addition to Equation (1) a mass momentum transfer equation can be written expressing the balance of momentum between the impacting car and standing cars:

$$\frac{W_A V_A}{g} = \frac{W_A + W_B + W_F}{g} V_{AB} \qquad (2)$$

Solving this equation for V_{AB}, we get

$$V_{AB} = \frac{W_A}{W_A + W_B + W_F} \; V_A \tag{3}$$

Substituting Equation (3) into Equation (1), and replacing the deflection, d, by $\frac{F}{K}$, where K is the coupler spring constant, and dividing both sides of the equation by $W_A^{\;2}$ we obtain

$$\frac{F^2}{W_A^{\;2}} = \frac{K \; V_A^{\;2}}{W_A \; g} \left(1 - \frac{W_A}{W_A + W_E + W_F}\right) \tag{4}$$

Multiplying the numerator and denominator of the right side of Equation (4) by g and extracting the square root, the acceleration, G_o in units of acceleration of gravity, experienced by W_A is obtained as follows:

$$G_o = \frac{F}{W_A} = \frac{V_A}{g} \sqrt{1 - \frac{W_A}{W_A + W_B + W_F}} \; \sqrt{\frac{Kg}{W_A}} \tag{5}$$

In the above expression, $\frac{Kg}{W_A} = \omega^2_{\;N}$, i.e., the natural frequency of the impacting car consists of a lumped mass W_A and the spring constant k, provided by the coupler spring attached to the mass. The other square root term can be considered as the fraction representing the amount of the initial velocity returned to the impacting car. It can then be calculated that the velocity applied to the container transportation car is given by

$$V_B = V_A \sqrt{1 - \frac{W_A}{W_A + W_B + W_F}} \tag{6}$$

This equivalent velocity will be used in estimating the acceleration provided by the impact to the container car.

2. Container Car Acceleration

A. Impact With No Bottoming of Coupler

Equation (5) developed above provides the acceleration felt by the car upon impact if coupler damping is assumed to be not present and coupler bottoming has not occurred. Equation (5) can be expressed simply in terms of velocity and natural frequency.

$$G_o = \frac{V_E \, \omega_N}{g} \qquad (7)$$

If an amount β of the critical damping is present at the coupler, the acceleration felt by the car will be from Reference 3.

$$G = \frac{V_E \, \omega_N}{g} \, e^{-\beta \omega_N t_p} \quad = G_o \, e^{-\beta \omega_N t_p} \qquad (8)$$

where the time, t_p corresponding to maximum acceleration is:

$$t_p = \frac{1}{\omega_N \sqrt{1 - \beta^2}} \tan^{-1} \frac{(1 - 4\beta^2) \quad (1 - \beta^2)^{\frac{1}{2}}}{\beta \, (3 - 4 \beta^2)}$$

The coupler deflection corresponding to these accelerations will be given by

$$d_o = \frac{GW}{K} \qquad (9)$$

The frequency of oscillation, under this type of impact will be obtained by the lumped car weight and the coupler spring constant.

B. Impact With Bottoming of Coupler

When the coupler bottoms, the behavior of the car coupler system will be the same as that of a system possessing a bilinear elasticity. The spring will deflect up to a closure deflection d_s with a spring rate K_o and beyond this deflection with a much higher spring rate K_B. In Reference (3) it is shown that for the case of bottoming impact, the acceleration can be obtained in terms of the acceleration and deflection which would be experienced if bottoming had not occurred. The equation is:

$$G = G_o \sqrt{\frac{K_B}{K_o} + \left(\frac{d_s}{d_o}\right)^2 \left[1 - \frac{K_B}{K_o}\right]} \qquad (10)$$

where G_o and d_o are given by either Equation (7) or (8) and (9).

In this type of impact, the excitation felt by the car will consist of a low and a high frequency acceleration. These two frequencies will be related to the spring constants of the coupler (K_o) and that of car bed frame (K_B) and, the bottoming deflection of the coupler (d_s) and the deflection (d_o) which would exist if bottoming had not occurred. The relations providing these frequencies are given in Reference (3) in terms of the frequency which would exist if bottoming had not occurred, as follows:

$$\frac{\omega_o}{\omega_L} = \frac{2}{\pi} \sin^{-1} \frac{d_s}{d_o} + \sqrt{\frac{K_o}{K_B}} \left[1 - \frac{2}{\pi} \tan^{-1} \sqrt{\frac{K_o}{K_B \left(\frac{d_o^2}{d_s^2} - 1\right)}}\right] \qquad (11)$$

and

$$\frac{\omega_o}{\omega_H} = \sqrt{\frac{K_o}{K_B}} \left[1 - \frac{2}{\pi} \tan^{-1} \sqrt{\frac{K_o}{K_B \left(\frac{d_o^2}{d_s^2} - 1\right)}}\right] \qquad (12)$$

where ω_H is the high frequency, ω_L is the low frequency and ω_o the frequency which would exist without bottoming.

3. Car Body Transmissibility

It will be assumed, conservatively, that the impact load experienced by the car frame at the coupler will be transmitted undiminished to the container mounting. This assumption is based on a conclusion in Reference (4) that no significant difference in g loading was noted at different locations of the test car during impacts.

4. Container Transmissibility and Inner Container Acceleration

A. Impact With No Bottoming of Coupler

The inner container accelerations will be calculated by modifying the accelerations to the outer container Equation (7) or (8) by the transmissibility for a viscous damped system. The transmissibility is given by

$$T = \sqrt{\frac{1 + \left(2 \beta \frac{\omega}{\omega_n}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2 \beta \frac{\omega}{\omega_n}\right]^2}} \qquad (13)$$

In this expression, ω is the forcing frequency of the excitation which in this case is the natural frequency of the car considered as a single degree of freedom lumped mass attached to a spring. The other parameters, β and ω_n, are characteristics of the container design.

B. Impact With Bottoming

For this case, the transmissibility will be calculated also by the use of Equation (13). This relation will be employed to find two transmissibilities. One for the low frequency acceleration, obtained from Equation (7) or (8) and the other for the high frequency acceleration, obtained from Equation (10). In order to obtain these transmissibilities, the forcing frequency, ω, to be used, will be: ω_L, obtained from Equation (11) for the low frequency excitation, and ω_H, obtained from Equation (12) for the high frequency excitation. The magnitude of the resultant acceleration R produced by the two excitations A_L and A_H will be obtained by the use of the cosine law for the addition of two harmonic functions. Thus

$$R = \sqrt{A_L^2 + A_H^2 + A_L A_H \cos\theta} \tag{14}$$

where the angle θ will be the difference between the phase angles, θ_H and θ_L of the high and low frequencies accelerations, i.e.

$$\theta = \tan^{-1}\left|\frac{\left[2\beta\dfrac{\omega_H}{\omega_N}\right]}{\left[1-\left|\dfrac{\omega_H}{\omega_N}\right|^2\right]} - \frac{\left[2\beta\dfrac{\omega_L}{\omega_N}\right]}{\left[1-\left|\dfrac{\omega_L}{\omega_N}\right|^2\right]}\right|$$

III. CALCULATIONS

1. Car Weights

The weight of the container car, W_A, including the container was 71,200 lbs. The weight of the three stationary cars were 51,200, 52,000, and 50,400 lbs. respectively, providing a stationary total weight of 153,600 lbs. The equivalent braking weight is therefore $W_F = \mu W_B = .35 \times 153{,}600 = 53{,}800$ lbs. With the relations indicated in Section II, it is possible now to calculate the acceleration of the system. However, the parameter to be used must first be established. These are the car weights, spring constants, natural frequencies, and equivalent impact speeds.

The total equivalent stationary weight will be $W_A + W_B + W_F = 71{,}200 + 153{,}600 + 53{,}800 = 278{,}600$ lbs.

2. <u>Equivalent Impact Velocity</u>

The equivalent impact velocity to the container car is from Equation (6)

$$V_E = V_A \sqrt{1 - \frac{71.2}{278.6}} = .863\ V_A \qquad (16)$$

3. <u>Spring Constant and Natural Frequencies</u>

The spring constant upon impact is provided by two couplers in series. The spring constant for the coupler will be estimated from data given in Reference 5 for the energy absorption and travel of the coupler during impact, as specified by AAR. The draft gear must have a minimum capacity of 18,000 ft-lb and the travel of the gear must be not less than 2.5 in. From this data, the spring constant of a single coupler being deflected is

$$K_o = \frac{2E}{d^2} = \frac{18.000 \times 12 \times 2}{(2.5)^2} = 75 \times 10^3 \text{ lb/in} \qquad (17)$$

For two couplers in series the spring constant will be

$$K = \frac{K_o}{2} = \frac{2E}{2d^2} = \frac{18,000 \times 12}{2.5^2} = 35 \times 10^3 \text{ lb/in} \qquad (18)$$

and the couplers relative displacement before bottoming is

$$d_s = 2 \times 2.5 = 5 \text{ in} \qquad (19)$$

The natural frequency for the car acting on a single coupler can then be calculated as

$$\omega_o = \sqrt{\frac{Kg}{W_A}} = \sqrt{\frac{75 \times 10^3 \times 386}{71,200}} = 20.2 \text{ rad/sec} \qquad (20)$$

corresponding to 3.2 cps. For the car acting on two couplers

$$\omega_N = \sqrt{\frac{Kg}{W_A}} = \sqrt{\frac{35 \times 10^3 \times 386}{71.2 \times 10^3}} = 13.75 \text{ rad/sec} \qquad (21)$$

corresponding to 2.2 cps.

4. Car and Outer Container Input Acceleration

A. Input With No Bottoming of Coupler

The magnitude of the acceleration experienced by the container and car where no bottoming of the coupler occurs will be obtained by the use of Equation (7) or (8) together with Equation (11) and (21). For no damping present at the coupler, Equation (7) will reduce to

$$G_o = \frac{V_E \omega_N}{g} = \frac{.863\ (13.75)\ V_A}{32.2} = \frac{.863 \times 13.75 \times \frac{5280}{3600} V_A}{32.3} \qquad (22)$$

$$= .54\ V_A$$

where V_A is expressed in mph.

For a 10% critical damping present in the system, Equation (8) plus the relation for the time for peak magnitude, reduces to

$$G = .86\ G_o = .464\ V_A \qquad (23)$$

The deflections corresponding to Equation (22) and (23) will be from Equation (9)

$$d_o = \frac{G_o W}{K} = .54\ V_A\ \frac{(71.2 \times 10^3)}{35 \times 10^3} = 1.1\ V_A \qquad (24)$$

and

$$d_o = \frac{GW}{K} = .464\ V_A\ \frac{(71.2 \times 10^3)}{35 \times 10^3} = .944\ V_A \qquad (25)$$

The results of these equations for several impact speeds have been shown in Figure 1

B. Input With Bottoming of Coupler

In order to apply equation (9) it is necessary to estimate the spring constant K_3 of the car body. Reference (6) shows a graph of applied force versus body yield. From this graph it can be estimated that the car body spring constant is

$$K_B = 1310 \times 10^3 \text{ lb/in} \tag{26}$$

Substituting Equation (16), (18), (19), (22 or 23), (24) and (26) into Equation (10), the following equations are obtained for acceleration with bottoming

a. For No Damping at the Coupler

$$G_o = .54\, V_A \sqrt{\frac{1310 \times 10^3}{35 \times 10^3} + \left(\frac{5}{1.1\, V_A}\right)^2 \left|1 - \frac{1310 \times 10^3}{35 \times 10^3}\right|}$$

$$= .54 \times 6.12\, V_A \sqrt{1 - \frac{20.1}{V_A^2}} \tag{27}$$

$$= 3.3\, V_A \sqrt{1 - \frac{20.1}{V_A^2}} = 3.3 \cdot \sqrt{V_A^2 - 20.1}$$

b. For 10% critical damping at the coupler

$$G = 2.84 \sqrt{V_A^2 - 20.1} \tag{28}$$

Equations (27) and (28) applicable when bottoming has occurred, will be valid for velocities producing $d_c > d_s$. From Equation (16) and (24), the velocity corresponding to these deflections will be

$$V_A > \frac{d_c}{1.1} > \frac{d_s}{1.1} > \frac{5}{1.1} > 4.55 \text{ mph}$$

and when 10% damping is included,

$$V_A > \frac{d_s}{.944} = 5.3 \text{ mph}$$

The values from Equation (27) and (28) are shown in Figure 1.

5. Container Transmissibility - Accelerations at Inner Container

A. Impact With No Bottoming of Coupler

The transmissibility for impact speeds less than V = 4.55 mph for no coupler damping and V = 5.3 with 10% coupler damping will be obtained from Equation (13) where the following substitutions have been made:

ω_N = 11.9 - the design natural frequency of the inner container with its content

β = .35, the fraction of critical viscous damping included in the suspension. This is also a design value.

ω = 20.2 rad/sec, the excitation frequency resulting from the oscillation of the car when a force is applied to the car coupler. This value was obtained from Equation (20).

With these values, the transmissibility for impact speeds lower than those indicated above, will be

$$= \frac{\sqrt{1 + \left[2\,(.35)\,\frac{20.2}{11.9}\right]^2}}{\sqrt{\left[1 - \left(\frac{20.2}{11.9}\right)^2\right]^2 + \left[2 \times .35 \times \frac{20.2}{11.9}\right]^2}} = .70$$

The accelerations to the inner container resulting from this transmission are plotted in Figure 3.

B. Impact With Bottoming of Coupler

Since during this impact, low and high frequency accelerations as obtained from Equation (22) or (23) and (27) or (28) respectively, are present at the outer container, and the transmissibility is a function of the forcing frequency, the high and low frequencies during this impact have to be found. In Figure 2, the frequencies are indicated. The equations used are Equation (11) and (12) which, with the substitution for $K_o = 35 \times 10^3$ lb/in, $K_B = 1310 \times 10^3$ lb/in and $\omega_o = 20.2$, reduce to

$$\frac{20.2}{\omega_H} = .1634 - .1041 \tan^{-1} \sqrt{\frac{2.67 \times 10^{-2}}{\left(\frac{d_o}{5}\right)^2 - 1}} \qquad (30)$$

and

$$\frac{20.2}{\omega_L} = .637 \sin \frac{5}{d_o} + \frac{20.2}{\omega_H} \qquad (31)$$

Having obtained ω_L and ω_H as in Figure 2, the transmissibilities are obtained by using Equation (13) with the design parameters of the container and with the forcing frequency $\omega = \omega_H$ and ω_L. Also the relative transmitted accelerations A_H and A_L and the phase angles are obtained from Equation (15). The resultant accelerations at the inner container obtained from Equation (14) are shown plotted in Figure 3.

IV. RESULTS AND CONCLUSIONS

Figure 1 shows the acceleration loads felt by the car bed and by the outer container at various impact speeds. The graph indicates also the reduction in acceleration provided by an assumed 10% viscous critical damping in the car coupler. The discontinuity in the curves indicates the speed at which the coupler bottoms. This change in the slope of the acceleration is confirmed by test results reported in the literature (Reference 7).

Figure 3 shows the accelerations at the inner container resulting from the attenuation of the accelerations of Figure 1 through the container suspension. The effect of the coupler damping on the magnitude of the resultant acceleration at the inner container is also shown. Superimposed as data points on these curves are the results of impact tests performed by the container designer. The comparison indicates that the calculated values agree with the upper bound values obtained from the tests. The comparison also suggests that 20% viscous damping at the coupler would have represented the condition of the test container car coupler. The extrapolated acceleration curve for 20% damping has been shown in Figure 3 to show direct comparison of test and calculation results.

The following conclusions can be drawn from the analysis:

1. The accelerations to be expected at the inner container for various impact speeds can be predicted by this type of analysis. A 20 - 25% viscous damping should be taken as a representative value of the damping provided by a railroad car coupler.

2. If the 20% value for viscous damping of the coupler is used, impact speeds higher than about 7.5 mph will produce inner container accelerations greater than 1.8 g and car bed acceleration above 20 g.

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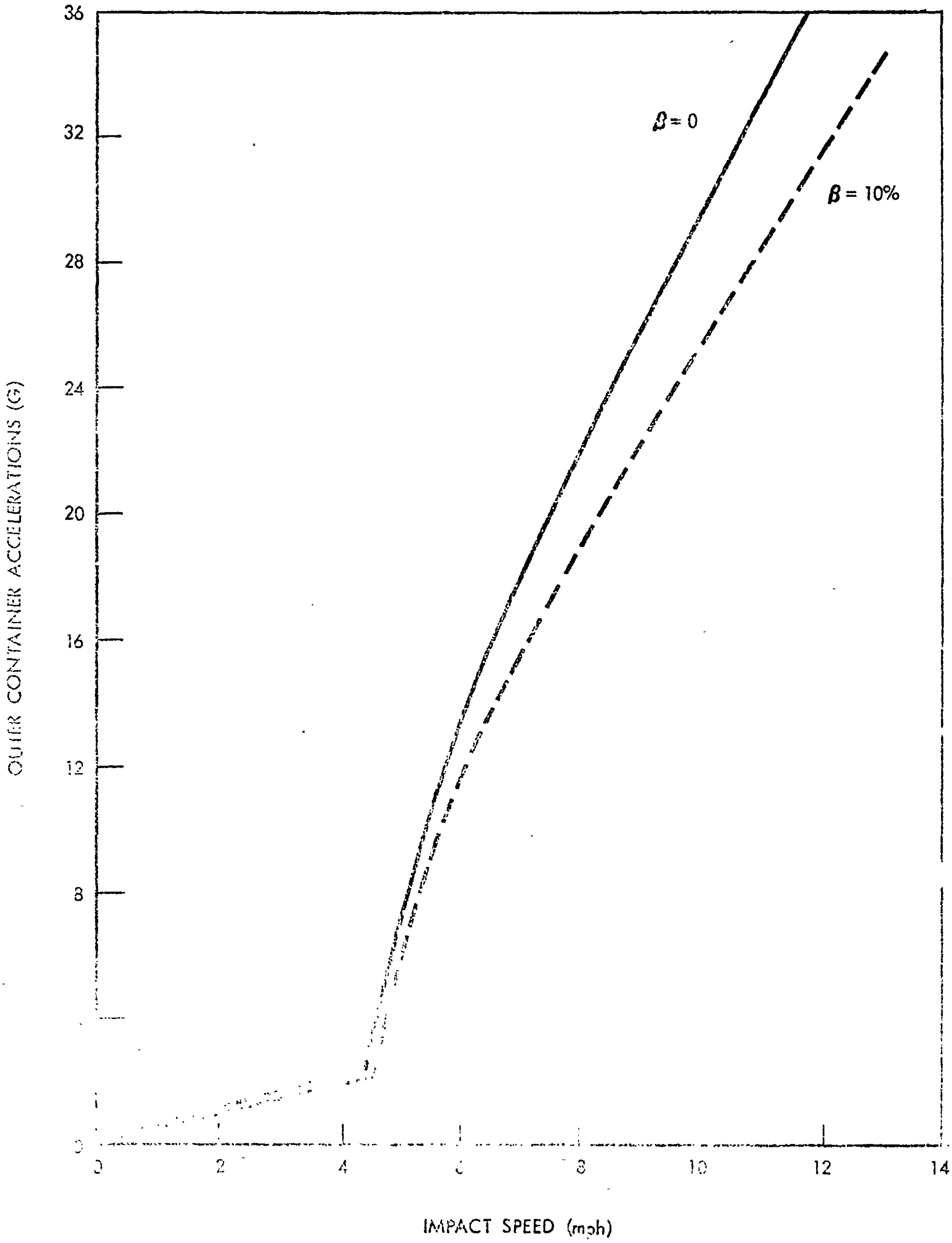

Figure 1. Car and outer container accelerations vs car impact speed for coupler with zero and 10% viscous critical damping

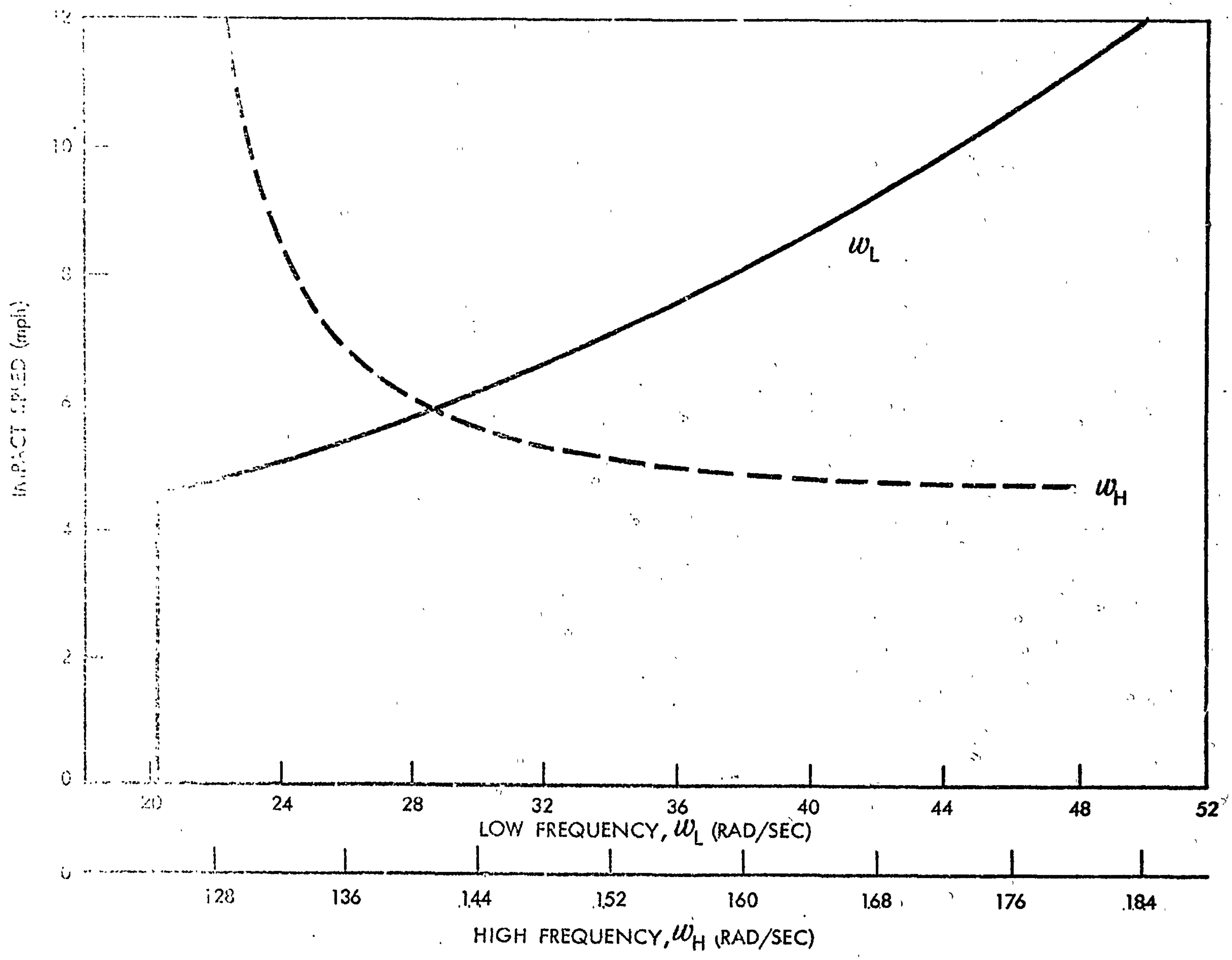

Figure 2. Car and outer container frequencies vs impact speed

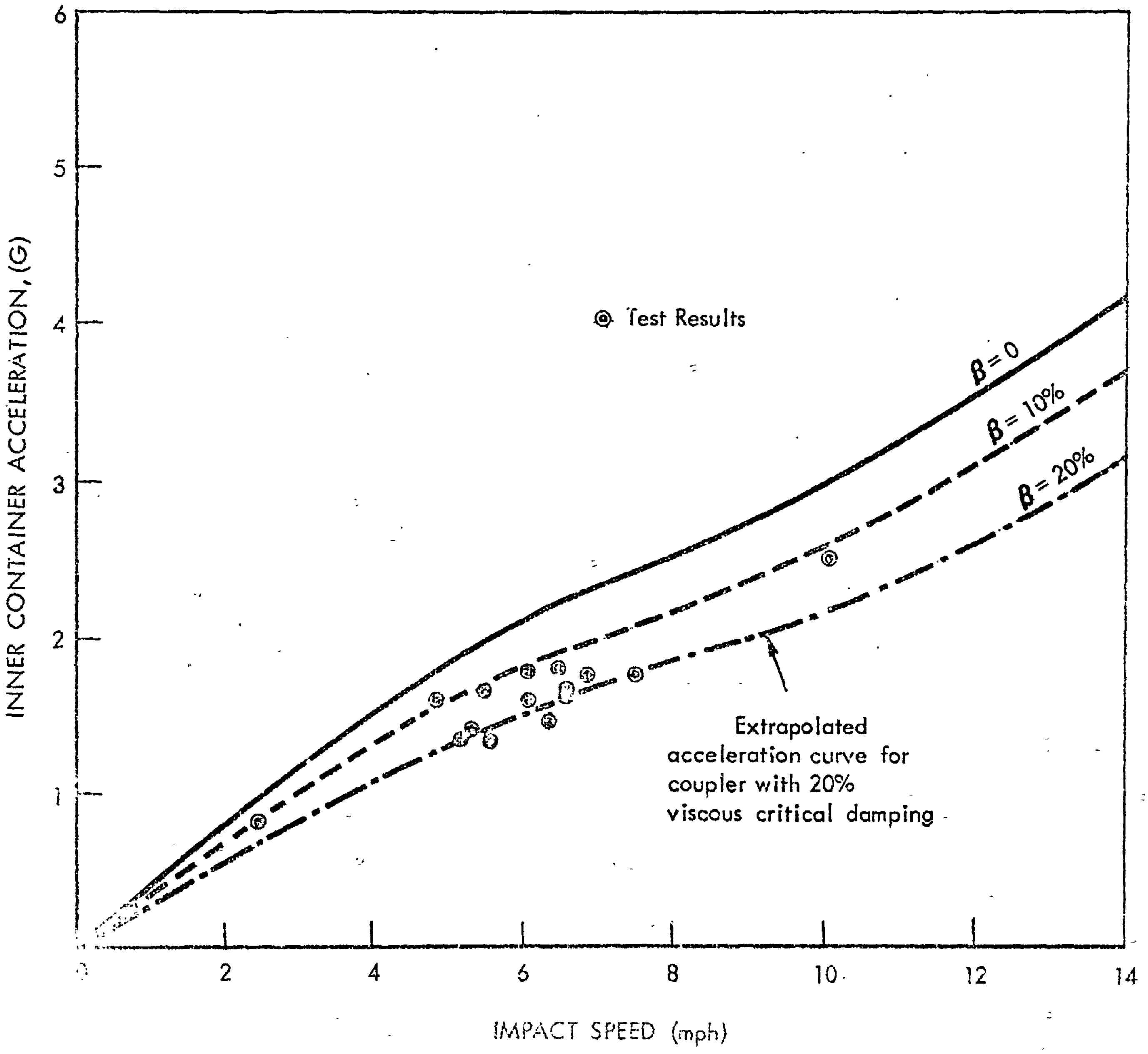

Figure 3. Inner container acceleration vs car impact speed