NASA TECHNICAL
MEMORANDUM

NASA TM X-53386

February 1, 1966

FACILITY FORM 602

N66-19510	
(ACCESSION NUMBER)	(THRU)
49	1
(PAGES)	(CODE)
TMX-53386	12
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

NASA TM X-53386

A MONTE CARLO PROGRAM FOR TRANSMISSION PROBABILITY CALCULATIONS INCLUDING MASS MOTIONS

By James O. Ballance
Aero-Astrodynamics Laboratory

GPO PRICE $ ______

CFSTI PRICE(S) $ ______

Hard copy (HC) 2.00

Microfiche (MF) .50

ff 653 July 65

NASA

*George C. Marshall
Space Flight Center,
Huntsville, Alabama*

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

TECHNICAL MEMORANDUM X-53386

A MONTE CARLO PROGRAM FOR TRANSMISSION PROBABILITY CALCULATIONS INCLUDING MASS MOTIONS

By

James O. Ballance

ABSTRACT

A Monte Carlo computer program which can be used to calculate the transmission probabilities for two cylindrical tubes in series is described, as well as the technique of adding directed mass motion to the random thermal motion of the molecules. The results for a simple straight cylindrical duct are compared to other solutions, and angle-of-attack effects are examined. Complex systems for which no adequate solutions have previously been found are easily analysed by this method. The Fortran instructions are listed along with a typical program solution.

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

TECHNICAL MEMORANDUM X-53386

February 1, 1966

A MONTE CARLO PROGRAM FOR TRANSMISSION PROBABILITY CALCULATIONS INCLUDING MASS MOTIONS

By

James O. Ballance

AERODYNAMICS DIVISION
AERO-ASTRODYNAMICS LABORATORY

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Cont'd)

TABLE

TECHNICAL MEMORANDUM X-53386

A MONTE CARLO PROGRAM FOR TRANSMISSION PROBABILITY CALCULATIONS INCLUDING MASS MOTIONS

SUMMARY

19510

A Monte Carlo computer program which can be used to calculate the transmission probabilities for two cylindrical tubes in series is described, as well as the technique of adding directed mass motion to the random thermal motion of the molecules. The results for a simple straight cylindrical duct are compared to other solutions, and angle-of-attack effects are examined. Complex systems for which no adequate solutions have previously been found are easily analysed by this method. The Fortran instructions are listed along with a typical program solution.

Ouena

I. INTRODUCTION

While the properties of rarefied gas flow have been studied in detail for many years, there still exists, in many respects, a real lack of understanding of the physics of the problem. Gross properties such as the transmission probability (the probability that, if a molecule enters a system at point 1, it will exit the system at point 2), the spatial distribution at, near, or in the system, etc., have been determined for very simple systems (i.e., cylindrical ducts, rectangular ducts, infinitely wide parallel plates, etc.) when the density is sufficiently low that the system is in the free molecular flow regime. However, the condition for free molecular flow is not exact since all the parameters which affect the flow properties are not known. The Knudsen number is normally defined as the rates of the mean free path (λ) to the characteristic dimension (L) of the system. When the Knudsen number is large (Kn = 10 or higher), the system is said to be in free molecular flow. But if one is considering a cylindrical duct as a system, it is not known if the characteristic length is the diameter of the duct, the length of the duct, or some dimension resulting from a combination of these values. For very large mean free paths and relatively small length-to-diameter ratios for the duct, the exact Knudsen number is not important. When the mean free path of the molecule is near the characteristic length ($10 \geq K \geq .1$), the flow regime is called transition flow, for which no sufficient description is presently available.

These problems are greatly magnified when one considers complex systems such as orifice-restricted tubes, baffles in tubes, elbows, etc., and flow where there is relative motion between the system and the gas. There is a wealth of information in the literature about this problem with most of the solutions anchored on the excellent work of Clausing [1]. His approach and techniques have been followed with some improvements in accuracy. It was not until Davis in 1959 [2] that any significantly different approach to the problem was developed. In a series of extremely useful papers [2,3,4,5], Davis and his co-workers made a significant contribution to the field of vacuum system design. Because of the obvious success of Davis' approach and the belief that this approach could be used for solutions of many problems, the writer has developed programs very similar to those of Davis to look at several types of problems. Some results of these programs have been published [6,7]. These programs consider complex geometrical systems, relative mass motion, reflection coefficients, spatial distributions, etc. Since many requests for information about these programs have been received, it was decided to present in detail the program for a simple system. This paper describes the Fortran computer program for the Monte Carlo calculations of the transmission probability for two co-axial cylinders connected in series with provisions for relative mass motion directed along the axis of the system or at any arbitrary angle with respect to the axis. Only the basic program will be described in detail. Modifications which have been incorporated to determine flux distributions, spatial distributions, capture coefficients, etc., will be only briefly mentioned since these aspects can be added to the basic program as the user desires. The program has not been refined. It was not the intent of this study to develop an optimum, time-saving computer solution. A skilled programmer could easily reduce the computing time. However, it was felt that the 8 minutes it takes for the present computer solution on a CDC-3200 (or 2 to 4 minutes on an IBM-7094) was reasonable. Most of the effort has been spent on the physics of the problem.

II. DESCRIPTION OF THE PROGRAM

A. ASSUMPTIONS

The following basic assumptions of the flow parameters of the molecules are consistent with most approaches:

1. The molecules enter the cylindrical duct with a direction proportional to the cosine of the angle between the normal to the entrance plane and the direction of the molecule. If the molecule strikes a wall, it leaves the wall in a direction proportional to the cosine of the angle between the normal to wall at the point of collision and the direction of the molecule.

2. The mean free path is sufficiently large such that collisions between molecules can be ignored. (Modifications to this basic program are not limited by this assumption; however, no description of these modifications will be made in this paper).

B. MODEL DESCRIPTION

The model for this study is shown in Figure 1, where we see the two cylindrical tubes in series with a common axis. For convenience, the radius of tube 1 is given the value of one. The configuration to be studied is then described in input parameters for the following parameters. (The symbols enclosed by parentheses refer to the parameters in the Fortran program, Figure 3):

1. The length of tube 1, (Q).
2. The length of tube 2, (QL2).
3. The radius of the entrance orifice to tube 1, (A).
4. The radius of the exit orifice of tube 1, (B).
5. The diameter of tube 2, (CL2).
6. The radius of the exit orifice of tube 2, (DL2).

In Figure 1, tube 2 is shown to be larger in diameter than tube 1. The program is not restricted to this configuration and can be used with tube 2 having any diameter. If QL2 is zero, tube 2 is not considered by the program.

C. THE MONTE CARLO PROCEDURE

In the Monte Carlo procedures, molecular histories are generated for a large number of molecular paths through the system. The properties of each molecule are recorded depending on the parameters of interest. This paper will consider only the tabulations of the number of molecules which exit the system at either exit plane as a function of the number of collisions with the walls (both the tube walls and the orifice plates).

A flow diagram for the computer program is shown in Figure 2; the Fortran print-out for the program is shown in Figure 3. The history for a molecule begins with instruction 2010. (The program instructions preceding number 2010 merely performs bookkeeping functions, presets counters, reads the input parameters, etc.) The initial coordinates of the molecule on the entrance plane are given by (with the axis of the cylinders being the positive X axis)

$$Y_1 = 1 - 2R_1$$

$$Z_1 = 1 - 2R_2,$$

where R_1 and R_2 are random numbers. (There are numerous methods to calculate pseudo-random numbers in the literature.) These coordinates can fall anywhere within the square bounded by $y = \pm A$, and $z = \pm A$. The next instruction selects only those points within the circle $Y_1^2 + Z_1^2 = A^2$. Instructions 2011 through 2142 calculate a cosine-bias angle.

If mass motion is to be considered, the instructions from 2555 through 2141 are used to calculate the random thermal speed of the molecule. This is explained more fully in the next section. Direction cosines for the molecular trajectory are determined from 2556 through 2020. Instruction 2024 tests to see if the molecule hits the exit plane of the tube. If it does not hit this plane, it must hit the wall of tube between X = 0 and X = L(Q). The point of collision is calculated by instructions 3026 through 3035. At this point, the molecule is reflected diffusely from the tube wall by the instructions 3039 up to 3045. The instruction before 3045 senses U1 to see which direction the molecule will go. Instructions 3045 to 3052 and 3060 to 3063 tests to see if the molecule hits the end planes of the cylinder. If it does not, the x coordinate of the point of collision with the tube wall is calculated by 3068 to 3642. The remainder of the program repeats these tests for molecules in tube 2.

D. SPEED DISTRIBUTION

Although the procedure for combining the relative mass motion to the random thermal motion is straightforward, it needs some explanation. In the program, the position of a molecule at the entrance plane of the system is determined, and the direction cosines for its motion considering only the cosine bias are calculated. For no mass motion, the molecules whose direction cosine U1, because of its thermal velocity, is negative will not enter the system. This is not necessarily so when there is mass motion. The magnitude of the thermal speed of the molecule is chosen by the instructions starting at 2555 in the program. Figure 5 presents a histogram of the thermal speed as selected by this procedure. From this distribution, one can determine the most probable speed to be 1.414, the root-mean-square speed to be 1.777, and the average or mean speed to be 1.596. Also shown in Figure 5 is the theoretical Maxwellian-Boltzman distribution. One can see that this calculation satisfactorily approximates the Maxwellian distribution.

Using the direction cosines, one can then determine the magnitude of each component of the speed. Mass motion can then be added to the thermal speed and new direction cosines for the actual path of the molecule relative to the moving tube. One small consideration must be made, however. This consideration is concerned with the relationship of the thermal speed of the molecules entering the system to the thermal speed of the molecules in the reservoir. From the kinetic theory of gases, one knows that the number of molecules striking the small area is given by

$$\frac{1}{4} V \, dN_V,$$

where V is the speed of the molecule and

$$dN_V = \frac{4N}{\sqrt{\pi}} \beta^3 V^2 e^{-\beta^2 V^2} dV$$

$$\beta = \frac{m}{2kT} .$$

The most probable speed of the molecules is found by setting the first derivative of function above equal to zero.

$$\frac{d}{dV} \left[\frac{1}{4} V \frac{4N}{\sqrt{\pi}} \beta^3 V^2 e^{-\beta^2 V^2} \right] = 0$$

From this we find that the most probable speed of the molecules leaving the reservoir is given by

$$\left(V_{exiting} \right)_{mp} = \sqrt{3/2} \left(V_{reservoir} \right)_{mp} .$$

The distribution shown in Figure 5 is for the molecules exiting the reservoir (or entering the tube). Since the speed ratio is defined as the ratio of the mass motion to the most probable motion of the gas molecules in the reservoir, the mass motion vector which is added to the thermal speed vector must be expressed in terms of the reservoir speed. Thus, the mass motion speed vector is given by

$$\frac{1.414\ S}{\sqrt{3/2}} = 1.155\ S,$$

where S = speed ratio.

Figure 6 shows a histogram for the speed distribution with mass motion. The solid curve represents the Monte Carlo solution for the general expression of the velocity distribution

$$\frac{d^3N_V}{N} = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}\left[(V_x - SI)^2 + V_y^2 + V_z^2\right]}\ dV_x\ dV_y\ dV_z$$

where N = 10,000 and $SI = 8\left(V_{reservoir}\right)_{mp}$.

The root mean square speed was found to be 9.384 and $\bar{V}$, 9.330. The dashed curve shows the distribution of molecules that actually enter the tube for the same mass motion. It is seen that, although the rms speed and the mean speed are approximately equal, the actual distributions are very different. This difference is a result of the cosine distribution of the entering molecules. In Figure 6, the peak on the program distribution at V = 8.5 results from the addition of the mass motion to those molecules at the entrance plane which were traveling opposite to the direction of mass motion, while the peak at V = 10.0 results from the addition of mass motion with the molecules which were traveling in the same direction as the mass motion.

III. RESULTS

The transmission probabilities for cylindrical ducts at various speed ratios have been calculated by Pond [8], by deLeeuw and Rothe [9], and by Hughes [10]. While Pond presents his data directly as transmission probabilities, it is quite difficult to accurately obtain values from his figures. Rothe and deLeeuw present their data as a pressure ratio; some simple calculations are required to convert the data into transmission probabilities. The results of this operation are presented in Table 1 and in Figures 7 through 9. Figures 10 through 15 present data from the Monte Carlo solutions with the solid line representing the values interpolated from the UTIA data. It is quite noticeable that the Monte Carlo data for all L/A values are slightly lower than the UTIA data below speed ratio of 5. For the zero speed ratio data, the Monte Carlo values are consistently within ±5 per cent of other solutions. For speed ratios up to 4, the difference between the Monte Carlo data and the UTIA data is between 5 and 10 per cent; however, the Monte Carlo data are always the smaller. No explanation of this can be given at this time.

All the previous results are for the case where the mass motion is directed along the axis of the tube. In general, the mass motion will be at any angle with respect to the axis of the tube. Defining the angle of attack to be the angle between the direction of mass motion and the axis of the tube, Figure 16 presents the Monte Carlo data for tubes of L/A = 2, and speed ratios of 2, at various angles of attack. It is interesting to note that, at an angle of attack of 90 degrees, the transmission probability (0.35) is <u>not</u> the same as the transmission probability for zero speed ratio (0.51).

In all the data presented so far, only a single tube has been considered. Normally, a measuring system in free molecule flow consists of a gauge volume connected to the free stream by an orifice-restricted tube or a series of tubes. The techniques for calculating the transmission probability for composite systems with zero mass motion have been examined by several investigators [7,11,12]. To see if these same approaches could be used for composite systems with mass motions, Monte Carlo calculations were made for an orifice-restricted tube of L/A = 8, A_1/A_2 = .5, and for various speed ratios. So that any differences which might appear would be the result of the procedure, the Monte Carlo transmission probability values for a tube of L/A = 4 were used in the calculations. Figure 17 presents the results of these calculations. Deviations up to approximately 15 per cent are observed between the calculated values and the Monte Carlo values. Tube lengths of L/A = 2 and 4 were also examined (although not presented in this report). Deviations from the theoretical values for these tubes were quite small, i.e., 2 to 4 per cent. The standard techniques for coupling orifices and tubes in free molecular flow may be used for mass motion studies. The deviation from these values increased with increasing tube length, but decreased with increasing mass motion.

For the case of two tubes with transmission probabilities α_1 and α_2 corresponding to tube L_1/A_1 and L_2/A_2, respectively, Reference 7 shows that the total transmission probability for zero mass motion is given by

$$\frac{1}{\alpha_T} = \frac{1}{\alpha_1} + \frac{A_1{}^2}{A_2{}^2\alpha_2} - \frac{A_1{}^2}{A_2{}^2} .$$

Assuming this equation to be valid for the case where there is mass motion such that $\alpha(L/A)$ becomes $\alpha(L/A,S)$, calculations were made for two values of L/A and various speed ratios and are compared with Monte Carlo data for the same parameters in Figure 19. There is excellent agreement between the calculated values and the Monte Carlo values.

Figure 20 presents a comparison of Monte Carlo solutions for transmission probabilities for cylindrical ducts and angles of attack with data from Reference 10 for the same configuration. Again, there is excellent agreement between the two methods.

IV. CONCLUSIONS

A simple Monte Carlo computer program similar to the one developed by Davis has been modified to permit investigation of the influence of mass motion with random orientation with respect to the tube. Comparison of results from this program to numerical solutions of other investigations show the agreement of the different methods to be excellent. Using the inherent advantages of this program, such as orifice studies, series of tubes, etc., the general coupling equations developed for zero mass flow appear to be applicable to flows with relative mass motion.

After this comparison of numerical methods to Monte Carlo methods for simple tubes, a great amount of confidence may be placed in attempting investigations of parameters such as reflection coefficients, thermal and accommodation coefficients, etc. Also rather simple modifications to this program allow one to examine ducts of other types, i.e., elliptical and conical.

TABLE I

TRANSMISSION PROBABILITIES FOR CYLINDRICAL DUCTS AT VARIOUS SPEED RATIOS

(CALCULATED FROM UNIVERSITY OF TORONTO REPORT NO. 88,
DECEMBER 1962, BY J. H. DELEEUW AND D. E. ROTHE)

L/A	K(S,L/A)			
	S=0	S=0.02	S=0.05	S=0.1
.010021	.995015	.995073	.995179	.995317
.013338	.993375	.993433	.993630	.993760
.017752	.991202	.991355	.991456	.991753
.023628	.988324	.988477	.988668	.989040
.031449	.984520	.984767	.985043	.985482
.041859	.979501	.979747	.980111	.980788
.055714	.972901	.973239	.973775	.974507
.074155	.964256	.964684	.965299	.966416
.098701	.952988	.953595	.954368	.955765
.131371	.938402	.939091	.940191	.941927
.174854	.919688	.920629	.921947	.924150
.232731	.895950	.897040	.898725	.901429
.309765	.866283	.867671	.869681	.872967
.412298	.829888	.831538	.833904	.837829
.548769	.786257	.788124	.790854	.795436
.730410	.735367	.737397	.740476	.745562
.972176	.677841	.679974	.683233	.688666
1.293971	.614986	.617159	.620498	.626050
1.722267	.548766	.550917	.554137	.559610
2.292342	.481741	.483769	.486810	.491991
3.051106	.416713	.418547	.421327	.426000
4.061021	.355790	.357391	.359827	.363959
5.405230	.299834	.301212	.303291	.306845
7.194348	.249045	.250213	.251962	.254952
9.575701	.203677	.204633	.206081	.208559
12.745267	.164051	.164837	.166018	.168025
16.963960	.130320	.130944	.131882	.133499
22.578970	.102309	.102799	.103545	.104813
30.052592	.079535	.079916	.080503	.081495
40.000000	.061367	.061667	.062114	.062879

TABLE I CONTINUED

TRANSMISSION PROBABILITIES FOR CYLINDRICAL DUCTS AT VARIOUS SPEED RATIOS

(CALCULATED FROM UNIVERSITY OF TORONTO REPORT NO. 88,
DECEMBER 1962, BY J. H. DELEEUW AND D. E. ROTHE)

L/A	K(S,L/A)			
	S=0.2	S=0.3	S=0.5	S=0.75
.010021	.995636	.995930	.996414	.997009
.013338	.994209	.994534	.995239	.995994
.017752	.992247	.992786	.993668	.994687
.023628	.989720	.990392	.991619	.992945
.031449	.986405	.987248	.988868	.990612
.041859	.981939	.983122	.985209	.987524
.055714	.976090	.977638	.980400	.983435
.074155	.968454	.970379	.974047	.978019
.098701	.958367	.960922	.965667	.970845
.131371	.945314	.948589	.954683	.961429
.174854	.928441	.932566	.940359	.949078
.232731	.906855	.912034	.921860	.932991
.309765	.879561	.886005	.898220	.912209
.412298	.845703	.853442	.868328	.885554
.548769	.804512	.813570	.831181	.851933
.730410	.755763	.765950	.786133	.810390
.972176	.699656	.710758	.732956	.760292
1.293971	.637380	.648912	.672337	.701856
1.722267	.570796	.582290	.605929	.636333
2.292342	.502600	.513549	.536385	.566302
3.051106	.435683	.445744	.466882	.494972
4.061021	.372523	.381433	.400314	.425685
5.405230	.314215	.321907	.338272	.360475
7.194348	.261150	.267640	.281523	.300456
9.575701	.213679	.219073	.230612	.246424
12.745267	.172202	.176594	.186016	.198984
16.963960	.136841	.140364	.147934	.158386
22.578970	.107458	.110245	.116238	.124537
30.052592	.083566	.085744	.090446	.096955
40.000000	.064482	.066169	.069811	.074866

TABLE I CONTINUED

TRANSMISSION PROBABILITIES FOR CYLINDRICAL DUCTS AT VARIOUS SPEED RATIOS

(CALCULATED FROM UNIVERSITY OF TORONTO REPORT NO. 88, DECEMBER 1962, BY J. H. DELEEUW AND D. E. ROTHE)

L/A	K(S,L/A)			
	S=1.0	S=1.5	S=2.0	S=3.0
.010021	.997460	.998153	.998587	.999066
.013338	.996636	.997533	.998118	.998727
.017752	.995520	.996726	.997500	.998314
.023628	.994043	.995648	.996680	.997738
.031449	.992060	.994198	.995579	.997046
.041859	.989455	.992289	.994109	.996107
.055714	.985974	.989744	.992171	.994793
.074155	.981379	.986356	.989582	.993026
.098701	.975260	.981871	.986122	.990741
.131371	.967202	.975905	.981549	.987666
.174854	.956595	.967981	.974281	.983536
.232731	.942654	.957496	.967353	.978117
.309765	.924480	.943676	.956601	.970900
.412298	.900963	.925521	.977487	.961324
.548769	.870840	.901780	.923601	.948489
.730410	.832991	.871117	.898954	.931629
.972176	.786407	.832080	.866865	.909231
1.293971	.730797	.783488	.825723	.879799
1.722267	.666936	.724985	.774219	.841413
2.292342	.597051	.657693	.712129	.792279
3.051106	.524404	.584387	.641035	.731374
4.061021	.452652	.509051	.564575	.659777
5.405230	.384325	.435216	.486945	.581021
7.194348	.320979	.365417	.411695	.499720
9.575701	.263678	.301455	.341523	.420331
12.745267	.213191	.244575	.278317	.346376
16.963960	.169876	.195418	.223168	.280303
22.578970	.133684	.154127	.176499	.223239
30.052592	.104149	.124756	.138040	.175498
40.000000	.080462	.093050	.106967	.136563

TABLE I CONTINUED

TRANSMISSION PROBABILITIES FOR CYLINDRICAL DUCTS AT VARIOUS SPEED RATIOS

(CALCULATED FROM UNIVERSITY OF TORONTO REPORT NO. 88,
DECEMBER 1962, BY J. H. DELEEUW AND D. E. ROTHE)

L/A	K(S,L/A)			
	S=4.0	S=5.0	S=10	S=20
.010021	.999324	.999422	.999728	.999854
.013338	.999078	.999232	.999621	.999817
.017752	.998780	.999003	.999504	.999755
.023628	.998319	.998668	.999334	.999668
.031449	.997809	.998212	.999126	.999556
.041859	.997074	.997655	.998812	.999407
.055714	.996051	.996861	.998422	.999218
.074155	.994749	.995837	.997901	.998948
.098701	.993071	.994415	.997208	.998606
.131371	.990710	.992590	.996293	.998146
.174854	.987687	.990126	.995078	.997530
.232731	.983614	.986862	.993430	.996716
.309765	.978170	.982532	.991253	.995628
.412298	.970961	.976744	.988376	.994182
.548769	.961334	.969042	.984521	.992262
.730410	.948588	.958846	.979401	.989701
.972176	.931699	.945256	.972597	.986288
1.293971	.909277	.927241	.963531	.981747
1.722267	.879753	.903438	.951487	.978051
2.292342	.841134	.872023	.935498	.967719
3.051106	.791366	.830957	.914288	.956995
4.061021	.729290	.778163	.886246	.942842
5.405230	.655847	.712652	.849368	.923969
7.194348	.574787	.635915	.801390	.899044
9.575701	.491180	.552293	.740265	.866123
12.745267	.410023	.467380	.665846	.822983
16.963960	.335220	.386354	.581363	.767235
22.578970	.269265	.313020	.493029	.697465
30.052592	.213006	.249408	.407378	.615228
40.000000	.166509	.195940	.329503	.526056

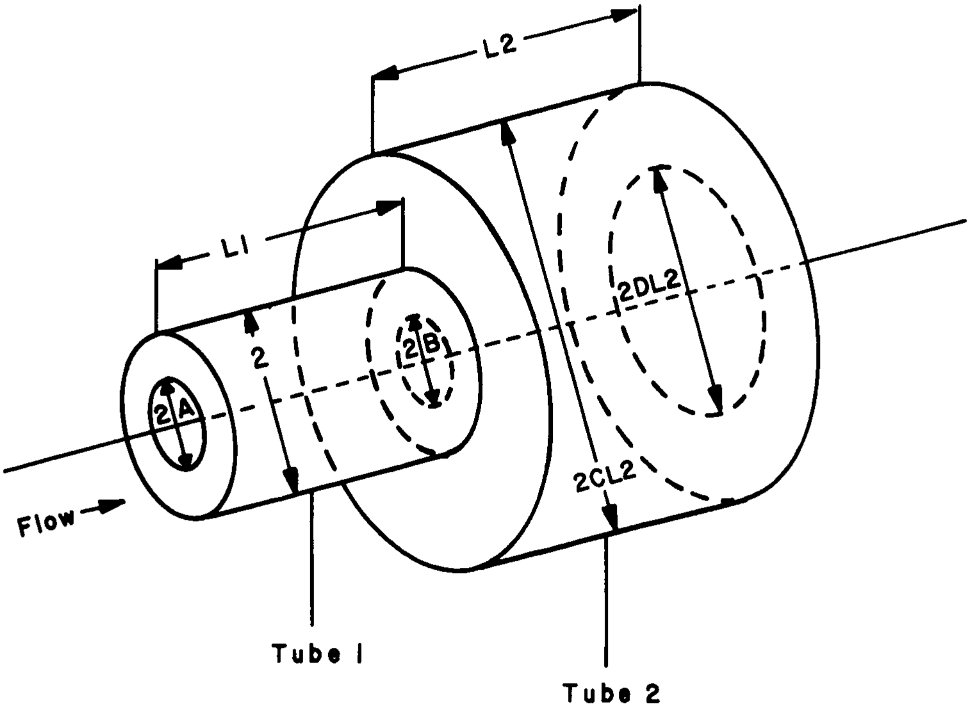


FIG. 1. MODEL FOR MONTE CARLO PROGRAM FOR TRANSMISSION PROBABILITY CALCULATION

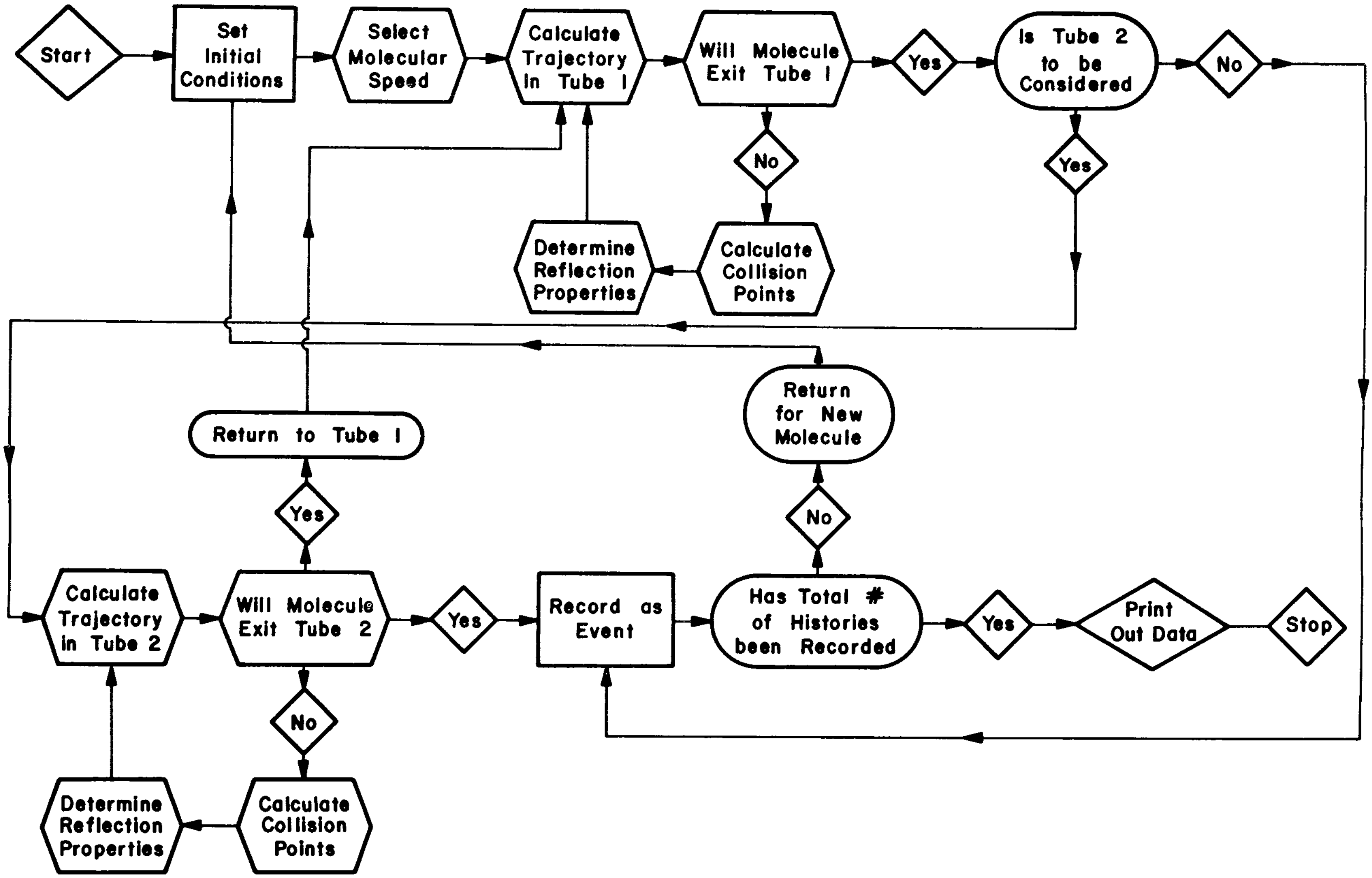


FIG. 2. COMPUTER FLOW DIAGRAM
FOR MONTE CARLO CALCULATIONS OF TRANSMISSION PROBABILITIES

```
C     TRANSMISSION PROBABILITIES FOR CYLINDRICAL DUCTS
      COMMON ISTART(2)
      DIMENSION COUNT(3,250)
    1 FORMAT(F9.4,5XF9.4,5XF9.4,5XF5.0)
    2 FORMAT(F9.4,5XF9.4,5XF9.4,5XF9.4)
3     FORMAT(1X9HCOLLISION 5X9HEXIT AT L7X9HEXIT AT 07X10HEXIT AT L2)
    4  FORMAT (2XI3, 12XF5.0, 12XF5.0, 12XF5.0)
    5 FORMAT(11X3HX=L2XF5.0,8X3HX=02XF5.0,8X4HX=L22XF5.0,7X3HOUT2XF5.0)
    6 FORMAT (F14.7)
    7 FORMAT (2H27,2X1HT2XF14.7,5X1HV2XF14.7,5X1HW2XF14.7)
    8 FORMAT (2H70,2X1HT2XF14.7,5X1HV2XF14.7,5X1HW2XF14.7)
    9 FORMAT (1H )
   37 FORMAT (5HX(IC), 2XF14.7, 5X8HCOLL NUM, 2XI3)
  151 FORMAT (F5.3)
  152 FORMAT(3HL1=F9.4,5X3HL2=F9.4,5X2HS=F9.4,5X5HTOTAL2XF5.0)
  153 FORMAT(1X2HA=F9.4,6X2HB=F9.4,5X2HC=F9.4,5X2HD=F9.4)
260   FORMAT(1X44HMONTE CARLO SOLUTIONS OF FREE MOLECULAR FLOW)
261   FORMAT (1X25HTHROUGH CYLINDRICAL DUCTS)
267    FORMAT(1X16HAND SPEED RATIOS)
268   FORMAT(1X15HNO SPEED RATIOS)
  270 FORMAT (1H1)
  271 FORMAT(I4)
272   FORMAT (1X25HSPEED RATIO = K(1.154700))
  604 FORMAT (6HX(IC1), F14.7,6HX(IC2),F14.7,8HCOLL NUM,I3)
  605 FORMAT(1HYF14.7,1HZF14.7,1HXF14.7,2HD1F14.7,2HD2F14.7)
  606 FORMAT(2HU1F14.7,2HU2F14.7,2HU3F14.7,2HAAF14.7,2HABF14.7,F14.7)
  611 FORMAT (5HX(IC), 2XF14.7, 7HCOLL NO, 2XI3)
650   FORMAT (O8, 5XO8)
 8700 FORMAT (F9.4)
8701   FORMAT(1X26HANGLE OF ATTACK IN DEGREES, F9.4)
      PI = 3.1415926536
 150  XL2T = 0.
7800  FORMAT (5X6HISTART, 5XO8, 5XO8)
      XOT = 0.
      XLT = 0.
      OUT = 0.
      ATST = 0.
      READ 1, Q, QL2, SI, TMAX
      READ 2, A, B, CL2, DL2
      READ 650, ISTART(1), ISTART(2)
      READ 271, NCOLL
      READ 8700 , ANGLE
      PRINT 270
      PRINT 260
      PRINT 261
      IF (SI) 265,266,265
```

FIGURE 3

FORTRAN PROGRAM FOR MONTE CARLO CALCULATIONS OF
FREE MOLECULAR FLOW THROUGH CYLINDRICAL DUCTS

```
 265 PRINT 267
     PRINT 272
     GO TO 269
 266 PRINT 268
 269 PRINT 9
     PRINT 9
     PRINT 152, Q, QL2, SI, TMAX
     PRINT 153, A, B, CL2, DL2
     PRINT 8701, ANGLF
     PRINT 9
     DO 500 J = 1,3
     DO 500 K = 1,NCOLL
 500 COUNT(J,K) = 0.
     AATAC = ANGLF*.0174532925
2010 Y1    = (1. - 2.*FLAT(ISTART))*A
     Z1    = (1. - 2.*FLAT(ISTART))*A
     D = Y1*Y1 + Z1*Z1
     ICOLL = 1
     IF (D - A*A ) 2011,2010,2010
2011 R1    = FLAT(ISTART)
     R2    = FLAT(ISTART)
     IF(R1 - R2) 2013,2011,2012
2012 CTHET = R1
     GO.TO 2142
2013 CTHET = R2
2142 STHET     = SQRTF(1. - CTHET**2)
     SP = 1.
     IF (SI) 2555,2556,2555
2555 XT = 0.
     YT = 0.
     ZT = 0.
     DO2557 J = 1,12
     XA = FLAT(ISTART)
     YA =.FLAT(ISTART)
2558 YT = YT + YA
     ZA = FLAT(ISTART)
2559 ZT = ZT + ZA
2557 XT = XT + XA
     VX = XT - 6.
     VY = YT - 6.
     VZ = ZT - 6.
     SP = SQRT(VX*VX + VY*VY + VZ*VZ)
     IF (FLAT(ISTART) - .5) 2141,2011,2556
2141 CTHET = -CTHET
2556 PSI = FLAT(ISTART)*2.*PI
     AA = CTHET* SP    + SI*1.154700*COSF(AATAC)
     AB =     STHET*COSF(PSI)*SP + SI*1.154700*SINF(AATAC)
     AC = STHET     * SINF(PSI)*SP
     DENO = SQRT(AA*AA + AB*AB + AC*AC)
     U1 = AA/DENO
     IF (U1    - 0.) 2011,2011,2998
2998 U2    = AB/DENO
```

FIGURE 3 CONTINUED

```
      U3    = AC/DENO
      ATST = ATST + 1.
 2021 DELTA = U2/U1
      DELTB = U3/U1
 2020 X1 = 0.
      XIXL = Q
      XIXO = 0.
      YIXL = DELTA*Q + Y1
      ZIXL = DELTB *Q + Z1
2024  IF (YIXL*YIXL + ZIXL*ZIXL - 1.) 4100,4100,3026
 3026 T = DELTA*Y1 + DELTB*Z1
      V = DELTA*DELTA + DELTB*DELTB
      W = Y1*Y1 + Z1*Z1 - 1.
 3029 P = SQRTF(T*T  - V*W)
      XIC    = (-T+P)/V + X1
 3034 IF (XIC   - Q)  3035,3035,3036
 3036 PRINT 37, XIC  , ICOLL
      GO TO 7115
 3035 YIC = DELTA*(XIC - X1) + Y1
      ZIC = DELTB*(XIC - X1) + Z1
 3039 STAU = ZIC
      CTAU = YIC
 3025 ICOLL = ICOLL + 1
      IF (ICOLL - NCOLL) 3341,7115,7115
3341  IF (XIC  )   3036,3036,3040
 3040 R1    = FLAT(ISTART)
      R2    = FLAT(ISTART)
      IF (R1 - R2)  3042,3040,3041
 3041 CTHET = R1
      GO TO 3043
 3042 CTHET = R2
 3043 STHET    = SQRTF(1. - CTHET**2)
 3044 PSI = FLAT(ISTART)*2.*PI
      U1 = SINF(PSI)*STHET
      U2 = -CTHET*CTAU - STHET*COSF(PSI)*STAU
      U3 = -CTHET*STAU + STHET*COSF(PSI)*CTAU
      DELTA = U2/U1
      DELTB = U3/U1
      IF (U1  )  3060,3045,3045
 3045 YIXL = DELTA*(Q - XIC) + YIC
      ZIXL = DELTB*(Q - XIC) + ZIC
      X1 = XIC
      Y1 = YIC
      Z1 = ZIC
3052  IF (YIXL*YIXL + ZIXL*ZIXL - 1.) 4100,4100,3068
 3060  YIXO = DELTA*(-XIC) + YIC
      ZIXO = DELTB*(-XIC) + ZIC
      XIXO = 0.
       X1 = XIC
      Y1 = YIC
      Z1 = ZIC
3063  IF(YIXO*YIXO + ZIXO*ZIXO - 1.) 5110,5110,3068
```

FIGURE 3 CONTINUED

```
3068 XIXO = 0.
3069 T = DELTA*Y1 + DELTB*Z1
     V = DELTA*DELTA + DELTB*DELTB
     IF (U1) 3691,3039,3692
3691 XIC = -ABSF((2.*T)/V) + X1
     GO TO 3034
3692 XIC =  ABSF((2.*T)/V) + X1
     GO TO 3034
4100 IF (YIXL*YIXL + ZIXL*ZIXL - B*B) 4101,4799,4799
4101 COUNT(1,ICOLL) = COUNT(1,ICOLL) + 1.
     XLT = XLT + 1.
     IF (QL2) 7120,7120,6170
4799 ICOLL = ICOLL + 1
     IF ( ICOLL - NCOLL)  4800,7115,7115
4800 R1 = FLAT(ISTART)
     R2 = FLAT(ISTART)
     IF(R1 - R2)   4801,4800,4802
4801 CTHET = - R2
     GO TO 4803
4802 CTHET = - R1
4803 STHET = SQRTF(1. - CTHET**2)
     PSI = FLAT(ISTART)*2.*PI
     U1 = CTHET
     U2 = STHET*COSF(PSI)
     U3 = STHET*SINF(PSI)
     DELTA = U2/U1
     DELTB = U3/U1
4804 YIXO = DELTA*(-XIXL) + YIXL
     ZIXO = DELTB*(-XIXL) + ZIXL
     X1 = XIXL
     Y1 = YIXL
     Z1 = ZIXL
     IF (YIXO*YIXO + ZIXO*ZIXO - 1.) 5110,5110,4805
4805 T = DELTA*YIXL + DELTB*ZIXL
     V = DELTA*DELTA + DELTB*DELTB
     W = YIXL*YIXL + ZIXL*ZIXL  - 1.
     XIC = (-T-SQRTF(T*T  - V*W))/V + XIXL
     GO TO 3035
5110 IF (YIXO*YIXO + ZIXO*ZIXO - A*A) 5111,5810,5810
5111 COUNT (2,ICOLL) = COUNT(2,ICOLL) + 1.
     XOT = XOT + 1.
     GO TO 7120
5810 ICOLL = ICOLL + 1
     IF (ICOLL - NCOLL)    5811,7115,7115
5811 R1     = FLAT(ISTART)
     R2     = FLAT(ISTART)
     IF (R1 - R2)   5812,5811,5813
5812 CTHET = R2
     GO TO 5814
5813 CTHET = R1
5814 STHET  = SQRTF(1. - CTHET**2)
     PSI = FLAT(ISTART)*2.*PI
```

FIGURE 3 CONTINUED

```
      U1 = CTHET
      U2 = STHET*COSF(PSI)
      U3 = STHET*SINF(PSI)
      Y1 = YIXO
      Z1 = ZIXO
      GO TO 2021
 6170 XL2 = XIXL + OL2
      YXL2 = DELTA*(XL2 - XIXL) + YIXL
      ZXL2 = DELTB*(XL2 - XIXL) + ZIXL
      IF (YXL2*YXL2 + ZXL2*ZXL2 - CL2*CL2) 6850,6171,6171
 6171 ICOLL = ICOLL + 1
      IF (ICOLL - NCOLL)  6172,7115,7115
 6172 T = DELTA*YIXL + DELTB*ZIXL
      V = DELTA*DELTA + DELTB*DELTB
      W = YIXL*YIXL + ZIXL*ZIXL - CL2*CL2
      R = SQRTF(T*T  -V*W)
      XIC2 = ((-T+R)/V) + XIXL
      X1 = XIXL
      Y1 = YIXL
      Z1 = ZIXL
 6720 YIC2 = DELTA*(XIC2 - X1) + Y1
      ZIC2 = DELTB*(XIC2 - X1 )+ Z1
      STAU = ZIC2/CL2
      CTAU = YIC2/CL2
 6173 R1 = FLAT(ISTART)
      R2 = FLAT(ISTART)
      IF (R1 - R2)    6175,6173,6174
 6174 CTHET = R1
      GO TO 6176
 6175 CTHET = R2
 6176 STHET  = SQRTF(1. - CTHET**2)
      PSI = FLAT(ISTART)*2.*PI
      U1 = SINF(PSI)*STHET
      U2 = -CTHET*CTAU - STHET*COSF(PSI)*STAU
      U2 = -CTHET*STAU + STHET*COSF(PSI)*CTAU
      DELTA = U2/U1
      DELTB = U3/U1
      IF (U1) 6177,6173,6178
 6177 YXON = DELTA*(XIXL - XIC2) + YIC2
      ZXON = DELTB*(XIXL - XIC2) + ZIC2
      IF (YXON*YXON + ZXON*ZXON - CL2*CL2) 6180,6180,6811
 6178 YXL2 = DELTA*(XL2 - XIC2) + YIC2
      ZXL2 =    DELTB *(XL2 - XIC2) + ZIC2
      IF(YXL2*YXL2 + ZXL2*ZXL2 - CL2*CL2) 6850,6811,6811
6180  IF (YXON*YXON + ZXON*ZXON - R*R) 6181,6181,6182
 6181 YIXL    = YXON
      ZIXL = ZXON
      GO TO 4804
 6182 ICOLL = ICOLL + 1
      IF (ICOLL - NCOLL) 6183,7115,7115
 6183 R1    = FLAT(ISTART)
      R2    = FLAT(ISTART)
```

FIGURE 3 CONTINUED

```
      IF (R1 - R2)  6184,6183,6185
 6185 CTHET = R1
      GO TO 6186
 6184 CTHET = R2
 6186 STHET = SQRTF(1. - CTHET**2)
       PSI = FLAT(ISTART)*2.*PI
      U1 = CTHET
      U2 = STHET*COSF(PSI)
      U3 = STHET*SINF(PSI)
      DELTA = U2/U1
      DELTB = U3/U1
      YIXL = YYON
      ZIXL = ZXON
      GO TO 6170
 6811 X1 = XIC2
      Y1 = YIC2
      Z1 = ZIC2
      ICOLL = ICOLL + 1
      IF (ICOLL - NCOLL)  6810,7115,7115
 6810 T = DELTA*Y1 + DELTB*Z1
      V = DELTA*DELTA + DELTB*DELTB
      IF ( U1)  6821,7115,6822
 6821 XIC2 = -ABSF((2.*T)/V) + X1
      GO TO 6720
 6822 XIC2 = ABSF((2.*T)/V) + X1
      GO TO 6720
6850  IF (YXL2*YXL2 + ZXL2*ZXL2 - DL2*DL2) 6860,6851,6851
 6851 ICOLL = ICOLL + 1
      IF (ICOLL - NCOLL)   6852,7115,7115
 6852 R1 = FLAT(ISTART)
      R2 = FLAT(ISTART)
      IF (R1 - R2)  6853,6852,6854
 6853 CTHET = R2
      GO TO 6855
 6854 CTHET = R1
 6855 STHET = SQRTF(1. - CTHET**2)
      PSI = FLAT(ISTART)*2.*PI
      U1 = - CTHET
      U2 = STHET*COSF(PSI)
      U3 = STHET*SINF(PSI)
      DELTA = U2/U1
      DELTB = U3/U1
      YXON = DELTA*(XIXL - XL2) + YXL2
      ZXON = DELTB*(XIXL - XL2) + ZXL2
      IF(YXON*YXON + ZXON*ZXON - CL2*CL2) 6180,6180,6856
 6856 T = DELTA*YXL2 +DELTB*ZXL2
      V = DELTA*DELTA + DELTB*DELTB
      W = YXL2*YXL2 + ZXL2*ZXL2 - CL2*CL2
      XIC2 = (-T-SQRTF(T**2 - V*W))/V + XL2
      Z1 = ZXL2
      Y1 = YXL2
      X1 = XL2
```

FIGURE 3 CONTINUED

```
     GO TO 6720
6860 COUNT (3,ICOLL) = COUNT(3,ICOLL) + 1.
     XL2T = XL2T + 1.
     GO TO 7120
7115 OUT = OUT + 1.
7120 IF (ATST - TMAX) 2010,7211,7211
7211 PRINT 3
     DO7140 J = 1,NCOLL
     K = J - 1
7140 PRINT 4, K, COUNT(1,J), COUNT(2,J), COUNT(3,J)
     PRINT 9
     PRINT 5, XLT, XOT, XL2T, OUT
     PRINT 9
     PRINT 7800, ISTART(1), ISTART(2)
9088 GO TO 150
     END
```

FIGURE 3 CONTINUED

```
MONTE CARLO SOLUTIONS OF FREE MOLECULAR FLOW
THROUGH CYLINDRICAL DUCTS
AND SPEED RATIOS
SPEED RATIO = K(1.154700)


1=    2.0000       L2=          0       S=     .4640      TOTAL    9999
A=    1.0000       B=    1.0000       C=          0      D=          0
ANGLE OF ATTACK IN DEGREES  40.0000

COLLISION       EXIT AT L          EXIT AT O          EXIT AT L2
    0              2203                0                  0
    1              1209             2126                  0
    2               853              975                  0
    3               533              542                  0
    4               319              320                  0
    5               178              180                  0
    6                98              118                  0
    7                70               74                  0
    8                42               45                  0
    9                25               17                  0
   10                15               11                  0
   11                11                6                  0
   12                 7                9                  0
   13                 2                3                  0
   14                 1                3                  0
   15                 1                1                  0
   16                 1                0                  0
   17                 0                0                  0
   18                 0                0                  0
   19                 0                0                  0
   20                 1                0                  0
   21                 0                0                  0
   22                 0                0                  0
   23                 0                0                  0
   24                 0                0                  0
   25                 0                0                  0
   26                 0                0                  0
   27                 0                0                  0
   28                 0                0                  0
   29                 0                0                  0
   30                 0                0                  0
   31                 0                0                  0
   32                 0                0                  0
   33                 0                0                  0
   34                 0                0                  0
   35                 0                0                  0
   36                 0                0                  0
   37                 0                0                  0
   38                 0                0                  0
   39                 0                0                  0
   40                 0                0                  0
   41                 0                0                  0
   42                 0                0                  0
   43                 0                0                  0
   44                 0                0                  0
   45                 0                0                  0
   46                 0                0                  0
   47                 0                0                  0
   48                 0                0                  0
   49                 0                0                  0

          X=L    5569          X=0    4430          X=L2       0         OUT         0
```

FIGURE 4 - TYPICAL COMPUTER RESULT

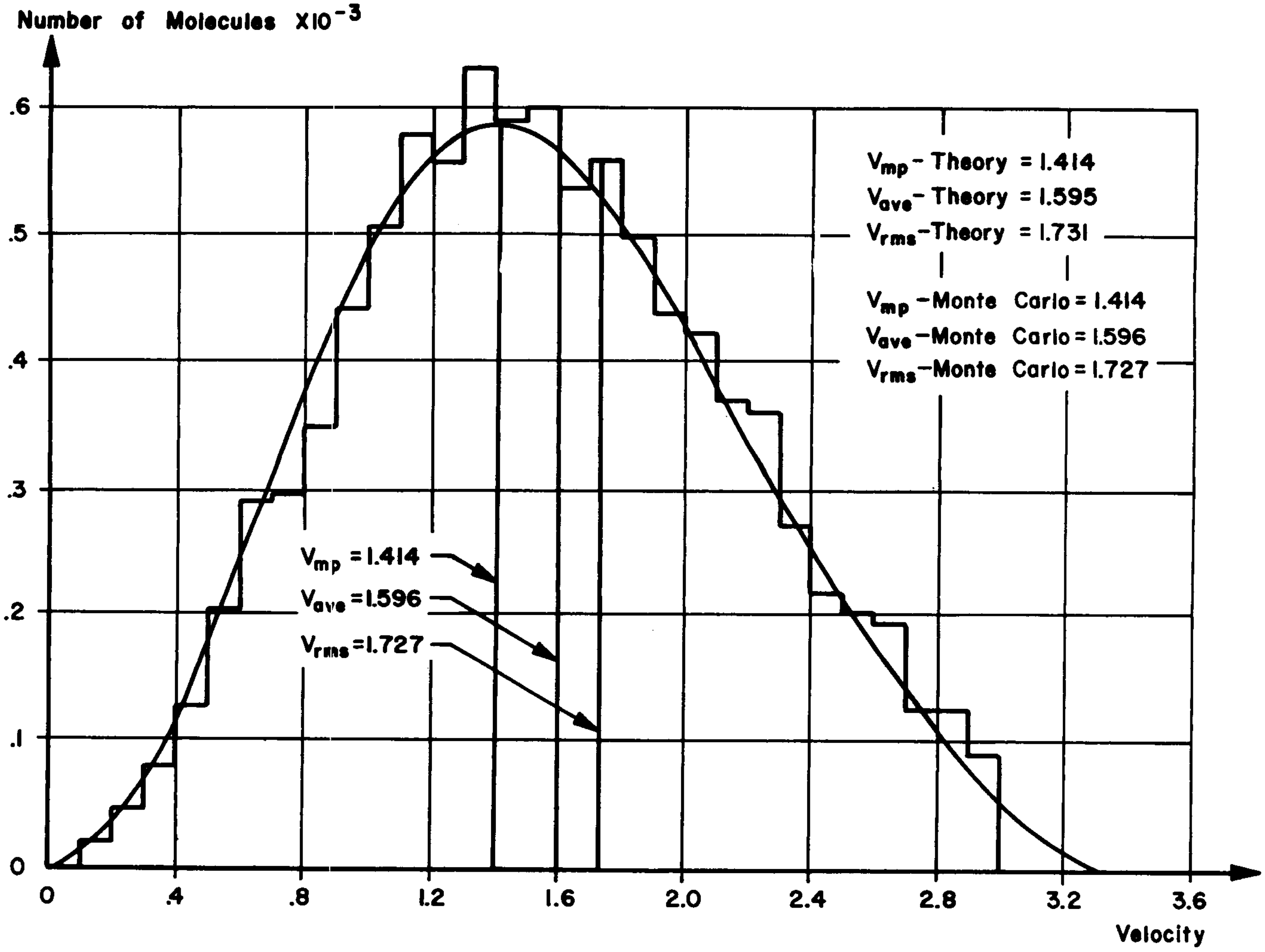


FIG. 5. DISTRIBUTION OF SPEEDS FROM MONTE CARLO CALCULATION

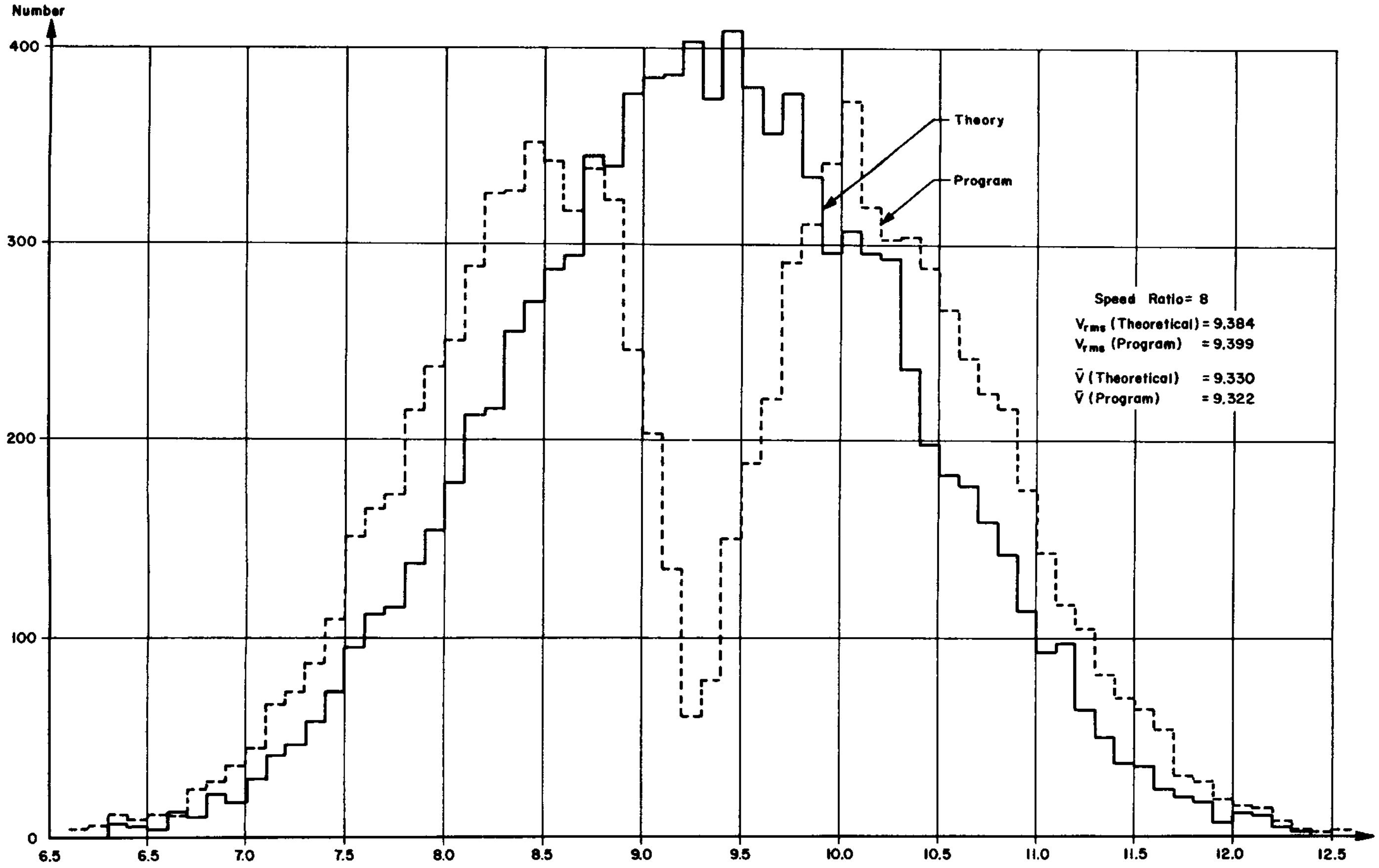
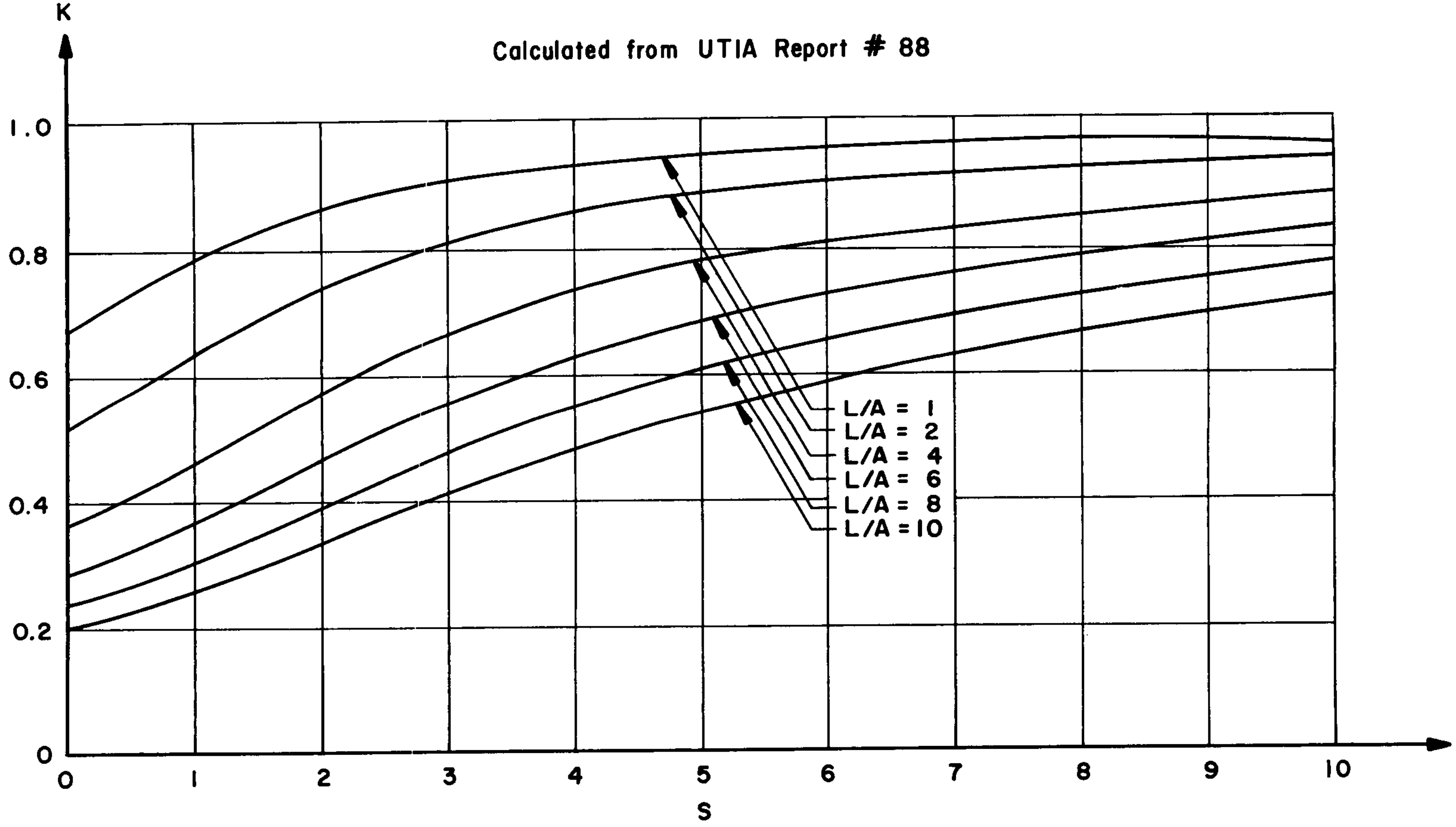
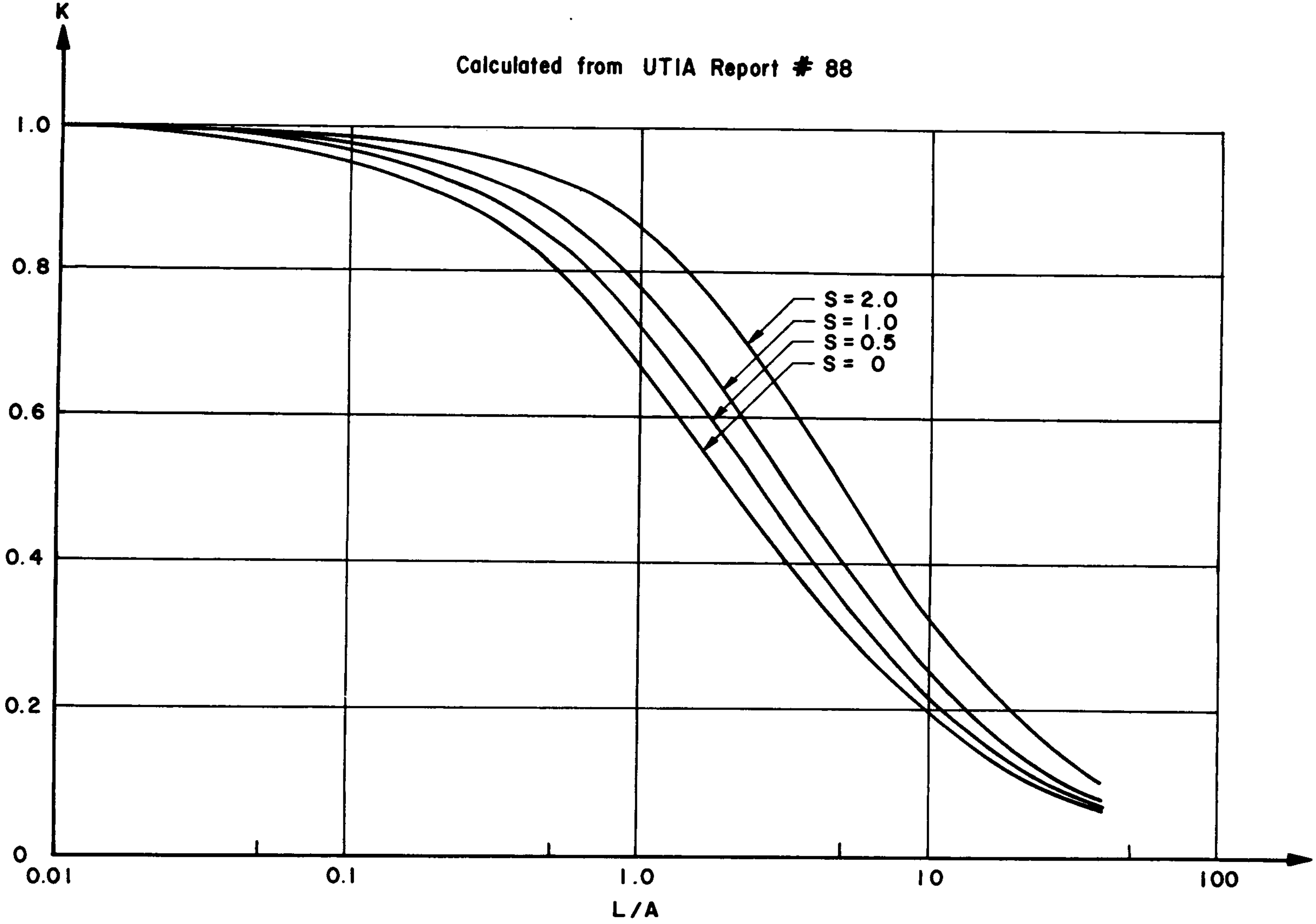


FIG. 6. HISTOGRAM FOR SPEED DISTRIBUTION WITH MASS MOTION



Calculated from UTIA Report # 88

FIGURE 7. TRANSMISSION PROBABILITES FOR CYLINDRICAL DUCTS AS A FUNCTION OF SPEED RATIO



Calculated from UTIA Report # 88

FIG. 8. TRANSMISSION PROBABILITIES FOR CYLINDRICAL DUCTS AS A FUNCTION OF LENGTH TO RADIUS RATIO, L/A, WITH SPEED RATIOS

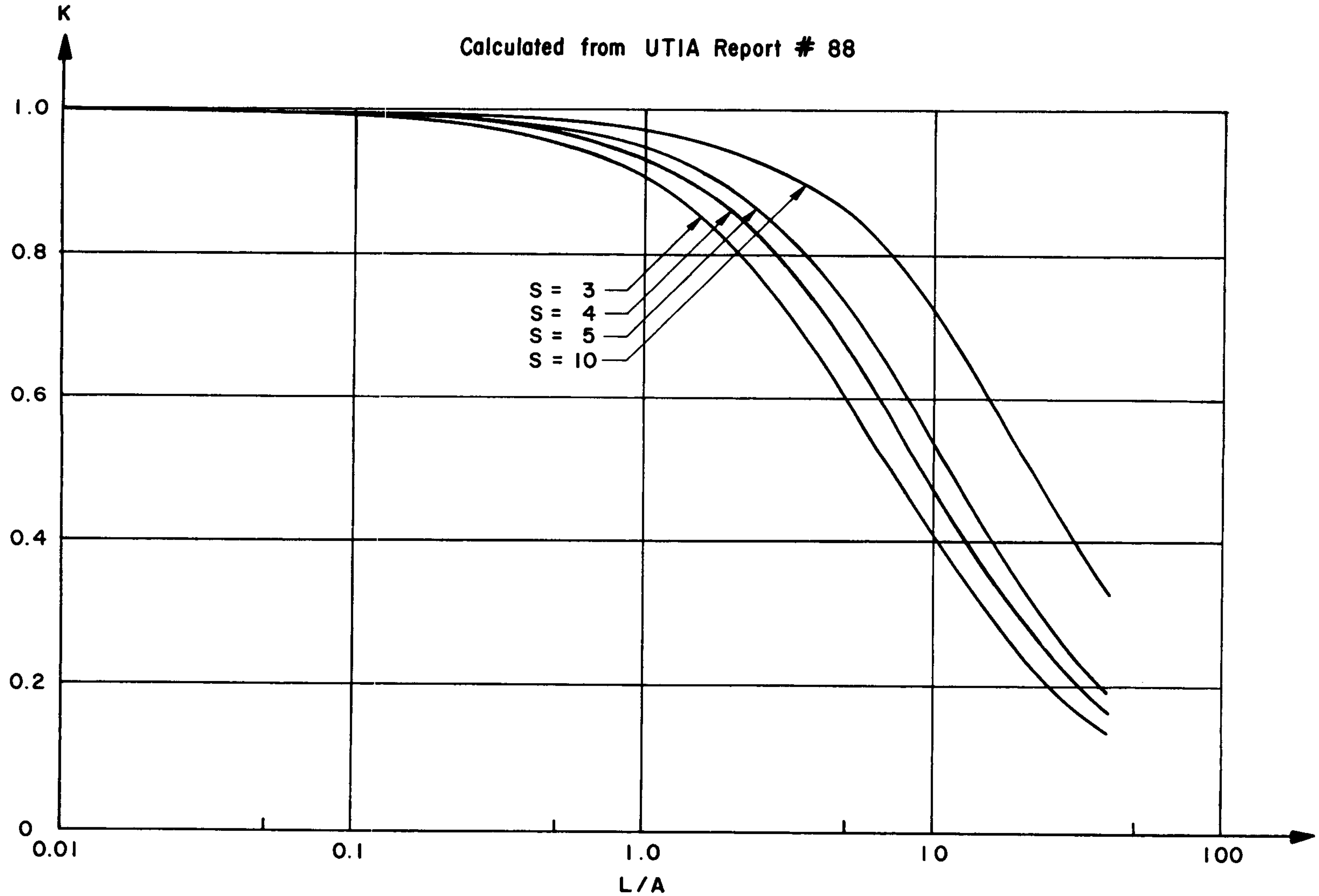

Calculated from UTIA Report # 88

FIG. 9. TRANSMISSION PROBABILITIES FOR CYLINDRICAL DUCTS AS A FUNCTION OF LENGTH TO RADIUS RATIO, L/A, WITH SPEED RATIOS

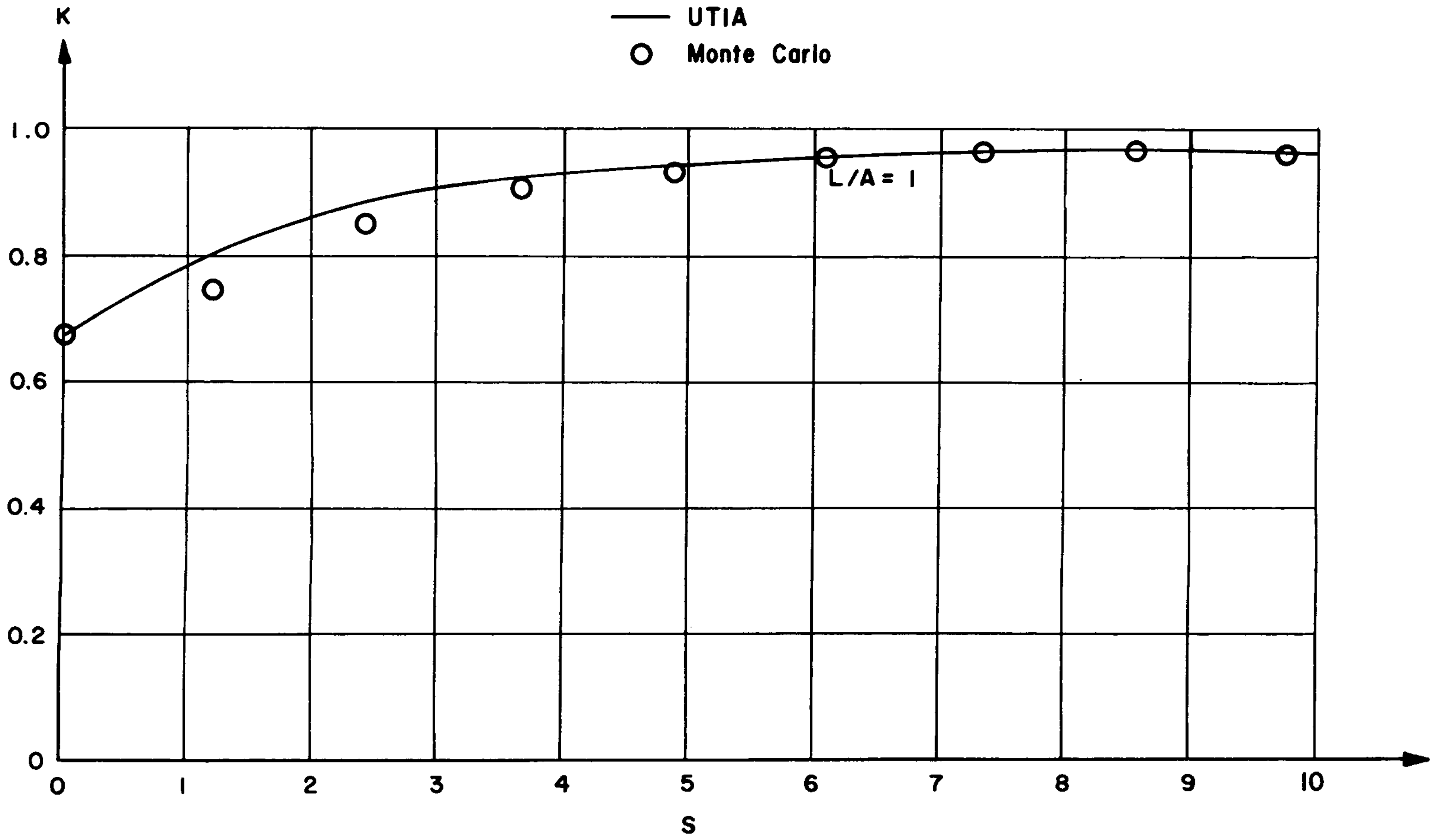


FIGURE 10. TRANSMISSION PROBABILITES FOR CYLINDRICAL DUCTS AS A FUNCTION OF SPEED RATIO

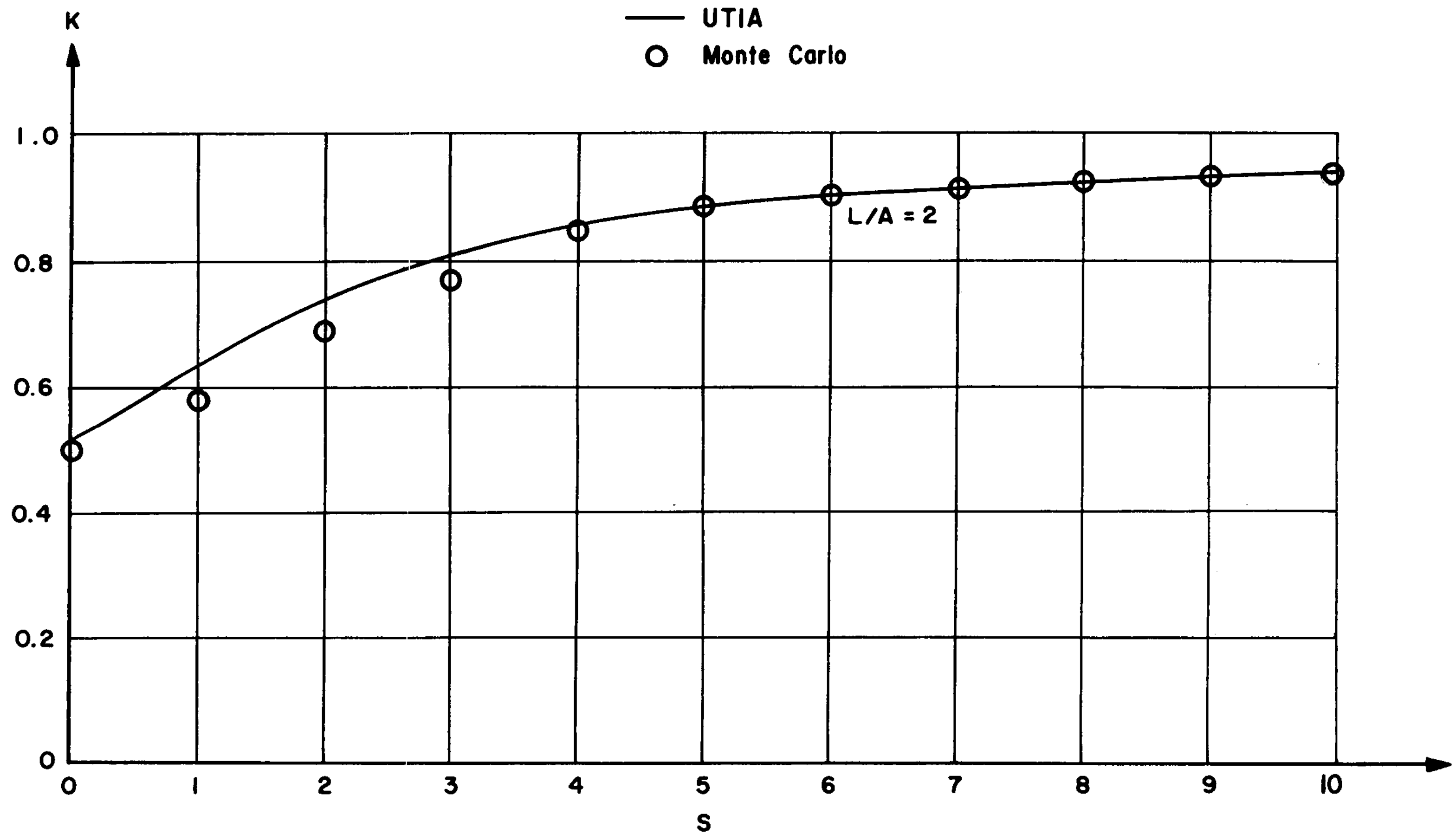


FIGURE II. TRANSMISSION PROBABILITES FOR CYLINDRICAL DUCTS AS A FUNCTION OF SPEED RATIO

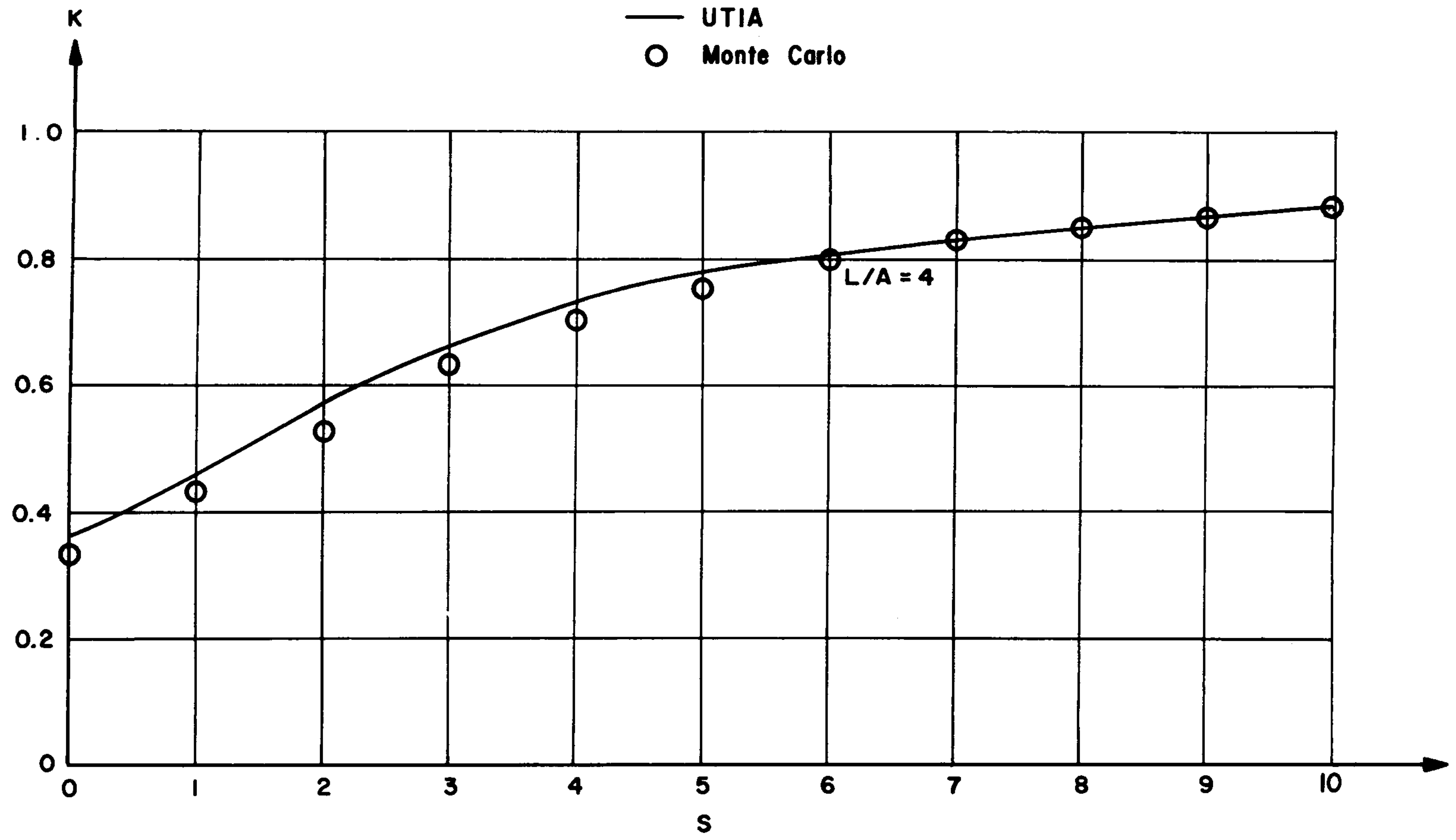


FIGURE 12. TRANSMISSION PROBABILITES FOR CYLINDRICAL DUCTS AS A FUNCTION OF SPEED RATIO

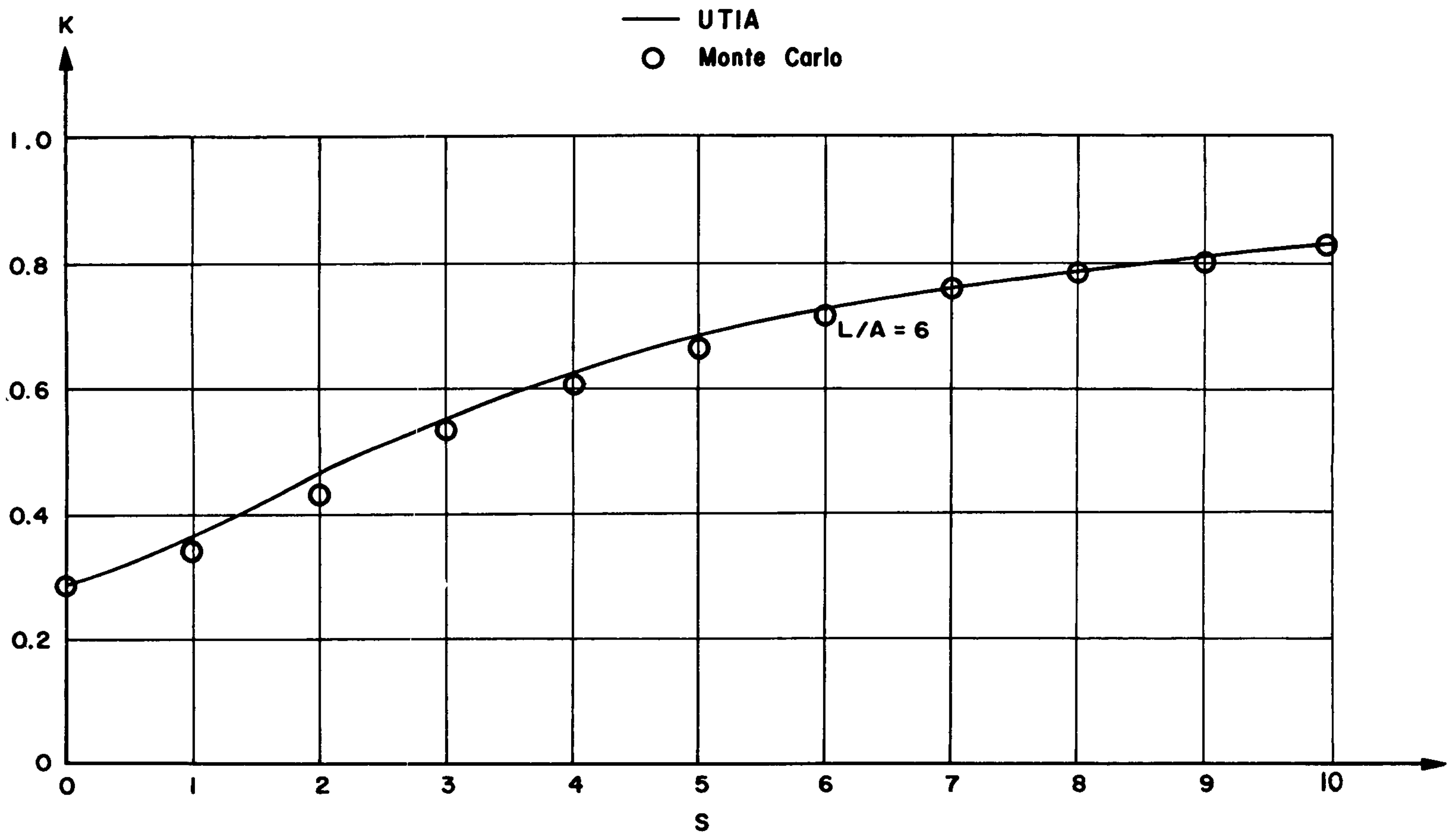


FIGURE 13. TRANSMISSION PROBABILITES FOR CYLINDRICAL DUCTS AS A FUNCTION OF SPEED RATIO

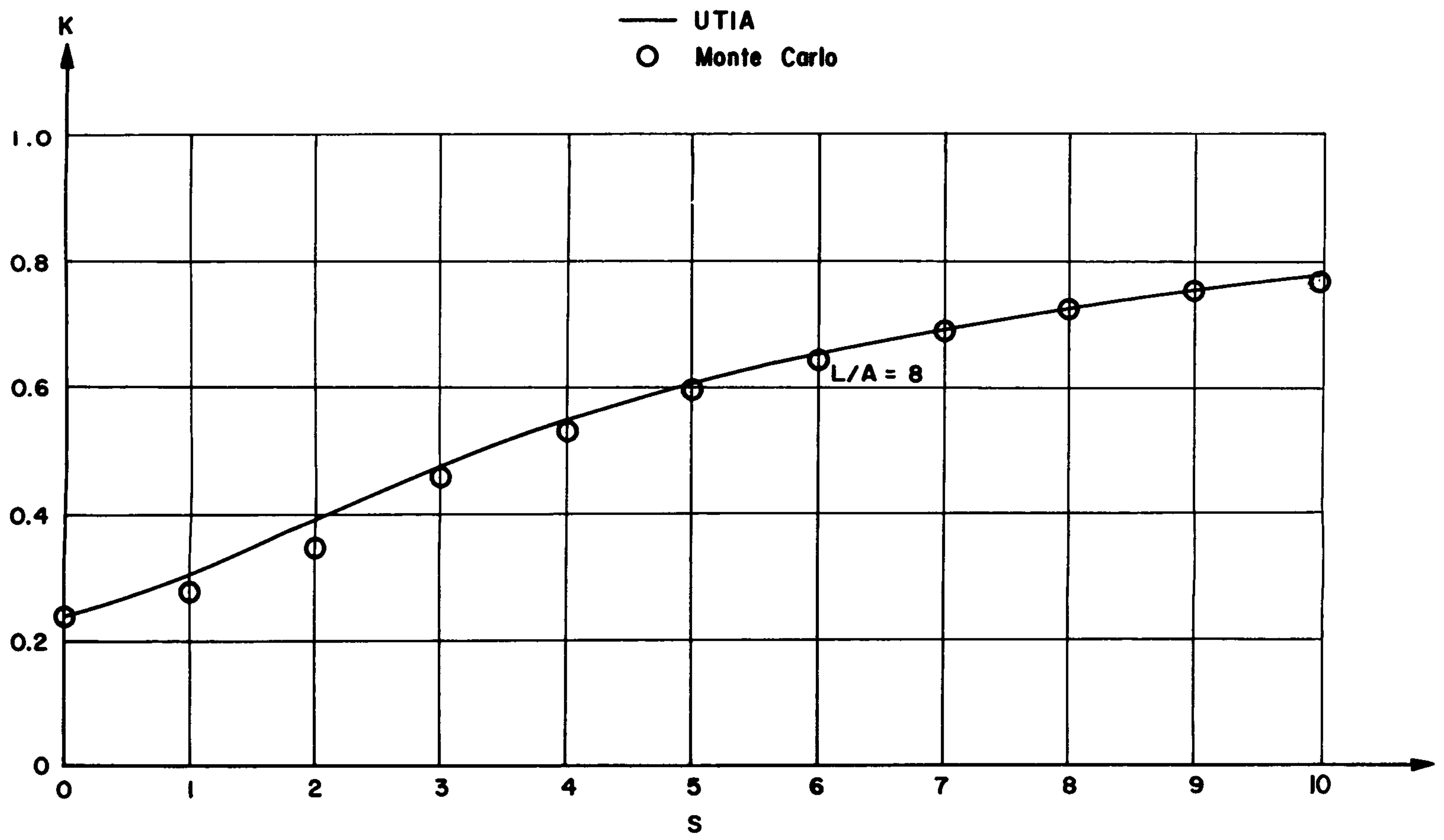


FIGURE 14. TRANSMISSION PROBABILITES FOR CYLINDRICAL DUCTS AS A FUNCTION OF SPEED RATIO

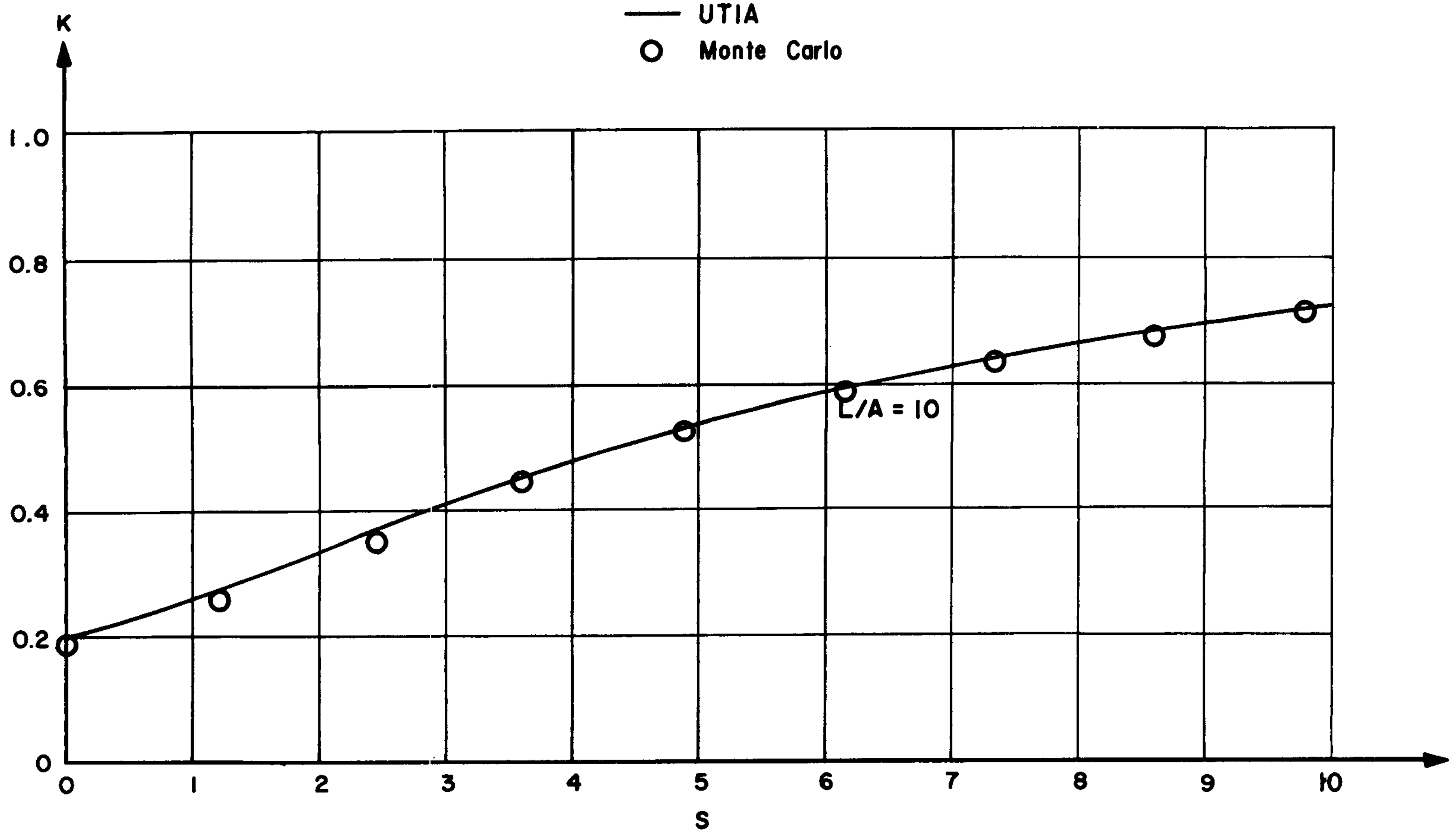


FIGURE 15. TRANSMISSION PROBABILITES FOR CYLINDRICAL DUCTS AS A FUNCTION OF SPEED RATIO

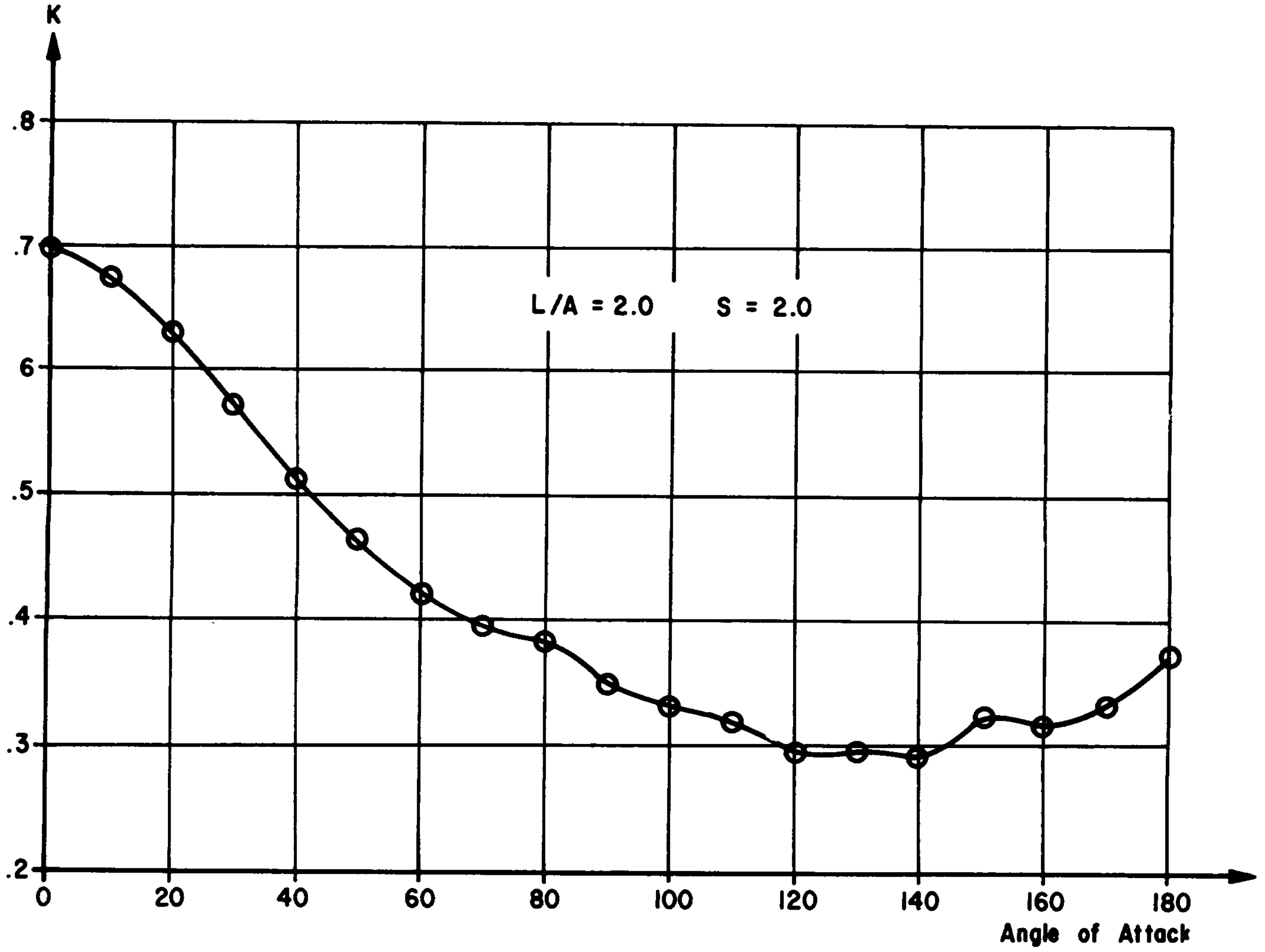


FIG. 16. TRANSMISSION PROBABILITIES FOR CYLINDRICAL DUCT AT A CONSTANT SPEED RATIO FOR VARIOUS ANGLES OF ATTACK

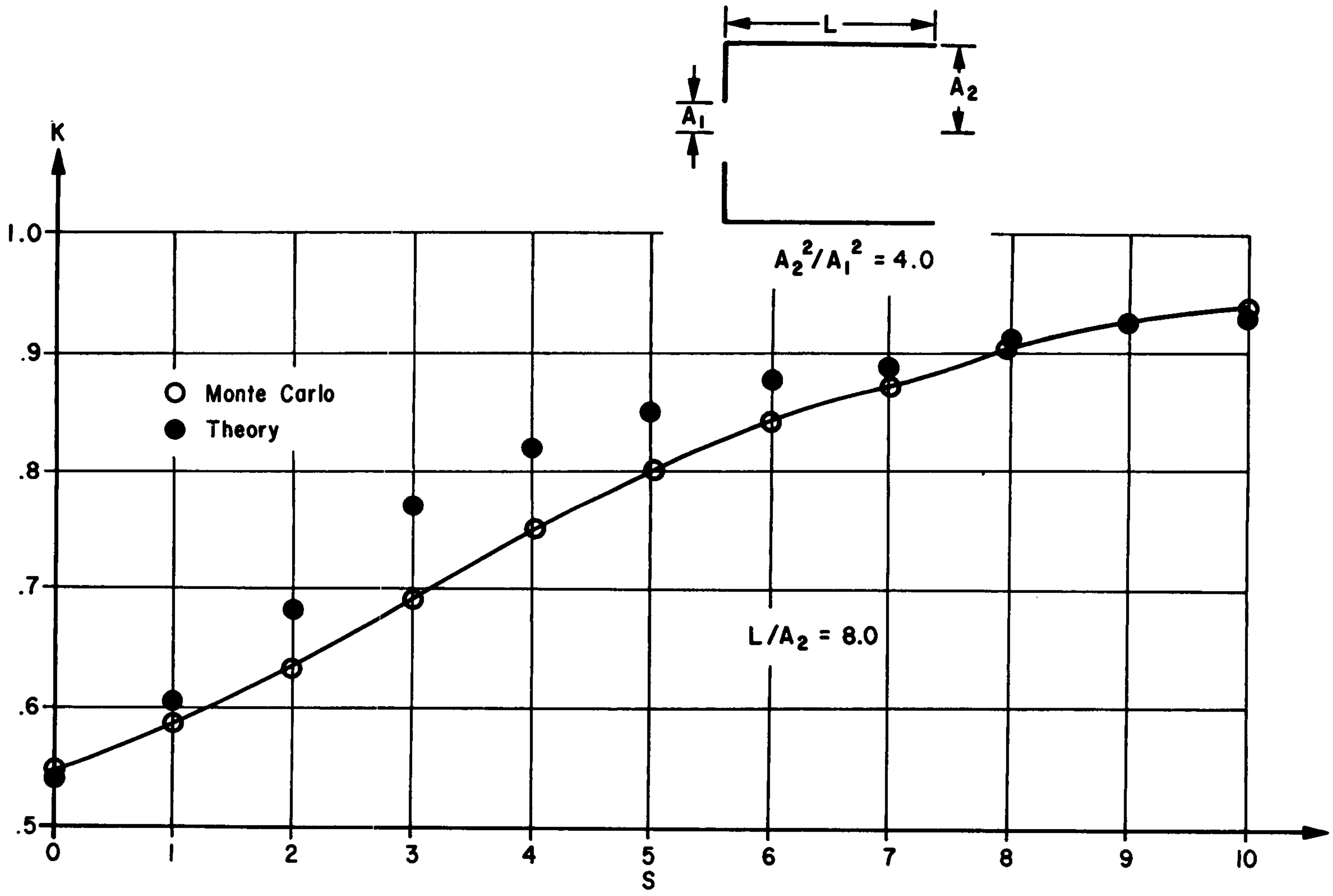


FIG. 17. TRANSMISSION PROBABILITIES FOR ORIFICE RESTRICTED CYLINDRICAL TUBE

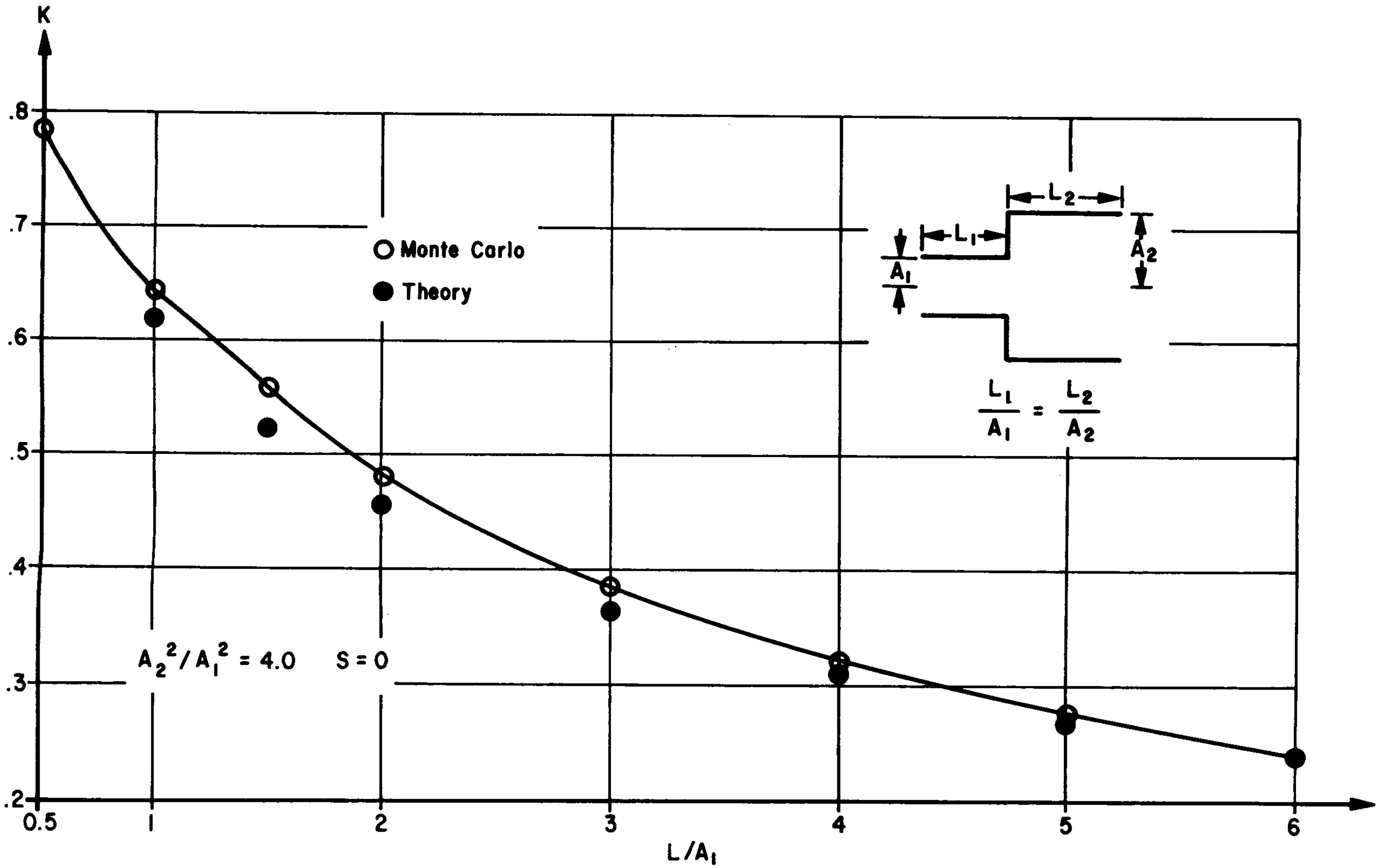


FIG. 18. TRANSMISSION PROBABILITIES FOR TWO CYLINDRICAL DUCTS IN SERIES

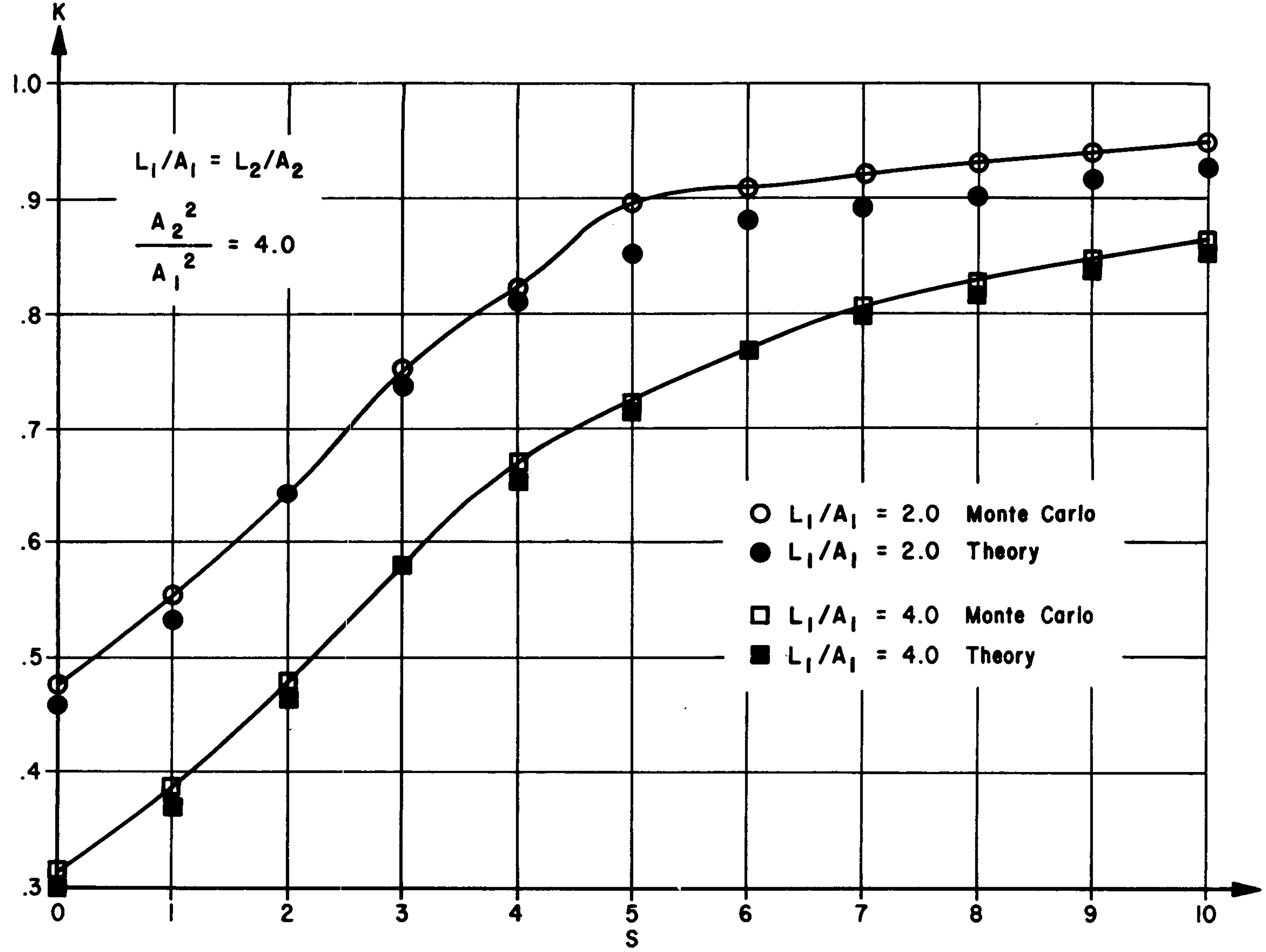


FIG. 19. TRANSMISSION PROBABILITIES FOR TWO DUCTS IN SERIES AT VARIOUS SPEED RATIOS

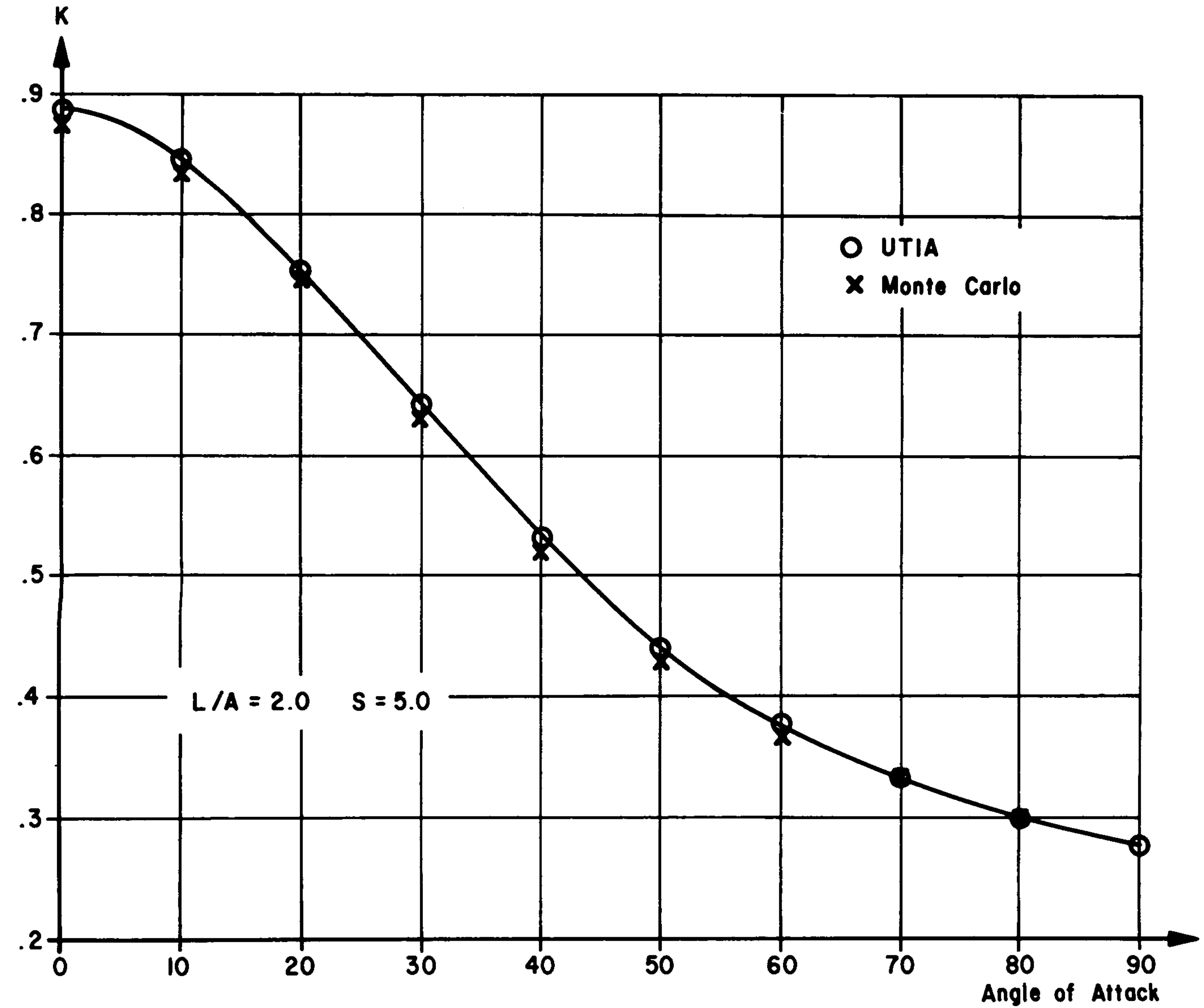


FIG. 20. TRANSMISSION PROBABILITIES FOR CYLINDRICAL DUCT AT A CONSTANT SPEED RATIO FOR VARIOUS ANGLES OF ATTACK

REFERENCES

1. Clausing, P., "Ülber die Strömung sehr verdunnter Gase durch Röhren von beliebiger Länge," Ann Plysik 12, p. 961, 1932.

2. Davis, D. H., "Monte Carlo Calculation of Molecular Flow Rates Through a Cylindrical Elbow and Pipes of Other Shapes," Journal of Applied Physics, Vol. 31, p. 1164, July 1960.

3. Levenson, L. L., N. Milleron, and D. H. Davis, "The Optimization of Molecular Flow Conductance," Transaction of the Seventh National Symposium on Vacuum Technology, Cleveland, Ohio, 1960.

4. Levenson, L. L., N. Milleron, and D. H. Davis, "Molecular Flow Conductance," Le Vide, No. 103, p. 42, January-February 1963.

5. Davis, D. H., L. L. Levenson, and N. Milleron, "Theoretical and Experimental Studies of Molecular Flow Through Short Ducts," Proceedings of the Second International Symposium on Rarefied Gas Dynamics, Berkeley, California, 1960.

6. Ballance, J. O., "Monte Carlo Solutions of Knudsen and Near-Knudsen Flow Through Infinitely Wide, Parallel and Skewed, Flat Plates," NASA TM X-53112, August 1964.

7. Ballance, J. O., "Transmission Probability Determination with Directed Mass Motion and with Mean Free Path Consideration," Third International Vacuum Congress, Stuttgard, Germany, June 1965.

8. Pond, H. L., "The Effects of Entrance Velocity on the Flow of a Rarefied Gas through a Tube," Journal of the Aerospace Sciences, Vol. 29, No. 8, August 1962.

9. deLeeuw, J. H. and D. E. Rothe, "A Numerical Solution for the Free-Molecular Impact-Pressure Probe Relations for Tubes of Arbitrary Length," UTIA Report No. 88, December 1962.

10. Hughes, P. C., "Theory for the Free Molecule Impact Probe at an Arbitrary Angle of Attack," UTIAS Report No. 103, May 1965.

11. Oatley, C. W., "The Flow of Gas Through Composite Systems at Very Low Pressures," British Journal of Applied Physics, Vol. 8, p. 15, January 1957.

12. Santeler, D. J., "Probability and Molecular Flow in Vacuum Systems," Proceedings of the Fifth Annual Symposium on Space Environmental Simulation, May 1964.

APPROVAL TM X-53386

A MONTE CARLO PROGRAM FOR TRANSMISSION PROBABILITY CALCULATIONS INCLUDING MASS MOTIONS

By James O. Ballance

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

W. Dahm

WERNER K. DAHM
Chief, Aerodynamics Division

E. D. GEISSLER
Director, Aero-Astrodynamics Laboratory

DISTRIBUTION

DIR

R-DIR

R-ASTR

Dr. Haeussermann

R-P&VE

Dr. Lucas
Mr. Riehl

R-RP

Dr. Stuhlinger
Mr. Heller
Dr. Shelton

R-AERO

Dr. Geissler
Mr. Vaughan
Mr. Scoggins
Mr. Dahm
Mr. Horn
Mr. Ballance (40)
Mr. Lindberg

MS-T

Mr. Roy Bland (6)

MS-IPL (8)
MS-IP
MS-H
HME-P
CC-P

EXTERNAL

Scientific and Technical Information Facility (25)
Attn: NASA Representative (S-AK/RKT)
P. O. Box 5700
Bethesda, Maryland

EXTERNAL

Langley Research Center
Langley Station
Hampton, Virginia

Lewis Research Center
21000 Brookpark
Cleveland, Ohio
Attn: Dr. Herman Mark
Mr. Lloyd Krause
Mr. Edward Richley
Library

Goddard Space Flight Center
Greenbelt, Maryland
Attn: Mr. George Newton
Library

JPL
4800 Oak Grove Drive
Pasadena, California

Ames Research Center
Moffett Field, California

Manned Spacecraft Center
Houston, Texas
Attn: William K. Roberts
Library

Arthur D. Little, Inc.
Cambridge, Massachusetts
Attn: Raymond Moore

Northrop Space Laboratories
Huntsville, Alabama
Attn: T. M. McCoy

EXTERNAL DISTRIBUTION (Continued)

AEDC
Arnold Air Force Station, Tennessee
Attn: Capt. George Mushalko

ARO, Inc.
AEDC
Arnold Air Force Station, Tennessee
Attn: Mr. George Kirby
Library

University of Illinois
Urbana, Illinois

Massachusetts Institute of Technology
Cambridge, Massachusetts
Attn: Dr. Robert Stickney
Dr. L. Trilling

Celestial Research Corporation
1015 Fremont Avenue
South Pasadena, California
Attn: Dr. Raymond Chuan
Mr. John Wainwright
Mr. Don Wallace

University of Alabama
University, Alabama
Attn: Dr. Walter Schaetzle

Georgia Institute of Technology
Atlanta, Georgia
Attn: Dr. A. B. Huang

Institute of Aerophysics
University of Toronto
Toronto, Canada
Attn: P. C. Hughes
Dr. G. N. Patterson

Cornell Aeronautical Laboratories, Inc.
Buffalo, New York

McDonnell Aircraft Corporation
P. O. Box 516
St. Louis, Missouri
Attn: E. S. J. Wang

University of California
Radiation Laboratory
Livermore, California

Lockheed Aircraft Corporation
Huntsville, Alabama
Attn: Mr. M. Culp
Mr. T. Cunningham
Mr. L. Morrison
Library

Brown Engineering Company
Huntsville, Alabama
Attn: Mr. D. Tarbell
Library

Heat Technology Laboratory
Huntsville, Alabama

University of Michigan
Ann Arbor, Michigan
Attn: Dr. A. G. Hansen
George Carignan
Hasso Niemann
Library