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Walter J. Köhnlein

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Smithsonian Institution
Astrophysical Observatory

Cambridge, Massachusetts 02138

DETERMINATION OF STATION COORDINATES FROM OPTICAL OBSERVATIONS OF ARTIFICIAL SATELLITES[1]

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Walter J. Köhnlein[2]

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Abstract.--Improvements of station coordinates and corrections to the nonzonal harmonics of the geopotential were computed from a combined dynamical and geometrical method using standard least-squares procedures.

Author

Optical tracking of artificial satellites against the star background leads to absolute space directions. In the geometrical method these directions are used, together with distance measurements, for relative position determinations of tracking stations. Absolute station positions are obtained by the dynamical method. The origin of the coordinate system coincides herein with the gravity center of the Earth – coefficients of first degree in the geopotential function are put to zero – while the scaling is introduced by the geocentric gravitational constant.

Extensive descriptions of the dynamical and geometrical method can be found in Izsak (1964) and Veis (1963b).[3] The following outlines are considered only for a simpler development of the theory used in the joint least-squares adjustment of both methods.

Dynamical method

By analyzing the motion of artificial satellites in the gravitational field of the Earth, Izsak determines the nonzonal harmonics of the geopotential together with corrections to the rectangular Cartesian station

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[2]Geodesist, Smithsonian Astrophysical Observatory, Cambridge, Massachusetts.

[3]For further literature, see references.

coordinates. If we call ΔU_i and ΔW_i the along-track and across-track residuals (Izsak, 1962), as obtained by a differential orbit improvement program (Gaposchkin, 1964), and similarly Δu_i and Δw_i the theoretically expected values

$$\begin{aligned} \Delta u_i &= \Delta u_i \left(C_{nm}, S_{nm}, \Delta X_j^{\nu}\right) , \\ \Delta w_i &= \Delta w_i \left(C_{nm}, S_{nm}, \Delta X_j^{\nu}\right) , \end{aligned} \qquad (n = 2, 3 \,..; 1 \leq m \leq n) \qquad (1)$$

wherein

C_{nm}, S_{nm} = nonzonal harmonic coefficients $(m \neq 0)$,

ΔX_j^{ν} = coordinate corrections to the j^{th} station $(\nu = 1, 2, 3)$,

then the most probable solution for C_{nm}, S_{nm} and ΔX_j^{ν} is obtained, in an overdetermined system with normal error distribution, by minimizing the weighted squares sum

$$\sum_i \left[p_i^u \left(\Delta U_i - \Delta u_i\right)^2 + p_i^w \left(\Delta W_i - \Delta w_i\right)^2 \right] \Rightarrow \text{minimum}. \qquad (2)$$

This is, however, equivalent to adjusting a linearized version of equation (1)

$$\underset{\sim}{A}\, \underset{\sim}{X} = \underset{\sim}{B} + \underset{\sim}{V} , \qquad (3)$$

with

$\underset{\sim}{A}$ = coefficient matrix of the linearized equation (1),
$\underset{\sim}{X}$ = solution vector $(C_{nm}, S_{nm}, \Delta X_j^{\nu})$,
$\underset{\sim}{B}$ = observation vector $(\Delta U_i, \Delta W_i)$,
$\underset{\sim}{V}$ = final residual vector $(\Delta u_i - \Delta U_i, \Delta w_i - \Delta W_i)$, in radians,

so that we get

$$\underset{\sim}{A}'\underset{\sim}{P}\underset{\sim}{A}\ \underset{\sim}{X} = \underset{\sim}{A}'\underset{\sim}{P}\underset{\sim}{B}, \quad (' \text{ sign for transposed matrix}), \tag{4}$$

or abbreviated

$$\underset{\sim}{N}_1\ \underset{\sim}{X}_1 = \underset{\sim}{M}_1 \quad , \tag{5}$$

with the obvious identities. The weight matrix $\underset{\sim}{P}$ is used in the adjustment computation for the along- and across-track residuals.

Geometrical method

Orbital effects such as air drag, solar radiation pressure, further parts of atmospheric refraction, etc., can be ignored if the satellite is observed simultaneously from pairs of tracking stations. If j is the one tracking station, then the other station k most likely appears in a direction $(\cos\beta_{jk})^{\nu}$ in which the weighted square sum of the distances to the "planes" i obtained by simultaneous observations is a minimum. In detail we have:

$$(\cos\alpha^{i}_{jk})_{\nu}\ (\cos\beta_{jk})^{\nu} = V^{i}_{jk}, \quad (\text{summation over the same indices } \nu \text{ with } \nu = 1, 2, 3) \tag{6}$$

with the condition

$$(\cos\beta_{jk})_{\nu}\ (\cos\beta_{jk})^{\nu} = 1 \ ; \tag{7}$$

herein are

$(\cos\alpha^{i}_{jk})^{\nu}$ = direction cosines of the normal vector to the observed plane i,

$(\cos\beta_{jk})^{\nu}$ = unknown direction cosines between the two stations j and k,

V^{i}_{jk} = residual, in radians,

or introducing approximate values $\bar{\beta}_{jk}$ we also can write equations (6) and (7)

$$\begin{aligned} \underset{\sim}{A}\,\underset{\sim}{Y} &= \underset{\sim}{B} + \underset{\sim}{V} \;, \\ \underset{\sim}{C}'\underset{\sim}{Y} &= 0 \;, \end{aligned} \tag{8}$$

with

$$\begin{aligned} Y^{\nu} &= (\Delta \cos \beta_{jk})^{\nu} \text{ and } (\cos \beta_{jk})^{\nu} = (\cos \bar{\beta}_{jk})^{\nu} + (Y)^{\nu} \;, \\ B^{i} &= -\,(\cos \alpha^{i}_{jk})_{\nu}\;(\cos \bar{\beta}_{jk})^{\nu} \;, \\ \underset{\sim}{C}' &= \left[(\cos \bar{\beta}_{jk})^{1},\; (\cos \bar{\beta}_{jk})^{2},\; (\cos \bar{\beta}_{jk})^{3}\right] \;. \end{aligned}$$

The minimization process finally gives:

$$\begin{bmatrix} \underset{\sim}{A}'\underset{\sim}{P}\underset{\sim}{A} & \underset{\sim}{C} \\ \underset{\sim}{C}' & 0 \end{bmatrix} \begin{bmatrix} \underset{\sim}{Y} \\ \lambda \end{bmatrix} = \begin{bmatrix} \underset{\sim}{A}'\underset{\sim}{P}\underset{\sim}{B} \\ 0 \end{bmatrix} \;, \tag{9}$$ [4]

wherein $\underset{\sim}{P}$ is the weight matrix used in the adjustment computation. Equation (9) is a subroutine of a space triangulation program developed by the author. To use these results for a combination of the geometrical method with the dynamical method, we have only to divide the matrix $\underset{\sim}{A}$ by the preliminary chord-distance $\bar{s}_{jk}$ and substitute for Y^{ν} the difference of the coordinate corrections $\Delta X^{\nu}_{k} - \Delta X^{\nu}_{j}$ (see also equation (1)). This step leads to the following normal matrix:

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[4] λ = Lagrange multiplier.

$$\begin{bmatrix} & \vdots & & \vdots & \\ \cdots & \frac{A'PA}{\bar{s}^2_{jk}} & \cdots & -\frac{A'PA}{\bar{s}^2_{jk}} & \cdots \\ & \vdots & & \vdots & \\ \cdots & -\frac{A'PA}{\bar{s}^2_{jk}} & \cdots & \frac{A'PA}{\bar{s}^2_{jk}} & \cdots \\ & \vdots & & \vdots & \end{bmatrix} \begin{bmatrix} \vert \\ \Delta X_j \\ \vert \\ \Delta X_k \\ \downarrow \end{bmatrix} = \begin{bmatrix} \vdots \\ -\frac{A'PB}{\bar{s}_{jk}} \\ \vdots \\ \frac{A'PB}{\bar{s}_{jk}} \\ \vdots \end{bmatrix} , \qquad (10)$$

which has the same structure of the solution vector as equation (4) and/or (5). The empty space in (10) is considered to be filled with zeros.

Combination of the geometrical and dynamical method

The simplest way to combine the geometrical and dynamical method in an adjustment computation is the immediate use of their individual results rather than their original observations. We consider for this purpose a system of observation equations

$$\underset{\sim}{A}\,\underset{\sim}{X} = \underset{\sim}{B} + \underset{\sim}{V} \; , \; \underset{\sim}{P} \; , \qquad (11)$$

with

$\underset{\sim}{A}$ = (linearized) coefficient matrix,

$\underset{\sim}{X}$ = solution vector,

$\underset{\sim}{B}$ = observation vector,

$\underset{\sim}{V}$ = residual vector,

$\underset{\sim}{P}$ = weight matrix,

and split (11) up into partitioned matrices

$$\begin{bmatrix} \underset{\sim}{A}_1 \\ \underset{\sim}{A}_2 \\ \cdot \\ \cdot \\ \underset{\sim}{A}_\mu \\ \cdot \\ \underset{\sim}{A}_n \end{bmatrix} \underset{\sim}{X} = \begin{bmatrix} \underset{\sim}{B}_1 \\ \underset{\sim}{B}_2 \\ \cdot \\ \cdot \\ \underset{\sim}{B}_\mu \\ \cdot \\ \underset{\sim}{B}_n \end{bmatrix} + \begin{bmatrix} \underset{\sim}{V}_1 \\ \underset{\sim}{V}_2 \\ \cdot \\ \cdot \\ \underset{\sim}{V}_\mu \\ \cdot \\ \underset{\sim}{V}_n \end{bmatrix} , \quad \begin{bmatrix} \underset{\sim}{P}_1 & \underset{\sim}{0} & \cdot & \underset{\sim}{0} & \cdot & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{P}_2 & \cdot & \underset{\sim}{0} & \cdot & \underset{\sim}{0} \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \underset{\sim}{0} & \underset{\sim}{0} & \cdot & \underset{\sim}{P}_\mu & \cdot & \underset{\sim}{0} \\ \cdot & \cdot & & \cdot & & \cdot \\ \underset{\sim}{0} & \underset{\sim}{0} & \cdot & \underset{\sim}{0} & \cdot & \underset{\sim}{P}_n \end{bmatrix} . \tag{12}$$

If we apply a least-squares procedure, equation (12) is transformed to

$$\left(\sum_{\mu=1}^{n} \underset{\sim}{A}'_\mu \underset{\sim}{P}_\mu \underset{\sim}{A}_\mu \right) \underset{\sim}{X} = \sum_{\mu=1}^{n} \underset{\sim}{A}'_\mu \underset{\sim}{P}_\mu \underset{\sim}{B}_\mu , \tag{13}$$

or shortened

$$\left(\sum_{\mu}^{n} \underset{\sim}{N}_\mu \right) \underset{\sim}{X} = \sum_{\mu}^{n} \underset{\sim}{M}_\mu . \tag{14}$$

Identifying $\mu = 1$ with the equation system (5) and $\mu > 1$ with the corresponding normal equations (10) we have herewith solved our problem. The solution vector $\underset{\sim}{X}$ can be finally written:

$$\underset{\sim}{X} = \underset{\sim}{X}_1 + \left(\sum_{\mu=1}^{n} \underset{\sim}{N}_\mu \right)^{-1} \left[\sum_{\mu=2}^{n} \underset{\sim}{M}_\mu - \left(\sum_{\mu=2}^{n} \underset{\sim}{N}_\mu \right) \underset{\sim}{X}_1 \right] . \tag{15}$$ [5]

In this joint least-squares procedure the scale and the position of the coordinate origin are, of course, the same as in the dynamical method.

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[5] assumming that we have simultaneous observations between n-1 pairs of stations.

Weighting of the two methods

In the previous section we assumed that the weights of the geometrical and dynamical methods are known. This is in most cases only true for the individual solutions. The weight proportion of both systems, however, may be determined from the standard deviations of their observations[6] which we call similar to the previous notation σ_1, $\sigma_2 \ldots \sigma_\mu \ldots \sigma_n$. If we normalize the standard deviations so that $\bar{\sigma}_1 = 1$, the weights of the dynamical system remain unchanged while the weights of the geometrical systems are multiplied by $\frac{1}{\bar{\sigma}_\mu^2}$, respectively. This procedure is then introduced in the combination, that is, in equations (12) to (15).

Additional remarks

The standard deviation of Izsak's final residuals has a magnitude of 7.9 seconds of arc. Compared herewith the individual geometrical solutions show smaller values, oscillating in the range between 1" and 2". This means that the mathematical models in both methods fit the geometrical part better than the dynamical one.[7] Orbital influences such as air drag, solar radiation pressure, secular perturbations of gravitational origin and parts of atmospheric refraction, etc., are empirically determined in the dynamical method, while these same effects are compensated or eliminated in the geometrical method. It is therefore of particular importance in the dynamical solution to have well-distributed observational material to zero out the above-mentioned influences.

In the joint adjustment computation we assumed a normal distribution of the final residuals, because only without a systematic distortion can

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[6] as obtained from the remaining residuals of the individual solutions.

[7] The camera observations in both methods are of the same accuracy.

one expect reasonable results in the sense of least-squares procedures.[8]

The combination gives the absolute positions of the tracking stations with the help of the dynamical part, fortified in their relative position by the geometrical solution.

Numerical results

1) Coordinate system

The origin of the rectangular Cartesian coordinate system coincides with the gravity center of the Earth while the x-axis (in the meridian plane through Greenwich), the y-axis and the z-axis (revolution axis pointing to the north pole) form a right-handed system.

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[8] Kaula points out in his paper (Kaula, 1965) that a longitudinal shift of the Baker-Nunn coordinates may occur in Izsak's dynamical solution. Such an effect could be eliminated by iterative use of the combined solution in the pure dynamical method (absolute orientation of the tracking network). But at the present there is no hint of such a systematic distortion which would show up especially in the g_2 component of the well-determined directions (1-10), (1-12) and (4-10). (See graphs.)

2) Observational material

a) Dynamical method

Station	No. of observations
1	3147
2	2983
3	3547
4	1930
5	1649
6	1126
7	1358
8	1861
9	1598
10	2237
11	2216
12	2592
Total no. of obs.	26244

b) Geometrical method

Station pairs	No. of simultaneous[9] observations
1-7	13
1-9	80
1-10	89
1-12	45
4-8	56
4-9	11
4-10	5
5-6	7
6-8	89
7-9	59
7-10	19
7-11	36
9-10	68
9-11	35
Total no. of obs.	612

See also Figure 2.

[9]The simultaneity was obtained by interpolation of sets of nearly simultaneous observations between pairs of stations (Veis, 1963b).

3) Numerical reference system

For the numerical computations the following reference system was introduced:

a) Normalized nonzonal harmonic coefficients

$\bar{C}_{22}$	0.173×10^{-5}	$\bar{S}_{22}$	-0.108×10^{-5}
$\bar{C}_{31}$	0.159×10^{-5}	$\bar{S}_{31}$	-0.720×10^{-7}
$\bar{C}_{32}$	0.373×10^{-6}	$\bar{S}_{32}$	-0.888×10^{-6}
$\bar{C}_{33}$	-0.125×10^{-6}	$\bar{S}_{33}$	0.130×10^{-5}
$\bar{C}_{41}$	-0.276×10^{-6}	$\bar{S}_{41}$	-0.260×10^{-6}
$\bar{C}_{42}$	0.115×10^{-6}	$\bar{S}_{42}$	0.556×10^{-6}
$\bar{C}_{43}$	0.485×10^{-6}	$\bar{S}_{43}$	-0.683×10^{-7}
$\bar{C}_{44}$	-0.911×10^{-7}	$\bar{S}_{44}$	0.583×10^{-6}

Izsak (1964, p. 2630) gives the normalization coefficients for a transformation to conventional harmonics.

b) Approximate station coordinates

Station	Approximate station coordinates		
	x	y	z
1	-1.535 742	-5.166 990	3.401 068
2	5.056 149	2.716 504	-2.775 837
3	-3.983 702	3.743 174	-3.275 648
4	5.105 621	-0.555 211	3.769 743
5	-3.946 671	3.366 331	3.698 867
6	1.018 216	5.471 113	3.109 599
7	1.942 795	-5.804 078	-1.796 972
8	3.376 903	4.403 994	3.136 280
9	2.251 821	-5.816 914	1.327 148
10	0.976 283	-5.601 393	2.880 257
11	2.280 587	-4.914 581	-3.355 465
12	-5.466 068	-2.404 281	2.242 204

length unit: 10^6 meters

4) Izsak's dynamical solution

Izsak used for the along-track and across-track residuals equal weights (Izsak, 1965b).

a) Improvements of the normalized nonzonal harmonic coefficients

$\Delta\bar{C}_{22}$	0.305×10^{-6}	$\Delta\bar{S}_{22}$	0.872×10^{-9}
$\Delta\bar{C}_{31}$	0.148×10^{-6}	$\Delta\bar{S}_{31}$	0.991×10^{-7}
$\Delta\bar{C}_{32}$	0.705×10^{-7}	$\Delta\bar{S}_{32}$	0.126×10^{-7}
$\Delta\bar{C}_{33}$	0.302×10^{-6}	$\Delta\bar{S}_{33}$	0.761×10^{-7}
$\Delta\bar{C}_{41}$	-0.111×10^{-6}	$\Delta\bar{S}_{41}$	-0.108×10^{-6}
$\Delta\bar{C}_{42}$	0.648×10^{-7}	$\Delta\bar{S}_{42}$	0.208×10^{-6}
$\Delta\bar{C}_{43}$	0.216×10^{-6}	$\Delta\bar{S}_{43}$	-0.577×10^{-7}
$\Delta\bar{C}_{44}$	-0.356×10^{-7}	$\Delta\bar{S}_{44}$	0.241×10^{-6}

These values have to be added to the harmonic coefficients in 3a).

b) Station corrections

Station	Coordinate improvements		
	Δx	Δy	Δz
1	5	4	- 6
2	3	3	- 2
3	-17	-17	5
4	2	1	-11
5	-11	-12	1
6	9	-10	- 5
7	-13	- 5	- 7
8	-14	4	-26
9	- 9	- 7	0
10	5	- 1	- 6
11	-10	- 4	1
12	- 1	-12	-11

length unit: meters

The station coordinates are obtained by adding the coordinate improvements to the approximate station coordinates of 3b).

5) Individual geometrical solutions

See equation (9). For weighting, we took the identity matrix.

Pairs of stations	Direction cosines		
	cos(x)[10]	cos(y)	cos(z)
1-7	0.553 300 1	-0.101 352 9	-0.826 792 9
1-9	0.867 355 2	-0.148 829 3	-0.474 915 7
1-10	0.965 436 9	-0.166 949 2	-0.200 148 9
1-12	-0.795 298 8	0.559 030 4	-0.234 488 4
4-8	-0.326 794 6	0.937 479 6	-0.119 738 2
4-9	-0.441 424 8	-0.813 881 8	-0.377 810 2
4-10	-0.627 485 7	-0.766 807 3	-0.135 160 6
5-6	0.915 233 9	0.388 005 5	-0.108 621 7
6-8	0.911 044 8	-0.412 179 1	0.010 286 0
7-9	0.098 444 7	-0.004 081 1	0.995 134 2
7-10	-0.202 183 4	0.042 399 5	0.978 429 4
7-11	0.185 004 1	0.487 136 8	-0.853 505 3
9-10	-0.631 058 3	0.106 627 0	0.768 372 3
9-11	0.006 028 1	0.189 211 9	-0.981 917 8

6) Weighting of the geometrical system against the dynamical system

For the individual adjustment computations to each observation the weight 1 was applied, assuming that the error distribution is normal. In the combination however we used the following weighting system (see also page 7).

[10] for simplicity cos(x), cos(y) and cos(z) are written instead of $(\cos \beta_{jk})^{\nu}$:

Method	Standard deviation of one observation	Normalization	Weights	Number of observations
dynamical	7".91	1.00	1	along- and across-track 52 488[11]
geometr.				fictitious observations
1-7	0".84	0.106	89[12]	1 157
1-9	1".36	0.172	34	2 720
1-10	1".26	0.159	39	3 471
1-12	1".89	0.239	18	810
4-8	1".55	0.196	26	1 456
4-9	1".29	0.163	38	418
4-10	0".79	0.100	100[12]	500
5-6	1".65	0.209	23	161
6-8	1".51	0.191	27	2 403
7-9	1".29	0.163	38	2 242
7-10	1".37	0.173	33	627
7-11	1".93	0.244	17	612
9-10	1".43	0.181	31	2 108
9-11	1".71	0.216	21	735

Multiplying the actual numbers of observations with the weights in the above table, we obtain a fictitious number of observations. Hence the sum of the dynamical observations corresponds to the sum of the simultaneous observations as 2.7:1.

7) Combination of the dynamical and geometrical method

The reference system is again 3a), 3b)

a) Improvements of the normalized nonzonal harmonic coefficients

$\Delta\bar{C}_{22}$	0.308×10^{-6}	$\Delta\bar{S}_{22}$	-0.519×10^{-9}
$\Delta\bar{C}_{31}$	0.151×10^{-6}	$\Delta\bar{S}_{31}$	0.105×10^{-6}
$\Delta\bar{C}_{32}$	0.533×10^{-7}	$\Delta\bar{S}_{32}$	0.364×10^{-8}
$\Delta\bar{C}_{33}$	0.324×10^{-6}	$\Delta\bar{S}_{33}$	0.864×10^{-7}

[11] Each observation leads to an along- and across-track residual, see page 2 .

[12] These weights are unrealistically high and will certainly drop with more observations.

$\Delta\bar{C}_{41}$	-0.111×10^{-6}	$\Delta\bar{S}_{41}$	-0.112×10^{-6}
$\Delta\bar{C}_{42}$	0.652×10^{-7}	$\Delta\bar{S}_{42}$	0.212×10^{-6}
$\Delta\bar{C}_{43}$	0.217×10^{-6}	$\Delta\bar{S}_{43}$	-0.633×10^{-7}
$\Delta\bar{C}_{44}$	-0.404×10^{-7}	$\Delta\bar{S}_{44}$	0.257×10^{-6}

b) Station corrections (see also Table 1)

Station	Coordinate improvements		
	Δx	Δy	Δz
1	- 1	- 1	-17
2	3	4	- 3
3	-17	-17	5
4	6	2	-16
5	-11	-13	1
6	9	-10	2
7	-16	- 8	- 3
8	-16	4	-25
9	3	2	11
10	5	1	- 1
11	- 9	- 4	2
12	- 2	-11	-10

length unit: meters

The coordinate improvements are of about the same magnitude and direction as already obtained from the dynamical solution. Point 9 was mostly influenced by the geometrical method, moving about 18 meters, followed by point 1 with 14 meters, etc., relative to the dynamical method. Stations 2 and 3 had no simultaneous observations and were therefore only slightly[13] varied or not at all. The tesseral harmonics $\Delta\bar{C}_{41}$ and $\Delta\bar{S}_{41}$ point out the highest accuracy followed by $\Delta\bar{C}_{31}$, $\Delta\bar{S}_{31}$, etc. An accuracy decrease takes place toward the sectorial terms in each section above (see also Izsak, 1964, p. 2628).

[13]by the correlation within the matrix-system.

8) Comparison of the combined solution with the individual solutions

a) Direction cosines

Pairs of stations	Solu-tion	Direction cosines cos(x)	cos(y)	cos(z)
1-7	C	0.553 294 3	-0.101 336 4	-0.826 798 9
	D	292 8	336 6	799 8
	G	300 1	352 9	792 9
	GG	299 9	338 5	794 9
1-9	C	0.867 351 9	-0.148 831 8	-0.474 920 8
	D	348 4	835 1	926 1
	G	355 2	829 3	915 7
	GG	354 9	830 6	915 6
1-10	C	0.965 435 5	-0.166 951 0	-0.200 154 4
	D	433 7	953 8	160 6
	G	436 9	949 2	148 9
	GG	437 0	949 3	148 4
1-12	C	-0.795 297 9	0.559 029 5	-0.234 493 6
	D	298 4	027 9	495 9
	G	298 8	030 4	488 4
	GG	--	--	--
4-8	C	-0.326 797 1	0.937 477 2	-0.119 750 1
	D	796 0	477 5	751 3
	G	794 6	479 6	738 2
	GG	--	--	--
4-9	C	-0.441 425 5	-0.813 878 8	-0.377 815 8
	D	425 6	878 1	817 3
	G	424 8	881 8	810 2
	GG	425 4	881 2	810 8

4-10	C	-0.627 485 5	-0.766 807 1	-0.135 162 1
	D	484 9	807 3	163 6
	G	485 7	807 3	160 6
	GG	484 5	808 4	159 2
5-6	C	0.915 236 4	0.387 998 4	-0.108 626 1
	D	236 3	998 2	627 4
	G	233 9	8 005 5	621 7
	GG	--	--	--
6-8	C	0.911 046 4	-0.412 175 3	0.010 295 3
	D	046 5	175 0	297 6
	G	044 8	179 1	286 0
	GG	--	--	--
7-9	C	0.098 440 5	-0.004 085 5	0.995 134 6
	D	436 0	089 3	135 0
	G	444 7	081 1	134 2
	GG	444 6	081 7	134 2
7-10	C	-0.202 180 3	0.042 401 6	0.978 429 9
	D	180 9	400 5	429 9
	G	183 4	399 5	429 4
	GG	182 4	404 4	429 4
7-11	C	0.184 994 9	0.487 136 2	-0.853 507 6
	D	994 2	135 7	508 0
	G	5 004 1	136 8	505 3
	GG	5 003 4	137 4	505 0
9-10	C	-0.631 055 3	0.106 625 8	0.768 375 1
	D	050 0	629 4	378 8
	G	058 3	627 0	372 3
	GG	059 3	625 4	371 8
9-11	C	0.006 029 5	0.189 212 5	-0.981 917 6
	D	031 9	214 7	917 1
	G	028 1	211 9	917 8
	GG	029 4	210 4	918 0

C = combined solution (dynamical and geometrical method)

D = dynamical method

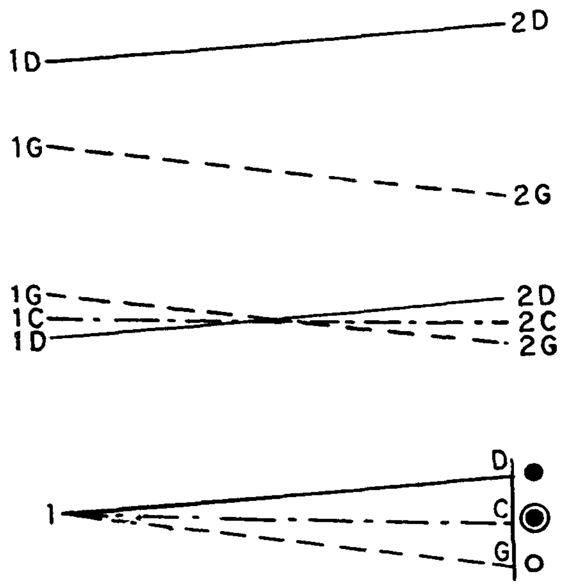
G = individual geometrical method

GG = geometrical method with plane conditions

The GG values were obtained from a combined solution of the dynamical and the geometrical method by applying particularly high weights (ca. 10,000) to the geometrical part. Hence simple directions without conditions result in the G values, while the others are adjusted according to the plane conditions which exist between sets of three stations.

b) Graphs (pages 22-24)

Using Veis's procedure (Veis, 1963b) we plotted the above results in planes perpendicular to the directions between pairs of tracking stations. A local coordinate system is defined by g_1 and g_2, wherein g_1 represents the horizon and g_2 the height in the opposite station as seen from the other. It



individual solutions (dynamical, geometrical)

combined solution

individual and combined solutions as shown in the graphs

Figure 1 .

may be pointed out that the graphs give only a distorted picture of the actual situation in the station positions. If we have, for example, a direction $(1\text{-}2)_D$ from the dynamical solution and a corresponding direction $(1\text{-}2)_G$ from the geometrical solution, then the adjustment computation will usually lead to a direction $(1\text{-}2)_C$ (Figure 1). In our graphs we show a magnified picture which is artificially produced by moving 1G, 1D, 1C into one point.

In Figure 3 the point for the combined solution lies somewhere between the dynamical and the geometrical solutions. The only exception seems to be direction (7-10). However, this result can be explained by the fact that only 19 observations stand against the total of observations from the surrounding points to stations 7 and 10, respectively.

The individual geometrical solution of (1-7) falls beyond the frame of the first graph. All the 13 "planes" were nearly parallel and led to a poor direction determination. Quite a reasonable result is obtained however from the geometrical solution (GG) which includes the triangle condition.

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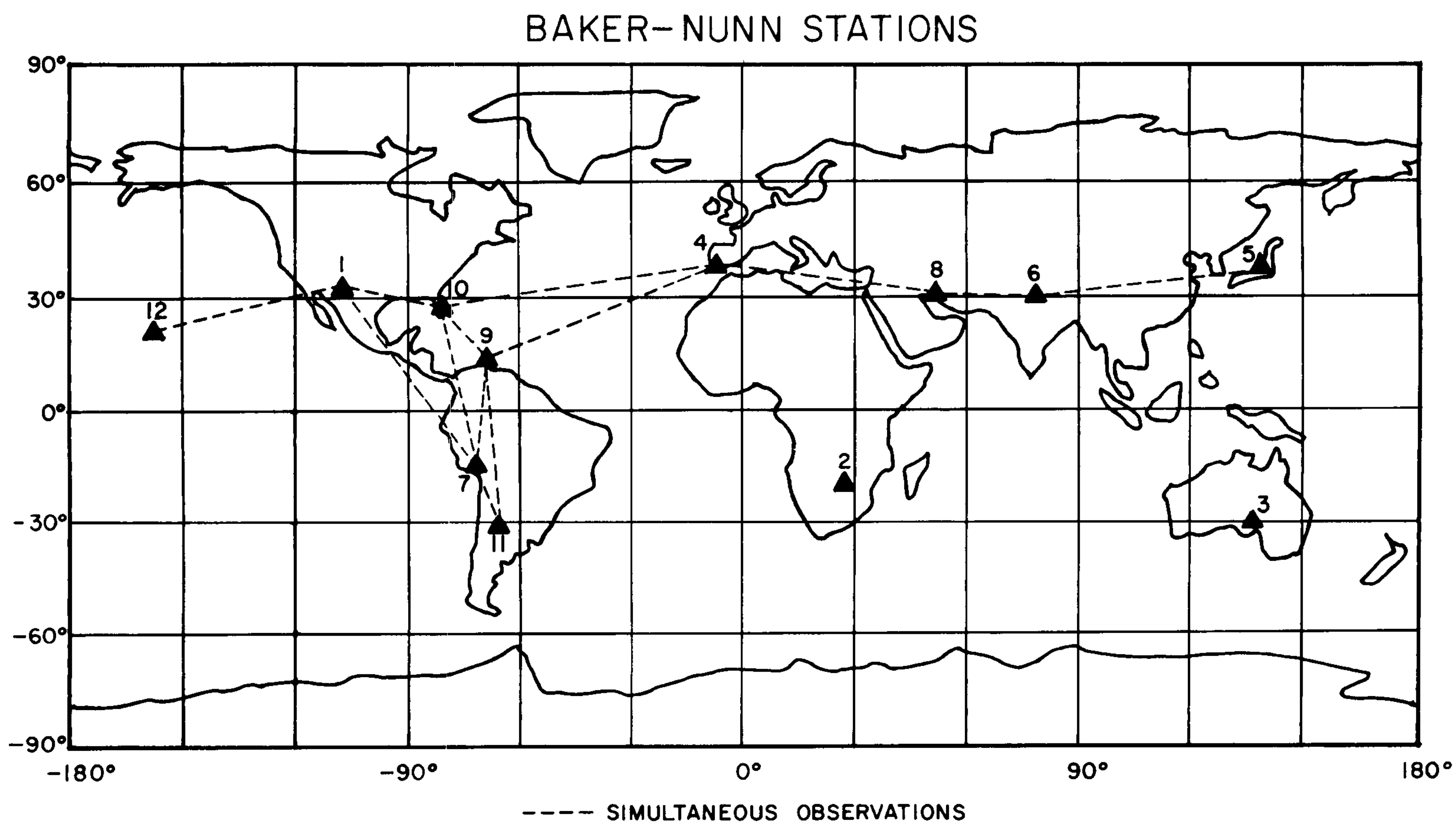
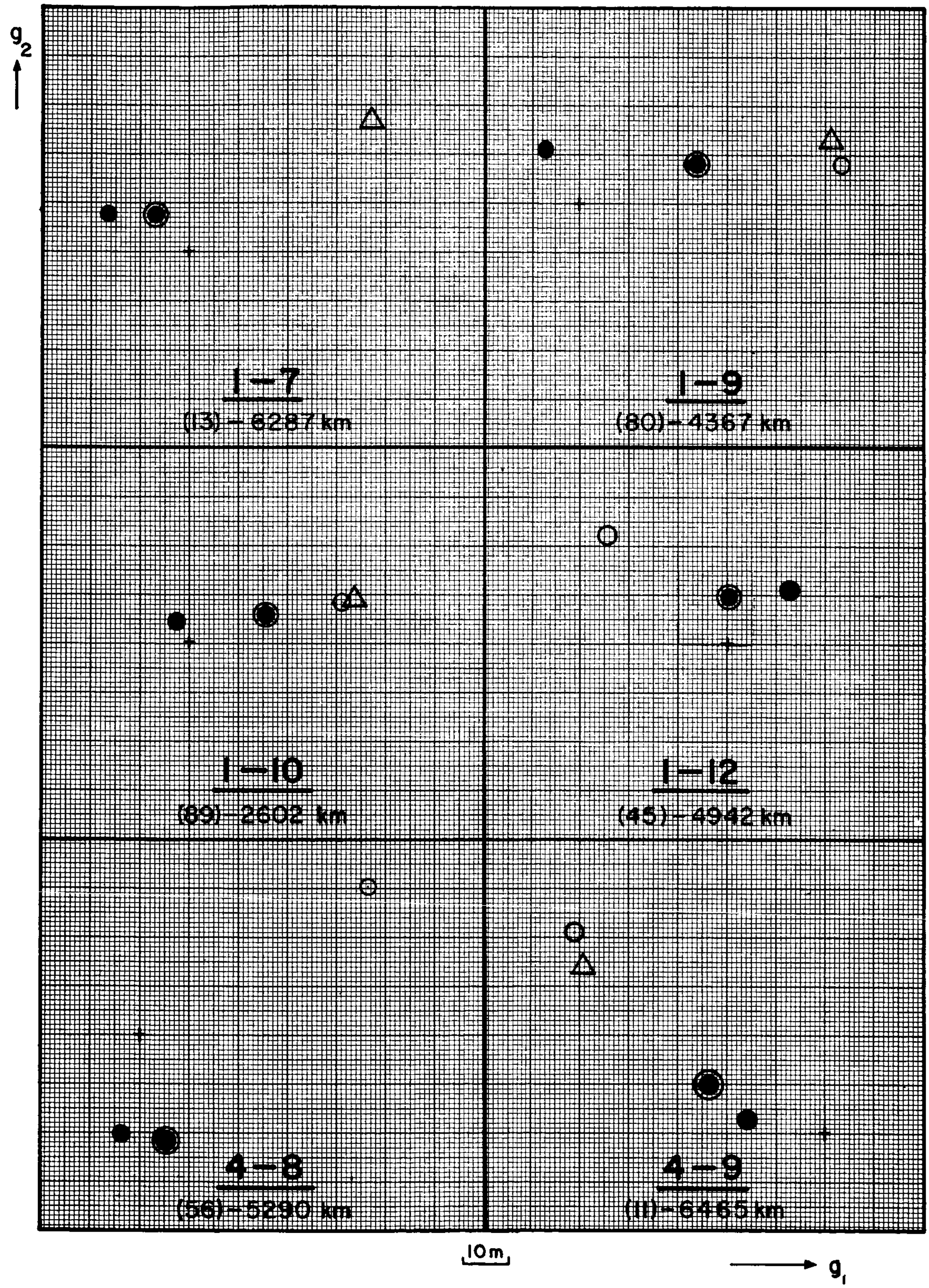
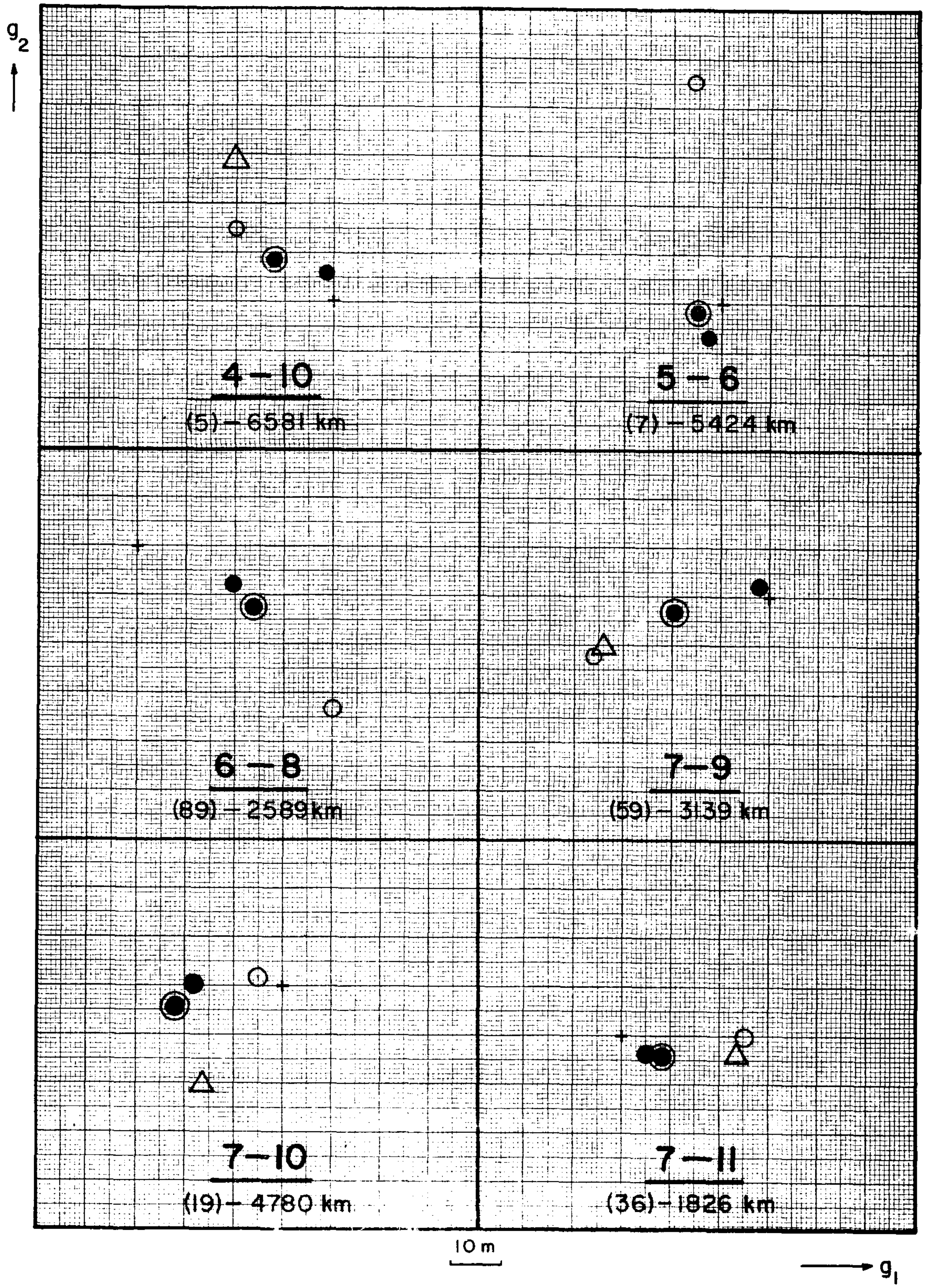


Figure 2. Locations of the Baker-Nunn stations.



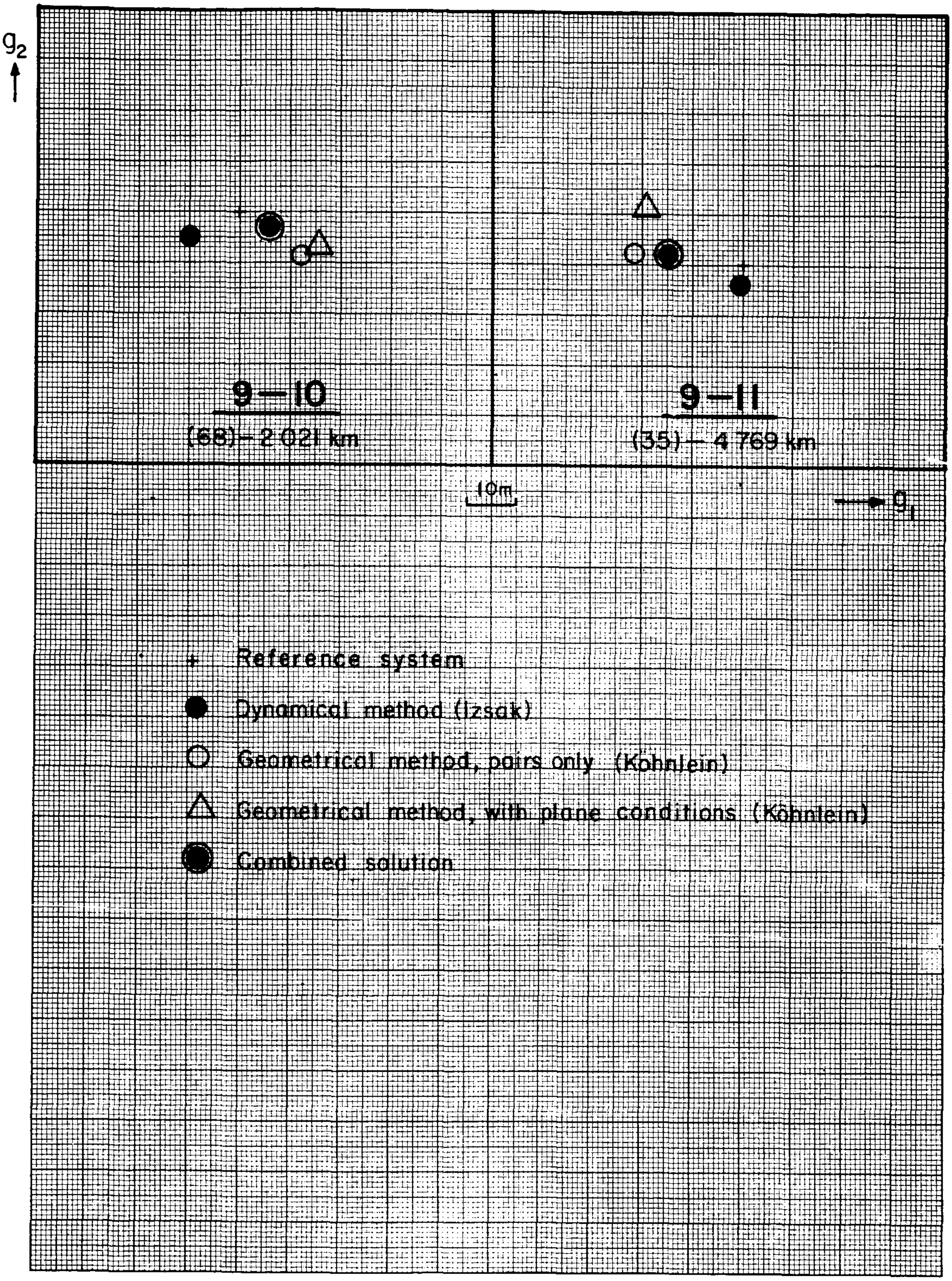
g_2
1—7
(13)—6287 km
1—9
(80)—4367 km
1—10
(89)—2602 km
1—12
(45)—4942 km
4—8
(56)—5290 km
4—9
(11)—6465 km
10 m
g_1

Figure 3a.



g_2
4–10
(5) – 6581 km
5 – 6
(7) – 5424 km
6 – 8
(89) – 2589 km
7–9
(59) – 3139 km
7–10
(19) – 4780 km
7 – 11
(36) – 1826 km
10 m
g_1

Figure 3b.



g_2
9—10
(68) — 2 021 km
9—11
(35) — 4 769 km
10m
g_1
+ Reference system
Dynamical method (Izsak)
Geometrical method, pairs only (Köhnlein)
Geometrical method, with plane conditions (Köhnlein)
Combined solution

Figure 3c.

Table 1.--Combination of the dynamical and geometrical methods.

	Station	x km	y km	z km
1.	Organ Pass	-1 535.743	-5 166.991	3 401.051
2.	Olifantsfontein	5 056.152	2 716.508	-2 775.840
3.	Woomera	-3 983.719	3 743.157	-3 275.643
4.	San Fernando	5 105.627	- 555.209	3 769.727
5.	Tokyo	-3 946.682	3 366.318	3 698.868
6.	Naini Tal	1 018.225	5 471.103	3 109.601
7.	Arequipa	1 942.779	-5 804.086	-1 796.975
8.	Shiraz	3 376.887	4 403.998	3 136.255
9.	Curaçao	2 251.824	-5 816.912	1 327.159
10.	Jupiter	976.288	-5 601.392	2 880.256
11.	Villa Dolores	2 280.578	-4 914.585	-3 355.463
12.	Maui	-5 466.070	-2 404.292	2 242.194

NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

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