THE DETERMINATION OF THE ORIENTATION OF A UNIFORMLY ROTATING OR RAPIDLY ROTATING VEHICLE UTILIZING THE OUTPUTS FROM SOLAR SENSORS AND A LATERAL MAGNETOMETER

by

Richard H. Ott, Jr., Henry M. Horstman,
and Richard M. Lahn

March 1966

Goddard Space Flight Center
Greenbelt, Maryland

TABLE OF CONTENTS

1.0 SUMMARY

19480

A general procedure is developed to determine the attitude of the longitudinal axis (spin axis) and a lateral axis (experiment axis) of a rotating vehicle from the output of solar sensors and a lateral magnetometer.

The orientation of the solar vector with respect to a rocket-axis system is obtained from solar sensors located both on the side and on the nose of the rocket. The lateral magnetometer is used to obtain the location of the component of the magnetic field vector along the lateral plane of the rocket. The solar vector is located in a horizon coordinate system from solar data obtained from "The American Ephemeris," and rocket radar inputs. The geomagnetic field vector in the horizon coordinate system is calculated from the Jensen and Cain field model using rocket radar data. The location of the longitudinal axis and a lateral (experiment) axis of the vehicle in both horizon coordinates and space fixed coordinates is calculated by the use of various coordinate transformations as discussed in references (3) and (4).

The method is programmed on an IBM 7094 computer. An explanation of the computer operations and a copy of the computer program is included in the report. A particular rocket is used as a sample case and the results are presented as time histories of the zenith and azimuth angles (horizon coordinates) and the Right Ascension and Declination (space fixed coordinates) of both the longitudinal and experiment axes.

author

2.0 INTRODUCTION

In order to translate and make use of the scientific information obtained from a rocket flight, it is often important that the attitude of the vehicle be known at a given time in flight. Various instruments can be included in the telemetry package to obtain this information such as; solar sensors, magnetometers, horizon sensors, and gyroscopes. This method utilizes solar sensors and a lateral magnetometer.

The solar sensor determines two angles which define two perpendicular planes each containing the sun. The intersection of the two planes defines a line pointing at the sun. The field of view of the sensor has the shape of an inverted pyramid with a typical angle between the opposite faces being 128 degrees. A total of 5 such sensors are necessary for a 4 Pi steradian coverage (see Figure 9). The lateral magnetometer simply measures the voltage output which is in turn proportional to the component of the magnetic field along that particular lateral axis.

It is necessary to locate at least two spatial vectors with respect to both the rocket and earth/space in order to obtain the orientation of the rocket with respect to earth/space. One vector is not sufficient as the results obtained are the same when the rocket is rotated about that vector.

The two vectors chosen in this procedure are the solar vector and the magnetic field vector. The method is therefore applicable only when the sun is within view of at least one solar sensor and the lateral component of the magnetic field is monitored by a lateral magnetometer.

3.0 COORDINATE SYSTEMS

It is necessary to describe the various coordinate systems referred to in the report. The three orthogonal unit vectors in each of the following right-handed systems are defined along the appropriate axes.

3.1 Rocket-axis Coordinate System (Figure 1)

$\bar{i}$ – experiment axis (an axis in the lateral rocket plane chosen as the reference axis

$\bar{j}$ – y axis (an axis in the lateral plane orthogonal to the experiment axis and the rocket longitudinal axis)

$\bar{k}$ – rocket longitudinal axis (spin axis of vehicle)

3.2 Horizon Coordinate System (Figures 5)

$\bar{i}$ – East axis (an axis in the local horizon plane in the east direction)

$\bar{j}$ – North axis (an axis in the local horizon plane in the north direction)

$\bar{k}$ – Vertical axis (an axis perpendicular to the horizon plane which is positive when measured upward)

3.3 Equatorial Coordinate System (Figure 5)

$\bar{i}$ – Equator axis (an axis along the equatorial plane perpendicular to the North Celestial Pole axis at the local meridian of the rocket.)

$\bar{j}$ – East axis (an axis in the equatorial plane orthogonal to the North Celestial Pole axis and the equator axis).

$\bar{k}$ – North Celestial Pole axis (an axis perpendicular to the equatorial plane measured from the center of the earth to the North Celestial Pole).

3.4 Space-Fixed Coordinate System (Figure 8)

$\bar{i}$ – Vernal equinox (an axis measured from the center of the earth towards the vernal equinox).

$\bar{j}$ – y* (an axis on the equitorial plane orthogonal to both the Vernal equinox and the North Celestial Pole axes).

$\bar{k}$ – North Celestial Pole axis (same as in 3.3).

4.0 SYMBOLS

A Azimuth angle

A_1 Lateral solar sensor angle (Case I sensor)

A_2 Longitudinal solar sensor angle (Case II sensor)

B_1 Lateral solar sensor angle (Case I sensor)

B_2 Longitudinal solar sensor angle (Case II sensor)

$\bar{E}$ Experiment axis vector

$\bar{H}$ Magnetic field vector

$\bar{i}, \bar{j}, \bar{k}$ Units vectors in right handed orthogonal coordinate systems

La Rocket latitude

Lo Rocket longitude

$\bar{R}$ Rocket longitudinal axis vector

$\bar{S}$ Solar vector

t_G Greenwich Sidereal time

$\bar{V}$ Arbitrary vector

X_1 Component of the solar vector along the experiment axis

X_2 Component of the magnetic field vector along the experiment axis

X_3 Component of the solar vector along the east axis

X_4 Component of the magnetic field vector along the east axis

x' Coordinate axis in lateral rocket plane

Y_1 Component of the solar vector along the y axis

Y_2 Component of the magnetic field vector along the y axis

Y_3 Component of the solar vector along the north axis

Y_4 Component of the magnetic field vector along the north axis

y Coordinate axis orthogonal to the Experiment axis and the rocket longitudinal axis

y' Coordinate axis orthogonal to the x' axis and the rocket longitudinal axis

Z Zenith angle

Z_1 Component of the solar vector along the rocket longitudinal axis

Z_2 Component of the magnetic field vector along the rocket longitudinal axis

Z_3 Component of the solar vector along the North Celestial Pole axis

Z_4 Component of the magnetic field vector along the North Celestial Pole axis

α Right Ascension

δ Declination

τ Hour angle

ϕ Azimuthal angle in the rocket lateral plane measured from the experiment axis

ϕ' Azimuthal angle in the rocket lateral plane measured from the x' axis (Case II sensor only)

ϕ_{S1} Angle between experiment and x' axis (Case I sensor)

ϕ_{S2} Angle between experiment and x' axis (Case II sensor)

θ Elevation angle from rocket lateral plane

All angles are in radians unless otherwise specified.

Subscripts

E Pertaining to the experiment axis vector

M Pertaining to the magnetic field vector

R Pertaining to the rocket longitudinal axis vector

S Pertaining to the solar vector

V Pertaining to an arbitrary vector

ER Pertaining to both the experiment axis vector and the rocket longitudinal axis vector

ZA Pertaining to a vector in the horizon coordinate system

$\epsilon\delta$ Pertaining to a vector in the equatorial coordinate system

5.0 METHOD OF ANALYSES

5.1 Solar Vector-Horizon Coordinate System (Figure 5)

The position of the solar vector with respect to the horizon coordinate system can be found from solar data in reference (1) and the rocket radar inputs; latitude, longitude, and altitude as a function of time.

5.2 Magnetic Field Vector — Horizon Coordinate System (Figure 6)

The magnetic field vector can be defined in the horizon coordinate system by use of the spherical harmonic analysis in reference (2). For a given latitude, longitude and altitude, the three earth-fixed components of the magnetic field vector can be obtained.

5.3 Solar Vector – Rocket-Axis System (Figures 1 and 2)

Each solar sensor measures the angle of incident sunlight with respect to the rocket-axis coordinate system along two perpendicular planes. For the sensor mounted along the side of the rocket, these planes will be the lateral plane and a plane containing the sensor and the vehicle longitudinal axes(Figure 1). For the sensor mounted on the nose of the vehicle, each plane contains the longitudinal axis and an arbitrary axis in the lateral plane. The lateral axes for the two planes are mutually perpendicular (Figure 2). Thus the solar vector can be totally defined with respect to the rocket-axis system by either type of sensor as long as the vector is within the viewing range of the sensor.

5.4 Magnetic Field Vector - Rocket-Axis System (Figure 3)

The lateral magnetometer measures the component of the external magnetic field along the lateral axis. The output of the magnetometer is sinusoidal for a rolling vehicle with the maximum or minimum corresponding to the alignment of the magnetometer along or opposite the projection of the magnetic field vector on the lateral plane. At this point in the analysis, only the angular location of the magnetic field vector in the rocket lateral plane (ϕ_M) is known. The elevation of this vector (θ_M) must be determined to totally define it in the rocket axis system. The magnetic field vector can then be totally defined in the rocket-axis system by equating the dot product of the solar and magnetic field vectors in the rocket-axis system with the known value of the dot product of the two vectors in the horizon coordinate system and subsequently obtaining the elevation angle of the magnetic field vector (θ_M).

5.5 Longitudinal and Experiment Axes — Horizon Coordinate System (Figure 5)

Two spatial vectors are at this point completely defined in the rocket-axis and horizon coordinate system. It is then necessary to obtain each rocket-axis unit vector in terms of horizon unit vectors. To do this, a new coordinate system is defined utilizing functions of the solar and magnetic field vectors as unit vectors. Each rocket coordinate axis can then be represented vectorially in this new coordinate system and each unit vector of this system is known in terms of the horizon system unit vectors. Thus each rocket-axis coordinate can be defined in the horizon coordinate system.

5.6 Longitudinal and Experiment Axes — Space Fixed Coordinates (Figures 5 and 8)

Each rocket system coordinate axis can be determined in space fixed coordinates by first converting to the equatorial coordinate system. This is obtained by utilizing the relationship between the unit vectors of the horizon and equatorial coordinate systems. A simple conversion can then be made to obtain the results in space fixed coordinates (right ascension and declination) from equatorial coordinates (hour angle and declination).

6.0 ANALYSIS

6.1 Solar Vector — Rocket-Axis System

The equations for obtaining the position of the solar vector in the rocket-axis coordinate system can be divided into the two following possible cases:

Case I. The solar sensor senses along an axis normal to the vehicle longitudinal axes (figure 1).

Case II. The sensor senses along the longitudinal axis (figure 2).

Case I.

$$\phi_S = \phi_{S1} + B_1 \tag{1}$$

The solar vector in a coordinate system consisting of the sensor axis ($\bar{i}$), rocket longitudinal axis ($\bar{k}$) and a third orthogonal axis ($\bar{j}$) is:

$$\overline{S} = \cos\theta_s \cos B_1\, \overline{i} + \cos\theta_s \sin B_1\, \overline{j} + \sin\theta_s \overline{k} \tag{2}$$

The component of the vector in the rocket-sensor plane is then:

$$\overline{S}_{ik} = \frac{\cos\theta_s \cos B_1 \overline{i} + \sin\theta_s \overline{k}}{\sqrt{\cos^2\theta_s \cos^2 B_1 + \sin^2\theta_s}} \tag{3}$$

Equation (3) is then dotted with the unit vector $\overline{i}$ to obtain $\cos A_1$

$$\cos A_1 = \overline{S}_{ik} \cdot \overline{i} = \frac{\cos\theta_s \cos B_1}{\sqrt{\cos^2\theta_s \cos^2 B_1 + \sin^2\theta_s}} \tag{4}$$

therefore

$$\sec^2 A_1 = 1 + \frac{\tan^2\theta_s}{\cos^2 B_1} \tag{5}$$

and

$$\theta_s = \tan^{-1}(\cos B_1 \tan A_1) \tag{6}$$

Case II. The solar vector in the coordinate system consisting of the rocket axis ($\overline{k}$), which is also the sensor axis, and the two orthogonal axes ($\overline{i}$, $\overline{j}$) along which the incident sunlight is measured is:

$$\overline{S} = \cos\theta_s \cos\phi'_s \overline{i} + \cos\theta_s \sin\phi'_s \overline{j} + \sin\theta_s \overline{k} \tag{7}$$

The component of the vector in the rocket, x' plane is:

$$\overline{S}_{ik} = \frac{\cos\theta_s \cos\phi'_s \overline{i} + \sin\theta_s \overline{k}}{\sqrt{\cos^2\theta_s \cos^2\phi'_s + \sin^2\theta_s}} \tag{8}$$

Equation 8 is then dotted with the unit vector $\overline{k}$

$$\cos(-A_2) = \overline{S}_{ik} \cdot \overline{k} = \frac{\sin\theta_s}{\sqrt{\cos^2\theta_s \cos^2\phi'_s + \sin^2\theta_s}} \tag{9}$$

then

$$\tan^2(-A_2) = \cot^2\theta_s \cos^2\phi'_s \tag{10}$$

The component in the rocket, y′ plane is:

$$\overline{S}_{ij} = \frac{\cos\theta_s \sin\phi'_s\,\overline{j} + \sin\theta_s\,\overline{k}}{(\cos^2\theta_s \sin^2\phi'_s + \sin^2\theta_s)^{1/2}} \tag{11}$$

Equation (11) is then dotted with $\overline{k}$

$$\cos B_2 = \overline{S}_{ij} \cdot \overline{k} = \frac{\sin\theta_s}{(\cos^2\theta_s \sin^2\phi'_s + \sin^2\theta_s)^{1/2}} \tag{12}$$

then

$$\tan^2 B_2 \quad \cot^2\theta_s \sin^2\phi'_s \tag{13}$$

ϕ'_s can be solved by combining equations (10) and (13)

$$\phi'_s = \tan^{-1}\left(\frac{-\tan B_2}{\tan A_2}\right) \tag{14}$$

where:		if:	
	$\phi'_s = \phi'_s$		$A_2 < 0,\ B_2 > 0$
	$\phi'_s = 180° + \phi'_s$		$A_2 > 0,\ B_2 > 0$
	$\phi'_s \quad 180° + \phi'_s$		$A_2 > 0,\ B_2 < 0$
	$\phi'_s = 360° + \phi'_s$		$A_2 < 0,\ B_2 < 0$

then

$$\phi_s \quad \phi'_s + \phi_{s2} \tag{15}$$

Equation (14) is substituted into (13) to obtain:

$$\theta_s = \tan^{-1}\left[\frac{1}{\tan^2 A_2 + \tan^2 B_2}\right]^{1/2} \tag{16}$$

The solar vector in the x, y, z coordinate system (figures 1 and 2) is:

$$\bar{S} = X_1 \bar{i}_{EXPERIMENT} + Y_1 \bar{j}_Y + Z_1 \bar{k}_{SPIN}$$

where:

$$X_1 = \cos \theta_S \cos \phi_S$$

$$Y_1 = \cos \theta_S \sin \phi_S$$

$$Z_1 = \sin \theta_S$$

θ_S and ϕ_S are obtained from equations (1) and (6) or (15) and (16) depending on whether case I or case II is applicable.

6.2 Magnetic Field Vector – Rocket-Axis System (Figure 3)

When the magnetic vector is aligned along the maximum projection of the magnetic vector on the lateral plane, a maximum voltage output is telemetered to the ground station. At this time, the magnetometer is sensing at $\phi_{M_{max}}$ which is the angle between the magnetometer axis and the reference axis. Conversely, when it is aligned opposite to this projection, a minimum output will result corresponding to $\phi_{M_{min}}$. A time history of ϕ_M can then be obtained by interpolation between the maximum and minimum values.

The magnetic vector in the rocket axis system is:

$$\bar{H} = X_2 \bar{i}_{EXPERIMENT} + Y_2 \bar{j}_Y + Z_2 \bar{k}_{SPIN} \tag{18}$$

where:

$$X_2 = \cos \theta_M \cos \phi_M$$

$$Y_2 = \cos \theta_M \sin \phi_M$$

$$Z_2 = \sin \theta_M$$

with θ_M yet to be determined.

6.3 Solar Vector - Horizon Coordinates (Figures 4 and 5)

The hour angle of the solar vector is obtained by noting figure 4.

$$t_S = \alpha_S - t_G - Lo \tag{19}$$

where α_S and t_G are found from reference (1) using the appropriate universal flight time and Lo is obtained from the rocket radar data.

Figure 5 is a representation of any vector, $\bar{V}$, in both a horizon and an equatorial coordinate system. It can be written as:

$$\bar{V}_{ZA} = \sin Z \sin A\ \bar{i}_{East} + \sin Z \cos A\ \bar{j}_{North} + \cos Z\ \bar{k}_{Vertical} \text{ (horizon)} \tag{20a}$$

$$V_{\epsilon\delta} = \cos \epsilon \cos \delta\ \bar{i}_{Equator} + \sin \epsilon \cos \delta\ \bar{j}_{East} + \sin \delta\ \bar{k}_{North\ Pole} \text{ (equatorial)} \tag{20b}$$

Each unit vector in one coordinate system can be represented by the unit vector in the other:

$$\bar{i}_{East} = \bar{j}_{East}$$

$$\bar{j}_{North} = -\sin La\ \bar{i}_{Equator} + \cos La\ \bar{k}_{North\ Pole} \tag{21a}$$

$$\bar{k}_{Vertical} = \cos La\ \bar{i}_{Equator} + \sin La\ \bar{k}_{North\ Pole}$$

$$\bar{i}_{Equator} = -\sin La\ \bar{j}_{North} + \cos La\ \bar{k}_{Vertical}$$

$$\bar{j}_{East} = \bar{i}_{East} \tag{21b}$$

$$\bar{k}_{North\ Pole} = \cos La\ \bar{j}_{North} + \sin La\ \bar{k}_{Vertical}$$

If the vector in question is the solar vector, the horizon components can be found by taking the dot product of equation (20b) and the unit vectors in (21a).

Then

$$\bar{S} = X_3\ \bar{i}_{East} + Y_3\ \bar{i}_{North} + Z_3\ \bar{k}_{Vertical} \tag{22}$$

where:

$$X_3 = \bar{S}_{\epsilon\delta} \cdot \bar{i}_{East} = \sin \epsilon_S \cos \delta_S$$

$$Y_3 = \bar{S}_{\epsilon\delta} \cdot \bar{j}_{North} = -\sin La \cos \epsilon_S \cos_S + \cos La \sin \delta_S$$

$$Z_3 = \bar{S}_{\epsilon\delta} \cdot \bar{k}_{Vertical} = \cos La \cos \epsilon_S \cos \delta_S + \sin La \sin \delta_S$$

and La is obtained from the rocket radar data.

6.4 Magnetic Field Vector – Horizon Coordinates (Figure 6)

The components of the magnetic field vector in the horizon coordinate system (X_4, Y_4, Z_4) are calculated using the Jensen and Cain, 48 term Expansion in Spherical Harmonics of the Magnetic Field in 1960 (reference 2). These equations have been programmed on the 7094 digital computer and are included as a subprogram of the general attitude program. The inputs to this subprogram are the vehicle's latitude, longitude, and altitude which are obtained from the rocket radar data.

The Magnetic field vector is then:

$$\bar{H} = X_4 \,\bar{i}_{East} + Y_4 \,\bar{j}_{North} + Z_4 \,\bar{k}_{Vertical} \tag{23}$$

6.5 Elevation of Magnetic Field Vector – Rocket-Axis System (Figure 3)

The dot product of two vectors is the same in any coordinate system; that is,

$$(\bar{S} \cdot \bar{H})_{Rocket\ Coordinates} = (S \cdot H)_{Horizon\ Coordinates}$$

Thus dotting equation (17) with (18), and (22) with (23) the following is obtained:

$$X_1X_2 + Y_1Y_2 + Z_1Z_2 = X_3X_4 + Y_3Y_4 + Z_3Z_4$$

or

$$\cos\theta_M \cos\phi_M X_1 + \cos\theta_M \sin\phi_M Y_1 + \sin\theta_M Z_1 = X_3X_4 + Y_3Y_4 + Z_3Z_4 \tag{24}$$

where:

θ_M is the only remaining unknown.

Equation (24) is solved for $\sin\theta_M$ and $\cos\theta_M$

$$\sin\theta_M = \frac{-B_{1M} \pm (B_{1M}^2 - 4A_M C_{1M})^{1/2}}{2A_M} \tag{25a}$$

$$\cos\theta_M = \frac{-B_{2M} \pm (B_{2M}^2 - 4A_M C_{2M})^{1/2}}{2A_M} \tag{25b}$$

where:

$$A_M = Z_1^2 + (X_1 \cos\phi_M + Y_1 \sin\phi_M)^2$$

$$B_{1M} = -2(X_3X_4 + Y_3Y_4 + Z_3Z_4)\, Z_1$$

$$B_{2M} = -2(X_3X_4 + Y_3Y_4 + Z_3Z_4)(X_1 \cos\phi_M + Y_1 \sin\phi_M)$$

$$C_{1M} = (X_3X_4 + Y_3Y_4 + Z_3Z_4)^2 - (X_1 \cos\phi_M + Y_1 \sin\phi_M)^2$$

$$C_{2M} = (X_3X_4 + Y_3Y_4 + Z_3Z_4)^2 - Z_1^2$$

Solving equation (25b) results in two solutions for $\cos\theta_M$. Since, by definition, θ_M must be either in the first or fourth quadrant (figure 3), $\cos\theta_M$ must be positive. It is possible for both values of $\cos\theta_M$ to be positive corresponding to a uniformly rotating case and an extreme coning case. The value of $\cos\theta_M$ corresponding to the extreme coning case can easily be eliminated by observing the continuity of the results. Once the correct value of $\cos\theta_M$ is known, the corresponding value of $\sin\theta_M$ from (25a) can be obtained.

6.6 Rocket Longitudinal and Experiment Vectors – Horizon Coordinates (Figures 5 and 7)

In order to obtain the Zenith and Azimuth angles of the rocket and experiment vectors, a new coordinate system is defined consisting of the unit vectors

$$[\bar{H}], \quad \left[\frac{\bar{S}\times\bar{H}}{(1-(\bar{S}\cdot\bar{H})^2)^{1/2}}\right], \quad \left[\frac{\bar{H}\times(\bar{S}\times\bar{H})}{(1-(\bar{S}\cdot\bar{H})^2)^{1/2}}\right]$$

These unit vectors can be represented in both the rocket-axis system and the horizon coordinate system. Thus the direction cosines of any arbitrary vector will be the same in either coordinate system. From figure 7, any vector in this new coordinate system becomes:

$$\bar{V} = (\bar{V}\cdot\bar{H})[\bar{H}] + \frac{\bar{V}\cdot(\bar{S}\times\bar{H})}{(1-(\bar{S}\cdot\bar{H})^2)^{1/2}}\left[\frac{\bar{S}\times\bar{H}}{(1-(\bar{S}\cdot\bar{H})^2)^{1/2}}\right] + \frac{(\bar{V}\cdot\bar{S})-(\bar{S}\cdot\bar{H})(\bar{V}\cdot\bar{H})}{(1-(\bar{S}\cdot\bar{H})^2)^{1/2}}\left[\frac{\bar{S}-(\bar{S}\cdot\bar{H})H}{(1-(\bar{S}\cdot\bar{H})^2)^{1/2}}\right] \tag{26}$$

The cross product of equations (17) and (18) yields:

$$\bar{S}\times\bar{H} = (Y_1Z_2 - Y_2Z_1)\,\bar{i}_{Experiment} + (Z_1X_2 - X_1Z_2)\,\bar{j}_Y + (X_1Y_2 - Y_1X_2)\,\bar{k}_{Rocket} \tag{27}$$

The rocket longitudinal and experiment vectors are then dotted with equations (17), (18), and (27)

$$\begin{aligned} \bar{R}\cdot\bar{S} &= Z_1 \\ \bar{R}\cdot\bar{H} &= Z_2 \\ \bar{R}\cdot(\bar{S}\times\bar{H}) &= X_1Y_2 - Y_1X_2 \end{aligned} \tag{28a}$$

$$\begin{aligned} \bar{E}\cdot\bar{S} &= X_1 \\ \bar{E}\cdot\bar{H} &= X_2 \\ \bar{E}\cdot(\bar{S}\times\bar{H}) &= Y_1Z_2 - Z_1Y_2 \end{aligned} \tag{28b}$$

Equations (28a) and (28b) are substituted into equation (26) to obtain the rocket longitudinal and experiment vectors.

$$\bar{R} = Z_2 [\bar{H}] + \frac{X_1Y_2 - Y_1X_2}{(1 - (\bar{S}\cdot\bar{H})^2)^{1/2}} \left[\frac{\bar{S} \times \bar{H}}{(1 - (\bar{S}\cdot\bar{H})^2)^{1/2}}\right] + \frac{Z_1 - (\bar{S}\cdot\bar{H})\, Z_2}{(1 - (\bar{S}\cdot\bar{H})^2)^{1/2}} \left[\frac{\bar{S} - (\bar{S}\cdot\bar{H}) H}{(1 - (\bar{S}\cdot\bar{H})^2)^{1/2}}\right] \tag{29a}$$

$$\bar{E} = X_2 [\bar{H}] + \frac{Y_1Z_2 - Z_1Y_2}{(1 - (\bar{S}\cdot\bar{H})^2)^{1/2}} \left[\frac{\bar{S} \times \bar{H}}{(1 - (\bar{S}\cdot\bar{H})^2)^{1/2}}\right] + \frac{X_1 - (\bar{S}\cdot\bar{H})\, X_2}{(1 - (\bar{S}\cdot\bar{H})^2)^{1/2}} \left[\frac{\bar{S} - (\bar{S}\cdot\bar{H})\, \bar{H}}{(1 - (\bar{S}\cdot\bar{H})^2)^{1/2}}\right] \tag{29b}$$

The cross product of equations (22) and (23) yields:

$$\bar{S} \times \bar{H} = (Y_3Z_4 - Z_3Y_4)\, \bar{i}_{East} + (Z_3X_4 - X_3Z_4)\, \bar{j}_{North} + (X_3Y_4 - Y_3X_4)\, \bar{k}_{Vertical} \tag{30}$$

The components of the rocket longitudinal and experiment vectors in the horizon coordinate system can be obtained by dotting each vector (as denoted by both equations (29a), (29b) and equation (20a)) into each horizon unit vector using equations (22), (23), and (30)

$$\bar{R}\cdot\bar{i}_{East} = \sin Z_R \sin A_R = Z_2X_4 + \frac{(X_1Y_2 - Y_1X_2)(Y_3Z_4 - Z_3Y_4)}{1 - (\bar{S}\cdot\bar{H})^2}$$

$$+ \frac{[Z_1 - (\bar{S}\cdot\bar{H})\, Z_2]\, [X_3 - (\bar{S}\cdot\bar{H})\, X_4]}{1 - (\bar{S}\cdot\bar{H})^2} \tag{31a}$$

$$\bar{R}\cdot\bar{j}_{North} = \sin Z_R \cos A_R = Z_2Y_4 + \frac{(X_1Y_2 - Y_1X_2)(Z_3X_4 - X_3Z_4)}{1 - (\bar{S}\cdot\bar{H})^2}$$

$$+ \frac{[Z_1 - (\bar{S}\cdot\bar{H})\, Z_2]\, [Y_3 - (\bar{S}\cdot\bar{H})\, Y_4]}{1 - (\bar{S}\cdot\bar{H})^2} \tag{31b}$$

$$\bar{R}\cdot\bar{k}_{Vertical} = \cos Z_R = Z_2Z_4 + \frac{(X_1Y_2 - Y_1X_2)(X_3Y_4 - Y_3X_4)}{1 - (\bar{S}\cdot\bar{H})^2}$$

$$+ \frac{[Z_1 - (\bar{S}\cdot\bar{H})\, Z_2]\, [Z_3 - (\bar{S}\cdot\bar{H})\, Z_4]}{1 - (\bar{S}\cdot\bar{H})^2} \tag{31c}$$

$$\bar{E} \cdot \bar{i}_{East} = \sin Z_E \sin A_E = X_2 X_4 + \frac{(Y_1 Z_2 - Z_1 Y_2)(Y_3 Z_4 - Z_3 Y_4)}{1 - (\bar{S} \cdot \bar{H})^2}$$

$$+ \frac{[X_1 - (\bar{S} \cdot \bar{H}) X_2]\ [X_3 - (\bar{S} \cdot \bar{H}) X_4]}{1 - (\bar{S} \cdot \bar{H})^2} \tag{32a}$$

$$\bar{E} \cdot \bar{j}_{North} = \sin Z_E \cos A_E = X_2 Y_4 + \frac{(Y_1 Z_2 - Z_1 Y_2)(Z_3 X_4 - X_3 Z_4)}{1 - (\bar{S} \cdot \bar{H})^2}$$

$$+ \frac{[X_1 - (\bar{S} \cdot \bar{H}) X_2]\ [Y_3 - (\bar{S} \cdot \bar{H}) Y_4]}{1 - (\bar{S} \cdot \bar{H})^2} \tag{32b}$$

$$\bar{E} \cdot \bar{k}_{vertical} = \cos Z_E = X_2 Z_4 + \frac{(Y_1 Z_2 - Z_1 Y_2)(X_3 Y_4 - Y_3 X_4)}{1 - (\bar{S} \cdot \bar{H})^2}$$

$$+ \frac{[X_1 - (\bar{S} \cdot \bar{H}) X_2]\ [Z_3 - (\bar{S} \cdot \bar{H}) Z_4]}{1 - (\bar{S} \cdot \bar{H})^2} \tag{32c}$$

where:

$$\bar{S} \cdot \bar{H} = X_3 X_4 + Y_3 Y_4 + Z_3 Z_4$$

From equations (31c) and (32c), the Zenith angle of the experiment or rocket longitudinal axis is:

$$Z_{ER} = \tan^{-1} \left[\frac{(1 - \cos^2 Z_{ER})^{1/2}}{\cos Z_{ER}} \right] . \tag{33}$$

From Figure 5, if:

$$Z_{ER} > 0 \qquad Z_{ER} = Z_{ER}$$

$$Z_{ER} < 0 \qquad Z_{ER} = 180^\circ + Z_{ER}$$

Equation (33) is substituted into (31b) and (31c) for the rocket longitudinal vector or (32b) and (32c) for the experiment vector to obtain:

$$\sin A_{ER} = \frac{(\sin Z_{ER} \sin A_{ER})}{\sin Z_{ER}} \tag{34}$$

$$\cos A_{ER} = \frac{(\sin Z_{ER} \cos A_{ER})}{\sin Z_{ER}} \tag{35}$$

The Azimuth of the rocket or experiment vector is obtained by dividing equation (34) by (35)

$$A_{ER} = \tan^{-1} \left[\frac{\sin A_{ER}}{\cos A_{ER}}\right] = \tan^{-1} (\tan A_{ER}) \tag{36}$$

if:

$\tan A_{ER} > 0,\ \sin A_{ER} > 0$	then:	$A_{ER} = A_{ER}$
$\tan A_{ER} < 0,\ \sin A_{ER} > 0$		$A_{ER} = 180^\circ + A_{ER}$
$\tan A_{ER} > 0,\ \sin A_{ER} < 0$		$A_{ER} = 180^\circ + A_{ER}$
$\tan A_{ER} < 0,\ \sin A_{ER} < 0$		$A_{ER} = 360^\circ + A_{ER}$

6.7 Rocket Longitudinal and Experiment Vectors – Equatorial Coordinates (Figures 4 and 5)

By taking the dot product of the rocket longitudinal and experiment vectors in horizon coordinates (equation 20a) and each equatorial unit vector (equation 21b), the components of the experiment and rocket longitudinal vectors in equatorial coordinates can be obtained.

$$\bar{V}_{ER} \cdot \bar{i}_{Equator} = \cos \epsilon_{ER} \cos \delta_{ER} = -\sin La \sin Z_{ER} \cos A_{ER} + \cos La \cos Z_{ER} \tag{37a}$$

$$\bar{V}_{ER} \cdot \bar{j}_{East} = \sin \epsilon_{ER} \cos \delta_{ER} = \sin Z_{ER} \sin A_{ER} \tag{37b}$$

$$\bar{V}_{ER} \cdot \bar{k}_{North\ Pole} = \sin \delta_{ER} = \cos La \sin Z_{ER} \cos A_{ER} + \sin La \cos Z_{ER} \tag{37c}$$

The declination of the experiment and rocket longitudinal vector can be obtained from equation (37c)

$$\delta_{ER} = \tan^{-1}\left[\frac{\sin \delta_{ER}}{(1 - \sin^2 \delta_{ER})^{1/2}}\right] \text{ where } -90^\circ \leq \delta_{ER} \leq 90^\circ \tag{38}$$

The hour angle can be found by substituting equation (38) into (37a) and (37b).

$$\sin \epsilon_{ER} = \frac{(\cos \delta_{ER} \sin \epsilon_{ER})}{\cos \delta_{ER}} \tag{39}$$

$$\cos \epsilon_{ER} = \frac{(\cos \delta_{ER} \cos \epsilon_{ER})}{\cos \delta_{ER}} \tag{40}$$

Therefore dividing equation (39) by (40)

$$\epsilon_{ER} = \tan^{-1}\left[\frac{\sin \epsilon_{ER}}{\cos \epsilon_{ER}}\right] = \tan^{-1}(\tan \epsilon_{ER})$$

if:

$$\begin{array}{lll}
\tan \epsilon_{ER} > 0,\ \sin \epsilon_{ER} > 0 & \text{then:} & \epsilon_{ER} = \epsilon_{ER} \\
\tan \epsilon_{ER} < 0,\ \sin \epsilon_{ER} > 0 & & \epsilon_{ER} = 180^\circ + \epsilon_{ER} \\
\tan \epsilon_{ER} > 0,\ \sin \epsilon_{ER} < 0 & & \epsilon_{ER} = 180^\circ + \epsilon_{ER} \\
\tan \epsilon_{ER} < 0,\ \sin \epsilon_{ER} < 0 & & \epsilon_{ER} = 360^\circ + \epsilon_{ER}
\end{array} \tag{41}$$

The Right Ascension can then be obtained from figure 4.

$$\alpha_{ER} = \epsilon_{ER} + t_G + Lo \tag{42}$$

7.0 COMPUTER PROGRAMMING

The procedure has been written in a program for an IBM 7094 computer in Fortran IV. It could also be applied to other computers with only a few format changes. A copy of the computer program is shown below along with an explanation of the data inputed in the particular sample case, the results of which are shown in Appendix B.

Tract Program Inputs

The Tract Program (Fortran IV), which determines the body orientation of the rocket, has been designed to run on the Moonlight System (IBM 7040-7094 direct Couple system).

Two BCD tapes are inputed to the program and are set up on tape units 18 and 19.

I. Input Tapes

A. Tape 18 contains the telemetry data and each record should contain time, latitude, longitude, and altitude in this order. The units are secs. (after launch), degrees, and feet, respectively.

B. Tape 19 contains the sunsensor data and each record should contain time (after launch), the eye or sensor number which was viewing at this time (1, 2, 3, or 4), the lateral solar sensor angle, and the longitudinal solar sensor angle (see Figures 1 and 2). The units are secs. none (integer), degrees, and degrees, respectively.

II. Input Cards

A. CARD 1 - Format (2I5, 7F10.5) Nine values are to be read in.

(1) if this value is nonzero the program will go through a Calcomp plotting sequence yielding an output tape at 200 B.P.I. which can be plotted on an uncoupled Calcomp Plotter,

(2) this value determines the number of cards to be read in section B,

(3) the initial time where the program will start computing,

(4) the final time the program will execute,

(5) the constant time increment for computation (the three above values are in secs),

(6) the angle between the experiment and 'x' axis, Case I sensor, Eye #1,

(7) the angle between the experiment and 'x' axis, Case I sensor, Eye #2,

(8) the angle between the experiment and 'x' axis, Case I sensor, Eye #3,

(9) the angle between the experiment and 'x' axis, Case II sensor (the four above values are in degrees).

B. The number of cards to be read in this section equals the number in position two of Card 1. Each card contains two values (2F10.5), time after launch (secs), and the angle between the experiment axis and the maximum or minimum (alternates) projection of the magnetic vector (degrees), respectively.

C. Seven cards of three numbers each (3F10.5) are read in this section. The times are read in hours, minutes, and seconds, respectively, on the card.

Card 1 – Greenwich sidereal time of launch day.

Card 2 – Greenwich sidereal time of following day.

Card 3 – Right ascension of sun for launch day.

Card 4 – Right ascension of sun for following day.

Card 5 – Declination of sun for launch day.

Card 6 – Declination of sun for following day.

Card 7 – Universal time of launch.

III. For Calcomp Plotting the following additional cards should be added.

Card 1 – A string of Hollerith characters to be printed as the title at the top of the graph. The characters should be centered in the 72 columns of the punched card.

Card 2 – One integer value specifying the number of plots (I5). The following cards should be added in sequence for each plot, e.g., if there are three plots there should be three sets of the following cards.

Card 3 – A string of Hollerith characters to be printed as the label for the abscissa of the graph. The characters should be centered in the 72 columns of the punched card.

Card 4 – A string of Hollerith characters to be printed as the label for the ordinate of the graph. The characters should be centered in the 72 columns of the punched card.

Card 5 – Eight integer values (7I5); (1) any non zero value, (2) I-Printed values of the ordinate are 10.**I times the actual values, (3) J-Printed values of the ordinate have J digits following the decimal point, (4) K-Printed values of the abscissa are 10.**K times the actual values, (5) M-Printed values of the abscissa have M digits following the decimal point, (6) number of horizontal grid lines, (7) number of vertical grid lines, (8) NC-number of curves on this plot (maximum 8).

Card 6 – Can contain from two to ten integer values (10I5) depending upon NC (#8, Card 5); (1) integer corresponds to the variable to be plotted as the abscissa for the plot, (2) (10) the integer values corresponding to the variable to be plotted as the ordinate or ordinates for the plot, e.g., if there are four curves on this plot then there are four ordinate integer values.

Note – The variables which can be plotted and their integer designations follow:

Integer Symbol	Variable Computed
1	Time (sec)
2	Azimuth, Rocket Longitudinal Axis (deg)
3	Azimuth, Experiment Axis (deg)
4	Zenith, Rocket Longitudinal Axis (deg)
5	Zenith, Experiment Axis (deg)
6	Right Ascension, Rocket Longitudinal Axis (HRS)
7	Right Ascension, Experiment Axis (HRS)
8	Declination, Rocket Longitudinal Axis (degrees)
9	Declination, Experiment Axis (deg)

Card 7 – Six values (6F10.5), (1) Spacing between horizontal grid lines, (2) Spacing between vertical grid lines (inches), (3) maximum value to be printed for the abscissa, (4) minimum values to be printed for the abscissa, (5) maximum value to be printed for the ordinate, (6) minimum values to be printed for the ordinate.

A copy of the computer program follows:

```
$JOB  1091C003 405LAHN
$SETUPIBFTC     18 TD           SAVE  REEL         000001BCD
$SETUPIBFTC     19 TD           SAVE  REEL         000001BCD
$EXECUTE        IBJOB
$IBJOB          GO,MAP,SOURCE
$IBFTC TRACT    NOLIST,NOREF,NODECK,M94
      DIMENSION T1(600),PHIM1(600),XHR(7),XMIN(7),SEC(7),T(7),NSCALE(5),
     XT2(100),XLAT1(1000),XLONG1(1000),ALT1(1000),PHIS1(1000),
     XTHETA1(1000),CPLVAR(9,1000),AA(12),BB(12),CC(12),BCDX(10),
     XEPSIL(2),
     XCOSAZM(2),SINAZM(2),TANAZM(2),AZM(2),SINDEL(2),CODSIE(2),
     XIO(8),
     XCODCOE(2),DELTA(2),COSDEL(2),SINEPS(2),COSEPS(2),TANEPS(2),
     XCOSZEN(2),SIZCOA(2),SIZSIA(2),ZEN(2),SINZEN(2),ALPHA(2),XH(2),
     XXM(2),XS(2),IX(2),NX(2)
    1 FORMAT (F10.2,I10,2F10.2)
    2 FORMAT(4E12.5)
    3 FORMAT(12A6)
    4 FORMAT(10I5)
    5 FORMAT(6F10.5)
    6 FORMAT(2F10.5)
    7 FORMAT(3F10.5)
    8 FORMAT(2I5,7F10.5)
    9 FORMAT(10X,4HTIME,10X,14HZENITH(ROCKET),10X,15HAZIMUTH(ROCKET),
     X10X,18HZENITH(EXPERIMENT),10X,19HAZIMUTH(EXPERIMENT))
   10 FORMAT(9X,F6.2,10X,F7.3,1X,3HDEG,14X,F7.3,1X,3HDEG,15X,F7.3,1X,
     X3HDEG,18X,F7.3,1X,3HDEG)
   15 FORMAT(19X,23HRIGHT ASCENSION(ROCKET),4X,19HDECLINATION(ROCKET),
     X4X,27HRIGHT ASCENSION(EXPERIMENT),4X,23HDECLINATION(EXPERIMENT))
   16 FORMAT(19X,I2,3HHRS,2X,I2,3HMIN,2X,F6.3,3HSEC,8X,F7.3,1X,3HDEG,
     X9X,I2,3HHRS,2X,I2,3HMIN,2X,F6.3,3HSEC,12X,F7.3,1X,3HDEG)
   17 FORMAT(10X,8HFOR TIME,2X,F6.2,2X,61HTHE COSINE OF THE AZMITH IS GR
     XEATER THAN 1. COMPUTATION STOPS)
   19 FORMAT(1H1)
   20 FORMAT(1H0)
   21 FORMAT(41H BOTH ROOTS NO GOOD, POINT SKIPPED, TIME=,F10.5)
   22 FORMAT(41H TWO NEGATIVE ROOTS, POINT SKIPPED, TIME=,F10.5)
   23 FORMAT(41H TWO POSITIVE ROOTS, POINT SKIPPED, TIME=,F10.5)
   30 FORMAT(41H  RAD21 IS NEGATIVE, POINT SKIPPED, TIME=,F10.5)
   31 FORMAT(41H  RAD22 IS NEGATIVE, POINT SKIPPED, TIME=,F10.5)
      DATA (BCDX(J),J=1,10)/O120000000000,O150000000000,O160000000000,
     XO170000000000,O320000000000,O350000000000,O360000000000,
     XO370000000000,O520000000000,O550000000000/
      NOUNT=0
      NOUNT1=1
      NOUNT2=0
      READ(5,8) NPLOT,N1,TIN,TFIN,TINCRE,SSA,SSB,SSC,SSD
      READ(5,6) (T1(J),PHIM1(J),J=1,N1)
      READ(5,7)(XHR(J),XMIN(J),SEC(J),J=1,7)
      WRITE(6,19)
      JUMP=0
C SUNSENSOR ROUTINE
C TWO CASES ARE POSSIBLE
C CASE I IS A LATERALLY MOUNTED SENSOR
C CASEII IS A LONGITUDINALLY MOUNTED SENSOR
      CONV=57.295779
      PI=3.14159265
      PHIS1A=SSA/CONV
      PHIS1B=SSB/CONV
      PHIS1C=SSC/CONV
      DO 210 J=1,7
  210 T(J)=XHR(J)*3600.+XMIN(J)*60.+SEC(J)
```

```
      T(5)=T(5)/15.
      T(6)=T(6)/15.
   11 READ (19,1)  TIME,NEYE,C,D
      IF(TIME-TIN)11,12,12
   13 READ(19,1) TIME,NEYE,C,D
   12 TL=TIME
      IF(TL-TFIN)14,14,90
   14 A=C/CONV
      B=D/CONV
      T1(NOUNT1)=TL
      TANA=SIN(A)/COS(A)
      TANB=SIN(B)/COS(B)
C   TEST FOR CASE NUMBER
      IF (NEYE.EQ.4) GO TO 66
C   COMPUTATIONS FOR CASE I
   54 THETAS=ATAN(COS(B)*TANA)
      IF (NEYE-2) 51,52,53
   51 PHIS=B+PHIS1A
      GO TO 80
   52 PHIS=PHIS1B+B
      GO TO 80
   53 PHIS=PHIS1C+B
      GO TO 80
C   COMPUTATIONS FOR CASE II
   66 THETAS=ATAN(1./SQRT(TANA**2+TANB**2))
      PHISP=ATAN(-TANB/TANA)
      IF(A.LT.0.) GO TO 67
      PHISP=PI+PHISP
      GO TO 70
   67 IF(B.LT.0.) GO TO 69
      PHISP=PHISP
      GO TO 70
   69 PHISP=2.*PI+PHISP
   70 PHIS=PHISP+SSD
C   X1,Y1,Z1, ARE COMPUTED(SOLAR VECTOR, ROCKET AXIS SYSTEM)
   80 PHIS1(NOUNT1)= PHIS
      THETA1(NOUNT1)=THETAS
      NOUNT1=NOUNT1+1
      GO TO 13
   90 NOUNT1=NOUNT1-1
      NT2CNT=1
      TFIN2=TFIN+.2
      TIN2=TIN-.2
   25 READ (18,2) T21,XLAT2,XLONG2,XALT2
      IF(TIN2-T21)26,26,25
   26 READ(18,2) T21,XLAT2,XLONG2,XALT2
      IF(T21-TFIN2)27,28,28
   27 T2(NT2CNT)=T21
      XLAT1(NT2CNT)=XLAT2
      XLONG1(NT2CNT)=XLONG2
      ALT1(NT2CNT)=XALT2
      NT2CNT=NT2CNT+1
      GO TO 26
   28 N2=NT2CNT-1
   98 XNOUNT=NOUNT2
      TD=TIN+XNOUNT*TINCRE
      IF(TD.GT.TFIN) GO TO 800
      NOUNT2=NOUNT2+1
      CALL TIMCOR(TD,T1,PHIS1,NOUNT1,PHIS2)
      CALL TIMCOR(TD,T1,THETA1,NOUNT1,THETA2)
  100 X1=COS(THETA2)*COS(PHIS2)
      Y1=COS(THETA2)*SIN(PHIS2)
      Z1=SIN(THETA2)
C     MAGNETIC FIELD COMPONENT SECTION
C THIS SECTION COMPUTES X4,Y4,Z4 WITH LATITUDE,LONGITUDE, AND ALTITUDE IMPUTS
C MAGNETIC FIELD VECTOR, EARTH-FIXED AXIS SYSTEM
```

```
      CALL TIMCOR(TD,T2,XLAT1,N2,XLAT)
      CALL TIMCOR(TD,T2,XLONG1,N2,XLONG)
      CALL TIMCOR(TD,T2,ALT1,N2,ALT)
      XLONG=-XLONG
      ALT=ALT/3280.833
      CALL FIELD(XLAT,XLONG,ALT,BT,BP,BR,B2)
      X4=BP/B2
      Y4=-BT/B2
      Z4=BR/B2
C THIS SECTION LOCATES THE SOLAR VECTOR RELATIVE TO AN EARTH-FIXED
C AXIS SYSTEM
C NUMBER =NUMBER OF TIMES THROUGH PROGRAM
C T(2) - GREENWICH SIDERIAL TIME OF FOLLOWING DAY (0 HRS. UT)
C T(2) - GREENWICH SIDERIAL TIME OF FOLLOWING DAY (OHRS. UT)
C T(3) - RIGHT ASCENSION OF SUN FOR LAUNCH DAY (0 HRS. UT)
C T(4) - RIGHT ASCENSION OF SUN FOR FOLLOWING DAY (0 HRS. UT)
C T(5) - DECLINATION OF SUN FOR LAUNCH DAY (0 HRS. UT)
C T(6) - DECLINATION OF SUN FOR FOLLOWING DAY (0 HRS. UT)
C T(7)- UNIVERSAL TIME OF LAUNCH
C TL- TIME AFTER LAUNCH
C COMPUTES GREENWICH SIDERIAL TIME, RIGHT ASCENSION, HOUR ANGLE,
C DECLINATION, OF THE SUN AND X3,Y3,Z3
C CONVERTS ALL IMPUTTED TIMES TO SECONDS
      CONST1= (T(7)+TL)/(24.*3600.)
      TG=(T(1)+CONST1*(T(2)-T(1))+T(7)+TL)/240.
      ALPHAS=(T(3)+CONST1*(T(4)-T(3)))/240.
      EPSILS=(ALPHAS-TG-XLONG)/CONV
      DELTAS=(T(5)+CONST1*(T(6)-T(5)))/(CONV*240.)
      XLAT=XLAT/CONV
      X3=COS(DELTAS)*SIN(EPSILS)
      Y3=-SIN(XLAT)*COS(EPSILS)*COS(DELTAS)+COS(XLAT)*SIN(DELTAS)
      Z3=COS(XLAT)*COS(EPSILS)*COS(DELTAS)+SIN(XLAT)*SIN(DELTAS)
C       MAGNETOMETER SECTION
C   CALCULATES X2,Y2,Z2  WITH AZMITH IMPUT
      CALL TIMCOR(TD,T1,PHIM1,N1,DPHIM)
      PHIM=DPHIM/CONV
      PIE=2.*PI
      IF(PHIM-PIE)409,409,410
  410 PHIM=PHIM-PIE
  409 DOTP1=X3*X4+Y3*Y4+Z3*Z4
      A1M=Z1**2+(X1*COS(PHIM)+Y1*SIN(PHIM))**2
      B1M=-2.*Z1*DOTP1
      C1M=DOTP1**2-(X1*COS(PHIM)+Y1*SIN(PHIM))**2
      A2M=A1M
      B2M=-2.*DOTP1*(X1*COS(PHIM)+Y1*SIN(PHIM))
      C2M=DOTP1**2-Z1**2
      RAD21=B1M**2-4.*A1M*C1M
      IF(RAD21.GT.0.) GO TO 408
      NFORM=30
      GO TO 998
  408 RAD1=SQRT(RAD21)
      SINE1=(-B1M+RAD1)/(2.*A1M)
      SINE2=(-B1M-RAD1)/(2.*A1M)
      RAD22=B2M**2-4.*A2M*C2M
      IF(RAD22.GT.0.) GO TO 407
      NFORM=31
      GO TO 998
  407 RAD2=SQRT(RAD22)
      COSE1=(-B2M+RAD2)/(2.*A2M)
      COSE2=(-B2M-RAD2)/(2.*A2M)
      IF(COSE1.GT.0.) GO TO 302
      IF(COSE2.GT.0.) GO TO 305
      NFORM=22
      GO TO 998
  302 IF(COSE2.LT.0.) GO TO 304
      NFORM=23
```

```
      GO TO 998
  304 COTHET=COSE1
      GO TO 306
  305 COTHET=COSE2
  306 CHECK=COTHET**2+SINE1**2
      CHECK1=COTHET**2+SINE2**2
      CONE=1.-CHECK
      CONE1=1.-CHECK1
      TEST = 0.1
  700 TEST = TEST*0.1
      IF(ABS(CONE).LE.TEST) GO TO 710
      IF(ABS(CONE1).LE.TEST) GO TO 720
      NFORM=21
      GO TO 998
  710 IF(ABS(CONE1).LE.TEST) GO TO 700
  307 SITHET=SINE1
      GO TO 310
  720 SITHET=SINE2
  310 X2=COTHET*COS(PHIM)
      Y2=COTHET*SIN(PHIM)
      Z2=SITHET
      A6=(X1*Y2-Y1*X2)/(1.-DOTP1**2)
      A7=(Y1*Z2-Z1*Y2)/(1.-DOTP1**2)
      B6=(Z1-DOTP1*Z2)/(1.-DOTP1**2)
      B7=(X1-DOTP1*X2)/(1.-DOTP1**2)
      C1=Y3*Z4-Z3*Y4
      C2=Z3*X4-X3*Z4
      C3=X3*Y4-Y3*X4
      D1=X3-DOTP1*X4
      D2=Y3-DOTP1*Y4
      D3=Z3-DOTP1*Z4
      SIZSIA(1)=Z2*X4+A6*C1+B6*D1
      SIZCOA(1)=Z2*Y4+A6*C2+B6*D2
      COSZEN(1)=Z2*Z4+A6*C3+B6*D3
      COSZEN(2)=X2*Z4+A7*C3+B7*D3
      IF(COSZEN(2).LT.1.)GO TO 599
      NFORM=17
      GO TO 998
  599 SIZSIA(2)=X2*X4+A7*C1+B7*D1
      SIZCOA(2)=X2*Y4+A7*C2+B7*D2
      DO 630 J=1,2
      ZEN(J)=ATAN((SQRT(1.-COSZEN(J)**2))/COSZEN(J))
      IF(ZEN(J))600,601,601
  600 ZEN(J)=PI+ZEN(J)
  601 SINZEN(J)=SIN(ZEN(J))
      SINAZM(J)=SIZSIA(J)/SINZEN(J)
      COSAZM(J)=SIZCOA(J)/SINZEN(J)
      TANAZM(J)=SINAZM(J)/COSAZM(J)
      AZM(J)=ATAN(TANAZM(J))
      IF(TANAZM(J).GT.0.) GO TO 604
      IF(SINAZM(J).GT.0.) GO TO 603
      AZM(J)=2.*PI+AZM(J)
      GO TO 610
  603 AZM(J)=PI+AZM(J)
      GO TO 610
  604 IF(SINAZM(J).GT.0.) GO TO 610
      AZM(J)=PI+AZM(J)
  610 CODCOE(J)=COS(XLAT)*COSZEN(J)-SIN(XLAT)*SINZEN(J)*COSAZM(J)
      CODSIE(J)=SINZEN(J)*SINAZM(J)
      SINDEL(J)=COS(XLAT)*SINZEN(J)*COSAZM(J)+SIN(XLAT)*COSZEN(J)
      DELTA(J)=ATAN((SINDEL(J))/(SQRT(1.-SINDEL(J)**2)))
      COSDEL(J)=COS(DELTA(J))
      SINEPS(J)=CODSIE(J)/COSDEL(J)
      COSEPS(J)=CODCOE(J)/COSDEL(J)
      TANEPS(J)=SINEPS(J)/COSEPS(J)
      EPSIL(J)=ATAN(TANEPS(J))
```

```
      IF(TANEPS(J).GT.0.) GO TO 614
      IF(SINEPS(J).GT.0.) GO TO 613
      EPSIL(J)=2.*PI+EPSIL(J)
      GO TO 620
  613 EPSIL(J)=PI+EPSIL(J)
      GO TO 620
  614 IF(SINEPS(J).GT.0.) GO TO 620
      EPSIL(J)=PI+EPSIL(J)
  620 EPSIL(J)=EPSIL(J)*CONV
      AZM(J)=AZM(J)*CONV
      ZEN(J)=ZEN(J)*CONV
      DELTA(J)=DELTA(J)*CONV
      ALPHA(J)=TG+XLONG+EPSIL(J)
      IF(ALPHA(J).LT.360.) GO TO 625
      ALPHA(J)=ALPHA(J)-360.
  625 XH(J)=ALPHA(J)/15.
      IX(J)=XH(J)
      XXI=IX(J)
      XM(J)=(XH(J)-XXI)*60.
      NX(J)=XM(J)
      XXN=NX(J)
      XS(J)=(XM(J)-XXN)*60.
  630 CONTINUE
      IF(NPLOT.EQ.0) GO TO 631
      NOUNT=NOUNT+1
      CPLVAR(1,NOUNT)=TD
      CPLVAR(2,NOUNT)=AZM(1)
      CPLVAR(3,NOUNT)=AZM(2)
      CPLVAR(4,NOUNT)=ZEN(1)
      CPLVAR(5,NOUNT)=ZEN(2)
      CPLVAR(6,NOUNT)=XH(1)
      CPLVAR(7,NOUNT)=XH(2)
      CPLVAR(8,NOUNT)=DELTA(1)
      CPLVAR(9,NOUNT)=DELTA(2)
  631 WRITE(6,9)
      WRITE(6,10) TIME,ZEN(1),AZM(1),ZEN(2),AZM(2)
      WRITE(6,15)
      WRITE(6,16) IX(1),NX(1),XS(1),DELTA(1),IX(2),NX(2),XS(2),DELTA(2)
      WRITE(6,20)
      JUMP=JUMP+6
      GO TO 999
  998 WRITE(6,NFORM) TD
      WRITE(6,20)
      JUMP=JUMP+3
  999 IF(JUMP.LE.59) GO TO 98
      WRITE(6,19)
      JUMP=0
      GO TO 98
  800 IF(NPLOT.EQ.0) GO TO 1000
      READ(5,3) (AA(J),J=1,12)
      READ(5,4) NPLTS
      CALL CPLTAP(10)
      DO 820 JPLOT=1,NPLTS
      READ(5,3) (BB(J),J=1,12)
      READ(5,3) (CC(J),J=1,12)
      READ(5,4)  ((NSCALE(J),J=1,5),NH,NV,JCPLT)
      READ(5,4) (IA,IO(N),N=1,JCPLT)
      READ(5,5) SBH,SBV,X2,X1,Y2,Y1
      CALL CPLOT1(NSCALE,NH,SBH,NV,SBV)
      CALL CPLOT2(X2,X1,Y2,Y1)
      KK=1
      DO 810 JCPLT1=1,JCPLT
      NO=IO(JCPLT1)
      CALL CPLOT3(BCDX(KK),CPLVAR(IA,1),CPLVAR(NO,1),NOUNT)
  810 KK=KK+1
  820 CALL CPLOT4(72,BB(1),72,CC(1),72,AA(1))
```

```
      CALL ENDPLT
 1000 STOP
      END
$IBFTC TIMC    LIST,REF,DECK,M94
      SUBROUTINE TIMCOR  (TIM,Y,THET,MT,ANS)
C     MT=NUMBER OF VALUES IN TABLE
C     ANS=VALUE OF THET CORRESPONDING TO TIM IN T VALUES
      DIMENSION  Y(400),THET(400)
      ATIM=TIM-Y(1)
      IF (ATIM)3,4,5
    4 ANS=THET(1)
      GO TO 50
    3 KT=1
   99 FORMAT(38H VALUE IS OUTSIDE RANGE OF TABLE,TIME=,E16.8,7H LIMIT=,
     XE16.8)
   97 WRITE(6,99) TIM,Y(KT)
      GO TO  51
    5 BTIM=TIM-Y(MT)
      IF (BTIM) 6,7,8
    8 KT=MT
      GO TO 97
    7 ANS=THET(MT)
      GO TO 50
    6 DO 47 K=2,MT
      CTIM=TIM-Y(K)
      IF(CTIM) 45,46,47
   46 ANS=THET(K)
      GO TO 50
   45 DELT=Y(K)-Y(K-1)
      TDEL=TIM-Y(K-1)
      REL=TDEL/DELT
      ANS=THET(K-1)+(THET(K)-THET(K-1))*REL
      GO TO 50
   47 CONTINUE
      GO TO 50
   51 STOP
   50 RETURN
      END
$IBFTC FIELD   LIST,REF,DECK,M94
      SUBROUTINE FIELD(FLAT,FLONG,ALT,BT,BP,BR,B2)
C     DUMMY SUBROUTINE FOR SETTING UP MCILWAINS MAGNET
      DIMENSION V(3,3)
      V(1,2) = ALT/6371.2
      V(2,2) = (90.-FLAT)/57.2957795
      V(3,2) = FLONG/57.2957795
      AER = V(1,2)
      SIT = ABS (SIN (V(2,2)))
      SSQ = SIT**2
 1002 IF(V(3,2))1000,1001,1001
 1000 V(3,2) = V(3,2) + 6.283185307
      GO TO 1002
 1001 CALL MAGNET(AER,SIT,V(3,2),BR,BT,BP,B2,V(2,2))
      CONTINUE
      RETURN
      END
$IBFTC MAGN    LIST,REF,DECK,M94
      SUBROUTINE MAGNET (R,S,PHI,BR,BTHET,BPHI,BB,THET)                 MAGNT000
    3 FORMAT(1H0,5X,F14.7,1X,F14.7,1X,F14.7,1X,F14.7)
      DIMENSION DP(49),P(49),G(49),H(49),CONST(49),AOR(7),SP(7),CP(7)   MAGNT001
      IF(KIP)151,150,150                                                MAGNT002
  150 KIP=-1                                                            MAGNT003
      DO 152 N=1,49                                                     MAGNT004
      G(N)=0.0                                                          MAGNT005
  152 H(N)=0.0                                                          MAGNT006
C     JENSEN AND CAIN COEFFICIENTS FOR 1960 (JUNE 1962)                 MAGNT007
C     G(I) = G(N,M) AND H(I) = H(N,M) WHERE I = N+7*(M-1)               MAGNT008
```

```
      G( 2)= 3.04112050E-01                                              MAGNT009
      G( 9)= 2.14736858E-02                                              MAGNT010
      G( 3)= 2.40353671E-02                                              MAGNT011
      G(10)=-5.12533379E-02                                              MAGNT012
      G(17)=-1.33811969E-02                                              MAGNT013
      G( 4)=-3.15178651E-02                                              MAGNT014
      G(11)= 6.21300906E-02                                              MAGNT015
      G(18)=-2.48981333E-02                                              MAGNT016
      G(25)=-6.49565905E-03                                              MAGNT017
      G( 5)=-4.17943639E-02                                              MAGNT018
      G(12)=-4.52983660E-02                                              MAGNT019
      G(19)=-2.17947447E-02                                              MAGNT020
      G(26)= 7.00825405E-03                                              MAGNT021
      G(33)=-2.04395562E-03                                              MAGNT022
      G( 6)= 1.62556271E-02                                              MAGNT023
      G(13)=-3.44067606E-02                                              MAGNT024
      G(20)=-1.94470026E-02                                              MAGNT025
      G(27)=-6.08211374E-04                                              MAGNT026
      G(34)= 2.77533549E-03                                              MAGNT027
      G(41)= 6.96802467E-04                                              MAGNT028
      G( 7)=-1.95231736E-02                                              MAGNT029
      G(14)=-4.85326147E-03                                              MAGNT030
      G(21)= 3.21172428E-03                                              MAGNT031
      G(28)= 2.14128828E-02                                              MAGNT032
      G(35)= 1.05051275E-03                                              MAGNT033
      G(42)= 2.26829448E-04                                              MAGNT034
      G(49)= 1.11471358E-03                                              MAGNT035
      H( 9)=-5.79890501E-02                                              MAGNT036
      H(10)= 3.31240714E-02                                              MAGNT037
      H(17)=-1.57893822E-03                                              MAGNT038
      H(11)= 1.48696943E-02                                              MAGNT039
      H(18)=-4.07490158E-03                                              MAGNT040
      H(25)= 2.10318235E-04                                              MAGNT041
      H(12)=-1.18245456E-02                                              MAGNT042
      H(19)= 1.00057732E-02                                              MAGNT043
      H(26)= 4.30380863E-04                                              MAGNT044
      H(33)= 1.38503490E-03                                              MAGNT045
      H(13)=-7.95897466E-04                                              MAGNT046
      H(20)=-2.00044021E-03                                              MAGNT047
      H(27)= 4.59718859E-03                                              MAGNT048
      H(34)= 2.42063078E-03                                              MAGNT049
      H(41)=-1.21806522E-03                                              MAGNT050
      H(14)=-5.75830293E-03                                              MAGNT051
      H(21)=-8.73461401E-03                                              MAGNT052
      H(28)=-3.40604073E-03                                              MAGNT053
      H(35)=-1.18162456E-04                                              MAGNT054
      H(42)=-1.11623013E-03                                              MAGNT055
      H(49)=-3.24831891E-04                                              MAGNT056
      P(1)=1.0                                                           MAGNT057
      DP(1)=0.0                                                          MAGNT058
      SP(1)=0.0                                                          MAGNT059
      CP(1)=1.0                                                          MAGNT060
      CONST(9)=0.0                                                       MAGNT061
      CONST(16)=0.0                                                      MAGNT062
      DO 80 N=3,7                                                        MAGNT063
      FN=N                                                               MAGNT064
      DO 80 M=1,N                                                        MAGNT065
      FM=M                                                               MAGNT066
      I=N+7*(M-1)                                                        MAGNT067
   80 CONST(I)=((FN-2.0)**2-(FM-1.0)**2)/((FN+FN-3.0)*(FN+FN-5.0))       MAGNT068
  151 C=SQRT (ABS (1.0-S*S))                                             MAGNT069
      IF(THET-1.570796327) 154,154,156                                   MAGNT070
  156 C=-C                                                               MAGNT071
  154 AR=1./(1.+R)                                                       MAGNT072
  155 SP(2)=SIN (PHI)                                                    MAGNT073
      CP(2)=COS (PHI)                                                    MAGNT074
```

```
      AOR(1)=AR*AR                                                      MAGNT075
      AOR(2)=AR*AOR(1)                                                  MAGNT076
      DO 90 M=3,7                                                       MAGNT077
      N=M-1                                                             MAGNT078
      SP(M)=SP(2)*CP(N)+CP(2)*SP(N)                                     MAGNT079
      CP(M)=CP(2)*CP(N)-SP(2)*SP(N)                                     MAGNT080
   90 AOR(M)=AR*AOR(N)                                                  MAGNT081
      BR=0.0                                                            MAGNT082
      BTHET=0.0                                                         MAGNT083
      BPHI=0.0                                                          MAGNT084
      DO 32 N=2,7                                                       MAGNT085
      FN=N                                                              MAGNT086
      SUMR=0.0                                                          MAGNT087
      SUMT=0.0                                                          MAGNT088
      SUMP=0.0                                                          MAGNT089
      DO 33 M=1,N                                                       MAGNT090
      IF(N-M)87,88,87                                                   MAGNT091
   88 I=8*N-7                                                           MAGNT092
      L=I-8                                                             MAGNT093
      P(I)=S*P(L)                                                       MAGNT094
      DP(I)=S*DP(L)+C*P(L)                                              MAGNT095
      GO TO 89                                                          MAGNT096
   87 I=N+7*(M-1)                                                       MAGNT097
      J=I-1                                                             MAGNT098
      K=I-2                                                             MAGNT099
      P(I)=C*P(J)-CONST(I)*P(K)                                         MAGNT100
      DP(I)=C*DP(J)-S*P(J)-CONST(I)*DP(K)                               MAGNT101
   89 FM=M-1                                                            MAGNT102
      TS=G(I)*CP(M)+H(I)*SP(M)                                          MAGNT103
      SUMR=SUMR+P(I)*TS                                                 MAGNT104
      SUMT=SUMT+DP(I)*TS                                                MAGNT105
   33 SUMP=SUMP+FM*P(I)*(-G(I)*SP(M)+H(I)*CP(M))                        MAGNT106
      BR=BR+AOR(N)*FN*SUMR                                              MAGNT107
      BTHET=BTHET-AOR(N)*SUMT                                           MAGNT108
   32 BPHI=BPHI-AOR(N)*SUMP                                             MAGNT109
      BPHI=BPHI/S                                                       MAGNT110
      BB=SQRT (BR**2+BTHET**2+BPHI**2)                                  MAGNT111
      WRITE (6,3) BR,BTHET,BPHI,BB
      RETURN                                                            MAGNT112
      END                                                               MAGNT113
```

8.0 RESULTS AND CONCLUSIONS

The orientation of a rocket can be obtained in horizon or space-fixed coordinates by obtaining the spatial location of the solar and magnetic field vectors with respect to both a rocket-axis and horizon coordinate system, with the use of only a lateral magnetometer and solar sensors.

The accuracy of the procedure is increased as the spin rate of the vehicle increases or becomes closer to a constant value. It is also difficult to obtain a solution if the solar and magnetic field vector are nearly coincident in space at a given time since two distinct vector are required in the procedure.

Another limitation of the method is the accuracy of the various sources of data which are used. (1) The location of the magnetic vector in horizon coordinates is obtained from a 48 term expansion in spherical harmonics. This expansion becomes slightly less accurate at very high altitudes. Care must be taken to insure that the most recent magnetic data has been used in the expansion. (2) The location of the solar vector in horizon coordinates obtained from reference (2) is quite accurately known and any resulting error is negligible; however, both (1) and (2) require accurate radar data. The accuracy of this data depends of course on the particular system and smoothing procedure used. (3) The accuracy of the solar sensors which determines the location of the solar vector with respect to the rocket-axis coordinate system is dependent on the particular type of sensor used; however, the majority of the sensors use the sun as a point source which results in a minimum error of approximately one-half of a degree which corresponds to the width of the solar disc. This error can be reduced somewhat by smoothing the sensor data timewise. (4) The accuracy of the magnetometer which determines the orientation of the magnetic field vector in the rocket-axis system is a function of the precision of the instrument used. The accuracy of the method is not affected by any magnetic source in the rocket payload since this effects only the absolute amplitude of the lateral magnetometer output. The method requires only a knowledge of the timewise location of the maxima and minima of the output.

It should also be noted that the method assumes that the sun is within view of at least one solar sensor. It is usually difficult to have a sensor mounted at the tail of the rocket; therefore, no data can be obtained if the tail is facing the sun or the flight occurs during a time when the sun is not visible.

9.0 REFERENCES

(1) National Almanac Office, U.S. Naval Observatory: The American Ephemeris and National Almanac for the Year 1963. United States Government Printing Office, 1961.

(2) Cain, J. C., Hendrick, S., Daniels, W. E., Jensen, D. C.: Computation of the Main Geomagnetic Field from Spherical Harmonic Expansions. NASA X-611-64-316, 1964.

(3) Phillips, H. B.: Vector Analysis. John Wiley & Sons, Inc., c1933.

(4) Thomson, W. T.: Introduction to Space Dynamics. John Wiley & Sons, Inc. 1961.

APPENDIX A
Illustrations

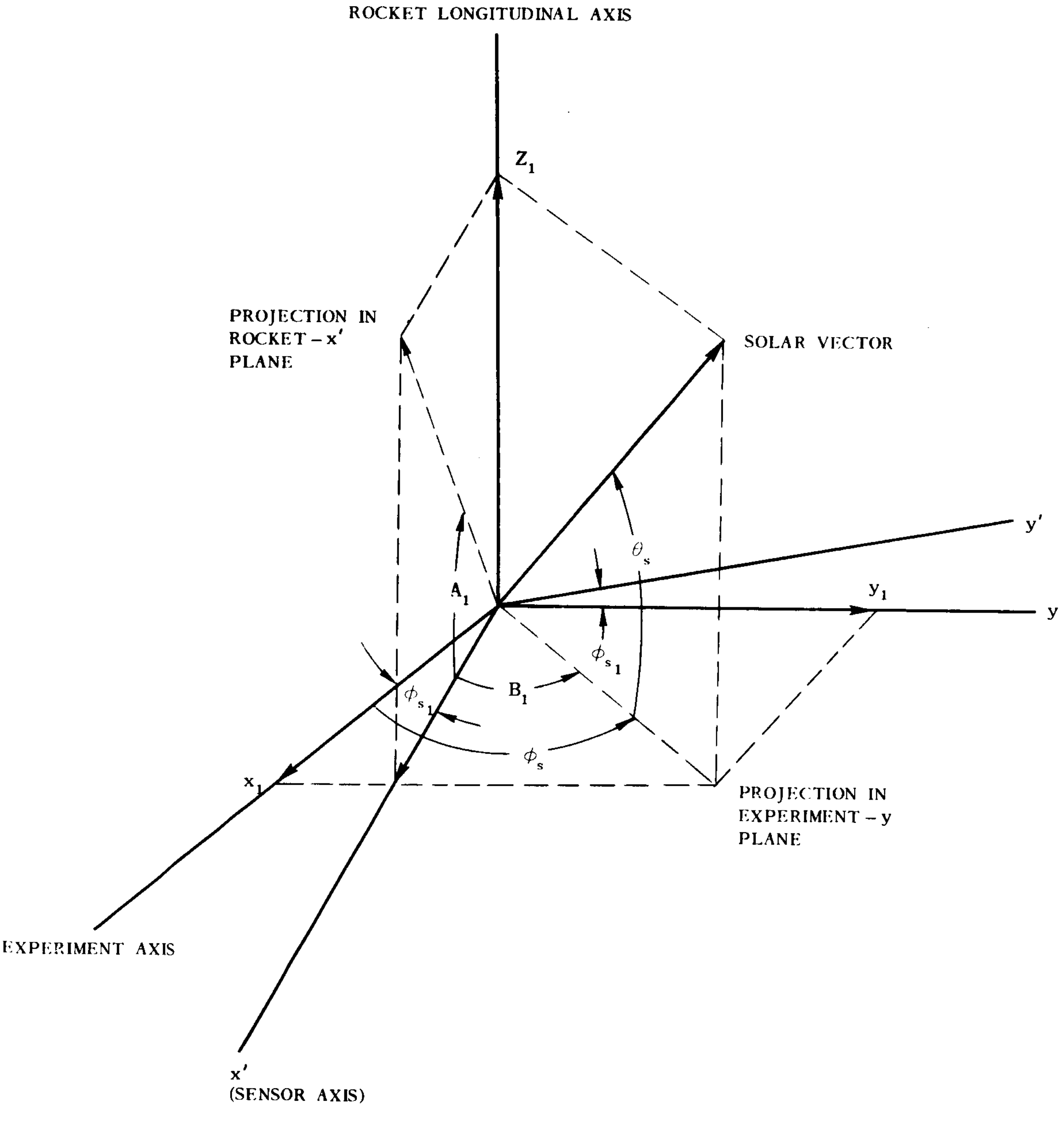


Figure 1–Location of Solar Vector in Rocket-Axis System using Case I Solar Sensor (Lateral Sensor).

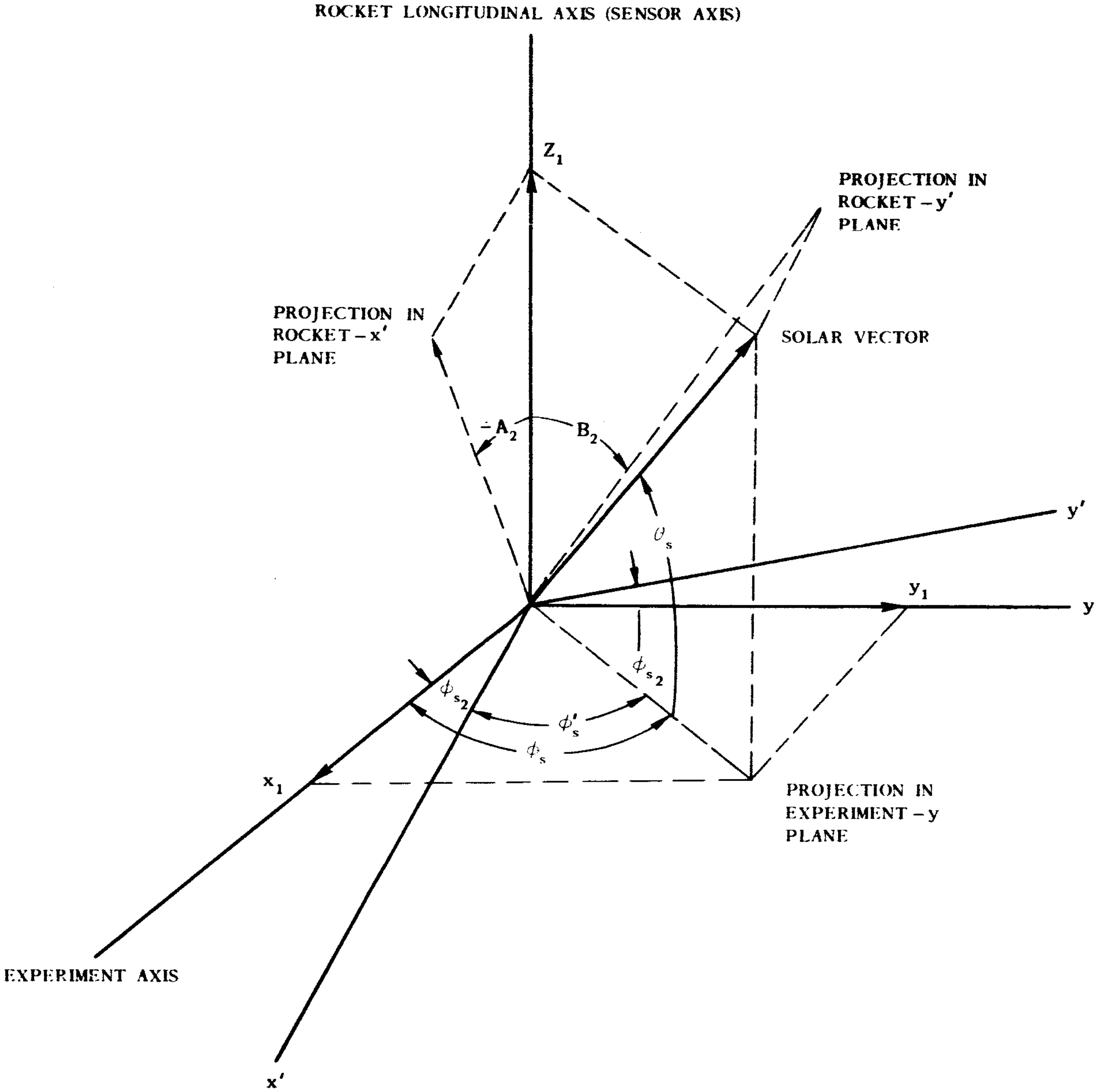


Figure 2–Location of Solar Vector in Rocket-Axis System using Case II Solar Sensor (Longitudinal Sensor).

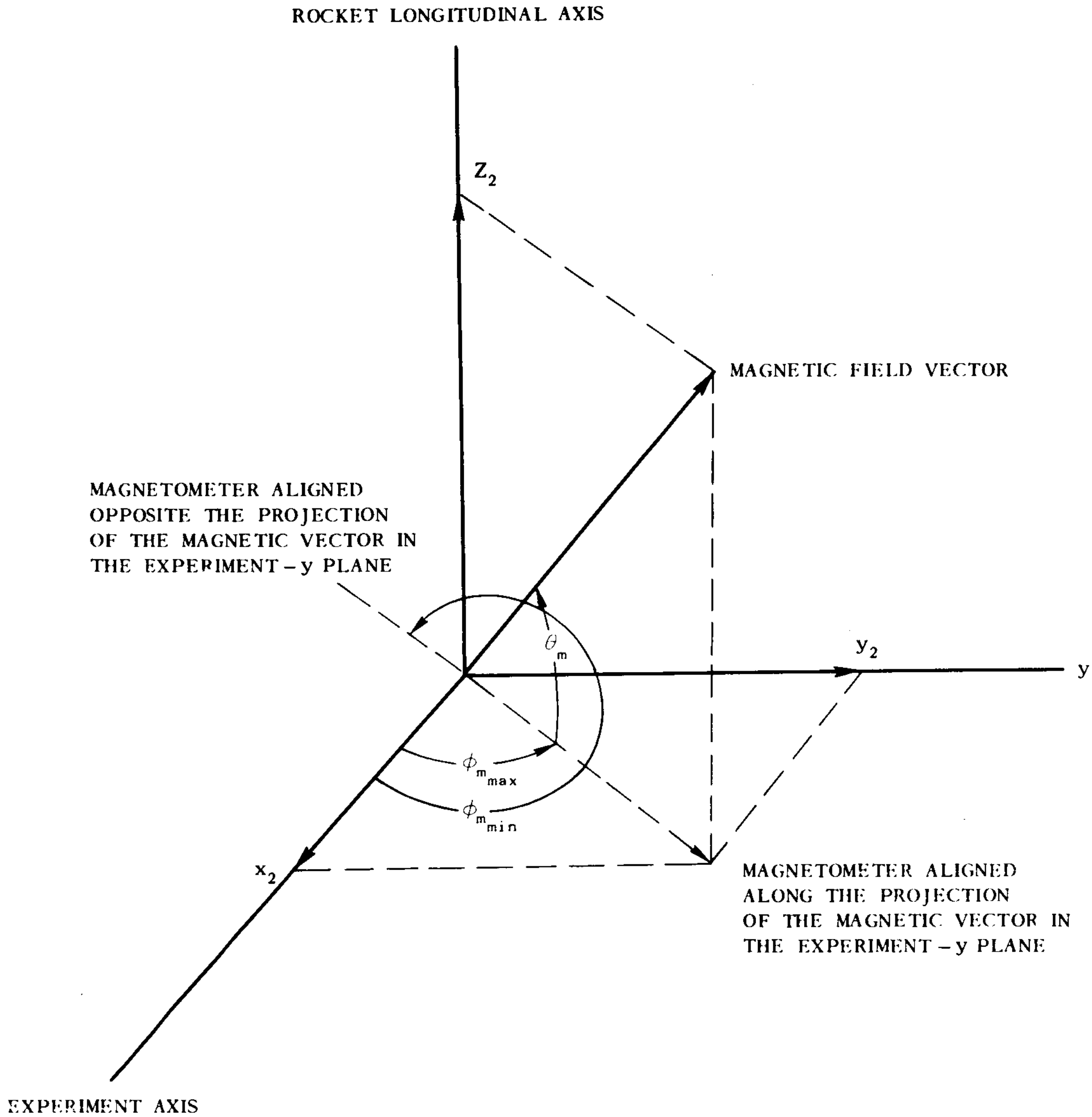


Figure 3–Location of Magnetic Field Vector in the Rocket-Axis System.

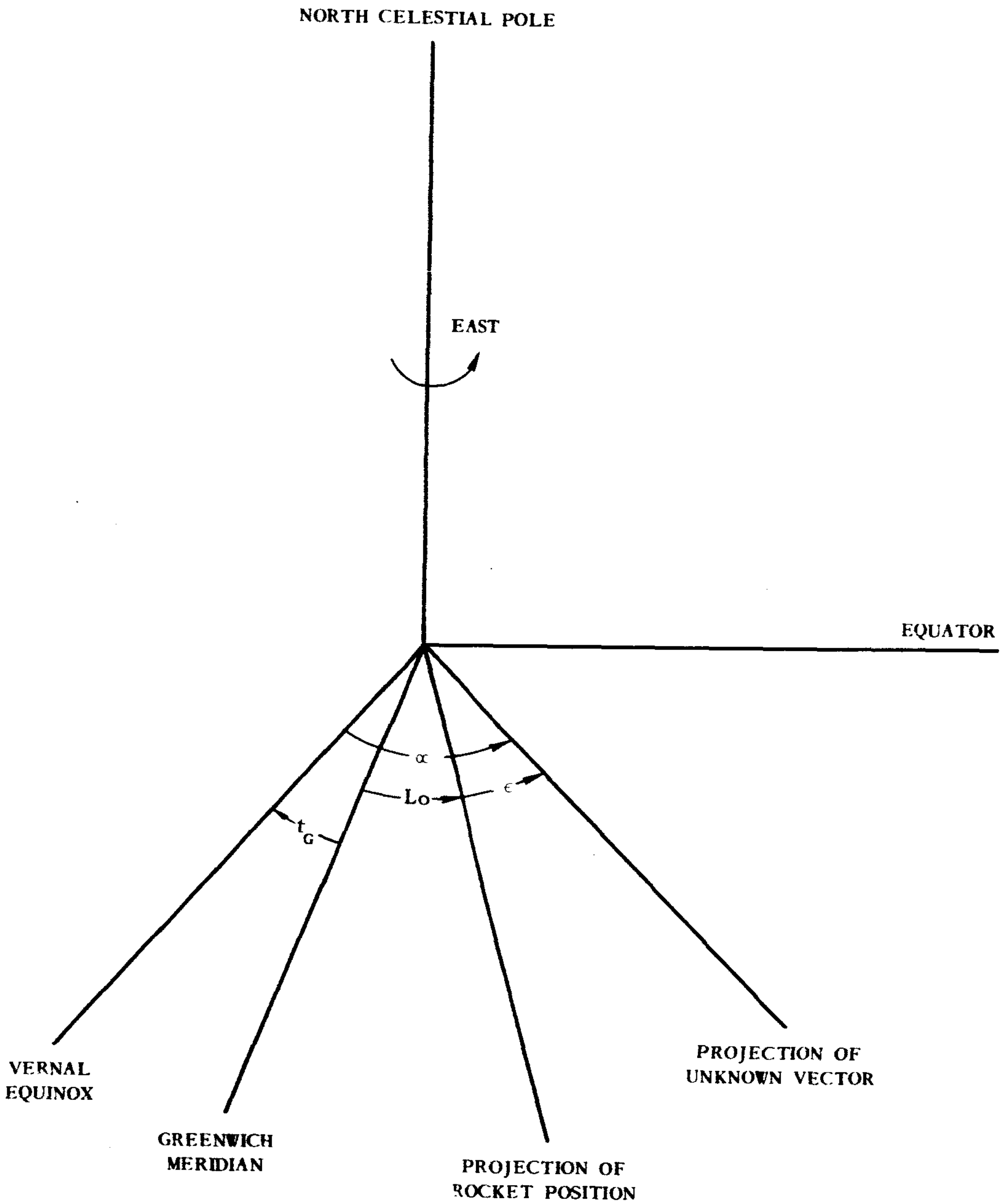


Figure 4–Location of a Vector in the Equatorial Plane.

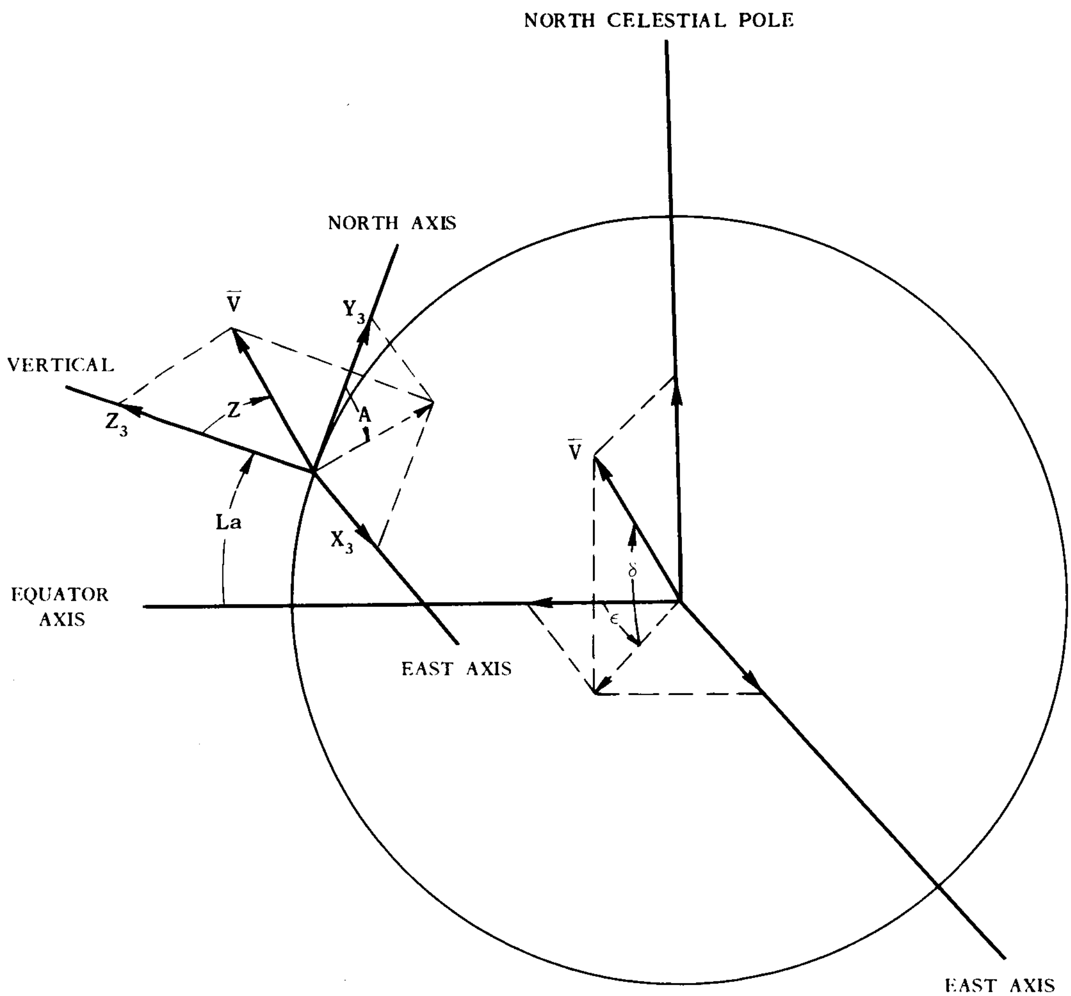


Figure 5—Transformation of a Vector between the Horizon and Equatorial Coordinate Systems.

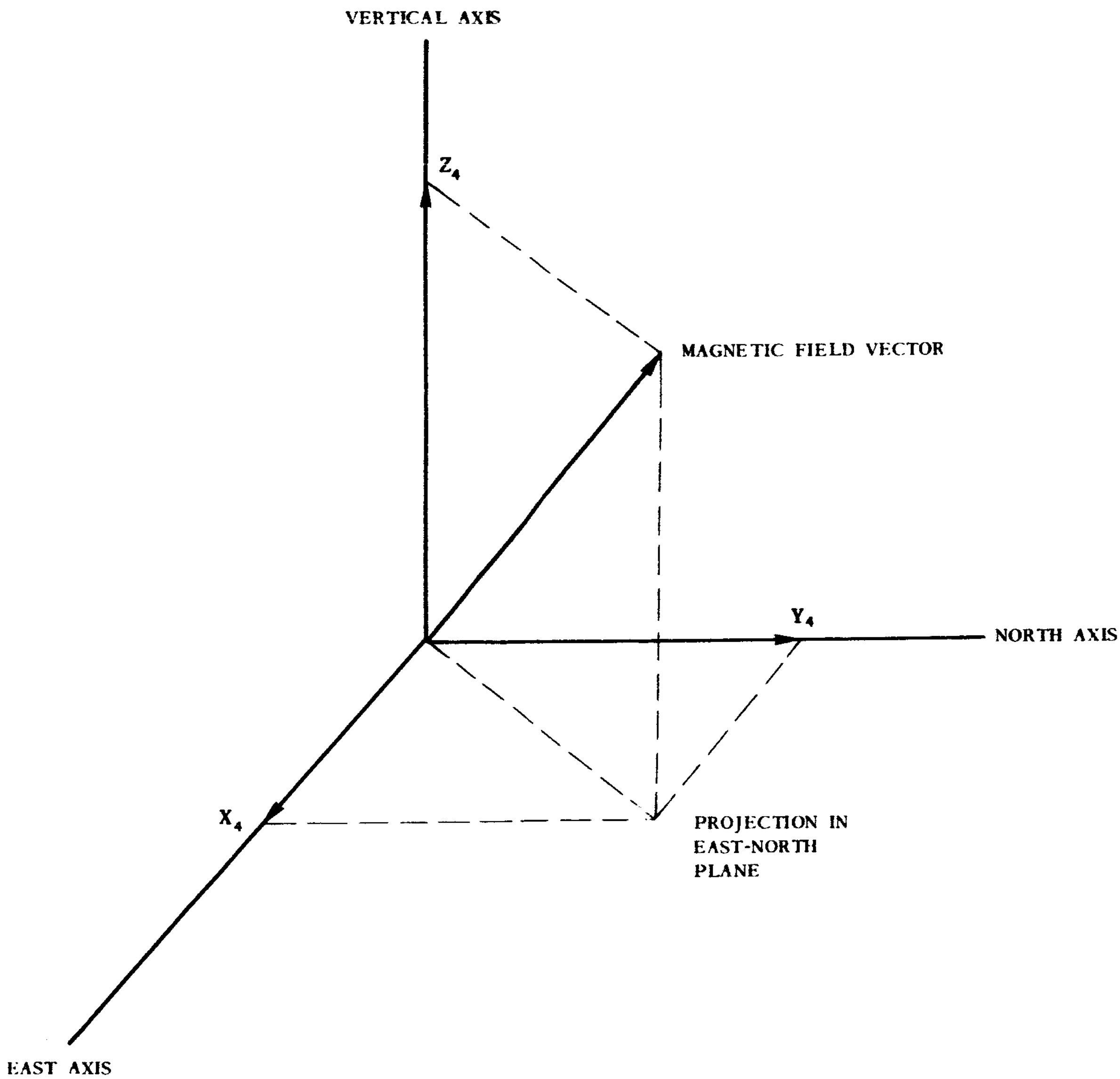


Figure 6–Location of the Magnetic Field Vector in the Horizon Coordinate System.

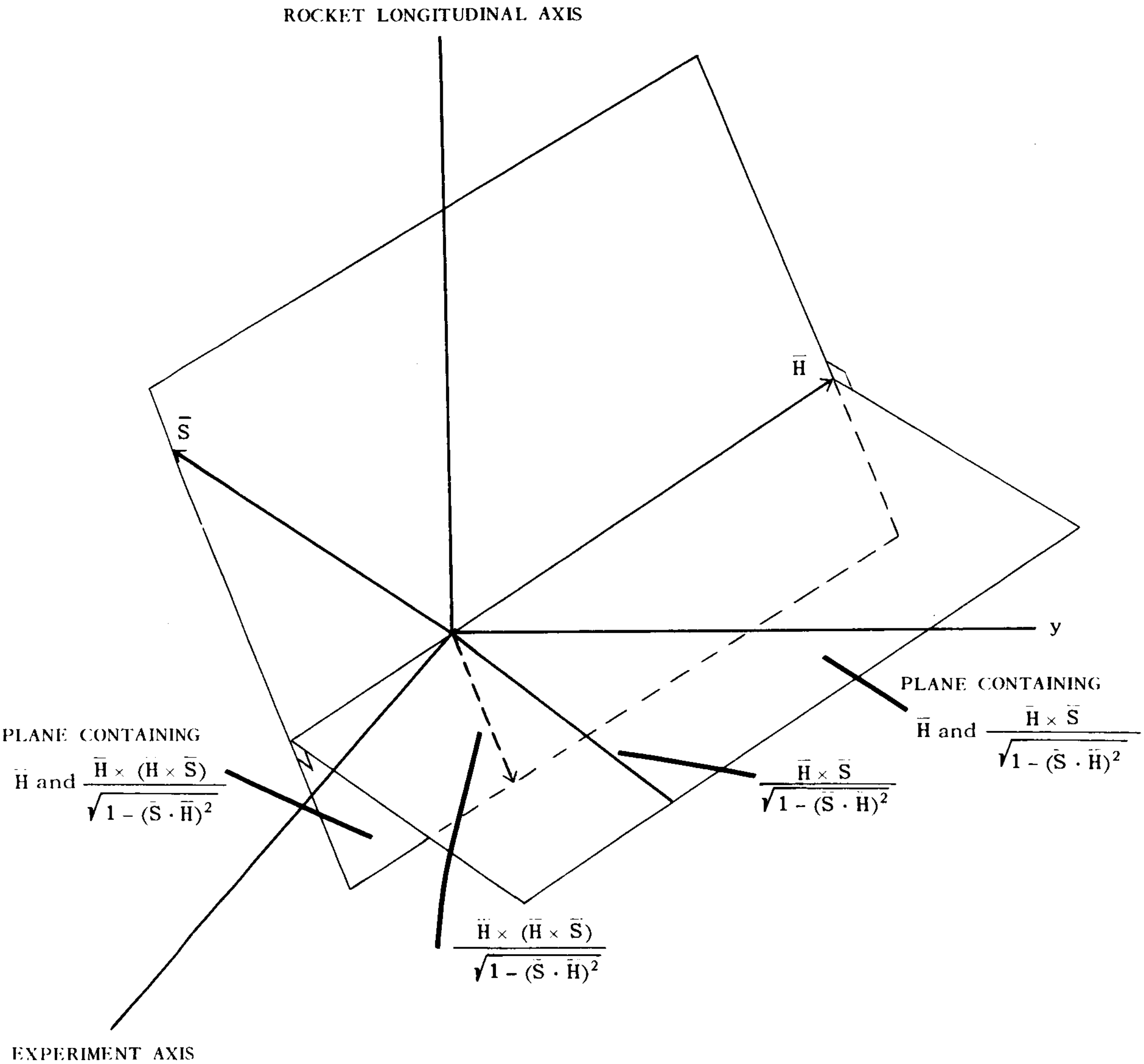


Figure 7–Representation of a Coordinate System consisting of the Unit Vectors:

$$[\bar{H}], \left[\frac{\bar{H} \times \bar{S}}{\sqrt{1 - (\bar{S} \cdot \bar{H})^2}}\right], \left[\frac{\bar{H} \times (\bar{H} \times \bar{S})}{\sqrt{1 - (\bar{S} \cdot \bar{H})^2}}\right].$$

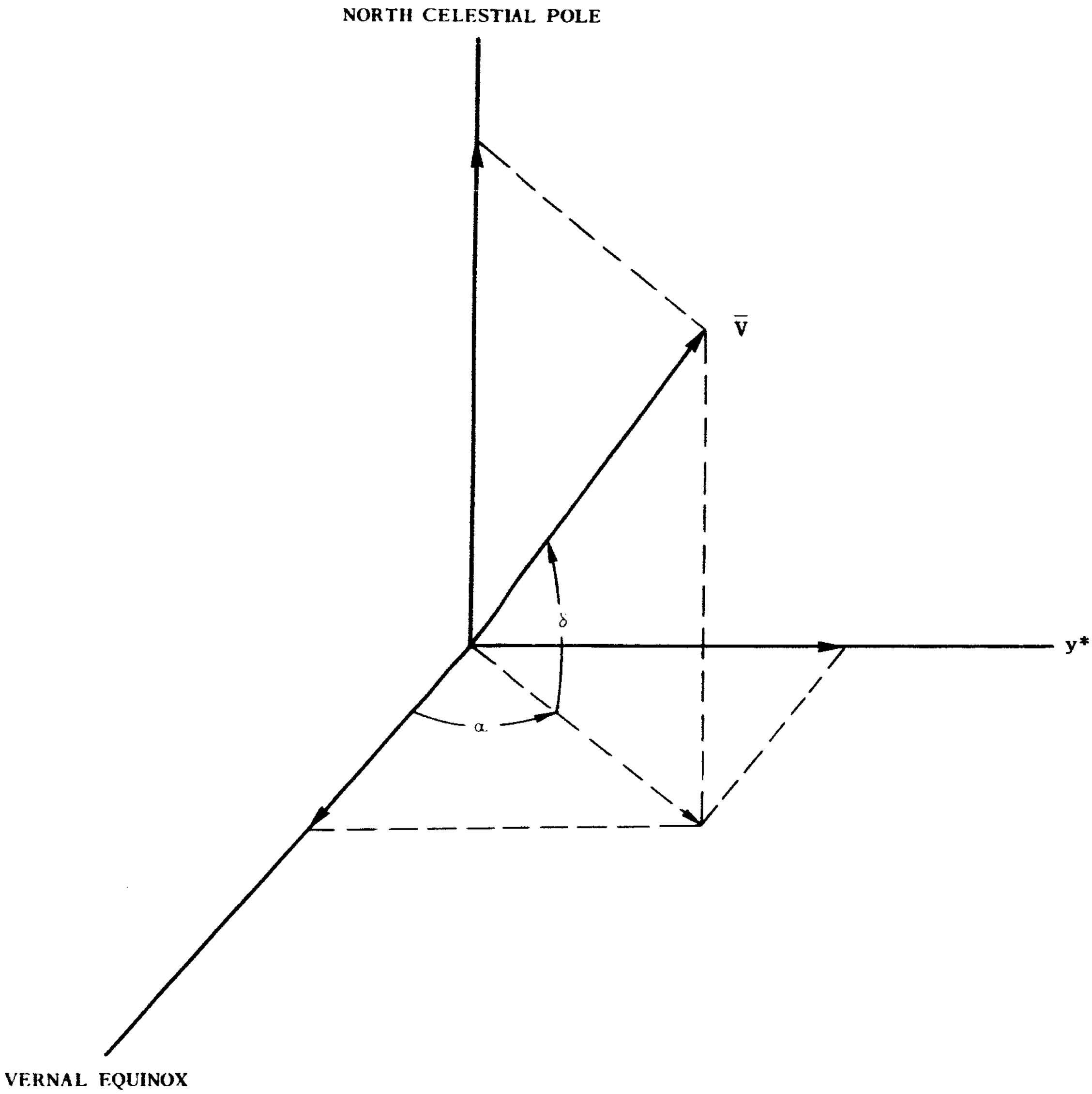


Figure 8—Location of a Vector in Space-Fixed Coordinate System.

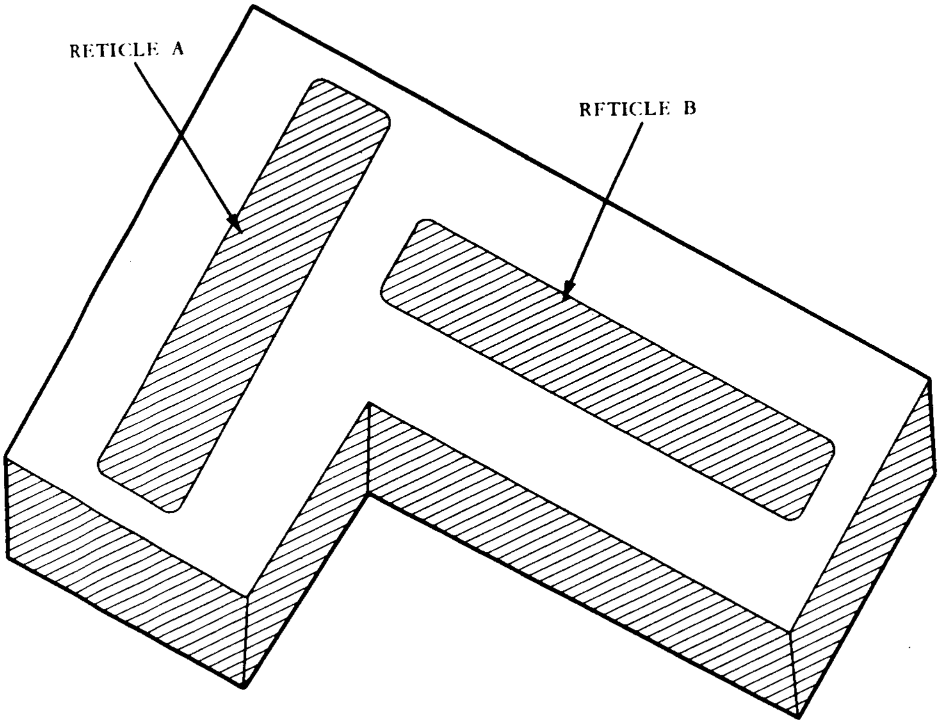


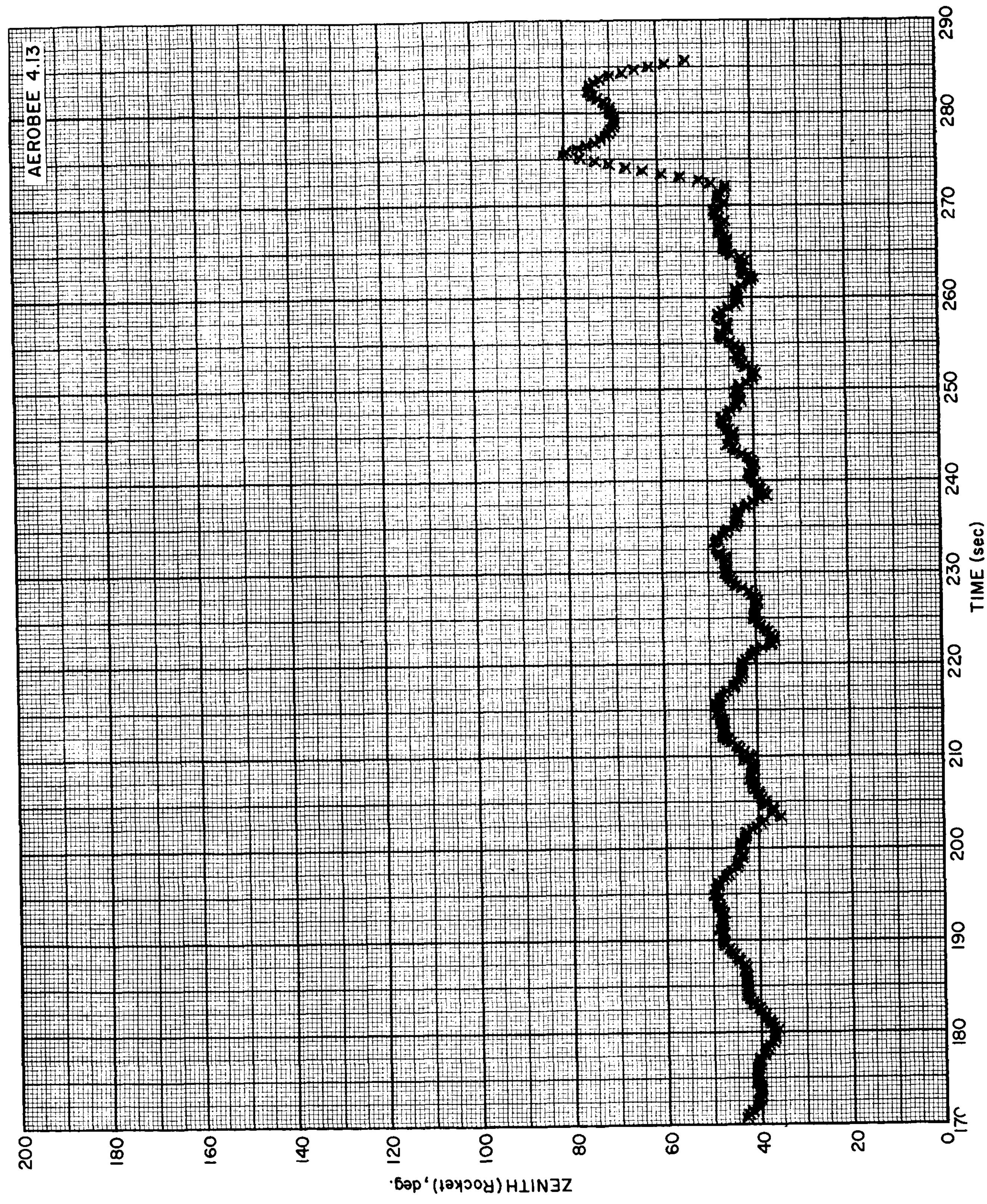
Figure 9–ADCOLE Type Solar Sensor

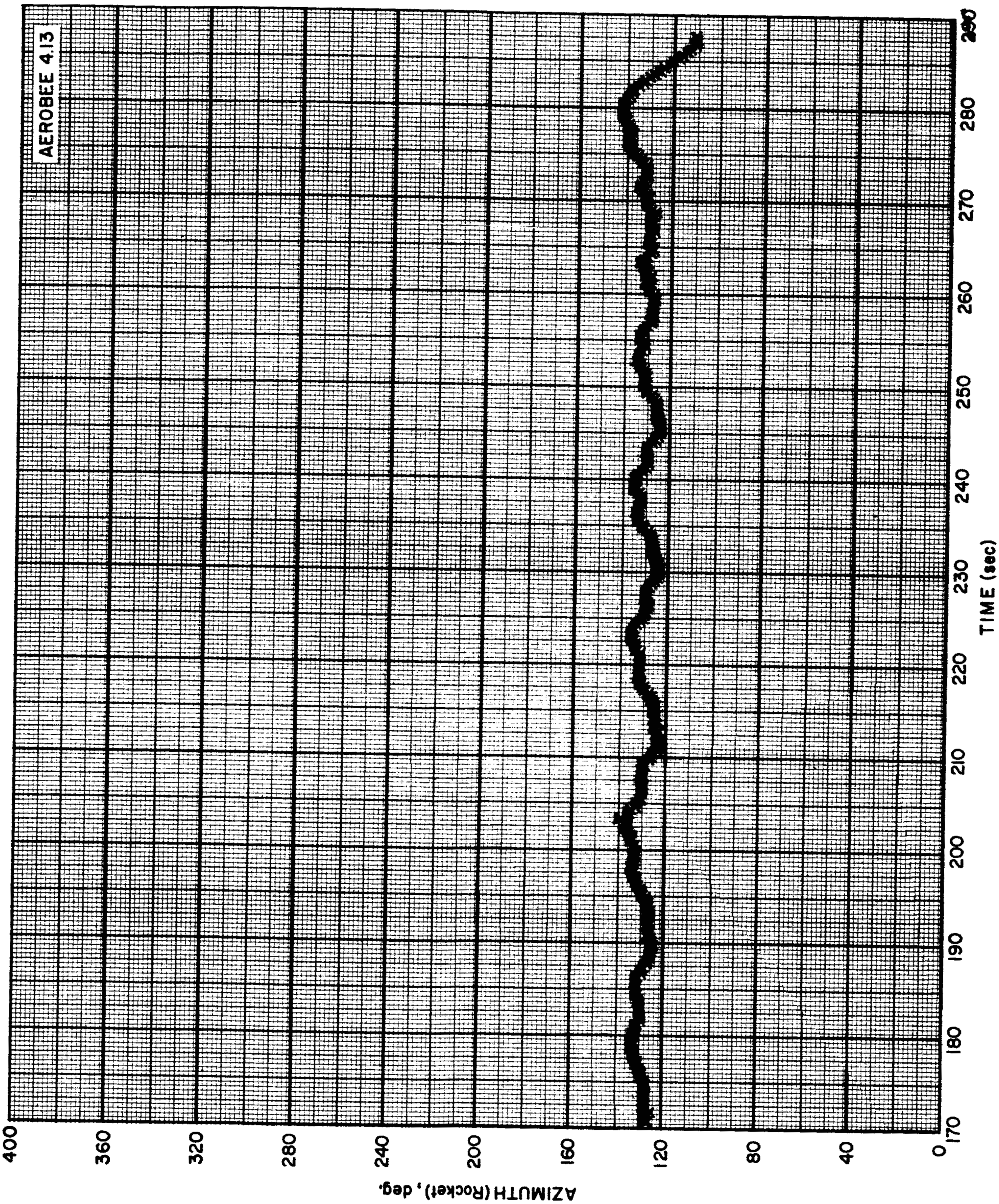
APPENDIX B

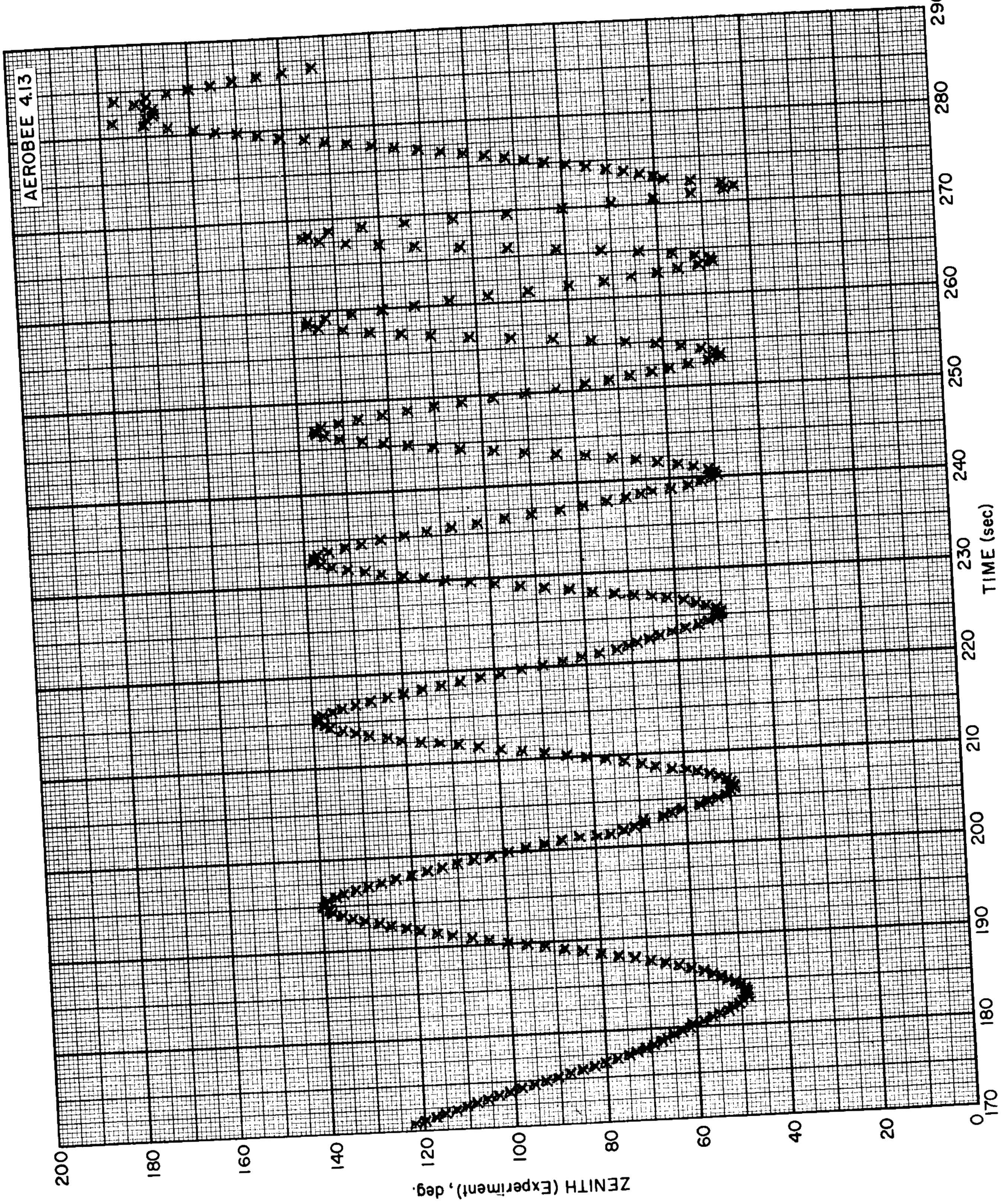
Sample Results

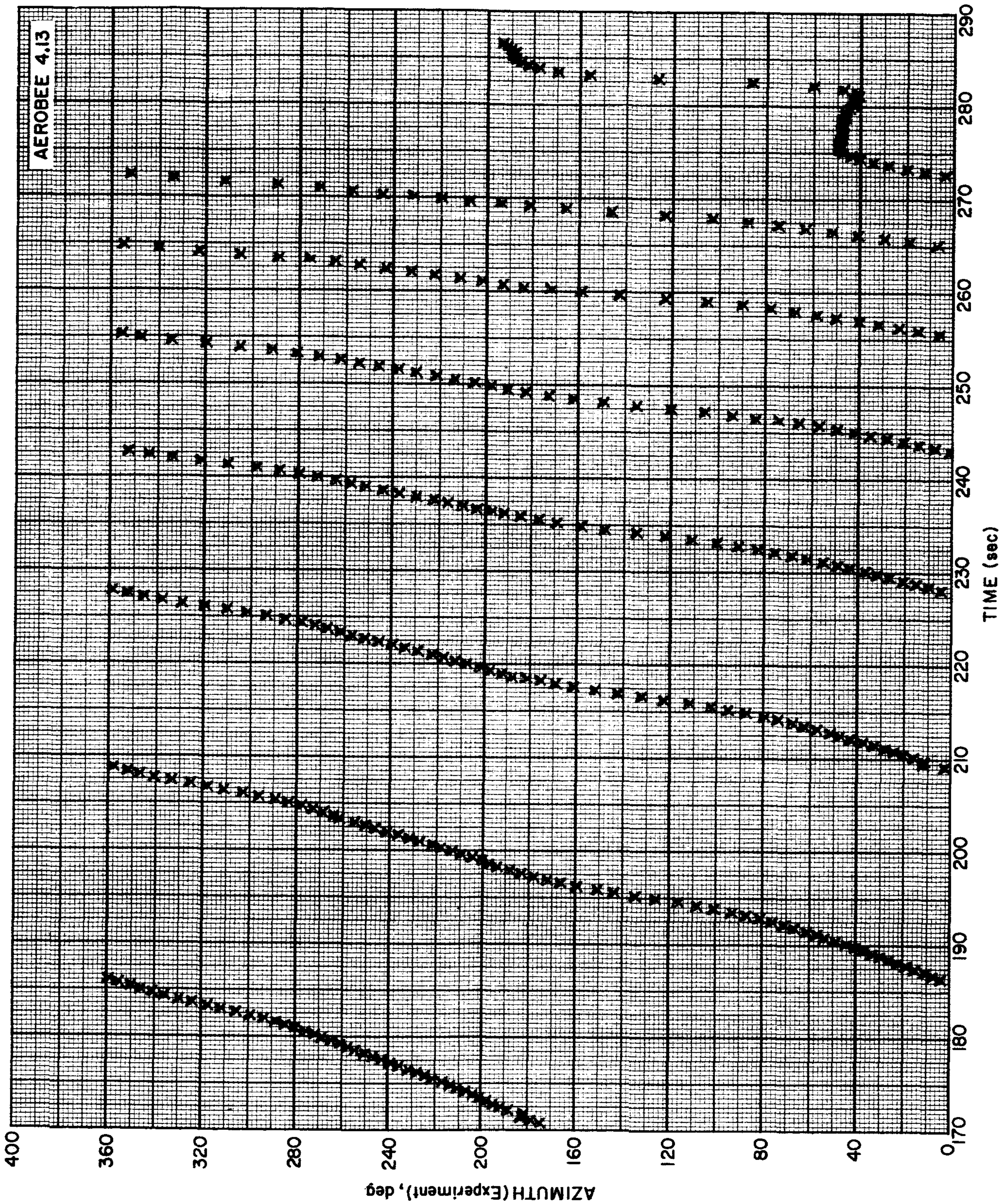
The procedure was applied to an Aerobee rocket and representative time histories of Azimuth and Zenith angles and Right Ascension and Declination for both the Experiment and Rocket Longitudinal axes are shown.

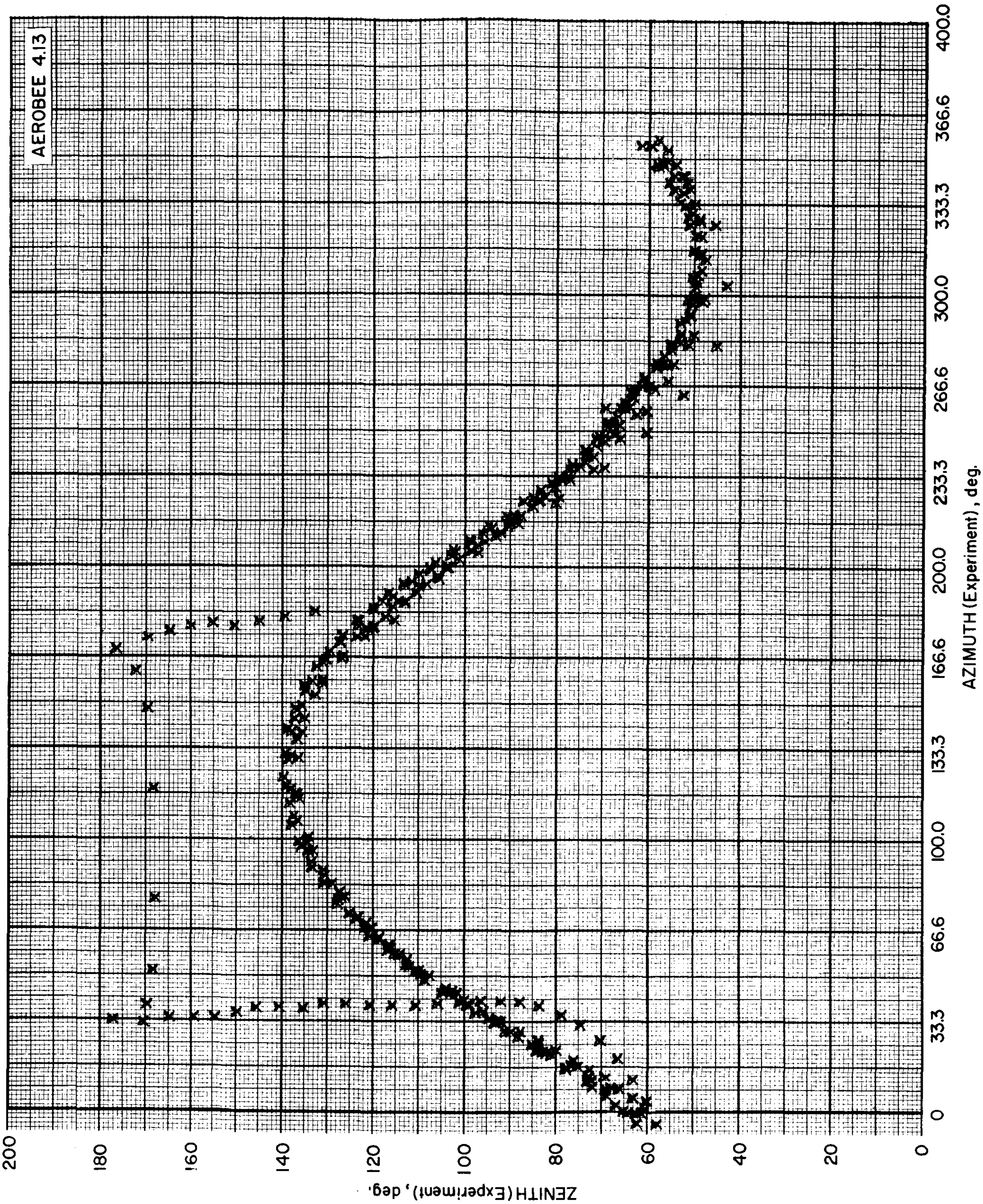
Aerobee 4.13 GP-GT was launched September 26, 1963. The rocket was de-spun from approximately 2.5 rps after burnout at which time an attitude control system was to have controlled the vehicle orientation. This system, however, did not function properly and the vehicle experienced a reverse roll rate of about 0.05 rps. A coning motion resulted with the coning and roll periods being approximately equal. The results presented describe the orientation during the reverse roll (170-300 seconds).


AEROBEE 4.13
TIME (sec)
ZENITH (Rocket), deg.
170
180
190
200
210
220
230
240
250
260
270
280
290
0
20
40
60
80
100
120
140
160
180
200

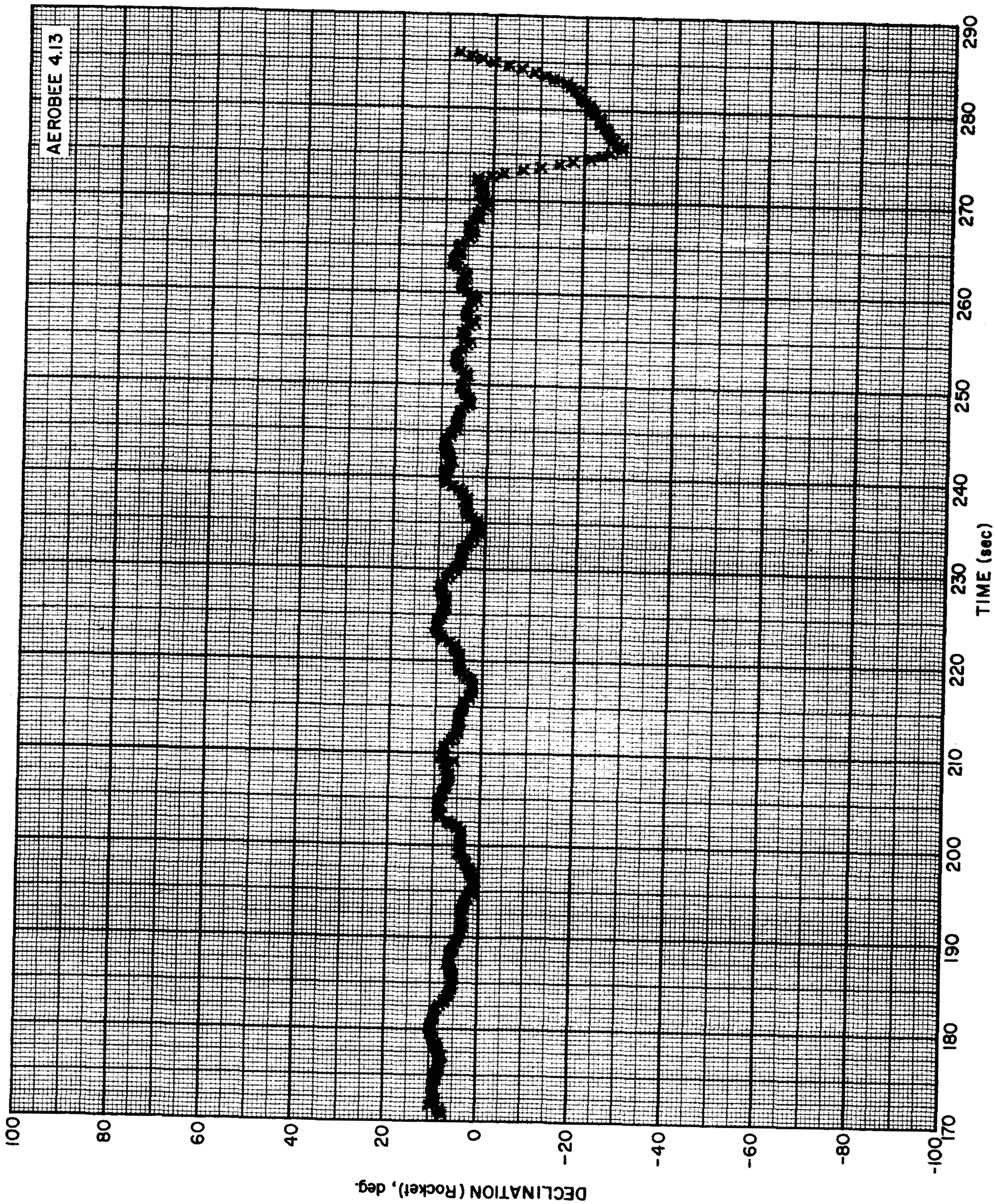

AEROBEE 4.13
TIME (sec)
170
180
190
200
210
220
230
240
250
260
270
280
290
AZIMUTH (Rocket), deg.
0
40
80
120
160
200
240
280
320
360
400


AEROBEE 4.13
TIME (sec)
170
180
190
200
210
220
230
240
250
260
270
280
290
ZENITH (Experiment), deg.
0
20
40
60
80
100
120
140
160
180
200

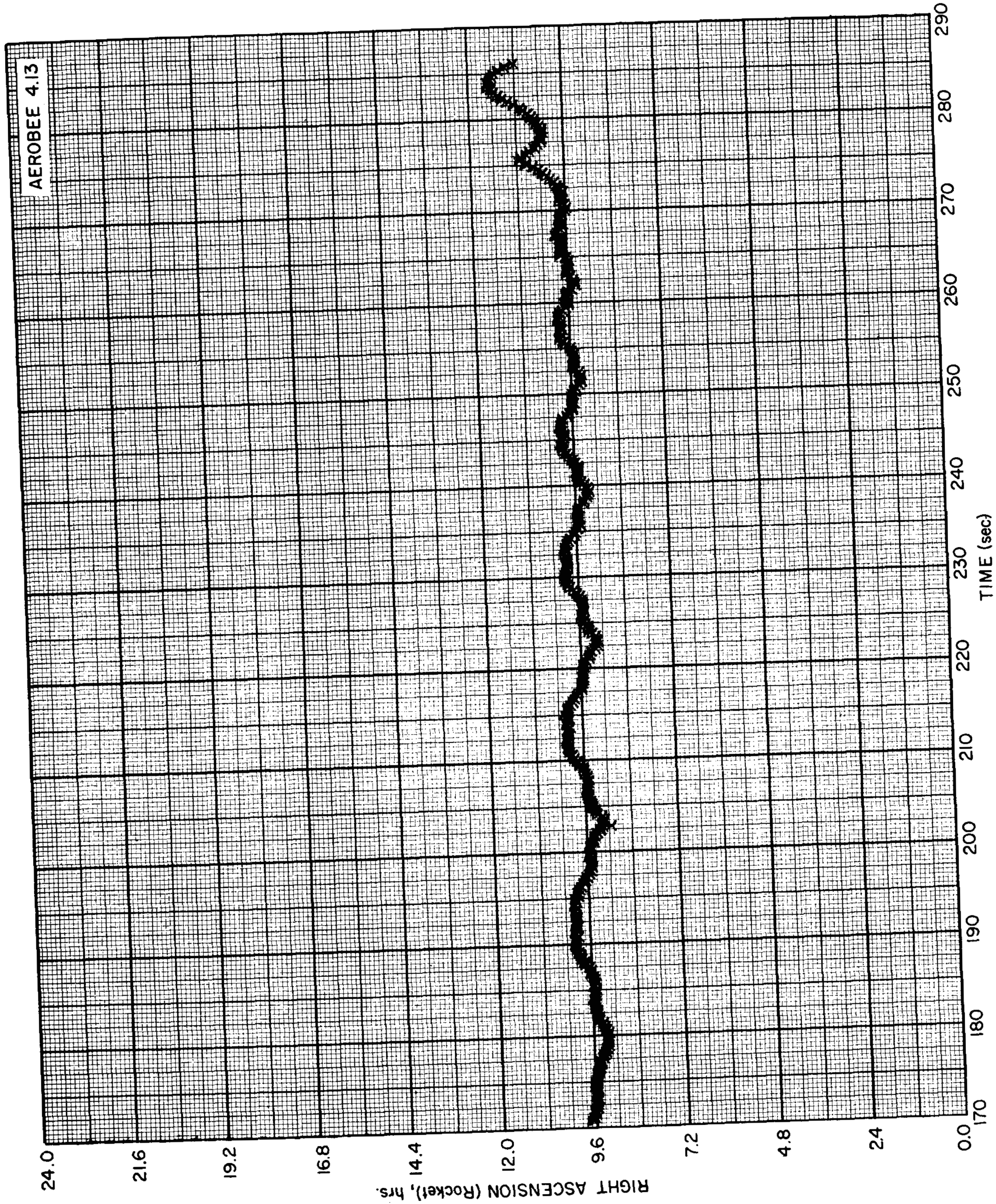

AEROBEE 4.13
TIME (sec)
170
180
190
200
210
220
230
240
250
260
270
280
290
AZIMUTH (Experiment), deg
0
40
80
120
160
200
240
280
320
360
400

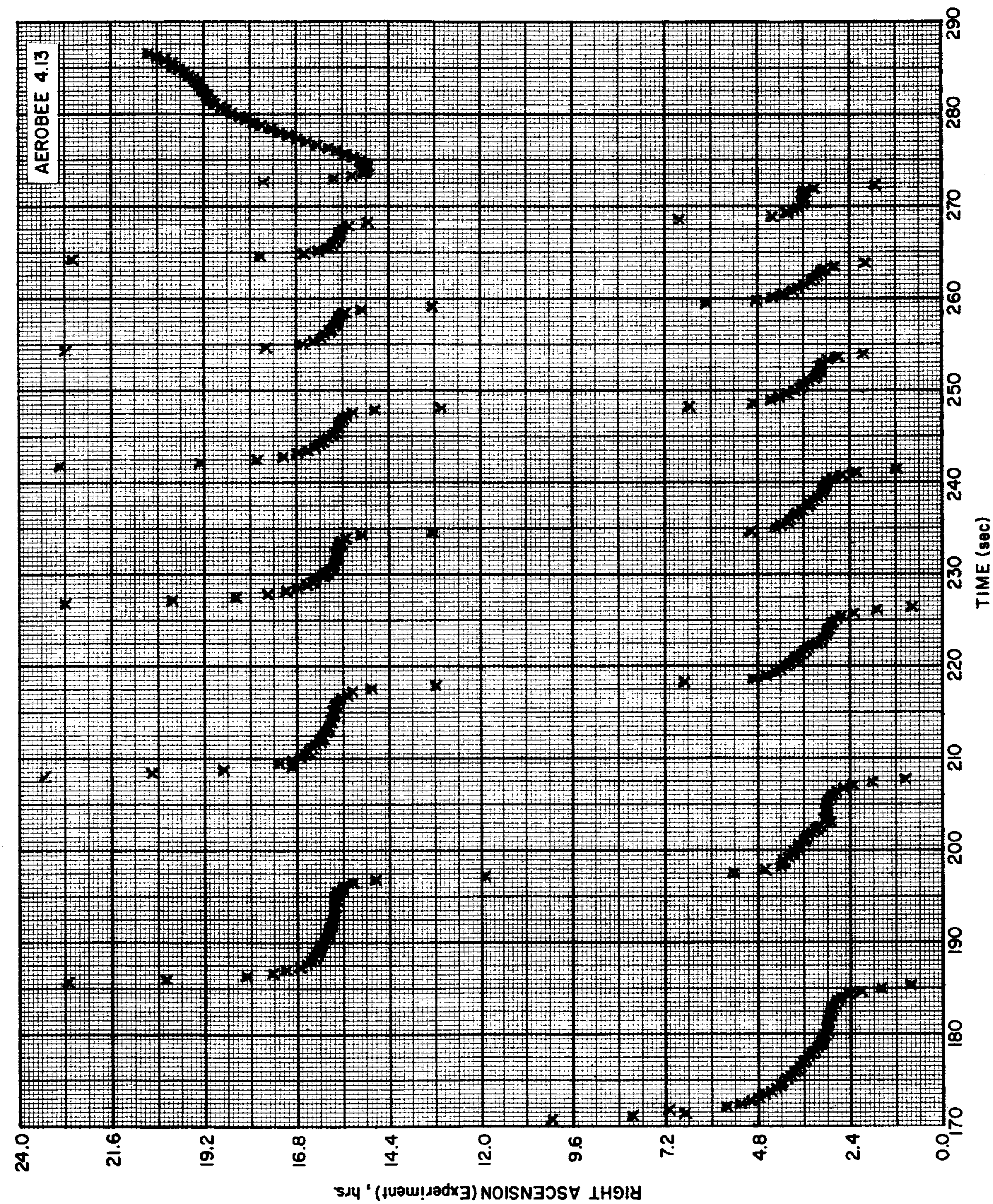


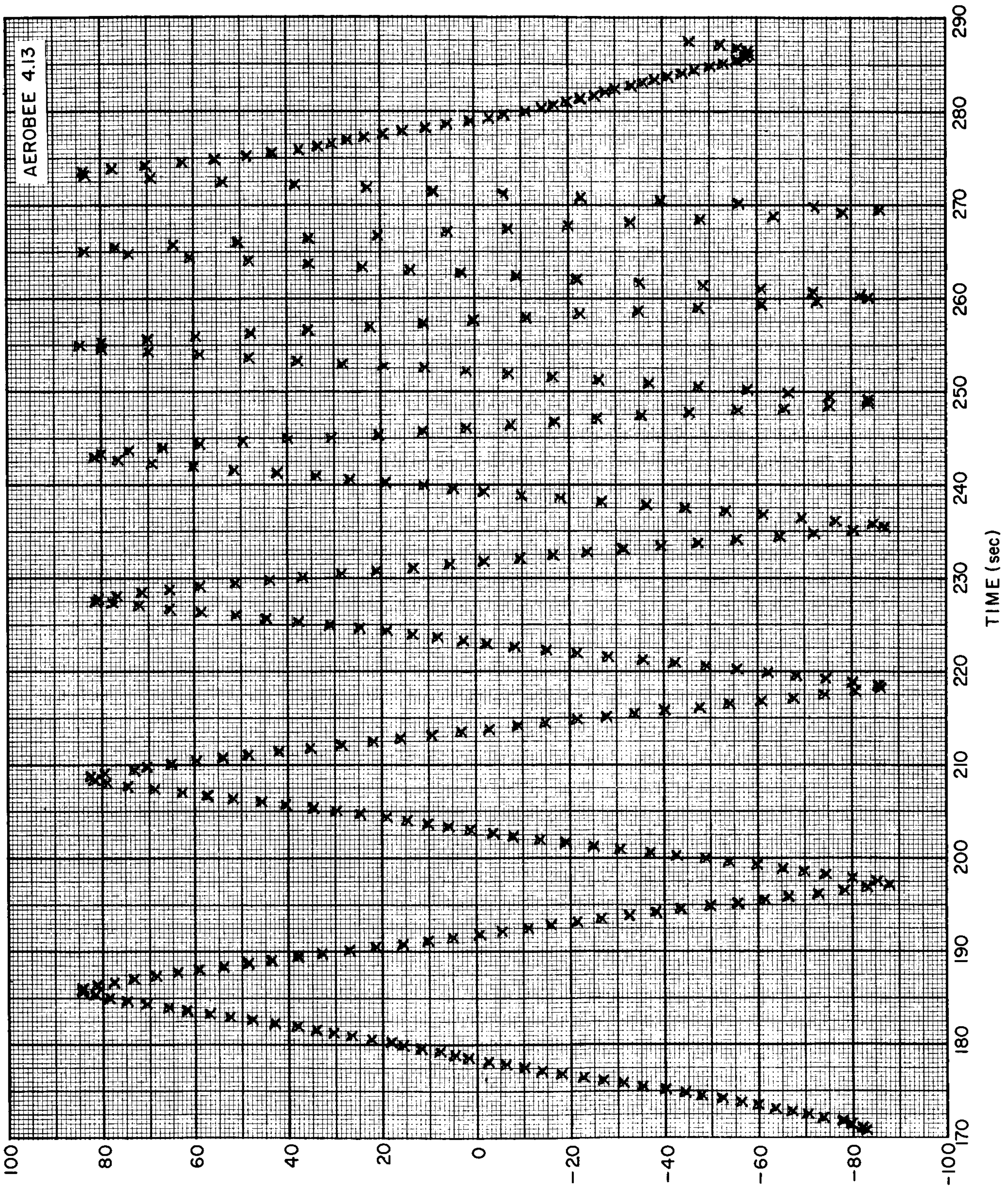
AEROBEE 4.13
AZIMUTH (Experiment), deg.
0
33.3
66.6
100.0
133.3
166.6
200.0
233.3
266.6
300.0
333.3
366.6
400.0
ZENITH (Experiment), deg.
200
180
160
140
120
100
80
60
40
20
0

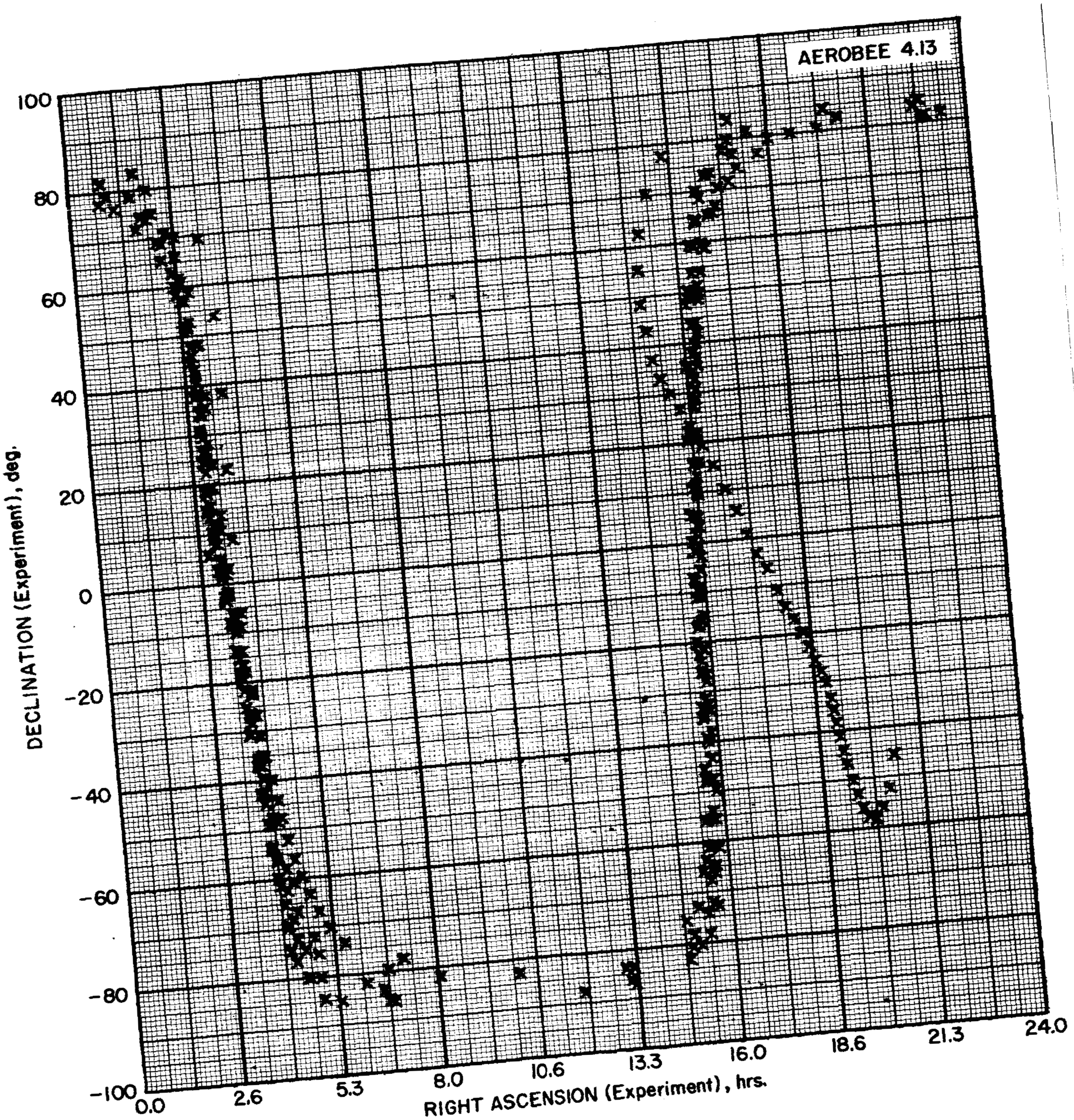


AEROBEE 4.13
TIME (sec)
DECLINATION (Rocket), deg.
170
180
190
200
210
220
230
240
250
260
270
280
290
100
80
60
40
20
0
-20
-40
-60
-80
-100


AEROBEE 4.13
TIME (sec)
170
180
190
200
210
220
230
240
250
260
270
280
290
RIGHT ASCENSION (Rocket), hrs.
0.0
2.4
4.8
7.2
9.6
12.0
14.4
16.8
19.2
21.6
24.0


AEROBEE 4.13
TIME (sec)
170
180
190
200
210
220
230
240
250
260
270
280
290
RIGHT ASCENSION (Experiment), hrs.
24.0
21.6
19.2
16.8
14.4
12.0
9.6
7.2
4.8
2.4
0.0


AEROBEE 4.13
TIME (sec)
DECLINATION (Experiment), deg.
170
180
190
200
210
220
230
240
250
260
270
280
290
100
80
60
40
20
0
-20
-40
-60
-80
-100


AEROBEE 4.13
DECLINATION (Experiment), deg.
100
80
60
40
20
0
-20
-40
-60
-80
-100
0.0
2.6
5.3
8.0
10.6
13.3
16.0
18.6
21.3
24.0
RIGHT ASCENSION (Experiment), hrs.