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DYNAMIC ANALYSIS OF A LITHIUM-BOILING POTASSIUM REFRACTORY METAL RANKINE CYCLE POWER SYSTEM FOR THE JET PROPULSION LABORATORY

(JPL Contract No. 951198)

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I. INTRODUCTION

This report contains a number of steady state and transient analyses of idealized models of the Jet Propulsion Laboratory Lithium-Boiling Potassium Rankine Cycle Space Power System.

The actual JPL power system is quite complex and the operator has many independent system parameters at his control. Dependent upon the method of control used, some of these may be: lithium heater power or lithium outlet temperature, lithium pump power, radiator area, potassium pump power or potassium pump outlet pressure, valve openings, preheater power level or preheater outlet temperature, and turboalternator load. In addition to these controllable parameters, there are many system constants which were determined at the time the system was designed and over which the operator has no control, such as insulation heat loss coefficients, orifice sizes, component heat capacities, fluid volume capacities, bearing lubricant flow rates, rotating machinery inertia and bearing and windage losses, and thermal and mechanical stress limitations. A power system with a nuclear reactor energy source would have in addition a temperature coefficient which was either positive or negative, a heat capacity, and more thermal and mechanical stress limitations.

System variables of interest over which the operator has no direct control may be lithium temperature in heater and boiler; potassium temperature in boiler, condenser and pump; turboalternator bearing temperature; potassium flow rate; turboalternator speed and power output; and others. The operator can only adjust his "parameters" to bring these system variables to their proper values.

A number of the processes which take place in a boiling liquid metal system are only approximately understood. For example, the heat and momentum transfer processes in the boiler are strongly affected by relatively subtle geometrical variations, so that with present knowledge, the prediction of the boiler performance may only be approximated. Similarly, the two phase flow through the orifice and turbine nozzle can only be estimated. It is clearly impossible to analyze precisely, or even to write the differential equations for the transient or steady state description of such a system in its entirety. Suitable idealizations must be made to allow its study by analysis. The analyses presented in this report are based on such idealizations which yield solutions which adequately represent the main features of the loop under construction.

Because of the extensive notation required in the various analyses presented, no general nomenclature section has been prepared. Instead, a separate notation is included with each analysis.

This report is submitted in fulfillment of the tasks outlined in JPL Contract No. 951198. In excess of 700 manhours were expended on these studies.

II. CONSIDERATION OF SOME OF THE UNIQUE ASPECTS OF LIQUID METAL RANKINE POWER SYSTEMS

The Jet Propulsion Laboratory Lithium, Boiling Potassium System is a Rankine power cycle with two flow circuits. The components listed in the direction of flow, are as follows. The principal lithium circuit consists of an electrical heater, centrifugal pump, and heat exchanger (the heat exchanger being an outside jacket on the potassium boiler tube). The principal potassium circuit consists of an electrical resistance preheater; a boiler tube in which the power is transferred from the lithium circuit; an orifice meter which may also be used as a flow restriction; the turboalternator; the condenser and subcooler; an electromagnetic pump; and a flow throttling valve near the pump outlet. Both of these circuits also contain hot traps, pressure transducers, electromagnetic flowmeters, and filling and emptying lines. In addition, the potassium circuit has a bypass line which can be used in place of the turboalternator, and a turboalternator bearing lubrication circuit. The condenser-subcooler, which rejects heat by radiation, has controllable shutters to vary the effective radiating view factor. The electrical lithium heater may be programmed to simulate a suitable nuclear reactor.

Ther are several modes of operation possible with this system. The turbine may be removed from the circuit by valving the potassium through the bypass, so that the primary flow restriction becomes the orifice flowmeter at the outlet of the boiler. The turbine nozzle or the orifice meter, whichever is the major restriction, may be operated either unchoked or choked. Principal controlled quantities as listed in the introduction, are lithium heater power, lithium pump power, potassium preheater power, potassium pump power, alternator load, radiator shutter position, and several valve positions. Automatic controls are available for some of these quantities, so that the lithium heater outlet temperature, the potassium preheater outlet temperature, and the pump outlet pressure may be maintained constant, at least in a time-averaged sense. The settling time of all of the controllers is short compared to the thermal time constants of the system, but rapid changes due to, for example, sudden boiling of superheated potassium liquid would of course not be followed accurately by the controllers.

The differences between this liquid metal power system and a conventional steam power plant are significant. The most striking difference is that in the liquid metal boiler there is no liquid hold-up. At any instant the flow into the boiler and the flow out of the boiler must be very nearly equal. One cannot, for example, close a valve on the boiler outlet, generate a vapor supply in the boiler, and then start the vapor flow at some desired value. This is the usual starting procedure for a steam power plant. The system with no vapor supply in the boiler must necessarily be started with less than critical flow in the turbine nozzles, and pass through critical in a continuous manner. In the subcritical region, the condenser and boiler conditions are coupled across the turbine nozzle, so that the boiler pressure and temperature may not be controlled independent of the condenser. In addition, in a system with very small fluid storage capacity in the boiler, minor variations in the rate of vapor generation, or feed pump flow rate, must be felt throughout the system even when the nozzles are choked, since boiler and condenser are coupled through the pump.

Another striking difference between the boiler of a Rankine cycle steam plant and a liquid metal heat exchanger-boiler system such as the JPL system is in the magnitude of the fluid to fluid temperature difference in the boiler and in the nature of the controlling heat transfer resistance in the boiler. In the conventional steam power plant the controlling resistance is on the hot gas side so that minor variations in the resistance between the wall and boiling fluid do not have much effect on the heat transfer rate. In the liquid metal to boiling liquid metal heat exchanger the controlling resistance to heat transfer is usually on the boiling side, so that variations in boiler pressure level, local vapor quality, and flow rate will likely have a much larger effect upon the power transfer in the boiler. The fluid to fluid temperature differences in the liquid metal heat exchanger are in the order of 100°F, while those in the boiling portion of a steam boiler are usually an order of magnitude higher. The small temperature difference in the liquid metal system increases the sensitivity of the boiler heat flux to minor variations in fluid temperatures.

Another difference between the steam power plant and the liquid metal system which increases the control problems of the latter is the low pressure difference between boiler and condenser, an order of magnitude lower than that of the steam power plant. The fluid mass flow rate is far more sensitive to pressure variations in the condenser and boiler than in the steam power plant since these variations represent a greater proportionate change.

The condenser also plays its role in the control problems of the liquid metal system. The steam power plant condenser has a very low absolute pressure so that large variations in the condensing power load and condenser temperature difference have only a minor effect upon the pressure level of the system. The same proportionate variation in condenser power in the space power system, which operates at significant temperature (and pressure) levels due to its necessity to reject heat by radiation, causes a much larger change in system pressure level.

Liquid metals frequently superheat to significant temperature levels above their normal boiling point before changing phase, even in flow systems with considerable agitation. The change of phase, when it occurs is accompanied by violent pressure fluctuations due to the rapid acceleration of the unvaporized component. Such behavior is never encountered in practical water boiling systems. If superheating occurs in the liquid metal system, it will cause significant instantaneous variations in boiler exit quality, liquid flow rate, pressure level, etc, which will affect all components of the system.

There are a number of other differences between the liquid metal space power system and the conventional steam power plant; those listed are the major ones.

It is clear that some significant simplifications and approximations must be made to idealize the liquid metal power system in order to perform any analysis. Idealization of complicated power systems yield solutions which can be used to aid in the design and interpretation of power system experiments. Further, a comparison of the theory with experimental behavior will make it possible to develop better models for predicting the performance of future power systems. A large portion of the work reported here was devoted to the development and evaluation of a suitable model for analysis.

In some portions of the analyses that are presented, a number of different processes were considered which lead to different mechanism functions; in others a somewhat arbitrary but reasonable choice of one function was chosen. One would not expect such arbitrary choices (for example, constant pump pressure rise, independent of flow rate) to have a major effect upon the operating characteristics of the system.

III. DISCUSSION OF MODEL SYSTEMS ANALYZED IN THIS REPORT

The procedure used to evolve a working model for the potassium circuit transients was to choose a simple model from experience, analyze it by steady state perturbations around the design point, then repeat the process by a more complex model embodying some of the features neglected initially. Comparison of the results determined whether the effects neglected in the simpler model were in fact, negligible. The model thus tested, was then subjected to transient analysis. An example of this process is the first two steady state analyses of section IV, which were called Model 1 and Model 3. It was concluded, after the comparison, that the additional effects included in Model 3, namely subcooling, preheating, gaseous friction pressure drop, and liquid level variation, did not change the performance of the system enough to warrant the increased mathematical complexity.

Model 2 is a system identical to that of Model 1 except that the turbine nozzle or orifice (whichever is providing the major gaseous pressure drop) is considered to be unchoked. The third analysis of section IV is a steady state analysis of the unchoked system including some additional effects in order to test their importance. This work also justified the omission of the additional effects included in the test analysis.

The presentation of results from such analyses in generalized form is quite difficult. There is an old saying that a function of one variable can be represented by a line, a function of two variables by a group of lines, a function of three variables by a volume of pages, and a function of four variables by a library. In some of these "simple" models the results are functions of five independent parameters. Since in general the independent parameters represent functions actually controllable by the system operator, a great variety of operating conditions are possible. The actual ones chosen for this report are some which are, in the opinons of the authors, likely to occur. Many other operating conditions are just as likely to occur as those chosen.

All of the model system analyses reported here have certain features in common, and all pertain to a particular potassium circuit configuration consisting of:

a) boiler
b) throttling nozzle
c) radiator
d) pump.

In the analyses which took the lithium circuit into consideration it was treated as a uniform temperature mass, which can be mathematically replaced by a boiler with a larger heat capacity. Equation 1 of section V-C and Equation 2 of section V-A, two analyses which exhibit this different point of view, are identical in form; only the values of the heat capacity in the equation are different. A table of transit lines for the lithium circuit at design flow rate is presented in Table 1. It may be seen that the idealization of uniform temperature of the lithium circuit or its equivalent, infinite circulation rate, is not an unreasonable one. The controlling thermal time constants of the potassium circuit are over an order of magnitude longer than the time for the lithium to flow completely around its circuit.

Other idealizations common to all analyses are

1) The turbine power is negligible compared to system total power.
2) The vapor quality at exit of boiler and entrance to condenser is always less than unity; i.e. pressure is always a function of temperature in the mixed phase region.
3) There are no heat losses from system except in radiator.
4) The condensing and wall resistances are negligible compared to radiation resistance.
5) The lithium and potassium temperatures in boiler are uniform.

6) The heat transfer conductance in the boiler is uniform.

7) The condensation process takes place at constant pressure.

8) The pump is isentropic.

9) Thermal delay times due to finite liquid flow velocities in the potassium circuit are negligible.

Other idealizations, specific to individual analyses are listed in Table 2.

Controlled parameters considered in all analyses were: lithium or boiler power input, radiator area (or view factor), and potassium pump pressure rise. Specific analyses also include as additional parameters preheater outlet temperature and liquid friction pressure drop.

In none of the analyses was the orifice specifically considered to be the controlling pressure drop. In order to obtain quantitative results for the orifice case, it is merely necessary to change the constant in the pressure drop equation.

The models which do not specifically include subcooling and preheating require that the unstated preheat be to saturation. The preheat power and output temperature are therefore both variable, and dependent upon the existing boiler inlet conditions. This simplification has a minor effect, as may be seen by comparison of the results of section IV-A and IV-B.

The assumption of constant latent heat of vaporization means that the Mollier diagram is distorted somewhat. Computed and actual qualities will disagree, but the energy flowing from the boiler to condenser is unaltered by this simplification.

The idealization that thermal delay times due to finite liquid flow velocities in the potassium circuit are negligible is not a particularly good one. The transit time for the potassium from the condenser outlet to the boiler inlet is 40 seconds. There is however no simple way to account for this delay in the analysis, and since the preheat power is a small portion of the total, variations in the preheat power which lag the principal power variations by a small amount should not affect the characteristics of the system significantly. Transit times through various portions of the potassium circuit are given in Table 3.

Table 1.

LITHIUM CIRCUIT DESIGN LIQUID FLOW TRANSIT TIMES

Heater inlet to heater outlet	2.9 sec
Heater outlet to bottom of level indicator	.7 sec
Bottom of level indicator to pump	.2 sec
Pump to inlet of boiler	1.8 sec
Boiler inlet to boiler outlet	.4 sec
Boiler outlet to heater inlet	.7 sec
Total transit time around lithium circuit	6.7 sec

Table 2.

SPECIFIC IDEALIZATIONS APPROPRIATE TO INDIVIDUAL ANALYSES PRESENTED

	Steady State Analyses			Transient Analyses			
	Section IV			Section V			
Function	IV-A	IV-B	IV-C	V-A	V-B	V-C	V-D
Lithium circuit temperature distribution	not req'd	not req'd	not req'd	uniform	uniform	not req'd	uniform
Lithium circuit heat input rate	parameter	parameter	not req'd	parameter	parameter	not req'd	parameter
Heat rate input to boiler	variable	variable	parameter	variable	variable	parameter	variable
Boiler heat transfer conductance	constant	constant	constant	constant	linearized variable	constant	linearized variable
Latent heat of vaporization potassium	constant	variable	variable	constant	constant	not req'd	constant
Vapor pressure-temperature relationship	actual	actual	linearized	linearized & exponential	linearized	linearized	linearized
Friction and orifice pressure drops	lumped w/nozzle	considered	lumped w/nozzle	lumped w/nozzle	lumped w/nozzle	not req'd	lumped w/nozzle
Nozzle pressure drop type	choked (sonic)	choked (sonic)	unchoked (subsonic)	choked (sonic)	choked (sonic)	not stated necessarily unchoked	unchoked
Nozzle pressure drop based on	vapor flow alone	vapor flow alone	vapor flow alone	vapor flow only	vapor flow only	not req'd	vapor flow only
Potassium radiation heat loss temperature dependence	4th power	4th power	1st power	1st power	1st power	1st power	1st power
Condensate liquid level	fixed	variable	variable	fixed	fixed	not req'd	fixed
Subcooling	none	variable	variable	none	none	not stated	none
Preheating (in preheater)	not stated necessarily to saturation	constant temp. output	none	same as IV-A	same as IV-A	none	same as IV-A
Preheat required in boiler	none	variable	variable	none	none	not stated	none

Table 2. (continued)

	Steady State Analyses			Transient Analyses			
	Section IV			Section V			
Function	IV-A	IV-B	IV-C	V-A	V-B	V-C	V-D
Liquid heat capacity	none req'd	constant	constant	none req'd	none req'd	none req'd	none req'd
Boiler liquid level	constant	variable	variable	fixed	fixed	fixed	fixed
Liquid friction	not considered	not considered	not considered	parameter	parameter	not considered	parameter

Table 3.

POTASSIUM CIRCUIT DESIGN
LIQUID FLOW TRANSIT TIMES

Condenser outlet to pump inlet	8.8 sec
Through pump, inlet to outlet	1.1 sec
Pump outlet to trap inlet	1.9 sec
Trap inlet to trap outlet	27.6 sec
Trap outlet to boiler inlet	.8 sec
Total time from condenser to boiler	40.2 sec

IV-A. SIMPLIFIED STEADY STATE MODEL FOR POTASSIUM CIRCUIT (Model 1)

A simplified steady state model of the potassium circuit was analyzed at off design conditions in order to realize the variation in various control parameters. The model consists of a boiler, a choked nozzle, a condenser and a pump. There is no subcooling or superheating; the potassium is in a saturated condition throughout the loop. There are no heat losses, therefore, the heat input into the boiler equals the energy radiated by the condenser. The heat transfer coefficient for the boiler is taken as a mean value which is constant.

The equations which represent the idealized simple model are:

1. Boiler,

$$q_B = hA\,(T_{Li} - T_1) \tag{1}$$

where hA, a constant which is evaluated at design condition

T_{Li}, lithium temperature in boiler

T_1, potassium temperature in boiler

q_B, energy input into the boiler

2. Choked Nozzle (Fliegner's formula)

$$Wg = C_2 \frac{P_1}{\sqrt{T_1}} \tag{2}$$

where Wg, gas flow rate into choked nozzle from boiler and dependent only upon boiler conditions

P_1, potassium vapor pressure in boiler

C_2, a constant

3. Condenser

$$q_C = C_1\,T_2^{\,4} \tag{3}$$

where q_C, energy radiated by condenser which is equal to q_B

C_1, a function of radiator area

T_2, potassium temperature in condenser

4. Pump

$$\Delta P = P_1 - P_2 \tag{4}$$

where ΔP, pressure drop across pump

P_2, potassium vapor pressure in condenser

5. Gas Flow Rate in Boiler

$$Wg = W_T \left(\frac{q_B}{h_v W_T} \right) = \frac{q_B}{h_v} \tag{5}$$

where W_T, total flow rate

h_v, heat of vaporization

Given that $q_C = q_B$, we have the following system of equations:

$$C_2 \frac{P_1}{\sqrt{T_1}} = \frac{hA(T_{Li} - T_1)}{h_v} = C_1 \frac{T_2^4}{h_v} = Wg \tag{6}$$

Normalizing equation (6) by the design values $(Wg)_d$, $(P_1)_d$, $(C_1)_d$, $(T_2)_d$, and $(h_v)_d$, we have

$$\frac{C_2 (P_1)_d}{(T_2)_d} \frac{\bar{P}_1}{\sqrt{\bar{T}_1}} = \left[\frac{hA\,(T_2)_d}{(h_v)_d} \right] \frac{\left(\bar{T}_{Li} - \bar{T}_1 \right)}{\bar{h}_v} = \left[\frac{(C_1)_d\,(T_2^{\,4})_d}{(h_v)_d} \right] \frac{\bar{C}_1\, \bar{T}_2^{\,4}}{\bar{h}_v} = (Wg)_d\, \overline{Wg} \tag{7}$$

where $\bar{T}_1 = T_1/(T_2)_d$; $\bar{T}_{Li} = T_{Li}/(T_2)_d$; $\bar{T}_2 = T_2/(T_2)_d$; $\bar{P}_1 = P_1/(P_1)_d$; $\bar{h}_v = h_v/(h_v)_d$;

$\bar{C}_1 = C_1/(C_1)_d$; $\overline{Wg} = Wg/(Wg)_d$; $\overline{\Delta P} = \Delta P/(\Delta P)_d$

The design conditions are:

$(q_B)_d = (q_C)_d = 102,400$ Btu/hr

$(T_{Li})_d = 2460°R$ (8)

$(T_1)_d = 2360°R$

$(P_1)_d = 112$ psi

$(T_2)_d = 1960°R$

$(P_2)_d = 24.5$ psi

From the above set of design conditions, we have

$h_d A_d = 1024$ Btu/hr°F

$$(C_1)_d \, (T_2^{\,4})_d = 102{,}400 \text{ Btu/hr} \tag{9}$$

$(Wg)_d = 136$ lb/hr

$(C_2)_d = 59$ lb°R$^{1/2}$/hr psi

Substituting the design values (8) and (9) into equation (7) gives

$$1.097 \frac{\bar{P}_1}{\sqrt{\bar{T}_1}} = 19.6 \frac{(\bar{T}_{Li} - \bar{T}_1)}{\bar{h}_v} = \frac{\bar{C}_1 \bar{T}_2^{\,4}}{\bar{h}_v} = \bar{W}g \tag{10}$$

The system of equation (10) was analyzed for various values of the gas flow rate, $\bar{W}g$, and the radiation area, $\bar{C}_1$. The results of the analysis are presented in Figures 1, 2, and 3.

The criterion that the lithium temperature must not exceed 2100°F sets an upper limit on the gas flow rate, as shown in Figure 1, at 1.25 times the design value or 170 lb/hr. The bounds on the radiation area, $\bar{C}_1$, and the pump pressure, $\overline{\Delta P}$, are shown more clearly in Figure 2. Here we see that the lithium temperature requires the upper limit on $\overline{\Delta P}$ to decrease with decreasing condenser radiation area. Also, as the condenser radiation area decreases to a value of 70% or less of the design area, the condenser temperature becomes an uncontrollable function of the condenser radiation area. This condition results because at the lower condenser radiation areas, the condenser temperature becomes equal to and exceeds the boiler temperature.

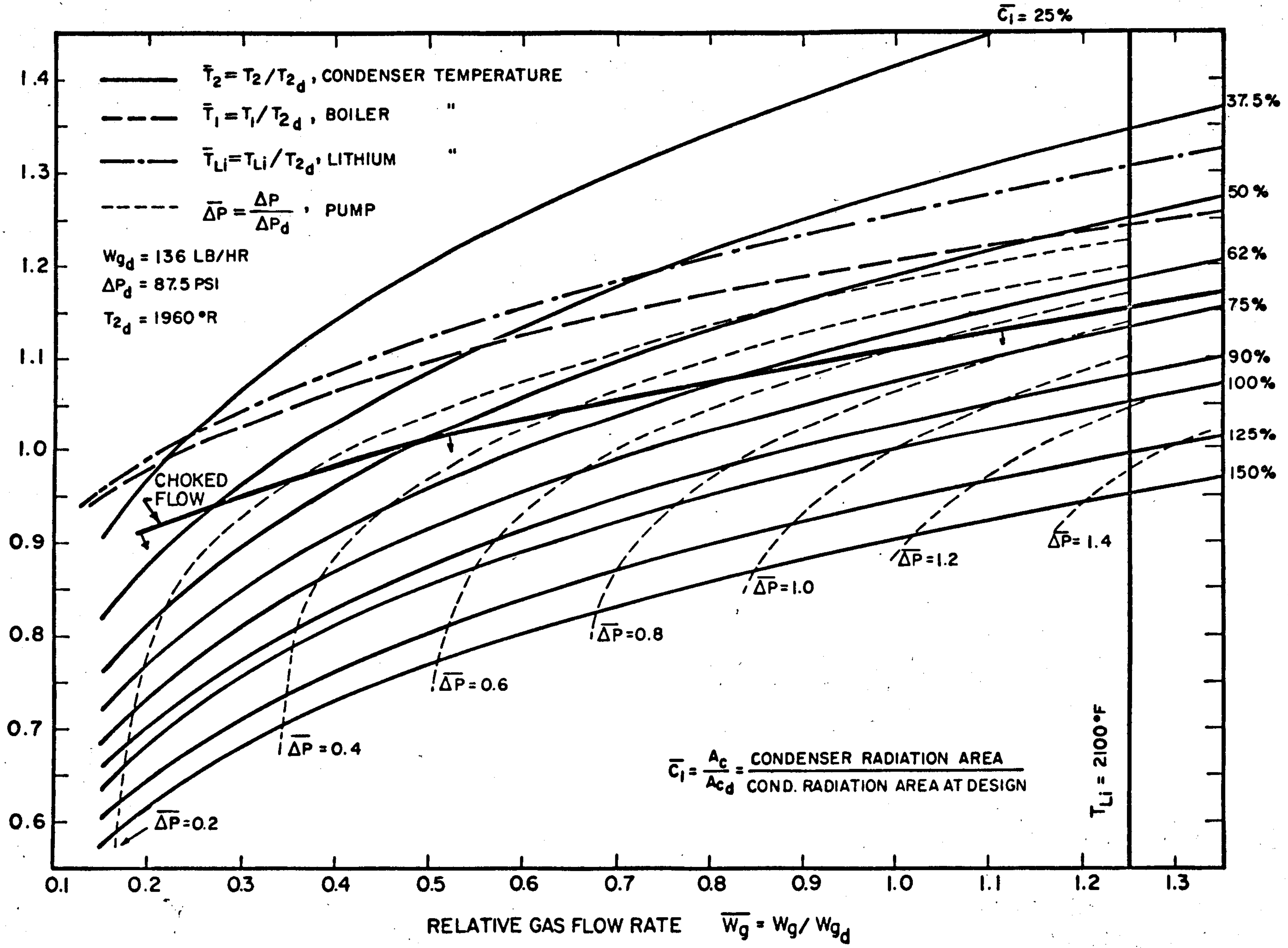


Figure 1. JPL potassium circuit - steady state model.

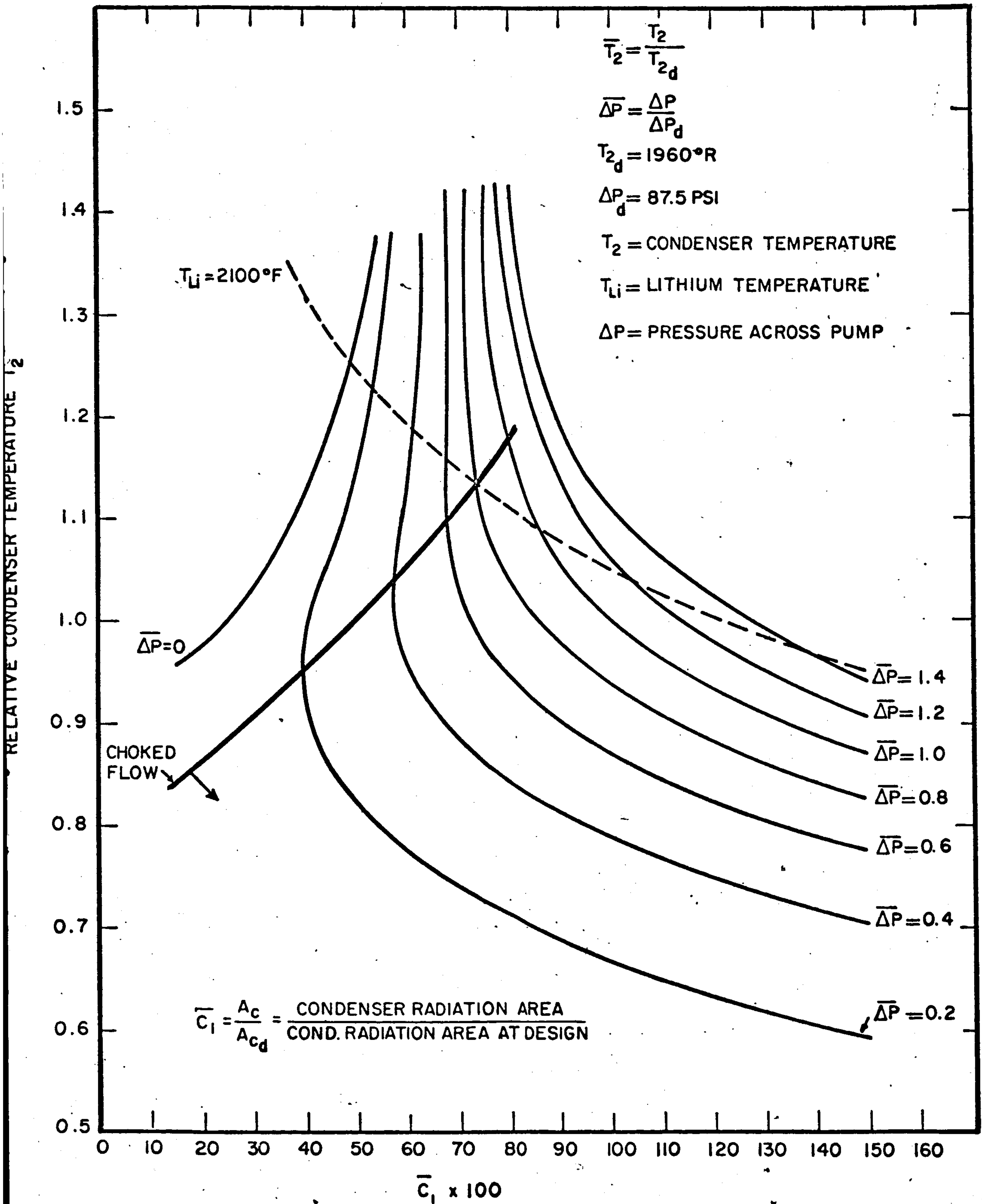


Figure 2. JPL potassium circuit - steady state model

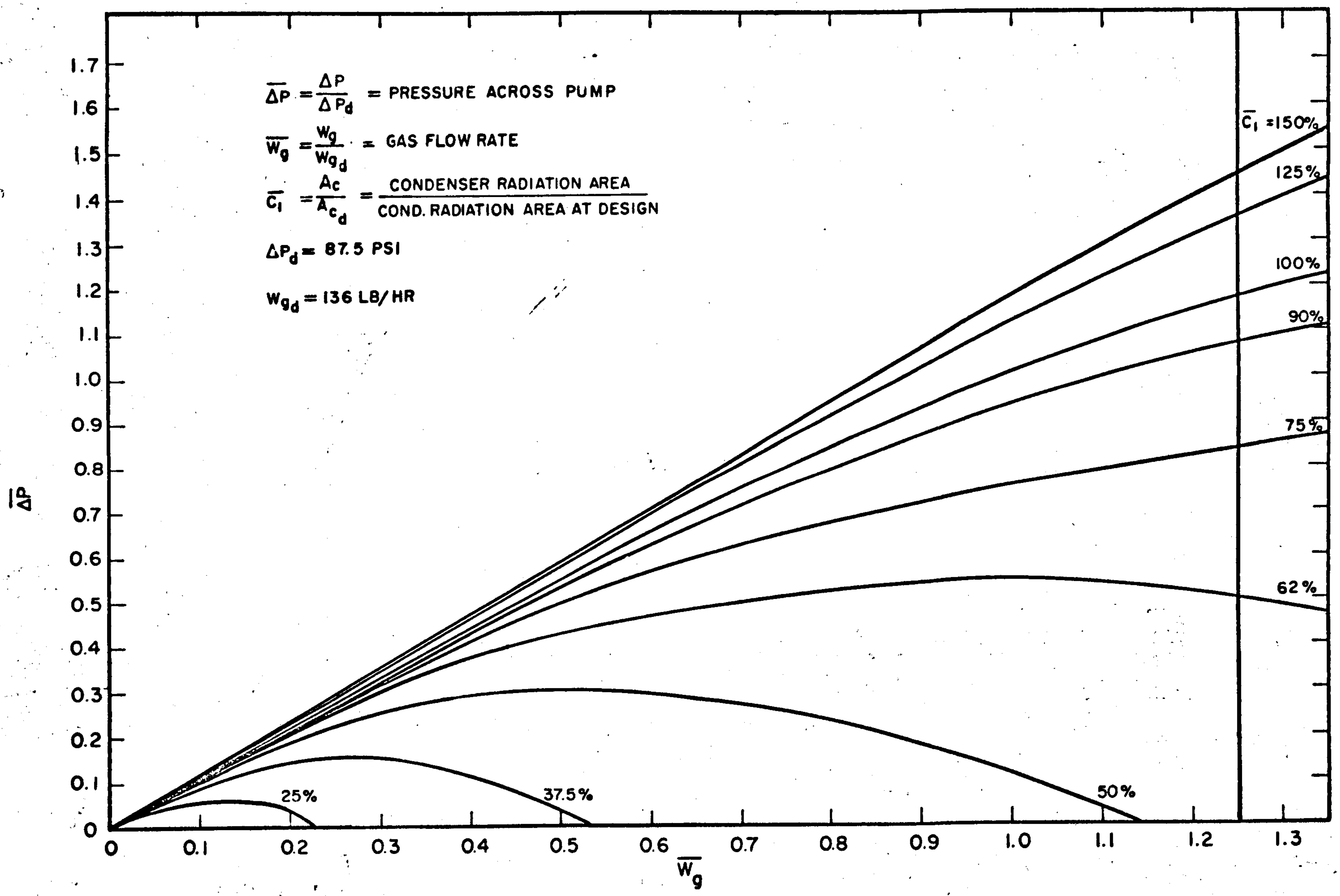


Figure 3. JPL potassium circuit - steady state model.

IV-B. STEADY STATE MODEL 3 FOR POTASSIUM CIRCUIT

Introduction

Steady state model III for the potassium circuit was analyzed at off design conditions in order to realize the variation in various control parameters. Model III is a refined version of model I, in that model III includes subcooling, preheating, liquid levels and frictional pressure drops. To review, model III is for a choked turbine nozzle, assumes no heat losses and assumes a constant heat transfer coefficient. In addition, the preheat temperature is held constant; the reason for doing so will be explained in the discussion.

Discussion

In formulating model III a difficulty arose in selecting a quantity that could be held constant (this condition must be imposed since there is one more unknown than there are equations). Fixing the energy input into the preheater and heat exchanger confined the calculations to a narrow range of control parameters which does not appear to be a natural constraint for the potassium circuit. Fixing the lithium temperature gave similar problems. However, a constant preheat temperature did not limit the calculations in any way over a wide range of flow conditions. Therefore, it appears that the preheat temperature is a natural quantity to hold constant in a liquid metal circuit.

The minimum number of parameters that have to be specified in order to solve the problem are four. In the mechanics of solving the problem, the solution was obtained much easier when the following parameters were specified: 1) the gas flow rate into the choked nozzle, 2) the preheat temperature, 3) the liquid flow rate, 4) the condenser shutter position. However, in presenting the results, the pressure head is specified instead of the gas flow rate into the choked nozzle. The calculations were performed for a limited range of the four parameters since to do otherwise would require an electronic computer. The values assigned to the four parameters for which the results are presented are: 1) values of liquid flow rate which range from 0.8 to 1.2 times the design flow rate, 2) a range of condenser shutter positions which give a radiating area equal to 0.7 to 1.5 times the design radiating area, 3) two values of the pump head which are the design pressure head and 0.9 times the design pressure head, and 4) a preheat temperature equal to 1700°F.

The circuit diagram used in model III is shown in Figure 1. The lengths and areas used in making the calculations are:

ℓ_{76} = heat exchanger length = 2.89 ft

ℓ_{23} = condenser height = 3.02 ft

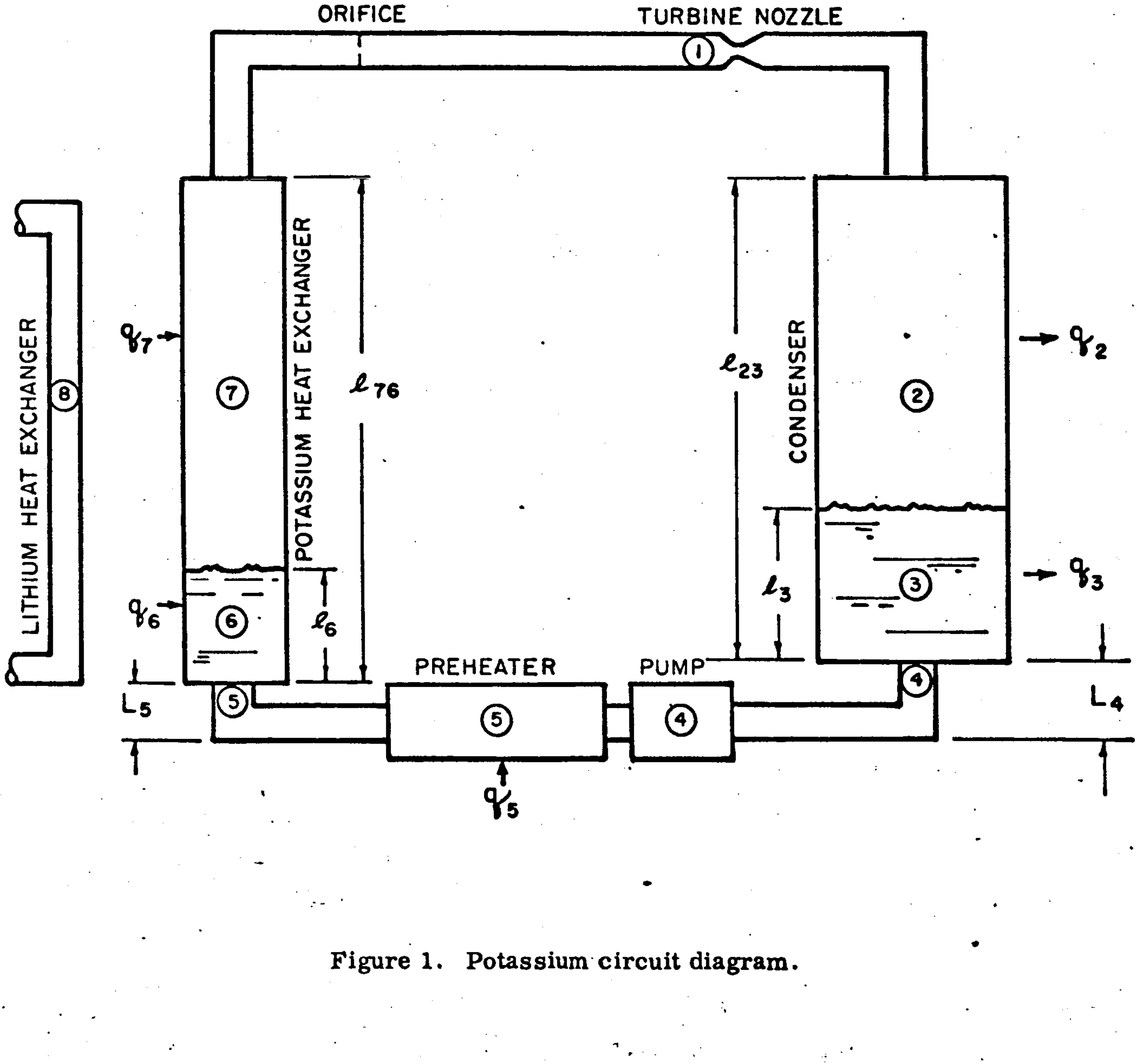


Figure 1. Potassium circuit diagram.

A_{23} = radiator area (3/4 of condenser area) = 9.41 ft^2

A_{76} = heat exchanger area = 0.378 ft^2

The values of the constants used in making the calculations are (refer to the section 4 for a definition of the constants):

$C_1 = 64.23$ lb(°R)$^{1/2}$ / hr (psi)

$C_2 = 8.7985 \times 10^{-10}$ Btu/hr ft^2 °R^4

$f_L = 3.56 \times 10^{-5}$ psi/(lb/hr)^2

$f_7 + f_1 = 1.504 \times 10^{-3}$ psi / (lb/hr)^2

h_L = 4660 Btu/hr ft^2 °F

h_7 = 2583 Btu/hr ft^2 °F

C_p = 0.208 Btu/lb °F

ρ_L = 40.75 lb/ft^3

The design values for model III are:

T_2 = 1500°F

T_4 = 1194°F

T_7 = 1922°F

T_8 = 2022°F

q_5 = 4.61 kilowatts

q_6 = 2 kilowatts

q_7 = 28 kilowatts

ℓ_6 = 0.81 inches

ℓ_3 = 4 inches

ΔP = 96.94 psi

W_L = 150 lb/hr

The results are presented in Figures 2 through 10. The results of model III and model I are compared in Figure 2. From this we see that refining model I did not significantly change the response of the condenser temperature to a changing radiating area. The same held true for the boiler and lithium temperatures. Therefore, model I although being highly simplified, contains the main features for describing the performance of a liquid metal circuit. Figures 3 through 10 present the variation of the various quantities versus the radiation area and are for the most part self-explanatory. The cut-off for choked flow is given in each figure for a specific heat ratio of 1.67 and only serves to locate the general area for turbine choking. Since the choking condition is calculated from ideal gas equations which do not hold, pursuing the precise location of choking would be of no avail. The liquid level in the condenser is not shown since it did not vary by more than 3% from the design level. The variation in the liquid level in the heat exchanger, as shown in Figure 7, is very slight, being only from 1/2 to 1 inch for a large variation in radiating area. The variation in the liquid flow rate and radiating area has a considerable effect on the preheat energy requirement (Figure 9). The selection of the preheat temperature level will have a significant effect on the preheat energy requirement.

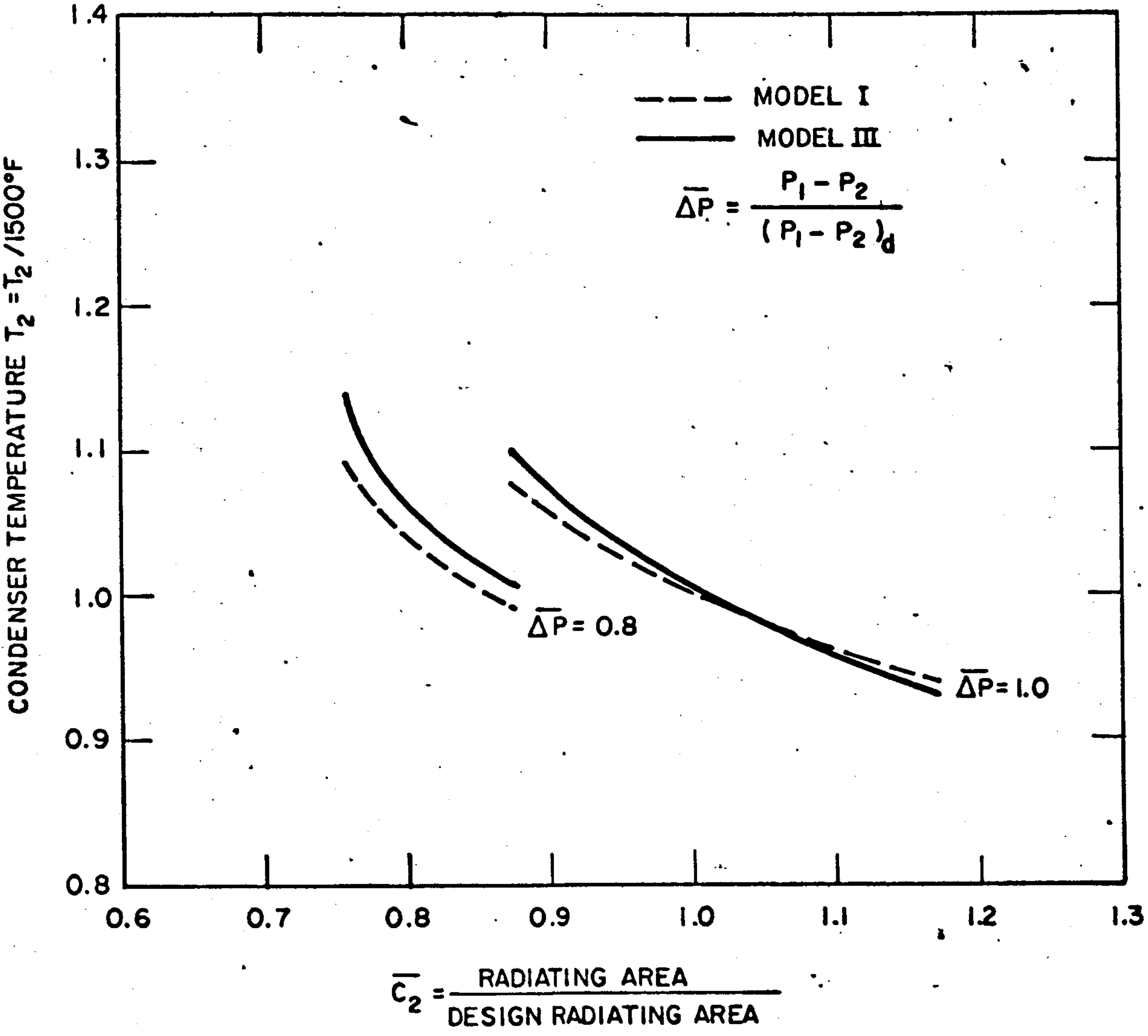


Figure 2. Comparison of condenser temperature for models I and III.

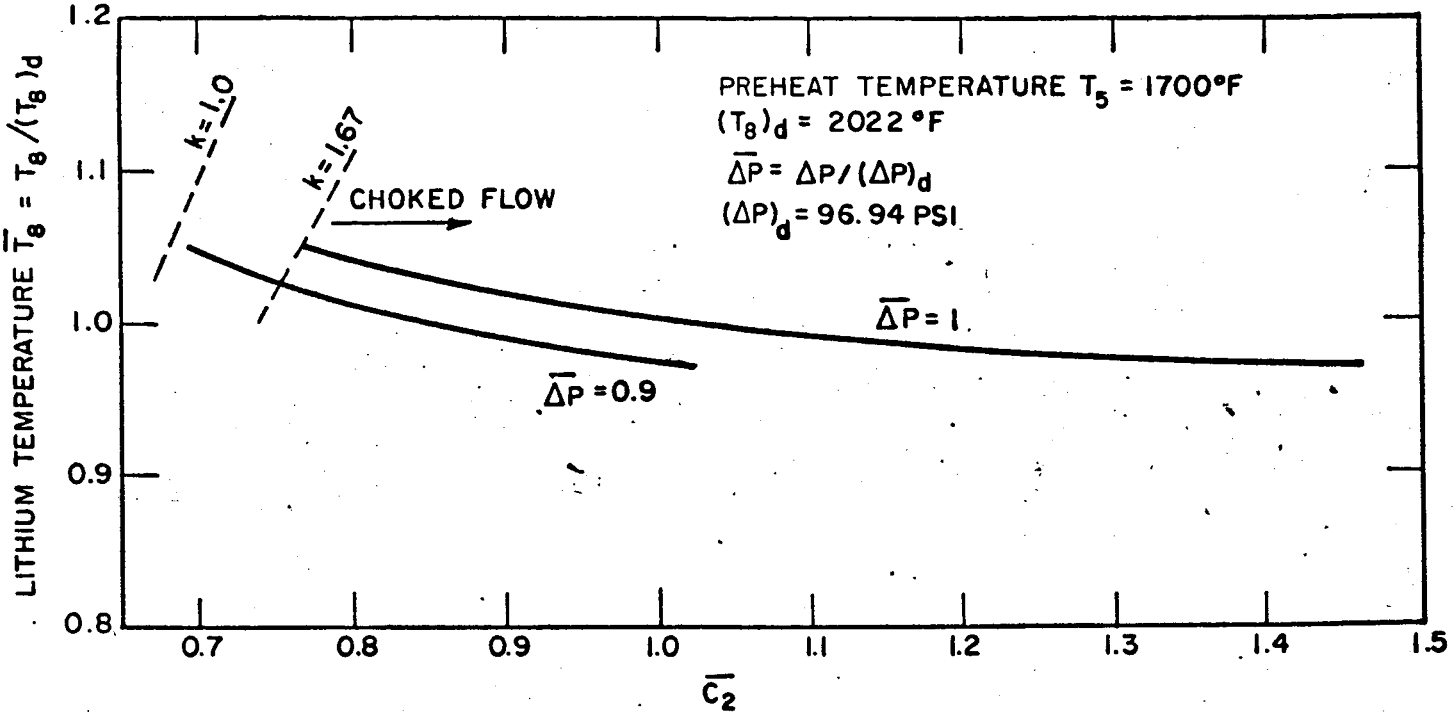


Figure 3.

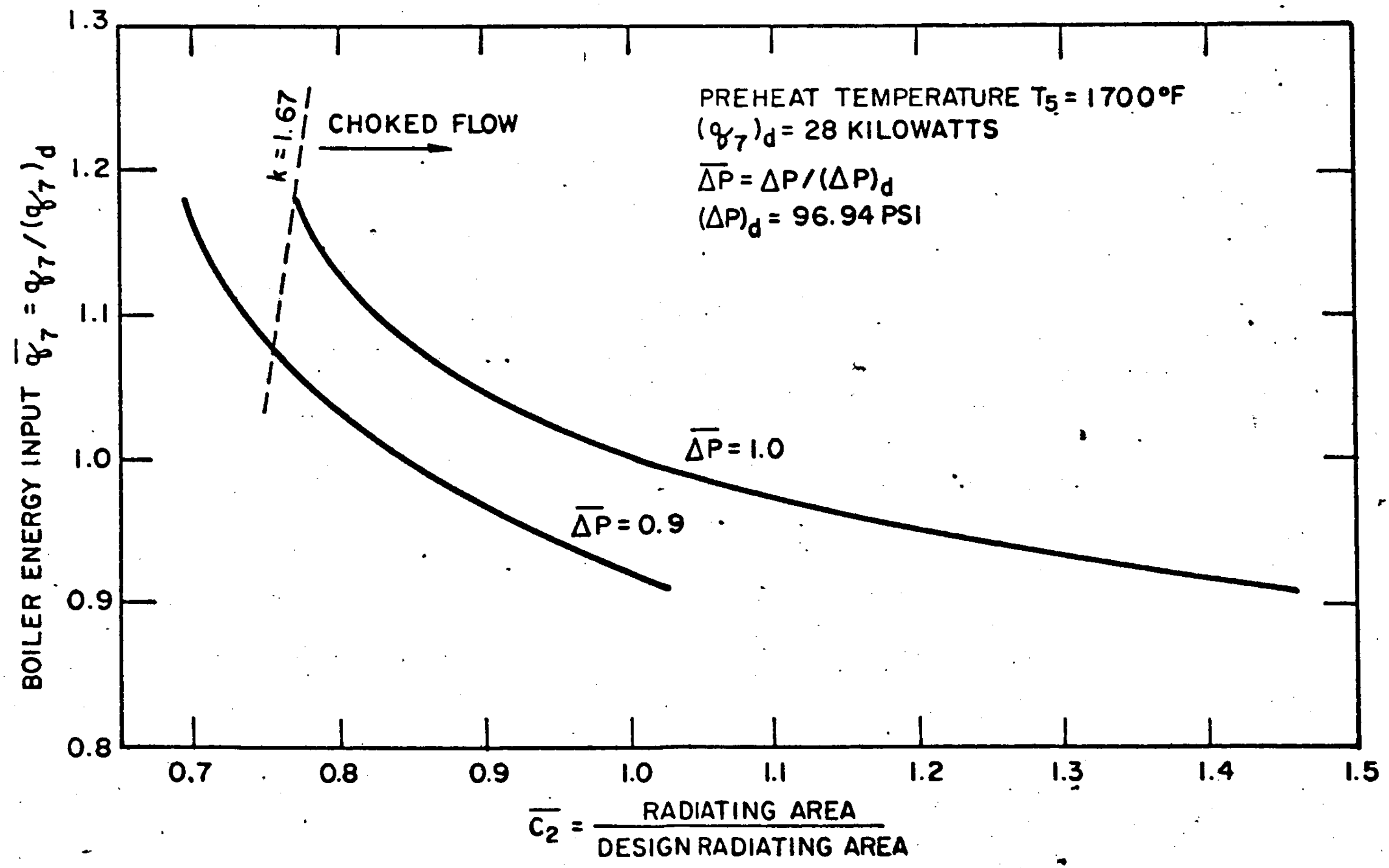


Figure 4.

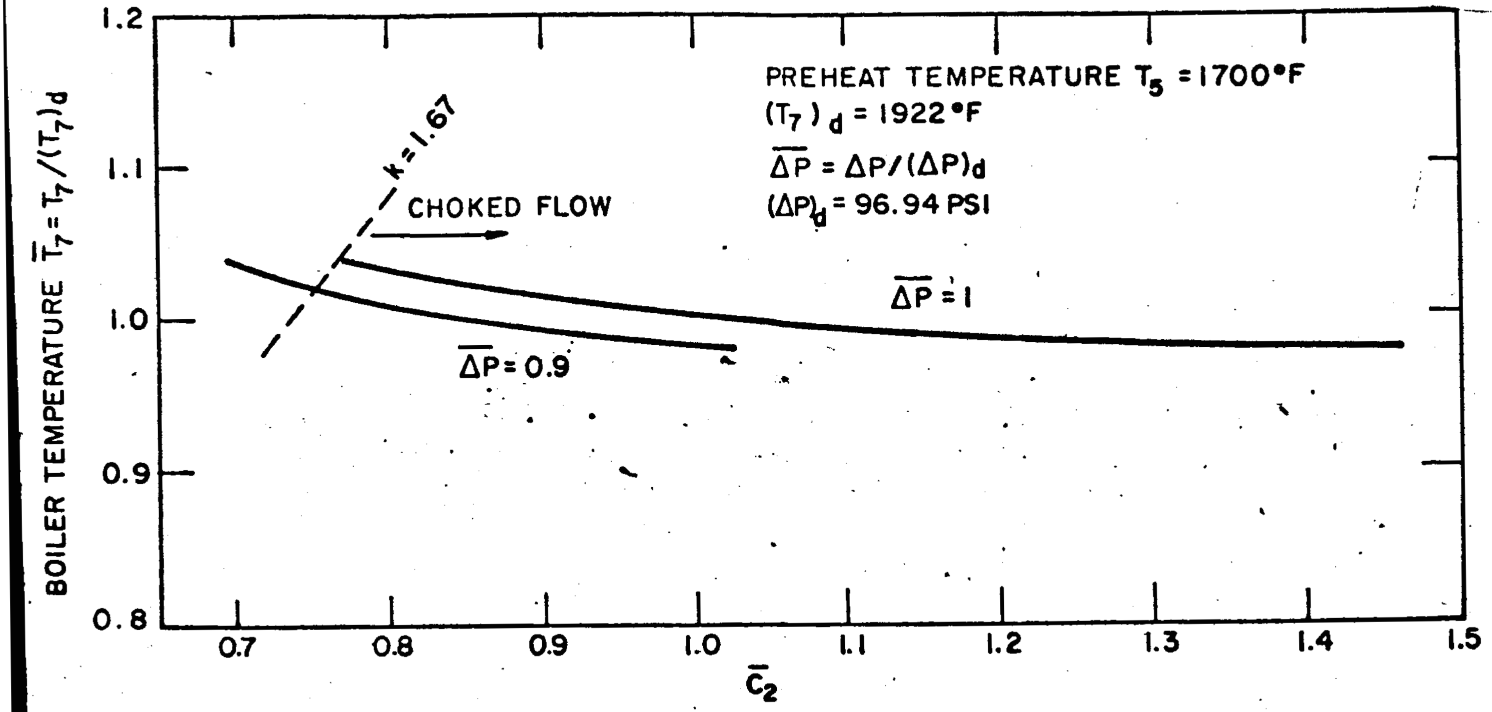


Figure 5.

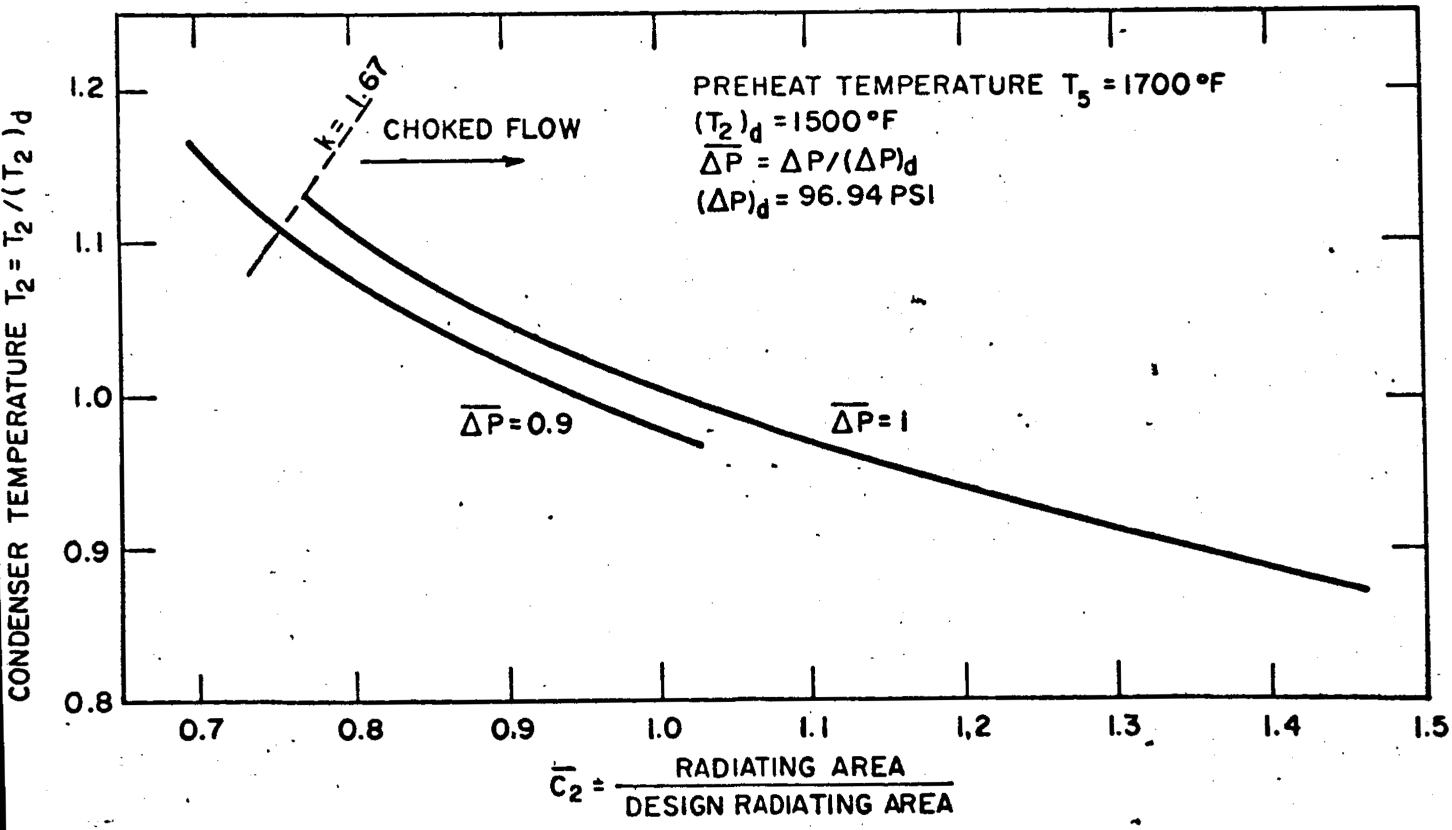


Figure 6.

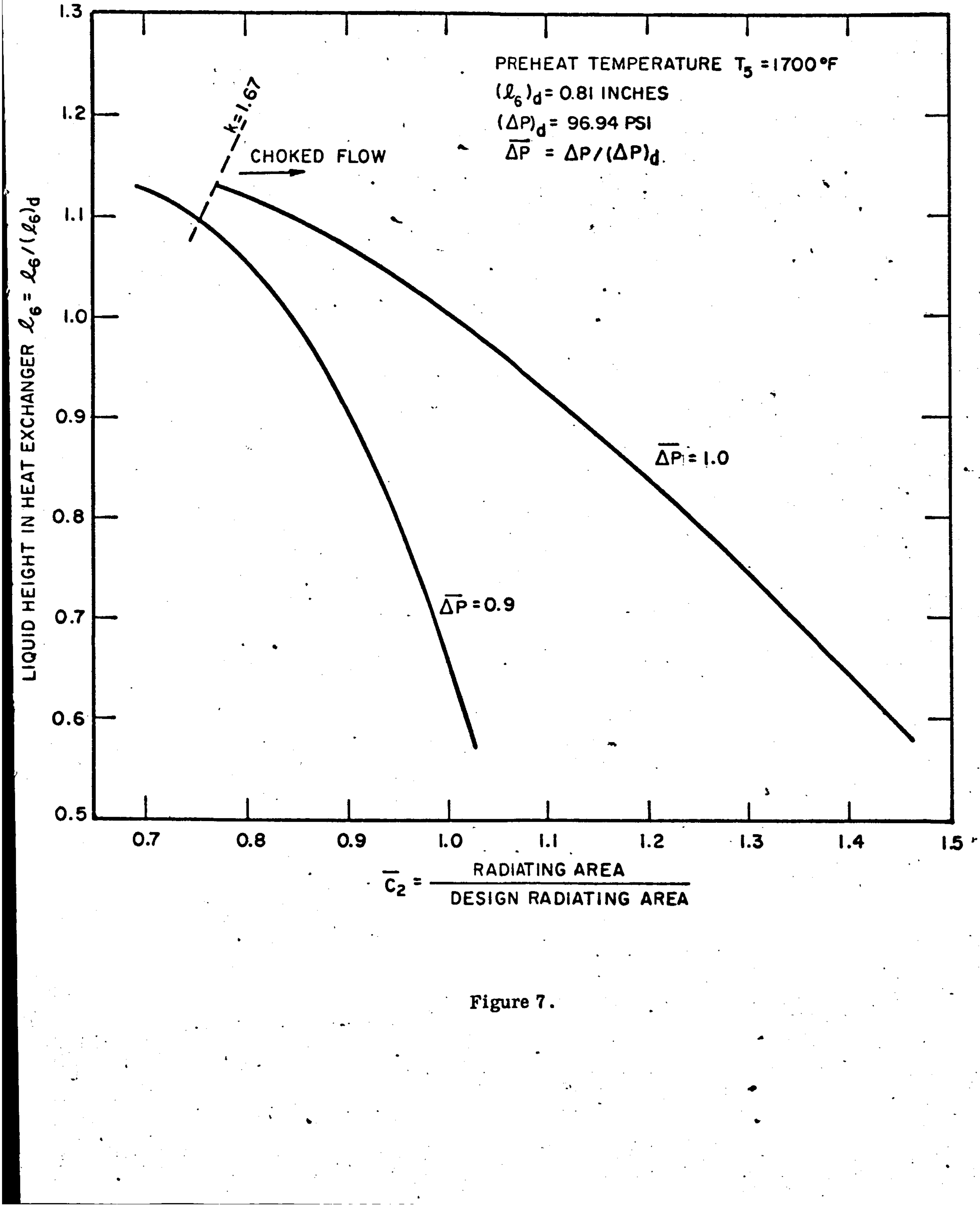
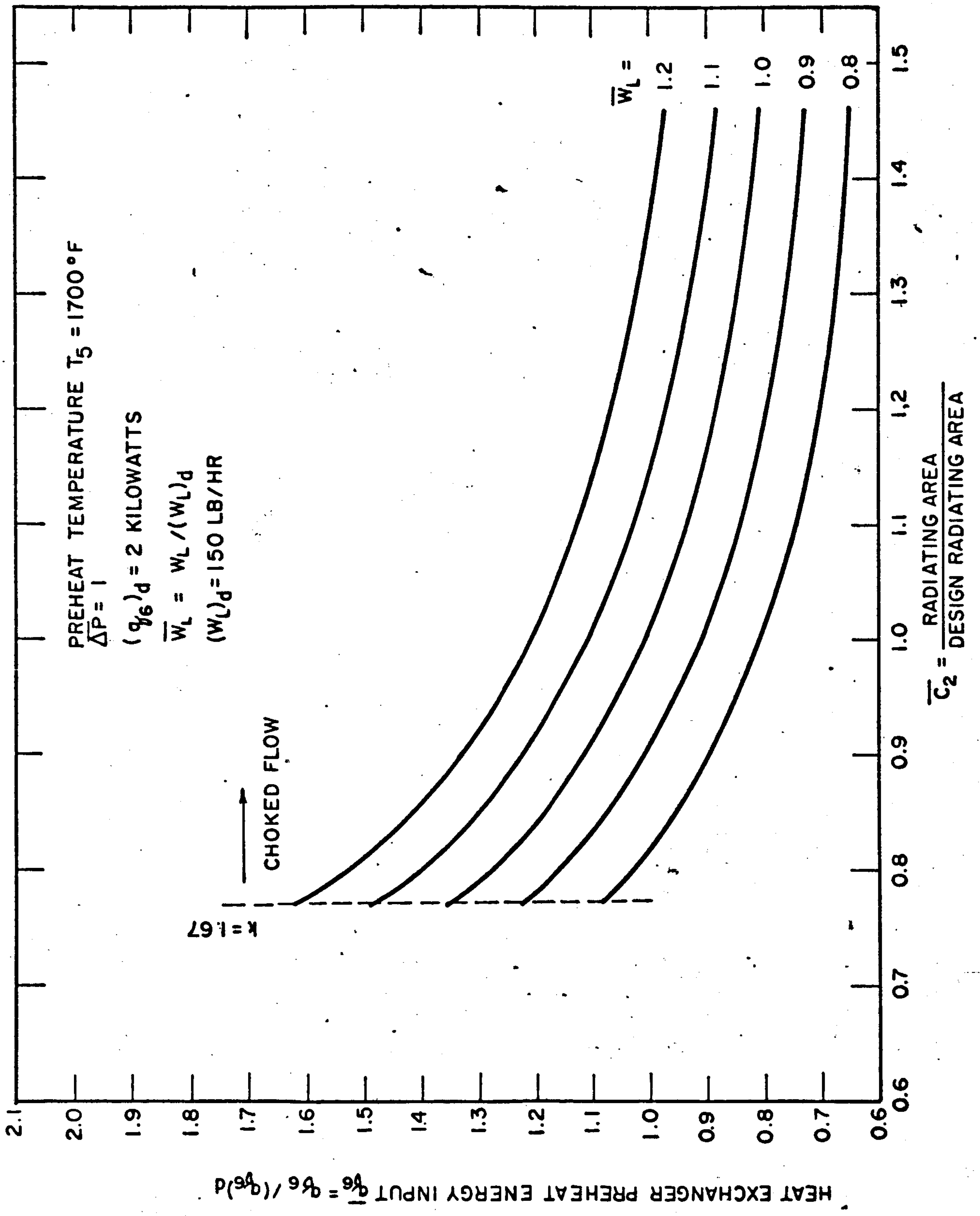

PREHEAT TEMPERATURE T_5 = 1700°F
$(\ell_6)_d$ = 0.81 INCHES
$(\Delta P)_d$ = 96.94 PSI
$\overline{\Delta P} = \Delta P/(\Delta P)_d$.
k = 1.67
CHOKED FLOW
$\overline{\Delta P}$ = 1.0
$\overline{\Delta P}$ = 0.9
LIQUID HEIGHT IN HEAT EXCHANGER $\ell_6 = \ell_6/(\ell_6)_d$
$\overline{C_2}$ = RADIATING AREA / DESIGN RADIATING AREA
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5

Figure 7.



PREHEAT TEMPERATURE T_5 = 1700°F
$\overline{\Delta P}$ = 1
$(q_6)_d$ = 2 KILOWATTS
$\overline{W_L} = W_L/(W_L)_d$
$(W_L)_d$ = 150 LB/HR
CHOKED FLOW
k = 1.67
$\overline{W_L}$ =
1.2
1.1
1.0
0.9
0.8
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
$\overline{C_2}$ = RADIATING AREA / DESIGN RADIATING AREA
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
HEAT EXCHANGER PREHEAT ENERGY INPUT $\overline{q_6} = q_6/(q_6)_d$

Figure 8.

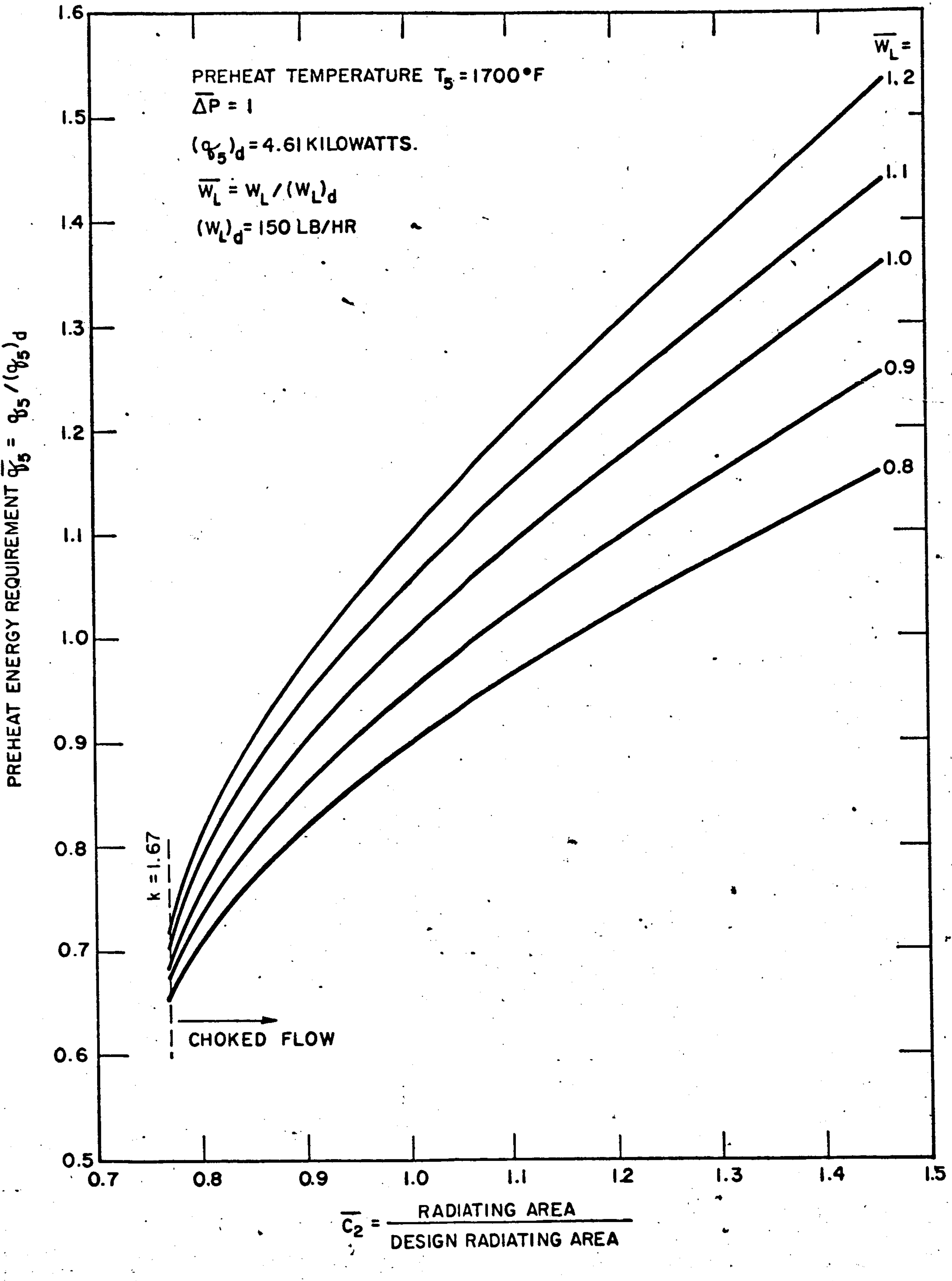

PREHEAT TEMPERATURE T_5 = 1700°F
$\overline{\Delta P}$ = 1
$(q_{\gamma 5})_d$ = 4.61 KILOWATTS.
$\overline{W_L} = W_L/(W_L)_d$
$(W_L)_d$ = 150 LB/HR
$\overline{W_L}$ =
1.2
1.1
1.0
0.9
0.8
k = 1.67
CHOKED FLOW
PREHEAT ENERGY REQUIREMENT $\overline{q_{\gamma 5}} = q_{\gamma 5}/(q_{\gamma 5})_d$
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
$\overline{C_2}$ = RADIATING AREA / DESIGN RADIATING AREA

Figure 9.

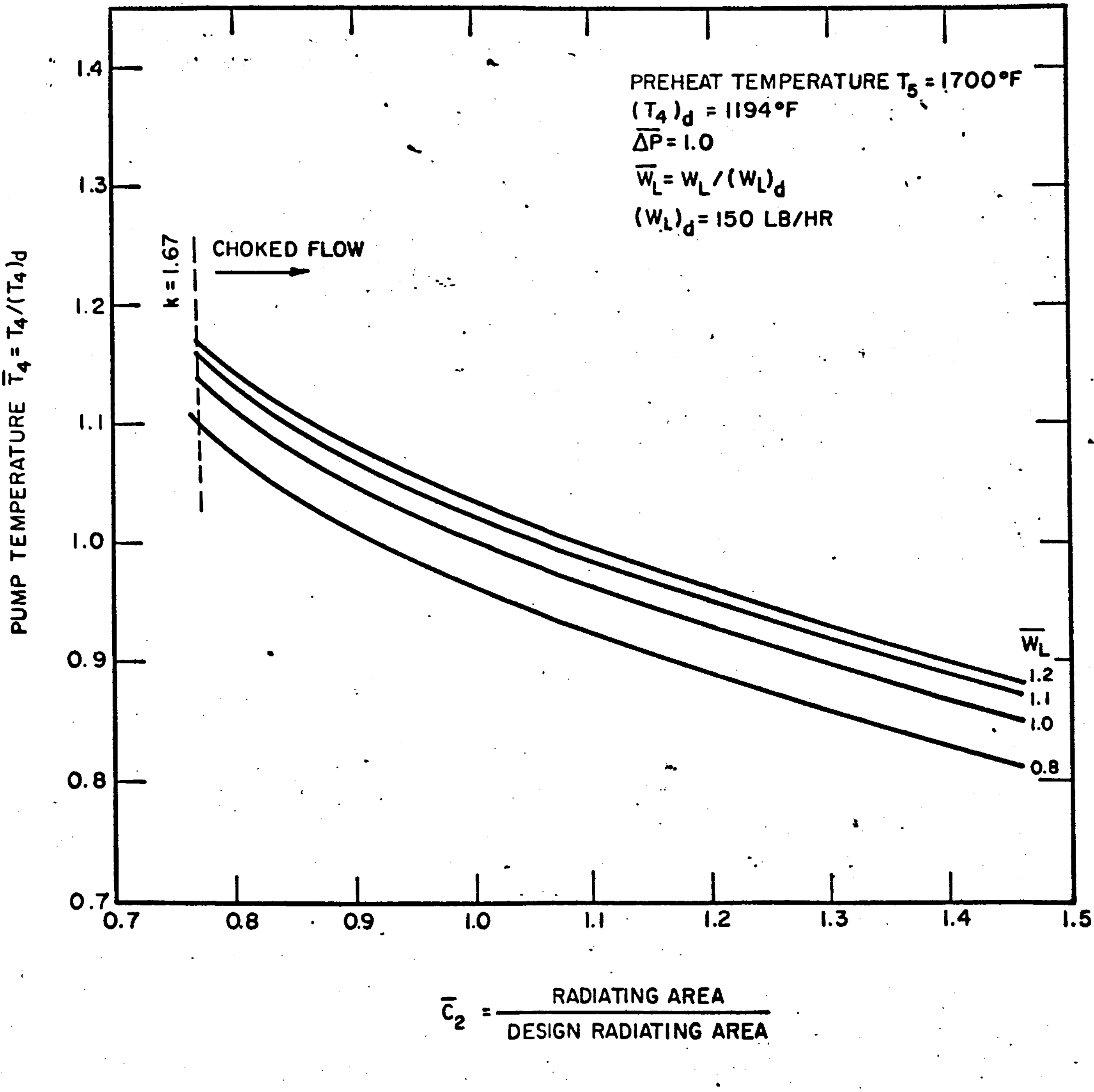

PREHEAT TEMPERATURE T_5 = 1700°F
$(T_4)_d$ = 1194°F
$\overline{\Delta P}$ = 1.0
$\overline{W_L} = W_L/(W_L)_d$
$(W_L)_d$ = 150 LB/HR
k = 1.67
CHOKED FLOW
PUMP TEMPERATURE $\overline{T_4} = T_4/(T_4)_d$
$\overline{C_2}$ = RADIATING AREA / DESIGN RADIATING AREA
$\overline{W_L}$
1.2
1.1
1.0
0.8
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5

Figure 10.

Equations

The following is a summation of the equations used to calculate the performance of the potassium circuit. The equations conform to the assumptions:

1. No heat additions or losses except in the heat exchanger, preheater and radiator.
2. The lithium temperature is constant throughout the length of the heat exchanger.
3. Only the gas phase effects the nozzle and orifice pressure drop.
4. The radiation coefficient is constant.
5. The condensing pressure drop is negligible.
6. The boiling temperature is uniform throughout the boiling length.
7. The pressure drop from the boiler to the nozzle can be approximated from the gas flow at the nozzle.
8. The boiler, nozzle and condenser are always in the mixed phase region.
9. The specific heat of the liquid is constant.
10. The pump is isentropic.

The equations are solved in the following order with the condition that the gas flow rate W_{g1}, liquid flow rate W_L, the preheat temperature T_5 and the shutter position C_2 are specified.

1. The pressure P_1 and temperature T_1 at the choked nozzle are given by Equations (1) and (2),

$$W_{g_1} = C_1 \frac{P_1}{\sqrt{T_1}} \tag{1}$$

T_1 = function of P_1 (vapor pressure curve). (2)

2. The pressure in the boiler, P_7, is obtained from Equation (3),

$$P_7 = P_1 + (f_7 + f_1) W^2_{g_1} \tag{3}$$

where $(f_7 + f_1)$ is the friction factor.

3. The temperature in the boiler, T_7, is obtained from the vapor pressure curve,

$$T_7 = \text{function of } P_7 \tag{4}$$

4. The quality at the choked nozzle, X_1, is given by Equation (5),

$$X_1 = \frac{W_{g_1}}{W_L} \tag{5}$$

5. The total enthalpy, H_T, in the mixed phase region is obtained from the Mollier diagram for given X_1 and T_1,

$$H_T = \text{function of } X_1 \text{ and } T_1 \tag{6}$$

6. The quality in the boiler, X_7, is obtained from the Mollier diagram for given H_T and T_7,

$$X_7 = \text{function of } H_T \text{ and } T_7 \tag{7}$$

7. The energy into the boiler is given by Equation (8),

$$q_7 = X_7 W_L h_v \tag{8}$$

where h_v = heat of vaporization

8. The lithium temperature T_8 is obtained from the transcendental Equation (9),

$$(T_8 - T_7) = \left(\frac{q_7}{h_7 A_{76}}\right) + \left(\frac{C_p W_L}{h_L A_{76}}\right) (T_8 - T_7) \ln\left(\frac{T_8 - T_5}{T_8 - T_7}\right) \tag{9}$$

where h_7 = boiler heat transfer coefficient

h_L = liquid heat transfer coefficient

C_p = specific heat

9. The heat exchanger preheat area, A_6, is given by Equation (10),

$$A_6 = A_{76} - \frac{q_7}{h_7 (T_8 - T_7)} \tag{10}$$

10. The heat exchanger preheat length, ℓ_6, is given by Equation (11),

$$\ell_6 = \ell_{76} \left(\frac{A_6}{A_{76}} \right) \tag{11}$$

11. The condenser temperature, T_2, is obtained from the transcendental Equation (12) with the help of the Mollier diagram,

$$T_2^{\,4} = \frac{X_2 W_L h_v}{C_2} \tag{12}$$

where C_2 = is a collection of radiation constants times the shutter radiating area

12. The pump temperature, T_4, is given by Equation (13),

$$T_4 = T_2 \sqrt[3]{\frac{1}{1 + \left(\frac{3 C_2 A_3}{C_p W_L} \right) \cdot T_2^{\,3}}} \tag{13}$$

where the liquid level in the condenser, ℓ_3, is taken constant (A_3 = constant) since this quantity varied no more than 3% for a wide range of flow conditions.

13. The preheat energy input, q_5, is given by Equation (14),

$$q_5 = C_p W_L (T_5 - T_4) \tag{14}$$

14. The pressure across the pump or pump head is given by Equation (15),

$$\Delta P = P_1 - P_2 + \rho_L \left\{ (L_5 - L_4) + (\ell_6 - \ell_3) \right\} + f_L W_L^{\,2} + (f_7 + f_1) W_{g_1}^{2} \tag{15}$$

where ρ_L = average density of the liquid

f_L = liquid friction factor

P_2 = condenser pressure which is obtained from the vapor pressure curve

Nomenclature

A	area
C_1	defined by Equation (1)
C_2	radiation constants times radiating area
C_p	constant specific heat
f_L	liquid friction factor
f_7+f_1	combined friction factor for station 7 and 1
h_7	boiler heat transfer coefficient
h_L	liquid heat transfer coefficient
h_v	heat of vaporization
H_T	total enthalpy
ℓ	length
P	pressure
ΔP	pressure rise across pump
q	energy flow rate
T	temperature
W_g	gas flow rate
W_L	liquid flow rate
X	quality
ρ_L	density of liquid

Subscripts

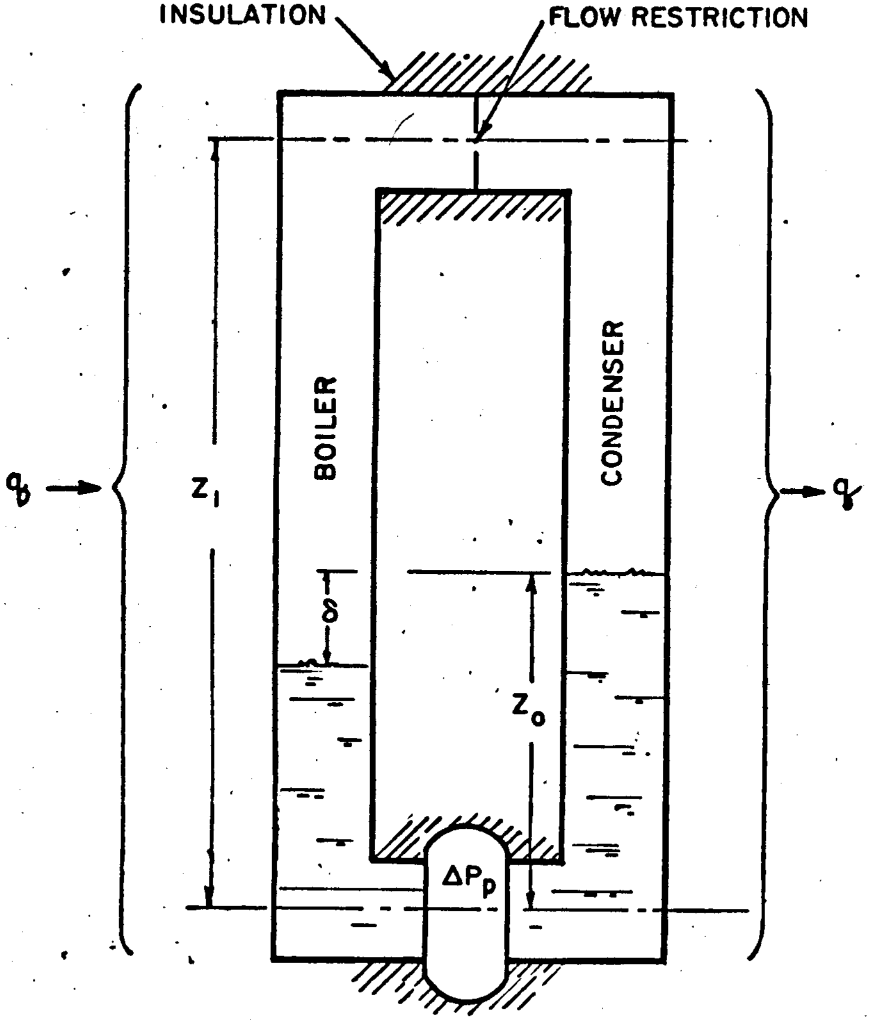
1	entrance to choked nozzle
2	condensation section of condenser
3	section of condenser filled with liquid

4	pump section
5	preheater
6	preheat section of boiler
7	boiling section of boiler
23	condenser
76	heat exchanger

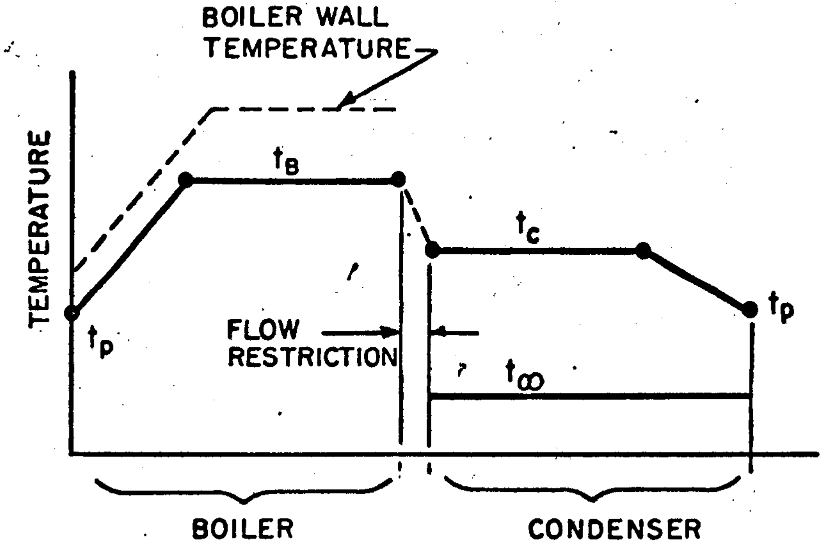
IV-C. STEADY STATE ANALYSIS OF AN IDEALIZED BOILER-CONDENSER SYSTEM WITH A CONSTANT, UNIFORM HEAT ADDITION AND AN UNCHOKED FLOW RESTRICTION

Introduction

A steady state analysis was made for an idealized boiler-condenser system in which heat was added to the boiler in a uniform manner with length. Further, the vapor flow through the flow restriction was postulated to be subsonic or unchoked. Also the pump heat was presumed to be a constant value. Preheating, boiling, condensation, and subcooling processes are included in the model. Schematic system and temperature diagrams are shown in Figure 1.


INSULATION
FLOW RESTRICTION
q_B
Z_1
BOILER
CONDENSER
δ
Z_0
ΔP_p
q

1A


BOILER WALL TEMPERATURE
TEMPERATURE
t_B
t_c
t_p
t_p
FLOW RESTRICTION
t_∞
BOILER
CONDENSER

1B

Analysis

The heat and momentum transfer equations which define the idealized system are presented below together with some qualifying postulates and conditions.

$$\Delta p_p + \delta w_\ell = K_o (x_B W)^2 \tag{1}$$

note: liquid and vapor pressure losses and acceleration pressure losses are neglected

$$K_o (x_B W)^2 = a(t_B - t_C) \tag{2}$$

note: a linearized vapor pressure is postulated, i.e. p = a t

$$q = W \left[c_{p_\ell} (t_B - t_p) + x_B L_B \right] \tag{3}$$

note: no heat is transferred from the pump or its entrance and discharge ducts

$$q = W \left[c_{p_\ell} (t_C - t_p) + x_C L_C \right] \tag{4}$$

$$q = h_r p_c (z_1 - z_o)(t_C - t_\infty) + h_r p_c (z_o) \left(\frac{t_C + t_p}{2} - t_\infty \right) \tag{5}$$

note: radiation transfer from the condenser is expressed in terms of the first power temperature equation with a variable conductance, h_r

$$q = W x_B L_B \left(\frac{z_1}{z_1 - z_o + \delta} \right) \tag{6}$$

$$W c_{p_\ell} (t_C - t_p) = h_r p_c z_o \left(\frac{t_C + t_p}{2} - t_\infty \right) \qquad (7)$$

Equations (1) to (7) represent a system of seven equations in seven unknowns (δ, x_B, x_C, W, t_C, t_B, and t_p). The uniform heat addition to the boiler, q, the pump pressure delivered, Δp_p, and the system geometry are given.

The method of solution was as follows:

a) From Equations (1) and (6) obtain δ and x_B W.

b) From Equation (3) obtain $W c_{p_\ell} (t_B - t_p)$.

c) From Equation (2) obtain $t_B - t_C$.

d) For a trial t_p calculate t_C from Equation (5).

e) From $t_B - t_C$ obtain t_B.

f) From $W c_{p_\ell} (t_B - t_p)$ obtain W.

g) From Equation (7) calculate t_p and repeat steps d), e), f) (i.e. iterate).

h) From x_B W obtain x_B.

i) From Equations (3) and (4) obtain x_C.

Two Specific Solutions

It was desired to establish the changes in system variables as change in one of the system parameters occurred. For example, the differences in the system variables were calculated for a 10% change in the radiator shutter opening (i.e. h_r was changed from 7.75 Btu/hr ft^2 °F to 7.0 Btu/hr ft^2) for the following system parameters: $q = 120{,}000$ Btu/hr

$c_{p_\ell} = 0.21$ Btu/lb °F

$w_\ell = 39 \text{ lb/ft}^3$

$\Delta p_p = 12{,}600 \text{ lb/ft}^2$

$a = 31.7 \text{ lb/ft}^2 \,°\text{F}$

$L_B = 754 \text{ Btu/lb}$

$L_C = 822 \text{ Btu/lb}$

$A_r = 9.4 \text{ ft}^2$

$p_c = 3.1 \text{ ft}$

$A_B = 0.38 \text{ ft}^2$

$z_1 = 3.0 \text{ ft}$

$z_o = 0.33 \text{ ft}$

$$h_r = \frac{q}{A_r(t_C - t_\infty)}$$

$t_\infty = 200°\text{F}$

$K_o = 1.5 \text{ hr}^2/\text{lb ft}^2$

The results for $h_r = 7.75 \text{ Btu/hr ft}^2 °\text{F}$ are:

$x_B W = 92 \text{ lb/hr}$

$W = 263 \text{ lb/hr}$

$x_B = 0.35$

$x_C = 0.52$

$\delta = -0.63 \text{ ft}$

$t_C = 1628°\text{F}$

$t_B = 2028°\text{F}$

$t_p = 1437°\text{F}$

The results for h_r = 7.0 Btu/hr ft^2 °F are:

$x_B W$ = 92 lb/hr

W = 263 lb/hr

x_B = 0.35

x_C = 0.52

δ = - 0.63 ft

t_C = 1763 °F

t_B = 2163 °F

t_p = 1573 °F

Nomenclature

A_B	heat transfer area of boiler
A_r	heat transfer area of radiator or condenser
a	constant in linearized vapor pressure equation, $p = at$
c_{p_ℓ}	heat capacity of liquid coolant
h_r	radiation heat transfer conductance of radiator
K_o	flow rate coefficient of unchoked flow constriction
L_B	latent heat of vaporization in boiler
L_C	latent heat of vaporization in condenser
p_c	perimeter of the radiation area of the radiator
q	uniform and constant heat addition to boiler
t_B	mean or saturation temperature of the working fluid in the boiler
t_C	saturation temperature of the condensate in the condenser
t_p	temperature of the subcooled liquid in the pump
t_∞	mean radiant temperature of the surroundings
W	total coolant flow rate
w_ℓ	weight density of the liquid coolant
x_B	vapor quality at exit of boiler
x_C	vapor quality at entrance of condenser
z_o	liquid level in the condenser

z_1	elevation of top of boiler and condenser
Δp_p	constant pump pressure rise developed
δ	deflection of liquid lievel in boiler (see Figure 1)

V-A. TRANSIENT ANALYSIS OF A SIMPLE MODEL OF THE JPL BOILING POTASSIUM POWER SYSTEM WITH A CHOKED NOZZLE (Model 1)

This analysis describes a simple model of the JPL Boiling Potassium Power System, operating in the region in which the turbine nozzle (or flow orifice in the case in which the turbine bypass is opened) is choked. Under choking conditions the nozzle flow is dependent upon the upstream conditions only. The model cycle is made up of four thermodynamic processes, which are: 1) constant pressure boiling to a quality less than unity; 2) constant enthalpy throttling to the condenser pressure; 3) constant pressure condensation; isentropic pressure rise across the pump.

To simplify the analysis the following idealizations have been made:

1) The nozzle is choked, and its pressure drop depends only on the gaseous component of the flow.

2) Subcooling in the condenser and preheating in the boiler are negligible.

3) The pump pressure rise is constant.

4) The power removed by the turbine is negligible. Thus the two circuits containing either the turboalternator or the bypass line are identical, except in the value of nozzle discharge coefficient used.

5) Heat losses and additions are confined to the condenser and lithium heater respectively.

6) There is significant thermal energy storage capacity only in the lithium loop and condenser.

7) There is no significant gas storage anywhere in the system. The gas generation rate in the boiler equals the gas condensation rate in the condenser at any instant.

8) The lithium circuit is considered to be a heat storage reservoir with a uniform temperature and an electrical input. The uniform lithium temperature is equivalent to an assumption that the circulation rate is infinite.

9) The boiler is idealized as a heat exchanger with a constant mean heat transfer coefficient.

Other approximations, idealizations and assumptions will be made as they are required.

An energy balance on the condenser yields

$$w_g = \frac{C_2}{h_{fg}} T_2^4 + \frac{C_c}{h_{fg}} \frac{dT_2}{dt} \tag{1}$$

where the condenser temperature is assumed uniform at any instant, fluid to wall temperature drop has been neglected, and the inlet fluid is always considered to be in the mixed phase.

An energy balance on the lithium circuit-boiler assembly yields

$$q_e = hA\,(T_{Li} - T_1) + C_{Li} \frac{dT_{Li}}{dt} \tag{2}$$

an energy balance on the boiler yields

$$w_g h_{fg} = hA\,(T_{Li} - T_1) \tag{3}$$

The properties of the turbine nozzle are such that

$$w_g = C_1 \frac{p_1}{T_1^{\frac{1}{2}}} \tag{4}$$

The pump equation is

$$p_{out} - p_{in} = \text{constant} = \Delta p_p \tag{5}$$

The liquid flow rate governing equation is

$$p_2 + \Delta p_p = p_1 + M_L \frac{dv_L}{dt} + C_f w_L^2 \qquad (6)$$

where the approximation is made that all of the gaseous pressure drop is in the turbine nozzle, all of the mass is in the liquid phase, compressibility effects are negligible, and changes in liquid elevation are negligible.

It has been shown previously (see Reference 1) that for the particular power system considered here, the flow transients have declined to the 1% level in the order of 0.1 seconds, and since it will be seen that all other system time constants are several orders of magnitude larger than this, flow accommodation will be assumed instantaneous, so that

$$p_2 + \Delta p_p = p_1 + C_f w_L^2 \qquad (7)$$

or

$$\Delta p_p = \Delta p_n + C_f w_L^2$$

where Δp_n denotes $p_1 - p_2$, the nozzle pressure drop.

The design gas flow rate is given by four expressions, which are compatible with the idealizations. These expressions are:

$$w_{g_d} = \frac{C_1 p_{1d}}{T_{1_d}^{\frac{1}{2}}} = \frac{hA}{h_{fg}} \left(T_{Lid} - T_{1d} \right) = \frac{C_{2d} T_{2d}^4}{h_{fg}} = \frac{q_{ed}}{h_{fg}} \qquad (8)$$

where the subscript d denotes the design condition. The governing equations may be normalized on the design conditions. In normalized form, they are:

Lithium circuit -

$$\frac{q_e}{T_{1d} hA} = \bar{T}_{Li} - \bar{T}_1 + \frac{d\bar{T}_{Li}}{d\bar{t}} \qquad (9)$$

where

$$\bar{T}_{Li} = \frac{T_{Li}}{T_{1d}}$$

$$\overline{T}_1 = \frac{T_1}{T_{1d}}$$

$$\overline{t} = \frac{t}{\tau_1}$$

$$\tau_1 = \frac{C_{Li}}{hA}$$

Boiler -

$$\overline{w}_g = \frac{T_{1d}}{\Delta T_d}\left(\overline{T}_{Li} - \overline{T}_1\right) \tag{10}$$

where

$$\overline{w}_g = \frac{w_g}{w_{gd}}$$

$$\Delta T_d = T_{Lid} - T_{1d}$$

Nozzle -

$$\overline{w}_g = \frac{\overline{p}_1}{\overline{T}_1^{\frac{1}{2}}} \tag{11}$$

where

$$\overline{p}_1 = \frac{p_1}{p_{1d}}$$

Condenser -

$$\overline{w}_g = C_2 \overline{T}_2^{\,4} + \frac{d\overline{T}_2}{d\overline{t}} \tag{12}$$

where

$$\overline{C}_2 = \frac{C_2}{C_{2d}}$$ to allow for changes in condenser shutter position,

$$\bar{T}_2 = \frac{T_2}{T_{2d}}$$

$$\hat{t} = \frac{t}{\tau_2}$$

and

$$\tau_2 = \frac{C_c}{C_{2d} T_{2d}^3}$$

An analytical expression for the vapor pressure - temperature relationship is required to solve this system of equations. An exponential form has been chosen, with both the temperature and temperature normalized on the design conditions, to be compatible with the other relationships.

Two relationships are thus required, one for the boiler, and one for the condenser.

Boiler -

$$\bar{p}_1 = e^{\frac{T_{1d}}{T_A}(\bar{T}_1 - 1)} \qquad (13)$$

where

$$T_A = 315°F$$

$$T_{1d} = 2360°R$$

Condenser -

$$\bar{p}_2 = e^{\frac{T_{2d}}{T_B}(\bar{T}_2 - 1)} \qquad (14)$$

where

$$T_B = 220°F$$

$$T_{2d} = 1960°R$$

In order to solve this system analytically it is necessary to linearize the boiler vapor pressure relationship. To the first order in the (small) quantity $(T_1 - 1)$,

$$\bar{p}_1 \approx 1 + \frac{T_{1d}}{T_A} (\bar{T}_1 - 1)$$

$$\bar{T}_1^{\frac{1}{2}} \approx 1 + \frac{(\bar{T}_1 - 1)}{2}$$

to simplify the notation, let $(\bar{T}_1 - 1) = x$. Then in linearized form the nozzle relationship is

$$\bar{w}_g = \frac{1 + \frac{T_{1d}}{T_a} x}{1 + \frac{x}{2}} \tag{15}$$

Equate expressions (10) and (15) to eliminate $\bar{w}_g$, and solve for x

$$x = \frac{1 - \frac{T_{1d}}{\Delta T_d} (\bar{T}_{Li} - 1)}{\left[\frac{T_{1d}}{2\Delta T_d} (\bar{T}_{Li} - 1) - \frac{T_{1d}}{\Delta T_d} - \frac{T_{1d}}{T_A} \right]} \tag{16}$$

Introduce some additional simplifying notation

$$\bar{T}_{Li} - 1 = y$$

$$\frac{T_{1d}}{T_A} = r$$

$$\frac{T_{1d}}{\Delta T_d} = s$$

then

$$x = \frac{1 - sy}{\frac{s}{2} y - s - r} \tag{17}$$

In the same notation, the lithium circuit equation is

$$\frac{dy}{d\bar{t}} + y - x = Q \tag{18}$$

where

$$Q = \frac{q_e}{hAT_{1d}}$$

In this equation, x can be replaced to yield a differential equation for y. The resulting equation is

$$\frac{dy}{d\bar{t}} + y + \frac{1 - sy}{s + r - \frac{s}{2}y} = Q \tag{19}$$

One of the terms in this equation is of the order of 2%, and will be neglected. The reasoning is as follows: At the design point,

$$y = \frac{\Delta T_d}{T_{1d}} = \frac{100°F}{2360°R} = \frac{1}{23.6}$$

$$s = \frac{T_{1d}}{\Delta T_d} = 23.6$$

$$r = 7.48$$

then

$$\frac{1}{s + r - \frac{s}{2}y} = \frac{1}{23.6 + 7.48 - \frac{1}{2}}$$

1/2 is neglected with respect to 31. The equation is then approximately

$$\frac{dy}{dt} + \frac{r}{s + r}y = Q - \frac{1}{s + r} \tag{20}$$

The initial condition is

$\bar{t} = 0, \quad y = y_d, \quad Q = \text{constant for } \bar{t} > 0$

In order that the system start at the design condition.

The solution, assuming in impulsive change of the electrical input Q, at t = 0, is

$$\frac{y_f - y}{y_f - y_d} = e^{-\frac{r\bar{t}}{r+s}} \tag{21}$$

where y_d is the design condition, and y_f is the final value of y at $t \rightarrow \infty$, given by

$$y_f = \frac{(r + s)\,Q - 1}{r} \tag{22}$$

This defines the lithium temperature as a function of time.

Notice that the time constant for the lithium circuit-boiler-nozzle combination is considerably modified from the lithium circuit-boiler time constant $\tau_1 = C_{Li}/hA$, and is now $[(r + s)/r)]\,\tau_1$, where $(r + s)/r = 4.16$. The temperature of the potassium in the boiler may now be written

$$x = \frac{x_f\,(1 - e^{-\tilde{t}})}{1 + \frac{x_f}{2}\,e^{-\tilde{t}}} \tag{23}$$

where

$$\tilde{t} = \frac{r}{r + s}\,\bar{t}$$

and

$$x_f = \frac{s y_f - 1}{r + s\,(1 - \frac{y_f}{2})}$$

To the first order, the boiler pressure is as before

$$\bar{p}_1 = 1 + rx$$

The gas flow rate through the nozzle is, to the first order

$$\bar{w}_g = \bar{w}_{gf} - (\bar{w}_{gf} - 1)\,e^{-\tilde{t}} \tag{24}$$

where

$$\bar{w}_{gf} = \frac{1 + r \cdot x_f}{1 + \frac{1}{2} x_f}$$

or

$$\bar{w}_{gf} = \frac{Q}{Q_d} \quad .$$

The condenser, which under the assumptions can affect the boiler only through the pressure at the inlet of the pump and the corresponding liquid flow rate, has the non-linear equation

$$\bar{w}_{gf} - (\bar{w}_{gf} - 1)\, e^{-\tilde{t}} = \bar{C}_2 \bar{T}_2^{\,4} + \frac{d\bar{T}_2}{d\hat{t}} \tag{25}$$

There are two time bases in this equation, but they have a common zero and are linearly related, so one can write

$$\hat{t} = k\tilde{t} \qquad \text{where } k = \frac{r + s}{r}\,\frac{\tau_1}{\tau_2} \tag{26}$$

then

$$\bar{w}_{gf} - (\bar{w}_{gf} - 1)\, e^{-\tilde{t}} = \bar{C}_2 \bar{T}_2^{\,4} + \frac{1}{k}\,\frac{d\bar{T}_2}{d\tilde{t}} \tag{27}$$

Again allow only small excursions in T_2, so that this equation may be linearized. The condenser pressure excursions are some eight times larger than the temperature excursions, so the condenser pressure expression is not linearized.

In linear form, Equation (27) becomes

$$\frac{dz}{d\hat{t}} + 4k\,\bar{C}_2\, z = k\,\bar{w}_{gf} - k\,\bar{C}_2 - k\,(\bar{w}_{gf} - 1)\, e^{-\tilde{t}} \tag{28}$$

where

$$z = \bar{T}_2 - 1 \quad .$$

The initial condition need not be the design point, so at $t < 0$ $z = z_i$, $\bar{C}_2 = \bar{C}_{2i}$. Since $\bar{C}_2$ is a function of radiator shutter position. An impulsive change in $\bar{C}_2$ at $\tilde{t} = 0$ is allowed. The solution to Equation (28) subject to these boundary conditions is

$$z = z_f - \frac{k(\bar{w}_{gf} - 1)}{4k\bar{C}_2 - 1} e^{-\tilde{t}} - \left[(z_f - z_i) - \frac{k(\bar{w}_{gf} - 1)}{4k\bar{C}_2 - 1} \right] e^{-4k\bar{C}_2 \tilde{t}} \qquad (29)$$

where

$$z_f = \frac{\bar{w}_{gf} - \bar{C}_2}{4\bar{C}_2}$$

$$z_i = \frac{1 - \bar{C}_{2i}}{4\bar{C}_{2i}}$$

For the special case $4k\bar{C}_2 = 1$, where the solution given by Equation (29) has a singularity, the solution is

$$z = z_f - k(\bar{w}_{gf} - 1)\tilde{t} e^{-\tilde{t}} - (z_f - z_i) e^{-\tilde{t}} \qquad (30)$$

The condenser pressure is given by

$$\bar{p}_2 = e^{\frac{T_{2d}}{T_B} z} \qquad (31)$$

The pressure drop across the nozzle is

$$p_{1d} \left[\bar{p}_1 - \frac{p_{2d}}{p_{1d}} \bar{p}_2 \right] = \Delta p_n \qquad (32)$$

This relation is easily evaluated from the forgoing equations.

The variation in liquid flow rate with time can now be computed. Normalize the flow rate equation on the design boiler pressure p_{1d}. Then

$$\Delta\bar{p}_p = \Delta\bar{p}_n + \bar{C}_f \bar{w}_L^2 \qquad (33)$$

where

$$\Delta\bar{p}_n = \frac{p_1 - p_2}{p_{1d}}$$

$$\bar{w}_L = w\left(\frac{C_{fd}}{p_{1d}}\right)^{\frac{1}{2}}$$

$$\bar{C}_f = \frac{C_f}{C_{fd}}$$

C_{fd} is the design liquid frictional coefficient, and C_f is a modified friction coefficient due to, for example, a partially closed valve.

The pump pressure rise is assumed fixed, so

$$\bar{w}_L = \left(\frac{1}{\bar{C}_f}\right)^{\frac{1}{2}} \left[\Delta\bar{p}_p - \Delta\bar{p}_n\right]^{\frac{1}{2}} \qquad (34)$$

where $\Delta\bar{p}_n$ is a function of time.

This completes the system of equations required to give the properties of the model. Two specific examples have been computed to demonstrate the manner in which the variables change with time, assuming an impulsive change in the parameters q_e and C_2 at $\tilde{t} = 0$. The specific examples are ones in which both the electrical input and the condenser shutter position are changed to give a final nozzle pressure drop identical to that which existed at the beginning. Thus the initial and final liquid flow rates will also be identical.

First Example

Initially $q_e = q_{ed}$

$\bar{C}_2 = 1$

finally $q_e = 1.05\, q_{ed}$

$\bar{C}_2 = 0.959$

Second Example

Initially $q_e = q_{ed}$

$\bar{C}_2 = 1$

Finally $q_e = 1.11\, q_{ed}$

$\bar{C}_2 = .90$

Numerical values which were used to make the calculations are

$T_{1d} = 2360°R$ $p_{1d} = 7.65\text{ atm} = 112.3\text{ psi}$

$T_{2d} = 1960°R$ $p_{2d} = 1.67\text{ atm} = 24.5\text{ psi}$

$T_{Li} = 2460°R$

$T_A = 315°F$

$T_B = 220°F$

$C_{Li} = 4.6\text{ Btu/°F}$

$q_{ed} = 30\text{ kw} = 1.025 \times 10^5\text{ Btu/hr}$

$hA = 1.023 \times 10^3\text{ Btu/hr°F}$

$\tau_1 = 16.2\text{ sec}$

$C_c = 3.58\text{ Btu/°F}$

$\tau_2 = 246\text{ sec}$

$h_{fg} = 850\text{ Btu/lb}$

$C_1 = 52.1\text{ lbm in}^2/\text{lb}_f\text{ hr }(°R)^{\frac{1}{2}}$

$C_{2d} = 6.93 \times 10^{-9}\text{ Btu/hr°R}^4$

The effective time constant of the lithium loop-boiler-nozzle system is 67.2 sec. Graphs of the variables for example 1, are shown in Figures 1 and 2. Graphs of the variables for example 2, are shown in Figures 3 and 4.

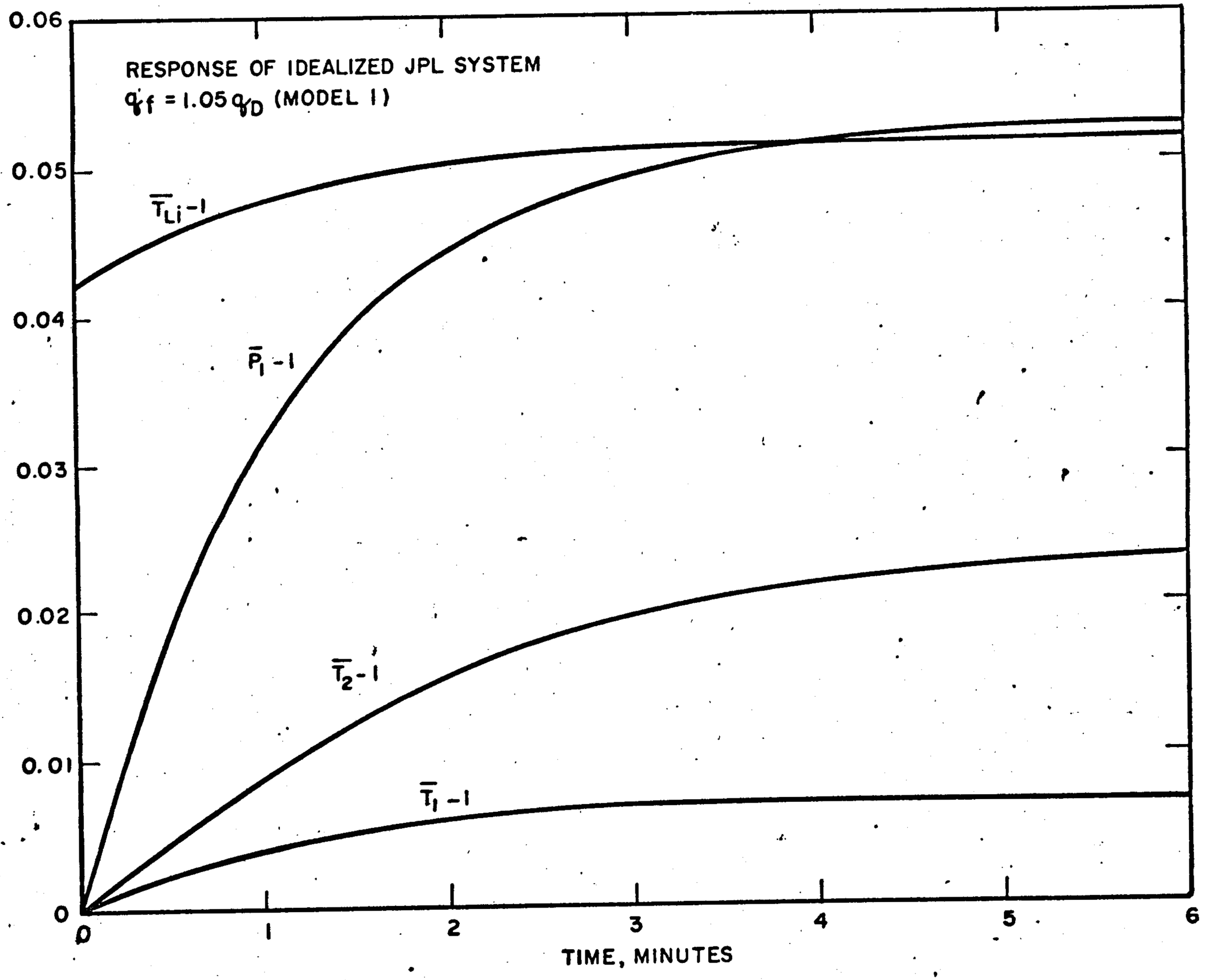

RESPONSE OF IDEALIZED JPL SYSTEM
$q_f = 1.05 q_D$ (MODEL 1)
$\overline{T}_{Li}-1$
$\overline{P}_1-1$
$\overline{T}_2-1$
$\overline{T}_1-1$
0.06
0.05
0.04
0.03
0.02
0.01
0
0
1
2
3
4
5
6
TIME, MINUTES

Figure 1.

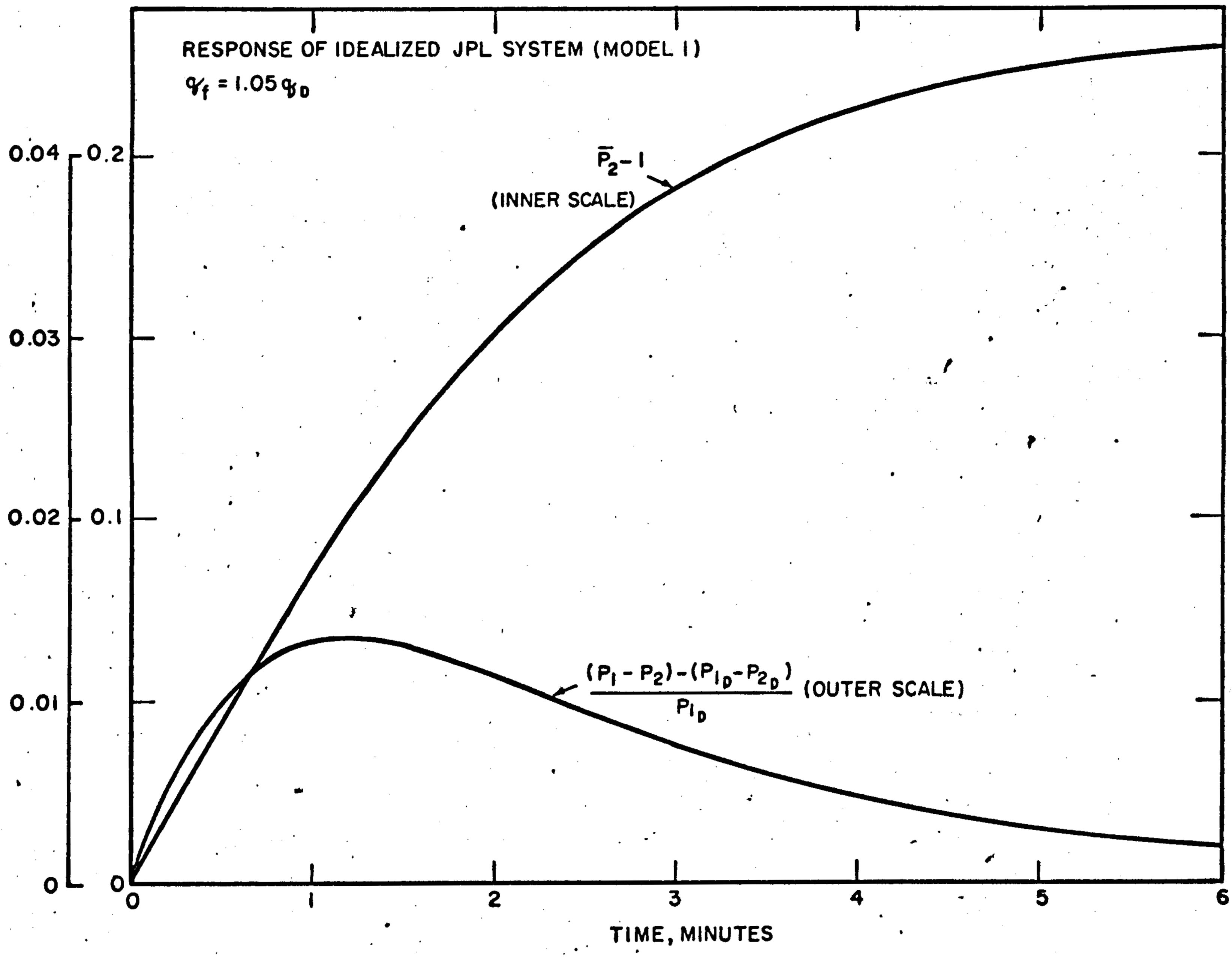

RESPONSE OF IDEALIZED JPL SYSTEM (MODEL I)
$q_f = 1.05 q_D$
$\bar{P}_2 - 1$
(INNER SCALE)
$\frac{(P_1 - P_2) - (P_{1_D} - P_{2_D})}{P_{1_D}}$ (OUTER SCALE)
0.04
0.03
0.02
0.01
0
0.2
0.1
0
0
1
2
3
4
5
6
TIME, MINUTES

Figure 2.

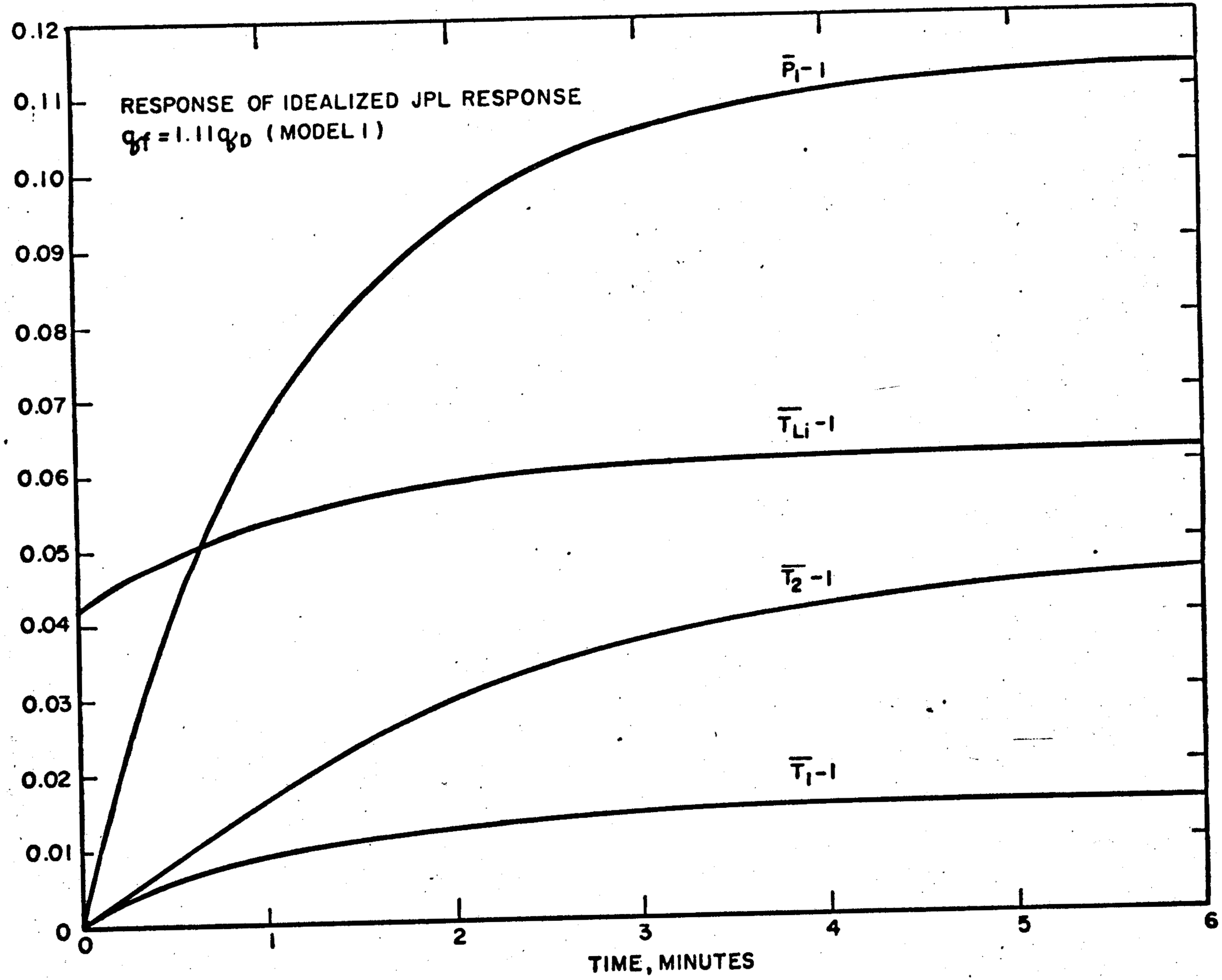

RESPONSE OF IDEALIZED JPL RESPONSE
$q_{\gamma f} = 1.11 q_{\gamma D}$ (MODEL I)
$\bar{P}_1 - 1$
$\overline{T_{Li}} - 1$
$\overline{T_2} - 1$
$\overline{T_1} - 1$
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
1
2
3
4
5
6
TIME, MINUTES

Figure 3.

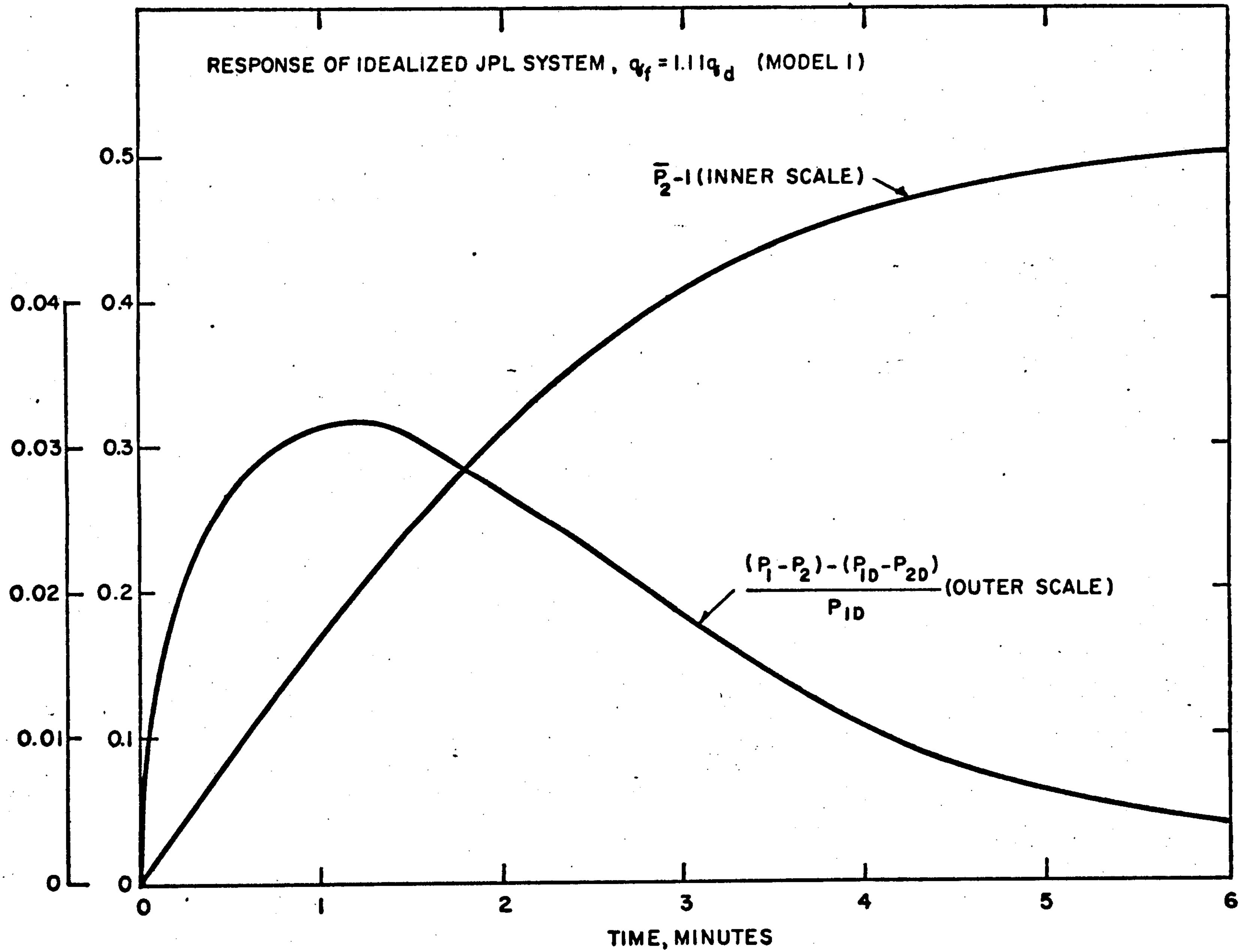

RESPONSE OF IDEALIZED JPL SYSTEM, $q_f = 1.11 q_d$ (MODEL I)
$\bar{P}_2 - 1$ (INNER SCALE)
$\frac{(P_1 - P_2) - (P_{1D} - P_{2D})}{P_{1D}}$ (OUTER SCALE)
0.5
0.4
0.3
0.2
0.1
0
0.04
0.03
0.02
0.01
0
0
1
2
3
4
5
6
TIME, MINUTES

Figure 4.

Calculation of Liquid Flow Rate for the Examples

In the calculations for steady state pressure drop as presented in Reference (1), it was shown that

$$C_{fd}\, w_{Ld}^{\;2} \approx 0.8 \text{ psi}$$

so that in the present model

$$\Delta p_{pd} - \Delta p_{nd} = 0.8 \text{ psi}$$

In the dimensionless sytem,

$$\bar{w}_{Ld}^{\;2} = \frac{\Delta p_p}{p_{1d}} - \frac{\Delta p_n}{p_{1d}}$$

$$= .72 \times 10^{-2} \qquad (35)$$

Excursions from the design flow rate are given by

$$\bar{w}_L^{\;2} - \bar{w}_{Ld}^{\;2} = -\frac{1}{\bar{C}_f}\left[\frac{p_1 - p_2}{p_{1d}} - \frac{p_{1d} - p_{2d}}{p_{1d}}\right]$$

Since the pump pressure rise is fixed, the function in the brackets on the right hand side of this equation is plotted for the two examples in Figures 2 and 4. Notice that it is a positive quantity, and that therefore $\bar{C}_{fd}\,\bar{w}_{Ld}^{2} > C_f\,\bar{w}_L^{\;2}$. Thus the flow rate decreases during the pressure surge. Notice also, that if $C_f = 1$, so that the design liquid friction coefficient applies, that the magnitude of pressure surge is greater than the design value of $\bar{C}_f\,\bar{w}_L^{\;2}$ (Equation (35)) so that the flow reverses. The model breaks down under such conditions.

Once concludes that if this model applies, that even quite moderate excursions from the design power output will cause large excursions in liquid flow rate and boiler exit quality. A simple cure for the possible flow reversals would be to increase the liquid friction coefficient, so that the liquid pressure drop exceeds the design value. The pump pressure rise will thus be larger, but the quantity $(p_1 - p_2)$ will remain the same since it is controlled solely by the temperature levels. This statement may be justified as follows:

In Equation (33), $\Delta\bar{p}_p$ may be found for any value of $\bar{C}_f$, such that

$$\Delta p_p = \Delta\bar{p}_{nd} + \bar{C}_f \bar{w}_{Ld}^2$$

then one can write

$$\left(\frac{w_L}{w_{Ld}}\right)^2 = \frac{\Delta\bar{p}_p - \Delta\bar{p}_n}{\bar{C}_f (\bar{w}_{Ld})^2}$$

or since

$$\Delta p_p - \Delta p_{pd} = \bar{w}_{Ld}^2 (\bar{C}_f - 1)$$

then by substitution and rearrangement

$$\left(\frac{w_L}{w_{Ld}}\right)^2 = 1 - \frac{\Delta\bar{p}_n - \Delta\bar{p}_{nd}}{\bar{C}_f (\bar{w}_{Ld})^2} \qquad (36)$$

It has already been shown that for power increases,

$$\Delta p_n > \Delta p_{nd}$$

The numerator on the right hand side of Equation (36) does not depend on C_f, so there is always a $\bar{C}_f$ large enough that $(w_L/w_{Ld})^2$ be greater than zero. In fact, within the limitations of the pump pressure rise, the flow rate can be made as close to constant as desired by increasing $\bar{C}_f$ to a value much larger than unity. The flow rate variation for the two examples has been plotted in Figure 5, using a value of $\bar{C}_f$ of 10.

Discussion

There are several obvious weaknesses in this model. One is the complete lack of coupling from the condenser to the boiler. Since the heat transfer in the boiler and flow rate through the nozzle are both considered to be independent of the liquid flow rate, no change in the condenser affects the boiler temperatures or pressures. This is unrealistic for operating conditions where the quality in the boiler is high enough that the heat transfer coefficient has begun to decrease. At

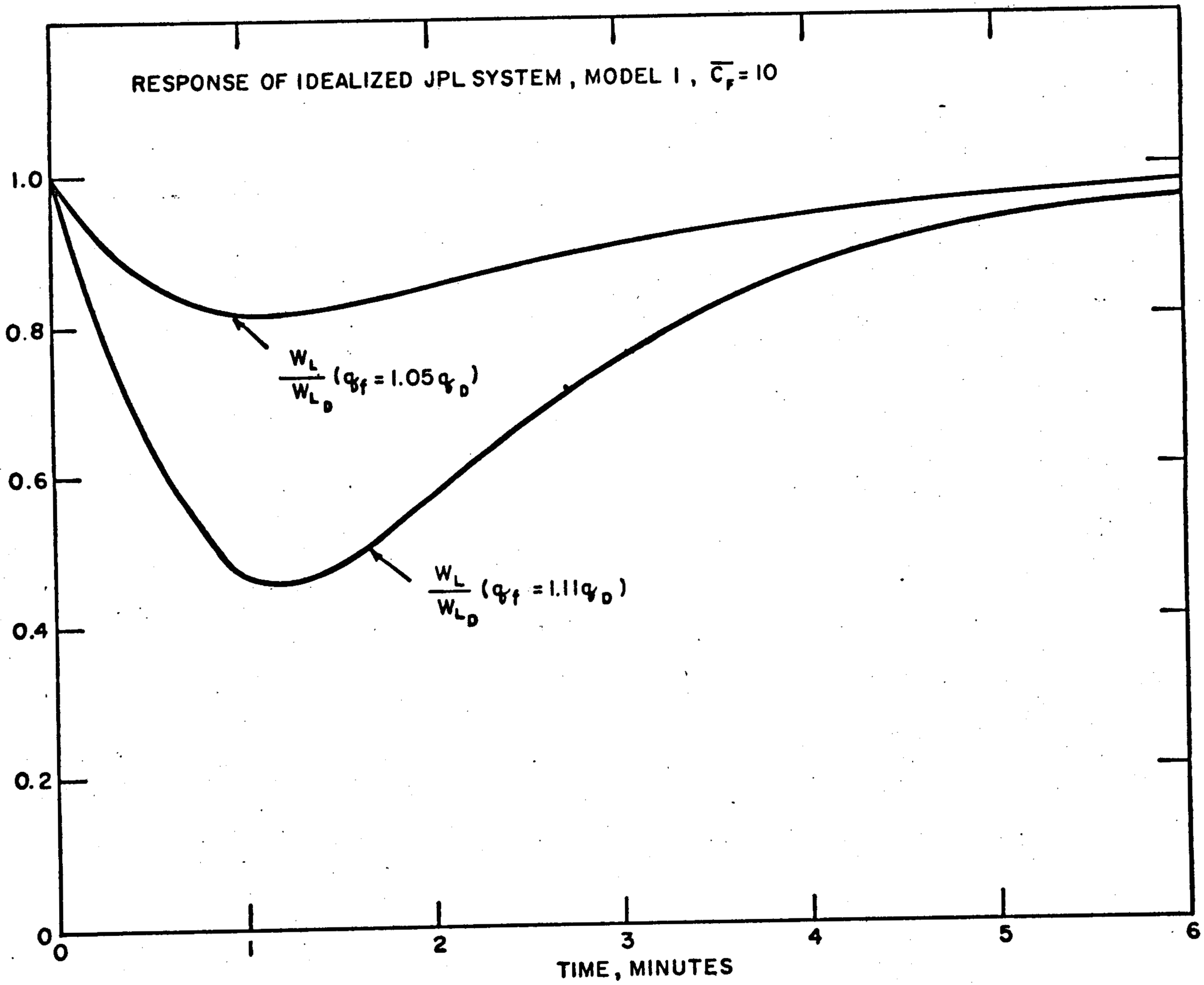

RESPONSE OF IDEALIZED JPL SYSTEM, MODEL 1, $\overline{C_F} = 10$
$\frac{W_L}{W_{L_D}} (q_{\theta f} = 1.05 q_{\theta D})$
$\frac{W_L}{W_{L_D}} (q_{\theta f} = 1.11 q_{\theta D})$
1.0
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
TIME, MINUTES

Figure 5.

fairly low qualities this approximation should be fair, and one would expect this model to represent the actual case performance of the system at least qualitatively in the low quality region. Certainly there is little doubt that flow transients will occur. The transients exhibited by this model are not violent; the thermal time constants of the system model are too long to generate violent transients.

References

1. Poppendiek, H.F., Livett, R.K., Sabin, C.M., Greene, N.D., MacDonald, F.R., "Steady State and Transient Performance Analysis of a Binary Rankine Cycle Utilizing Potassium and Lithium as Working Fluids," (JPL Contract No. 950946) GLR-31, 1964.

Nomenclature

A	Boiler heat transfer area
C_c	heat capacity of condenser assembly
C_f	liquid friction coefficient
$\bar{C}_f$	C_f/C_{fd}
C_{Li}	heat capacity of lithium circuit assembly
C_1	nozzle discharge coefficient
C_2	lumped radiation constant of condenser
$\bar{C}_2$	C_2/C_{2d}
e	base of natural logarithms
h	boiler heat transfer coefficient
h_{fg}	latent heat of vaporization of potassium
k	$\left(\frac{r+s}{r}\right)\left(\tau_1/\tau_2\right)$
M_L	mass of liquid in system
p_1	boiler absolute pressure
$\bar{p}_1$	p_1/p_{1d}
p_2	condenser ablsolute pressure
$\bar{p}_2$	p_2/p_{2d}
q_e	electrical power input to lithium heater
Q	dimensionless electrical power input $q_e/T_{1d}hA$
r	T_{1d}/T_A
s	$T_{1d}/\Delta T_d$
t	time

$\bar{t}$	t/τ_1
$\hat{t}$	t/τ_2
$\tilde{t}$	$\left(\frac{r}{r+s}\right)\bar{t}$
T_A	proportionality constant in Equation (13)
T_B	proportionality constant in Equation (14)
T_{Li}	lithium temperature
$\bar{T}_{Li}$	T_{Li}/T_{Lid}
T_1	boiler temperature (potassium temperature)
$\bar{T}_1$	T_1/T_{1d}
T_2	condenser temperature
$\bar{T}_2$	T_2/T_{2d}
v_L	liquid velocity
w_g	mass flow rate of gas
w_L	mass flow rate of liquid (flow rate through pump)
x	$\bar{T}_1 - 1$
y	$\bar{T}_{Li} - 1$
z	$\bar{T}_2 - 1$

Subscripts

d	design point
f	final state ($t = \infty$)
i	initial state ($t \leq 0$)

Greek Symbols

Δp_p	pump pressure rise

Δp_n	$p_1 - p_2$, the nozzle pressure drop
ΔT_d	$T_{Lid} - T_{1d}$
τ_1	C_{Li}/hA
τ_2	$C_c/(C_{2d}T_{2d}^3)$

V-B. TRANSIENT ANALYSIS OF A SIMPLE MODEL OF THE JPL BOILING POTASSIUM POWER SYSTEM WITH A CHOKED NOZZLE AND VARIABLE HEAT TRANSFER CONDUCTANCE

A previous simple analysis of the JPL Boiling Potassium Power System (Reference 1) indicated that rather large flow rate variations could occur with small changes in the controlled parameters. The analysis of Reference 1 was carried out with the idealization of constant boiler heat transfer conductance, which allowed the boiler and condenser to be analysed separately. It is known, however, that under some circumstances boiling heat transfer conductances are strong functions of the vapor quality in the boiler, and one would expect that the large predicted flow rate variations and resultant quality variation indicated by the analysis of Reference 1 would affect the heat transfer coefficient. The possibility of positive feedback from condenser to boiler exists, since an increase in power causes a decrease in flow rate and a resulting increase in quality (or decrease in conductance). If the proper time relationship between these effects existed, flow oscillations would occur. The model has therefore been analysed for the case in which the heat transfer conductance is variable.

The system model considered in this analysis is identical to that of Reference 1, with the exception that the heat transfer conductance varies according to a simple analytic model. Although the modification to the model is minor, considerable complexity is added to the mathematics since the boiler and condenser are now coupled through the liquid flow rate, which affects the heat transfer conductance.

The Heat Transfer Coefficient Model

A number of sets of boiling potassium heat transfer coefficient data show the logarithm of the heat transfer coefficient to decrease approximately linearly from the liquid value at a quality of zero, to the gaseous value at a quality of one. Figure 1, which was taken from Reference 2, shows this variation. In agreement with these experimental data, the model heat transfer coefficient used in the present analysis is assumed to have a linear variation in semilog coordinates. This leads to the following expression for the heat transfer coefficient:

$$\frac{h}{h_L} = \left(\frac{h_g}{h_L}\right)^X \tag{1}$$

where X is the vapor quality

h_g is the value of h for fully established flow at $X = 1$

h_L is the value of h for fully established flow at $X = 0$

The quantity h in Equation (1) is the heat transfer coefficient at X, and is therefore a "local" value.

The average heat transfer coefficient, zero to X, is given by

$$\frac{h}{h_L} = \left[\frac{1}{X \ln\left(\frac{h_L}{h_g}\right)}\right]\left[1 - \left(\frac{h_g}{h_L}\right)^X\right] \tag{2}$$

where h is the average value for the entire boiler, and X is the exit quality. For convenience in the following problem synthesis, the average h will be referred to the final steady state value attained at $t = \infty$, after a given perturbation.

The perturbation will be assumed small, so that

$$X_f - X_i \ll X_f.$$

Then h may be written approximately as

$$\frac{h}{h_f} = \frac{1}{X_f + \epsilon}\left[X_f + \frac{h_L}{h_f}\left(\frac{h_g}{h_L}\right)^{X_f} \epsilon\right] \tag{3}$$

where ϵ is the (small) perturbation in X from the final value X_f. When the final quality is greater than the initial quality, ϵ is negative, and decreases to zero as time increases.

The group of constants $\frac{h_L}{h_f}\left(\frac{h_g}{h_L}\right)^{X_f}$ is evaluated with the constants of Figure 1.

These constants are:

$$h_g = 67 \text{ Btu/hr ft}^2 \text{ °F}$$

$$h_L = 8000 \text{ Btu/hr ft}^2 \text{ °F}$$

The design average heat transfer coefficient, from Equation (3) is

$$h_d = 1920 \text{ Btu/hr ft}^2 \text{ °F}$$

at

$$X_d = X_f = 0.85$$

to simplify notation, define

$$b = \frac{h_L}{h_f}\left(\frac{h_g}{h_L}\right)^{X_f}$$

so that

$$\frac{h}{h_f} = \frac{1 + b\frac{\epsilon}{X_f}}{1 + \frac{\epsilon}{X_f}} \tag{4}$$

Equation (4) may be linearized, neglecting terms in ϵ^2 and higher. Thus

$$\bar{h} = 1 - (1 - b)\frac{\epsilon}{X_f} \tag{5}$$

where

$$\bar{h} = \frac{h}{h_f}$$

and the approximation has been made that

$$\frac{1}{1 + \frac{\epsilon}{X_f}} \approx 1 - \frac{\epsilon}{X_f}$$

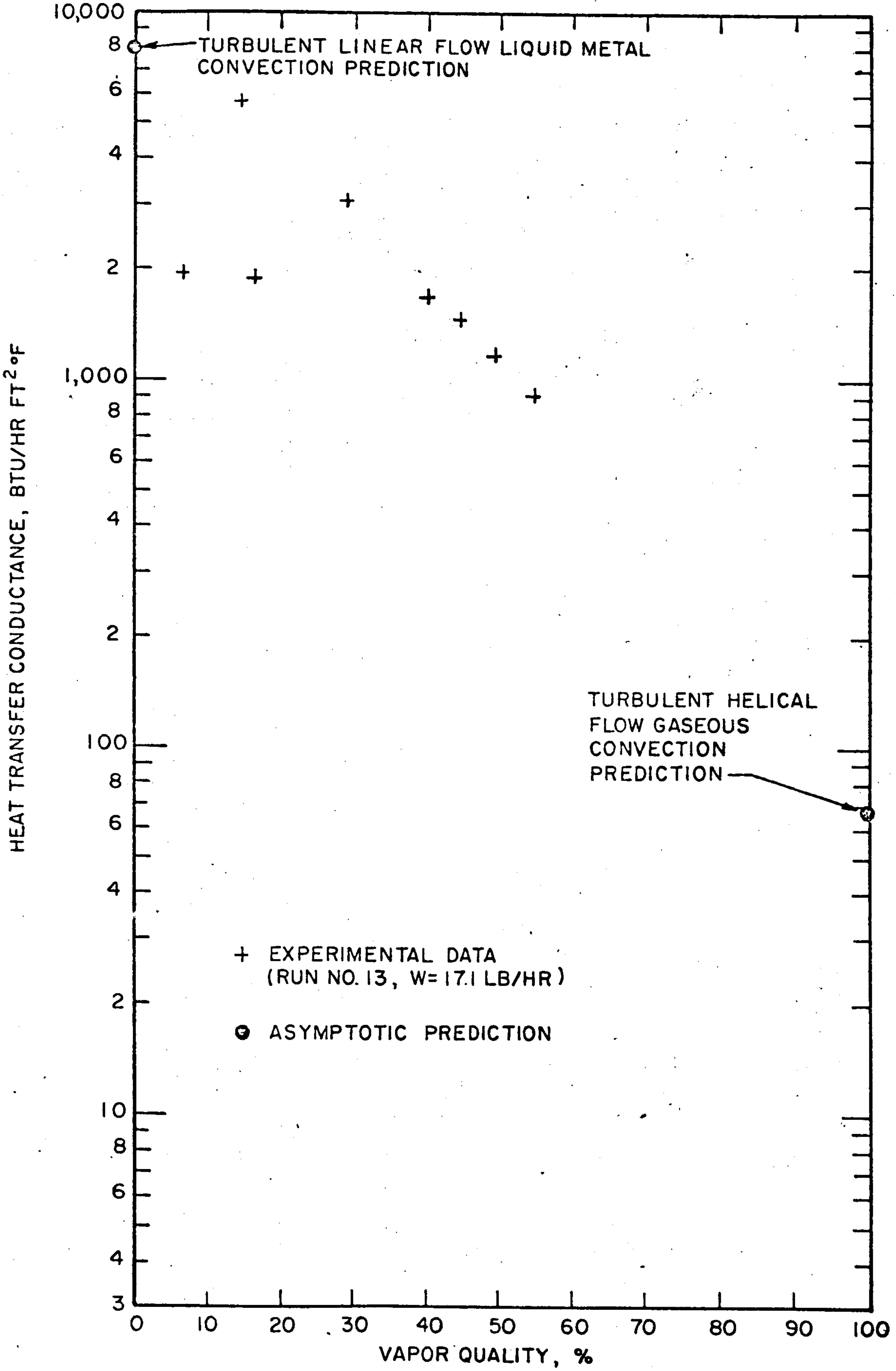


Figure 1. Potassium helical, forced flow boiling data (Run #13).

The remaining equations, taken from Reference 1, are:

Lithium circuit-boiler assembly

$$C_{Li} \frac{dT_{Li}}{dt} + hA\,(T_{Li} - T_1) = q_e$$

Potassium side of boiler

$$hA\,(T_{Li} - T_1) = w_g h_{fg}$$

Turbine nozzle

$$w_g = C_1 \frac{p_1}{T_1^{\frac{1}{2}}}$$

Condenser

$$\frac{C_c}{h_{fg}} \frac{dT_2}{dt} + \frac{C_2}{h_{fg}} T_2^{\,4} = w_g$$

Pump

$$\Delta p_p = \text{constant}$$

Momentum equation

$$\Delta p_p = p_1 - p_2 + C_f w_L^{\,2}$$

where, as before, w_L is the total flow rate.

The vapor pressure-temperature relationship is given by two relationships Equation (13) and (14) of Reference 1.

In Reference 1, the quantities x, y, and z, the potassium temperature in the boiler, the lithium temperature, and the condenser temperature variables respectively, were used. These are not small quantities in the strict sense, since the initial condition may be displaced from the design condition by the radiator shutter position or other parameters. Since it will be necessary to linearize all equations to obtain a solution in the present problem, the variables must be made truly small. The working variables will therefore be changed to

$$x_1 = x - x_f$$

$$y_1 = y - y_f \qquad (6)$$

$$z_1 = z - z_f$$

with the restrictions that

$$x_i - x_f << 1$$

$$y_i - y_f << 1$$

$$z_i - z_f << 1$$

The subscripts i & f denote the initial and final states. In this notation, the linear boiler and condenser pressure-temperature relationships are

$$\bar{p}_1 = \bar{p}_{1_f} \left(1 + a_1 x_1\right) \qquad (7)$$

and

$$\bar{p}_2 = \bar{p}_{2_f} \left(1 + a_2 z_1\right) \qquad (8)$$

where $a_1 = \frac{T_{1d}}{T_A}$

$$a_2 = \frac{T_{2d}}{T_B}$$

$$\bar{p}_{1_f} = e^{a_1 x_f}$$

$$\bar{p}_{2_f} = e^{a_2 z_f}$$

As before, the governing equations are normalized on the design condition. In the new notation, the system governing equations are as follows. Lithium circuit-boiler assembly temperature equation:

$$\frac{dy_1}{d\bar{t}} + \frac{h_f}{h_d}(y_1 - x_1) = Q(1 - b)\frac{\epsilon}{X_f} \tag{9}$$

Vapor generation in the boiler:

$$\frac{h_f}{h_d} s(y_1 - x_1) = \bar{w}_g - \bar{w}_{g_f} + \bar{w}_{g_f}(1 - b)\frac{\epsilon}{X_f} \tag{10}$$

Gas flow rate through the nozzle:

$$\bar{w}_g - \bar{w}_{g_f} = \bar{w}_{g_f}\left[a_1 - \frac{1}{2(1 + x_f)}\right]x_1 \tag{11}$$

Condenser temperature:

$$\frac{dz_1}{d\bar{t}} + 4\frac{\tau_1}{\tau_2}\bar{C}_2 z_1 = \frac{\tau_1}{\tau_2}\left\{\bar{w}_g - \bar{w}_{g_f}\right\} \tag{12}$$

Compatible steady state relations are:

$$Qs = \bar{w}_{g_f} = s\frac{h_f}{h_d}\left[y_f - x_f\right] = \frac{\bar{p}_{1_f}}{(1 + x_f)^{\frac{1}{2}}} = \bar{C}_2(1 + 4z_f) \tag{13}$$

The quantities ϵ and x_1 may be eliminated between these equations to yield a pair of simultaneous linear differential equations in y_1 and z_1. This proceeds as follows:

From the definition of X and ϵ, it may be shown that to first order in the flow rate perturbations, (which are of the same order as ϵ)

$$\epsilon = X_f\left[\frac{\bar{w}_g - \bar{w}_{g_f}}{\bar{w}_{g_f}} - \frac{\bar{w}_L - \bar{w}_{L_f}}{\bar{w}_{L_f}}\right] \tag{14}$$

where $\bar{w}_L$ is $\frac{w_L}{w_{L_d}}$

The quantity $\frac{\overline{w}_g - \overline{w}_{g_f}}{\overline{w}_{g_f}}$ may be taken directly from Equation (11). The liquid flow rate disturbance must be computed. From the momentum equation,

$$\left(\frac{w_L}{w_{L_f}}\right)^2 = \frac{\Delta p_p - (p_1 - p_2)}{C_f w^2_{L_f}} \tag{15}$$

Since the pump pressure rise is fixed for t > 0,

$$\Delta p_p = p_{1_f} - p_{2_f} + C_f w^2_{L_f}$$

so that Equation (15) becomes

$$\left(\frac{w_L}{w_{L_f}}\right)^2 = \frac{(p_{1_f} - p_1) - (p_{2_f} - p_2)}{C_f w^2_{L_f}} + 1$$

By introduction of the pressure temperature relationships, one obtains

$$\left(\frac{w_L}{w_{L_f}}\right)^2 = \frac{- a_1 x_1 + \frac{p_{2_d}}{p_{1_d}} a_2 z_1}{\frac{1}{p_{1_d}} (C_f w^2_{L_f})} + 1$$

This is of the order of unity plus a small perturbation so that

$$\frac{w_L}{w_{L_f}} \sim 1 + \frac{\frac{p_{2_d}}{p_{1_d}} a_2 z_1 - a_1 x_1}{\frac{2}{p_{1_d}} (C_f w^2_{L_f})}$$

Then in the first order approximation

$$\frac{w_L - w_{L_f}}{w_{L_f}} = \left(\frac{p_{2_d}}{p_{1_d}} \frac{a_2}{F} z_1 - \frac{a_1}{F} x_1\right) \tag{16}$$

where F is the equilibrium liquid friction pressure drop, given by

$$F = \frac{2\, C_f\, w^2_{L_f}}{p_{1_d}}$$

$$= 2\, \bar{C}_f \left(\frac{w_{L_f}}{w_{L_d}}\right)^2 \left(\frac{C_{fd}\, w^2_{L_d}}{p_{1_d}}\right) .$$

By combination of Equations (10), (11), (14), and (16), a relationship for x_1 in terms of y_1 and z_1 results. This is

$$x_1 = \left(\frac{b_2 s}{b_6}\right) y_1 + \left(\frac{\bar{w}_{g_f}\, b_1\, b_4}{b_6}\right) z_1 \qquad (17)$$

where the notation is

$$b_1 = (1 - b)$$

$$b_2 = \frac{h_f}{h_d}$$

$$b_3 = \left[a_1 - \frac{1}{2(1 + x_f)}\right]$$

$$b_4 = \frac{p_{2_d}}{p_{1_d}} \quad \frac{a_2}{F}$$

$$b_5 = \frac{a_1}{F}$$

$$b_6 = \left\{b_2 s + \bar{w}_{g_f}\left[b_3 + b_1 (b_3 + b_5)\right]\right\}$$

The perturbation in boiler exit quality is given by

$$\epsilon = X_f \left\{ \left[(b_3 + b_5) \frac{b_2 s}{b_6} \right] y_1 + \left[(b_3 + b_5) \frac{\bar{w}_{g_f} b_1 b_4}{b_6} - b_4 \right] z_1 \right\} \tag{18}$$

The relationships (17) and (18) may be introduced into the lithium circuit Equation (9), and rearranged to yield

$$\frac{dy_1}{d\bar{t}} = - \left[\frac{b_2 b_3}{b_6} Qs \right] y_1 - \left[\frac{b_1 b_3 b_4 Q^2 s}{b_6} \right] z_1 \tag{19}$$

Relationships (11) and (17) may be introduced into the condenser temperature relationship Equation (12), and rearranged to yield

$$\frac{dz_1}{d\bar{t}} = \left[\frac{b_1 b_3 b_4 b_7 Q^2 s^2}{b_6} - 4 b_7 \bar{C}_2 \right] z_1 + \left[\frac{b_2 b_3 b_7 s^2 Q}{b_6} \right] y_1 \tag{20}$$

where

$$b_7 = \frac{\tau_1}{\tau_2}$$

Equations (19) and (20) represent a pair of similtaneous linear first order differential equations for y_1 and z_1. The final conditions by definition are $y_1 = z_1 = 0_1$ and the initial conditions are

$\bar{t} = 0 \quad y_1 = y_i - y_f$

$\qquad\quad z_1 = z_i - z_f$

These equations contain four controllable parameters, corresponding to the four system controls, these are: electrical power to the lithium, throttling of potassium flow at the outlet of the pump, pump pressure rise, and radiator shutter position.

Equations (19) and (20) are of the form

$$\left. \begin{aligned} \frac{dy_1}{d\bar{t}} &= Ay_1 + Bz_1 \\ \frac{dz_1}{d\bar{t}} &= Cz_1 + Dy_1 \end{aligned} \right\} \tag{21}$$

The solution of such a set as (21) is accomplished by assuming solutions of the form (see for example Reference 3)

$$\left.\begin{aligned} y_1 &= \alpha\, e^{\lambda t} \\ z_1 &= \beta\, e^{\lambda t} \end{aligned}\right\} \qquad (22)$$

These are substituted into the differential equations and the exponential terms are cancelled to yield two simultaneous algebraic equations for α and β. These algebraic equations are:

$$\left.\begin{aligned} \alpha\lambda &= \alpha A + \beta B \\ \beta\lambda &= \beta C + \alpha D \end{aligned}\right\} \qquad (23)$$

It can be shown that non-trivial solutions for α and β exist only when the determinant of their coefficients is zero, and this leads to the characteristic equation for the present problem

$$\lambda^2 - (C + A)\lambda + (A C - B D) = 0 \qquad (24)$$

The time constants are thus

$$\lambda_{1,2} = \frac{A + C}{2} \pm \frac{1}{2}\left[(A + C)^2 - (AC - DB)\right]^{\frac{1}{2}} \qquad (25)$$

or

$$\lambda_{1,2} = \frac{A + C}{2} \pm \frac{A - C}{2}\left[1 + \frac{4\,DB}{(A - C)^2}\right]^{\frac{1}{2}}$$

Periodic solutions exist for the cases where these roots have imaginary components. The solutions are functions of four parameters; Δp_p, $\overline{C}_2$, $\overline{C}_f$, and q_e. It is virtually impossible to present a function of four independent parameters in a compact manner, and no attempt at a complete presentation has been made. Instead, two specific examples, one containing only exponentials, and one containing periodic components as well, have been worked out in order to show how the solutions are obtained. In both of these examples, the equilibrium (final) state coincides with the design power input and shutter position, and the pump pressure rise and friction pressure drop have been chosen so that the design liquid flow rate is also obtained at steady state. The two examples differ in that the first is for the design liquid friction coefficient and the second is for 10 times the liquid friction coefficient. The system constants are the same as those of Reference 1. Thus,

$$T_A = 315\ °F$$

$$T_B = 220°F$$

$$T_{1_d} = 2360°R$$

$$T_{2_d} = 1960°R$$

$$T_{Li_d} = 2460°R$$

$$p_{1_d} = 7.65\ \text{atm}$$

$$p_{2_d} = 1.67\ \text{atm.}$$

$$s = 23.6$$

$$\frac{\tau_1}{\tau_2} = 0.0658$$

$$b = 0.075$$

$$\bar{C}_2 = 1$$

$$Q_d = 1/s$$

$$\frac{C_{f_d}\, w^2_{L_d}}{p_{1_d}} = 0.72 \times 10^{-2}$$

$$\left(\frac{w_{L_f}}{w_{L_d}}\right)^2 = 1$$

$$\bar{w}_{g_f} = 1$$

$$x_f = z_f = 0$$

From the above conditions, the constants of the analysis common to both examples are

$$a_1 = 7.5$$

$$a_2 = 8.9$$

$$\frac{p_{2_d}}{p_{1_d}} = 0.218$$

$$F = 2\bar{C}_f \left(\frac{w_{L_f}}{w_{L_d}} \right)^2 \frac{C_{f_d} \, w^2_{L_d}}{p_{1_d}} = 1.44 \times 10^{-2} \, \bar{C}_f$$

$$b_1 = (1 - b) = 0.925$$

$$b_2 = \frac{h_f}{h_d} = 1$$

$$b_3 = \left[a_1 - \frac{1}{2(1 + x_f)} \right] = 7.0$$

$$b_4 = \frac{p_{2_d}}{p_{1_d}} \, \frac{a_2}{F} = \frac{134.9}{\bar{C}_f}$$

$$b_5 = \frac{a_1}{F} = \frac{520}{\bar{C}_f}$$

$$b_7 = 0.0658$$

For the first example, in which $\bar{C}_f = 1$:

$$b_6 = 518$$

$$A = -\frac{b_2 b_3}{b_6} Q_d s = -0.0135$$

$$B = -\frac{b_1 b_3 b_4}{b_6} Q_d^2 s = -0.07145$$

$$C = \frac{b_1 b_3 b_4 b_7 Q_d^2 s^2}{b_6} - 4 b_7 \bar{C}_2 = -0.1522$$

$$D = \frac{b_2 b_3 b_7 s^2 Q_d}{b_6} = 0.021$$

The roots of the characteristic equation are

$$\lambda_1 = -0.0253$$

$$\lambda_2 = -0.1404$$

Both roots are real and negative, so the system is definitely stable. The solution is therefore

$$\begin{aligned} y_1 &= \alpha_1 e^{\lambda_1 \bar{t}} + \alpha_2 e^{\lambda_2 \bar{t}} \\ z_1 &= \beta_1 e^{\lambda_1 \bar{t}} + \beta_2 e^{\lambda_2 \bar{t}} \end{aligned} \tag{26}$$

The constants α_1, α_2, β_1, β_2, are evaluated from the initial conditons. At $\bar{t} = 0$,

$$y_1 = y_i - y_f = \alpha_1 + \alpha_2$$

$$\begin{aligned} \frac{dy_1}{d\bar{t}} &= Ay_1 + B z_1 \\ &= A(y_i - y_f) + B(z_i - z_f) \\ &= \alpha_1 \lambda_1 + \alpha_2 \lambda_2 \end{aligned}$$

z_f is zero, since that is the design point.

$$y_f = y_d = \frac{1}{s}$$

The system starts from an equilibrium state before the disturbance is imposed at

$\bar{t} = 0$. From the equilibrium relationships, Equation (13), and making the approximation that the disturbance is very small, so that $h_i/h_d \approx 1$,

$$y_i = Q_i + x_i$$

$$z_i = \frac{1}{4}(Q_i s - 1)$$

From Equation (17),

$$x_i = \frac{b_2 s}{b_6}\left(y_i - \frac{1}{s}\right) + \frac{b_1 b_4}{b_6}(z_i - 0)$$

$$= \left[\frac{b_2 + \frac{b_1 b_4}{4}}{b_6 - b_2 s}\right]\left[Q_i s - i\right]$$

$$= 0.0651\ (Q_i s - 1)$$

Also

$$y_i - y_f = Q_i - \frac{1}{s} + x_i$$

$$= 0.1075\ (Q_i s - 1)$$

From these relationships,

$$\alpha_1 = -\ 0.034\ (Q_i s - 1)$$

$$\alpha_2 = +\ 0.141\ (Q_i s - 1)$$

By the same process,

$$\beta_1 = -\ 0.007\ (Q_i s - 1)$$

$$\beta_2 = +\ 0.257\ (Q_i s - 1)$$

The solutions may therefore be written

$$\left.\frac{y_1}{(Q_i s - 1)} = -\ (0.034)\ e^{-(0.025)\bar{t}} + (0.141)\ e^{-(0.140)\bar{t}}\ \right\} \qquad (27)$$

$$\left. \frac{z_1}{(Q_i s - 1)} = -\,(0.007)\,e^{-(0.025)\bar{t}} + (0.257)\,e^{-(0.140)\bar{t}} \right) \qquad (27)$$

From Equations (16) and (17), the liquid flow rate may be written,

$$\frac{w_L - w_{L_f}}{w_{L_f}} = b_4 \left(1 - \frac{b_1 b_5}{b_6}\right) z_1 - \frac{b_2 b_5 s}{b_6}\, y_1$$

$$= 9.1\, z_1 - 23.7\, y_1$$

so that

$$\frac{1}{(Q_i s - 1)} \frac{w_L - w_{L_f}}{w_{L_f}} = +\,(.7407)\,e^{-0.026\bar{t}} - (.9202)\,e^{-0.140\bar{t}} \qquad (28)$$

The initial displacement of the flow rate from the design value is

$$\frac{w_{L_i} - w_{L_f}}{w_{L_f}} = -\,(.1795)\,(Q_i s - 1)$$

If the initial state is above the design value the liquid flow rate is below the design value.

Graphs of the time histories of y_1, z_1 and $\frac{w_L - w_{L_f}}{w_{L_f}}$ from Equations (27) and (28) are shown in Figure 2.

For the second example, in which $\bar{C}_f = 10$,

$b_4 = 13.49$

$b_5 = 52.08$

$b_6 = 85.1$

and

$A = -\,0.08225$

$B = -\,0.04349$

$C = -\,0.1957$

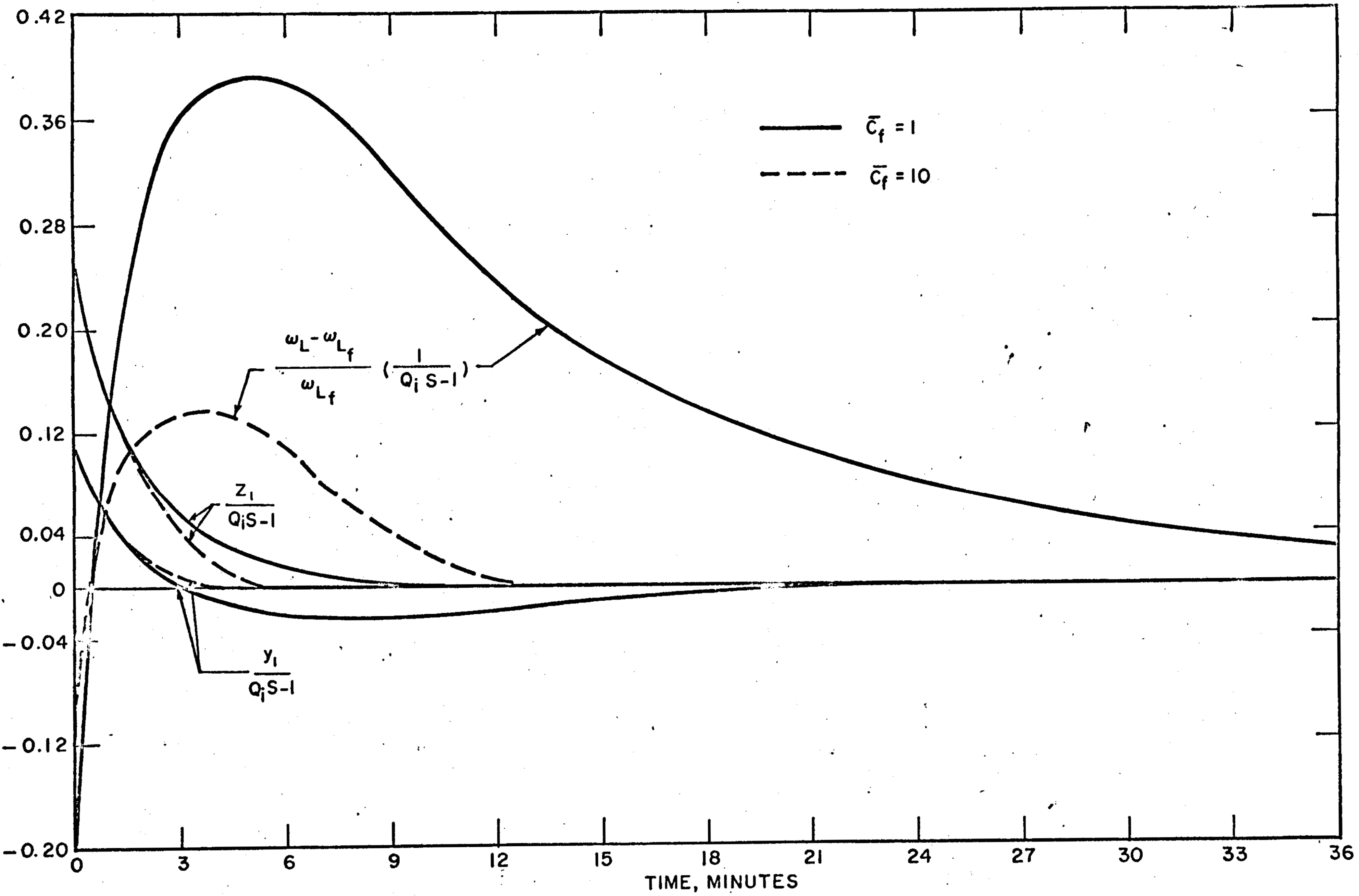


Figure 2. Response of idealized JPL Power System to a small disturbance at t = 0.

$D = -0.1277$

These constants yield the roots

$$\lambda_{1,2} = -(0.13895) \pm (0.04837)\,i$$

$$= \gamma \pm i\,\nu \qquad (29)$$

The roots are complex, so that the solution will be a combination of exponential and periodic functions. Thus

$$\left.\begin{aligned} y_1 &= e^{\gamma \bar{t}}\,(\alpha_1 \cos \nu \bar{t} + \alpha_2 \sin \nu \bar{t}) \\ z_1 &= e^{\gamma \bar{t}}\,(\beta_1 \cos \nu \bar{t} + \beta_2 \sin \nu \bar{t}) \end{aligned}\right\} \qquad (30)$$

The constants α_1, α_2, β_1, β_2, are evaluated from the initial conditions as in the preceeding example. The initial values in this case are

$$x_i = (0.0668)\,(Q_i s - 1)$$

$$y_i - y_f = (0.109)\,(Q_i s - 1)$$

$$z_i - z_f = (0.250)\,(Q_i s - 1)$$

The constants are therefore

$\alpha_1 = (0.109)\,(Q_i s - 1)$

$\alpha_2 = (-0.097)\,(Q_i s - 1)$

$\beta_1 = (0.250)\,(Q_i s - 1)$

$\beta_2 = -(0.00527)\,(Q_i s - 1)$

The solutions are

$$\left.\begin{aligned} \frac{y_1}{(Q_i s - 1)} &= e^{-(0.139)\bar{t}}\left[(0.109) \cos (0.0484)\bar{t} - (0.097) \sin (0.0484)\bar{t}\right] \\ \frac{z_1}{(Q_i s - 1)} &= e^{-(0.139)\bar{t}}\left[(0.250) \cos (0.0484)\bar{t} - (0.0053) \sin (0.0484)\bar{t}\right] \end{aligned}\right\} (31)$$

The liquid flow rate varies as

$$\frac{w_L - w_{L_d}}{w_{L_d}} = 5.85\, z_1 - 14.4\, y_1$$

$$\frac{1}{(Q_i s - 1)} \; \frac{w_L - w_{L_d}}{w_{L_d}} = e^{-(0.139)\bar{t}} \left[- 0.11 \cos (0.0484)\bar{t} + 1.38 \sin (0.0484)\bar{t} \right] \tag{32}$$

The initial displacement of the flow rate from the design value is therefore

$$\frac{w_{L_i} - w_{L_d}}{w_{L_d}} = (- 0.11)\,(Q_i s - 1)$$

Graphs of y_1, z_1, and $(w_L - w_{L_d})/w_{L_d}$ as functions of time from Equations (31) and (32) are shown in Figure 2.

Conclusions

The interaction of the condenser with the boiler is evident from Figure 2 and Equations (26) and (31), since the lithium temperature-time history is now a sum of two exponentials rather than the simple solution of Reference 1. The flow rate shows essentially the same characteristic "surge" during the transient as was indicated in Reference 1.

Periodic terms appear in the solution example two, showing that flow oscillations can indeed occur.. In the case cited this conclusion is probably academic, since the damping is very large and the overshoot of both the temperatures and the flow rate is negligible. It may well be, however, that different combinations of the four free constants lead to cases in which the damping is not large compared to the period of the oscillatory terms. Specific interesting cases would have to be computed to test the possiblity of significant oscillations.

References

1. "Transient Analysis of a Simple Model of the JPL Boiling Potassium Power System with a Choked Nozzle" (Model 1), C. M. Sabin, June 1965. (Geoscience Ltd.)

2. "Quarterly Technical Report on High Acceleration Field Heat Transfer for Auxiliary Space Nuclear Power Systems," (AEC Contract No. AT(04-3)-409) for Period Dec 1, 64 to Feb 28, 65, H.F. Poppendiek, N.D. Greene, C. M. Sabin, L.V. Feigenbutz, R.K. Livett, P.E. Schwartz, and D.J. Connelly. (Geoscience Ltd.)

3. "Advanced Calculus", W. Kaplan, Addison Wesley Publishing Co., Inc. 1952.

Nomenclature

A	boiler heat transfer area
A	constant in Equation (21)
a_1	constant in Equation (7)
a_2	constant in Equation (8)
B	constant in Equation (21)
b	constant in Equation (4)
b_1	$1 - b$
b_2	h_f/h_d
b_3	$\left[a_1 - \frac{1}{2(1 + x_f)}\right]$
b_4	$\left(p_{2d}/p_{1d}\right)\left(a_2/F\right)$
b_5	a_1/F
b_6	$\left\{ b_2 s + \overline{w}_{gf}\left[(b_3 + b_s)\right]\right\}$
b_7	τ_1/τ_2
C	constant in Equation (21)
C_f	liquid friction coefficient for potassium loop
$\overline{C}_f$	C_f/C_{fd}
C_{Li}	lumped lithium circuit-boiler heat capacity
C_1	constant in the turbine nozzle flow equation
C_c	lumped condenser heat capacity
C_2	radiation heat transfer constant
$\overline{C}_2$	dimensionless radiation heat transfer constant, a controlled parameter
D	constant in Equation (21)

F	$2\bar{C}_f \left(\frac{w_{Lf}}{w_{Ld}}\right)^2 \left(\frac{C_{f_d} w^2_{Ld}}{p_{1d}}\right)$
h	average heat transfer conductance in boiler
h_g	heat transfer conductance in boiler for $X = 1$
h_L	heat transfer conductance in boiler for $X = 0$
$\bar{h}$	h/h_f
h_{fg}	latent heat of vaporization of potassium
i	$\sqrt{-1}$
p_1	potassium vapor pressure in boiler
p_2	potassium vapor pressure in condenser
$\bar{p}_1$	p_1/p_{1d}
$\bar{p}_2$	p_2/p_{2d}
Q	dimensionless electrical power input $q_e/(T_{1d}\, h_d A)$
q_e	electrical power input to lithium
s	$\frac{T_{1d}}{T_{Lid} - T_{1d}}$
t	time
$\bar{t}$	t/τ_1
T_1	potassium temperature in boiler
T_2	potassium temperature in condenser
T_{Li}	lithium temperature
$\bar{T}_1$	T_1/T_{1d}
$\bar{T}_2$	T_2/T_{2d}

$\overline{T}_{Li}$	T_{Li}/T_{1d}
T_A	proportionality constant in Equation (7)
T_B	proportionality constant in Equation (8)
w_g	vapor flow rate out of boiler
$\overline{w}_g$	w_g/w_{gd}
w_L	total mass flow rate of potassium (liquid flow into boiler)
$\overline{w}_L$	w_L/w_{Ld}
X	vapor quality at boiler exit
x	$(T_1/T_{1d}) - 1$
x_1	$x_i - x_f$
y	$(T_{Li}/T_{1d}) - 1$
y_1	$y_i - y_f$
z	$(T_2/T_{2d}) - 1$
z_1	$z_i - z_f$

Greek Symbols

α	solution coefficients (Equation (22))
β	
γ	real part of λ
Δp_p	pump pressure rise
ϵ	small perturbation in boiler exit quality
λ	system time constants (Equation (24))
ν	imaginary part of λ
τ_1	$C_{Li}/h_d A$

τ_2	$C_c/(C_{2d}\ T_{2d}^3)$

Subscripts

d	design condition
f	equilibrium condition at $t = \infty$
i	initial condition at $t = 0$

V-C. TRANSIENT HEAT TRANSFER MODEL OF THE JPL BOILER-CONDENSER SYSTEM HAVING THERMAL COUPLING

Introduction

The calculation of the individual time constants of the boiler and condenser of the JPL power system indicates which component controls in transient performance. However, it is of interest to derive equations which define the transient behavior of a coupled boiler-condenser system. The present analysis is based on the following postulates:

Idealized System

1. Heat rate addition to the boiler is constant
2. A mean, constant potassium heat transfer resistance exists in the boiler
3. The heat addition to the boiling, circulating potassium in the boiler is transferred to the condenser where the heat is stored and lost by radiation (no heat is lost from the ducts between the boiler and condenser)
4. The controlling heat transfer resistance in the condenser is the radiation resistance (i.e. the temperature drops across the condensate layer and the metal walls are small and negligible compared to the radiation temperature drop)
5. The constant head or pressure rise developed by the pump is all lost across an adiabatic flow restriction between the boiler and condenser
6. At time equal to zero the system temperatures are constant. Suddenly, the thermal resistance of the condenser is changed from R_{c_o} to R_c.

Derivation

The following equations define the coupled boiler-condenser system. A heat rate balance on the boiler yields,

$$q = C_b \frac{dt_b}{d\theta} + \frac{t_b - t_{f_b}}{R_b} \tag{1}$$

A similar heat rate balance on the condenser is,

$$\frac{t_b - t_{f_b}}{R_b} = C_c \frac{dt_c}{d\theta} + \frac{t_c}{R_c} \tag{2}$$

The total constant head or pressure rise developed by the pump is simply related to the vapor pressures in the boiler and condenser,

$$\Delta p_p = p_{f_b} - p_{f_c} \tag{3}$$

From a linearized vapor pressure-temperature expression (i.e. $\frac{dp}{dt} = a$), and the postulate that $p_{f_c} = p_c$, Equation (3) can be expressed as,

$$\Delta p_p = a(t_{f_b} - t_c) \tag{4}$$

Upon substituting Equation (4) into Equations (1) and (2) one obtains two simultaneous equations in two unknowns (namely t_b and t_c),

$$q = C_b \frac{dt_b}{d\theta} + \frac{t_b - \frac{\Delta p_p}{a} - t_c}{R_b} \tag{5}$$

$$\frac{t_b - \frac{\Delta p_p}{a} - t_c}{R_b} = C_c \frac{dt_c}{d\theta} + \frac{t_c}{R_c} \tag{6}$$

The solution of this system is accomplished by 1) adding these two equations, 2) solving for t_c from Equation (5) and then differentiating t_c with respect to θ and 3) substituting t_c and $dt_c/d\theta$ into the sum of Equations (5) and (6).

The resulting equation is,

$$P_1 = \frac{d^2 t_b}{d\theta^2} + K_1 \frac{dt_b}{d\theta} + K_2 t_b \tag{7}$$

where

$$P_1 = \frac{q\left(1 + \frac{R_b}{R_c}\right) + \frac{\Delta p_p}{aR_c}}{C_c\, C_b\, R_b}$$

$$K_1 = \frac{C_b + C_c + \frac{C_b R_b}{R_c}}{C_c\, C_b\, R_b}$$

$$K_2 = \frac{1}{C_c\, C_b\, R_c\, R_b}$$

The solution of Equation (7) is,

$$t_b = C_1 e^{\left(-\frac{K_1}{2} + \sqrt{\left(\frac{K_1}{2}\right)^2 - K_2}\right)\theta} + C_2 e^{\left(-\frac{K_1}{2} - \sqrt{\left(\frac{K_1}{2}\right)^2 - K_2}\right)\theta} + \frac{P_1}{K_2} \tag{8}$$

where constants C_1 and C_2 are to be determined.

The boundary value problem is defined by the following initial conditions:

At $\theta = 0$,

$$t_b = q(R_b + R_{c_o}) + \frac{\Delta p_p}{a} = t_{b_o} \tag{9}$$

$$t_c = q\,R_{c_o} = t_{c_o} \tag{10}$$

$$\frac{dt_b}{d\theta} = \frac{q}{C_b} - \frac{\left(t_{b_o} - t_{c_o} - \frac{\Delta p_p}{a}\right)}{R_b\,C_b} \tag{11}$$

Upon substituting Equations (9), (10), and (11) into the solution for t_b and its derivative and then solving that simultaneous set of equations, the constants C_1 and C_2 are found to be,

$$C_2 = \frac{\left[q(R_b + R_{c_o}) + \frac{\Delta p_p}{a} - \frac{P_1}{K_2}\right]\left(-\frac{K_1}{2} + \sqrt{\left(\frac{K_1}{2}\right)^2 - K_2}\right)}{2\sqrt{\left(\frac{K_1}{2}\right)^2 - K_2}} \tag{12}$$

$$C_1 = q(R_b + R_{c_o}) + \frac{\Delta p_p}{a} - \frac{P_1}{K_2} - C_2 \tag{13}$$

The solution for t_c can be found in the same manner as the outline given above for the solution for t_b.

Typical Problem and Concluding Remarks

This transient analysis was applied to the JPL boiler-condenser system using the following representative system parameters including a 25 percent sudden increase in radiator resistance:

$R_b = 2.0 \times 10^{-3}$ hr °F/Btu

$R_{c_o} = 1.2 \times 10^{-2}$ hr°F/Btu

$R_c = 1.5 \times 10^{-2}$ hr °F/Btu (25% change in R_{c_o})

$C_b = 0.126$ Btu/°F

$C_c = 7.56$ Btu/°F

$q = 1.02 \times 10^{5}$ Btu/hr

$\Delta p_p = 1.26 \times 10^{4}$ lb/ft^2

$a = 31.7$ lb/ft^2°F

The results for this illustrative problem are shown in Figure 1.

Further extensions of the above transient thermal analysis can be made. For example, the boiler can be represented by two heat exchangers, the first of which has a low thermal resistance (representing nucleate boiling) and the second one having a high thermal resistance (representing film boiling). This lumped parameter method of representation can also be used to evaluate non linear problems.

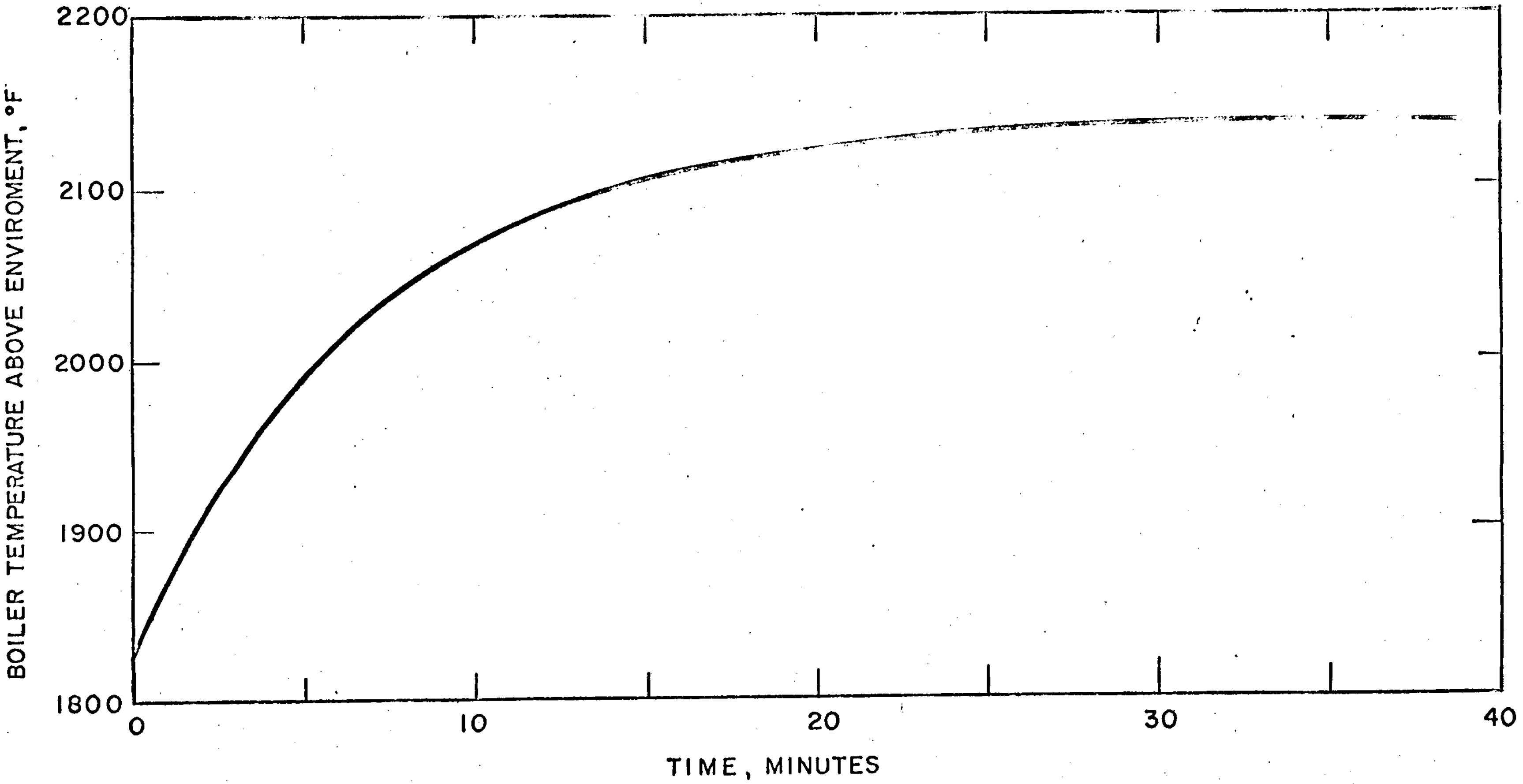


Figure 1. Transient boiler history for the illustrative problem.

Nomenclature

a	the vapor pressure derivative with respect to temperature (a constant)
C_b	thermal capacity of the boiler
C_c	thermal capacity of the condenser
q	constant heat rate addition to the boiler
p_{f_b}	vapor pressure in the boiler
p_{f_c}	vapor pressure in the condenser
R_b	mean potassium heat transfer resistance in the boiler
R_c	mean radiation heat transfer resistance for the condenser (radiator)
t_b	boiler temperature above the environment temperature
t_{f_b}	mean potassium temperature above the environment temperature in the boiler
t_c	condenser temperature above the environment temperature
t_{f_c}	mean potassium temperature above the environment temperature in the condenser
θ	time
Δp_p	constant pressure rise generated by the pump

V-D. TRANSIENT ANALYSIS OF A SIMPLE MODEL OF THE JPL BOILING POTASSIUM POWER SYSTEM WITH AN UNCHOKED NOZZLE (Model 2)

Introduction

This study pertains to a system identical to the system of Reference 1 except that the nozzle pressure drop for an unchoked nozzle now applies. Two cases are considered; the case of constant boiling heat transfer conductance, and the case of variable heat transfer conductance. The variable heat transfer conductance is taken to be identical to that of Reference 2.

In the choked nozzle cases considered in References 1 and 2 the condenser could affect the boiler only by changing the liquid flow rate through the pump, the pump pressure rise being constant. In the unchoked nozzle case the gaseous flow through the nozzle is also affected by condenser conditions, since the back pressure provided by the condenser appears in the nozzle flow expression. Thus both of the present solutions will follow the form of Reference 2, since both pertain to coupled systems.

Datum State Considerations

The design operating point for the power system has no significance in the present case, since a pressure ratio p_1/p_2 anywhere near design would choke the nozzle. Therefore, although the same general notation will be used here as was used in the two analyses of Model 1, the subscript d will now apply to an arbitrary datum operating level, rather than to the design operating point. The only restriction on the datum state is that the pressure ratio across the nozzle must be less than critical. The analysis will be carried out for small perturbations about this operating point. Since the datum is arbitrary, no loss of generality is incurred by assuming the datum and final states to coincide.

Analytic Expression for the Nozzle Flow

The vapor phase flow rate through the nozzle, which is assumed to control the pressure drop, as before, is given by

$$w_g = \rho_{g_n} A_n V_n \qquad (1)$$

where the subscript n applies to the vena contracta.

If the nozzle is adiabatic, then the velocity in the vena contracta is (see for example Reference 3 page 79)

$$V_n = [2\, c_p\, (T_1 - T_n)]^{\frac{1}{2}} \tag{2}$$

assuming the velocity at the exit of the boiler is negligible so that T_1 is a total temperature.

If there is negligible pressure recovery downstream of the vena contracta (normally less than 10%) then

$$T_n = T_2$$

where T_2 is the condenser temperature as before.

The density of the gas in the vena contracta will be taken as

$$\rho_{g_n} = \frac{p_2}{RT_2} \tag{3}$$

It has been found that a better fit to the vapor-pressure-temperature relationship than those used in References 1 and 2 may be obtained by the expression

$$\frac{p}{p_o} = \left(\frac{T}{T_o}\right)^{k_1} \tag{4}$$

where p_o = 14.7 psia

T_o = 1850 °R

k_1 = 8.24

This relationship is applicable within a few percent over the temperature range 1850 °R to 2550 °R. See Figure 1. Thus the boiler and condenser pressures may be written

$$\bar{p}_1 = \bar{T}_1^{\,k_1} \tag{5}$$

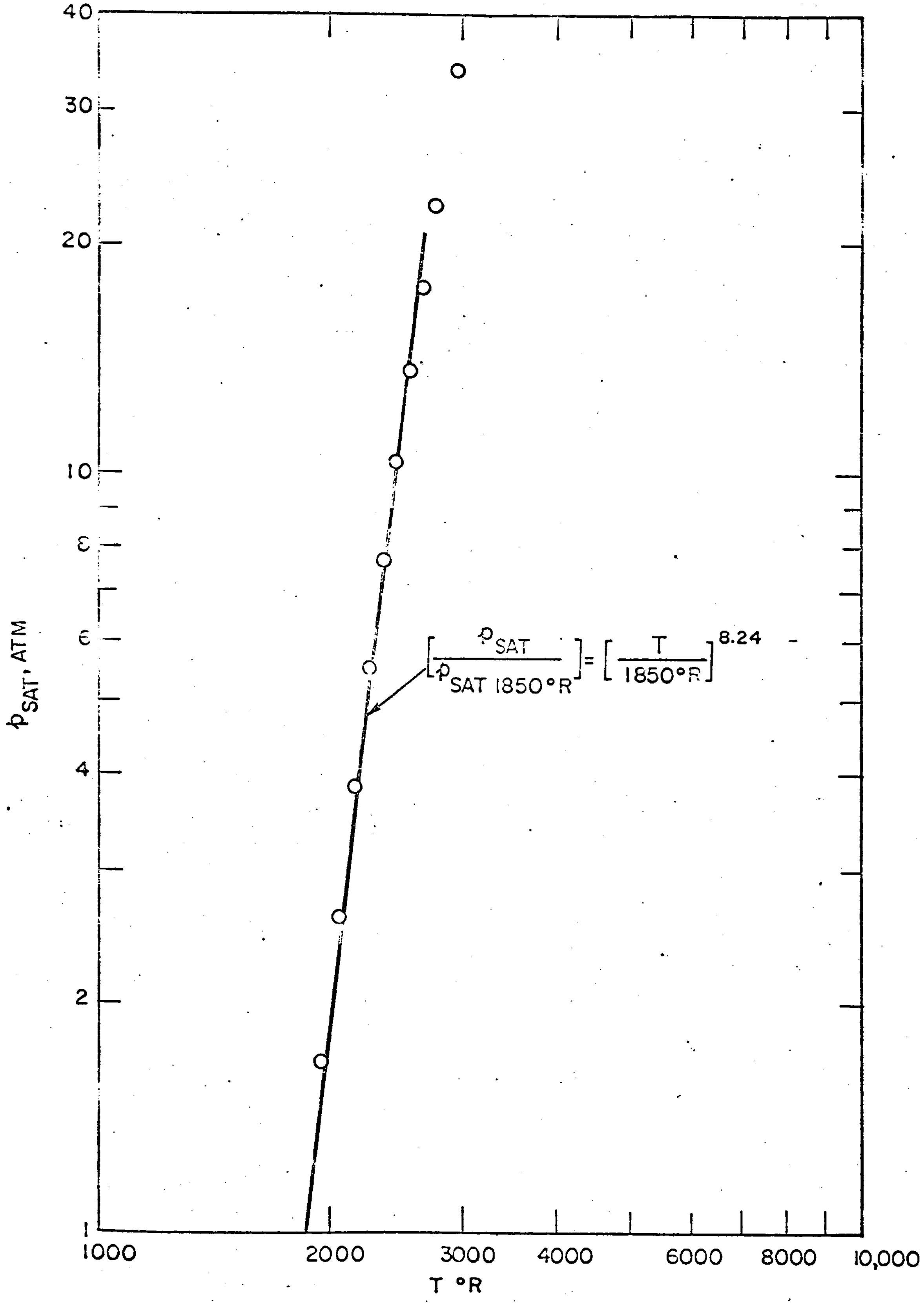


Figure 1. Comparison of fitted function to experimental potassium vapor pressure-temperature relationship.

and

$$\overline{p}_2 = \overline{T}_2^{\,k_1} \tag{6}$$

The vapor flow rate through the nozzle, from Equations (1), (2), (3), (5), and (6) may be written

$$\overline{w}_g = \frac{\overline{T}_2^{\,(k_1-1)}}{\left[1 - \frac{T_{2d}}{T_{1d}}\right]^{\frac{1}{2}}} \left[\overline{T}_1 - \frac{T_{2d}}{T_{1d}}\,\overline{T}_2\right]^{\frac{1}{2}} \tag{7}$$

The same small perturbation notation as used before is introduced, in which

$x = \overline{T}_1 - 1$

$x_1 = x - x_f$

$y = \frac{T_{Li}}{T_{1d}} - 1$

$y_1 = y - y_f$

$z = \overline{T}_2 - 1$

$z_1 = z - z_f$

but now, since the datum and final states coincide

$x_f = z_f = 0$

$\overline{w}_{gf} = 1$

Denote T_{2d}/T_{1d} by k_2. In the small perturbation approximation, Equation (7) may be written

$$\overline{w}_g - 1 \approx k_3\, x_1 + k_4\, z_1 \tag{8}$$

where k_3 and k_4 are given by

$$k_3 = \frac{1}{2(1 - k_2)}$$

$$k_4 = \frac{2(k_1 - 1)(1 - k_2) - k_2}{2(1 - k_2)}$$

and all terms in x_1 and z_1 higher than the first order have been neglected.

Equation (8) is the only new equation. The other equations describing the system transients are; Lithium-Loop Boiler assembly temperature:

$$\frac{dy_1}{dt} + \frac{h_f}{h_d}(y_1 - x_1) = Q(1 - b)\frac{\epsilon}{X_f} \tag{9}$$

Vapor Generation in the Boiler:

$$\frac{h_f}{h_d} s(y_1 - x_1) = \bar{w}_g - \bar{w}_{gf}\left[1 - (1 - b)\frac{\epsilon}{X_f}\right] \tag{10}$$

Condenser Temperature:

$$\frac{dz_1}{dt} + 4\frac{\tau_1}{\tau_2}\bar{C}_2 z_1 = \frac{\tau_1}{\tau_2}(\bar{w}_g - \bar{w}_{gf}) \tag{11}$$

Boiler Exit Quality Variation:

$$\epsilon = X_f\left(\frac{w_g - w_{gf}}{w_{gf}} - \frac{w_L - w_{Lf}}{w_{Lf}}\right) \tag{12}$$

Liquid Flow Rate:

$$\frac{w_L - w_{Lf}}{w_{Lf}} = \frac{1}{F}\left(\frac{p_{1f} - p_1}{p_{1d}}\right) - \frac{1}{F}\left(\frac{p_{2f} - p_2}{p_{2f}}\right)\frac{p_{2d}}{p_{1d}} \tag{13}$$

In the present cases, by definition,

$$\frac{h_f}{h_d} = \bar{C}_2 = 1$$

and as before, $Qd = y_f = \frac{1}{s}$ at $\bar{t} = \infty$

Equations (9) through (13) are solved simultaneously to yield the description of the system variables as a function of time. The solution procedure is as follows:

1) with the aid of the vapor pressure-temperature relationships, rewrite Equation (13), to yield

$$\bar{w}_L - 1 = \frac{1}{F}\left[- k_1 x_1 + \frac{p_{2d}}{p_{1d}} k_1 z_1\right] \tag{14}$$

where

$$\bar{w}_L = \frac{w_L}{w_{Ld}}$$

2) combine Equations (8), (12), and (14) to yield

$$\frac{\epsilon}{X_d} = \left(k_3 + \frac{k_1}{F}\right) x_1 + \left(k_4 - \frac{p_{2d}}{p_{1d}}\right) z_1 \tag{15}$$

3) combine Equations (8), (10), and (15), solve for x_1, to yield

$$x_1 = k_5 z_1 + k_6 y_1 \tag{16}$$

where

$$k_5 = \frac{k_4 - (1 - b)\left(k_4 - \frac{p_{2d}}{p_{1d}} \frac{k_1}{F}\right)}{s + k_3 - (1 - b)\left(k_3 + \frac{k_1}{F}\right)}$$

$$k_6 = - \frac{s}{s + k_3 - (1 - b)\left(k_3 + \frac{k_1}{F}\right)}.$$

4) by use of Equation (16), eliminate x_1 from the Lithium circuit temperature

equation. To accomplish this combine (9), (10), and (16). The result may be written

$$\frac{dy_1}{d\bar{t}} = -\frac{k_3 k_6}{s} y_1 - \frac{k_3 k_5 + k_4}{s} z_1 \tag{17}$$

or

$$\frac{dy_1}{d\bar{t}} = Ay_1 + Bz_1 \tag{18}$$

5) After the same manner, combining Equations (8), (11), and (16), the condenser temperature equation may be written

$$\frac{dz_1}{d\bar{t}} = \frac{\tau_1}{\tau_2} (k_3 k_5 + k_4 - 4) z_1 + \frac{\tau_1}{\tau_2} (k_3 k_6) y_1 \tag{19}$$

or

$$\frac{dz_1}{d\bar{t}} = C z_1 + D y_1 \tag{20}$$

Equations (18) and (20) are identical to those given as Equation (21) of Reference 2. The appropriate initial conditions for the present case are $t = 0$, $y = y_i$, $z = z_i$. These are two values slightly different from the datum state and correspond to some equilibrium conditions for $\bar{t} < 0$, at which $\bar{C}_2 \left\{\begin{matrix}=\\ \neq\end{matrix}\right\} 1$, $Q \left\{\begin{matrix}=\\ \neq\end{matrix}\right\} Q_d$, etc. The method of obtaining solutions to this equation system is identical to that given in Reference 2 and will not be repeated here.

As stated previously, the two cases h = constant, and h = variable, are equivalent to $(1 - b) = 0$, and $(1 - b) > 0$, respectively.

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VI. SIMPLIFIED EQUATIONS WHICH DEFINE THE NUCLEAR REACTOR (LITHIUM HEATER) AND POWER SYSTEM COMPONENTS

Introduction

It is important to be able to define the transient nuclear-thermal coupling between the nuclear reactor and the various components of the power system. For example, it is desirable to be able to predict the transient response of a nuclear reactor which supplies the power for a Rankine cycle system when changes in turbo-alternator demand or radiator heat transfer area are imposed.

General Equations

The well known reactor kinetics equations which define the transient behavior of a nuclear reactor are

$$\frac{dP}{d\Theta} = \frac{\delta k_{ep} P}{\ell} + \sum_i \lambda_i C_i \tag{1}$$

$$\frac{dC_i}{d\Theta} = - \lambda_i C_i + \frac{k_e \beta_i P}{\ell} \tag{2}$$

where P, power

Θ, time

δk_{ep}, prompt excess multiplication

ℓ, total neutron cycle time

λ_i, decay constant of i^{th} emitter

C_i, modified concentration of i^{th} emitter

k_e, effective multiplication factor

β_i, fraction of delayed neutrons of i^{th} emitter

The heat transfer equations that describe the remaining components of the power system are

1. Fuel Elements

$$\frac{dt_e}{d\Theta} = \frac{P - (UA)_r (t_e - t_r)}{C_e} \qquad (3)$$

where P, fission heat flow rate

$(UA)_r (t_e - t_r)$, coolant heat flow rate

2. Reactor Coolant

$$\frac{dt_r}{d\Theta} = \frac{(UA)_r (t_e - t_r) - CM (t_{ci} - t_{co})}{C_r} \qquad (4)$$

where $(UA)_r (t_e - t_r)$, heat flow from fuel elements to coolant

$CM (t_{ci} - t_{co})$, heat flow to the heat exchanger

3. Primary Coolant Circuit (Lithium)

$$\frac{dt_c}{d\Theta} = \frac{CM (t_{ci} - t_{co}) - UA (t_c - t_s)}{C_c} \qquad (5)$$

where $UA (t_c - t_s)$, heat transfer to secondary circuit

$CM (t_{ci} - t_{co})$, net heat flow rate from reactor

4. Secondary Coolant Circuit (Potassium)

$$\frac{dt_s}{d\Theta} = \frac{UA (t_c - t_s) - P_1}{C_s} \qquad (6)$$

where P_1, turbine power

$UA (t_c - t_s)$, heat exchanger heat flow

All mean temperatures in the above equations are averages of inlet and outlet values. Finite flow times accounting for the fluids to flow from one component to another can be added in these equations for further refinement. The solutions of the above set of equations must be accomplished numerically or by analog methods.

Specific Solutions

A number of specific solutions of the transient reactor-power system can be found in the literature. For example (Reference 1), consider the case where the turbine power demand suddenly changes from P_0 to P_1, a new power level. For this situation, the reactor coolant would decrease in temperature at the rate

$$\frac{dt}{d\Theta} = \frac{P - P_1}{C} \tag{7}$$

The reactor power level would increase because of the negative temperature coefficient. Using a one delayed-neutron group model, differentiating Equation (1), substituting $\frac{dC}{d\Theta}$ from Equation (2) and C from Equation (1), yields

$$\frac{d^2P}{d\Theta^2} + \left(\lambda - \frac{\delta k_{ep}}{\ell}\right)\frac{dP}{d\Theta} - \left[\frac{d(\delta k_{ep})}{\ell\, d\Theta} + \frac{\lambda\, \delta k_e}{\ell}\right] P = 0 \tag{8}$$

where $k_e = 1 - |\alpha| t$

$\delta k_e = - |\alpha| t$

$\delta k_{ep} = - \left(\beta + |\alpha| t\,(1 - \beta)\right)$

$\frac{d(\delta\, k_{ep})}{d\Theta} = - |\alpha| \frac{dt}{d\Theta} (1 - \beta)$

α, temperature coefficient

Equation (8) can be linearized and combined with Equation (7) yielding,

$$\frac{1}{a}\frac{d^2P}{d(\lambda\tau)^2} + \frac{dP}{d(\lambda\tau)} + P = P_1 \tag{9}$$

where $a = \frac{\alpha P_o}{\beta\lambda C}$

The solution of Equation (9), which can be used to estimate transient reactor power performance for the subject boundary value problem, is

$$P = (P_o - P_1)\, e^{-\frac{a\tau}{2}} \left(\cos\omega\tau + \frac{a}{2\omega}\sin\omega\tau\right) + P_1 \tag{10}$$

where τ, is measured in units of the mean delayed neutron lifetime and $\omega = \sqrt{a - \frac{a^2}{4}}$

Transfer Function Method

Transient response of nuclear reactor systems have successfully been analyzed using a steady state network with sinusoidal driving functions. For example, the reactor can be treated as an element in a control system and a transfer function applied to it, the transfer function being equal to the ratio of output to input for a sinusoidal change in the multiplication constant δk. Single and multi-group delayed neutron groups can be considered (Reference 2). The transfer functions are actually Laplace-transforms of the ratio of output to input functions.

The various components of a nuclear power system can be represented by transfer functions that are coupled together and the equivalent transfer function of the total system deduced. From this result, the amplitude of the output to input ratio and the phase shift between the output and input can be plotted as a function of exitation frequency. Stability features can be obtained from these curves. Also, general information on the absolute stability and degree of stability can be inferred using the amplitude and phase lag results in a Nyquist diagram. Typical examples can be found in Reference 2.

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VII. SYSTEM SIMULATION MODEL AND GOVERNING EQUATIONS

A simplified circuit for the JPL power system is shown in Figure 1. Suitable idealizations for this circuit, which are not as restrictive as Models 1 and 2 of sections IV and V are as follows.

1. Heat addition only in preheater and lithium heater.
2. Heat removal only in radiator and in shaft work of turbine.
3. No heat generation in electromagnetic pump. (Heating actually done by pump can be lumped with the preheater if desired).
4. Liquid friction is concentrated in the valve at the preheater outlet.
5. Liquid level variations are negligible.
6. Instantaneous temperature of entire lithium circuit is uniform.
7. Boiling process is constant pressure.
8. Condensation process is constant pressure.
9. Potassium vapor is always less than 100% quality. Thus, $p = p(T)$ in the mixed phase region.
10. The fluid, piping, and insulation in the potassium loop outside the boiler and radiator have negligible thermal storage capacity.
11. Pressure and flow accommodation is instantaneous. Thus the fluid inertia is negligible.
12. Variations in fluid mass storage in different parts of the circuit are negligible.
13. The potassium pump pressure rise is independent of liquid flow rate.
14. Mixed phase pressure drops are dependent upon the vapor phase only.

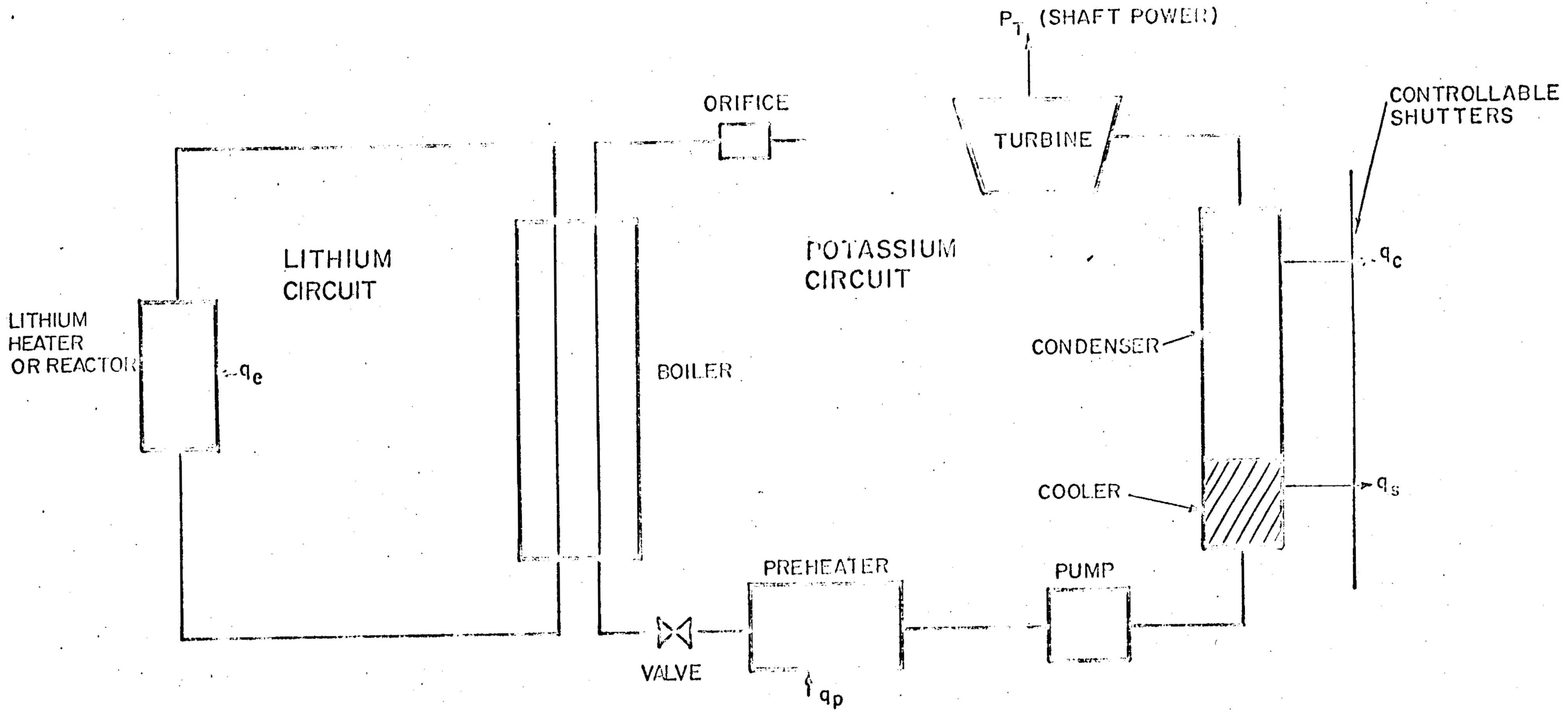

Figure VII-1. Simplified circuit for simulation.

15. Preheating in the boiler is negligible. All heat added in the boiler goes into vaporizing liquid.
16. Liquid specific heat is independent of temperature.
17. Pressure recovery from the fluid momentum in the nozzle is negligible.
18. Potassium vapor may be represented by the equation of state of a perfect gas.

In the analyses of sections IV and V, lithium heater power, radiator shutter position, pump pressure rise, valve position, and preheat temperature were treated as controllable parameters, constant in time. Any or all of these quantities may instead be treated as time functions, whose variation is specified by the operator. With the foregoing idealizations, the thermal behavior of the lithium circuit boiler assembly may be represented by

$$q_e = hA\,(T_{Li} - T_1) + C_{Li}\,\frac{dT_{Li}}{dt} \tag{1}$$

The notation, unless otherwise defined, is consistent with that of section IV-A.

In general, q_e, T_{Li}, and T_1 are functions of time. The mean boiler heat transfer conductance h is a function of boiler exit quality. C_{Li} is the heat capacity of the entire lithium circuit-boiler assembly, and may include the reactor as well, depending upon the location at which q_e is defined.

The condenser governing equation is

$$w_\ell\, X_2\, h_{fg_2} = C_2\, T_2^{\,4} + C_c\,\frac{dT_2}{dt} \tag{2}$$

X_2 and h_{fg_2} are the quality and latent heat of vaporization at the entrance to the condenser. C_2 is a parametric function of shutter position and is also a function of time if finite-time motions of the shutters are considered. C_2 and C_c are based on the radiator area above the liquid level. w_ℓ is the entire mass flow rate.

The pressure drop equation is

$$\Delta p_p = C_f w_\ell^2 + p_1 - p_2$$

$$= C_f w_\ell^2 + C_o w_g^2 + p_n - p_2 \qquad (3)$$

where $C_f w_\ell^2$ is the liquid pressure drop

$C_o w_g^2$ is the orifice pressure drop

p_n is the pressure before the turbine nozzle

and Δp_p, the pump pressure rise, can be a function of time.

There are two alternative expressions for nozzle pressure drop, dependent upon whether the pressure ratio p_2/p_n is greater or smaller than critical (approximately 0.55).

For unchoked flow, $\frac{p_2}{p_n} > 0.55$,

$$w_g = \rho g_n A_n V_n$$

$$= \frac{p_2}{R T_2} A_n \left[2C_p (T_n - T_2)\right]^{\frac{1}{2}} \qquad (4)$$

where T_n is the nozzle inlet temperature (downstream of the orifice)

A_n is the nozzle minimum area corrected for boundary layer effects if considered. In the case in which the turboalternator is not in the circuit, A_n would be the orifice vena contracta area, and T_n would be identically T_1.

For choked flow, $\frac{p_2}{p_1} < 0.55$,

$$w_g = C_1 \frac{p_n}{\sqrt{T_n}} \qquad (5)$$

C_1 can take on two values, dependent upon whether the orifice or turbine nozzle is the controlling restriction.

Turboalternator

In section III-C of Reference 1 it was shown that the time constant of the turboalternator of this power system is in the order of 3 seconds, which is at least an order of magnitude less than the thermal time constants of the boiler-condenser system. The approximation may therefore be made that the inertia of the rotating machinery is negligible. If in addition the turbine is operating near design speed, then its thermal efficiency will be nearly independent of the flow throughput. Its shaft power will be approximately in direct proportion to the flow throughput. The turboalternator may thus be replaced by

$$\frac{P_T}{w_\ell} = \text{const} \qquad (6)$$

for the thermal analysis of the circuit. Since this constant enthalpy change is in the order of three percent of the maximum enthalpy difference around the potassium circuit, it was neglected altogether in the analyses of sections IV and V.

Subcooling Portion of Radiator

A number of approximations for the transient behavior of the pump temperature due to subcooling in the radiator are possible. One of these is given below, based on an averaged heat loss. This averaging has no effect at all upon the transient behavior of the boiler-condenser portions of the circuit, since in the system of idealizations the preheat power is exactly that required to bring the liquid to saturation at the entrance to the boiler. The pump temperature, if desired, can be computed after w_ℓ and T_2 are determined from other considerations. The steady-state pump temperature is given by Equation (13) of section IV-B.

$$w_\ell C_p T_2 + \frac{C_3}{2} T_2^4 + \frac{C_s}{2}\frac{dT_2}{dt} + \frac{C_s}{2}\frac{dT_p}{dt} + w_\ell C_p T_p - \frac{C_3}{2} T_p^4 \qquad (7)$$

where C_3, the subcooler radiation constant, is a function of shutter position

T_p is the pump temperature

C_s is the thermal capacity of the liquid-filled portion of the radiator

w_ℓ and T_2 are time variables, but not unkowns in this equation.

Equation (7) is equivalent to

$$\phi_1 (t) = \phi_2 (t) T_p + \frac{C_s}{2} \frac{dT_p}{dt} + \phi_3 T_p^4$$

where ϕ_1, ϕ_2, and ϕ_3 are known functions.

The preheater thermal equation is

$$q_p = w_\ell C_p (T_1 - T_p) \qquad (8)$$

The pump temperature T_p, and the preheat power are time variable. The preheat power is not independently variable however, but depends upon T_1, T_p, and w_ℓ. C_p is the liquid specific heat. This equation and the subcooling relationship are unnecessary to the solution of the boiler-condenser transient behavior.

In addition to the preceding equations, there are required the following expressions.

$$w_g h_{fg} = h A (T_{Li} - T_1) \qquad (9)$$

in which h_{fg}, the latent heat of vaporization of potassium, is a function of T and is to be evaluated at T_1.

$$p = p (T) \qquad (10)$$

An acceptable working relationship for this function is given by Equation (4) of section V-D.

$$h = h (X) \qquad (11)$$

A reasonable estimate of this functional dependence is given by Equation (2) of section V-B.

The relationship between the vapor quality latent heat product in boiler and condenser, based on the stated idealizations, is

$$X_2 h_{fg_2} - X_1 h_{fg_1} = C_p (T_1 - T_2) - \frac{P_T}{w_\ell} \qquad (12)$$

The vapor flow rate at any point is of course

$$w_g = X w_\ell \qquad (13)$$

An approximate working relationship for h_{fg}, based information from Reference 2 of this section is

$$h_{fg} = A - BT \qquad (14)$$

for T in °F $\quad A = 1017$ Btu/lb $\quad B = 0.118$ Btu/(lb°F)

for T in °R $\quad A = 1071$ Btu/lb $\quad B = 0.118$ Btu/(lb°R)

The system constants required for the preceding equations are tabulated below. For the most part they are taken from other work reported in this document, and where applicable that work is cited.

Quantity	Symbol	Value	Source
Boiler area	A	$.378\ \text{ft}^2$	section IV-B
Lithium circuit-boiler heat capacity	C_{Li}	4.6 Btu/°F	section V-A
Condenser radiation constant (design point)	C_2	7.36×10^{-9} Btu/(hr°R^4)	computed from data of section IV-B
Condenser heat capacity	C_c	3.18 Btu/°F	computed from data of sections IV-B & V-A
Liquid friction coefficient (design point)	C_f	$.36 \times 10^{-4}$ psi/(lb/hr)^2	section V-A
Orifice pressure loss coefficient (unchoked)	C_o	$.80 \times 10^{-3}$ psi/(lb/hr)^2	section II B Reference 1
Turbine Nozzle Area (actual)	A_n	$0.023\ \text{in}^2$	section II F, Ref. 1
Turbine Nozzle choked discharge coefficient	C_1	$59\ \frac{\text{lb}}{\text{hr psi}}\ {}^\circ\text{R}^{\frac{1}{2}}$	section IV-A

Quantity	Symbol	Value	Source
Fluid enthalpy drop across turbine	P_T/w_ℓ	32.5 Btu/lb	computed from section III C of Reference 1
Subcooler radiation constant (design point)	C_3	$.915 \times 10^{-9}$ Btu/(hr°R^4)	computed from data of section IV-B
Subcooler heat capacity	C_s	.396 Btu/°F	computed from data of sections IV-B & V-A

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