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A CRITICAL REVIEW OF THE USE OF DOPPLER FREQUENCY FOR RANGE AND RANGE RATE MEASUREMENTS

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SUMMARY

A derivation of the range rate equation (integrated Doppler equation) is presented. The derivation is based on phase and time rather than on the Doppler frequency shift. The phase method describes the physical situation and the measuring process better than the Doppler method. This greatly simplifies the derivation of equations and provides abetter insight into the measuring process. As a result, the confidence in system analysis is increased and better subsystem specifications can be written which ultimately will lead to improved system performance.

CONTENTS

A CRITICAL REVIEW OF THE USE OF DOPPLER FREQUENCY FOR RANGE AND RANGE RATE MEASUREMENTS

1. INTRODUCTION

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The Doppler frequency has been used for many years as the basis for range rate measurements and the integrated Doppler frequency has been used with great success for range measurements. To cast any doubts on the superiority of the use of Doppler frequency for range and range-rate measurements, therefore, almost sounds like blasphemy. In spite of this, let us consider the matter for a moment. Frequency is nothing but the time derivative of phase and the tracking equipment cannot measure frequency, it can only measure phase and finite phase differences. The measured "frequency" is, therefore, the ratio of finite phase and time differences and is not a true derivative. Consequently, what is measured is only an approximation of frequency. The Doppler frequency should, be regarded as a mathematical artifice, an aide for the process of thinking, rather than a measurable physical quantity. In a recent paper it has been pointed out that the Doppler frequency is a function of the time derivative of propagation delay. Again, the tracking equipment cannot measure derivatives, only finite differences.

Generally, results are obtained more accurately and in a more straightforward manner if the primary quantities are used than if derived quantities are used. These thoughts have been the motivation to try to interpret the measurements made by the tracking equipment in terms of the primary quantities phase and propagation delay, instead of, as is customary, in the derived quantity frequency. The results are presented in this paper.

author

2. RANGE AND RANGE RATE WITHOUT DOPPLER FREQUENCY

Let us consider a case of practical importance. A space vehicle is tracked from the ground by means of transmitting an electromagnetic wave from the ground and measuring the two-way propagation delay as shown in Figure 1. The time variation of the wave may be written as $\exp\{j\phi\}$ where ϕ is the phase of the wave and is a function of time.

At time t_1 the transmitted phase from the ground is $\phi(t_1)$ and arrives at the vehicle at time $t = t_1 + \tau_{11}$. The wave is reflected or instantaneously retransmitted from the vehicle and is received at the ground at time $t_2 = t_1 + \tau_{11} + \tau_{21}$. A finite time interval T_1 later the phase $\phi(t_1 + T_1)$ is transmitted from the ground. Due to the movement of the vehicle the propagation time has changed. The phase $\phi(t_1 + T_1)$ will therefore be received by the vehicle at time $t_1 + T_1 + \tau_{12}$ and at the ground receiver at time $t_1 + T_1 + \tau_{12} + \tau_{22}$. The times of transmission and reception are summarized in Table 1.

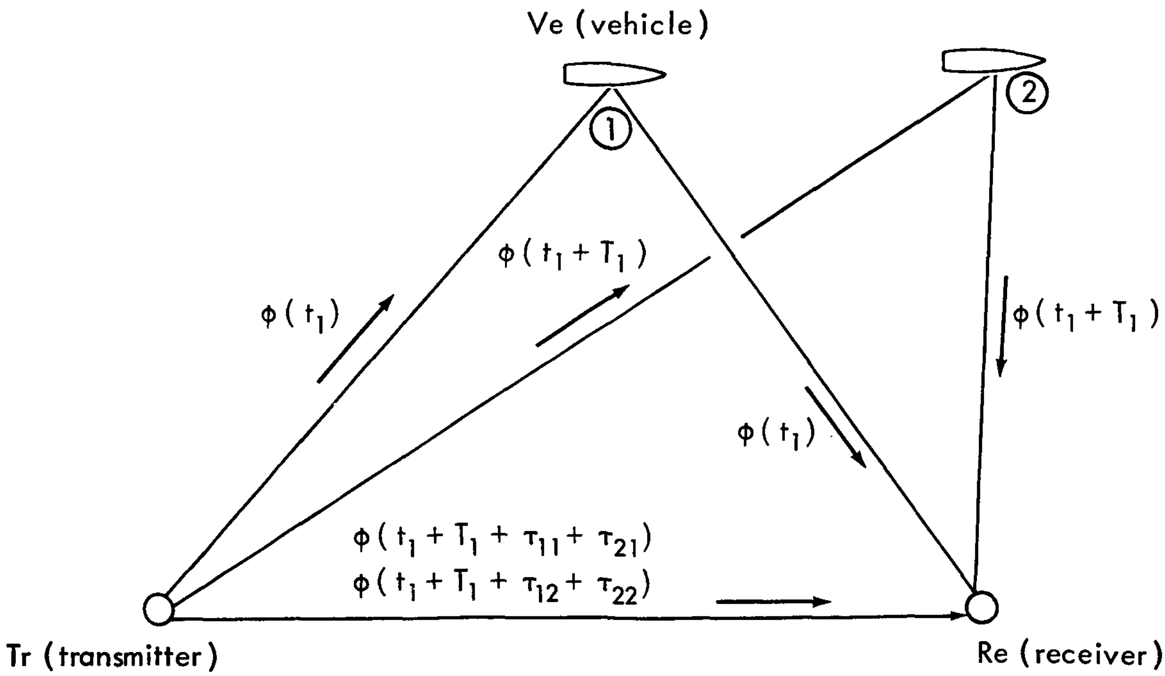

Figure 1

Table 1

Phase	Time of Transmission	Received at Vehicle	Received at Ground Receiver
$\phi(t_1)$	t_1	$t_1 + \tau_{11}$	$t_1 + \tau_{11} + \tau_{21}$
$\phi(t_1 + T_1)$	$t_1 + T_1$	$t_1 + T_1 + \tau_{12}$	$t_1 + T_1 + \tau_{12} + \tau_{22}$

The equipment on the ground is capable of measuring phase differences and can therefore measure

$$\phi_1 = \phi(t_1 + T_1) - \phi(t_1) \tag{1}$$

If the transmitter and receiver are close together, we can, without delay, feed the transmitted wave directly to the measuring equipment. The phase difference ϕ_2 transmitted by this direct path and measured during the same time interval as ϕ_1 is measured, is

$$\phi_2 = \phi(t_1 + T_1 + \tau_{12} + \tau_{22}) - \phi(t_1 + \tau_{11} + \tau_{21}) \tag{2}$$

Of interest is the difference $\Delta\phi$ between ϕ_2 and ϕ_1

$$\Delta\phi = \phi_2 - \phi_1$$

$$= \left\{\phi(t_1 + T_1 + \tau_{12} + \tau_{22}) - \phi(t_1 + T_1)\right\} - \left\{\phi(t + \tau_{11} + \tau_{21}) - \phi(t_1)\right\} \tag{3}$$

The two quantities $\Delta\phi$ and ϕ_2 contain all the information the equipment at the receiver is capable of obtaining. The problem is now to interpret these quantities in terms of range rate or change in range. In order to do this, we have to know how propagation delay is related to range and phase, to time. In the ideal case of vacuum and a perfect oscillator we have

$$r = c\tau \tag{4}$$

and

$$\phi = \omega_t t \tag{5}$$

Hence,

$$\Delta\phi = \omega_t\left\{\tau_{22} - \tau_{21} + \tau_{12} - \tau_{11}\right\} \tag{6}$$

and

$$\phi_2 = \omega_t\left\{T_1 + \tau_{22} - \tau_{21} + \tau_{12} - \tau_{11}\right\} \tag{7}$$

The time required for the vehicle to move from point (1) to point (2) in Figure 1 is

$$T = t_1 + T_1 + \tau_{12} - \left(t_1 + \tau_{11}\right) = T_1 + \tau_{12} - \tau_{11} \tag{8}$$

and after substitution of eq (8) into eq (7) we obtain

$$\phi_2 = \omega_t \left\{T + \tau_{22} - \tau_{21}\right\} \tag{9}$$

where ω_t is the transmitted angular frequency.

With

$$\Delta r_1 = c\left(\tau_{12} - \tau_{11}\right) \tag{10}$$

$$\Delta r_2 = c\left(\tau_{22} - \tau_{21}\right) \tag{11}$$

we obtain from eqs (6) and (9)

$$\Delta r_1 + \Delta r_2 = \frac{c\Delta\phi}{\omega_t} \tag{12}$$

$$\Delta r_2 + cT = \frac{c\phi_2}{\omega_t} \tag{13}$$

The average range rate is defined as

$$\dot{r}_{a1} = \frac{\Delta r_1}{\Delta T} \tag{14}$$

$$\dot{r}_{a2} = \frac{\Delta r_2}{T} \tag{15}$$

Elimination of ω_t between eqs (12) and (13) yields

$$\frac{\dot{r}_{a1} + \dot{r}_{a2}}{\dot{r}_{a2} + c} = \frac{\Delta\phi}{\phi_2} \tag{16}$$

or

$$\dot{r}_{a1} + \left(1 - \frac{\Delta\phi}{\phi_2}\right) \dot{r}_{a2} = c\,\frac{\Delta\phi}{\phi_2} \tag{17}$$

Eq (17) expresses $\dot{r}_{a1}$ and $\dot{r}_{a2}$ as a function of $\Delta\phi$ and ϕ_2, which are directly measureable by the receiver equipment in contrast to the Doppler equation which expresses dr_1/dt and dr_2/dt as a function of the non-measurable instantaneous Doppler frequency.

Eq (17) is referred to as the integrated Doppler equation and may be derived from the "instantaneous" Doppler equation by integration (Reference 1).

3. DISCUSSION

It has been shown in this report that the same results are achieved independently of whether the phase and delay time concept or the Doppler equation is used. This is reassuring, but it also raises these questions: Why use the phase method; are there any advantages? The author believes that the phase method has the following advantages:

1. Simplicity. Using Doppler, we first have to take the derivative of the phase and then integrate it again. Great care has to be taken in using the correct times and limits of integration. Otherwise, errors of relative magnitude $\dot{r}/c$ are easily introduced. Using the phase method reduces considerably the possibility of making errors.

2. The effects of special relativity are automatically taken care of. This is due to the fact that the analysis is based on phase and time. The phase is invariant and all time observations are made in the same reference frames. Relativistic time translations are not therefore called for.

3. The phase method describes the physical situation better, especially with respect to the measuring process, than the Doppler method does. The enhanced understanding of the process is of great value for error analysis. Especially the effects of oscillator drift are sometimes not analyzed correctly if the Doppler method is used. The phase method makes the correct analysis rather obvious.

4. It is obvious from the phase analysis that the average range rate $\dot{r}_a$ and not the instantaneous range rate $\dot{r}$ is measured. Note that $\dot{r}_a$ is the ratio of finite differences and $\dot{r}$ is a derivative.

5. In the Doppler method the quantity ϕ_2 is considered to be the product of ω_t and T_2. One error source, ϕ_2, is thereby split up into two error sources, ω_t and T_2. In the equipment ω_t and T_2 are handled separately and the error sources are, therefore, not only conceptually, but actually increased. If the phase method is employed, constant $\Delta\phi$ and constant ϕ_2 will correspond to the Doppler counting methods of constant N and constant T respectively.

It is apparent from the above discussion that the phase method provides better insight into the measuring process. As a result, the confidence in system analysis is increased and subsystem specifications can be improved, ultimately leading to a better system performance.

REFERENCE

1. Kruger, B., "The Doppler Equation in Range and Range Rate Measurement," Report X-507-65-385, GSFC, October 8, 1965.