Direct Manual Calculations of Ionospheric Parameters using a Single-Polynomial Analysis

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Two extensions of the basic polynomial analysis are described. Firstly the polynomial representation of the real height curve is modified to include a parabolic peak; this greatly increases the accuracy of calculations near the peak of the ionospheric layers. Secondly the analytic expression for the real height curve is used to obtain expressions for the height of the peak (h_m) and the scale height at the peak (H). Integration of the real height curve also gives an expression for the effective sub-peak thickness of the ionosphere (T), defined as the total amount of ionization below the peak divided by the density at the peak. These three expressions are then used to obtain coefficients relating h_m, H and T to the virtual heights at any required frequencies.

84 sets of coefficients are given for analysing h'(f) records taken anywhere in the world, with critical frequencies between 1.5 and 20 Mc/s. These coefficients give the values of h_m, H, T and the real heights of reflection directly in terms of the virtual heights at 5 or 6 different frequencies. For calculations on a single ionospheric layer the accuracy is extremely high, being equivalent to that obtained with a lamination analysis using more than 50 points. For calculations on the day-time F layer coefficients are provided with alternative sampling points chosen to reduce the effects of the E and F1 layer cusps, so that the height, scale height and total content of the F layer can be determined with an accuracy equivalent to a lamination analysis using more than 20 points. Finally coefficients including an extraordinary-ray correction for the underlying ionization are given, to enable accurate calculations on the night-time F layer. [redacted]

1. Introduction

The most valuable technique of ionospheric research is the use of sweep-frequency virtual height records (ionograms) to determine the variation of the electron density with height. For many years ionograms have been recorded at the rate of several thousand per day at observatories all over the world. However only a very small proportion of these records are fully analysed, because of the length and complexity of the analysis. The purpose of the present paper is to give a simple, rapid method of analysing these records to obtain the main ionospheric parameters: the height of the peaks of the different layers, the scale height at the peak and the total number of electrons below the peak. These parameters are obtained directly from the virtual heights measured at five or six frequencies. The accuracy is equal to that obtained by carrying out a complete profile calculation using a lamination analysis with more than twenty points. The real heights of reflection at the measured frequencies can also be obtained if required, giving five or six points on the electron density profile. The basis of the method is described in sections 2 and 3, while section 4 gives 84 sets of coefficients which can be used to analyse day-time or night-time ionograms taken anywhere in the world.

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The analysis of ionograms consists basically of converting an observed h'(f) curve, which gives the virtual height of reflection h' as a function of the wave frequency f, into an N(h) curve giving the variation of the electron density N with height h. These two curves are related by

$$h'(f) = \int_0^{h_r} \mu' dh \tag{1}$$

where the group refractive index μ' is a complicated function of f, N and the strength and direction of the magnetic field. h_r is the height of reflection of the wave of frequency f, and depends on f, N and (for the extraordinary ray only) the strength of the magnetic field.

There is no analytic solution of (1), giving N(h) in terms of h'(f). Consequently the first method of analysing ionograms was to assume various model N(h) curves, and compare the h'(f) curves calculated from these models with those observed experimentally. Later Budden (1954) suggested an N(h) model consisting of a large number of linear segments. The integral in (1) can then be replaced by a finite sum, and the sizes of the successive segments determined from the virtual heights of the waves reflected at the ends of the segments. This gives the 'lamination' method of analysis which is widely used at present (Thomas, 1959). It gives good accuracy when a large number of segments are used, but the calculations then become rather lengthy and an electronic computer is generally employed.

The development of the polynomial method (Titheridge, 1959, 1961) was an attempt to obtain a more efficient process for the analysis of ionograms. The N(h) profile was represented by a single polynomial, of the form

$$h = \alpha_1 + \alpha_2 f_N^2 + \alpha_3 f_N^3 + \alpha_4 f_N^4 + \ldots + \alpha_n f_N^n \qquad (2)$$

where f_N is the plasma frequency, proportional to the square root of the electron density. The number of terms (n) in the polynomial is taken equal to the number of frequencies at which the virtual heights are to be measured. Coefficients can then be obtained giving the real heights of reflection at these frequencies directly in terms of the measured virtual heights. This gives n points on the N(h) curve, the calculated real heights lying on the polynomial of lowest degree which is consistent with the observed virtual heights. Because of the assumption of a single, smooth real height curve the number of points needed for an accurate analysis is much less than in the normal lamination method; typically the number of points required for a given accuracy is reduced by a factor of five for day-time ionograms, and a factor of ten at night.

The polynomial approach was also suggested independently by Unz (1961) and by Knecht, van Zandt and Watts (1962). Unz envisaged a lengthy, manual calculation of the coefficients α_i, but no results were given and the process he suggests must be modified slightly before useful results can be obtained, since

(1) The real heights finally obtained include an "arbitrary additive constant" which is not determined. This meaningless result is caused by an error in the expressions for the virtual heights, which should include the constant term a_o.

(2) The direct expansion of h in powers of f_N, including a term in f_N^1, can give unstable results.

This instability occurs because the virtual heights are not observed at low frequencies. Consequently there are an infinite number of possible real height curves, corresponding to different heights and gradients near $f_N = 0$, which would give the observed virtual heights. As in all methods of analysis some assumption must be made about the behaviour of the real height curve at low frequencies if a single, physically reasonable result is to be obtained. In the polynomial analysis it has been found sufficient to require that the real height curve is horizontal, i.e. $dh/df_N = 0$, at $f_N = 0$.

If a single-polynomial analysis is used with ten or more points the solution becomes unstable whatever starting assumptions are made. The instability is revealed by the occurrence of large positive and negative values in the calculated polynomial coefficients, so that the real heights are obtained as the difference of comparatively large multiples of the virtual heights. It is caused by the oscillations which occur in an approximating polynomial fitted through a large number of points. With the polynomial approach, however, a comparatively small number of points is often sufficient for accurate results. The method described in the present paper uses the virtual heights measured at five or six different frequencies. This is sufficient for accurate profiles of the night-time F layer, and gives a good representation of the overall shape of the day-time ionosphere since the results are equivalent to a lamination analysis using more than 20 points. For a detailed profile of the day-time ionosphere many more points are needed, and the real height curve must be represented by a series of overlapping polynomials (Titheridge, 1961, 1965).

2. The Addition of a Parabolic Peak to the Polynomial Assumption

The polynomial type of analysis has so far been based on expressions similar to that given in equation 2. It is not, however, necessary to take h as a polynomial in f_N, and many other expressions for the real height curve can be used. In general, a 'polynomial' type analysis can be based on the assumption that the real height curve is given by

$$h = \alpha_1 \phi_1(f_N) + \alpha_2 \phi_2(f_N) + \ \ldots \ + \alpha_n \phi_n(f_N) \qquad (3)$$

where ϕ_1, ϕ_2, ...ϕ_n are any real, differentiable functions of f_N.

A simple polynomial (or lamination) representation of the real height curve cannot be accurate near the peak of the ionosphere, where dh/df_N becomes infinite. Consequently most methods of analysis become increasingly innaccurate as the peak of a layer is approached, and give only an approximate result for the height of the peak. To reduce this error a function

$$\phi_p = 1 - (1 - f_N^2/f_c^2)^{\frac{1}{2}}$$

is added to the normal polynomial model of the real height curve, which then becomes

$$h = \alpha_1 + \alpha_2 f_N^2 + \alpha_3 f_N^3 + \ldots \alpha_{n-1} f_N^{n-1} + \alpha_p \{1-(1-f_N^2/f_c^2)^{\frac{1}{2}}\} \qquad (4)$$

The function ϕ_p corresponds to a parabolic layer with unit semithickness and a critical frequency f_c, so that dh/df_N now becomes infinite at $f_N = f_c$.

The real height model should become approximately parabolic near $f_N=f_c$, to agree with theoretical calculations of the shape of the peak. To ensure this the profile given by the first n-1 terms must become horizontal at $f_N=f_c$. This is achieved by adding a suitable term in f_N^n. Thus for a six point analysis the form finally assumed for the real height curve is

$$h = \alpha_1 + \alpha_2(x^2-\tfrac{1}{3}x^6) + \alpha_3(x^3 - \tfrac{1}{2}x^6) + \alpha_4(x^4 - \tfrac{2}{3}x^6) + \alpha_5(x^5 - \tfrac{5}{6}x^6) + \alpha_p\{1 - (1 - x^2)^{\frac{1}{2}}\} \qquad (5)$$

where $x = f_N/f_c$. The real height curve is therefore represented by a parabolic layer plus a correcting polynomial, the polynomial having $dh/df_N = 0$ at $f_N = 0$ (to reduce the variations in the amount of underlying ionisation) and $dh/df_N=0$ at $f_N=f_c$ (to ensure that the peak is approximately parabolic).

Equation (5) is used to derive a set of polynomial coefficients, giving the real heights at six frequencies (i.e. at six values of x) in terms of the virtual heights at those frequencies. The procedure is basically similar to that employed in the simple polynomial analysis (Titheridge, 1961), and is outlined below.

Differentiating equation (5) gives

$$dh = \sum_{j=2}^{5} j\alpha_j(x^{j-1}-x^5)dx + \alpha_p x(1-x^2)^{-\frac{1}{2}}dx$$

so that equation (1) becomes

$$h'(f_i) = \alpha_1 + \sum_{j=2}^{5} j\alpha_j\int_0^{x_i}\mu'(x^{j-1}-x^5)\,dx + \alpha_p\int_0^{x_i}\mu' x(1-x^2)^{-\frac{1}{2}}dx$$

where $x_i = f_i/f_c$. Substituting $t^2 = 1-f_N^2/f_i^2 = 1 - x^2/x_i^2$

we get $xdx = -x_i^2 tdt$ and

$$h'(f_i) = \alpha_1 + \sum_{j=2}^{5} j\alpha_j x_i^2\int_0^1\mu' t(x^{j-2}-x^4)\,dt + \alpha_p x_i^2\int_0^1\mu' t(1-x^2)^{-\frac{1}{2}}dt$$

or

$$h'(f_i) = b_{i1}\alpha_1 + b_{i2}\alpha_2 + b_{i3}\alpha_3 + b_{i4}\alpha_4 + b_{i5}\alpha_5 + b_{i6}\alpha_p \qquad (6)$$

where $b_{i1} = 1.0$,

$$b_{ij} = jx_i^2 \int_0^1 \mu' t(x^{j-2} - x^4)\,dt \qquad \text{for} \qquad j = 2, 3, 4 \text{ and } 5$$

and $b_{i6} = x_i^2 \int_0^1 \mu' t(1-x^2)^{-\frac{1}{2}} dt.$

These values of b_{ij} must be calculated at each of the six frequencies f_i at which the virtual heights are measured. The integrals can be calculated with sufficient accuracy, for magnetic dip angles up to 80 degrees, using a 12 point Gaussian relation. The integrands are well-behaved and the 72 values of $\mu' t$ (12 for each frequency) are calculated from the standard expressions (Shinn and Whale, 1952).

For each frequency f_i, equation (6) gives the virtual height in terms of the real height parameters α_j. The set of six equations can be written in matrix notation as $\underline{h'} = \underline{B\alpha}$ where $\underline{B}$ is the 6 x 6 matrix with elements b_{ij}, and $\underline{h'}$ and $\underline{\alpha}$ are 6 x 1 column matrices. Inverting the matrix B we get the set of six equations

$$\underline{\alpha} = \underline{B}^{-1}\,\underline{h'} \qquad (7)$$

giving the real height parameters α_j in terms of the observed virtual heights. Applying equation 5 for each value of x_i then gives the set of equations

$$\underline{h} = \underline{A}\,\underline{\alpha} = (\underline{A}\,\underline{B}^{-1})\,\underline{h'} \qquad (8)$$

where $\underline{A}$ is a 6 x 6 matrix with elements a_{ij} such that $a_{i1} = 1.0$, $a_{ij} = x_i^2 - jx_i^6/6$ for j = 2 to 5, and $a_{i6} = 1-(1-x_i^2)^{\frac{1}{2}}$. The product of the matrices $\underline{A}$ and $\underline{B}^{-1}$ therefore gives the required polynomial coefficients, relating the real heights at the six frequencies f_i to the virtual heights at those frequencies.

3. Direct Calculations of the Height of the Peak, the Scale Height at the Peak and the Sub-peak Content

3.1. Calcualtion of h_m

The polynomial approach is not restricted to the determination of the real heights of reflection corresponding to a series of measured virtual heights. The method employs an analytical expression for the entire real height curve, and can therefore be used to determine any required features of this curve. In particular, any number of real heights can be calculated at any required frequencies by using these frequencies in the matrix $\underline{A}$. If n virtual heights are measured and m real heights are required, this gives an m x n matrix $\underline{A}$ and an m x n matrix of coefficients $\underline{A}\,\underline{B}^{-1}$.

One of the most important ionospheric parameters is the height h_m of the peak of an ionospheric layer. This is the real height at the frequency $f = f_c$, so that coefficients for calculating h_m are obtained by putting $x_i = 1$ in the matrix $\underline{A}$. Thus if the virtual heights are measured at six frequencies f_i, the height of the peak of the layer is given by

$$h_m = \beta_1 h'(f_1) + \beta_2 h'(f_2) + \beta_3 h'(f_3) + \beta_4 h'(f_4) + \beta_5 h'(f_5) + \beta_6 h'(f_6) \qquad (9)$$

where the coefficients β are obtained by multiplying the matrix $\underline{B}^{-1}$ by the line matrix $(1, 2/3, \tfrac{1}{2}, 1/3, 1/6, 1)$.

3.2. Calculation of the Scale Height H

The polynomial terms in the expression for the real height curve (equation 5) give $dh/df_N = 0$ at $f_N = f_c$. The curvature at the peak of the layer is therefore produced entirely by the last term in equation (5), so that the peak is approximately parabolic with a scale height of $\frac{1}{2}\alpha_p$. That is, the scale height at the peak of the layer is given in terms of the coefficients α by

$$H = 0.\alpha_1 + 0.\alpha_2 + 0.\alpha_3 + 0.\alpha_4 + 0.\alpha_5 + \tfrac{1}{2}\alpha_p \qquad (10)$$

or $\quad H = (0,0,0,0,0,\frac{1}{2}).\ \underline{\alpha}$

The column matrix $\underline{\alpha}$ is given in terms of the observed virtual heights by $\underline{\alpha} = \underline{B}^{-1}\underline{h}'$. Coefficients giving the scale height H in terms of the observed virtual heights are therefore obtained by multiplying $\underline{B}^{-1}$ by the line matrix $(0,0,0,0,0,\frac{1}{2})$.

3.3. The Calculation of the Total Electron Content up to the Peak of the Layer

The total number of free electrons below the peak of an ionospheric layer is most conveniently calculated in terms of the effective thickness (T) of the layer. This is defined as the ratio of the sub-peak electron content to the peak density, so that

$$T = \frac{1}{N_m} \int_{h_o}^{h_m} N \, dh \qquad (11)$$

where h_o is the height of the bottom of the ionosphere (at $N = 0$) and N_m is the electron density at the peak of the layer. Integrating by parts gives

$$T = h_m - \frac{1}{N_m} \int_0^{N_m} h \, dN$$

or
$$T = h_m - 2\int_0^1 h \, x \, dx$$

where $x = f_N/f_c = (N/N_m)^{\frac{1}{2}}$ as before.

From equation (5) we have

$$h = \alpha_1 + \sum_{j=2}^{5} \alpha_j (x^j - jx^6/6) + \alpha_p \{1 - (1-x^2)^{\frac{1}{2}}\}$$

and
$$h_m = \alpha_1 + \sum_{j=2}^{5} \alpha_j (1 - j/6) + \alpha_p$$

so that

$$T = \sum_{j=2}^{5} \alpha_j \{1 - \frac{j}{6} - 2 \int_0^1 (x^{j+1} - jx^7/6) \, dx\} + 2\alpha_p \int_0^1 x(1-x^2)^{\frac{1}{2}} dx$$

$$= \sum_{j=2}^{5} j(\frac{1}{j+2} - \frac{1}{8})\alpha_j + \frac{2}{3} \alpha_p$$

or
$$T = 0.\alpha_1 + \frac{1}{4} \alpha_2 + \frac{9}{40} \alpha_3 + \frac{1}{6} \alpha_4 + \frac{5}{56} \alpha_5 + \frac{2}{3} \alpha_p \qquad (12)$$

Coefficients giving the effective thickness T in terms of the observed virtual heights are therefore obtained by multiplying the matrix $\underline{B}^{-1}$ by the line matrix (0,1/4,9/40,1/6,5/56,2/3).

4. World-wide Coefficients for the Analysis of Ionograms

4.1. The Effect of the Earth's Magnetic Field

The calculated coefficients depend on the frequencies involved and on the strength and direction of the magnetic field. When the frequencies used are fixed submultiples of the critical frequency, it can be shown that the coefficients depend only on the ratio of the critical frequency f_c to the gyrofrequency f_H and on the magnetic dip angle I. Coefficients calculated for a sufficiently wide range of values of f_c/f_H and of I can therefore be used for analysing ionograms taken anywhere in the world.

To examine the effect of changes in f_c/f_H and in I, the virtual height curve corresponding to a parabolic layer at the equator was analysed using coefficients calculated for a wide variety of different conditions. The results are shown in figure 1, which gives the amount by which the calculated heights were less than the true height. The reduction is expressed as a percentage of the true height measured from the bottom of the layer. The curves shown apply to a frequency of $0.9f_c$, where f_c is the critical frequency of the parabolic layer, but the percentage reduction is approximately the same at all frequencies above $0.5f_c$.

The values of I and of f_c/f_H used for the coefficients of table 1 are shown by the crosses in figure 1. These values are spaced at intervals giving about 2% change in the heights calculated from a fixed h'(f) curve. Records taken under any conditions can therefore be analysed with a maximum error, through using the nearest set of coefficients, of about one percent. When interpolating between the given values of I the 'nearest' coefficients should be chosen from figure 1 to give approximately the correct heights; that is to give the correct reduction from the no-field height. Thus to analyse records taken at a dip angle of 40 degrees with a critical frequency of $8f_H$, the coefficients calculated for $I = 35^\circ$ and $f_c = 6f_H$ should be used.

The percentage errors in the calculated values of scale height H and effective thickness T are about twice the errors in the calculated heights. Thus for a layer with a semithickness of 100 km the maximum error in the calculated heights (through using the nearest set of coefficients) is about 1 km, while the maximum errors in H and T are about 2% or 1 km. An accuracy greater than this is seldom justified for the analysis of normal ionograms. Accurate coefficients for any required conditions can, however, be obtained by linear interpolation between the points marked in figure 1. If a series of records from one or more stations is to be analysed exact coefficients calculated for any required frequencies will be supplied by the author. The computer program used takes about six minutes to calculate and print each set of coefficients in the form of tables 1, 2 and 3.

4.2. Coefficients for General Use (Table 1)

Table 1 gives five-point coefficients calculated for the values of I and of f_c/f_H marked in figure 1. These coefficients are suitable for general calculations on the E and F layers under any conditions. In all cases the virtual heights are measured at frequencies of $.15f_c$, $.44f_c$, $.68f_c$, $.87f_c$ and $.98f_c$, where f_c is the critical frequency of the layer. When the ionograms have a logarithmic frequency scale, the required virtual heights can be rapidly obtained by using a transparent scale with six vertical lines corresponding to the ratios .15, .44, .68, .87, .98, and 1.0. The scale is adjusted so that the 1.0 line falls on the critical frequency of the layer considered, and the heights where the other lines cross the virtual height trace are recorded.

This procedure is identical to that used with the five-point Kelso-Schmerling coefficients. The frequency ratios used in the present work are in fact based on the Schmerling coefficients (Schmerling and Ventrice 1959), since calculations with a wide range of different frequency intervals showed clearly that these coefficients provide the optimum sampling points for accurate calculations near the peak of the layer. The coefficients were however modified in two ways for the present purposes:

(1) The highest frequency has been limited to $.98f_c$, although the five-point Kelso coefficients give an upper frequency of $.988f_c$ at the equator. Measurements nearer the critical frequency do improve the theoretical accuracy of the calculations, but the experimental errors are greatly increased by slight uncertainties in the critical frequency and hence in the reading frequency. A fixed upper limit of $.98f_c$ appears a reasonable compromise.

(2) A fixed series of reading points is used under all conditions. This enables a single transparent scale to be used with all records, and permits interpolation between the different sets of coefficients given in table 1 to obtain coefficients for intermediate values of I and of f_c/f_H.

The coefficients labelled h_m in table 1 give the factors by which the five measured virtual heights are multiplied before being summed to give the height of the peak of the layer. Thus for $I = 55^\circ$ and $f_c = 5f_H$ we have

$$h_m = .199h'_1 + .192h'_2 + .224h'_3 + .158h'_4 + .227h'_5$$

where h'_1, h'_2, h'_3, h'_4 and h'_5 are the measured virtual heights at $f/f_c = .15$, .44, .68, .87, and .98 respectively. Similarly the scale height at the peak of the layer is

$$H = -.015h'_1 - .063h'_2 + .061h'_3 - .309h'_4 + .326h'_5$$

and the effective sub-peak thickness is

$$T = -.211h'_1 - .101h'_2 + .007h'_3 + .091h'_4 + .214h'_5 \ .$$

The last five rows of coefficients in each set are used for obtaining the real heights of reflection at the measured frequencies. Thus the real height of reflection at $f = .68f_c$ is

$$h_3 = .307h'_1 + .397h'_2 + .340h'_3 - .054h'_4 + .010h'_5 \ .$$

For single layer h'(f) records the accuracy obtained near the peak of the layer is similar to that from a lamination analysis using about 50 points. Because of the inclusion of a parabolic peak in the assumed form of the real height curve, the errors in h_m are much less than for a Kelso type analysis using 10 points. The accuracy with which the real heights of reflection are obtained varies considerably. At the lowest frequency ($.15f_c$) the accuracy is of course very poor, since there is no measure of the amount of ionization below this height. The accuracy increases rapidly at higher frequencies, becoming very good above $0.7f_c$.

4.3. Coefficients for F Layer Calculations During the Day (Table 2)

In investigations of the day-time F layer, the presence of a large cusp on the h'(f) record greatly reduces the accuracy of real height calculations using a small number of points. This cusp is caused by the presence of the E layer, with a critical frequency of the order of one third of the critical frequency of the F layer. The coefficients of table 2 have been calculated to provide increased accuracy under these conditions. An additional reading point at $f = .35f_c$ is included to double the accuracy near the cusp. The virtual heights are measured at the six frequencies $.15f_c$, $.35f_c$, $.55f_c$, $.75f_c$, $.9f_c$ and $.98f_c$, where f_c is the critical frequency of the F layer. The tabulated coefficients are then used to obtain the values of h_m, H and T as before. Thus for a dip angle of 55^o at $f_c = 6.5f_H$ we have

$$h_m = .186h'_1 + .098h'_2 + .183h'_3 + .202h'_4 + .100h'_5 + .231h'_6$$

$$H = -.030h'_1 + .034h'_2 - .123h'_3 + .134h'_4 - .406h'_5 + .390h'_6$$

and $$T = -.222h'_1 + .014h'_2 - .185h'_3 + .174h'_4 - .021h'_5 + .240h'_6$$

The virtual height (h'_2) at $.35f_c$ will commonly be near the E layer cusp, but since the corresponding coefficients are only about half as large as the other coefficients, the errors caused by the cusp are no larger than for a 12 point Kelso type analysis.

Many day-time ionograms also show a second cusp, due to the F1 layer, at a frequency of about 1.4 times the critical frequency of the E layer. Errors caused by this cusp can also be reduced by the increased number of reading points in table 2. It is, however, desireable to avoid measuring virtual heights too near a cusp. To make this possible the frequencies in table 2 fall about half-way between the frequencies of table 1, over most of the frequency range. Cusps can therefore be avoided, in most cases, by choosing the appropriate table. To facilitate this a transparent scale with both the 5-point ratios for table 1 and the 6-point ratios for table 2 marked on it, in different colors, can be used for reading the ionograms. Table 2 should be used for most of the ionograms, but when this entails measuring a virtual height near a cusp the five virtual heights for table 1 should be measured instead.

In all cases where a measurement could be influenced by a cusp, the virtual height curve should be smoothed before the virtual height is measured. If this smoothing is carried out over a frequency range equal to the separation of the reading points, maintaining the same total area under the h'(f) curve, the errors caused by the cusp are considerably reduced.

4.4. Coefficients for F layer Calculations at Night (Table 3)

During the night, the E region ionization cannot be observed directly on normal h'(f) records. Consequently the effect of this ionization is generally neglected when calculating the height and shape of the F layer. This neglect can cause quite large errors in the calculated height of the peak of the F layer, particularly near sunset, and even larger errors in the calculated values of scale height and sub-peak content. There are in fact an infinite number of real height curves, corresponding to different amounts of E region ionization, which would give the observed ordinary ray virtual height curve. The different real height curves will, however, give quite different values for the virtual height of the extraordinary ray (h'_x) at low frequencies. A single value of h'_x can therefore be used to resolve the ambiguity in the analysis of night-time ionograms, and give reasonably correct results for the height and shape of the F layer and the sub-peak electron content.

The coefficients of table 3 have been designed for this purpose. The virtual heights of the ordinary ray are measured at a series of frequencies similar to those used in table 1, except that the lowest frequency (f_1) is not allowed to fall below 1 Mc/s. The virtual height of the extraordinary ray is also measured, at the frequency f_x which is reflected at the same real height as the ordinary ray of frequency f_1. The coefficients of table 3 then give the multiples of h'_1, h'_x, h'_2, h'_3, H'_4h and h'_5 which must be summed to obtain h_m, H, T and the real heights of reflection.

To obtain the right correction for the underlying ionization the values of f_1 and f_x must not be changed as the critical frequency varies. Thus the first set of coefficients in table 3 are used for f_c/f_H between about 2.0 and 3.5, at dip angles of up to 10 degrees. As f_c varies the values of f_2, f_3, f_4 and f_5 vary in proportion, but the values $f_1 = 1.0f_H$ and $f_x = 1.62f_H$ must not be altered. Because of this restriction the errors due to using the nearest set of coefficients can become much larger than the figure of 1% given for tables 1 and 2. For accuract calculations at any station a set of coefficients calculated for that station should therefore be obtained from the author.

5. Conclusions

The coefficients of tables 1, 2 and 3 were calculated on an IBM 1620 computer using eight significant figures throughout. The calculation and printing of each set of coefficients takes about five minutes using a twelve point integration relation. The coefficients are correctly rounded to four decimal places, giving an overall accuracy of about 2 parts in 10^4.

Coefficients are given for a range of values of I, f_c and f_H such that ionograms taken anywhere in the world can be analysed with a maximum error, through using the nearest set of coefficients, of about 2 kilometres. The coefficients are calculated by assuming a mathematical model for the real height curve, consisting of a parabolic layer plus a correcting polynomial. This enables a very accurate fit to the peak of any normal ionospheric layer and, since the polynomial used is of the lowest possible degree, provides the smoothest real height curve consistent with the observations. The accuracy of the results obtained when analysing a single-layer ionogram is therefore very high, since a normal, smooth virtual height curve implies a very smooth real height curve.

The accuracy of the coefficients was tested using some highly accurate h'(f) curves calculated by Professor W. Becker for parabolic and cosine layers. These curves have a critical frequency of 6Mc/s and were given for f_H = 1.20 Mc/s at I = 29° and for f_H = 1.18 Mc/s at I = 67°. Coefficients of the form used in table 1 were calculated for these conditions and used to analyse the h'(f) curves for the parabolic layer. The resulting values of h_m, H, T and the real heights of reflection were almost exactly correct, the mean error being less than 1 part in 10^4 and the maximum error in any quantity less than 2 parts in 10^4. This is equal to the rounding off error in the printed coefficients, and demonstrates their basic accuracy.

The virtual height curve for the cosine layer provides a more realistic test, since this layer has a point of inflection at 1.5 Mc/s, a peak which is not parabolic, and cannot be represented by a finite polynomial. The errors in the real heights and in the values of h_m, H and T calculated using the 5-point coefficients at I = 67° are shown in table 4. The errors obtained at I = 29° differ by less than 0.1 km from those at I = 67°, showing that the same parabola-plus-polynomial representation of the cosine layer is obtained at both dip angles. The only significant error is at the lowest frequency. The 4.6 km error at this point is caused by the low-density ionization in the cosine layer, which gives a difference of 14.5 km in the real and virtual heights at 0.9Mc/s.

The real heights of the cosine layer were also calculated using a 30 point lamination analysis, with a parabolic peak fitted through the last three calculated heights. The analysis was started from 0.9 Mc/s, and the errors in the calculated heights are shown in table 4. It is seen that for determining the height and shape of the peak of this layer, the five-point analysis is an order of magnitude more accurate than the 30 point lamination analysis. It also provides a much better estimate of h_m than the 5 and 10 point Kelso-Schmerling coefficients, the results of which are shown at the bottom of table 4.

The accuracy attainable in practice will generally be limited by the ionograms. The precautions necessary to obtain consistent results from ionograms with incomplete or obscured traces are similar to those required by most other methods of analysis. They have been discussed by Piggott and Rawer (1961), and chapter S2 of this reference should be studied before any analysis program is undertaken.

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Table 4. The errors (in kilometres) in the calculated heights of the cosine layer at $f_H = 1.18$ Mc/s and $I = 67^o$.

Frequency in megacycles	0.90	2.64	4.08	5.22	5.88	h_m	H	T
Virtual height in kilometres	133.6	199.3	268.2	360.8	552.2			
Real height in kilometres	119.1	158.0	195.2	234.4	274.5	300.0	64.4	100.0
Error in value calculated from 5-point coefficients	+4.6	+1.1	+0.6	+0.5	+0.5	+0.7	+0.1	−0.2
Error in value from 30-point lamination analysis	+14.5	+4.1	+2.7	+2.4	+5.1	+8.0	+7.8	
5-pt Schmerling coefficients						−10.5		
10-pt Schmerling coef icients						−5.6		

Figure 1. The effect of the magnetic field on the calculated heights of a parabolic layer. The virtual height curve corresponding to a parabolic layer at the equator was analysed using coefficients calculated for different magnetic dip angles and different values of f_c/f_H. The amount by which the calculated heights (at $f = 0.9f_c$) were less than the true height of the parabolic layer is shown as a percentage of the true height. The circles show the values of f_c/f_H and of dip angle for which coefficients are given in table 1.

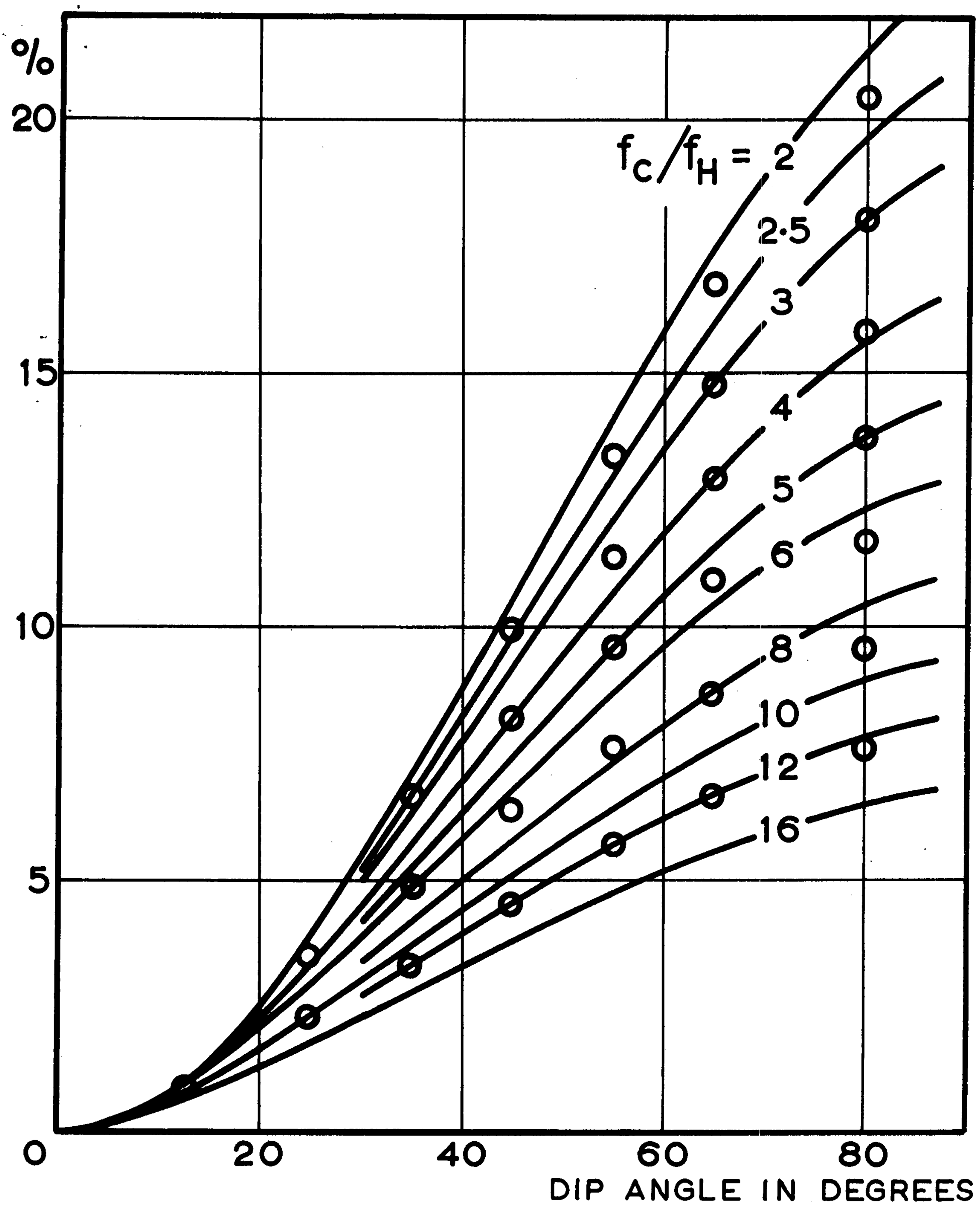

FIGURE 1. THE EFFECT OF THE MAGNETIC FIELD ON THE CALCULATED HEIGHTS OF A PARABOLIC LAYER. The virtual height curve corresponding to a parabolic layer at the equator was analysed using coefficients calculated for different magnetic dip angles and different values of f_c/f_H. The amount by which the calculated heights (at $f = 0.9f_c$) were less than the true height of the parabolic layer is shown as a percentage of the true height. The circles show the values of f_c/f_H and of dip angle for which coefficients are given in table 1.

Table 1. Five-point coefficients for calculating the height of the peak, the scale height at the peak, the slab thickness and the real heights of reflection of ionospheric layers with critical frequencies between $1.6f_H$ and $16f_H$.

DIP = 13 DEGREES, FC = 8.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1928	.1743	.2225	.0933	.3172
H	-.0081	-.0901	.1440	-.4880	.4422
T	-.1923	-.1217	.0078	.0055	.3007
.150	1.2211	-.4350	.3883	-.2267	.0522
.440	.5000	.5734	-.1121	.0487	-.0101
.680	.2886	.3626	.3902	-.0510	.0096
.870	.2276	.2145	.3027	.2679	-.0128
.980	.1970	.2074	.1741	.2168	.1447

DIP = 25 DEGREES, FC = 3.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1966	.1807	.2254	.0997	.2976
H	-.0100	-.0862	.1304	-.4503	.4160
T	-.1970	-.1154	.0131	.0174	.2819
.150	1.2277	-.4479	.4016	-.2354	.0541
.440	.5148	.5580	-.1112	.0483	-.0099
.680	.2972	.3700	.3730	-.0496	.0093
.870	.2328	.2228	.3033	.2532	-.0121
.980	.2016	.2125	.1818	.2687	.1354

DIP = 25 DEGREES, FC = 10.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1933	.1770	.2222	.1063	.3012
H	-.0095	-.0848	.1276	-.4553	.4220
T	-.1953	-.1188	.0068	.0219	.2854
.150	1.2290	-.4520	.4069	-.2386	.0547
.440	.5076	.5676	-.1150	.0502	-.0103
.680	.2909	.3690	.3821	-.0518	.0098
.870	.2287	.2172	.3082	.2584	-.0125
.980	.1981	.2082	.1800	.2771	.1366

DIP = 35 DEGREES, FC = 2.5 FH

F/FC =	.150	.440	.680	.870	.980
HM	.2002	.1876	.2280	.1107	.2735
H	-.0122	-.0804	.1115	-.4030	.3841
T	-.2027	-.1084	.0177	.0348	.2587
.150	1.2368	-.4665	.4217	-.2488	.0568
.440	.5316	.5411	-.1112	.0484	-.0099
.680	.3061	.3793	.3540	-.0485	.0091
.870	.2379	.2315	.3061	.2359	-.0113
.980	.2060	.2174	.1913	.2615	.1238

DIP = 35 DEGREES, FC = 6.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1955	.1823	.2241	.1195	.2786
H	-.0114	-.0788	.1080	-.4103	.3925
T	-.2003	-.1135	.0094	.0408	.2636
.150	1.2389	-.4729	.4299	-.2536	.0577
.440	.5214	.5546	-.1167	.0512	-.0104
.680	.2971	.3783	.3665	-.0516	.0097
.870	.2319	.2239	.3130	.2432	-.0119
.980	.2010	.2114	.1891	.2729	.1256

DIP = 35 DEGREES, FC = 12.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1935	.1791	.2197	.1216	.2860
H	-.0106	-.0790	.1115	-.4252	.4033
T	-.1977	-.1159	.0027	.0399	.2709
.150	1.2380	-.4711	.4272	-.2511	.0570
.440	.5127	.5643	-.1183	.0517	-.0105
.680	.2920	.3737	.3778	-.0536	.0100
.870	.2293	.2178	.3142	.2512	-.0125
.980	.1987	.2082	.1837	.2806	.1288

DIP = 45 DEGREES, FC = 2.4 FH

F/FC =	.150	.440	.680	.870	.980
HM	.2035	.1950	.2300	.1271	.2445
H	-.0147	-.0728	.0870	-.3459	.3462
T	-.2095	-.1005	.0205	.0587	.2307
.150	1.2497	-.4936	.4524	-.2694	.0609
.440	.5502	.5229	-.1122	.0490	-.0099
.680	.3150	.3904	.3339	-.0483	.0090
.870	.2427	.2402	.3113	.2164	-.0106
.980	.2102	.2221	.2023	.2558	.1096

DIP = 45 DEGREES, FC = 4.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1990	.1900	.2268	.1346	.2496
H	-.0138	-.0717	.0847	-.3536	.3544
T	-.2072	-.1054	.0133	.0636	.2357
.150	1.2517	-.4996	.4600	-.2738	.0617
.440	.5406	.5356	-.1174	.0516	-.0104
.680	.3065	.3896	.3454	-.0510	.0095
.870	.2370	.2334	.3175	.2232	-.0111
.980	.2054	.2166	.2004	.2661	.1115

DIP = 45 DEGREES, FC = 6.7 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1960	.1855	.2223	.1390	.2573
H	-.0129	-.0713	.0865	-.3682	.3659
T	-.2041	-.1092	.0054	.0647	.2432
.150	1.2518	-.5006	.4605	-.2730	.0613
.440	.5300	.5482	-.1206	.0531	-.0107
.680	.2994	.3855	.3587	-.0536	.0100
.870	.2330	.2258	.3209	.2320	-.0117
.980	.2020	.2119	.1954	.2759	.1148

DIP = 45 DEGREES, FC = 12.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1940	.1819	.2173	.1384	.2685
H	-.0118	-.0726	.0939	-.3909	.3814
T	-.2004	-.1122	-.0014	.0600	.2540
.150	1.2482	-.4928	.4506	-.2655	.0595
.440	.5191	.5598	-.1215	.0532	-.0107
.680	.2938	.3789	.3724	-.0554	.0103
.870	.2303	.2189	.3205	.2427	-.0123
.980	.1996	.2085	.1880	.2841	.1199

DIP = 55 DEGREES, FC = 2.3 FH

F/FC =	.150	.440	.680	.870	.980
HM	.2070	.2029	.2313	.1462	.2126
H	-.0172	-.0641	.0607	-.2842	.3048
T	-.2169	-.0911	.0217	.0863	.2000
.150	1.2660	-.5278	.4924	-.2964	.0658
.440	.5702	.5032	-.1131	.0494	-.0097
.680	.3245	.4016	.3135	-.0486	.0090
.870	.2479	.2489	.3173	.1958	-.0098
.980	.2147	.2269	.2134	.2511	.0939

DIP = 55 DEGREES, FC = 3.5 FH

F/FC =	.150	.440	.680	.870	.980
HM	.2019	.1971	.2281	.1535	.2194
H	-.0161	-.0635	.0596	-.2954	.3153
T	-.2140	-.0970	.0141	.0902	.2066
.150	1.2677	-.5335	.4993	-.2998	.0664
.440	.5586	.5182	-.1188	.0523	-.0103
.680	.3147	.4003	.3270	-.0515	.0095
.870	.2414	.2411	.3239	.2041	-.0104
.980	.2091	.2208	.2110	.2625	.0967

DIP = 55 DEGREES, FC = 5.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1987	.1924	.2243	.1575	.2270
H	-.0151	-.0633	.0614	-.3094	.3264
T	-.2109	-.1013	.0071	.0910	.2140
.150	1.2676	-.5341	.4993	-.2986	.0659
.440	.5482	.5309	-.1223	.0539	-.0106
.680	.3074	.3970	.3396	-.0539	.0099
.870	.2371	.2339	.3275	.2124	-.0109
.980	.2056	.2159	.2068	.2718	.0999

DIP = 55 DEGREES, FC = 7.5 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1962	.1881	.2195	.1589	.2373
H	-.0140	-.0640	.0667	-.3297	.3409
T	-.2071	-.1051	-.0001	.0883	.2240
.150	1.2649	-.5287	.4916	-.2923	.0645
.440	.5368	.5438	-.1246	.0548	-.0108
.680	.3008	.3913	.3537	-.0561	.0103
.870	.2337	.2263	.3288	.2227	-.0116
.980	.2027	.2117	.2003	.2808	.1045

DIP = 55 DEGREES, FC = 12.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1944	.1845	.2148	.1556	.2507
H	-.0128	-.0662	.0768	-.3570	.3591
T	-.2031	-.1085	-.0059	.0804	.2370
.150	1.2588	-.5152	.4748	-.2803	.0619
.440	.5251	.5558	-.1248	.0547	-.0108
.680	.2953	.3839	.3675	-.0573	.0105
.870	.2312	.2197	.3268	.2346	-.0122
.980	.2004	.2088	.1920	.2880	.1108

DIP = 66 DEGREES, FC = 2.2 FH

F/FC =	.150	.440	.680	.870	.980
HM	.2109	.2115	.2328	.1677	.1771
H	-.0195	-.0544	.0329	-.2176	.2586
T	-.2250	-.0800	.0219	.1175	.1657
.150	1.2866	-.5707	.5443	-.3317	.0715
.440	.5921	.4812	-.1132	.0493	-.0094
.680	.3350	.4131	.2924	-.0495	.0089
.870	.2535	.2578	.3241	.1736	-.0089
.980	.2194	.2321	.2252	.2468	.0765

DIP = 66 DEGREES, FC = 3.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.2058	.2057	.2303	.1737	.1845
H	-.0183	-.0544	.0331	-.2299	.2695
T	-.2220	-.0862	.0154	.1200	.1728
.150	1.2875	-.5747	.5489	-.3333	.0716
.440	.5804	.4962	-.1188	.0521	-.0099
.680	.3252	.4120	.3054	-.0520	.0094
.870	.2470	.2504	.3302	.1819	-.0095
.980	.2139	.2262	.2230	.2573	.0796

DIP = 66 DEGREES, FC = 4.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.2022	.2005	.2270	.1778	.1925
H	-.0172	-.0545	.0349	-.2445	.2813
T	-.2187	-.0914	.0088	.1205	.1807
.150	1.2869	-.5749	.5480	-.3311	.0711
.440	.5692	.5101	-.1231	.0543	-.0104
.680	.3170	.4092	.3185	-.0544	.0098
.870	.2421	.2430	.3344	.1906	-.0100
.980	.2099	.2209	.2193	.2669	.0831

DIP = 66 DEGREES, FC = 5.5 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1991	.1955	.2225	.1799	.2029
H	-.0161	-.0551	.0393	-.2642	.2960
T	-.2147	-.0963	.0017	.1185	.1908
.150	1.2843	-.5700	.5406	-.3246	.0697
.440	.5568	.5246	-.1265	.0558	-.0107
.680	.3093	.4043	.3330	-.0567	.0102
.870	.2379	.2350	.3368	.2009	-.0106
.980	.2064	.2159	.2135	.2765	.0877

DIP = 66 DEGREES, FC = 8.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1966	.1907	.2169	.1790	.2168
H	-.0149	-.0567	.0478	-.2914	.3153
T	-.2102	-.1008	-.0056	.1123	.2043
.150	1.2786	-.5578	.5244	-.3125	.0673
.440	.5434	.5394	-.1285	.0566	-.0109
.680	.3022	.3970	.3489	-.0586	.0105
.870	.2345	.2268	.3365	.2137	-.0114
.980	.2034	.2116	.2050	.2859	.0941

DIP = 66 DEGREES, FC = 12.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1948	.1869	.2119	.1738	.2326
H	-.0137	-.0597	.0602	-.3233	.3365
T	-.2057	-.1047	-.0109	.1016	.2197
.150	1.2699	-.5390	.5012	-.2962	.0640
.440	.5309	.5521	-.1284	.0563	-.0109
.680	.2967	.3885	.3635	-.0594	.0107
.870	.2319	.2199	.3332	.2271	-.0121
.980	.2011	.2087	.1955	.2931	.1016

DIP = 80 DEGREES, FC = 2.2 FH

F/FC =	.150	.440	.680	.870	.980
HM	.2141	.2195	.2353	.1924	.1387
H	-.0207	-.0437	.0054	-.1492	.2081
T	-.2331	-.0686	.0205	.1527	.1285
.150	1.3120	-.6232	.6123	-.3775	.0764
.440	.6132	.4603	-.1139	.0492	-.0087
.680	.3439	.4249	.2738	-.0515	.0088
.870	.2577	.2652	.3334	.1516	-.0079
.980	.2231	.2361	.2380	.2449	.0578

DIP = 80 DEGREES, FC = 3.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.2082	.2124	.2326	.1982	.1486
H	-.0193	-.0442	.0071	-.1661	.2226
T	-.2287	-.0767	.0134	.1538	.1382
.150	1.3116	-.6251	.6116	-.3739	.0758
.440	.5975	.4800	-.1208	.0527	-.0094
.680	.3315	.4233	.2902	-.0543	.0093
.870	.2499	.2563	.3403	.1621	-.0086
.980	.2166	.2292	.2350	.2571	.0621

DIP = 80 DEGREES, FC = 3.9 FH

F/FC =	.150	.440	.680	.870	.980
HM	.2045	.2070	.2289	.2014	.1582
H	-.0182	-.0448	.0099	-.1832	.2363
T	-.2245	-.0828	.0070	.1527	.1476
.150	1.3097	-.6215	.6038	-.3666	.0746
.440	.5842	.4961	-.1254	.0550	-.0099
.680	.3224	.4201	.3044	-.0566	.0097
.870	.2448	.2485	.3441	.1717	-.0091
.980	.2125	.2238	.2306	.2666	.0664

DIP = 80 DEGREES, FC = 5.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.2020	.2023	.2243	.2027	.1687
H	-.0173	-.0457	.0142	-.2024	.2512
T	-.2204	-.0878	.0008	.1496	.1579
.150	1.3057	-.6130	.5912	-.3569	.0731
.440	.5723	.5101	-.1287	.0566	-.0103
.680	.3154	.4156	.3174	-.0585	.0101
.870	.2413	.2413	.3456	.1815	-.0097
.980	.2096	.2193	.2247	.2752	.0711

DIP = 80 DEGREES, FC = 6.5 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1998	.1977	.2185	.2022	.1818
H	-.0165	-.0471	.0207	-.2266	.2695
T	-.2165	-.0926	-.0056	.1441	.1707
.150	1.2990	-.5994	.5742	-.3453	.0714
.440	.5608	.5233	-.1315	.0581	-.0107
.680	.3095	.4095	.3307	-.0602	.0105
.870	.2385	.2340	.3449	.1930	-.0105
.980	.2072	.2151	.2167	.2840	.0771

DIP = 80 DEGREES, FC = 9.0 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1971	.1923	.2114	.1984	.2007
H	-.0155	-.0499	.0324	-.2624	.2954
T	-.2122	-.0978	-.0127	.1336	.1890
.150	1.2885	-.5792	.5515	-.3299	.0691
.440	.5479	.5378	-.1340	.0594	-.0111
.680	.3036	.4005	.3466	-.0614	.0108
.870	.2353	.2255	.3418	.2088	-.0114
.980	.2042	.2106	.2055	.2940	.0858

DIP = 80 DEGREES, FC = 12.5 FH

F/FC =	.150	.440	.680	.870	.980
HM	.1945	.1874	.2064	.1907	.2210
H	-.0144	-.0541	.0479	-.3021	.3227
T	-.2083	-.1020	-.0179	.1195	.2087
.150	1.2782	-.5597	.5289	-.3139	.0665
.440	.5361	.5501	-.1341	.0591	-.0112
.680	.2981	.3906	.3622	-.0618	.0109
.870	.2321	.2180	.3375	.2246	-.0122
.980	.2011	.2071	.1949	.3015	.0955

Table 2. Six-point coefficients for calculations on the daytime F layer, for critical frequencies between $2f_H$ and $20f_H$.

DIP = 13 DEGREES, FC = 8.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1761	.1129	.1409	.2294	.0137	.3271
H	-.0367	.0617	-.1770	.2471	-.6247	.5296
T	-.2072	.0095	-.1980	.1680	-.1012	.3288
.150	1.4073	-.8152	.7163	-.5399	.3100	-.0784
.350	.6242	.3390	.0829	-.0855	.0532	-.0138
.550	.3557	.2175	.5157	-.1470	.0772	-.0190
.750	.2619	.0941	.3792	.2721	-.0070	-.0003
.900	.2082	.1032	.2156	.2416	.2479	-.0165
.980	.1903	.0919	.2042	.1469	.2436	.1230

DIP = 25 DEGREES, FC = 4.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1799	.1116	.1515	.2254	.0256	.3060
H	-.0358	.0559	-.1669	.2272	-.5780	.4976
T	-.2114	.0106	-.1913	.1718	-.0888	.3091
.150	1.4191	-.8412	.7413	-.5610	.3234	-.0816
.350	.6422	.3167	.0905	-.0917	.0570	-.0147
.550	.3653	.2211	.5031	-.1488	.0787	-.0194
.750	.2672	.0982	.3852	.2547	-.0043	-.0009
.900	.2123	.1039	.2253	.2382	.2364	-.0162
.980	.1939	.0927	.2114	.1498	.2380	.1143

DIP = 25 DEGREES, FC = 14.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1773	.1099	.1473	.2242	.0308	.3105
H	-.0353	.0566	-.1667	.2252	-.5858	.5060
T	-.2092	.0097	-.1965	.1676	-.0852	.3136
.150	1.4209	-.8466	.7489	-.5680	.3271	-.0823
.350	.6333	.3285	.0859	-.0889	.0555	-.0143
.550	.3581	.2207	.5123	-.1513	.0798	-.0196
.750	.2631	.0933	.3871	.2630	-.0061	-.0005
.900	.2091	.1017	.2201	.2443	.2414	-.0166
.980	.1911	.0908	.2069	.1497	.2458	.1157

DIP = 35 DEGREES, FC = 3.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1845	.1097	.1646	.2199	.0418	.2795
H	-.0347	.0487	-.1539	.2015	-.5193	.4578
T	-.2167	.0125	-.1839	.1762	-.0727	.2845
.150	1.4347	-.8762	.7760	-.5909	.3425	-.0861
.350	.6646	.2886	.1007	-.1001	.0623	-.0160
.550	.3771	.2250	.4886	-.1519	.0811	-.0199
.750	.2736	.1025	.3931	.2333	-.0009	-.0017
.900	.2174	.1046	.2370	.2343	.2226	-.0160
.980	.1982	.0934	.2200	.1532	.2319	.1033

DIP = 35 DEGREES, FC = 7.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1808	.1072	.1593	.2190	.0491	.2847
H	-.0339	.0493	-.1535	.1979	-.5273	.4675
T	-.2139	.0109	-.1909	.1718	-.0675	.2896
.150	1.4381	-.8864	.7898	-.6036	.3497	-.0876
.350	.6536	.3034	.0949	-.0968	.0605	-.0155
.550	.3672	.2258	.4998	-.1552	.0827	-.0202
.750	.2676	.0966	.3967	.2433	-.0030	-.0012
.900	.2127	.1014	.2310	.2428	.2286	-.0165
.980	.1941	.0906	.2145	.1542	.2419	.1047

DIP = 35 DEGREES, FC = 15.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1788	.1067	.1542	.2172	.0494	.2936
H	-.0340	.0514	-.1557	.2031	-.5470	.4822
T	-.2110	.0101	-.1946	.1656	-.0680	.2979
.150	1.4352	-.8793	.7822	-.5963	.3441	-.0860
.350	.6418	.3187	.0888	-.0923	.0577	-.0148
.550	.3606	.2229	.5101	-.1560	.0826	-.0201
.750	.2646	.0921	.3946	.2547	-.0052	-.0007
.900	.2102	.1003	.2240	.2466	.2358	-.0169
.980	.1921	.0894	.2100	.1510	.2495	.1080

DIP = 45 DEGREES, FC = 3.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1886	.1062	.1789	.2122	.0659	.2482
H	-.0330	.0398	-.1372	.1673	-.4483	.4115
T	-.2225	.0147	-.1782	.1805	-.0502	.2556
.150	1.4570	-.9289	.8311	-.6400	.3739	-.0932
.350	.6889	.2576	.1129	-.1112	.0694	-.0177
.550	.3880	.2296	.4759	-.1581	.0855	-.0209
.750	.2791	.1054	.4050	.2104	.0027	-.0025
.900	.2216	.1041	.2493	.2331	.2080	-.0160
.980	.2016	.0931	.2286	.1577	.2292	.0898

DIP = 45 DEGREES, FC = 5.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1850	.1038	.1739	.2116	.0719	.2539
H	-.0323	.0407	-.1376	.1655	-.4582	.4218
T	-.2197	.0131	-.1848	.1764	-.0461	.2611
.150	1.4601	-.9378	.8431	-.6508	.3798	-.0944
.350	.6779	.2725	.1070	-.1076	.0673	-.0171
.550	.3784	.2303	.4866	-.1609	.0867	-.0211
.750	.2733	.0998	.4080	.2203	.0006	-.0021
.900	.2171	.1010	.2436	.2408	.2140	-.0164
.980	.1977	.0904	.2235	.1583	.2385	.0916

DIP = 45 DEGREES, FC = 9.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1822	.1026	.1676	.2098	.0739	.2639
H	-.0321	.0431	-.1397	.1697	-.4795	.4386
T	-.2160	.0117	-.1901	.1692	-.0453	.2705
.150	1.4584	-.9335	.8389	-.6460	.3753	-.0929
.350	.6633	.2920	.0988	-.1016	.0636	-.0162
.550	.3692	.2281	.4992	-.1623	.0868	-.0210
.750	.2688	.0938	.4071	.2339	-.0021	-.0015
.900	.2135	.0989	.2354	.2466	.2225	-.0170
.980	.1949	.0883	.2177	.1556	.2483	.0952

DIP = 45 DEGREES, FC = 16.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1803	.1034	.1612	.2095	.0693	.2763
H	-.0325	.0463	-.1443	.1805	-.5079	.4579
T	-.2126	.0105	-.1926	.1626	-.0496	.2819
.150	1.4501	-.9130	.8161	-.6250	.3611	-.0893
.350	.6496	.3100	.0913	-.0953	.0596	-.0151
.550	.3627	.2247	.5088	-.1609	.0853	-.0206
.750	.2659	.0904	.4019	.2473	-.0047	-.0009
.900	.2112	.0988	.2273	.2492	.2307	-.0172
.980	.1930	.0880	.2128	.1518	.2543	.1001

DIP = 55 DEGREES, FC = 3.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1927	.1018	.1948	.2023	.0942	.2142
H	-.0310	.0306	-.1190	.1302	-.3727	.3620
T	-.2288	.0181	-.1736	.1850	-.0252	.2245
.150	1.4846	-.9954	.9019	-.7039	.4149	-.1020
.350	.7148	.2236	.1281	-.1254	.0786	-.0197
.550	.3992	.2330	.4653	-.1668	.0914	-.0220
.750	.2847	.1069	.4188	.1863	.0068	-.0035
.900	.2257	.1027	.2622	.2319	.1937	-.0162
.980	.2051	.0919	.2381	.1612	.2287	.0751

DIP = 55 DEGREES, FC = 4.3 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1892	.0997	.1895	.2022	.0985	.2209
H	-.0304	.0319	-.1202	.1304	-.3852	.3735
T	-.2258	.0162	-.1797	.1808	-.0223	.2309
.150	1.4865	-1.0011	.9099	-.7108	.4180	-.1025
.350	.7031	.2395	.1214	-.1209	.0759	-.0190
.550	.3896	.2338	.4754	-.1687	.0920	-.0221
.750	.2790	.1018	.4209	.1967	.0046	-.0030
.900	.2213	.0998	.2565	.2390	.1999	-.0166
.980	.2014	.0893	.2331	.1615	.2374	.0773

DIP = 55 DEGREES, FC = 6.5 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1862	.0984	.1828	.2015	.1004	.2307
H	-.0302	.0342	-.1226	.1344	-.4056	.3898
T	-.2220	.0144	-.1852	.1741	-.0213	.2400
.150	1.4849	-.9968	.9057	-.7059	.4131	-.1010
.350	.6884	.2596	.1124	-.1141	.0716	-.0179
.550	.3793	.2328	.4873	-.1697	.0918	-.0220
.750	.2740	.0959	.4205	.2101	.0018	-.0024
.900	.2175	.0976	.2484	.2455	.2080	-.0170
.980	.1982	.0872	.2271	.1599	.2467	.0809

DIP = 55 DEGREES, FC = 10.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1839	.0985	.1759	.2009	.0978	.2431
H	-.0304	.0373	-.1265	.1432	-.4329	.4093
T	-.2181	.0127	-.1887	.1666	-.0238	.2513
.150	1.4780	-.9790	.8856	-.6865	.3994	-.0975
.350	.6730	.2805	.1029	-.1064	.0667	-.0167
.550	.3716	.2300	.4983	-.1688	.0905	-.0216
.750	.2703	.0914	.4166	.2246	-.0011	-.0017
.900	.2146	.0967	.2396	.2498	.2165	-.0173
.980	.1959	.0863	.2212	.1566	.2542	.0857

DIP = 55 DEGREES, FC = 16.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1818	.1000	.1685	.2017	.0897	.2583
H	-.0310	.0412	-.1327	.1577	-.4677	.4326
T	-.2144	.0110	-.1906	.1598	-.0310	.2652
.150	1.4657	-.9484	.8518	-.6557	.3793	-.0927
.350	.6577	.3007	.0939	-.0987	.0618	-.0155
.550	.3651	.2266	.5072	-.1660	.0882	-.0211
.750	.2674	.0889	.4095	.2394	-.0040	-.0011
.900	.2123	.0973	.2309	.2516	.2255	-.0175
.980	.1940	.0865	.2159	.1524	.2592	.0920

DIP = 66 DEGREES, FC = 2.7 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1985	.0973	.2142	.1904	.1248	.1748
H	-.0289	.0207	-.0988	.0907	-.2876	.3039
T	-.2369	.0239	-.1679	.1919	.0006	.1884
.150	1.5167	-1.0750	.9866	-.7825	.4662	-.1119
.350	.7470	.1794	.1498	-.1455	.0916	-.0224
.550	.4149	.2343	.4529	-.1776	.0989	-.0233
.750	.2929	.1092	.4336	.1566	.0125	-.0047
.900	.2318	.1016	.2780	.2275	.1775	-.0163
.980	.2102	.0910	.2505	.1633	.2269	.0581

DIP = 66 DEGREES, FC = 3.7 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1943	.0951	.2078	.1914	.1284	.1831
H	-.0283	.0224	-.1009	.0923	-.3033	.3179
T	-.2332	.0211	-.1746	.1873	.0032	.1962
.150	1.5180	-1.0786	.9925	-.7871	.4671	-.1118
.350	.7331	.1989	.1409	-.1391	.0876	-.0214
.550	.4033	.2363	.4636	-.1788	.0988	-.0232
.750	.2860	.1038	.4356	.1689	.0098	-.0041
.900	.2265	.0985	.2715	.2358	.1844	-.0167
.980	.2057	.0881	.2446	.1643	.2361	.0611

DIP = 66 DEGREES, FC = 5.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1912	.0937	.2012	.1915	.1298	.1926
H	-.0281	.0246	-.1035	.0961	-.3226	.3334
T	-.2294	.0188	-.1799	.1813	.0043	.2049
.150	1.5165	-1.0741	.9879	-.7814	.4615	-.1103
.350	.7186	.2192	.1312	-.1315	.0828	-.0202
.550	.3932	.2364	.4742	-.1791	.0983	-.0230
.750	.2807	.0985	.4356	.1818	.0070	-.0035
.900	.2225	.0962	.2642	.2425	.1917	-.0170
.980	.2024	.0860	.2388	.1636	.2446	.0646

DIP = 66 DEGREES, FC = 7.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1883	.0933	.1934	.1914	.1284	.2051
H	-.0282	.0276	-.1072	.1036	-.3491	.3533
T	-.2251	.0164	-.1843	.1736	.0030	.2164
.150	1.5107	-1.0582	.9700	-.7635	.4482	-.1072
.350	.7020	.2424	.1198	-.1223	.0769	-.0188
.550	.3835	.2351	.4855	-.1784	.0969	-.0227
.750	.2761	.0933	.4328	.1969	.0037	-.0028
.900	.2190	.0947	.2550	.2482	.2003	-.0173
.980	.1996	.0846	.2321	.1613	.2529	.0694

DIP = 66 DEGREES, FC = 10.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1859	.0941	.1853	.1917	.1232	.2198
H	-.0285	.0310	-.1123	.1153	-.3817	.3762
T	-.2209	.0141	-.1871	.1654	-.0013	.2297
.150	1.5000	-1.0306	.9394	-.7345	.4283	-.1025
.350	.6852	.2656	.1085	-.1128	.0708	-.0174
.550	.3753	.2324	.4959	-.1762	.0948	-.0222
.750	.2723	.0894	.4270	.2130	.0004	-.0021
.900	.2161	.0945	.2452	.2522	.2096	-.0176
.980	.1972	.0842	.2256	.1578	.2598	.0753

DIP = 66 DEGREES, FC = 15.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1836	.0961	.1770	.1933	.1133	.2366
H	-.0293	.0354	-.1195	.1319	-.4200	.4015
T	-.2166	.0119	-.1886	.1577	-.0093	.2450
.150	1.4840	-.9915	.8966	-.6942	.4016	-.0964
.350	.6681	.2885	.0979	-.1032	.0646	-.0159
.550	.3680	.2287	.5053	-.1720	.0915	-.0215
.750	.2690	.0871	.4184	.2298	-.0030	-.0013
.900	.2135	.0955	.2354	.2544	.2190	-.0177
.980	.1952	.0847	.2196	.1538	.2644	.0824

DIP = 80 DEGREES, FC = 2.6 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.2038	.0915	.2349	.1777	.1600	.1320
H	-.0260	.0112	-.0772	.0498	-.1985	.2406
T	-.2449	.0310	-.1647	.2008	.0287	.1490
.150	1.5541	-1.1738	1.0938	-.8851	.5324	-.1215
.350	.7795	.1329	.1750	-.1702	.1077	-.0250
.550	.4291	.2346	.4442	-.1920	.1085	-.0244
.750	.2999	.1093	.4516	.1263	.0189	-.0059
.900	.2367	.0989	.2946	.2241	.1620	-.0163
.980	.2145	.0885	.2636	.1654	.2281	.0399

DIP = 80 DEGREES, FC = 3.3 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1999	.0897	.2289	.1793	.1619	.1402
H	-.0256	.0130	-.0797	.0522	-.2139	.2540
T	-.2410	.0279	-.1702	.1961	.0308	.1565
.150	1.5550	-1.1742	1.0943	-.8826	.5280	-.1206
.350	.7654	.1531	.1650	-.1620	.1023	-.0238
.550	.4176	.2373	.4534	-.1915	.1073	-.0241
.750	.2932	.1048	.4531	.1384	.0159	-.0053
.900	.2318	.0964	.2886	.2318	.1680	-.0166
.980	.2104	.0862	.2583	.1666	.2356	.0429

DIP = 80 DEGREES, FC = 4.2 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1969	.0886	.2222	.1801	.1626	.1496
H	-.0254	.0151	-.0825	.0560	-.2325	.2693
T	-.2370	.0249	-.1747	.1898	.0319	.1651
.150	1.5539	-1.1675	1.0851	-.8714	.5187	-.1188
.350	.7504	.1748	.1536	-.1526	.0962	-.0225
.550	.4070	.2389	.4625	-.1905	.1059	-.0238
.750	.2878	.1002	.4528	.1511	.0128	-.0047
.900	.2279	.0944	.2815	.2385	.1745	-.0168
.980	.2073	.0844	.2524	.1665	.2430	.0465

DIP = 80 DEGREES, FC = 5.5 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1941	.0883	.2138	.1803	.1614	.1621
H	-.0256	.0177	-.0861	.0624	-.2578	.2893
T	-.2325	.0215	-.1786	.1816	.0315	.1765
.150	1.5482	-1.1488	1.0627	-.8499	.5037	-.1159
.350	.7332	.2000	.1398	-.1413	.0891	-.0209
.550	.3968	.2395	.4720	-.1889	.1042	-.0235
.750	.2831	.0957	.4499	.1659	.0094	-.0040
.900	.2245	.0931	.2723	.2446	.1827	-.0172
.980	.2044	.0832	.2451	.1649	.2511	.0513

DIP = 80 DEGREES, FC = 7.5 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1910	.0890	.2032	.1805	.1568	.1795
H	-.0261	.0211	-.0914	.0736	-.2941	.3168
T	-.2277	.0177	-.1823	.1719	.0278	.1926
.150	1.5353	-1.1142	1.0267	-.8185	.4328	-.1120
.350	.7139	.2284	.1244	-.1287	.0813	-.0193
.550	.3871	.2385	.4824	-.1869	.1022	-.0232
.750	.2787	.0915	.4436	.1840	.0054	-.0032
.900	.2210	.0926	.2602	.2503	.1934	-.0176
.980	.2015	.0826	.2361	.1616	.2601	.0581

DIP = 80 DEGREES, FC = 10.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1879	.0901	.1931	.1815	.1492	.1982
H	-.0267	.0247	-.0979	.0884	-.3342	.3456
T	-.2238	.0149	-.1855	.1638	.0209	.2098
.150	1.5190	-1.0766	.9909	-.7870	.4615	-.1079
.350	.6972	.2522	.1123	-.1187	.0749	-.0179
.550	.3796	.2356	.4923	-.1847	.1001	-.0229
.750	.2749	.0882	.4358	.2017	.0019	-.0025
.900	.2178	.0927	.2489	.2543	.2043	-.0180
.980	.1986	.0825	.2281	.1577	.2674	.0656

DIP = 80 DEGREES, FC = 15.0 FH

F/FC =	.150	.350	.550	.750	.900	.980
HM	.1839	.0920	.1810	.1851	.1331	.2249
H	-.0278	.0310	-.1098	.1147	-.3942	.3861
T	-.2191	.0130	-.1902	.1557	.0061	.2345
.150	1.4976	-1.0294	.9449	-.7415	.4299	-.1014
.350	.6769	.2787	.1013	-.1085	.0681	-.0165
.550	.3707	.2285	.5070	-.1804	.0963	-.0222
.750	.2698	.0839	.4247	.2255	-.0024	-.0016
.900	.2134	.0929	.2357	.2575	.2189	-.0183
.980	.1950	.0821	.2199	.1523	.2740	.0767

Table 3. Coefficients including an extra-ordinary ray correction for underlying ionization, for critical frequencies between about $2f_H$ and $10f_H$.

DIP= 5 DEGREES, FC= 2.5 FH, F1= 1.00 FH, FX= 1.62 FH

F/FC =	(F1)	(FX)	.600	.750	.900	.980
HM	.5640	-.2724	.1706	.1837	.0229	.3313
H	-.0171	.0449	-.2445	.3424	-.6729	.5472
T	-.4706	.3041	-.3165	.2870	-.1485	.3445
F1	1.6[illegible]	-.8074	.2335	-.1784	.0806	-.0191
.600	1.0067	-.4961	.6686	-.2651	.1090	-.0251
.750	.7558	-.3936	.4789	.1272	.0424	-.0108
.900	.6303	-.3178	.2716	.1603	.2776	-.0219
.980	.5741	-.2903	.2579	.0673	.2704	.1206

DIP= 5 DEGREES, FC= 5.0 FH, F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.500	.700	.880	.980
HM	.3776	-.1267	.1233	.2416	.0560	.3282
H	-.0663	.1073	-.2209	.2831	-.5899	.4866
T	-.4123	.2270	-.1884	.1062	-.0513	.3189
F1	1.9556	-.9813	.0431	-.0263	.0113	-.0024
.500	.8545	-.3238	.5507	-.1152	.0425	-.0087
.700	.5874	-.2631	.3917	.2954	-.0131	.0017
.880	.4511	-.1741	.1911	.2797	.2657	-.0135
.980	.4038	-.1656	.2023	.1446	.2754	.1395

DIP= 15 DEGREES, FC= 2.5 FH, F1= 1.00 FH, FX= 1.62 FH

F/FC =	(F1)	(FX)	.600	.750	.900	.980
HM	.5874	-.2817	.1545	.1966	.0198	.3234
H	-.0221	.0458	-.2346	.3294	-.6504	.5319
T	-.4921	.3131	-.2876	.2681	-.1348	.3334
F1	1.7490	-.8353	.1722	-.1311	.0591	-.0140
.600	1.0532	-.5137	.6209	-.2335	.0949	-.0218
.750	.7916	-.4070	.4477	.1437	.0323	-.0083
.900	.6583	-.3285	.2504	.1761	.2634	-.0197
.980	.5994	-.3000	.2382	.0848	.2588	.1187

DIP= 15 DEGREES, FC= 5.0 FH, F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.500	.700	.880	.980
HM	.3871	-.1343	.1265	.2416	.0599	.3192
H	-.0708	.1086	-.2138	.2723	-.5700	.4737
T	-.4266	.2376	-.1822	.1059	-.0448	.3101
F1	2.0138	-1.0336	.0333	-.0203	.0087	-.0019
.500	.8808	-.3417	.5410	-.1133	.0418	-.0085
.700	.6061	-.2757	.3915	.2895	-.0132	.0018
.880	.4636	-.1833	.1937	.2799	.2594	-.0132
.980	.4150	-.1738	.2030	.1485	.2718	.1355

DIP= 25 DEGREES, FC= 2.5 FH, F1= 1.00 FH, FX= 1.62 FH

F/FC =	(F1)	(FX)	.600	.750	.900	.980
HM	.6277	-.2953	.1192	.2260	.0140	.3084
H	-.0316	.0469	-.2149	.3023	-.6052	.5024
T	-.5340	.3282	-.2298	.2301	-.1059	.3114
F1	1.8596	-.8817	.0424	-.0305	.0133	-.0031
.600	1.1357	-.5414	.5228	-.1677	.0653	-.0147
.750	.8558	-.4272	.3837	.1800	.0108	-.0032
.900	.7074	-.3443	.2055	.2120	.2345	-.0150
.980	.6435	-.3140	.1958	.1238	.2360	.1150

DIP= 25 DEGREES, FC= 4.0 FH, F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.540	.720	.890	.980
HM	.5290	-.2445	.1268	.2354	.0497	.3036
H	-.0537	.0743	-.1931	.2528	-.5513	.4710
T	-.5301	.3196	-.1747	.1368	-.0500	.2983
F1	2.1200	-1.1325	.0222	-.0146	.0064	-.0015
.540	1.0914	-.5215	.5215	-.1298	.0492	-.0108
.720	.7751	-.3983	.3838	.2448	-.0058	.0004
.890	.6141	-.2990	.1968	.2579	.2446	-.0144
.980	.5519	-.2725	.1957	.1501	.2534	.1215

DIP= 25 DEGREES, FC= 7.0 FH, F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.450	.680	.870	.980
HM	.2170	.0018	.1402	.2482	.0897	.3031
H	-.1708	.2177	-.2036	.2372	-.5139	.4334
T	-.3517	.1806	-.1770	.0690	-.0123	.2914
F1	2.1173	-1.1199	.0039	-.0020	.0009	-.0002
.450	.6195	-.0851	.5265	-.0871	.0323	-.0062
.680	.4515	-.1658	.4075	.3275	-.0245	.0037
.870	.2922	-.0462	.2008	.3124	.2521	-.0113
.980	.2792	-.0755	.2133	.1671	.2812	.1346

DIP= 35 DEGREES, FC= 2.5 FH. F1= 1.00 FH, FX= 1.62 FH

F/FC =	(F1)	(FX)	.600	.750	.900	.980
HM	.6795	-.3100	.0657	.2701	.0073	.2874
H	-.0438	.0476	-.1866	.2613	-.5397	.4611
T	-.5914	.3461	-.1474	.1739	-.0614	.2802
F1	2.0120	-.9370	-.1541	.1228	-.0573	.0136
.600	1.2457	-.5726	.3806	-.0707	.0212	-.0043
.750	.9391	-.4493	.2918	.2355	-.0216	.0045
.900	.7705	-.3614	.1382	.2681	.1927	-.0082
.980	.7000	-.3291	.1319	.1826	.2045	.1101

DIP= 35 DEGREES, FC= 4.0 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.540	.720	.890	.980
HM	.5591	-.2644	.1203	.2422	.0623	.2805
H	-.0615	.0734	-.1712	.2182	-.4949	.4361
T	-.5721	.3446	-.1448	.1215	-.0229	.2737
F1	2.2653	-1.2353	-.0551	.0387	-.0176	.0040
.540	1.1678	-.5643	.4728	-.1071	.0392	-.0084
.720	.8274	-.4277	.3648	.2463	-.0129	.0021
.890	.6513	-.3215	.1848	.2728	.2252	-.0127
.980	.5850	-.2922	.1814	.1694	.2444	.1120

DIP= 35 DEGREES, FC= 7.0 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.450	.680	.870	.980
HM	.2267	-.0095	.1487	.2432	.1079	.2829
H	-.1732	.2107	-.1822	.2059	-.4683	.4071
T	-.3706	.1931	-.1686	.0649	.0088	.2723
F1	2.2727	-1.2670	-.0089	.0048	-.0021	.0004
.450	.6414	-.0969	.5178	-.0893	.0334	-.0064
.680	.4735	-.1835	.4141	.3171	-.0249	.0038
.870	.3023	-.0551	.2053	.3173	.2414	-.0112
.980	.2896	-.0840	.2141	.1739	.2817	.1247

DIP= 45 DEGREES, FC= 2.4 FH. F1= 1.00 FH, FX= 1.62 FH

F/FC =	(F1)	(FX)	.600	.750	.900	.980
HM	.7796	-.3396	-.0449	.3514	-.0110	.2644
H	-.0554	.0485	-.1568	.2133	-.4610	.4114
T	-.6780	.3772	-.0350	.0963	-.0018	.2413
F1	2.2278	-.9867	-.4720	.3557	-.1628	.0380
.600	1.4492	-.6322	.1242	.1010	-.0557	.0136
.750	1.0882	-.4921	.1232	.3405	-.0772	.0174
.900	.8876	-.3952	.0067	.3688	.1292	.0030
.980	.8052	-.3593	.0099	.2816	.1562	.1064

DIP= 45 DEGREES, FC= 3.5 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.540	.720	.890	.980
HM	.6929	-.3453	.0495	.2851	.0625	.2553
H	-.0607	.0707	-.1588	.1853	-.4301	.3937
T	-.6843	.4194	-.0815	.0884	.0158	.2422
F1	2.4609	-1.3130	-.2495	.1534	-.0667	.0149
.540	1.4621	-.7356	.2821	-.0061	-.0037	.0012
.720	1.0185	-.5417	.2637	.2909	-.0394	.0080
.890	.8037	-.4126	.1024	.3279	.1857	-.0071
.980	.7197	-.3728	.1056	.2245	.2198	.1033

DIP= 45 DEGREES, FC= 6.0 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.470	.690	.880	.980
HM	.3353	-.1017	.1506	.2422	.1177	.2560
H	-.1159	.1337	-.1433	.1512	-.4111	.3854
T	-.4487	.2503	-.1501	.0718	.0283	.2484
F1	2.4531	-1.4257	-.0434	.0247	-.0111	.0024
.470	.8586	-.2868	.4935	-.0933	.0354	-.0074
.690	.5933	-.2728	.4082	.2909	-.0235	.0040
.880	.4136	-.1473	.2016	.3150	.2304	-.0132
.980	.3785	-.1493	.2018	.1933	.2692	.1065

DIP= 55 DEGREES, FC= 2.3 FH. F1= .90 FH, FX= 1.53 FH

F/FC =	(F1)	(FX)	.600	.750	.900	.980
HM	.7250	-.2833	-.0357	.3587	.0041	.2312
H	-.0626	.0398	-.1154	.1540	-.3719	.3561
T	-.6555	.3227	.0406	.0360	.0554	.2006
F1	2.1979	-.9080	-.6058	.4974	-.2367	.0551
.600	1.3661	-.5318	.0858	.1394	-.0785	.0190
.750	1.0266	-.4126	.1070	.3532	-.0960	.0218
.900	.8309	-.3300	.0038	.3923	.0963	.0068
.980	.7528	-.2995	.0045	.3101	.1377	.0945

DIP= 55 DEGREES, FC= 3.0 FH. F1= 1.00 FH, FX= 1.62 FH

F/FC =	(F1)	(FX)	.540	.720	.890	.980
HM	.6312	-.2724	.0484	.2900	.0800	.2227
H	-.0566	.0517	-.1245	.1379	-.3531	.3446
T	-.6246	.3346	-.0355	.0644	.0535	.2077
F1	2.2976	-1.0923	-.3539	.2271	-.1009	.0223
.540	1.3509	-.5841	.2236	.0223	-.0169	.0041
.720	.9350	-.4276	.2397	.2918	-.0490	.0100
.890	.7340	-.3246	.0918	.3440	.1598	-.0051
.980	.6561	-.2926	.0923	.2465	.2079	.0898

DIP= 55 DEGREES, FC= 4.0 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.540	.720	.890	.980
HM	.6287	-.3073	.0968	.2607	.0957	.2254
H	-.0743	.0670	-.1186	.1338	-.3608	.3529
T	-.6709	.3987	-.0650	.0725	.0510	.2137
F1	2.6198	-1.4656	-.2831	.2004	-.0920	.0205
.540	1.3435	-.6554	.3459	-.0420	.0097	-.0016
.720	.9449	-.4891	.3095	.2629	-.0351	.0070
.890	.7353	-.3681	.1444	.3184	.1781	-.0082
.980	.6597	-.3332	.1389	.2188	.2265	.0893

DIP= 55 DEGREES, FC= 6.0 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.470	.690	.880	.980
HM	.3564	-.1225	.1573	.2377	.1403	.2307
H	-.1137	.1229	-.1210	.1173	-.3568	.3514
T	-.4800	.2725	-.1367	.0645	.0558	.2240
F1	2.6618	-1.6107	-.0812	.0467	-.0211	.0046
.470	.9081	-.3211	.4779	-.0929	.0352	-.0072
.690	.6296	-.3016	.4111	.2820	-.0255	.0043
.880	.4355	-.1659	.2028	.3228	.2178	-.0129
.980	.3987	-.1660	.2004	.2014	.2709	.0945

DIP= 66 DEGREES, FC= 2.2 FH. F1= .90 FH, FX= 1.53 FH

F/FC =	(F1)	(FX)	.600	.750	.900	.980
HM	.8413	-.3069	-.1841	.4684	-.0223	.2036
H	-.0766	.0394	-.0749	.0935	-.2765	.2951
T	-.7604	.3480	.1986	-.0764	.1386	.1516
F1	2.4618	-.9412	-1.0423	.8216	-.3875	.0876
.600	1.6071	-.5812	-.2579	.3769	-.1880	.0431
.750	1.2024	-.4472	-.1230	.5044	-.1758	.0393
.900	.9674	-.3566	-.1755	.5314	.0119	.0214
.980	.8751	-.3232	-.1595	.4424	.0747	.0905

DIP= 66 DEGREES, FC= 2.7 FH. F1= 1.00 FH, FX= 1.62 FH

F/FC =	(F1)	(FX)	.570	.740	.900	.980
HM	.7714	-.3219	-.0546	.3692	.0402	.1958
H	-.0728	.0448	-.0766	.0788	-.2754	.3011
T	-.7496	.3800	.1035	-.0100	.1148	.1614
F1	2.5368	-1.1298	-.7418	.5332	-.2603	.0619
.570	1.5706	-.6483	-.0368	.1923	-.1026	.0249
.740	1.1190	-.4803	.0635	.3853	-.1139	.0263
.900	.8874	-.3749	-.0321	.4308	.0810	.0077
.980	.8032	-.3402	-.0285	.3477	.1375	.0803

DIP= 66 DEGREES, FC= 3.5 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.540	.720	.890	.980
HM	.7868	-.3804	-.0224	.3310	.0886	.1963
H	-.0792	.0620	-.0910	.0895	-.2823	.3010
T	-.8141	.4653	.0611	.0046	.1097	.1733
F1	2.8679	-1.4856	-.6474	.4064	-.1798	.0386
.540	1.7060	-.8170	.0324	.1278	-.0631	.0139
.720	1.1788	-.5924	.1267	.3521	-.0820	.0168
.890	.9178	-.4498	.0011	.4093	.1213	.0003
.980	.8209	-.4050	.0088	.3058	.1891	.0804

DIP= 66 DEGREES, FC= 4.5 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.500	.700	.880	.980
HM	.5764	-.2814	.1075	.2554	.1444	.1978
H	-.0814	.0771	-.1027	.0950	-.2905	.3025
T	-.6569	.3907	-.0603	.0457	.0935	.1872
F1	2.8215	-1.6380	-.3009	.1805	-.0789	.0158
.500	1.3650	-.6588	.3171	-.0282	.0057	-.0008
.700	.9096	-.4809	.3248	.2786	-.0390	.0069
.880	.6850	-.3414	.1444	.3468	.1730	-.0078
.980	.6093	-.3103	.1436	.2278	.2488	.0808

DIP= 66 DEGREES, FC= 7.0 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.450	.680	.870	.980
HM	.2780	-.0653	.1753	.2233	.1762	.2125
H	-.1562	.1652	-.1115	.1017	-.3146	.3154
T	-.4397	.2404	-.1369	.0435	.0873	.2053
F1	2.9569	-1.9017	-.0854	.0472	-.0210	.0040
.450	.7263	-.1498	.4905	-.0973	.0369	-.0067
.680	.5628	-.2593	.4318	.2890	-.0285	.0042
.870	.3446	-.0933	.2147	.3369	.2080	-.0108
.980	.3338	-.1223	.2148	.1930	.2906	.0901

DIP= 80 DEGREES, FC= 2.2 FH. F1= .90 FH, FX= 1.53 FH

F/FC =	(F1)	(FX)	.600	.750	.900	.980
HM	.9099	-.3148	-.2799	.5504	-.0347	.1691
H	-.0861	.0368	-.0341	.0312	-.1770	.2293
T	-.8418	.3611	.3400	-.1863	.2266	.1005
F1	2.6898	-.9825	-1.4164	1.1346	-.5419	.1164
.600	1.7615	-.6012	-.5048	.5634	-.2796	.0606
.750	1.3129	-.4594	-.2823	.6203	-.2434	.0518
.900	1.0488	-.3648	-.2964	.6417	-.0612	.0319
.980	.9476	-.3302	-.2707	.5475	.0242	.0815

DIP= 80 DEGREES, FC= 2.6 FH. F1= 1.00 FH, FX= 1.62 FH

F/FC =	(F1)	(FX)	.600	.750	.900	.980
HM	.8668	-.3460	-.1685	.4827	-.0033	.1682
H	-.0842	.0410	-.0437	.0370	-.1883	.2382
T	-.8448	.4066	.2696	-.1548	.2157	.1078
F1	2.7563	-1.1780	-1.2284	1.0543	-.5159	.1117
.600	1.6752	-.6578	-.2665	.4202	-.2193	.0482
.750	1.2508	-.5042	-.1141	.5264	-.2023	.0434
.900	.9989	-.4006	-.1672	.5675	-.0233	.0248
.980	.9034	-.3630	-.1548	.4773	.0599	.0772

DIP= 80 DEGREES, FC= 3.2 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.600	.750	.900	.980
HM	.9370	-.4336	-.1570	.4778	.0011	.1748
H	-.0928	.0517	-.0464	.0393	-.2015	.2497
T	-.9517	.5153	.2679	-.1664	.2218	.1131
F1	3.1072	-1.5300	-1.2517	1.0964	-.5389	.1171
.600	1.8009	-.8184	-.2157	.3970	-.2103	.0465
.750	1.3490	-.6298	-.0862	.5234	-.1994	.0429
.900	1.0786	-.5014	-.1526	.5662	-.0149	.0240
.980	.9770	-.4550	-.1423	.4719	.0691	.0793

DIP= 80 DEGREES, FC= 4.0 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.540	.720	.890	.980
HM	.7024	-.3465	.0608	.2900	.1320	.1613
H	-.0776	.0545	-.0650	.0462	-.2133	.2552
T	-.7731	.4471	.0351	.0059	.1420	.1431
F1	3.0105	-1.6865	-.5969	.4325	-.2015	.0419
.540	1.5235	-.7348	.1890	.0474	-.0324	.0073
.720	1.0620	-.5408	.2362	.2963	-.0668	.0131
.890	.8193	-.4073	.0880	.3800	.1227	-.0026
.980	.7350	-.3680	.0832	.2805	.2061	.0631

DIP= 80 DEGREES, FC= 5.0 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.500	.700	.880	.980
HM	.5343	-.2568	.1422	.2350	.1776	.1677
H	-.0814	.0693	-.0762	.0552	-.2295	.2626
T	-.6401	.3780	-.0628	.0416	.1235	.1597
F1	3.0553	-1.8712	-.3156	.2065	-.0930	.0180
.500	1.2538	-.5806	.3738	-.0637	.0200	-.0034
.700	.8533	-.4439	.3660	.2531	-.0342	.0057
.880	.6325	-.3073	.1763	.3402	.1670	-.0087
.980	.5665	-.2823	.1690	.2221	.2585	.0662

DIP= 80 DEGREES, FC= 7.0 FH. F1= 1.20 FH, FX= 1.80 FH

F/FC =	(F1)	(FX)	.450	.680	.870	.980
HM	.3033	-.0904	.1837	.2162	.2012	.1860
H	-.1397	.1405	-.0881	.0669	-.2600	.2804
T	-.4633	.2565	-.1246	.0351	.1164	.1800
F1	3.2396	-2.1581	-.1265	.0708	-.0317	.0058
.450	.7573	-.1691	.4800	-.0997	.0380	-.0066
.680	.5987	-.2890	.4365	.2797	-.0303	.0044
.870	.3630	-.1090	.2170	.3434	.1961	-.0106
.980	.3527	-.1380	.2146	.1990	.2945	.0773