ELECTRON BEAM COMPRESSION WITH
ELECTRIC AND MAGNETIC FIELDS

by

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INTRODUCTION

Electron beams are very important in microwave devices such as amplifiers and oscillators, in radar and communication equipment, plasma interaction devices, micro-welding devices for welding small parts, and in high power electron tubes. For microwave device and micro-welding applications, electron beams with small diameters and large current and power densities are required. Electron beams of this type may be produced by a compression mechanism employing both electrostatic and magnetic fields. This paper presents design and test information on a cylindrically focused beam with a current density of 2000 amps/cm². The beam entrance conditions and the magnetic field intensity at the cathode are critical parameters in the gun design. Under proper conditions, the beam produced is well focused with little ripple.

ANALYSIS

The beam compression system consists of an electrostatically convergent electron gun with a subsequent magnetic compression region as shown in Fig. 1. The magnetic field in the transition region \( 0 < z < \ell \) is assumed to be

\[
B_z = \frac{B_0 - B_k}{2} \left( 1 - \cos \frac{\pi z}{\ell} \right) + B_k
\]

In the region \( x > \ell \), \( B_z = B_0 \), and in the region \( z < 0 \), a constant magnetic field is assumed, \( B_z = B_k \), where \( B_k \) is the cathode flux density.

The beam rotates as it enters the transition region because flux is cut during the electrostatic compression process. The angular velocity of the beam in the transition region is given by Busch's theorem

\[
\dot{\theta} = \frac{e}{2m} \left( \frac{B_k r_k^2}{B z - \frac{B_k r_k^2}{r^2}} \right)
\]

where \( B_k \) is the flux density at \( z = 0 \), and \( r_k \) is the radius at the cathode.

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Fig. 1. Electron beam compression system.
Once the angular velocity is known, the equations of motion for the edge electron may be written in a convenient form. The normalized equations of motion are

\[
\frac{d^2\sigma}{dT^2} = \frac{\beta}{\sigma \left( \frac{d\sigma}{dT} \right)} - \frac{\alpha}{2} \sigma \left[ 1 - \cos \frac{Z}{\ell} + \frac{B_k}{B_0} \left( 1 + \cos \frac{Z}{\ell} \right)^2 \right]^2
\]

\[
+ \frac{2\alpha B_k}{r_k} \frac{r_k^4}{B_0^2 r_0^4} \frac{1}{3}
\]

\[
\frac{dZ}{dT} = \left\{ 1 - \sigma^2 \left( \frac{d\sigma}{dT} \right)^2 - \frac{\pi \alpha}{2} \left[ \frac{2\alpha \sigma}{\pi} \left[ 1 - \cos \frac{Z}{\ell} \right]^2 \right. \right.
\]

\[
+ \frac{B_k}{B_0} \left( 1 + \cos \frac{Z}{\ell} \right) - \frac{4 B_k}{B_0} \frac{r_k^2}{r_0^2} \left( \frac{1}{3} \right)^2 \left[ \left. \left[ d \right] \right] \right]^{1/2}
\]

where

\[
\sigma = \frac{r}{r_0}
\]

\[
r_o = \text{final beam radius}
\]

\[
Z = \pi z/r_o
\]

\[
T = \pi u_o t/r_o
\]

\[
u_o = \text{axial beam velocity}
\]

\[
\alpha = \frac{1}{2} \left( \frac{e B_0 r_o}{2\pi \mu u_o} \right)^2
\]

\[
\beta = \frac{1}{2} \left( \frac{\omega r}{\pi u_o} \right)^2
\]

\[
\omega_p = \text{beam plasma frequency}
\]
The above equations were programmed for an IBM 1620 computer using the Runge-Kutta-Gill method. The value of perveance used was $2.4 \times 10^{-6}$. Figure 2 shows some trajectory plots for the outer electron for various values of cathode flux. The final magnetic field $B_0$ is 7,500 gauss, and these curves show that the cathode must be well shielded to obtain a near-Brillouin beam.

Figure 3 shows a plot of $2\theta/\omega$, where $\omega = eB/m$ is the electron cyclotron frequency. The computer results were used to formulate a set of design curves giving the required entrance slope and radius for a smoothly focused beam.

The equations of motion have been modified to account for initial thermal velocities at the cathode. The thermal velocity terms appear in the equations in the same way the cathode flux term does. Curves have been published which give the per cent of charge lying outside a given radius for a given beam voltage, current, and temperature. For a typical case of 1000:1 convergence, 30 per cent of the beam current lies outside of the Brillouin-focused diameter.

**EXPERIMENTAL RESULTS**

A high perveance and high area compression electron gun designed by Frost, Purl, and Johnson was selected for use in the magneto-electric compression system. Beam perveance and profile measurements were made on the new gun in a demountable beam tester. In order to make tests in a solenoid, a hard tube with an internal pole piece was designed from the curves obtained in the computer study. The tube was designed to operate at 10 kv and 2.4 amps with a final beam diameter of .0136 inches in a magnetic field of 7,500 gauss.

The first beam tester was operated with the beam impinging directly on a flat copper collector. A measure of the beam diameter was achieved by measuring the erosion pattern on the collector as shown on Fig. 4. The erosion pattern was slightly smaller than the .0136 inch diameter calculated value, but this was probably due to the decrease in current density near the beam edges.

A second test was conducted in which the beam was allowed to burn

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Fig. 2. Plot of the normalized radius vs. Z with different values of flux at the cathode.
Fig. 3. Plot of $2\theta / \omega_c$ vs. $Z$ with different values of flux at the cathode.
Fig. 4. Beam impression on a flat copper collector. The theoretically calculated beam diameter of 13.6 mills is shown.
holes through a stack of thin carbon-paper foils. Four foils made of carbon paper were placed in the tube perpendicular to the beam axis and between the electron gun and the final collector. The beam burned holes in the four carbon foils of diameters .028, .024, .024, and .023 inches.

Some experimental results by Brewer have indicated that the Pierce-Walker theory for thermal spreading effects in electron beams is fairly accurate up to about 95 per cent of the enclosed beam current. The Pierce-Walker curve is shown in Fig. 5 as the solid curve, and the Brewer experimental curve as the dashed curve. The vertical axis is the normalized current outside a given radius. The vertical dotted line in Fig. 5 represents 0.024 inches, the experimentally determined beam diameter. Calculations of the carbon foil temperature from information given by the Pierce-Walker curve shows that the vaporization temperature of carbon paper would not be reached. If the Brewer curve is used for the calculation, the calculated temperature at the edge of the hole is near the vaporization temperature. This experiment shows that the beam is fairly well focused at the expected diameter, and tends to verify the Brewer curve for the thermal tail.

CONCLUSIONS

A combined electrostatic and magnetostatic compression system may be employed to yield electron beams of very small diameters and very large current and power densities. Analytical results have been obtained which give design curves for a minimum-ripple beam. The effects of small design errors in initial beam slope and radius have been investigated, as well as effects due to thermal velocities and cathode flux.

Experimental results have been obtained on a perveance 2.4 x 10^{-6} beam operating at 10 kv. A final beam diameter of .013 inches was obtained in a magnetic field of 7,500 gauss. An area compression of 1350:1 was obtained with a final current density of 2,000 amps per square centimeter. The beam size was detected by copper collector erosion patterns and holes burned in carbon-paper discs.

Fig. 5. Plot of the normalized current outside a given normalized radius.