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MISSION ANALYSIS REPORT - MAR 2

THE PERTURBATIONS OF A SYNCHRONOUS SATELLITE RESULTING FROM THE GRAVITATIONAL FIELD OF A TRIAXIAL EARTH

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SUMMARY

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This paper presents a study of the nature of the 24 hour synchronous satellite perturbations due to the earth's triaxiality. Equations representing the drift due to these perturbations are developed. It is found that the radial drift is linear and the longitudinal drift is parabolic. Both have superimposed on these primary curves, oscillations with one day periods. This analysis also verifies the existence of only two dynamically stable points, where no drift due to the earth's triaxiality will occur. These are located at opposite ends of the equatorial minor axis. author

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LIST OF SYMBOLS

A,B,C,D,E,F	coefficients of linearized perturbation equations
C_1, C_2, C_3, C_4	coefficients of the transient solution for drift
F_r, F_θ, F_ϕ	radial, longitudinal and latitudinal forces acting on a satellite
J_{20}, J_{22}	dimensionless gravitational constants
K_{22}	a dimensionless constant related to J_{22}
m	satellite mass
r	radial distance from the center of the earth to the satellite
r_s	synchronous radius
s	operator equivalent of $\frac{d}{d\tau}$
t	time
U	earth's gravitational potential
β	angle between Greenwich (zero longitude and the equatorial minor axis
$\beta_1, \beta_2, \beta_3$	coefficients of the characteristic equation for radial and longitudinal drift
γ	angle between the equatorial minor axis and the projection of the satellite radius vector
γ_o	initial value of γ
Δ	transient solution for drift
θ	inertial longitude of satellite
θ_E	inertial longitude of equatorial minor axis
θ_{E_o}	inertial value of θ_E
$\dot{\theta}_E$	earth's rate of rotation
μ	EARTH [redacted] gravitational constant

λ	geographic longitude
τ	dimensionless time
ϕ	inertial latitude of satellite

INTRODUCTION

It has been shown (Refs. 1 and 2) that there are only two dynamically stable points for 24 hour synchronous satellites. These are in the vicinity of 57° east longitude and 123° west longitude in the geographic system. Satellites positioned at locations other than these will drift toward the nearest point of stability. In order to obtain maximum global coverage using the synchronous communications satellite concept, it is necessary to maintain at least three satellite systems, which are equally spaced about the equator, in operation. This means that drift correction is necessary. General perturbation studies have shown the disturbances of satellite orbits to be caused primarily by the potential field of a triaxial earth. The indicated influence of the lunar and solar gravitational fields is small in comparison. It is the purpose of this paper to determine the nature of the 24 hour synchronous satellite perturbations due to the earth's triaxiality.

DEVELOPMENT

REFERENCE SYSTEM AND DEFINITIONS

An inertial reference system with axes X, Y, and Z as shown in Figure 1 is used in this paper. The origin and Z axis of the coordinate system coincide with the earth's center and polar axis respectively. The X-Y plane coincides with the equatorial plane and completes a right hand coordinate system. It is found convenient to specify the satellite position by the spherical coordinates θ, ϕ and r. The earth's equatorial minor axis lies at an angle,

$$\theta_{\epsilon} = \theta_{\epsilon_o} + \dot{\theta}_{\epsilon} t \qquad (1)$$

from the X axis, where

θ_{ϵ_o} = the initial angle between the X axis and the equatorial minor axis

$\dot{\theta}_{\epsilon}$ = the earth's rate of rotation about its polar axis

and, t = time

From Figure 2 the relationship of inertial to geographic longitude is:

$$\theta = \theta_{\epsilon} + \lambda \quad \text{[redacted]} \qquad (2)$$

where λ = geographic longitude

and, β = the angle between Greenwich (zero longitude) and the equatorial minor axis and is approximately 123° west longitude

Further the angle between the equatorial minor axis and the projection of the satellite radius vector, r, into the equatorial plane is given by

$$\gamma = \lambda - \beta \qquad (3)$$

EQUATIONS OF MOTION

The components of acceleration in terms of r, θ, and ϕ are:

$$a_r = \ddot{r} - r\dot{\theta}^2\cos^2\phi - r\dot{\phi}^2 \qquad (4)$$

$$a_\theta = \frac{1}{r\cos\phi}\frac{d}{dt}\left(r^2\dot{\theta}\cos^2\phi\right) \qquad (5)$$

$$a_\phi = \frac{1}{r}\frac{d}{dt}\left(r^2\dot{\phi}\right) + r\dot{\theta}^2\cos\phi\sin\phi \qquad (6)$$

If one assumes the influence of the lunar and solar gravitation fields to be small, the force components are determined from the earth's gravitational potential. The most recent formulation of this potential is:

$$U = \frac{\mu}{r} - \frac{J_{20}\mu R_o^2}{2r^3}\left(3\sin^2\phi - 1\right) + \frac{3J_{22}\mu R_o^2}{r^3}\cos^2\phi\cos 2\gamma \qquad (7)$$

where

μ = Universal gravitational constant

R_o = Mean equatorial radius of the earth

and J_{20} and J_{22} are dimensionless constants whose values are (Reference 2):

$J_{20} = 1.082 \times 10^{-3}$

$J_{22} = -5.35 \times 10^{-6}$

It is seen that Eq. (7) includes one each of the zonal and secterial harmonics. All higher order harmonics are assumed negligible.

From Eq. (7) the force components are:

$$F_r = m\frac{\partial U}{\partial r} = m\left[-\frac{\mu}{r^2} + \frac{3J_{20}\mu R_o^2}{2r^4}\left(3\sin^2\phi - 1\right) - \frac{9J_{22}\mu R_o^2}{r^4}\cos^2\phi\cos 2\gamma\right] \qquad (8)$$

$$F_\theta = \frac{m}{\cos\phi}\frac{1}{r}\frac{\partial U}{\partial \theta} = m\left(-\frac{6J_{22}\mu R_o^2}{r^4}\cos\phi\sin 2\gamma\right) \qquad (9)$$

and, $$F_\phi = m\frac{1}{r}\frac{\partial U}{\partial \phi} = m\left(-\frac{3J_{20}\mu R_o^2}{r^4}\sin\phi\cos\phi\right.$$

$$\left. -\frac{6J_{22}\mu R_o^2}{r^4}\cos\phi\sin\phi\cos 2\gamma\right) \qquad (10)$$

where

m = Mass of the satellite

By equating the corresponding acceleration and force expressions from Eqs. (4), (5), (6), (8), (9) and (10) the following equations of motion are obtained.

$$\ddot{r} - r\dot{\theta}^2\cos^2\phi - r\dot{\phi}^2 = -\frac{\mu}{r^2} + \frac{3J_{20}\mu R_o^2}{2r^4}\left(3\sin^2\phi\right.$$

$$\left.-1\right) - \frac{9J_{22}\mu R_o^2}{r^4}\cos^2\phi\cos 2\gamma \qquad (11)$$

$$\frac{1}{r\cos\phi}\frac{d}{dt}\left(r^2\dot{\theta}\cos^2\phi\right) = -\frac{6J_{22}\mu R_o^2}{r^4}\cos\phi\sin 2\gamma \qquad (12)$$

and, $$\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\phi}\right) + r\dot{\theta}^2\cos\phi\sin\phi = -\frac{3J_{20}\mu R_o^2}{r^4}\sin\phi\cos\phi$$

$$-\frac{6J_{22}\mu R_o^2}{r^4}\cos\phi\sin\phi\cos 2\gamma \qquad (13)$$

General perturbation studies have indicated the latitude perturbations to be periodic and having a magnitude of less than one degree. For purposes of this report, latitude is assumed equal to zero at all times. Therefore, Eq. (13) is eliminated and the analysis becomes two dimensional. Eqs. (11) and (12) reduce to:

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2} - \frac{3J_{20}\mu R_o^2}{2r^4} - \frac{9J_{22}\mu R_o^2}{r^4}\cos 2\gamma \qquad (14)$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = -\frac{6J_{22}\mu R_o^2}{r^4}\sin 2\gamma \qquad (15)$$

Since the primary concern is with deviations of a satellite from a synchronous orbit, it becomes advantageous to linearize Eqs. (14) and (15) in the vicinity of this orbit. Perturbations about the synchronous orbit will be radial, Δr, and longitudinal, $\Delta\gamma$. Now define:

$$\theta = \theta_E + \gamma_o + \Delta\gamma \qquad (16)$$

$$\gamma = \gamma_o + \Delta\gamma \qquad (17)$$

$$r = r_s + \Delta r \qquad (18)$$

where γ_o = the desired longitude difference between the equatorial minor axis and the satellite position

and, r_s = synchronous radius

Substitution of Eqs. (16), (17) and (18) into Eqs. (14) and (15)

and division by $r_s \dot{\theta}_E^2$ yields:

$$\frac{1}{\dot{\theta}_E^2}\left(\frac{\Delta\ddot{r}}{r_s}\right) - 1 - \frac{\Delta r}{r_s} - 2\frac{1}{\dot{\theta}_E}\Delta\dot{\gamma} = -\frac{1}{\dot{\theta}_E^2}\frac{\mu}{r_s^3}\left(1 - 2\frac{\Delta r}{r_s}\right)$$
$$- \frac{3 J_{20}\mu R_o^2}{2\dot{\theta}_E^2 r_s^5}\left(1 - 4\frac{\Delta r}{r_s}\right)$$
$$- \frac{9 J_{22}\mu R_o^2}{\dot{\theta}_E^2 r_s^5}\left(1 - 4\frac{\Delta r}{r_s}\right)\left(\cos 2\gamma_o - 2\Delta\gamma \sin 2\gamma_o\right) \qquad (19)$$

and,

$$\frac{1}{\dot{\theta}_E^2}\Delta\ddot{\gamma} + 2\frac{1}{\dot{\theta}_E}\left(\frac{\Delta\dot{r}}{r_s}\right)$$
$$= -\frac{6 J_{22}\mu R_o^2}{\dot{\theta}_E^2 r_s^5}\left(1 - 5\frac{\Delta r}{r_s}\right)\left(\sin 2\gamma_o + 2\Delta\gamma\cos 2\gamma_o\right) \qquad (20)$$

where small angle approximations have been made and terms of second order and greater have been neglected. A dimensionless time is now introduced as:

$$\tau = \dot{\theta}_E t \qquad (21)$$

In terms of this dimensionless time, Eqs. (19) and (20) reduce to:

$$\frac{d^2}{d\tau^2}\left(\frac{\Delta r}{r_s}\right) - C\left(\frac{\Delta r}{r_s}\right) - 2\frac{d}{d\tau}\Delta\gamma - D\Delta\gamma = A \tag{22}$$

$$2\frac{d}{d\tau}\left(\frac{\Delta r}{r_s}\right) - F\left(\frac{\Delta r}{r_s}\right) + \frac{d^2}{d\tau^2}\Delta\gamma + E\Delta\gamma = B \tag{23}$$

where

$$A = 1 - \frac{\mu}{\dot{\theta}_E^2 r_s^3} - \frac{3J_{20}\mu R_o^2}{2\dot{\theta}_E^2 r_s^5} - \frac{9J_{22}\mu R_o^2}{\dot{\theta}_E^2 r_s^5}\cos 2\gamma_o \tag{24}$$

$$B = -\frac{6J_{22}\mu R_o^2}{\dot{\theta}_E^2 r_s^5}\sin 2\gamma_o \tag{25}$$

$$C = 1 + \frac{2\mu}{\dot{\theta}_E^2 r_s^3} + \frac{6J_{20}\mu R_o^2}{\dot{\theta}_E^2 r_s^5} + \frac{36J_{22}\mu R_o^2}{\dot{\theta}_E^2 r_s^5}\cos 2\gamma_o \tag{26}$$

$$D = \frac{18J_{22}\mu R_o^2}{\dot{\theta}_E^2 r_s^5}\sin 2\gamma_o \tag{27}$$

$$E = \frac{12J_{22}\mu R_o^2}{\dot{\theta}_E^2 r_s^5}\cos 2\gamma_o \tag{28}$$

and

$$F = \frac{30J_{22}\mu R_o^2}{\dot{\theta}_E^2 r_s^5}\sin 2\gamma_o \tag{29}$$

By using operater notation, the variables of Eqs. (22) and (23) can be separated. The result is:

$$(S^4 + \beta_1 S^2 + \beta_2 S + \beta_3)\frac{\Delta r}{r_s} = AE + BD \tag{30}$$

and,

$$(S^4 + \beta_1 S^2 + \beta_2 S + \beta_3)\Delta\gamma = AF - BC \tag{31}$$

where

$$S = \frac{d}{d\tau} \qquad \beta_1 = 4 - C + E \tag{32}$$

$$\beta_2 = 2(D - F) \tag{33}$$

and,

$$\beta_3 = -CE - DF \tag{34}$$

Examination of Eqs. (24) thru (29) shows the following to be very good approximations.

$$A \cong 0 \quad (35)$$

$$B \cong -6 J_{22} \left(\frac{R_o}{r_s}\right)^2 \sin 2\gamma_o \quad (36)$$

$$C \cong 3.0 \quad (37)$$

$$D \cong 18 J_{22} \left(\frac{R_o}{r_s}\right)^2 \sin 2\gamma_o \quad (38)$$

$$E \cong 12 J_{22} \left(\frac{R_o}{r_s}\right)^2 \cos 2\gamma_o \quad (39)$$

and, $$F \cong 30 J_{22} \left(\frac{R_o}{r_s}\right)^2 \sin 2\gamma_o \quad (40)$$

Further, it is seen that:

$$\beta_1 \cong 1.0 \quad (41)$$

$$\beta_2 \cong 24 K_{22} \sin 2\gamma_o \quad (42)$$

$$\beta_3 \cong 36 K_{22} \cos 2\gamma_o \quad (43)$$

$$AE + BD \cong 0 \quad (44)$$

$$AF - BC \cong -18 K_{22} \sin 2\gamma_o \quad (45)$$

where $K_{22} = -J_{22} \left(\frac{R_o}{r_s}\right)^2$

Eqs. (30) and (31) then become:

$$\left[S^4 + S^2 + (24 K_{22} \sin 2\gamma_o) S + (36 K_{22} \cos 2\gamma_o)\right] \frac{\Delta r}{r_s} = 0 \quad (46)$$

$$\left[S^4 + S^2 + (24 K_{22} \sin 2\gamma_o) S + (36 K_{22} \cos 2\gamma_o)\right] \Delta\gamma = -18 K_{22} \sin 2\gamma_o \quad (47)$$

It is seen that the characteristic equation for both $\Delta\gamma$ and $\frac{\Delta r}{r_s}$ is:

$$\Delta = S^4 + \beta_1 S^2 + \beta_2 S + \beta_3 \tag{48}$$

Extensive numerical analysis of Eq. (48) for the values of β_1, β_2, and β_3 involved, has shown

$$\Delta = \left(S^2 + \beta_2 S + \beta_3\right)\left(S^2 - \beta_2 S + 1\right) \tag{49}$$

to be the solution as a product of two quadratics. The individual roots are:

$$S_1 = 12 K_{22} \sin 2\gamma_o + j \tag{50}$$

$$S_2 = 12 K_{22} \sin 2\gamma_o - j \tag{51}$$

$$S_3 = -12 K_{22} \sin 2\gamma_o + \sqrt{-36 K_{22} \cos 2\gamma_o} \tag{52}$$

$$S_4 = -12 K_{22} \sin 2\gamma_o - \sqrt{-36 K_{22} \cos 2\gamma_o} \tag{53}$$

For $0^\circ \leq \gamma_o < 45^\circ$, the characteristic solution is:

$$\begin{aligned} \Delta = {} & e^{(12 K_{22} \sin 2\gamma_o)\tau}\left(C_1 \sin\tau + C_2 \cos\tau\right) \\ & + C_3 e^{\left(-12 K_{22} \sin 2\gamma_o + \sqrt{-36 K_{22} \cos 2\gamma_o}\right)\tau} \\ & + C_4 e^{\left(-12 K_{22} \sin 2\gamma_o - \sqrt{-36 K_{22} \cos 2\gamma_o}\right)\tau} \end{aligned} \tag{54}$$

For $45^\circ \leq \gamma_o < 90^\circ$, the characteristic solution is:

$$\Delta = e^{(12 K_{22} \sin 2\gamma_o)\tau}\left(C_1 \sin\tau + C_2 \cos\tau\right)$$

$$+ e^{(-12 K_{22} \sin 2\gamma_o)\tau}\Big[C_3 \sin\sqrt{-36 K_{22}\cos 2\gamma_o}\,\tau$$

$$+ C_4 \cos\sqrt{-36 K_{22}\cos 2\gamma_o}\,\tau \tag{55}$$

The exponential terms of Eqs. (54) and (55) have very large time constants and can, therefore, be replaced by unity. Also, the transcendental functions corresponding to the C_3 and C_4 coefficients of Eq. (55) involve extremely small angles and can be replaced by small angle approximations. The solution for all values of γ_o then becomes:

$$\Delta = C_1 \sin\tau + C_2 \cos\tau + C_3\tau + C_4 \tag{56}$$

In order to evaluate the unknown coefficients, the initial conditions must be determined. It is assumed that a perfect injection into a synchronous orbit has been achieved. This implies:

$$\Delta r_o = \Delta\dot{r}_o = \Delta\gamma_o = \Delta\dot{\gamma}_o = 0 \tag{57}$$

Substitution of Eq. (57) into Eqs. (22) and (23) yields:

$$\Delta\ddot{r}_o = \Delta\dddot{\gamma}_o = 0 \tag{58}$$

$$\frac{d^2}{d\tau^2}\Delta\gamma_o = 6 K_{22} \sin 2\gamma_o \tag{59}$$

and, $$\frac{d^3}{d\tau^3}\left(\frac{\Delta r_o}{r_s}\right) = 12 K_{22} \sin 2\gamma_o \tag{60}$$

Evaluation of Eq. (56) and its derivatives for the initial conditions gives for the characteristic solutions:

$$(\Delta\gamma)_c = 6 K_{22} \sin 2\gamma_o (1 - \cos\tau) \tag{61}$$

and, $$\left(\frac{\Delta r}{r_s}\right)_c = 12 K_{22} \sin 2\gamma_o (\tau - \sin\tau) \tag{62}$$

There is no particular (steady state) solution for $\frac{\Delta r}{r_s}$.

For $\Delta\gamma$, the particular solution is:

$$(\Delta\gamma)_p = -(9 K_{22} \sin 2\gamma_o)\,\tau^2 \tag{63}$$

Finally, the total solutions for $\Delta\gamma$ and Δr, respectively, are:

$$\Delta\gamma = \frac{180}{\pi} \cdot 6K_{22} \sin 2\gamma_o \left(1 - \cos\dot{\theta}_\varepsilon t - 1.5\dot{\theta}_\varepsilon^2 t^2\right) \tag{64}$$

$$\Delta r = r_s \cdot 12K_{22} \sin 2\gamma_o \left(\dot{\theta}_\varepsilon t - \sin\dot{\theta}_\varepsilon t\right) \tag{65}$$

where real time has been substituted. As shown, the unit of $\Delta\gamma$ is degrees and the unit of Δr corresponds to that of r_s.

RESULTS AND DISCUSSION

For a truly synchronous condition to exist, the forcing functions of the coupled perturbation equations, Eqs. (30) and (31), must be zero. It follows immediately that A and B given by Eqs. (24) and (25), which are repeated below as Eqs. (66) and (67), must be zero.

$$A = 1 - \frac{\mu}{\dot{\theta}_\varepsilon^2 r_s^3} - \frac{3J_{20}\mu R_o^2}{2\dot{\theta}_\varepsilon^2 r_s^5} - \frac{9J_{22}\mu R_o^2}{\dot{\theta}_\varepsilon^2 r_s^5}\cos 2\gamma_o \tag{66}$$

$$B = -\frac{6J_{22}\mu R_o^2}{\dot{\theta}_\varepsilon^2 r_s^5}\sin 2\gamma_o \tag{67}$$

Obviously, for Eq. (67) to be zero, $\sin 2\gamma_o$ must also be zero. This gives four values of γ_o (0°, 90°, 180° and 270°) at which a truly synchronous condition exists. Blitzer (Ref. 3) has shown that the locations $\gamma_o = 0^\circ$ and 180° represent dynamically stable points while the other two points are statically stable. The value of r_s which makes Eq. (66) zero at the dynamically stable points is found to be:

$$r_s = 22752.292 \text{ nautical miles} \tag{68}$$

Shown as Figures 3 and 4 are plots of $\Delta\gamma$ and Δr, Eqs. (64) and (65), as functions of time for various values of γ_o. It is seen that the satellite motion is symmetrical about $\gamma_o = 45^\circ$. For the cases shown, the drift was also determined by numerical integration of Eqs. (11), (12) and (13). Agreement of the two methods was very good within the limits of the linearizations used in the perturbation equations. These limits are:

$$\Delta r << r_s \tag{69}$$

$$\Delta\gamma < 5^\circ \tag{70}$$

The limit on $\Delta\gamma$ is established by the limits of the small angle approximations which were used. Figure 5 is a plot of the differences between the two methods of solution for $\gamma_o = 45^o$ where the drift magnitudes are maximum. It should be realized that for longer drift periods Eqs. (64) and (65) represent only short time period drift. For example at values of γ_o in the vicinity of 90^o there is very little short time period drift. But since $\gamma_o = 90^o$ is only statically stable, it follows that long time period drift will be present.

It is noted that if the oscillations are ignored, Eqs. (64) and (65) are identical to equations developed by Frick and Garber (Reference 2). The period of the oscillation in both cases is:

$$\text{PERIOD} = \frac{2\pi}{\dot{\theta}_E} = 1 \text{ day} \tag{71}$$

When $\Delta\gamma$ is expressed in degrees, the maximum amplitude of its oscillating term is:

$$\text{AMPLITUDE} = \frac{180}{\pi} \cdot 6 J_{22} \left(\frac{R_o}{r_S}\right)^2 = .4208 \times 10^{-4} \text{ deg} \tag{72}$$

The amplitude of the oscillation in degrees is very small. However, in terms of linear position this amplitude becomes

$$\text{AMPLITUDE} = r_S \cdot 6 J_{22} \left(\frac{R_o}{r_S}\right)^2 = 101.5 \text{ ft.} \tag{73}$$

The maximum amplitude of the Δr oscillating term is:

$$\text{AMPLITUDE} = r_S \cdot 12 J_{22} \left(\frac{R_o}{r_S}\right)^2 = 203 \text{ ft} \tag{74}$$

It therefore seems that the oscillation terms are of sufficient magnitude to warrant consideration in drift correction studies.

Figure 6 shows the nature of the drift in each of the four geographic quadrants. The symmetry is readily apparent. It is pointed out that the magnitudes of drift are equal in all quadrants. Only the directions are different.

CONCLUSIONS

The purpose of this paper was achieved in that equations were developed which accurately determine drift for short time periods. Consequently the following conclusions can be stated.

1. The only locations at which truly synchronous conditions exist are in the vicinities of $\gamma_o = 0^o$ and 180^o at an orbit radius of 22752.292 nautical miles.

2. A realization of the presence of oscillatory motion in both the longitudinal and radial directions was achieved. This oscillation is of sufficient magnitude to warrant consideration in drift correction studies.

3. The symmetrical nature of the drift in the four geographic quadrants is such as to greatly reduce the analysis of different longitudes as potential satellite locations.

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3. Izsak, I. G., "A Determination of the Ellipticity of the Earth's Equator from the Motion of Two Satellites", Astronomical Journal, 66, No. 5, 1961

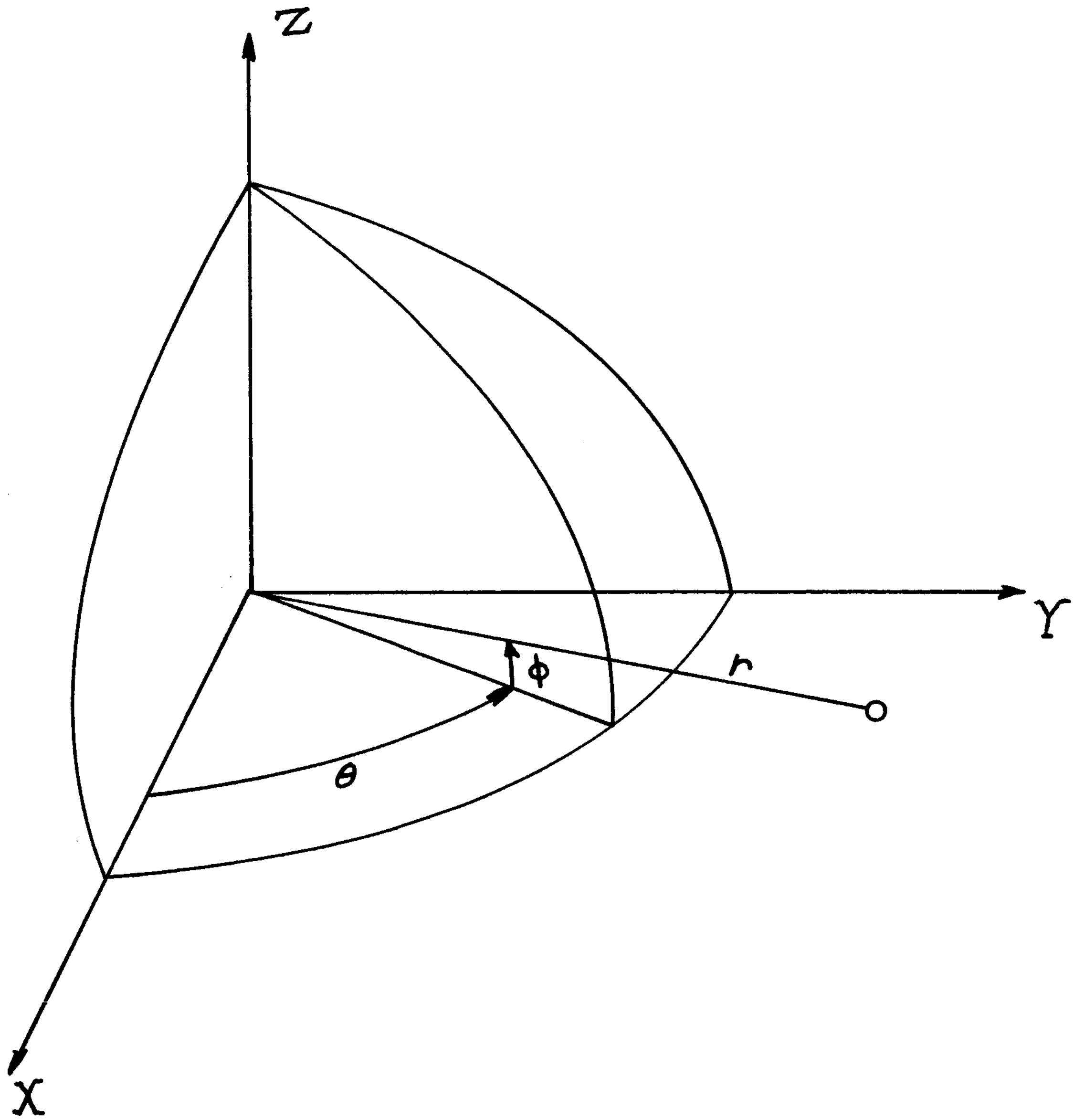

FIGURE 1. INERTIAL REFERENCE SYSTEM

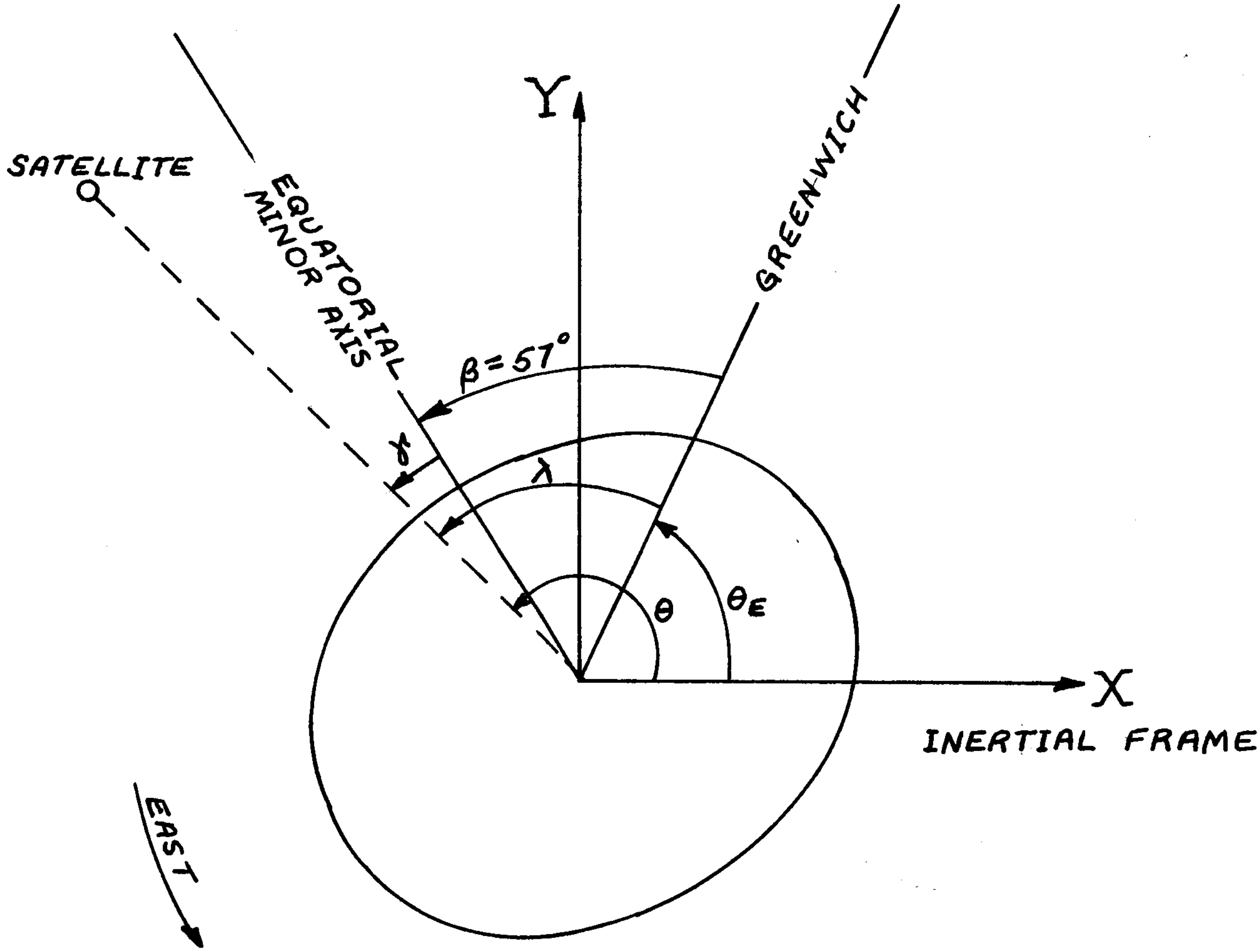

FIGURE 2. EARTH'S EQUATORIAL PLANE

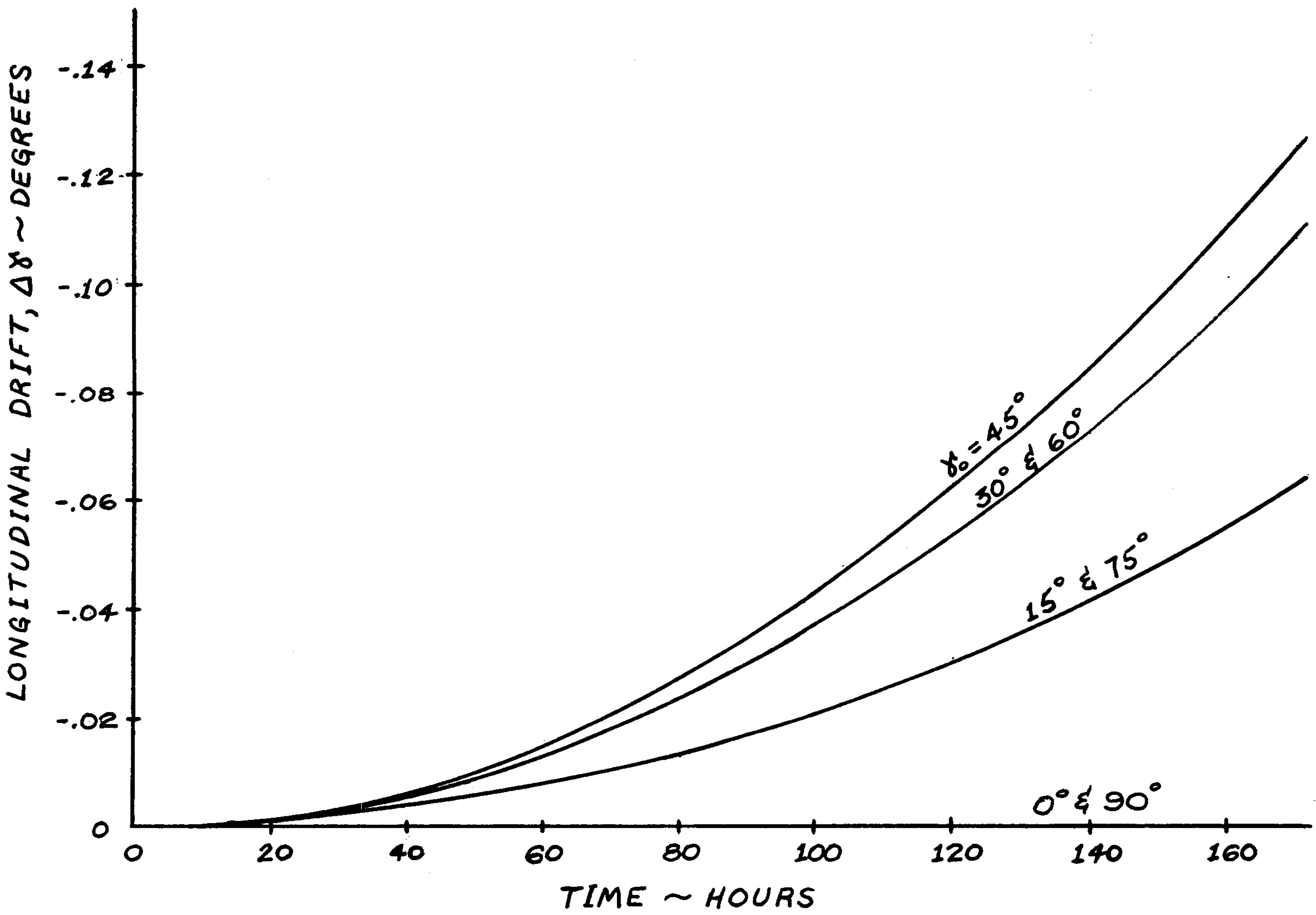

FIGURE 3. LONGITUDINAL DRIFT

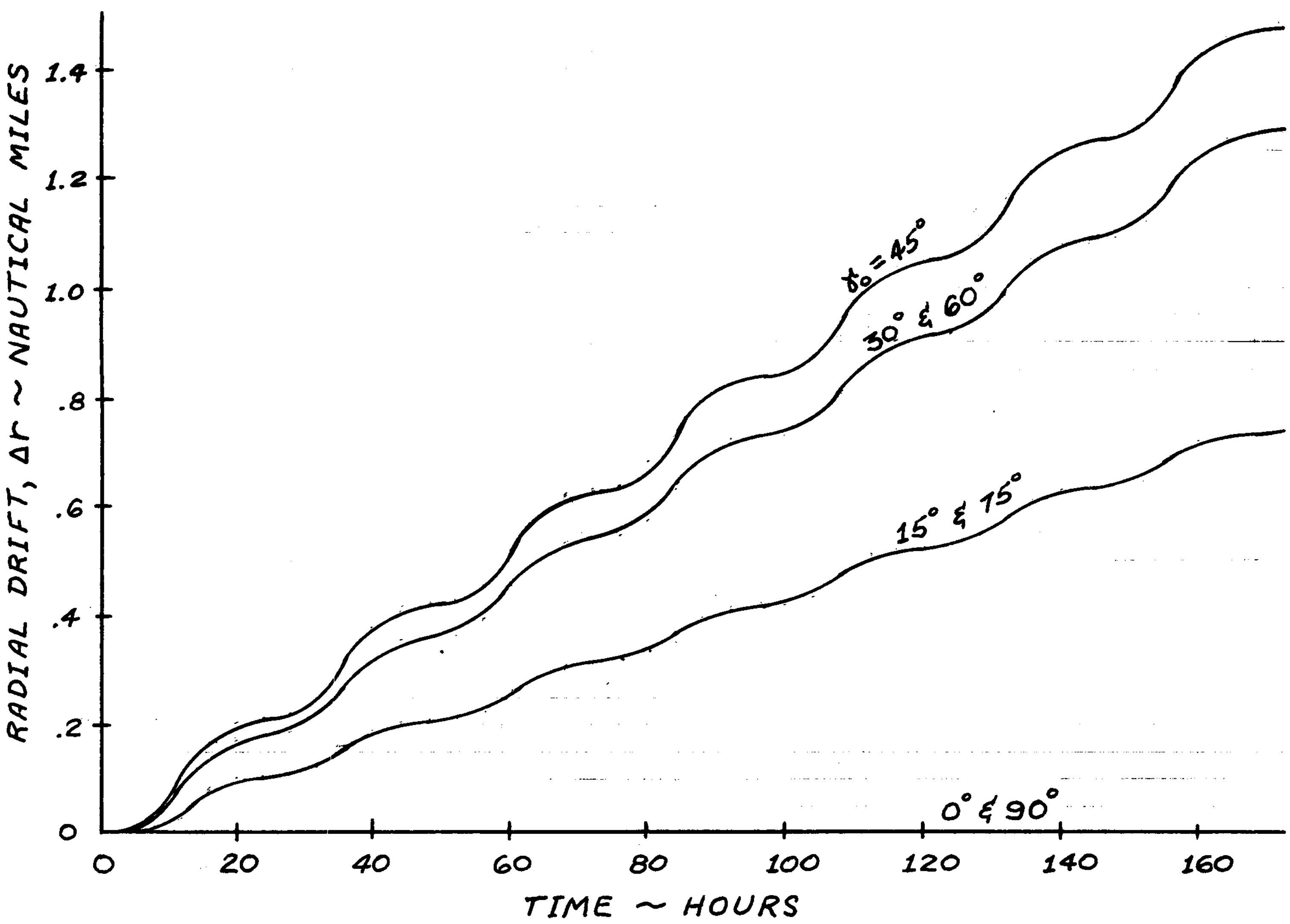

FIGURE 4. RADIAL DRIFT

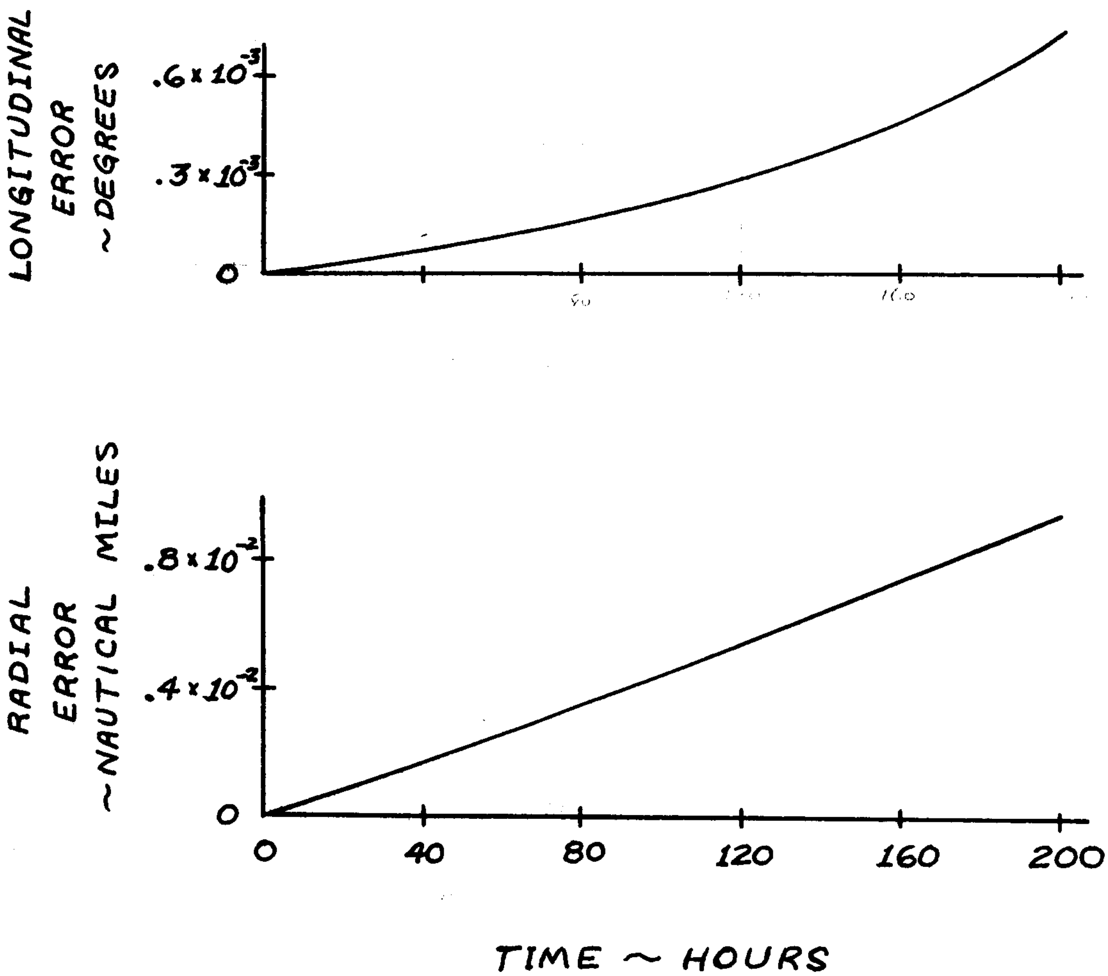

FIGURE 5. COMPARISON OF THE DRIFT RESULTS OF THE PERTURBATION STUDY WITH THE NUMERICAL INTEGRATION OF THE EQUATIONS OF MOTION AT $\gamma_o = 45°$. THE NUMERICAL INTEGRATION TECHNIQUE IS USED AS BASE.

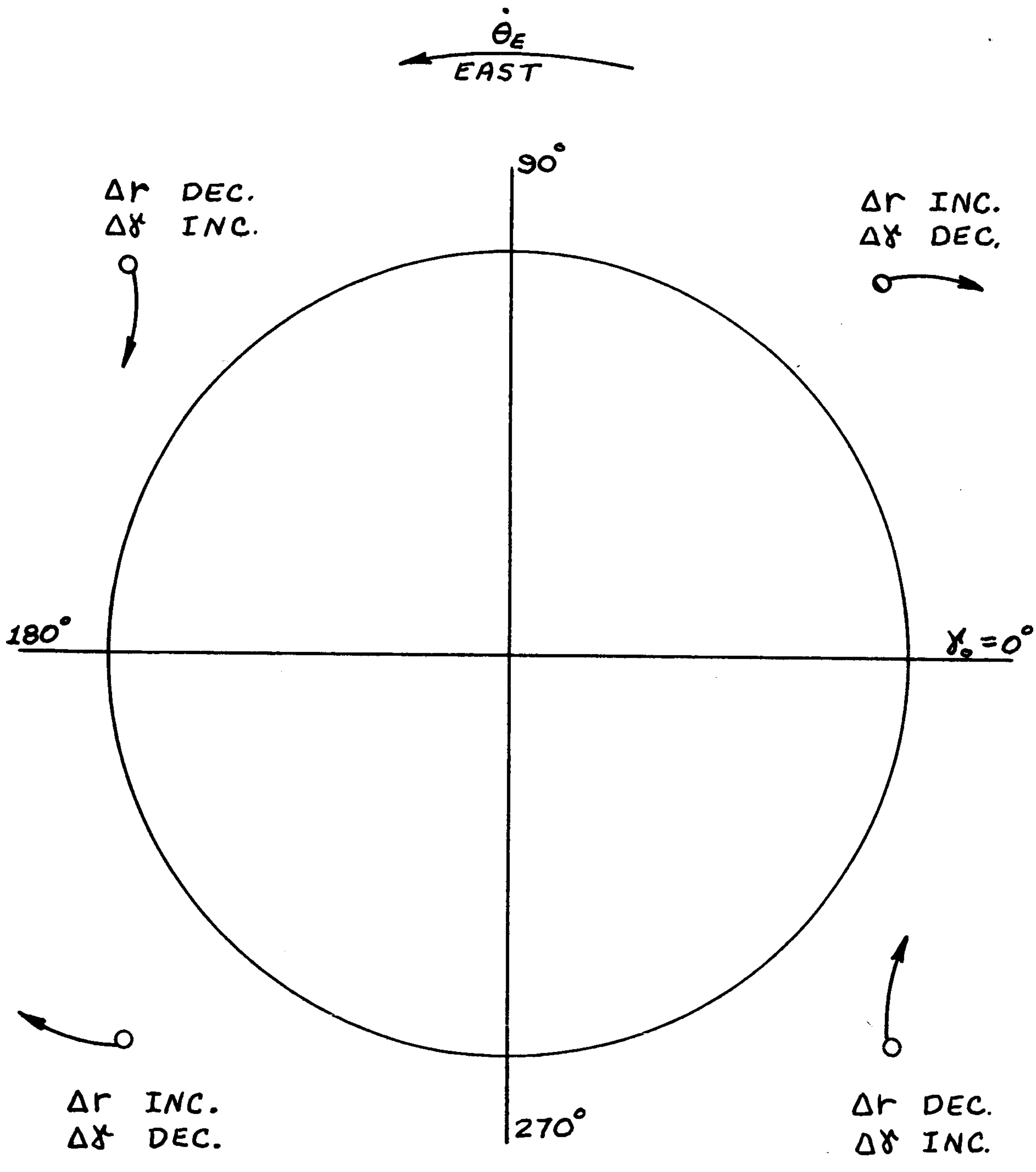

FIGURE 6. INITIAL SATELLITE MOTION