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STATIC STIFFNESS AND DYNAMIC ANGULAR STIFFNESS OF THE COMBINED HYDROSTATIC JOURNAL - THRUST BEARING

by

J. W. Lund

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ABSTRACT

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This report presents an analysis of the externally pressurized combined journal thrust bearing where the flow from the journal bearing feeds a double acting, annular thrust bearing. The analysis assumes a large number of feeder holes such that the two journal bearing feeding planes are treated as line sources. Furthermore, small displacements are assumed. The solution obtains results for flow, radial and axial stiffness and dynamic angular stiffness and damping, all for the concentric journal position. Two computer programs have been written and instructions for their use are included in the appendicies.

Two numerical examples are performed and the results are presented graphically.

author

INTRODUCTION

In the application of the externally pressurized bearing it frequently happens that both a journal and a thrust bearing is required. Then it may be advantageous to combine the two bearings such that the flow leaving the journal bearing feeds the thrust bearing thereby reducing the total flow consumption. No loss in stiffness or load is necessarily encountered if the supply pressure is increased to compensate for the relative reduction of pressure drop available to each bearing. For a gyro gimbal axis bearing this feature is obviously desirable. Hence, there is a need to establish an analysis for predicting the performance of the combined bearing and for selecting the optimum bearing and orifice dimensions. This is the purpose of the present report.

The combined bearing consists of a journal bearing fed through orifice or Millipore restricted feeding holes in two symmetrically located feeding places. At each end of the journal bearing is an annular thrust bearing with an annular pocket closest to the journal bearing. It is wanted to calculate the load, the flow, the stiffness and the angular stiffness of the combined bearing and the effect of the various bearing dimensions such as radial and axial clearance, pocket dimensions, etc. An exact analysis is prohibitive primarily due to the difficulties in an accurate feeder hole representation and the circumferential variation of the journal bearing film thickness. Furthermore, from a design viewpoint a simplified analysis which properly accounts for the essential parameters may be of more value than an exact, but complex and time consuming solution. The presented analysis assumes that the number of feeder holes is sufficiently large that the feeding planes become line sources. In addition it is assumed that both radial and axial displacements are small in comparison with the clearances or, in other words, the solution gives the stiffness for the concentric position. The calculation of the angular stiffness includes the effect of frequency upon the stiffness and the damping for use in a vibratory response investigation.

The analysis has been programmed for computer calculation. Two programs are written: one for the static stiffness and flow and one for the dynamic angular stiffness and damping. Instructions for use of the programs and data input forms are given in appendicies. Two numerical examples have been calculated: one for the static stiffness and flow as a function of bearing dimensions for the NASA AB-5 Millipore bearing and one for the dynamic angular stiffness, damping and resonant frequency of the NASA AB-3 Millipore bearing. Comparison with test results indicates that the analysis predicts too large a stiffness, but calculates the flow accurately. Furthermore, qualitatively the results are reasonable and hence, should be valid for design selection.

DISCUSSION

DISCUSSION

The present analysis follows the methods developed by MTI under a previous contract with NASA (Reference 1). Two simplifying assumptions are: a) line source feeding replaces localized source feeding, and b) displacements are small.

The first assumption allows the bearing to be solved independently of the feeder characteristic and the second assumption linearizes the equations involved. From a practical point of view, this method ensures a minimum of essential parameters and short time and low cost for obtaining results. The data are in a form which adapts itself readily to clear and easily interpreted design charts. This is of importance when a bearing design has to be selected.

It is possible to obtain a more accurate solution, at least in principle, where the gas is fed at discrete points and the static displacement is finite (see Refs. 2 and 3). Such an analysis is considerably more complicated and requires a larger number of parameters. Therefore, each design point must be calculated separately which is both time consuming and costly. A comprehensive understanding of parameter changes and their relative importance is not readily acquired and the effort involved in the theoretical study may approach the effort spent in a corresponding test program.

As a first step in establishing an analysis for the bearing configuration under study an analysis as complex as the one just outlined does not seem justified at present. Furthermore, the given configuration is rather special and should await a refined theoretical investigation until more experience has been obtained on simpler and more general geometries. There are even indications that it may be possible to establish a form of equivalence between line source and point source feeding in which case the accuracy of the present analysis could be improved without drastic changes.

The analysis is divided into two parts: one for the radial and axial static stiffnesses and one for the dynamic angular stiffness. The first part is intended primarily for design selection and optimization whereas the second part is undertaken to ensure that the combined journal-thrust bearing does not have an inherent loss in angular stiffness. Subsequent comparisons with test results have shown that the calculated stiffnesses may be as much as 50% larger than the measured values. This discrepancy is primarily caused by the assumption of line source feeding as discussed briefly in Ref. 2. The calculated flow and axial stiffness agrees well with experimental data. It should be noted that once the correct flow is obtained the calculated axial stiffness should be close to the measured value since no feeding takes place inside the thrust bearing.

RESULTS

The analysis is covered by two computer programs:

1. PN0109: Stiffness and Flow of the Hydrostatic Combined Journal-Thrust Bearing.
2. PN0135: Angular Dynamic Stiffness of the Hydrostatic Combined Journal-Thrust Bearing.

The instructions for using the programs and interpretating the output results are given in Appendix A and Appendix B. Input forms are also included. The analyzed bearing geometry is shown in Figure 1. It should be noted that in calculating the static stiffness (PN0109) the thrust bearing pocket is assumed to be so deep that the pressure is constant in the pocket. This assumption is not made in calculating the dynamic angular stiffness (PN0135).

The computer results from PN0109 may be given in actual dimenions (i.e. lbs/in, ccm/min. etc), if so desired. PN0135, on the other hand, gives only dimensionless results. The conversion to actual dimensions is described in Appendix B.

<u>1st Numerical Example:</u> Static stiffness and flow of NASA AB-5 Millipore Bearing. Referring to Fig. 1 the bearing dimensions are:

$L = 1.977$ inch
$L_1 = 1.253$ inch
$D = 2.1625$ inch
$R_2 = .65$ inch
$N = 48$

The radial clearance C, the axial clearance C_T and the inner pocket radius R_1 are treated as variables in order to select the optimum values. The gas is air at 70^{o}F, i.e.

viscosity, $\mu = 2.83 \cdot 10^{-9}$ lbs-sec/in^2
(gas constant) · (total temperature), $\mathcal{R}\,T = 342{,}500$ in.

The Millipore coefficients are as given for NASA AB-5, Sleeve 4, Appendix A, Page 31. The calculated results are shown graphically in Figs. 2 to 8 for radial clearance values of .25 to $1.25 \cdot 10^{-3}$ inch also for three values of the clearance ratio C_T/C and for three supply pressures. Three values of the inner pocket radius are used, i.e. R_1 = .76, .86 and .96 inch. The detailed calculations are only given for R_1 = .86 i.e. Figs. 2-7, but the variation with R_1 is summarized in Fig. 8 for equal radial and tangential stiffness for the optimum condition.

<u>2nd Numerical Example</u>: Dynamic Angular Stiffness and Damping of NASA AB-3 Millipore bearing. The bearing dimensions are (Fig. 1):

L = 1.183 inch
L_1 = .783 inch
D = 1.422 inch
R_1 = .585 inch
R_2 = .425 inch
C = .00071 inch
C_T = .00082 inch
C_H = .00107 inch
N = 48

The Millipore coefficients are as given for NASA AB-3, Appendix A, page 31. The gas is air at 70°F and the ambient pressure is 14.7 psia. Three supply pressure values are considered: $\Delta P = P_s - P_a$ = 5, 10 and 15 psig.

The angular stiffness and damping are calculated for three vibratory frequencies: 3.45, 250 and 500 CPS to evaluate the effect of gas compressibility due to frequency. These results are then used to compute the corresponding resonant frequency:

$$N_c = \frac{1}{2\pi}\sqrt{\frac{K_a}{I_T}}\ \sqrt{1-\left(\frac{B_a}{B_c}\right)^2}$$

where

N_c = Resonant frequency, CPS
K_a = angular stiffness, lbs-in/rad.
B_a = angular damping, lbs-in-sec/rad.

B_c = critical damping = $2\sqrt{K_a I_T}$, lbs-in-sec/rad
I_T = transverse mass moment of inertia, lbs-in-sec^2

and the log decrement δ:

$$\delta = 2\pi\left(\frac{B_a}{B_c}\right)\Big/\sqrt{1-\left(\frac{B_a}{B_c}\right)^2} = \log_e\left(\frac{x_1}{x_2}\right)$$

where x_1 and x_2 are the values of two successive amplitudes to the same side. The transverse mass moment of inertia is I_T = .477 g-cm-sec^2. The results are summarized in a table:

Angular Stiffness and Resonance of AB-3
Millipore Bearing

ΔP psig	Flow ccm/min	Radial Stiffn. kp/cm	Axial Stiffn. kp/cm	Freq. CPS	Angular Stiffn. kp·cm/rad	Angular Damping kp·cm·sec/rad	$\left(\frac{B_a}{B_c}\right)$	Log Decr. δ	$\frac{x_1}{x_2}$	Reson. Freq. CPS
				3.45	397	.618	.698	6.11	45	104
5	358	5,330	2,680	250	515	.45	.451	3.18	24	148
				500	705	.354	.304	2.05	7.8	185
				3.45	925	.601	.454	3.20	25	198
10	860	11,800	6,150	250	1,025	.44	.315	2.08	8.0	221
				500	1,190	.345	.228	1.47	4.35	245
				3.45	1,600	.59	.334	2.23	9.3	273
15	1545	19,200	10,450	250	1,680	.426	.239	1.54	4.7	289
				500	1,820	.332	.178	1.14	3.15	306

Comparison with test results shows that the calculated stiffness is up to 50 percent larger than the measured value. Hence, the resonant frequency is also too high but fair agreement is obtained with the calculated log decrement. Furthermore, the qualitative trend predicted by the analysis agrees with the experimental data.

A comparison between the combined journal-thrust bearing and the bearing, where journal and thrust bearing are separately fed, has been made on the basis of angular stiffness. For the same flow the combined bearing has a higher angular stiffness. The reason is that for the small operating flows the separately fed thrust bearing is very ineffective for angular displacement whereas in the combined bearing the thrust bearing becomes partly an extension of the journal bearing.

Finally an approximate stability calculation has been performed. Since the present analysis does not take the dynamic radial and axial stiffness into account use is made of the analysis of the dynamic journal load in Reference 1. The criterion for instability is that either the damping or the stiffness becomes negative. The calculation assumes that ΔP is kept constant while the ambient pressure is reduced until instability occurs. For the above AB-3 configuration the following results are obtained

ΔP psig	Ambient Pressure for Instability psia
5	4.0
10	6.8
15	9.1

ANALYSIS

The bearing geometry is shown in Figure 1. The bearing consists of a journal bearing fed from two feeding planes, each with 1/2 N equidistant restricted feeding holes around the circumference. The flow from the journal bearing feeds the two end thrust bearings. The thrust bearing is an annulus with a circumferential step such that the outer part forms a pocket whereas the inner part serves as a seal.

The supply pressure is denoted P_s, the downstream pressure from the feeding hole is P_i and the ambient pressure is P_a. The gas film thickness is h. Then the pressure in the gas film is described by Reynolds Equation (Ref.1):

(1) journal bearing: $$\frac{\partial}{\partial\theta}\left[h^3\frac{\partial P^2}{\partial\theta}\right]+\frac{\partial}{\partial\zeta}\left[h^3\frac{\partial P^2}{\partial\zeta}\right]=2\sigma\frac{\partial(Ph)}{\partial\tau}$$

(2) thrust bearing: $$\frac{1}{r}\frac{\partial}{\partial r}\left[r\,h_T^3\frac{\partial P^2}{\partial r}\right]+\frac{\partial}{r\partial\theta}\left[h_T^3\frac{\partial P^2}{r\partial\theta}\right]=2\sigma_T\frac{\partial(Ph)}{\partial\tau}$$

under the assumption of isothermal conditions and no rotation. The equations are dimensionless such that:

$$\zeta=\frac{\bar{\zeta}}{R}\qquad \tau=\omega t\qquad h=\frac{\bar{h}}{C}=1+\varepsilon\cos\theta\qquad P=\frac{\bar{P}}{P_a}$$

$$r=\frac{\bar{r}}{R_2}\qquad h_T=\frac{\bar{h}_T}{C_T}=1+\varepsilon_T$$

and

(3) $$\sigma=\frac{12\mu\omega}{P_a}\left(\frac{R}{C}\right)^2$$

(4) $$\sigma_T=\frac{12\mu\omega}{P_a}\left(\frac{R_2}{C_T}\right)^2=\sigma\left(\frac{R_2}{R}\frac{C}{C_T}\right)^2$$

The orifice feeding is approximated by a line source at $\zeta=0$ (See Fig. 1):

(5) $$h^3\left[-\frac{\partial P^2}{\partial \zeta}\Big|_{\zeta=0}+\frac{\partial P^2}{\partial \zeta_1}\Big|_{\zeta_1=\xi_1}\right]=\Lambda_t V m$$

where:

(6) $$\Lambda_t=\frac{6\mu N a^2\sqrt{\mathcal{R}T}}{P_a C^3}$$

(7) $$V=\frac{P_s}{P_a}$$

(8) $$m=\frac{M\sqrt{\mathcal{R}T}}{\pi a^2 P_s}=\nu\sqrt{\frac{2k}{k-1}}\left(\frac{P_i}{V}\right)^{1/k}\sqrt{1-\left(\frac{P_i}{V}\right)^{\frac{k-1}{k}}}$$

To solve Eqs.(1) and (2) with Eq. (5) as a boundary condition a first order perturbation solution is employed. The analysis is performed in two parts: a) static load ($\delta=0$) and b) dynamic, angular stiffness.

<u>Static Load ($\delta=0$)</u>

To obtain a first order perturbation solution set:

(9) $$P=P_0+\varepsilon P_1+\varepsilon_T P_2$$

Such that:

$$h^3=1+3\varepsilon\cos\theta$$
$$h_T^3=1\mp 3\varepsilon_T$$
$$P^2=P_0^2+2\varepsilon P_0P_1+2\varepsilon_T P_0 P_2$$

Substituting into Eqs. (1), (2) and (5) we get:

$$\left.\begin{aligned}
&(10)\quad \frac{\partial^2 P_0^2}{\partial\theta^2}+\frac{\partial P_0^2}{\partial\zeta^2}=0\\
&(11)\quad \frac{\partial^2(P_0P_1)}{\partial\theta^2}+\frac{\partial^2(P_0P_1)}{\partial\zeta^2}=0 \qquad \left(\text{since } \frac{\partial P_0^2}{\partial\theta}=0\right)\\
&(12)\quad \frac{\partial^2(P_0P_2)}{\partial\theta^2}+\frac{\partial^2(P_0P_2)}{\partial\zeta^2}=0
\end{aligned}\right\}\text{Journal bearing}$$

$$\left.\begin{aligned}
&(13)\quad \frac{1}{r}\frac{\partial}{\partial r}\left[r\,\frac{\partial P_0^2}{\partial r}\right] + \frac{\partial^2 P_0^2}{r^2\partial\theta^2} = 0\\
&(14)\quad \frac{1}{r}\frac{\partial}{\partial r}\left[r\,\frac{\partial (P_0P_1)}{\partial r}\right] + \frac{\partial^2 (P_0P_1)}{r^2\partial\theta^2} = 0 \qquad \left(\text{since } \frac{\partial P_0^2}{\partial\theta} = 0\right)\\
&(15)\quad \frac{1}{r}\frac{\partial}{\partial r}\left[r\,\frac{\partial (P_0P_2)}{\partial r}\right] + \frac{\partial^2 (P_0P_2)}{r^2\partial\theta^2} = 0
\end{aligned}\right\}\text{thrust bearing}$$

(16) $$-\left.\frac{\partial P_0^2}{\partial \zeta}\right|_{\zeta=0} + \left.\frac{\partial P_0^2}{\partial \zeta_1}\right|_{\zeta_1=\xi_1} = \Lambda_t V m_0$$

(17) $$\left.\frac{\partial (P_0P_1)}{\partial \zeta}\right|_{\zeta=0} - \left.\frac{\partial (P_0P_1)}{\partial \zeta_1}\right|_{\zeta_1=\xi_1} = \frac{3}{2}\Lambda_t V m_0 \cos\theta + \psi\,(P_0P_1)_i$$

(18) $$\left.\frac{\partial (P_0P_2)}{\partial \zeta}\right|_{\zeta=0} - \left.\frac{\partial (P_0P_2)}{\partial \zeta_1}\right|_{\zeta_1=\xi_1} = \psi\,(P_0P_2)_i$$

where m_o is given by Eq. (8) with $P_i = P_{o,i}$ and

(19) $$\psi = -\frac{\Lambda_t}{2P_{0i}}\left.\frac{\partial m}{\partial\left(\frac{P_i}{V}\right)}\right|_{P_i=P_{0i}}$$

The zero-order solution is:

(20) $\underline{1 \le r \le \gamma}$ $$P_0^2 = 1 + q_T \ln r$$

(21) $\underline{\gamma \le r \le \eta}$ $$P_0^2 = 1 + (q_T - q_H)\ln\gamma + q_H \ln r$$

(22) $\underline{0 \le \zeta \le \xi_2}$ $$P_0^2 = 1 + \left[\left(\frac{c}{c_T}\right)^3 \ln\gamma + \left(\frac{c}{c_H}\right)^3 \ln\left(\frac{\eta}{\gamma}\right) + \xi_2 - \zeta\right] q$$

(23) $\underline{0 \le \zeta_1 \le \xi_1}$ $$P_0^2 = 1 + \left[\left(\frac{c}{c_T}\right) \ln\gamma + \left(\frac{c}{c_H}\right)^3 \ln\left(\frac{\eta}{\gamma}\right) + \xi_2\right] q$$

where:

(24) $$q_H = \left(\frac{c}{c_H}\right)^3 q \qquad\qquad q_T = \left(\frac{c}{c_H}\right)^3 q$$

(25) $$q = \Lambda_t V m_0$$

Since the downstream feeder hole pressure P_i is given by Eq. (23) and m_o is given by Eq. (8), Eq. (25) may be solved for $P_{o,i}$ and, hence, for q.

To obtain the solution for $P_o P_1$ set:

(26) $$P_o P_1 = H \cos\theta$$

The solution becomes:

(27) $\underline{1 \le r \le \gamma}$ $$H = A_4 \left(r - \frac{1}{r}\right)$$

(28) $\underline{\gamma \le r \le \eta}$ $$H = A_3 \left(r + \frac{\varkappa}{r}\right)$$

(29) $\underline{0 \le \zeta \le \xi_2}$ $$H = -A_2 \left(\cosh\zeta + \vartheta \cdot \sinh\zeta\right) + \frac{3}{2} q \sinh\zeta$$

(30) $\underline{0 \le \zeta_1 \le \xi_1}$ $$H = -A_2 \frac{\cosh\zeta_1}{\cosh\xi_1}$$

where:

(31) $$\varkappa = \gamma^2 \cdot \frac{1 - \left(\frac{C_T}{C_H}\right)^3 \frac{\gamma^2+1}{\gamma^2-1}}{1 + \left(\frac{C_T}{C_H}\right)^3 \frac{\gamma^2+1}{\gamma^2-1}}$$

(32) $$\lambda = \left(\frac{C}{C_H}\right)^3 \frac{\eta^2 + \varkappa}{\eta^2 - \varkappa}$$

(33) $$\vartheta = \tanh\xi_1 + \psi$$

(34) $$A_2 = \frac{3}{2} q \frac{\sinh\xi_2 + \lambda(\cosh\xi_2 - 1)}{(1+\lambda\vartheta)\cosh\xi_2 + (\lambda+\vartheta)\sinh\xi_2}$$

(35) $$A_3 = \frac{\frac{3}{2} q \lambda}{\eta + \frac{\varkappa}{\eta}} \frac{\cosh\xi_2 + \vartheta \sinh\xi_2 - 1}{(1+\lambda\vartheta)\cosh\xi_2 + (\lambda+\vartheta)\sinh\xi_2}$$

(36) $$A_4 = \frac{2\gamma^2}{\gamma^2 - 1 + \left(\frac{C_T}{C_H}\right)^3 (\gamma^2+1)} A_3$$

Due to axial symmetry P_oP_2 is independent of θ. The solution is:

(37) $\underline{1 \le r \le \gamma}$ $\qquad P_oP_2 = B_4 \ln r$

(38) $\underline{\gamma \le r \le \eta}$ $\qquad P_oP_2 = B_3 \ln r + C_3$

(39) $\underline{0 \le \zeta \le \xi_2}$ $\qquad P_oP_2 = \left(\frac{\xi_1}{1+\xi_1\psi} + \zeta\right) B_2$

(40) $\underline{0 \le \zeta_1 \le \xi_1}$ $\qquad P_oP_2 = \frac{B_2}{1+\xi_1\psi}\zeta_1$

where:

$$B_4 = \frac{3}{2}q \frac{\left(\frac{C}{C_T}\right)^3\left(1-\frac{C_T}{C_H}\right)\ln\left(\frac{\eta}{\gamma}\right) + \left(\frac{C_H}{C_T}\right)^3\left(\xi_2 + \frac{\xi_1}{1+\xi_1\psi}\right)}{\ln\eta + \left[\left(\frac{C_H}{C_T}\right)^3 - 1\right]\ln\gamma + \left(\frac{C_H}{C}\right)^3\left(\xi_2 + \frac{\xi_1}{1+\xi_1\psi}\right)} \tag{41}$$

$$B_3 = \frac{3}{2}q \frac{\left(\frac{C_T}{C_H}\right)\left(\xi_2 + \frac{\xi_1}{1+\xi_1\psi}\right) - \left(\frac{C}{C_T}\right)^3\left(1-\frac{C_T}{C_H}\right)\ln\gamma}{\ln\eta + \left[\left(\frac{C_H}{C_T}\right)^3 - 1\right]\ln\gamma + \left(\frac{C_H}{C}\right)^3\left(\xi_2 + \frac{\xi_1}{1+\xi_1\psi}\right)} \tag{42}$$

$$C_3 = \frac{3}{2}q \ln\gamma \frac{\left(\frac{C}{C_T}\right)^3\left(1-\frac{C_T}{C_H}\right)\ln\eta + \left[\left(\frac{C_H}{C_T}\right)^3 - \frac{C_T}{C_H}\right]\left(\xi_2 + \frac{\xi_1}{1+\xi_1\psi}\right)}{\ln\eta + \left[\left(\frac{C_H}{C_T}\right)^3 - 1\right]\ln\gamma + \left(\frac{C_H}{C}\right)^3\left(\xi_2 + \frac{\xi_1}{1+\xi_1\psi}\right)} \tag{43}$$

$$B_2 = \frac{3}{2}q\frac{C_T}{C_H} - \left(\frac{C_H}{C}\right)^3 B_3 \tag{44}$$

The radial and axial load is obtained by integration:

$$W_r = -2P_aR^2\varepsilon\left[\int_0^{2\pi}\int_0^{\xi_1} P_1\cos\theta\, d\zeta_1 d\theta + \int_0^{2\pi}\int_0^{\xi_2} P_1\cos\theta\, d\zeta\, d\theta\right] \tag{45}$$

$$W_a = 2P_aR_2^2\varepsilon_T\left[\int_0^{2\pi}\int_1^{\gamma} P_2\, r dr d\theta + \int_0^{2\pi}\int_\gamma^{\eta} P_2\, r dr d\theta\right] \tag{46}$$

Hence, the dimensionless radial and axial stiffness become:

$$\frac{K_r}{\frac{1}{C}DL(P_s-P_a)} = \frac{\pi}{2(\xi_1+\xi_2)(V-1)}\left[\frac{A_2\tanh\xi_1}{P_{oi}} + \int_0^{\xi_2}\frac{A_2(\cosh\zeta + \nu\sinh\zeta) - \frac{3}{2}q\sinh\zeta}{P_o}d\zeta\right] \tag{47}$$

(48) $$\frac{K_a}{\frac{1}{C_T}\pi(R^2-R_2^2)(P_s-P_a)} = \frac{4}{(\eta^2-1)(V-1)}\left[B_4\int_1^{\gamma}\frac{r\ln r\,dr}{\sqrt{1+q_T\ln r}} + \int_{\gamma}^{\eta}\frac{r(B_3\ln r+C_3)\,dr}{\sqrt{1+(q_T-q_H)\ln\gamma+q_H\ln r}}\right]$$

The dimensionless total mass flow is:

(49) $$q = \frac{6\mu \Re T}{\pi P_a^2 C^3} M_{Total}$$

or in alternate form

(50) $$m_o = \frac{q}{\Lambda_t V} = \frac{M_{Total}\sqrt{\Re T}}{N \pi a^2 P_s}$$

The axial displacement causes a change in flow through the thrust bearings:

(51) $$\frac{6\mu \Re T}{\pi P_a^2 C^3}\Delta M = \pm\left[\left(\frac{C_T}{C}\right)^3 B_4 - \frac{3}{2}q\right]\varepsilon_T \qquad \begin{pmatrix} + \sim h = 1-\varepsilon_T \\ - \sim h = 1+\varepsilon_T \end{pmatrix}$$

Dynamic Angular Stiffness

Let the journal perform vibrations with angular displacement α and frequency ω around the concentric position such that the dimensionless gas film thickness becomes:

(52) journal bearing: $h = 1 - \frac{R}{C}\zeta_1 \alpha e^{i\tau}\cos\theta$

(53) thrust bearing: $h_T = 1 + \frac{R_2}{C_T} r\alpha e^{i\tau}\cos\theta$

for the right hand part of the bearing in Figure 1. Considering first the journal bearing, a first order perturbation in α is employed setting:

(54) $$P = P_o + \alpha e^{i\tau} P_1$$

i.e.,

$$P^2 = P_o^2 + 2\alpha e^{i\tau} P_o P_1$$

$$h^3 = 1 - 3\frac{R}{C}\zeta_1 \alpha e^{i\tau}\cos\theta$$

Introduce:

(55) $$P_o P_1 = \frac{R}{C} H(\zeta) \cos\theta$$

and subsitute into Eq. (1) to get:

(56) $$\frac{\partial^2 P_o^2}{\partial \zeta^2} = 0$$

(57) $$\frac{\partial^2 H}{\partial \zeta^2} - \left(1 + \frac{i\sigma}{P_o}\right) H = \frac{3}{2}\frac{\partial P_o^2}{\partial \zeta} - i\sigma P_o \zeta_1$$

Since $\frac{\partial P_o^2}{\partial \theta} = 0$ because of symmetry. The solution of Eq. (56) is given by Eqs.(22) and (23). Eq.(57) may be solved between the feeding planes:

(58) $\underline{0 \le \zeta_1 \le \xi_1}$ $$H = \left[H_o - \frac{i\sigma P_{oi}\xi_1}{1 + \frac{i\sigma}{P_{oi}}}\right] \frac{\sinh \zeta_1(\alpha + i\beta)}{\sinh \xi_1(\alpha + i\beta)} + \frac{i\sigma P_{oi}\zeta_1}{1 + \frac{i\sigma}{P_{oi}}}$$

where:

(59) $$\alpha + i\beta = \sqrt{1 + \frac{i\sigma}{P_{oi}}}$$

i.e.,

(60) $$\alpha = \sqrt{\frac{1}{2}\left[1 + \sqrt{1 + \left(\frac{\sigma}{P_{oi}}\right)^2}\,\right]}$$

(61) $$\beta = \sqrt{\frac{1}{2}\left[-1 + \sqrt{1 + \left(\frac{\sigma}{P_{oi}}\right)^2}\,\right]}$$

(62) $$H_o = H_{\zeta_1 = \xi_1}$$

and P_{oi} is given by Eq. (23)

The boundary condition for Eq. (57) then becomes:

$$\left.\frac{\partial H}{\partial \zeta}\right|_{\zeta=0} = -\frac{3}{2} q \xi_1 + \left.\frac{\partial H}{\partial \zeta_1}\right|_{\zeta_1 = \xi_1} + \psi H_o$$

or

(63) $$\left.\frac{dH}{d\zeta}\right|_{\zeta=0} = -\frac{3}{2} q \xi_1 - \frac{i\sigma P_{oi}}{1 + \frac{i\sigma}{P_{oi}}}\left[\xi_1(\alpha + i\beta)\coth \xi_1(\alpha + i\beta) - 1\right] + \left[(\alpha + i\beta)\coth \xi_1(\alpha + i\beta) + \psi\right] H_o$$

Eq. (57) may be written:

(64) $\underline{0 \leq \zeta \leq \xi_2}$ $\quad \dfrac{d^2H}{d\zeta^2} - EH = F$

where

(65) $$E = 1 + \frac{i\delta}{P_o}$$

(66) $$F = -\tfrac{3}{2}q - i\delta P_o(\xi_1 + \zeta)$$

Eq.(64) may be integrated twice to get:

(67) $$H = \int_0^{\zeta} (\zeta - x)G\,dx + \left.\frac{dH}{d\zeta}\right|_o \zeta + H_o$$

(68) $$G = EH + F$$

In finite difference form Eq. (67) becomes:

(69) $\underline{0 \leq n \leq m-1}$ $\quad H_{n+1} = H_n + (\Delta\zeta)^2\left[\tfrac{1}{2}G_o + G_1 + \text{-----} + G_n\right] + \Delta\zeta \left.\dfrac{dH}{d\zeta}\right|_o$

where

(70) $$\Delta\zeta = \frac{\xi_2}{m}$$

For computer calculations set:

(71) $$H_n = s_{rn} + is_{in} + (t_{rn} + it_{in})H_o$$

(72) $$(\Delta\zeta)^2\left[\tfrac{1}{2}G_o + G_1 + G_2 + \text{---} + G_n\right] = p_{rn} + ip_{in} + (q_{rn} + iq_{in})H_o$$

(73) $$a + ib = -\tfrac{3}{2}q\xi_1 - \frac{i\delta P_{oi}}{1 + \frac{i\delta}{P_{oi}}}\left[\xi_1(\alpha + i\beta)\coth\xi_1(\alpha + i\beta) - 1\right]$$

(74) $$c + id = (\alpha + i\beta)\coth\xi_1(\alpha + i\beta) + \psi$$

Then from Eqs.(22), (63) and (69)

(75) $$P_{on} = \sqrt{1+\left[\left(\frac{C}{C_T}\right)^3 \ln\gamma + \left(\frac{C}{C_H}\right)^3 \ln\left(\frac{\eta}{\gamma}\right) + \xi_2 - n\cdot\Delta\zeta\right] q}$$

(76) $$p_{rn} = p_{r,n-1} + (\Delta\zeta)^2\left[s_{rn} - \frac{\delta}{P_{on}} s_{in} - \frac{3}{2} q\right]$$

(77) $$p_{in} = p_{i,n-1} + (\Delta\zeta)^2\left[s_{in} + \frac{\delta}{P_{on}} s_{rn} - \delta P_{on}(\xi_1 + n\Delta\zeta)\right]$$

(78) $$q_{rn} = q_{r,n-1} + (\Delta\zeta)^2\left[t_{rn} - \frac{\delta}{P_{on}} t_{in}\right]$$

(79) $$q_{in} = q_{i,n-1} + (\Delta\zeta)^2\left[t_{in} + \frac{\delta}{P_{on}} t_{rn}\right]$$

(80) $$s_{r,n+1} = s_{rn} + p_{rn} + a\Delta\zeta$$

(81) $$s_{i,n+1} = s_{in} + p_{in} + b\Delta\zeta$$

(82) $$t_{r,n+1} = t_{rn} + q_{rn} + c\Delta\zeta$$

(83) $$t_{i,n+1} = t_{in} + q_{in} + d\cdot\Delta\zeta$$

To initiate the calculation set:

(84) $$p_{ro} = -\frac{1}{2}(\Delta\zeta)^2 \frac{3}{2} q$$

(85) $$p_{io} = -\frac{1}{2}(\Delta\zeta)^2 \delta P_{oi} \xi_1$$

(86) $$q_{ro} = \frac{1}{2}(\Delta\zeta)^2$$

(87) $$q_{io} = \frac{1}{2}(\Delta\zeta)^2 \frac{\delta}{P_{oi}}$$

(88) $$s_{ro} = s_{io} = t_{io} = 0$$

(89) $$t_{ro} = 1$$

Thus the calculation procedure is:

1) preset p_{ro}, p_{io} ---- t_{io}, from Eqs. (84) - (89)
2) set n=0 and calculate Eqs. (80) - (83)
3) advance n by 1 and repeat the cycle Eqs. (75) to (83) to n = m - 1.

Having completed the calculation the values of H and of $\frac{dH}{d\zeta}$ at $\zeta = \xi_2$ are:

(90) $$H_1 = H_{\zeta=\xi_2} = H_m = s_{rm} + i s_{im} + (t_{rm} + i t_{im}) H_o = a_1 + i b_1 + (c_1 + i d_1) H_o$$

(91) $$H_1' = \frac{dH}{d\zeta}\Big|_{\zeta=\xi_2} = \Delta\zeta \left[\tfrac{1}{2} G_0 + G_1 + \text{----} + G_{m-1} + \tfrac{1}{2} G_m\right] + a + ib + (c + id) H_o$$
$$= \frac{1}{\Delta\zeta}\left[p_{r,m-1} + i p_{i,m-1} + (q_{r,m-1} + i q_{i,m-1}) H_o\right] + \tfrac{1}{2}\Delta\zeta\left[\left(1 + \frac{i\sigma}{P_{om}}\right) H_1 - \tfrac{3}{2} q - i\sigma P_{om}(\xi_1 + \xi_2)\right] + a + ib + (c + id) H_o$$
$$= a_2 + i b_2 + (c_2 + i d_2) H_o$$

where H_o must be determined by combining the journal bearing with the thrust bearing as shown later.

The journal bearing moment is given by:

$$M_J = 2 P_a R^3 \alpha e^{i\tau} \int_0^{\xi_1+\xi_2} \int_0^{2\pi} P_1 \cos\theta \, \zeta_1 \, d\theta \, d\zeta_1$$

or in dimensionless form:

(92) $$\frac{M_J/\alpha}{\frac{1}{c} L D (P_s - P_a) L^2} = \frac{\frac{\pi}{8}}{(V-1)(\xi_1+\xi_2)^3} e^{i\tau} \left[\int_0^{\xi_1} \frac{H}{P_o} \zeta_1 \, d\zeta_1 + \int_0^{\xi_2} \frac{H}{P_o} \zeta \, d\zeta \right]$$

The first integral can beevaluated from Eq. (58).

(93) $$\int_0^{\xi_1} \frac{H}{P_o} \zeta_1 \, d\zeta_1 = \frac{1}{P_{oi} + i\sigma}\left[\left(H_o - \frac{i\sigma P_{oi}\xi_1}{1 + \frac{i\sigma}{P_{oi}}}\right)\left(\xi_1(\alpha + i\beta)\coth \xi_1(\alpha + i\beta) - 1\right) + i\,\tfrac{1}{3}\sigma P_{oi}\xi_1^3\right]$$

The second integral is evaluated by numerical integration.

The thrust bearing is treated analogous to the journal bearing. A first order perturbation in α is used:

$$P = P_0 + \alpha e^{i\tau} P_1$$

i.e.,

$$P^2 = P_0^2 + 2\alpha e^{i\tau} P_0 P_1$$

(94) $\underline{1 \leq r \leq \gamma}$ $$h^3 = 1 + 3 \frac{R_2}{C_T} r \alpha e^{i\tau} \cos\theta$$

(95) $\underline{\gamma \leq r \leq \eta}$ $$h^3 = 1 + 3 \frac{R_2}{C_H} r \alpha e^{i\tau} \cos\theta$$

Set:

(96) $\underline{1 \leq r \leq \gamma}$ $$P_0 P_1 = \frac{R_2}{C_T} H_T(r) \cos\theta$$

(97) $\underline{\gamma \leq r \leq \eta}$ $$P_0 P_1 = \frac{R_2}{C_H} H_H(r) \cos\theta$$

and substitute into Eq. (2) to get:

$$\frac{1}{r}\frac{d}{dr}\left[r \frac{dP_0^2}{dr}\right] = 0$$

(98) $\underline{1 \leq r \leq \gamma}$ $$\frac{1}{r}\frac{d}{dr}\left[r \frac{dH_T}{dr}\right] - \left(\frac{1}{r^2} + \frac{i\sigma_T}{P_0}\right) H_T = -\frac{3}{2}\frac{dP_0^2}{dr} + i\sigma_T P_0 r$$

(99) $\underline{\gamma \leq r \leq \eta}$ $$\frac{1}{r}\frac{d}{dr}\left[r \frac{dH_H}{dr}\right] - \left(\frac{1}{r^2} + \frac{i\sigma_H}{P_0}\right) H_H = -\frac{3}{2}\frac{dP_0^2}{dr} + i\sigma_H P_0 r$$

(100) $$\sigma_H = \sigma \cdot \left(\frac{R_2}{R}\frac{C}{C_H}\right)^2$$

and σ_T is given by Eq. (4). The solution for P_0 is given by Eq.(20) and (21). The boundary conditions for Eqs. (98) and (99) are:

(101) $\underline{r = \eta}$ $$H_{H,\eta} = \eta \frac{C_H}{C} H_1$$

(102) $\underline{r = \eta}$ $$\left.\frac{dH_H}{dr}\right|_\eta = -\left(\frac{C}{C_H}\right)^2 \left[\frac{3}{2} q \left(\frac{C}{C_H} + \xi_1 + \xi_2\right) + H_1'\right]$$

(103) $\underline{r = \gamma}$ $$H_{H,\gamma} = \frac{C_H}{C_T} H_{T,\gamma} = \frac{C_H}{C_T} H_2$$

(104) $\underline{r=\gamma}$ $$\left.\frac{dH_H}{dr}\right|_\gamma = \left(\frac{C_T}{C_H}\right)^2\left[-\frac{3}{2}q\left(\frac{C}{C_T}\right)^3\left(\frac{C_T}{C_H}-1\right)+\left.\frac{dH_T}{dr}\right|_\gamma\right] = a_H + b_H H_2'$$

(105) $\underline{r=1}$ $$H_T = 0$$

where

(106) $$H_2 = H_{T,r=\gamma} \qquad H_2' = \left.\frac{dH_T}{dr}\right|_{r=\gamma}$$

Let Eqs. (98) and (99) be written:

(107) $$\frac{1}{r}\frac{d}{dr}\left[r\,\frac{dH}{dr}\right] - EH = F$$

where:

(108) $$E = \begin{cases} \dfrac{1}{r^2} + \dfrac{i\sigma_T}{P_o} & \underline{1 \le r \le \gamma} \\[2ex] \dfrac{1}{r^2} + \dfrac{i\sigma_H}{P_o} & \underline{\gamma \le r \le \eta} \end{cases}$$

(109) $$F = \begin{cases} -\dfrac{3}{2}\dfrac{q_T}{r} + i\sigma_T P_o r & \underline{1 \le r \le \gamma} \\[2ex] -\dfrac{3}{2}\dfrac{q_H}{r} + i\sigma_H P_o r & \underline{\gamma \le r \le \eta} \end{cases}$$

(110) $$G = EH + F$$

Integrate Eq.(107) twice to get:

(111) $$H = \int_\gamma^r x \ln\left(\frac{r}{x}\right) G\,dx + \gamma \ln\left(\frac{r}{\gamma}\right)\left.\frac{dH}{dr}\right|_\gamma + H_\gamma$$

In finite difference form:

(112) $\underline{0 \le n \le m-1}$ $$H_{n+1} = H_n + \Delta r \cdot \ln\left(\frac{r_{n+1}}{r_n}\right)\left[\frac{1}{2}\gamma G_o + r_1 G_1 + \cdots + r_n G_n\right] + \gamma \ln\left(\frac{r_{n+1}}{r_n}\right)\left.\frac{dH}{dr}\right|_\gamma$$

where:

(113) $$\Delta r = \begin{cases} \dfrac{\gamma - 1}{m} & \underline{\gamma \ge r \ge 1} \\[2ex] \dfrac{\eta - \gamma}{m} & \underline{\gamma \le r \le \eta} \end{cases}$$

(114) $$r_n = \begin{cases} \gamma - n\cdot\Delta r & \gamma \geq r \geq 1 \\ \gamma + n\cdot\Delta r & \gamma \leq r \leq \eta \end{cases}$$

For computer calculations set:

(115) $$H_n = s_{rn} + i s_{in} + (t_{rn} + i t_{in}) H_2' + (v_{rn} + i v_{in}) H_2$$

(116) $$\Delta r\left[\tfrac{1}{2}\gamma G_0 + r_1 G_1 + \cdots + r_n G_n\right] = p_{rn} + i p_{in} + (q_{rn} + i q_{in}) H_2' + (w_{rn} + i w_{in}) H_2$$

(117) $$E = e_r + i e_i$$

(118) $$F = f_r + i f_i$$

(119) $$a = \begin{cases} 0 & \gamma \geq r \geq 1 \\ \gamma a_H = -\frac{3}{2} q_T \left(\frac{C_T}{C_H}\right)^2 \left(\frac{C_T}{C_H} - 1\right)\gamma & \gamma \leq r \leq \eta \end{cases}$$

(120) $$b = \begin{cases} \gamma & \gamma \geq r \geq 1 \\ \gamma b_H = \gamma \left(\frac{C_T}{C_H}\right)^2 & \gamma \leq r \leq \eta \end{cases}$$

Therefore:

(121) $$P_{on} = \begin{cases} \sqrt{1 + q_T \ln r_n} & \gamma \geq r \geq 1 \\ \sqrt{1 + (q_T - q_H)\ln\gamma + q_H \ln r_n} & \gamma \leq r \leq \eta \end{cases}$$

(122) $$e_{rn} = \frac{1}{r_n^2}$$

(123) $$e_{in} = \begin{cases} \dfrac{\sigma_T}{P_{on}} \\ \dfrac{\sigma_H}{P_{on}} \end{cases}$$

(124) $$f_{rn} = \begin{cases} -\frac{3}{2}\frac{q_T}{r_n} & \gamma \geq r \geq 1 \\ -\frac{3}{2}\frac{q_H}{r_n} & \gamma \leq r \leq \eta \end{cases}$$

(125) $$f_{in} = \begin{cases} \sigma_T P_{on} r_n & \gamma \geq r \geq 1 \\ \sigma_H P_{on} r_n & \gamma \leq r \leq \eta \end{cases}$$

(126) $$p_{rn} = p_{r,n-1} + \Delta r \cdot r_n \left(s_{rn} e_{rn} - s_{in} e_{in} + f_{rn} \right)$$

(127) $$p_{in} = p_{i,n-1} + \Delta r \cdot r_n \left(s_{rn} e_{in} + s_{in} e_{rn} + f_{in} \right)$$

(128) $$q_{rn} = q_{r,n-1} + \Delta r \cdot r_n \left(t_{rn} e_{rn} - t_{in} e_{in} \right)$$

(129) $$q_{in} = q_{i,n-1} + \Delta r \cdot r_n \left(t_{rn} e_{in} + t_{in} e_{rn} \right)$$

(130) $$w_{rn} = w_{r,n-1} + \Delta r \cdot r_n \left(v_{rn} e_{rn} - v_{in} e_{in} \right)$$

(131) $$w_{in} = w_{i,n-1} + \Delta r \cdot r_n \left(v_{rn} e_{in} + v_{in} e_{rn} \right)$$

(132) $$s_{r,n+1} = s_{rn} + \ln\left(\frac{r_{n+1}}{r_n}\right)\left(p_{rn} + a\right)$$

(133) $$s_{i,n+1} = s_{in} + \ln\left(\frac{r_{n+1}}{r_n}\right) p_{in}$$

(134) $$t_{r,n+1} = t_{rn} + \ln\left(\frac{r_{n+1}}{r_n}\right)\left(q_{rn} + b\right)$$

(135) $$t_{i,n+1} = t_{in} + \ln\left(\frac{r_{n+1}}{r_n}\right) q_{in}$$

(136) $$v_{r,n+1} = v_{rn} + \ln\left(\frac{r_{n+1}}{r_n}\right) w_{rn}$$

(137) $$v_{i,n+1} = v_{in} + \ln\left(\frac{r_{n+1}}{r_n}\right) w_{in}$$

To initiate the calculations set:

(138) $$p_{ro} = \tfrac{1}{2} \Delta r \gamma f_{ro}$$

(139) $$p_{io} = \tfrac{1}{2} \Delta r \gamma f_{io}$$

(140) $$q_{ro} = q_{io} = 0$$

(141) $$w_{ro} = \begin{cases} \tfrac{1}{2} \Delta r \gamma e_{ro} & \underline{\gamma \geq r \geq 1} \\ \tfrac{1}{2} \Delta r \gamma e_{ro} \frac{C_H}{C_T} & \underline{\gamma \leq r \leq \eta} \end{cases}$$

(142) $$W_{io} = \begin{cases} \frac{1}{2}\Delta r \gamma e_{io} & \underline{\gamma \geq r \geq 1} \\ \frac{1}{2}\Delta r \gamma e_{io}\frac{C_H}{C_T} & \underline{\gamma \leq r \leq \eta} \end{cases}$$

(143) $$S_{ro} = S_{io} = t_{ro} = t_{io} = v_{io} = 0$$

(144) $$V_{ro} = \begin{cases} 1 & \underline{\gamma \geq r \geq 1} \\ \frac{C_H}{C_T} & \underline{\gamma \leq r \leq \eta} \end{cases}$$

The calculation procedure is :

1) preset P_{ro}, P_{io}, --- V_{ro} according to Eqs. (138) to (144)

2) set n=0 and calculate Eq.(132) to (137)

3) advance n by 1 and repeat the cycle of Eqs. (121 to (137) to n=m-1.

The calculation for $\gamma \geq r \geq 1$ is performed first and results in:

(145) $$H_{T,r=1} = H_m = s_{rm} + i s_{im} + (t_{rm} + i t_{im}) H_2' + (v_{rm} + i v_{im}) H_2 = 0$$

i.e.,

(146) $$H_2' = -\frac{s_{rm} + i s_{im} + (v_{rm} + i v_{im}) H_2}{t_{rm} + i t_{im}} = a_3 + i b_3 + (c_3 + i d_3) H_2$$

The calculation for $\gamma \leq r \leq \eta$ yields:

(147) $$H_{H,\eta} = s_{rm} + i s_{im} + (t_{rm} + i t_{im}) H_2' + (v_{rm} + i v_{im}) H_2 = a_4 + i b_4 + (c_4 + i d_4) H_2$$

(148) $$\left.\frac{dH}{dr}\right|_{r=\eta} = \frac{1}{\eta}\left[\Delta r\left(\tfrac{1}{2}\gamma G_0 + r_1 G_1 + \cdots + r_{m-1} G_{m-1} + \tfrac{1}{2}\eta G_m\right) + \gamma(a_H + b_H H_2')\right]$$
$$= \frac{1}{\eta}\left\{p_{r,m-1} + i p_{i,m-1} + (q_{r,m-1} + i q_{i,m-1}) H_2' + (w_{r,m-1} + i w_{i,m-1}) H_2 + \tfrac{1}{2}\eta\Delta r\left[(e_{rm} + i e_{im}) H_{H,\eta} + f_{rm} + i f_{im}\right] + \gamma(a_H + b_H H_2')\right\}$$
$$= a_5 + i b_5 + (c_5 + i d_5) H_2$$

To determine H_2 Eqs. (101) and (102) apply. Substituting from Eqs. (90), (91), (147) and (148) gives:

(149) $$a_4+ib_4+(c_4+id_4)H_2 = \eta\frac{C_H}{C}\left[a_1+ib_1+(c_1+id_1)H_0\right]$$

(150) $$a_5+ib_5+(c_5+id_5)H_2 = -\left(\frac{C}{C_H}\right)^2\left[\frac{3}{2}q\left(\frac{C}{C_H}+\xi_1+\xi_2\right)+a_2+ib_2+(c_2+id_2)H_0\right]$$

Eqs. (149) and (150) may be solved for H_0 and H_2. Hence, H can be calculated by back substitution for the complete bearing. The thrust bearing moment is given by:

$$M_T = -2P_a R_2^3 \alpha e^{i\tau}\int_1^{\eta}\int_0^{2\pi} P_1 r\cos\theta\, r d\theta\, dr$$

or in dimensionless form:

(151) $$\frac{M_T/\alpha}{\frac{1}{C}LD(P_s-P_a)L^2} = \frac{\frac{\pi}{8}}{(V-1)(\xi_1+\xi_2)^3}e^{i\tau}\frac{\frac{C}{C_T}}{\eta^4}\left[-\int_1^{\gamma}\frac{H_T}{P_0}r^2 dr - \frac{C_T}{C_H}\int_{\gamma}^{\eta}\frac{H_H}{P_0}r^2 dr\right]$$

The integrals are evaluated numerically.

The total bearing moment M_B is the sum of Eqs. (92) and (151). It may be expressed by:

(152) $$\frac{M_B/\alpha}{\frac{1}{C}LD(P_s-P_a)L^2} = (m_{Br}+im_{Bi})e^{i\tau} = m_B\cos(\omega t+\varphi)$$

where:

(153) $$m_B = \sqrt{m_{Br}^2+m_{Bi}^2}$$

(154) $$\varphi = \tan^{-1}\left(\frac{m_{Bi}}{m_{Br}}\right)$$

Hence, φ is the phase angle by which the moment leads the angular displacement.

Since the angular displacement is $\alpha e^{i\tau}$ the angular velocity is $i\omega\alpha e^{i\tau}$ i.e.,

(155) $$\frac{M_B}{\frac{1}{C}LD(P_s-P_a)L^2} = m_{Br}\,\alpha e^{i\tau} + \frac{m_{Bi}}{\omega}i\omega\alpha e^{i\tau}$$

Therefore, the angular spring coefficient K_a and the angular damping coefficient B_a can be expressed by:

$$\frac{K_a}{\frac{1}{c} L D (P_s - P_a) L^2} = m_{Br} = m_B \cos\varphi \tag{156}$$

$$\frac{\omega B_a}{\frac{1}{c} L D (P_s - P_a) L^2} = m_{Bi} = m_B \sin\varphi \tag{157}$$

The last equation may also be written:

$$\frac{B_a}{\mu L^3 \left(\frac{R}{C}\right)^3} = \frac{24(V-i)}{\sigma} m_{Bi} \tag{158}$$

Eqs. (156) and (157) may be used to determine the response of the journal due to an external excitation.

CONCLUSIONS

1. The combined hydrostatic journal-thrust bearing may be optimized in the same way as the conventional journal and thrust bearing.

2. For equal flow the combined journal-thrust bearing has more angular stiffness than if the bearings were separated and fed individually.

3. The simplified line source analysis gives too high stiffness values but predicts the flow correctly. The qualitative trend of the calculated values agrees with experience.

4. The presented dynamic analysis may be adapted to stability calculations. One numerical example is given.

5. Resonant frequency and damping can be calculated from the presented dynamic analysis. Hence, a vibratory response calculation may be performed.

RECOMMENDATIONS

1. The analysis may be improved by establishing an equivalence between line source feeding and point source feeding. Such an investigation should first be performed on a simpler configuration (for instance, a thrust bearing) and then generalized to be adapted to the present configuration.

2. The combined journal-thrust bearing in connection with Millipore restricted feeding has a very low flow consumption for the attained stiffness. However, even if the stiffness per unit of flow has been greatly reduced the overall stiffness is probably too low. Obviously, the stiffness can be increased by a larger supply of flow but in the present design instability sets a limit. Hence, it should be investigated how the design and the feeder restriction may be modified to ensure higher stiffness without incurring instability.

REFERENCES

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2. MTI-63TR44: "Static and Dynamic Stiffness and Damping of the Source Fed, Annular Thrust Bearing," J. Lund. Prepared under Contract No. NAS 8 -5154 for NASA, Huntsville, Ala. Oct.1, 1963.

3. MTI-63TR42: "The Externally Pressurized 360° Hydrostatic Gas Journal Bearing at Arbitrary Eccentricity Ratio," By R. J. Wernick. Perpared under Contract No. NAS 8 -5154, Huntsville, Ala. Oct. 1, 1963.

APPENDIX A

PN0109: "Stiffness and Flow of Hydrostatic Combined Journal-Thrust Bearing"

The IBM 1620 computer program PN0109 calculates the static stiffness and the flow for the externally pressurized combined journal-thrust bearing shown in Figure 1. Note, that the program assumes that the outer portion of the thrust bearing is a pocket with uniform pressure, i.e. set $C_H = \infty$ in the analysis and in Figure 1.
Below follow the instructions for using the program.

INPUT

CARD 1 (FORMAT 5·(1XE13.6))

The first 6 cards could conveniently be considered a part of the program deck itself. Only because changes may be desired occasionally have they been made a part of the input. However, the 6 cards should always be filed together with the program deck. Note that the first 6 cards are only given once, i.e. only for the first set of input. They should not be repeated.

Word 1 This is the ratio of specific heats k, i.e. k = 1.4 for air
Word 2 to 5 These are 4 experimentally evaluated coefficients: a_o, B,C and D used to establish an equation for the flow through the Millipore restrictor as:

$$M = \frac{\pi a^2 P_a}{\sqrt{\mathcal{R}T}} \left[C P_a P_i^2 + P_i + \frac{D}{P_a} \right] \frac{(V-P_i)(V-P_i+B)}{V-P_i+a_o}$$

where:

M	=	mass flow through one Millipore restricted hole, lbs-sec/in
P_a	=	ambient pressure, psia
V	=	pressure ratio P_s/P_a
P_s	=	supply pressure, psia
P_i	=	downstream pressure divided by P_a
πa^2	=	effective flow area, in^2 (evaluated from test data)
$\mathcal{R}T$	=	(gas constant)·(total temperature), in^2/sec^2

Examples of values of coefficients:

	NASA AB-5 Sleeve 4 (62 ccm/min at $\Delta P=15, P_a=14.6$)	NASA AB-5 Endplate No.2 (59 ccm/min at $\Delta P=15, P_a=14.6$)	NASA AB-3 (32.5 ccm/min at $\Delta P=15, P_a=14.7$)
$a =$	$1.183 \cdot 10^{-3}$	$1.161 \cdot 10^{-3}$	$.985 \cdot 10^{-3}$
$a_o =$	3.8918	3.859	7.296
$B =$	1.406	1.2674	2.088
$C(psi^{-1}) =$	.1007	.108	.1007
$D(Psia) =$	-.335	-.40	-.335

Since both C and D have dimension it is actually not possible to calculate a dimensionless performance of the Millipore bearing by this program.

The only justification for the given equation is that it fits the experimental data very well. From a formal point of view one would expect the Millipore restrictor to behave as a laminar restrictor such that the flow could be written:

$$M = \frac{\pi a^4 P_a^2}{16 \mu R T \ell} \left(V^2 - P_i^2\right) \qquad \frac{lbs \cdot sec}{in}$$

where

ℓ = equivalent length of flow passages, inch.

and the other symbols are explained above. This expression has been tried and it was found that the ratio a^4/ℓ depends strongly on the ambient pressure. Hence, there seems no immediate advantage of abandoning the first equation.

<u>CARD 2 to 6 (FORMAT 5·(1XE13.6))</u>

These 5 cards give 25 values of the vena contracta coefficient ν. ν is assumed to be 1 for the choked orifice. For pressure ratios larger than the critical ratio ν becomes a decreasing function of the pressure ratio P_i/V. The interval from $P_i/V = (P_i/V)_{critical}$ to $P_i/V=1$ is subdivided into 25 divisions. ν is given at every point starting from the first point after $(P_i/V)_{critical}$ and with the 25'th value at $P_i/V=1$. As an example the list below gives the vena contracta coefficient as evaluated for a NASA AB-5 orifice bearing. The gas is air:

$\frac{P_i}{V}$	ν	$\frac{P_i}{V}$	ν	$\frac{P_i}{V}$	ν
.54715	.9985	.69810	.9623	.84905	.9130
.56602	.9955	.71697	.9570	.86792	.9046
.58489	.9915	.73584	.9516	.88679	.8950
.60376	.9871	.75471	.9460	.90566	.8836
.62263	.9824	.77358	.9402	.92453	.8690
.64149	.9776	.79244	.9341	.94339	.8477
.66036	.9726	.81131	.9276	.96226	.8105
.67923	.9675	.83018	.9206	.98113	.7360
				1.00000	.6000

The number of significant figures is not justified from the test data and serves only the purpose of ensuring a smooth derivative when used in the program.

CARD 7

Any text may be given in column 2-52. This card is used to identify the particular calculation.

CARD 8 (FORMAT 6·(1XI4),13XE13.6)

Word 1 is denoted NV. If a dimensionless calculation is performed (i.e. MDIM=0) NV gives the number of pressure ratios V in the V-list, see later. V is the ratio between supply and ambient pressure, both in psia. In this case NV$\leq$20. If a calculation with dimensions is performed (MDIM=$\pm$1) NV gives the number of pairs of P_s,P_a in the V-list. Since the V-list can take a maximum of 20 values NV$\leq$10 in this case.

Word 2 is denoted LMB. It gives the values of words in the Λ_s-list for both types of calculation. Note: LMB$\leq$20.

Word 3 is denoted MDIM. It is used for two purposes. If MDIM=0 the calculation is dimensionless and the input must be given accordingly.

If MDIM $\neq 0$ the calculation is with dimensions. If MDIM=-1 the Λ_s-list (see later) gives values of the orifice radius, a, whereas word 1, card 10, gives the radial clearance C. If MDIM=+1 the Λ_s-list gives values of C and word 1, card 10, specifies a. This feature allows for a convenient way of optimizing the bearing stiffness (i.e. to obtain the stiffness as a function of either clearance or orifice radius) without an excessive amount of input data.

Word 4 If 0, the feeding holes are orifice restricted. If 1, the feeding holes are Millipore restricted

Word 5 The bearing and the feeder hole flow are matched by a numerical interpolation as described in principle in the analysis, page . The difference between the two flows is calculated step by step as a function of the pressure ratio across the feeder. When the difference changes sign a final interpolation is used to establish the actual pressure ratio. The steps in this calculation will be given in the output if word 5 is 1. If the word is 0 no such output is given. Only in exceptional cases are these results of any value, i.e. in general word 5=0.

Word 6 If 0, one or more input cases follow the present one. If 1, this is the last set of input.

Word 7 is a "fudge" factor, denoted pressure correction factor, by which the parameter ψ is multiplied. For no effect, set the word equal to 1.0 which is the general case.

As a further explanation it should first be noted that ψ expresses the slope of the orifice flow versus orifice pressure ratio relationship at the pressure ratio corresponding to the concentric position. Since the analysis is based on the assumption that the displacement away from the concentric position is small it is seen that the magnitude of ψ is an important factor in determining the circumferential pressure variation and, therefore, the load. When the pressure ratio across the feeder becomse close to 1, ψ becomes very large and experience has indicated that the orifice equation as used in the present program is not accurate enough. Hence, the program provides for a correction factor. Under the above stated conditions a pressure correction factor of 2 has been used occasionally but the practice lacks any formal justification.

DIMENSIONLESS CALCULATION (MDIM=0)

CARD 9 (FORMAT 5·(1XE13.6))

Word 1 is L_1/D where L_1 is distance between feeding planes and D is the journal diameter.

Word 2 is L_2/D where L_2 is the total length outside the feeding planes, i.e. $L_2 = L - L_1$ where L is the overall journal bearing length.

Word 3 is $\eta = R/R_2$ where $R = D/2$ and R_2 is the inner radius of the thrust bearing. Note = $\eta \neq 1$.

Word 4 is $\gamma = R_1/R_2$ where R_1 is the inner radius of the thrust bearing pocket. If it is desired to calculate the journal bearing without the thrust bearing set $\gamma = 1$ and η may have any value as long as $\eta \neq 1$.

Word 5 is C_T/C where C_T is the thrust bearing clearance and C is the radial journal bearing clearance. If no thrust bearing is desired set $C_T/C = 1$.

V-LIST (FORMAT 5·(1XE13.6))

The cards making up the V-list (maximum 4 cards, 5 words per card) give the values of the pressure ratio $V = P_s/P_a$. P_s is the supply pressure, psia and P_a is the ambient pressure, psia.

Λ_s-LIST (FORMAT 5·(1XE13.6))

The Λ_s-list gives the values of the feeding parameter Λ_s. With 5 words per card a maximum of 4 cards can be given. Λ_s is defined as:

$$\Lambda_s = \frac{\Lambda_t}{V} = \frac{6\mu N a^2 \sqrt{\mathcal{R}T}}{P_s C^3}$$

where $\mathcal{R}T$ is in^2/sec^2. ($\sqrt{\mathcal{R}T} = 1.15\cdot 10^4$ for air at 70^oF)

CALCULATION WITH DIMENSIONS (MDIM= $\pm$ 1)

CARD 9 (FORMAT 5·(1XE13.6))

Word 1 gives the journal diameter D , inch

Word 2 gives the length between feeding planes L_1, inch

Word 3 gives $L_2=(L-L_1)$ where L is the total journal bearing length, inch.

Word 4 gives the inner radius of the thrust bearing pocket R_1, inch.
Noted: $R_1 \neq D/2$.

Word 5 gives the inner thrust bearing radius R_2, inch. If the thrust bearing is not wanted, set $R_1 = R_2$ but such that $R_1 \neq D/2$. Note: $R_2 \neq 0$.

CARD 10 (FORMAT 5·(1XE13.6))

Word 1 is used for two purposes. If MDIM=-1, word 1 is the radial journal bearing clearance C, inch. If MDIM=+1, word 1 is the orifice radius a, inch. An explanation is given in the comments to the Λ_s- list below.

Word 2 is the thrust bearing clearance C_T, inch.

Word 3 is the total number of orifices (both feeding planes), denoted N. It is in floating point.

Word 4 is the gas viscosity μ, lbs-sec/in^2

Word 5 is the gas constant times the total temperature $\mathcal{R}T$, inch.

V-LIST (FORMAT 5·(1XE13.6))

The V-list gives the pairs of (P_s,P_a) where P_s is the supply pressure, psia, and P_a is the ambient pressure,psia,i.e.the list: P_s, P_a, P_s, P_a, ---. Since the V-list can contain a maximum of 20 items a maximum of 10 pairs may be listed.

Λ_s-LIST (FORMAT 5·(1XE13.6))

If MDIM=-1, the Λ_s-list gives values of the orifice radius a, inch. If MDIM=+1, the list gives values of the radial journal bearing clearance C_1 inch. The purpose of this option is to facilitate design calculations.

In the initial design stages most of the bearing dimensions are given and it remains to select either the clearance or the orifice radius such that the bearing is optimized, i.e. that the stiffness is maximum for the given supply pressure. Hence, the Λ_s-list allows for calculating the stiffness as a function of either a or C with only one set of input data.

OUTPUT

The output values are identified by text of which many is self-explanatory except for the following comments:

MO is the dimensionless orifice flow m_o, see Eq. (8) where $P_i = P_{o,i}$. Note, that the total bearing flow $M_T = N\pi a^2 P_s m_o / \mathcal{R} T$ lbs-sec/in.

POCKET PR is the pressure in the thrust bearing pocket divided by the ambient pressure.

ORIF. PR. is $P_{o,i}$, i.e. the orifice downstream pressure divided by the ambient pressure, (concentric journal).

V.C.COEFF is the vena contracta coefficient ν used to calculate m_o

SLOPE is Ψ as given by Eq. (19).

Q2 is B_4 as given by Eq. (41) with $C_H = \infty$

END INT is the value of the dimensionless integral in Eq. (47) $(C_H = \infty)$

CTR INT is the value of $A_2 \tanh \xi_1 / P_{o,i}$ in Eq. (47) $(C_H = \infty)$

LAND INT is the value of the first integral in Eq. (48) $(C_H = \infty)$

POCKET INT is the value of the second integral in Eq. (48) $(C_H = \infty)$

P1 gives the value of the perturbed pressure P_1 at the orifice. P_1 is defined by Eq. (9) and is made dimensionless with respect to P_a. The value of P_1 may be used to estimate the maximum eccentricity ratio for which the calculation is valid. Since the total pressure at the orifice is $P_i = P_{oi} + \epsilon P_1 \cos\theta$ and the pressure can neither exceed the supply pressure nor go below the ambient pressure, then

$$\epsilon_{max} = \begin{cases} \dfrac{P_{oi} - 1}{P_1} \\ \text{or} \\ \dfrac{V - P_{oi}}{P_1} \end{cases}$$

DIM.J-FRC is the dimensionless journal bearing stiffness by Eq. (47)
DIM.T-FRC is the dimensionless thrust bearing stiffness by Eq. (48)
DIM.FLW is the dimensionless mass flow q, see Eq. (25). The total bearing flow can then be computed from Eq. (49).
D(FLW) is the dimensionless flow change through the thrust bearings, see Eq. (51).
PO/V is the pressure ratio across the orifice
J-LOAD is the journal bearing load, lbs, for $\epsilon=1$ based on a linear load-displacement curve.
J-STIFF is the journal bearing stiffness. lbs/in.
TH-LOAD is combined thrust bearing load, lbs, at $\epsilon_T = 1$ based on a linear load-displacement curve.
TH-STIFF is the combined thrust bearing stiffness, lbs/in.
WT.FLOW is the total bearing mass flow, lbs/sec.
D(WT.FLW) is the change in mass flow through the thrust bearings, i.e. ΔM from Eq. (51), lbs/sec.
SCFM is the bearing flow in SCFM
D(SCFM) is ΔM in SCFM
CCM/M is the bearing flow in ccm/min
D(CCM/M) is ΔM in ccm/min.

OPERATING INSTRUCTIONS

The program is written in FORTRAN 1 for the IBM 1620 computer, 40,000 bits storage. The program deck does not include the subroutines. The input and output are given on cards.

INPUT FORM

PN0109: "STIFFNESS AND FLOW OF HYDROSTATIC COMBINED JOURNAL-THRUST BEARING"

Card 1 FORMAT 5·(1XE13.6)

__________.1 k, Ratio of Specific Heats (k=1.4 for air)

__________.2 a_o

__________3.B

__________4.C psi^{-1}

__________5.D psi

(items 2–5) Millipore coefficients, see input instructions. These coefficients not used in the orifice bearing calculation but required in input.

Card 2 to 6 FORMAT 5·(1XE13.6)

These 5 cards give the 25 values of the vena contracta coefficient ν such that ν_n applies at: $\left(\frac{Pi}{V}\right)_{critical} + \left(1-\left(\frac{Pi}{V}\right)_{critical}\right)\cdot\frac{n}{25}$, n=1--25. For details, see instructions. Note: Card 1-6 are only given with first set of input.

Card 7 Text, Col. 2-52

Card 8 FORMAT 6·(1XI4), 13XE13.6

_____ 1. NV: No. of pressure ratios V or of pairs of (P_s,P_a). NV $\leq$ 20(for V), $\leq$ 10(for P_s,P_a)

_____ 2. LMB: Number of Λ_s'S or of a/C in list. LMB $\leq$ 20

_____ 3. MDIM: -1: Λ_s-list is a; item 1, Card 10 is C
0: Dimensionless calculation, Λ_s-list is Λ_s
+1: Λ_s-list is C; item 1, Card 10 is a

_____4. 0: Orifice restriction 1: Millipore restriction

_____5. 0: No output of flow interpolation 1: Output given of flow interpolation.

_____6. 0: More input follows 1: last set of input

__________7. Pressure correction factor. In most cases equal to 1.0

DIMENSIONLESS CALCULATION

(MDIM=0)

Card 9 FORMAT 5·(1XE13.6)

__________ 1. L_1/D: Ratio of distance between feeding planes over journal diameter

__________ 2. L_2/D: Ratio of total length outside feeding planes over journal diameter

__________ 3. $\eta = \frac{R}{R_2}$: Ratio between journal radius and inner radius of thrust bearing

__________ 4. $\gamma = R_1/R_2$: Ratio between pocket radius and inner radius of thrust bearing

__________ 5. C_T/C: Ratio between thrust bearing clearance and radial journal bearing clearance

V-LIST: FORMAT 5·(1XE13.6)

List as many pressure ratios V as given by NV, 5 values per card

Λ_s-LIST FORMAT 5·(1XE13.6)

List as many feeding parameter Λ_s as given by LMB, 5 values per card

CALCULATION WITH DIMENSIONS

(MDIM=± 1)

Card 9 FORMAT 5·(1XE13.6)

__________ 1. D: Journal diameter, inch

__________ 2. L_1: Length between feeding planes, inch

__________ 3. L_2: Total length outside feeding planes, inch ($L_2 = L_{total} - L_1$)

__________ 4. R_1: Inner pocket radius, inch

__________ 5. R_2: Inner thrust bearing radius, inch

Card 10 FORMAT 5·(1XE13.6)

________ 1. If MDIM=-1: Item is radial journal bearing clearance C, inch.

If MDIM=+1: Item is orifice radius a, inch.

________ 2. C_T: Thrust bearing clearance, inch

________ 3. N: Total number of feeding holes

________ 4. μ: Viscosity, lbs-sec/in^2

________ 5. $\mathcal{R}$ T: (Gas constant)(Total temperature), lbs-in/lbs.

V-LIST FORMAT 5·(1XE13.6)

List as many pairs of ($P_s P_a$) as given by NV, 5 words per card. Maximum 4 cards.

Λ_s-List FORMAT 5·(1XE13.6)

List as many words as given by LMB, 5 words per card. Maximum 4 cards.

If MDIM=-1, the list means orifice radius a, inch

If MDIM=+1, the list means radial journal bearing clearance C, inch

APPENDIX B

PN0135: Angular Dynamic Stiffness of Hydrostatic Combined Journal-Thrust Bearing

The IBM 704 computer program PN0135 calculates the dynamic angular stiffness and damping of the externally pressurized combined journal-thrust bearing shown in Fig. 1.

Input

The first 6 cards are as described in the input instructions for PN0109. The format is (1P5E14.6). For a particular calculation the first 6 cards are only given as input once and not repeated in each following input set.

Card 7 and 8

Any text may be given in column 2-72. These cards are used for identification purpose.

Card 9   FORMAT (8I5))

Word 1, is NCTC, the number of C_T/C - ratios in the C_T/C-list, Fig.1.NCTC$\leq$ 50.
Word 2, is NCH, the number of C_T/C_H - ratios in the C_T/C_H-list, Fig.1.NCH$\leq$50.
Word 3, is NVEE, the number of pressure ratios V in the V-list, NVEE$\leq$50.
Word 4, is NLAM, the number of feeding parameters Λ_s in the Λ_s-list. NLAM$\leq$100.
Word 5, is NSIG, the number of frequency numbers σ in the σ-list; NSIG$\leq$100
Word 6, is NFD. If NFD=0, the bearing is orifice restricted. If NFD=1, the bearing is Millipore restricted.
Word 7, is MT, the number of finite difference subdivisions in both journal bearing and thrust bearing. MT=10 or even smaller should be adequate. MT$\leq$50
Word 8, is INP. If INP=0, one or more input sets follow the present set. If NFD=1, the present input set is the last one.

Card 10 (FORMAT (1P4E15.7))

Word 1, is L_2/D. $L_2=L-L_1$ where L is the total journal bearing length and L_1 is the distance between feeding planes. D is the journal diameter. Note: $L_2/D \neq 0$.

Word 2 is L_1/D. L_1/D may be zero if there is only a single feeding plane.

Word 3 is R/R_2 where R is the journal radius (R=D/2) and R_2 is the inner thrust bearing radius, see Fig. 1 Note: $R/R_2 \neq 1$.

Word 4 is R_1/R_2 where R_1 is the inner radius of the pocket in the thrust bearing. If it is desired to calculate the journal bearing without a thrust bearing set $R_1/R_2=1$, $R/R_2 = 1.001$ and $C_T/C_H = 50$.

C_T/C-LIST (FORMAT (1P4E15.7))

Give as many values of C_T/C as specified by NCTC (word 1, card 9) 4 values per card. A maximum of 50 values is allowed.
C_T is the thrust bearing clearance at the land and C is the radial journal bearing clearance, see Fig. 1; Note: $C_T/C \neq 0$.

C_T/C_H-LIST (FORMAT 1P4E15.7))

Give as many values of C_T/C_H as specified by NCH (word 2, card9) 4 values per card. A maximum of 50 values is allowed.
C_T is the thrust bearing clearance at the land and C_H is the clearance of the pocket in the thrust bearing (i.e.pocket depth plus C_T), see Fig. 1. Note: $C_T/C_H \neq 0$.

V-LIST (FORMAT (1P4E15.7))

Give as many values of the pressure ratio V as specified by NVEE (word 3, card 9) 4 values per card, a maximum of 50 values is allowed.
$V=P_s/P_a$ where P_s is the supply pressure, psia, and P_a is the ambient pressure, psia.

Λ_s-LIST (FORMAT 1P4E15.7))

Give as many values of the feeding parameter Λ_s as specified by NLAM (word 4, card 9), 4 values per card. A maximum of 100 values is allowed.
$\Lambda_s = \Lambda_t/V$ so that

$$\Lambda_s = \frac{6\mu N a^2 \sqrt{RT}}{P_s C^3}$$

where:

μ = gas viscosity, lbs-sec/in^2 (μ=2.83·10^{-9} for air at 70°F)

$\mathcal{R}T$ = (gas constant)(Total temperature), in^2/sec^2 ($\mathcal{R}T$=1.322·10^8 for air at 70°F i.e. $\sqrt{\mathcal{R}T}$= 1.15·10^4 for air at 70°F)

N = total number of feeding holes (includes both feeding planes)

a = orifice radius, inch

C = radial journal bearing clearance, inch

P_s= supply pressure, psia.

<u>σ-LIST (FORMAT (1P4E15.7))</u>

Give as many values of the frequency number σ as specified by NSIG (word 5, card 9), 4 values per card. A maximum of 100 values is allowed. The frequency number is defined as:

$$\sigma = \frac{12\mu\omega}{P_a}\left(\frac{R}{C}\right)$$

where:

μ = gas viscosity, lbs-sec/in^2

ω = vibratory frequency, rad/sec

P_a= ambient pressure, psia

R = journal bearing radius, inch

C = journal bearing radial clearance, inch

<u>OUTPUT</u>

The output first lists the input of Card 9 and the Millipore and orifice coefficients. Thereafter follows the results of each calculation. Each output value is labeled and an explanation is given below except where the description is self-explanatory.

<u>L/D</u> is the ratio of total journal bearing length to journal diameter,

$L/D = L_1/D + L_2/D$

<u>ORIF.PR.RATIO</u> is the pressure ratio across the orifice P_{oi}/V

<u>DIM.MOMENT</u> is the dimensionless total dynamic moment $M_B/\frac{1}{C}LD(P_s-P_a)L^2\alpha)$ where M_B is the total dynamic moment, lbs/in (journal and thrust bearing) and α is the angular displacement, radians, see Eqs. (152) and (153).

PHASE ANGLE is the angle, degrees, by which the total dynamic moment M_B leads the angular displacement, i.e. φ in Eq. (154)

DIM.STIFFNESS is the dimensionless dynamic stiffness $K_a/(\frac{1}{C}LD(P_s-P_a)L^2)$ where K_a is the stiffness in lbs-in/rad, see Eq. (156)

DIM.DAMPING 1 is the dimensionless damping $\omega B_a/(\frac{1}{C}LD(P_s-P_a)L^2)$ where B_a is the damping in lbs-in-sec/rad, see Eq.(157)

DIM.DAMPING 2 is another form of the dimensionless damping, namely $B_a/(\mu L^3(\frac{R}{C})^3)$. The advantage of this form is that it is independent of frequency explicity. Note, that for $\sigma = 0$ this item is given the arbitrary value zero which is in general incorrect. To evaluate the damping at $\sigma = 0$, set $\sigma = .05$ or some other small value.

DIM.DAMPING 3 is again another form of the dimensionless damping, namely $B_a N \alpha^2 \sqrt{\mathcal{R}T}/(L^3 D^3 (P_s-P_a))$. The same comments as above apply.

DIM.JRNL.MMT is the dimensionless dynamic moment contributed by the journal bearing $M_J/(\frac{1}{C}LD(P_s-P_a)L^2\alpha)$ where M_J is the journal bearing moment, lbs-in, see Eq.(92)

DIM.THR.MMT.1 is the dimensionless dynamic moment contributed by the thrust bearing $M_T/(\frac{1}{C}LD(P_s-P_a)L^2\alpha)$ where M_T is the thrust bearing moment, lbs-in, see Eq. (151). Note: the moment derives from two thrust bearings.

DIM.THR.MMT.2 is another form of the dimensionless thrust bearing moment, namely $M_T/(\frac{1}{C_T}\pi(R_2^2-R_2^2)(P_s-P_a)R_1^2\alpha)$, to facilitate comparison with previously obtained thrust bearing calculations.

JRNL. PHASE ANG is the phase angle by which the journal bearing moment M_J leads the angular displacement, i.e. analogous to φ in Eq. (154).

THR.PHASE ANG is the phase angle by which the thrust bearing moment H_T leads the angular displacement, i.e. analogous to φ in Eq. (154).

Q is the flow parameter q, see Eq. (25). It may be used to calculate the total mass flow:

$$M_{total} = \frac{\pi C^3 P_a^2}{6\mu \mathcal{R}T} q$$

where $\mathcal{R}T$ is in inch ($\mathcal{R}T$= 342,500 in. for air at 70°F)

DIM.FLOW is another form of the dimensionless flow. It is actually the dimensionless orifice mass flow m_o, see Eq. (8). Hence, the total mass flow becomes:

$$M_{total} = \frac{N\pi a^2 P_s}{\sqrt{RT}} m_o \quad \frac{lbs \cdot sec}{in}$$

QH is the pocket flow parameter q_H, see Eq. (24)

QT is the thrust bearing land flow parameter q_T, see Eq. (24)

ORIF.FLW.FCT is the orifice flow-pressure parameter Ψ, see Eq. (19)

VENA CONTR. is the vena contracta coefficient. For the Millipore restriction it is set equal to zero.

RE(HO) is $\Re\{H_o\}$, see Eq. (62) and (149)-(150)

IM(HO) is $\Im\{H_o\}$, see Eq. (62) and (149)-(150)

RE(H2) is $\Re\{H_2\}$, see Eq. (106) and (149)-(150)

IM(H2) is $\Im\{H_2\}$, see Eq. (106) and (149)-(150)

RE(H2P) is $\Re\{H_2'\}$, see Eq. (106) and (146)

IM(H2P) is $\Im\{H_2'\}$, see Eq. (106) and (146)

In general, the only results of interest are the dynamic stiffness and the damping coefficient.

OPERATING INSTRUCTIONS

The program is written in FORTRAN 2 for the IBM 704 computer with 32,767 word storage. Input is on cards and output is given on tape 3. The program does not use sense switches. However, the MTI-deck is compiled at LSTG8G, i.e. sense switch 1 and 2 down. The final stop is at 5153_8. The computer time may be estimated from

Time = 2.NCTC.NCH.NVEE.NLAM.NSIG seconds

This estimate is based on MT=10. In addition, allow approximately 1 minute for reading in the program deck.

INPUT FORM

PN0135: "ANGULAR DYNAMIC STIFFNESS OF HYDROSTATIC COMBINED JOURNAL-THRUST BEARING for IBM 704 Computer

Card 1 FORMAT (1P5E14.6)

__________ 1. k, Ratio of specific heats

__________ 2. a_o
__________ 3. B
__________ 4. $C \cdot P_a$
__________ 5. D/P_a

Millipore coefficients, see input instructions. These coefficients do not apply for orifice calculation but is required in the input

Card 2-6 FORMAT (1P5E14.6)

These 5 cards give 25 values of the vena contracta coefficient ν such that ν_n applies at: $(\frac{Pi}{V})_{critical} + \left(1-(\frac{Pi}{V})_{critical}\right)\frac{n}{25}$, n=1,2---25.

For details, see instructions.

Card 7 Text Col. 2-72

____

Card 8 Text Col. 2-72

____

Card 9 FORMAT (8I5)

__________ 1. NCTC, number of C_T/C-values in C_T/C-List, NCTC$\leq$50
__________ 2. NCH, number of C_T/C_H-values in C_T/C_H-List. NCH$\leq$50
__________ 3. NVEE, number of V-values in V-list, NVEE$\leq$50
__________ 4. NLAM, number of Λ_s Values in Λ_s-list. NLAM $\leq$ 100.
__________ 5. NSIG, number of σ-values in σ-list, NSIG$\leq$ 100
__________ 6. NFD. NFD=0, Orifice restriction. NFD=1, Millipore restriction
__________ 7. MT. number of finite difference subdivisions. MT$\leq$50
__________ 8. 0: More input follows 1: last input set

Card 10 FORMAT (1P4E15.7)

__________ 1. L_2/D. Ratio of journal bearing length outside feeding planes to journal diameter.

__________ 2. L_1/D. Ratio of length between feeding planes to journal diameter

______________ 3. R/R_2 Ratio of journal radius to inner radius of thrust bearing.

______________ 4. R_1/R_2 Ratio of inner pocket radius to inner radius of thrust bearing.

<u>C_T/C-List</u> FORMAT (1P4E15.7)

List as many C_T/C-values as specified by NCTC, 4 values per card. $C_T/C \neq 0$.

<u>C_T/C_H-List</u> FORMAT (1P4E15.7)

List as many C_T/C_H-values as specified by NCH, 4 values per card. $C_T/C_H \neq 0$.

<u>V-List</u> FORMAT (1P4E15.7)

List as many pressure ratios $V=P_s/P_a$ as specified by NVEE, 4 values per card.

<u>Λ_s-List</u> FORMAT (1P4E15.7)

List as many feeding parameters Λ_s as specified by NLAM, 4 values per card

<u>σ-List</u> FORMAT (1P4E15.7)

List as many frequency numbers σ as specified by NSIG, 4 values per card

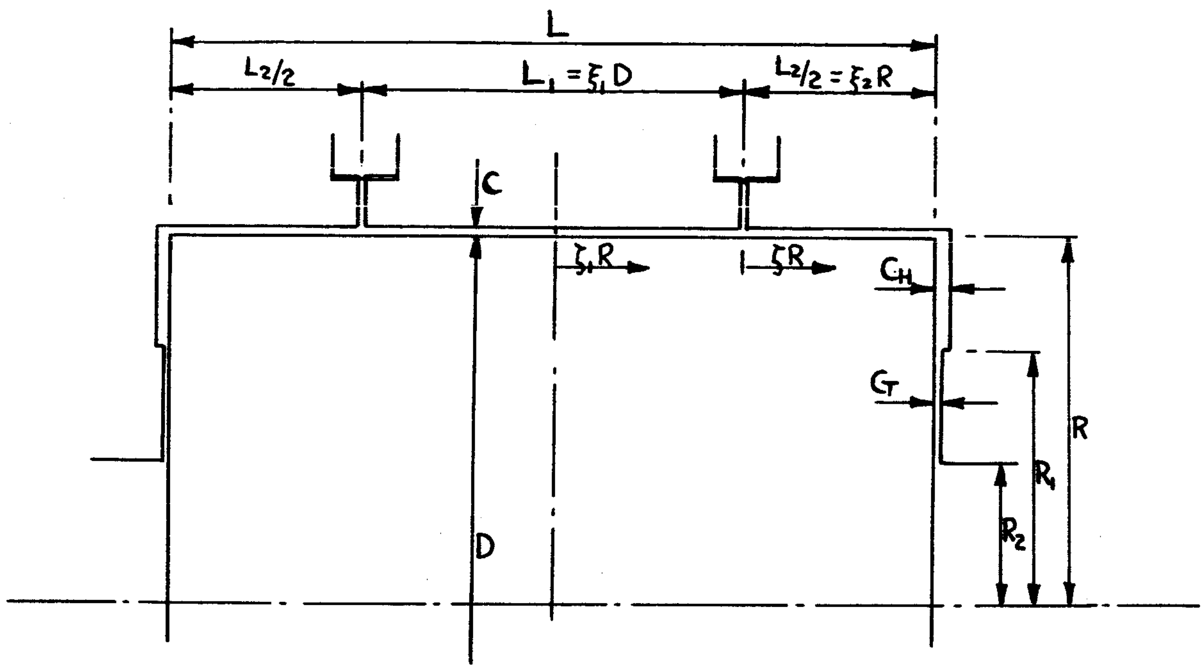

Fig. 1

GEOMETRY OF COMBINED JOURNAL-THRUST BEARING

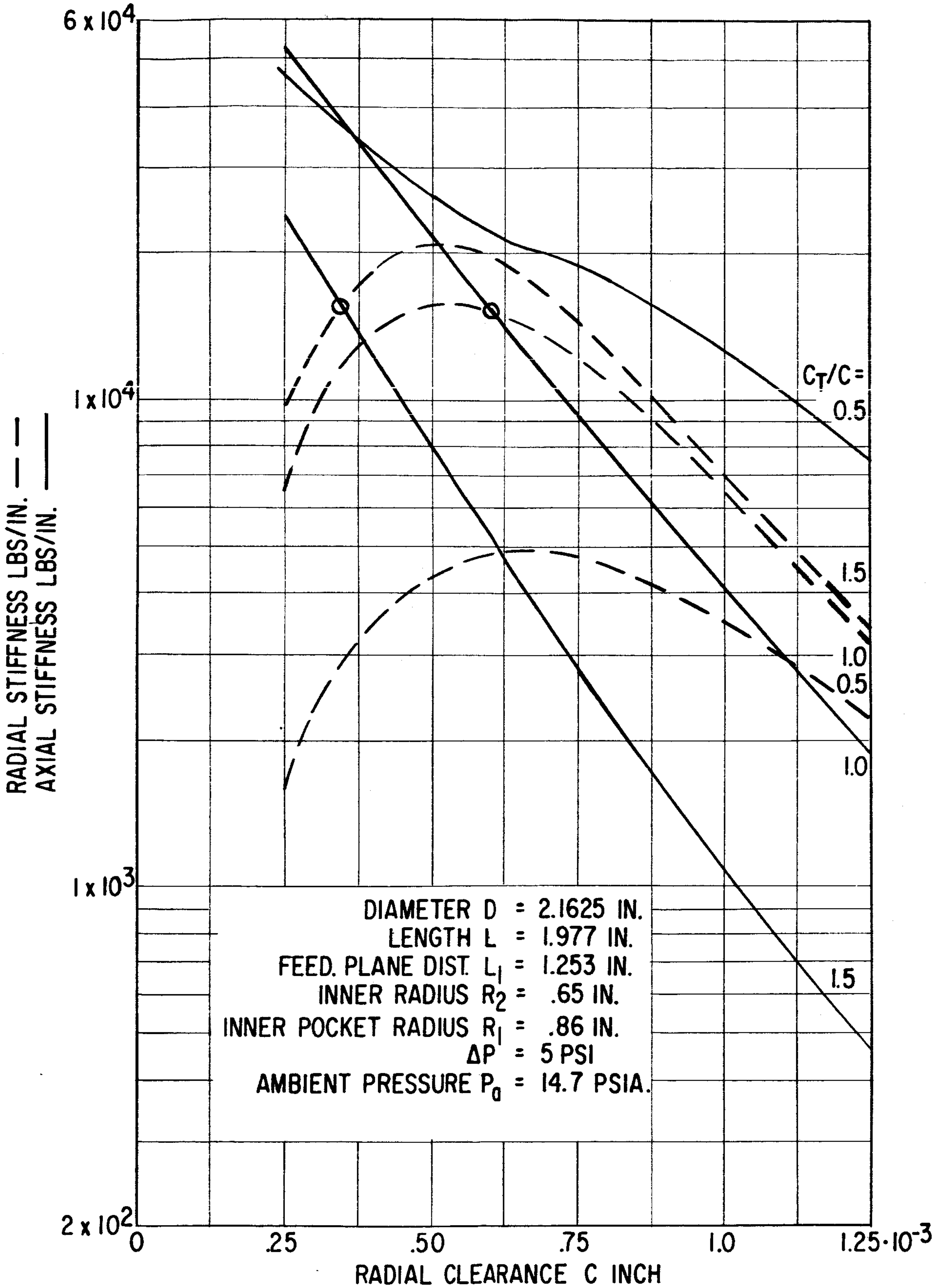

FIG. 2 NASA AB-5 MILLIPORE COMBINED JOURNAL-THRUST BEARING

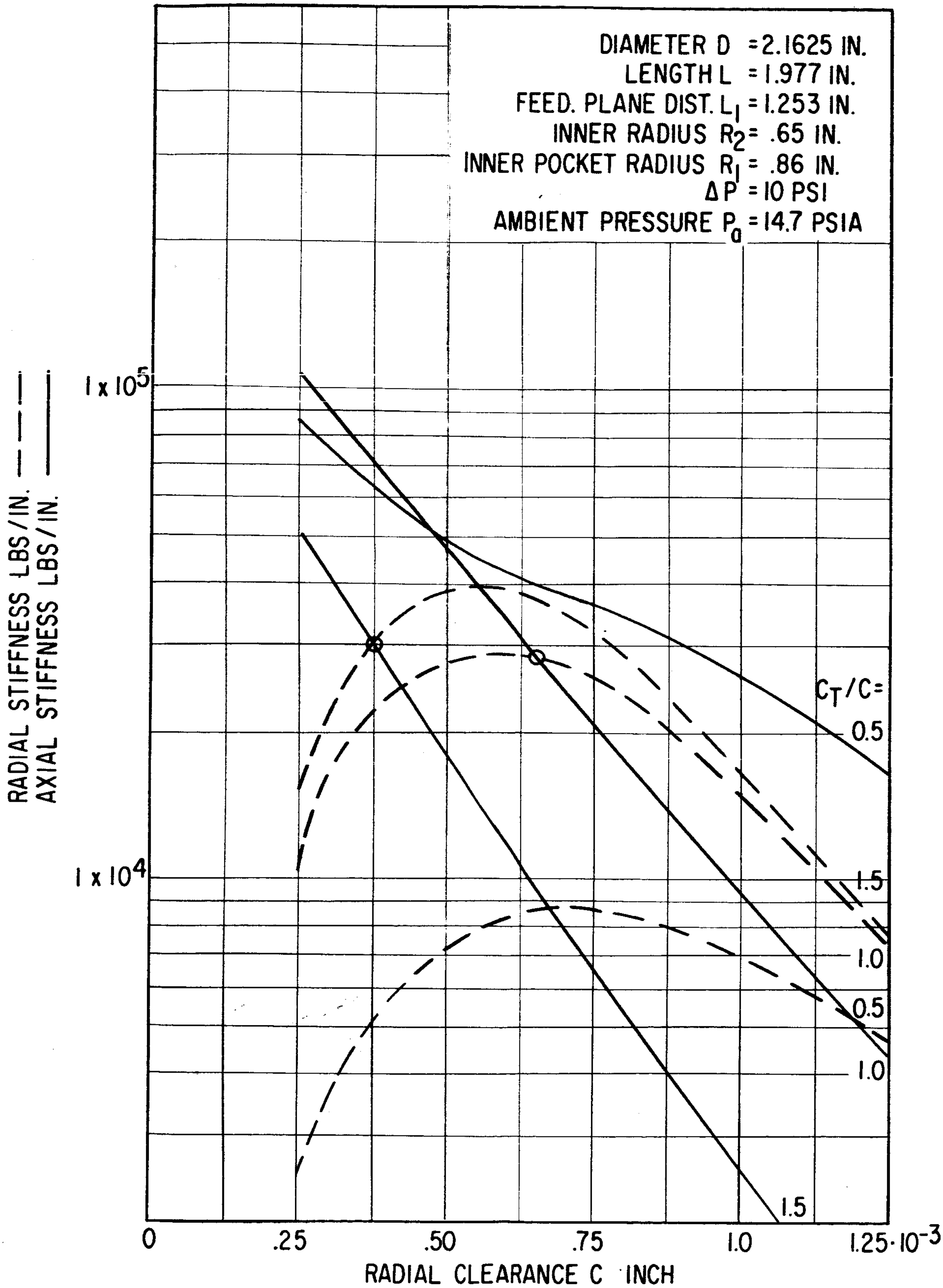

FIG. 3 NASA AB-5 MILLIPORE COMBINED JOURNAL-THRUST BEARING

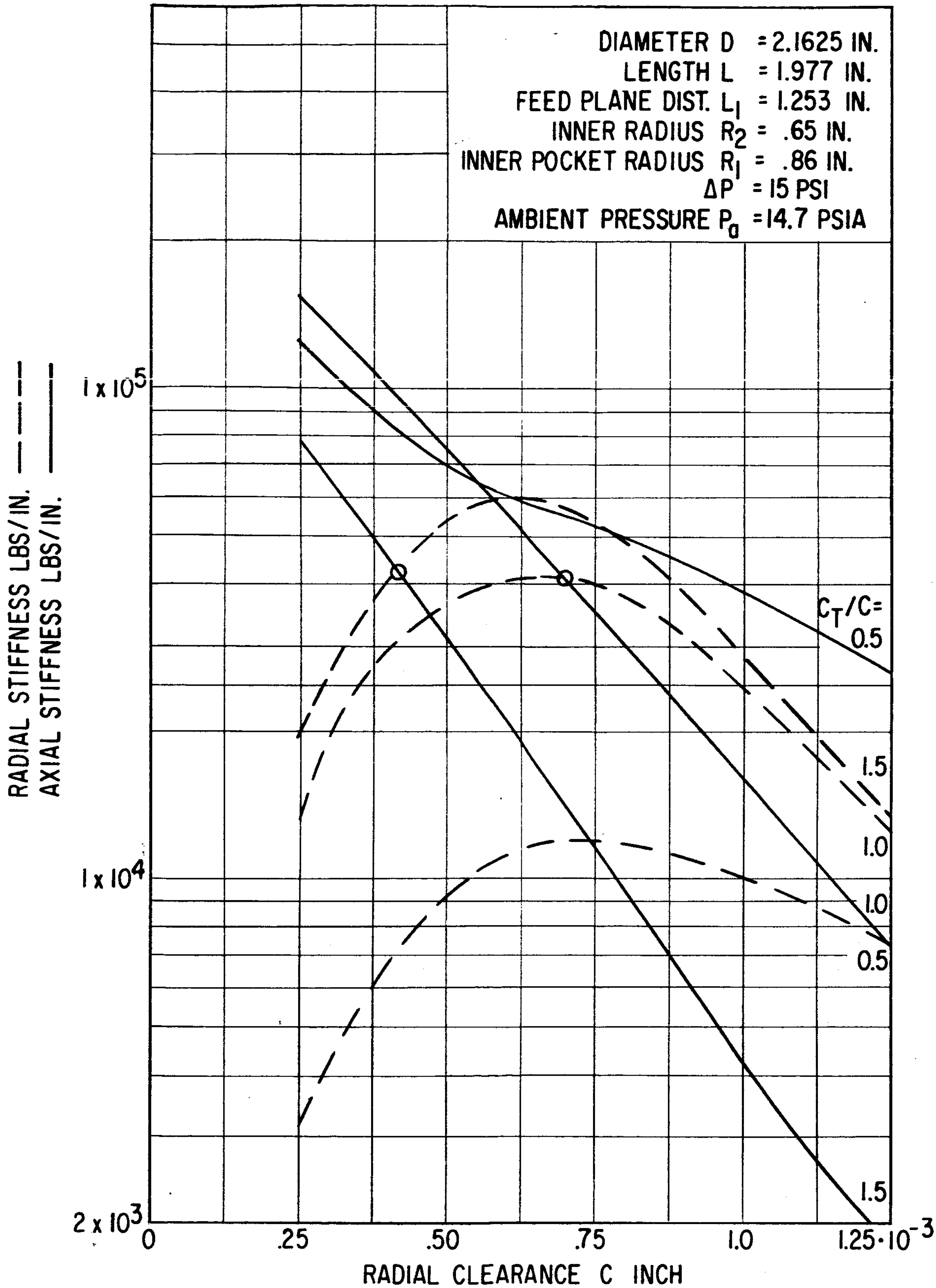

FIG. 4 NASA AB-5 MILLIPORE COMBINED JOURNAL - THRUST BEARING

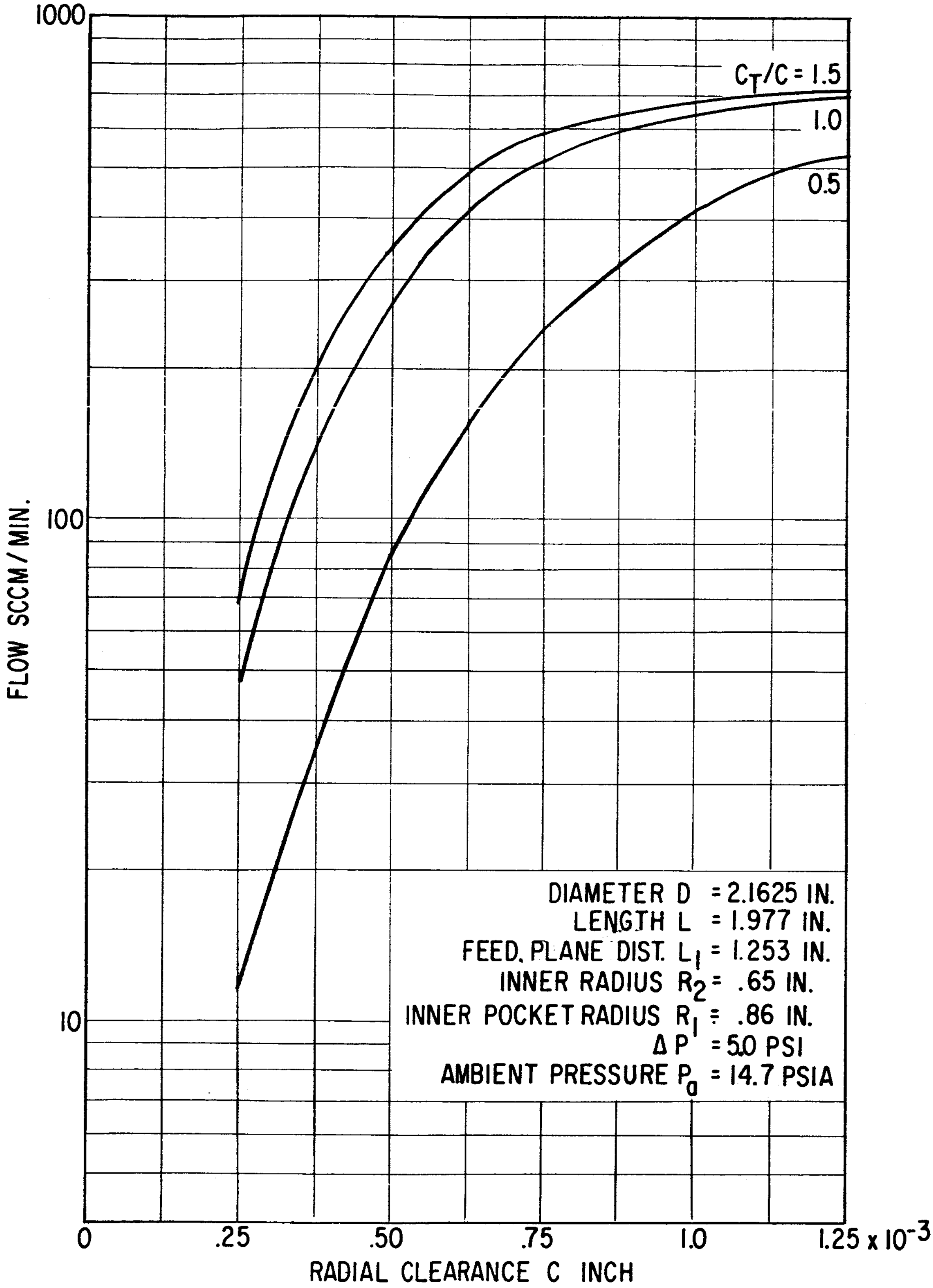

FIG. 5 NASA AB-5 MILLIPORE COMBINED JOURNAL - THRUST BEARING

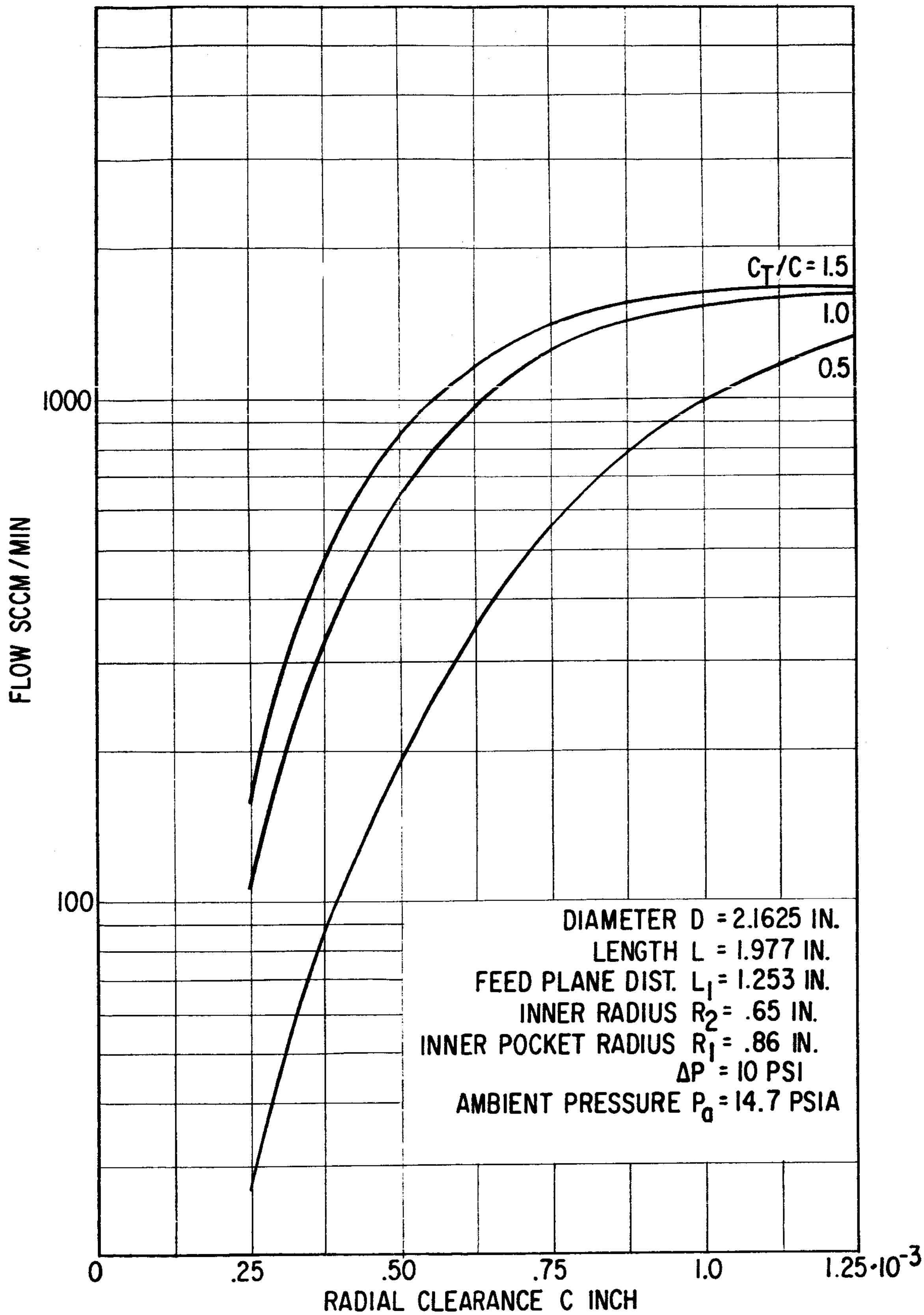

FIG.6 NASA AB-5 MILLIPORE COMBINED JOURNAL-THRUST BEARING

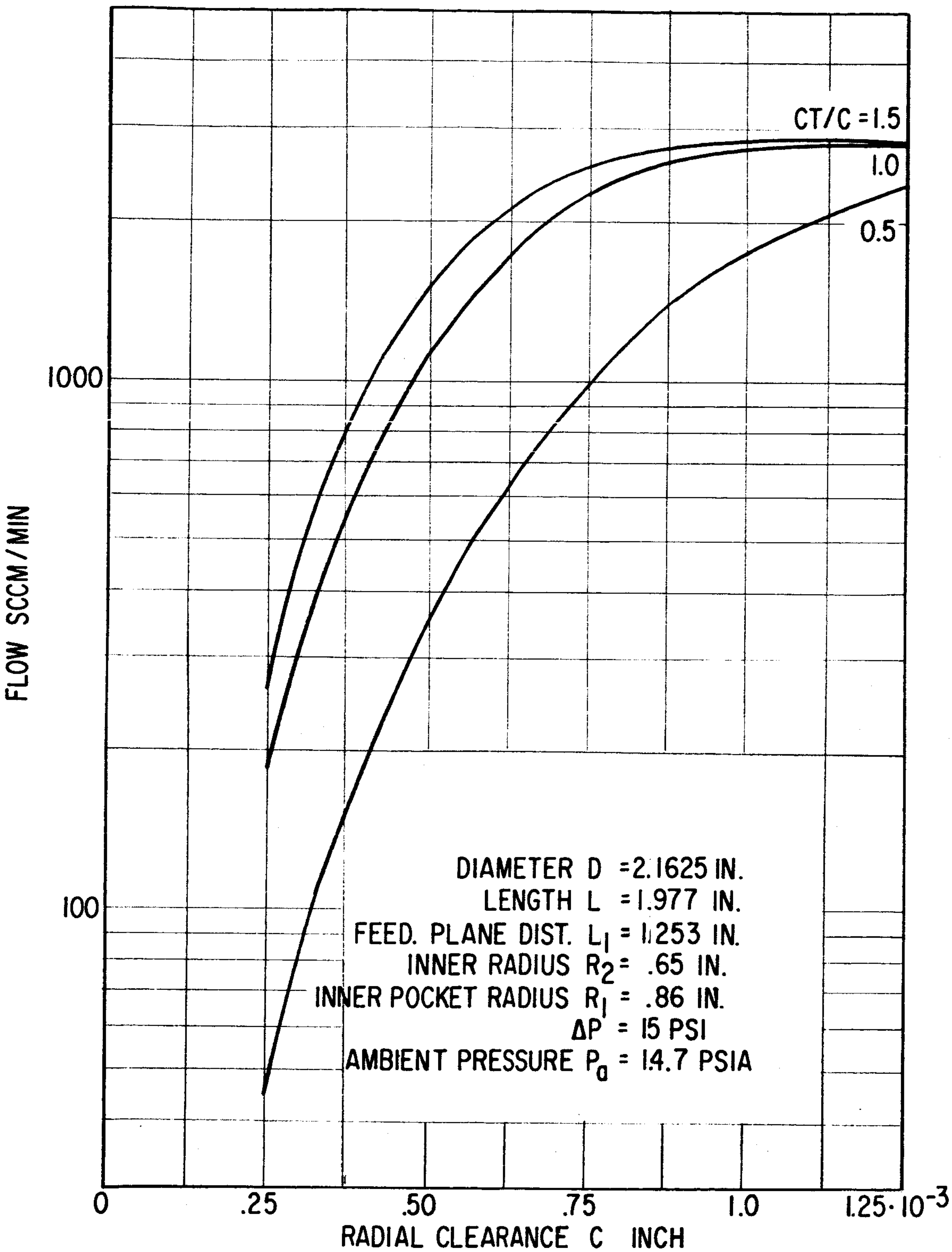

FIG. 7 NASA AB-5 MILLIPORE COMBINED JOURNAL-THRUST BEARING

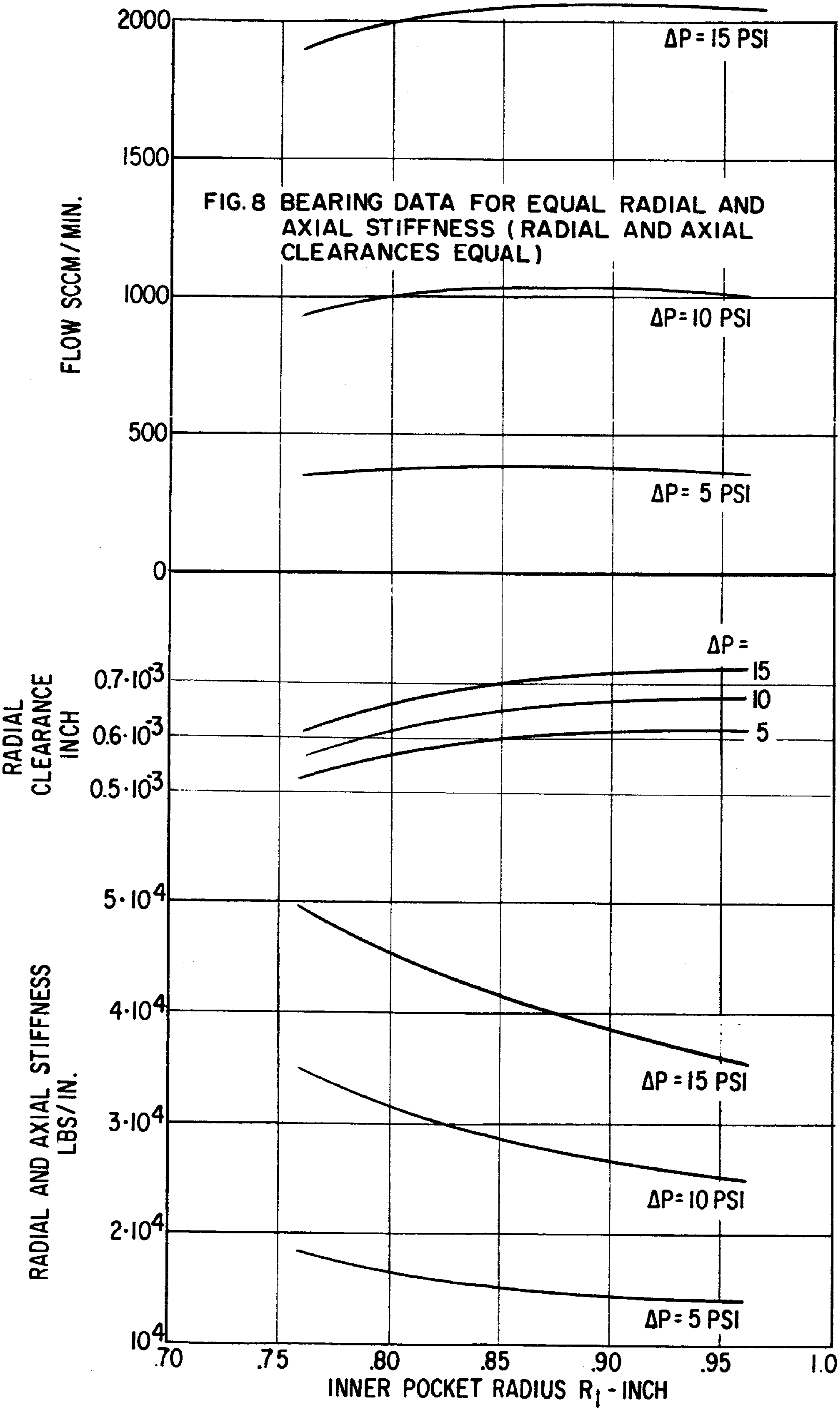

FIG. 8 BEARING DATA FOR EQUAL RADIAL AND AXIAL STIFFNESS (RADIAL AND AXIAL CLEARANCES EQUAL)

NOMENCLATURE

a = Orifice radius, inch

a_o,B,C,D= Millipore coefficients

B_a = Angular damping coefficient, lbs-in-sec/rad

C = Radial journal bearing clearance, inch

C_H = Clearance in thrust bearing pocket, inch

C_T = Clearance at thrust bearing land, inch

D = Journal bearing diameter, inch

h = Filmthickness divided by radial clearance

h_T = Thrust bearing filmthickness divided by C_T

K_a = Axial stiffness, lbs/inch

K_a = Angular stiffness, lbs-inch/rad.

K_r = Radial stiffness, lbs/inch

k = Ratio of specific heats

L = Journal bearing length, inch

L_1 = Distance between feeding planes, inch

L_2 = $L-L_1$, i.e.length outside feeding planes, inch

M = Mass flow, lbs-sec/in.

M_T = Total mass flow, lbs-sec/in.

M_J = Dynamical journal bearing moment, lbs-in.

M_T = Dynamical thrust bearing moment, lbs-in.

M_B = Total dynamical moment, lbs-in.

m = Dimensionless orifice flow, Eq. (8)

m_o = Dimensionless orifice flow in concentric position

N = Total number of feeding holes

P = Gas film pressure, atm.

P_a = Ambient pressure, psia

P_s = Supply pressure, psia

P_o = Film pressure for concentric position divided by P_a

P_1 = Pressure due to radial displacement, divided by P_a

P_2 = Pressure due to axial displacement, divided by P_a

P_i = Downstream orifice pressure, divided by P_a

P_{oi} = Downstream orifice pressure for concentric position,divided by P_a

q = Dimensionless flow parameter

q_H = Dimensionless flow parameter for thrust bearing pocket

q_T = Dimensionless flow parameter for thrust bearing

R = Journal radius, inch

R_1 = Inner pocket radius, inch

R_2 = Inner thrust bearing radius, inch

$\mathcal{R} T$ = (Gas Constatn)(Total temperature), in^2/sec^2

r = Radial coordinate for thrust bearing, divided by R_2

t = Time, seconds

V = P_s/P_a, pressure ratio

W_a = Axial load, lbs.

W_r = Radial load, lbs.

α = Angular displacement, radians

α, β = Coefficients given by Eq. (60) and (61)

γ = R_1/R_2

ϵ = Journal bearing eccentricity ratio

ϵ_T = Axial displacement divided by C_T

ζ, ζ_1 = Axial coordinates for journal bearing, divided by R. See Fig. 1

η = R/R_2

θ = Circumferential angular coordinate, radians

Λ_t = Feeding parameter ,see Eq. (6)

Λ_s = Λ_t/V

μ = Gas viscosity, lbs-sec/in^2

ν = Vena contracta coefficient

ξ_1 = L_1/D

ξ_2 = L_2/D

σ = Frequency number, see Eq. (3)

τ = ωt

ψ = Orifice parameter, see Eq. (19)

ω = Frequency, rad/sec