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The directional distribution of the reflected thermal radiation from surfaces of varying roughness is explored experimentally. Measurements were made of the plane-polarized components of the reflected radiation as well as of the mixed radiation (containing all components of polarisation). The test materials ineluded magnesium oxide ceramic and aluminum coated ground glass, thereby permitting a study of reflection in the presence and in the absence of subsurface scattering. The angle of incidence was varied from 10 to $87^{\circ}$, while the angle of reflection extended from 0 to $89^{\circ}$. The roughness of the test surfaces ranged from optically smooth to $5.8 \mu$, while the measurements were performed monochromatically at a wavelength of $0.5 \mu$. The measured directional distributions affirm that the diffuse limit (Lambert's cosine law) does not hold when roughened surfaces are illuminated at moderate to large angles of incidence. Rather, it is found that a maximum in the distribution of the reflected intensity occurs at reflection angles larger than the apocular-ray direction. The contributions of the s- and p-components of polarization to this off-specular peak are delineated. The degree of polarisation imparted by reflection at surfaces of varying roughness Is also investigated. It is show that the absence or presence of sub-surface scattering is an important factor. The various finding a of the experiments are subjected to interpretation by a model of the reflection process which pictures the surface as being composed of elementary mirror-like facets.

## INTRODUCTION

This paper is concerned with the offects of surface roughness and angle of incidence on the directional diatribution and polarization of reflected thermal radiation. There are two limiting cases which are widely regarded as bounds for the directional reflection characteristice of real surfaces, these are diffuse reflection and epecular reflection. It has been verified both experimentally and analytically, ${ }^{1-6}$ that for any fixed wavelength, specular reflection is approached as the surface roughness decreases. On the other hand, recent experiments ${ }^{7}$ have show that at a fixed wavelength, the diffuse limit is approached with inoreasing surface roughness only when the angle of incidence is near-normal. At moderate and large angles of incidence, the diffuse limit is not approached as the surface roughness increases. Instead, one finds a maximum in the distribution of the reflected intensity at a reflection angle (relative to the normal) larger than the apecular angle.

One of the aims of this research is to provide new information on the aforementioned off-specular maxim which contributes to their understanding. Specific consideration is given to the contributions of the separate couponents of polarization to the directional distributions and to the off-specular maxima. The experiments were performed using a metal and a nonmetal which had widely different
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polarisation obarecteriatics. For oach meterial, test surfaces of varioua roughnesses were prepered and investigated.

The extent of the polarization inparted by the reflection of unpolarized incident radistion from a given metorial dopends on the ourface condition, the angle of incidence, and the wavelength. For optionily amooth ourfaces, the degree of polarization at any angle of reflection is fully specified by the fresmol equations. The effect of surface roughness and angle of incidence on the degree of polarization is explored experimentaliy as part of this investigation.

In perforsing the exporiments, the test aurfaces were illuminated by a narrow beam of radiation incilined at e pre-selected angle relative to the surface normal. The reflected radiation was collected in a pre-selected angular direction in the plane of incidence, whereupon it was passed through a polarizer and then into spectroneter for wavelangth resolution. The angle of incidence ranged from 10 to $87^{\circ}$, while the angle of reflection was varied from 0 to $89^{\circ}$. All reflection measurements were performed at a wavelength $\lambda=0.5 \mu$. The test surfaces ware either of magnesium odide ceramic, a weskiy-absorbing, internaliyscattering dielectric, or of evaporated aluminma on a glass substrate. A total of aine test aurfaces were employed, ranging in roughness from a hlgh polish to a root-mean-square value of $5.8 \mu$. Further datails of the experimental method will be described efter appropristw heckground literiature is disoussed.

The exdatence of off-apecular maxdm was observed as early as 1903 by Thaler. ${ }^{8}$ Such peaks have been inherent in the reflection data of many investigators since that time, but in the majority of ceses the phenomenon went unde~ tected or was not discussed. A survey of pertinent contributions to the subject is presented eisewhere. A physical model.purporting to explain the off-speculer maxima was first proposed by Pokrowaki. ${ }^{9}$ The Pokrowski model, in common with ${ }^{8}$ F. Thaler, Annalen, der Phystk, vol. 11, 1903, p. 996.
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most subsequent ones, postulated specular reflection obeying Fresnel's equations from small mirror-like facets on the surface, plue a diffuse scattering that originates either on the surface or internal to the material. Pokrowski attributed his measured off-specular peaks to reflection from the facets. Later, Schulz ${ }^{10}$ modified the Pokrowaki model by giving a atnitieticel diatribution of alopea to the mirror-like facets on the surface.

More recently, Middleton and Mungali ${ }^{l l}$ observed off-specular peaks in the anguler distribution of light reflected from snow and ice surfaces. To explain the experimental findings, an analytical model of the reflection proceas was proposed. This model was similar to that of Pokrowski-Schula, but included amitiplicative factor to account for the incomplete illumination of the mirror-like facets due to shadowing by adjacent facets. Thus formulatod, the model predicts certain trends characteristic of the experimental data of Middleton and Mungall.

Within the knowledge of the present authors, there have been no prior measurements of angular distributions of the plane-polarized reflection compononts, nor have there been attempts to relate such information to the off-specular peaks.

Experimental information on the state of polarization of radiation reflected from rough surfaces would be of general value in fostering an understanding of the reflection process at such surfaces. Hovever, to be useful in this connection, the measurements would have to axtand ovar a wide range of inetdence and reflection angles. Much of the literature on polarization by reflection does not meet this requirement. ${ }^{\text {12-16 }}$ The recent work of Gorodinakii, ${ }^{17}$ however, covered such an
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angular range.
Gorodinskii studied the degree of polarization of visible light after reflection from roughened samplea of dark glass, but did not measure angular distributions. Such glass reduces the offect of internal acattaring, with a resulting fominance of the role of surface reflection. Curves portraying the degree of polarization of light raflected in the apecular direction ware presented for a range of incidence angles. These resulta demonstrate that as the surface roughness decreases, the Fresnel limit is approached. Furthermore, for each of three angles of incidence, Gorodinskil show the dependence of the degree of polarization on the angle of reflection in the incident plane. For each incident angla, the curves for the various surface roughnesses peak at approximately the same reflection angle. The angular poaition correaponding to the peak is shown to be related to the polerizing angle ${ }^{18}$ of radiation incident upon and specularly reflected from the mirror-like facets of the roughened surface. Indeed, it is found that the sum of the aforementioned renection angle plus the angle of incidence 1s equal to twice the polarizing angle. This, the measurements of Gorodinskil sheri ame light on the mechaniam of reflection from a highly-abaorbing dielectric. The investigation of Gorodinakli and the findings resulting therefron correspond to a specific type of material. It is of interest to investigate differert types of materials with a view to diacerning in what ways (and why) the ciegree of polarization is altered.

## EXPERTHENTAL APPARATUS

A schematic diagran of the experimental apparatus is shown in Fig. 1 and a photograph is prosented in Fig. 2. The flow path of the radiation can be described with the aid of Fig. 2. The outprat of a radiation source A is focused by mirror $B$ onto the test surface $C$ (normal $\hat{N}$ ). Radiation reflected from the 18 The pojarizing (Brewster) angle is that angle at which the parallel-polarized component (parallel to the plane of incidence) reflected from a emooth surface is a ninimum.
teat surface in a pre-selected direction is collected by mirror $D$ and brought to - focus on the entrance silt of the spectrometer $F$. The resulting monochromatic output from the apectrometer is aensed by a detector located at a. a polarizer E is inserted into the beam to facilitate meamurmenta of the degree of polarisation.

The optioal aystem extermal to the apeotrometar includes amitiple-yoke device by which the directions of the incident and reflected beans can be varied independentily and with precision. Figure 2 showa a complete view of the entire test apparatus looking toward the apectrometer and the multiple-yoke, which, reapectively, appear at the left and at the right. The mitiple-yoke apparatus is capable of orienting the sample so that radiation reflected into any angular direco tion in the hemispherical space above the teat eurface oan be measured for any angle of incidence. 6 In Fig. 2, the device is shown in position for a masuremant out of the plane of incidence. ${ }^{19}$

The measurements reported here, however, were confined to the plane of incidance. Consequently, the anguler orientations of the incident and the reflected beams can each be characterised by a single coordinate angle, as shown in Fig. 1 The incident direction is epecified by the poiar angle $\Psi$, measured with respect to the surface normal. The direction of reflection ia characterized by the polar angle $\theta$, also measured from the surfece normel.

In order to vary the polar angles $\Psi$ and $\theta$, two turntables are used. In Fig. 1, the axes of these turntables are coarisl, lying perpendicular to the plane of the schematic diagram and in the plane of the sample surface. The radiant energy focused by mirror $B$ onto the aample surface is centered on the turntable axes. The larger of the two turntables supports the source A, nirror B, and sample C. It thus permits rotation of these components as unit in order to vary the reflection angle $\theta$. The smaller turntable rotetes only the sample in 19 The plane of incidence includes the incident beam and the surface normal.
order to vary the incidence angle $\psi$. The angular settings of the two turntables are read from greduated circles with vernier indicators.

Pertinent details of the optical system are as follows: The radiation source ${ }^{20}$ employed for meaaremente in the viaible region is a $5 / 8$-in. diamater fluorescent bulb maked to $2 / 2$ by $3 / 4$ in. Mirror $B$ is a spherical mirror with an aluminum first surface ( $d=1 \mathrm{in} ., f=8 \mathrm{inc}$ ); it subtends solid angle of $\pi / 1024$ steradians with respect to both the source and the test apecimen. The area of the specimen that is illuminated under normal incidence is $1 / 2 \mathrm{in}$. by $3 / 4$ in. The $1 / 2$ in. dimension is increased by the factor $(\cos \Psi)^{-1}$ for other angles of incidence $\Psi$. The collecting mirror $D$ is also apherical mirror with an alumimum first surface ( $\mathrm{d}=2 \mathrm{in}, \mathrm{f}=16 \mathrm{in}$ ) ; it subtenda a solid angle of $\pi / 1024$ steradians with respect to both the test sample and the entrance slit of the spectrometer. The entrance silt has a maximan width of 0.084 in . and is masked to a height of $5 / 16 \mathrm{in}$. The alit width was adjusted so that the radiant energy focused onto it by metrror $D$ almays filled the alit completely. The comercial sboet polariser I is mounted so that the plane of polarisation can be rotated through $90^{\circ}$ (E is not shown in Fif: 2).

The aforementioned finorescent scurce onde radiation consiating of a contimene enfesion spectrum peaking at $\lambda=0.5 \mu$ with a mperispoeed line apectrum. The meacuremente precented bere mere performed at $\lambda=0.5 \mu$, a region quite far removed spectrally from the aformentioned endasion lines. The apectrometar ia a Perikin-mimer model 122-0 from which the standard source aseandy had been removed. A fused-quarts prise and photomitiplier-tube detector were used for rem solving and senaing the radiant enery, reapectively.
tEST SPLCDEENS AD TESIR PRgPARATIOA
The two materiels employed in thia invastigation were solected on the basis
${ }^{0}$ in alternative infrared ecurce conaiating of a globar enclosed in a water-cooled jecket is available and is chown in Fig. 2.
of their reflection properties. The fused polycrystalline magnesium oxide ceramic is a white dielectric material characterized by weak absorption and strong internal scattering of Fisible iight. Trus, both surface and intermal reflection contribute. 17
This is in contrast to the dark-colored dieleatric studied by Oorodinsidi. Such
a dark material strongly absorbs light which is tranemitted through the curface, minimizing the role of internally-reflected rediation and acconturating aurface reflection. The second mierial amplojed bere, evaporeted alumimu on ground giasa, is similar to that of Corodinskil's dielectric in that reflection occurs essentialis at the surface, but possesses matalic reflection obaracteristics.

The fused polyerystalline mgnesium oxide is of high purity (99.9 percent) and was supplied by Honeymell Incorporated. It is the aame material employed in reforence 6. The four teat opecimans had surface dimansions $3 / 4 \mathrm{in}$. by $1 / 2 \mathrm{in}$. and were $1 / 4$ in. thick. The specimens were mormted in standard lucite metallurgical sample holders; a typical apecimen is shom in place on the apparatus in Mg. 2. Awiliary transadesion meecuremente at $\lambda=0.5 \mu$ shored that the $1 / 4$ in. thicknesr of the sample was sufficient to preclude effecte of the opecimen holder. ${ }^{21}$

Five ground glase diacs $11 / 4$ in. in diameter and $1 / 4$ in. thick were enaployed as subatrates for the aecond group of teet surfaces. These vere aimulaneoushy


411 test ourfaces were initially polishod Nat using a standerd optical polishing technique. Subsequently, a aimilar technique was used to roughen eight of the nine specimans with the grinding grits as liated in Table 1.

The surface roughness of the test opecimons was measured with high-preciaion stylus profilomater, the Taylor-Hobson Talysurf Model 3. The tip of the sensor was diamond stylus of $1.25 \mu$ ( 0.00005 in .) radius. A standard roughness-width cutoff length of 0.030 in . was used in determining the root-mean-square mechanical
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roughnesa values $\sigma_{m}$ listed in Table 1. The roughness measurements for the aluainum coated ground glass ware made after the coating had been applied.

## EXPSRDMENLAL EVALDATIOI OF THE BIANGULAR RESFIBCTANGS

Definition of the roflectance. The angular distribution and the degree of polariastion of reflected radiation were ovaluated in terms of the biangular reflectance. The designation biangular stens from the fact that two angular apecifioations are involved: The angle of illumdnation and the angle of reflection. The definition of the biangular reflectance uced in this atudy is in accord with that of other investigators. ${ }^{22-24}$

The biangular reflectance of the arixod radiation (containing all polarization components) is denoted hare by the symbol $\rho$. It is defined as the reflected intensity $d_{r}$ in the direction $\theta$ divided by the rediant energy de incident per undt time and unit area on the surface and coatained in a solid angle day inclinod In the direction $\psi$ relative to the aurface normal, that is

$$
\begin{equation*}
\rho(\Psi, \theta)=\frac{d I_{r}(\Psi, \theta)}{d e_{i}(\Psi)} \tag{1}
\end{equation*}
$$

When a pair of anglee appears within parentheees, the firat angle correaponds to the directim of the incident beam and the second angle corresponds to the firection of the roflectad bean. A single angle in prontheen is uged for quantities which have only one angular dependence.

The inteasity of radiation is defined as follows: Plrst, let de denote the radiant evergy per unit time and unit area containod within an infinitesimal solid angle $d u$ that is inclined at an angle $\beta$ relative to the ourface normal. Then, the intensity is given by the ratio of de to the product $\cos \beta$ d $\omega$. In

[^0]accordance with this definition, the intensity of the incident radiation is written as
\[

$$
\begin{equation*}
I_{i}(\Psi)=\frac{d e_{i}(\Psi)}{\cos \Psi d \omega_{i}} \tag{2a}
\end{equation*}
$$

\]

The intensity of the radiation reflected from acattering aurface is realietically amall quantity and is correopondingly expreased at

$$
\begin{equation*}
d I_{r}(\mu, \theta)=\frac{d^{2} e_{r}(\mu, \theta)}{\cos \theta d \omega_{r}} \tag{2b}
\end{equation*}
$$

The plane-polerised bianguler reflectanoen are reapectively denoted by $f_{0}$ and $P_{p}$. The aubseript is used to diatinguiah the component polarized perpendicular to the plane of incidence, the lotter $s$ atemang from the Cerman word for perpendicular, "eenkrecht." The subecript $p$ dietinguishea the component polarised paralial to the plane of incidence.

The biangular reflectances for the plane-polarized components are defined in a manner analogous to that for the mired radiation, that is

$$
\begin{equation*}
f_{s}(\Psi, \theta)=\frac{d I_{n, s}(\Psi, \theta)}{d e_{i}(\Psi) / 2}, \quad \rho_{p}(\Psi, \theta)=\frac{d I_{r, p}(\Psi, \theta)}{d e_{i}(\Psi) / 2} \tag{3}
\end{equation*}
$$

The quantitiea $d I_{r, s}$ and $d r_{r, p}$ are the refleoted intensities of the perpendicular and parallel-polerised componente. Furthermore, since the inoident beat is unpolarized, the incident radiant flux carried by each component is $\mathrm{de}_{\mathrm{i}} / 2$. Inecmach as $d I_{r}=d I_{r, B}+d I_{r, p}$, it follows from oquations (1) and (3) that

$$
\begin{equation*}
\rho(\Psi, \theta)=\frac{1}{2}\left[\rho_{5}(\psi, \theta)+\rho_{p}(\Psi, \theta)\right] \tag{L}
\end{equation*}
$$

By using the reflectances defined in equations (3), the degree of polarization of a reflected beem can be stated as

$$
\begin{equation*}
\text { Degree of Polarlantion }=\frac{p_{5}-p_{p}}{p_{S}+p_{p}} \tag{5}
\end{equation*}
$$

Inamach as $P_{0}$ and $P_{p}$ dapend on the anglea. $\psi$ and $\theta$, so also does the degree of polarisution.

In the present application of the foregoing definitions, the solid angle $d \omega_{i}$, subtended by the incident beam, is equal to the solid angle $d \omega_{r}$ of the reflected bean. Furthermore, all expresaions are applied monochromaticaliy.

Experimantal ovaluation of refloothpoel. The refleotance meemuremente perforsed during this inveatigation are of three types. First, the angular distalbutions of the plane-polarised and the mixed biangular reflectances were maanured relative to the sired biangular reflectanoe in the epeoviar direction. Thase masurments ware oarried cut for a renge of incidence angles. Second, the degree of polarisation was deteraned over a broad range of inoidence and reflection angles. Thind, in oerder to provide informetion on the absolute value of the reflectance, the dxed bianguiar ronlectance in the direction of the eurface normal was determined as a funotion of the angle of incidenoe. This biangular reflectance is tormod the ndxed normal biangular reflectance.

The exporimentel determanstion of the biangular roflectance is greatiy facilitated becase the opectrometer output is directly proportional to the intensity of the reflected rediant bean. This proportionality stems from the fact that both the solid angle $d \omega_{r}$ and the projectod area of the test opecimen, as viewed by the apectrometer, are not altered as the angle of reflectance is verifu. The constency of the projected area occura because the spectroneter entrance slit, which is always fully illuminated, hat a amiler area than the ilIurinated spot on the specimen.

In performing the masuremente, proper account must be thaken of the background radiation. This oonsists of emdesion from the sample, the optioa, and the surroundings. The level of the background rediation in the visible region of the spectrum was sinimised by performing the experiments in a darkened room. The background lavel was determined for each angular oriantation of intereat by shuttering mirror B (Fig. 1). Therefore, the ebsolute intensity of radiation
reflected from the test sample in the direction $\theta$, denoted by $\Delta I_{\mathbf{r}}(\psi, \theta)$, ia proportional to the detector output mims the corresponding background value. The $\Delta$-aymbol is used in place of the difforentiel $d$ to indicate a finite, experi-mentelly-detervined quantity.

For given test specimen, the rutio of the mosed biangular reflectance in an arbitrary direction $\theta$ to the adred biangular reflectance in the apecular direction is evaluated from equation (1) in terms of measured quantities as

$$
\begin{equation*}
\frac{\rho(\Psi, \theta)}{\rho(\Psi, \psi)}=\frac{\Delta I_{r}(\Psi, \theta)}{\Delta \cdot I_{r}(\varphi, \Psi)} \tag{6a}
\end{equation*}
$$

for given angle of incidence $\Psi$. The plane-polarised biangalar reflectances relative to the mixed biangular reflectance in the apecular direction are similarly evaluated from equations (1) and (3) as

$$
\begin{equation*}
\frac{\rho_{s}(\psi, \theta)}{\rho(\Psi, \psi)}=2 \cdot \frac{\Delta \operatorname{Ins}(\psi, \theta)}{\Delta I_{r}(\Psi, \psi)}, \frac{\rho_{p}(\Psi, \theta)}{\rho(\Psi, \psi)}=2 \cdot \frac{\Delta \operatorname{Inp}(\Psi, \theta)}{\Delta \operatorname{Ir}(\Psi, \psi)} \tag{6b}
\end{equation*}
$$

In any prism-type opectrometer, there is a polarising effect owing to differential reflection of the two compenants of polarieation at the priam faces. The extent of the polarisation due to the epactrometer was determined from auxiliary experiments, and an appropriate correotion was applied in evaluating equations (6b) However, as will be discuased later, the correction was not applied to equation (6a) in order to facilitate a display of the extent of the opectrometer polarisation.

The degree of polarisation was evalusted by introducing the corrected plane-polarized biangular reflectances into equation (5). Alternatively, the degree of polarisation could be determined by aimply masuring the relative intensities of the s- and p-compoents and correcting for spectrometer polarization. Thus,

$$
\begin{equation*}
\text { Degree of Polarigation }=\frac{1-\rho_{s} / \rho_{p}}{1+\rho_{s} / \rho_{p}} \tag{7}
\end{equation*}
$$

Bquation (7) was applied in those cases in which the angular distribution of reflected radiation was not moasured,

The wixed normal biangular reflectance oorresponding to various incidence angles was convenientiy meacured relative to a reference value of the normal bianguiar reflectance. The aboolute mendtude of the latter guantity was doterminod from an independant experimant.

## ANGOLAR DISIRIBUTIONS OF RIXED AND PLANR-POLARIZED RETLECTANC:S

The experimentaliy-determined anguiar diatributions of the mixed and planepolarized biangular reflectances are presented in Figs. 3 to 7. Among these, Figs. 3 to 6 contain results for magneaium ocide cerame, while Fig. 7 pertains to alumimin coated ground glaze. In all figures, the biangular reflectances are plotted relative to the correaponding maced blangular reflectance in the apecularray direction. The absciasa is the reflectance angle $\theta$. Figures 3 and 7 highlight the effect of varying incident angle for surfaces of fixed roughness. On the other hand, Figs. 4, 5, and 6 display information for various fixed incidence angles ( 45,60 , and $75^{\circ}$, reapectively) with surface roughness as curve parameter.

The off-gpecular peake. The effect of incidence angle $\psi$ on the distribution of reflected radiation can be conveniently exandned with the aid of Fig. 3. This figure presents the miced biangular reflectance distributions for a nagnesium oxde specimen of intermediate surface roughness ( $\sigma_{m}=0.76 \mu$ ). The trends in Fig. 3 are typical of matale as woll as of nometale when the surface roughness is comparable to or larger than the wavelength of the radiation ( $\sigma_{m} / \lambda \geqslant 1.0$ ). In the type of presentation amployed in Fig. 3, a perfectly diffuse surface (i.e., one which obeys Imabert's cosine iaw of reflection) would have a constant value of the relative biangular reflectance equal to 1.0 .

The results for near-normal incidance, $\Psi=10^{\circ}$, approach the diffuse limit. As the angle of incidence increasas, it is apparent that the corresponding
reflectance distributions differ marimedy from that for diffuse surface. For Incidence at $\Psi=45^{\circ}$, the distribution displays weak jocal naxima t the specular-reflaction angle $\left(\theta=45^{\circ}\right)$ and at approxdeataly $\theta=80^{\circ}$. is $\Psi$ uncmasan atill further, the madman in the vioinity of $\theta=80^{\circ}$ growa rupidiy, until.
 peak is quite different in shape from the sharp, epecular-reflection majams oscuring on amoother surface. In Fig. 6, such specular reflection pemex in
 that the difrues diatribution is appromened only when the angis of inciserom is noar normi.

Diatiributions of rdxed and planemolarized reflectancea. Inammen as the
 Incidence, more detailed consideration is given to the refiection distritnitione corregjendint to incidence at auch angles. Figures 4 through ? ineve besh prepared in this connection. Tha firgt three of these show the offact of varinis furinga roughiegs at fixot anglise of incidence $\psi=45,60$, and $75^{\circ}$ for mgnenium oxids. FHgu:e 7 pertains to siuminus comtad ground glase and will be diecusnsi later.

Sech of Fign. 4 to 6 dioplays the plene-poinrised and alxed biengular rea
 1.9. and $5.8 \mu$ ). The distritritione of the madre radiation ars shoun as biackeract circles throagh whteh aolid lines have been patad, and the plene-poisizad cop fonents are represented by open circles comocted by dashed lines. In all ensos, the couponenc (6), poiarised perpendiculer to the plane of incidence, lien sbove the solid curve. The combonent íp), polarized parailei to the piane of inetconce, liso below the solld erve. Accordige to aquation (4), the wixed rgflectanco should be tha evarage of the plansmpolarized compouand. In point of fact, the averege of that dahed diryeg is very nearily, but not quite, soinctiont whth the:
corresponding solid curve. This sidght disparity occurs becanbe the latidr wos not corrested for the polarization inturduced by the syyctrometer. The pronsantation of the vacorrected curve was pronsenal in inat it was desimed ite inv dicate to whet oxtent the weesuremente are sffactec by the apeotrometar-1isiagod polarization.

The ragulte for the smootibest surfane in Fige. L to 6. $\sigma_{\text {w }}=0.23 \mu$, rispiay


 be zero. Furthermore, the penk wehiti be ajametrica about $=\psi$, with seas-

 anguiar positions wey from the sponlu: region la not zero. In odatition, the peak is not gymantic, stowing sowowht larger reflectances in the region $\theta>\psi$. This skeving is presumbly due to the ofi-specular peat mechanima.

An axwenation of each of Figs. I to 6 revauls that the peak at tive spociar.

 spacular peak, the magituda of the offmpencular poais increasee whith larger suelea ge incidence. In oume cssas, the iptemetty at the off-apocuin: poak ia 3 or $L$ tises the intenaity in the rpecular-ray direction.
 prowide further informetior mbote tha railoction process. Uonsidering the smoothest surface, $\sigma_{m}-0.23 \mu$, it da tateroetigg to note that the contributhons of the s-and p-oxeperente to the wifoctaice in the spectular direction


optically smooth dielectric aurface is irradiated by an unpolarized basm incident at the polarizing anglo (Brewstar angle). For magneaium oxide, the Brewster angle is $60.2^{\circ}$, poilich is apprcaisately equal to the angle of incidence in Pig. 5. Corrempondizeiy, in thet oass, the papanzised ocmponent contributer very intile to the apecular peais. At an incidence angle $\psi=45^{\circ}$ (Fig. 4), the p-couponont contributas commathat rore, and at $\mathrm{Ya}_{\mathrm{a}} 75^{\circ}$ (F1g. 6), it contributes eignificentiy, es prodtctad by the eleotrampretio theory. For the rougher surfaces, the predictions of elactromegnetio thsory are not expected to apply. For such surfaces, the experimental datei show that wensver there is an inflection in the reflectanos distribution et the specular-ywy direction, the e-component sppears to be responarible.

In addition, other featiaree of the polarized components may be notod. In Fig. 4, the p-compoient remains almost constant with reflection angle 0 , while the g-component gives the shape to the mixed reflectance distribrtion. In Fige. 5 and 6, howezer, toth the s. and p-polerised somponerits contribute to the off apecular pesis. Mile is in good agresmant mith nodels of the reflection process that postuints the presence of nirror-1ite surface elesents. Prom auch model, it is expecter thet for the incidonce angie or $748.4\left(\Psi=45^{\circ}\right)$, inttio augmentation of p-polarized light world occur in the off-specuiar peak ragion. Thit is because the off-apesculey fizelic region of that igure inclucies anguiar atireotions corresponding to reflection at the Brewater angle from seriace facets. On the other hand, for the euglea of invidenes corresponding to Fige. 5 end 6, the offe opecular peak regior, does not include angular direstions correaponding to Srewnterangie reflantion fiom the fariots.

The affoct of incidence angle on the plane-polarised and mixed bianguiner refleotsence dis'ributions for slundmu sosted ground gless is show in Fig. ?. The figure wartains to e eurface with rcaghness $\sigma_{m}=2.8 \mu$. The data are prosented ie asmer fisediar to that used in the foregoing sigures. The fatare
that is inmediately observable in Fig. 7 is the accentuation of the off-specular peak with increasing angle of incidence. Both the s-and p-polarized components contribute to the off-specular peaks for all three inoidence angles. This finding is consiatent with the model which pictures the surface as consiating of mirrorlike facets. For, the aluminn coating, the Brewater angle is large ( $\sim 80^{\circ}$ ). In addition, the Brewster angle for motala correaponda to the angle of incidence for a minimin (rather than sero) rellectance of p-polarised light after apecular reflection. This minimum reflectance of the p-composent is typionily large (78 percent for aluminm). For the incidence angles $\psi=45$ and $60^{\circ}$, the region of the off-apecular peaks does not include angular directions correaponding to Brewaterangle reflection at the facets. In the caee of the $75^{\circ}$ incidence, Brewater-angle reflection at the facets may fall into the region of the off-apecular peaks. However, owing to the aforementioned large reflectance at the Brewster angle, this does not significantily diminish the contribution of the p-couponent.

As a final point in connection with Mg. 7, it is apparent that the relative biangular reflectance in the normal direction ( $0=0^{\circ}$ ) is considerably lower than for the surfaces of aimiler roughness in Figs. 4 to 6 . This may be made plausible by noting that for incidence at large angles on a rough surface, there would be considerable thedoung of the raughose relloye. then the reflection occure pri= marily at the surface, as with a motal, this causes an apparent low reflectance in the direction of the surface normal. On the other hand, for the magnesium axide cervic, there is considerable internal scattering of iight. Therefore, the shadowing of the roughness valleys is partially offset by the internaliyreflected light which re-emerges from the surface.

The definition of the biangular reflectance requires that the solid angles of incidence and reflection be infinitesimally amall. The solid angles of this experiment are indeed very small, but necesaamily finite. To assess whether or
not they are small enough to permit meaningful measurements of the piangular reflectance, som data were collected using a solid angle one-fourth as iarge. The results are represented by the crosses in Fig. 7 for incidence at $60^{\circ}$. Oood agreement is seen to exist between the date taiken with $\Delta \omega=\pi / 10<i 4$ and $\pi / 4096$.

The curves portraying the reflection diatributions in Fige, is tircough ? were terminsted at various 0 values less than $90^{\circ}$. The axperimental procedurs required that the eatrance slit of the apectromater be fully illuminated so that the portion of the epecimen aurfece Fiewed by that instrumant be coafined to the interior of the illuminated spot. This necessitated the ver of wide source and narrow entrance slit widths. The madman value of 0 at whith data could be taken without viewing too olose to the boundary of tive illuminated spot thus depended on incidence angle $\psi$, and the width of the aciroe, apecimen, and entrance alit. For a given incidence angle $\Psi$, only the entrance alit width was variable. The minime entrance slit width wes determaned by the minimen snergy level that oould be accuretely sensed by the spectrometar detector.

## morles of polarization

The degree of polarization of reflected light was evmiuated according to ef ther equation (5) or (7). Resuits for magnesium oxide are shoin in figs. 8 and 9 and for aluainum couted ground glass in Fig. 10. The abscises is the refleotion angle $\theta$.

Figures 8 and 20 display the degree of polarieation of apeculerly-roflectod light for surfacss of varying roughneas. The ourfaces ware illuminated at incidence angles $\psi$ ranging from 10 to $87^{\circ}$, and the degree of polerizstion of the specularly renected light $(\theta=\Psi)$ wa determined. Such experiments naturaily land themselves to ocmparison with the polarisation for opticsing smooth surfaces predicted from the Freanel equations.

The measured results for four megnesium oxide specimens ( $\sigma_{m}=0.23$, $0.76,2.9$, and $5.8 \mu$ ) are represented by the solid iinea in Fig. 8. The dashed line appearing in the figure represente the Freanel equationa, which were evaluated uaing the index of refraction of manemium ocide. 25 The experimontal curves anccessively approach the Froanel prodiction ta the Eurface roughnese deoreaces; this trand is in agreament with the work of oorodinabil. For the amoothest surface, $\sigma_{m}=0.23 \mu$, the peak of the curve lies at approidmately the Brewater angle of $60.2^{\circ}$. The peaks in the other experimatal curvee move to larger angles $O$ as the surface roughness increases. In contrast, Gorodinakil's curres for ground dark glass apecimens of varying roughness all poak at approcimatoly the Brewator angle. The dark glass is characterised by atrong internal absorption and weak internal acattering. The observed difference in the reault is attributed to the atrong, internal scattering in the magnosium aride ceranic.

A presentation of results aimilar to Fig. 8, but for aluminum costed ground glass specimens, is contained in Fig. 10n. The solid lines connect date points for surfaces having roughneases $\sigma_{m}=0.33,0.45,0.85$, and $2.8 \mu$. In addition, data points for a polished surface (polished glass conted with alumdmum) are ahown connected by a dashad curve. This dashed curve was culculated from the Preanel equations ${ }^{26}$ using the index of refrectica and extinction coelficient for an evaporated elumimum. Iilm at $\lambda=0.5 \mu .^{27}$ scoellent agreement is seen to exdst between the Freanel prediction and the axperimental date. As surface roughness decreases, the curves do not tend successively toward the Freanol curve. Instead, the surface with $\sigma_{m}=0.45 \mu$ show a greater degree of polurisation than that with $\sigma_{m}=0.85 \mu$, and the latter a greator dagree of polarisation than the $\sigma_{m}=2.8 \mu$ surface. However, the results for the $\sigma_{m}=0.33 \mu$ surface suggest 25 American Institute of Physics Handbook, socond edition, 1963, Moaraw-ByIl,
New York, N.Y., P. 6-12. 26 ${ }^{26}$ Handboch der Physik, vol. 20, Spr-inger-Verlag 080, Borlin, 1928, p. 240. 27 American Instituto of Phyaica Bandbook, second edition, 1963, McOraw-Fiill,

that a further decrease in aurface roughness would lead to an approach to the Fresnel curve. The just-deacribed behavior. with decreasing surface roughnesa is, at present, unoxplained.

It is interecting to note that all the ourves in Pig. 10 prak at approcimotely the Brewater angle of $80^{\circ}$. Thic ia in agreement with the findings of Gorodinskif. The common behavior is consistant with the fact that both the aludnum specimons of Fig. 10, and the dark glase specimens of Corodinskii are characterised by surface reflection in the absence of internal scattering.

The viriation of the degree of polarization with reflection angle $Q$ for various fired incidence angles. $\psi$ is shonn in Mgs. 9 and 100, respectively for magnoalum oxide and alwainum costed ground glass. In Fig. 9, results are given for incidence anglea of 45,60 , and $75^{\circ}$, with surface roughness as curve parameter. Fig. 10 b pertains to a arface with roughness $\sigma_{m}=2.8 \mu$, and the curve parameter is the incidence angle $\Psi$.

The information in Figs. 9 and 10b can be conveniently discussed by again taking the work of Gorodinskil as a point of departure. For a fixed incidence angle, his degree of polarization curves for three different surface roughnesses all exhibit a madimum at the same reflection angle $\theta$, which will be designated here as $\theta_{\text {max. }}$ Corodinakil used fixed incidence anglos $\Psi$ of 45,60 , and $70^{\circ}$. He presents the following experimentally-baced reletion between $\Psi$ and $\theta_{\max }$

$$
\begin{equation*}
\psi+\theta_{\max }-2 \theta_{B} \tag{8}
\end{equation*}
$$

where $\theta_{B}$ is the Brewster angle for dark glass. Equation (8) can also be derived frem the mirror-like facet model of surface reflection.

For each of the fixed incidence anglee of Fig. 9, the curves for the varicus surface roughnesses do not all exhibit a pronounoed peak at the same reflection anglo 0 . Using $Q_{B}=60.2^{\circ}$ for magnosium oxide, equation ( 8 ) would predict peaks at $75.4,60.4$, and $45.4^{\circ}$, reapectivaly for $\Psi=45,60$, and $75^{\circ}$. Clearly,
equation (8) does not deacribe the trends in Fig. 9. This ia attributed to the effect of internal reflections.

For the aluninum coated ground glass in Fig. 10b, better agreement with equation (8) is to be expeoted. Thie ie becauee internal ecattering does not affect the rellection process for the aluminum surfaces. The data points in Fig. 100 betmeen $\theta=80$ and $89^{\circ}$ for $\psi=75,80$, and $85^{\circ}$ are anitted to preserve clarity. Upen applying equation ( 8 ) for $\theta_{B}=80^{\circ}$ and recognizing that $\theta$ must be less than $90^{\circ}$, one obtains $\theta_{\text {max }}$ values of $90,90,85,80$, and $75^{\circ}$ when $\psi=45,60,75,80$, and $85^{\circ}$. The trende and general locations of the pesks in Fig. 100 am in agreement with the predictions from equation (8), thus further substantiating the model consisting of elementery merror-like surface facete.

## MORMAL BLANGTLAR REFISCTANCB

The mixad normal biangular reflectance for incidence at angle $\psi$ is prosented in Fig. IL. Four curvea for mgneatur ofide and ace for alwinum coated ground glass ere whow. Each of the curves correaponds to a particular aurface roughness $\sigma_{m}$. Th.s, the four magneaius oride curves eerve to intermrelate the biangular reflectance diatributions of Figs. 3 to 6. Similarly, the curve for alumam coated ground glass in Mig. 11 serves to inter-relate the data in Fig. 7.

The values of the normal biangular reflectance are plotted relative to reference quantities $I$ that are listed in the legend of Fig. 11 . The reference quantity for mansium oride is the mixed normal biangular reflectance correaponding to $10^{\circ}$ incidence oa the $\sigma_{m}=0.23 \mu$ surface. The results for alumanm costed ground glase are referred to the normal blangular reflectence for $10^{\circ}$ incidence.

Certain espeste of the reaulte in Fig. 21 are worthy of discussion. For magneaium oxide, the curves for the two intermedinte eurface roughneasea lie
between the curve for the smootheat and the roughest eurfaces. Also, the variation of the magnesium odide data with $\psi$ is mach less than that of the data for aluminum coated ground glass. This is explained by the stronger shadowng effect at large incidence angles experienced by the latter. The shadowing effect for magnesium oxide is neutralised by internal ocattering.

An important additional ifnding may be dectuced by applying the results of Fig. 11. The variation of the mixed normal biangular reflectance with $\Psi$ is generally amall for magnesium oxide. In particular, for the $\sigma_{m}=0.76 \mu$ apecimen, the ordinate decreases by about 16 parcent as $\psi$ ranges from 10 to $75^{c}$. In FIg. 3, however, the mixed biangular reflectance ratios at $0=0^{\circ}$ decrease by 86 percent for $\Psi$ in this range. Thue, the variation in the magnitude of the offapecular peak with incidence angle would be conoiderably amplified if Fig. 3 were to be replotted on an absolute baais. Sindiar remarks apply to the data shown in Figs. 4 to 7.

## AGNNOMIEDCEATI

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Table 1. Grite used in preparation of apecimens and romulting ras mechanical roughneas $\sigma_{\text {n }}$

| Specimen Material | Orit Dian | ter and Type | $\sigma_{\text {m }}$, Microns |
| :---: | :---: | :---: | :---: |
| Mag. Oxide | $5.0 \mu$ | alum. oxide | 0.23 |
| Mag. Oxide | $22.5 \mu$ | alum. axide | 0.76 |
| Mag. Oxide |  | alum. aride | 1.9 |
| Mag. Oocide | ${ }^{165} \mu$ | alum. oxide | 5.8 |
| Alum. Coated Ground Glass |  | polished | $<0.004$ |
| Alum. Coated Ground Glass | $3.0 \mu$ | slum. coxide | 0.33 |
| Alum. Coated Ground Gless | 5.0 | alum. oxide | 0.45 |
| Alum. Coated Ground Glass |  | alun, oxide | 0.85 |
| Alum. Costed Oround Glass | ${ }^{63} \mu$ | alum. axide | 2.9 |





Fig. 3 Angular distribution of mixed biangular reflectance for various incidence angles $\Psi$, magnesium oxide ceramic.


Fig. 4 Angular distributions of mixed and plane-polarized biangular reflectances, magnesium oxide ceramic. Angle of incidence $\Psi=45^{\circ}$.


Fig. 5 Angular distributions of mixed and plane-polarized biangular reflectances, magnesium oxide ceramic. Angle of incidence $\Psi=60^{\circ}$


Fig. 6 Angular distributions of mixed and plane-polarized biangular reflectances, magnesium oxide ceramic. Angle of incidence $\psi=75^{\circ}$.


Fig. 7 Angular distributions of mixe



Fig. 9 Angular distributions of degree of polarization for three fixed incidence angles $\psi$, magnesium oxide ceramic.


Fig. 10 Aluminum coated ground glass. (a) Degree of polarization in the specular direction $\theta=\psi$ as a function of incidence angle $\psi$. (b) Angular distribution of degree of polarization for several fixed incidence angles $\psi$.



[^0]:    ¿¿ट H. J. MeNfcholas, Journal of Research of the National Bureau of Standarde, rol. 1, 1928, p. 29.
    ${ }^{23}$.c. von Mragstein, Optik, rol. 12, 1955, p. 60.
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