SYNOPSIS--A band-stop filter with a very narrow stop band can be produced by placing two RC distributed notch filters with slightly different notch frequencies in cascade. Design data, including stop-bandwidth, stop-band attenuation, and frequencies at several attenuation levels, are given. Some experimental results are also included.

A single-frequency rejection network can be produced by connecting a lumped impedance either in series or in parallel with an RC transmission line. Such notch filters are useful in numerous situations in which either a single frequency or an extremely small frequency band is to be rejected. These notch filters are generally difficult to adjust and maintain since the notch frequency is quite sensitive to parameter variation.

There are situations in which rejection is permitted to take place over a finite band of frequency. These situations arise either when a band-stop characteristic is desired or a single-frequency as well as frequencies in the vicinity of this frequency may be rejected. These situations permit the use of a band-stop filter in place of a single-frequency notch filter and circumvents the need for exactly locating and maintaining the notch frequency.

A simple means for achieving transfer characteristics useful in
these situations is to place two notch filters with slightly different notch frequencies in cascade. One such arrangement is shown in Fig. 1a in which two identical transmission lines of different lengths ($\lambda_1$ and $\lambda_2$) are used. Both notch frequencies are obtained by series resistances ($R_1$ and $R_2$). If the line parameters are $r_0$ ohms and $c_0$ farads per meter, and if

$$R_i = 0.0561844 \ r_0 \lambda_i, \quad i = 1, 2$$

(1)

then two transmission zeros are produced at $\omega_{01}$ and $\omega_{02}$ where

$$\omega_{0i} = \frac{11.166\lambda_i}{r_0 c_0 \lambda_i^2}, \quad i = 1, 2$$

(2)

Further, it is clear that

$$\frac{\omega_{01}}{\omega_{02}} = \left( \frac{\lambda_2}{\lambda_1} \right)^2 = K^2$$

(3)

The open-circuit voltage transfer characteristic of such an arrangement will have the general shape of that shown in Fig. 1b. The characteristic of Fig. 1b is plotted for $K = 1.25$. For other values of $K$, the voltage characteristics will still have this general shape. The minimum attenuation between the two notches ($L_0$) may be considered as the stop-band attenuation. The other two frequencies at which this attenuation occur may be regarded as the stop-band edges. The nominal band center ($\omega_0$), which is usually defined as the geometric mean of
the band-edge frequencies, is generally different from the frequency corresponding to $L_0$ ($\omega_0$).

A quantitative study of this class of filters has been made and the result summarized in Fig. 2. Fig. 2a gives $L_0$ for various values of $K$. The stop-band edges are shown in dashed curves in Fig. 2b. Fig. 2b also gives the frequencies outside the stop band at which various attenuation levels occur. These frequencies are all given in terms of $\omega_0$.

From the information contained in Fig. 2 the approximate characteristic of such a filter for any value of $K$ can readily be estimated. For example, for $K = 1.05$, we have approximately

$L_0 = 77.4$ dB

Stop-band edges: $\omega_2 = 1.08 \omega_0$ and $\omega_1 = 0.94 \omega_0$

Band center $= \omega_0' = \sqrt{\omega_1 \omega_2} = 1.02 \omega_0$

Per-unit bandwidth $= (\omega_2 - \omega_1)/\omega_0' = 0.138$

Attenuation at $2\omega_0' = 34.6$ dB

Attenuation at $\omega_0'/2 = 29.3$ dB

Hence, such a filter has a band-stop characteristic of about 14% bandwidth with approximately 43-dB or more discrimination at one octave from the band center.

The computed characteristic of Fig. 1b has been verified by an experimental network with RC lines made of Teledeltos paper with 1900
ohms per square resistance and Mylar film with thickness of 3 mils. The actual $\omega_n$ corresponds to 110 Kc. The observed points are shown as dots in Fig. 1b.

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Footnote  
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References


Legends to Illustrations

Fig. 1  
- a Filter arrangement  
- b Typical frequency characteristic ($K = 1.25$)

Fig. 2  
- a Stop-band attenuation versus value of $K$  
- b Normalized band edges (dashed curves) and points of several attenuation levels versus value of $K$
Fig. la Filter arrangement
Fig. 1b Typical frequency characteristic \( (K = 1.25) \)
Fig. 2b  Normalized band edges (dashed curves) and points of several attenuation levels versus value of K