DESCRIPTION
OF EXPERIMENTAL OMEGA
POSITION LOCATION EQUIPMENT
(OPLE)

JANUARY 1966

GPO PRICE $ 

CFSTI PRICE(S) $ 

Hard copy (HC) 2.00
Microfiche (MF) 50

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND
DESCRIPTION OF EXPERIMENTAL
OMEGA POSITION LOCATION EQUIPMENT (OPLE)

Charles Laughlin
Gay Hilton
Roger Hollenbaugh
Richard Lavigne

January 1966

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland
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</table>
1. INTRODUCTION

The purpose of the Omega Position Location Equipment (OPLE) Experiment is to demonstrate the feasibility of using the Omega Navigational System in conjunction with synchronous satellites to establish a global location and data collection system. The OPLE concept can be applied to various platform user requirements such as oceanographic buoys, commercial aircraft, ocean vessels, animal migration studies, etc. However, the platform which presents the most stringent packaging requirements and, which is of primary concern in the OPLE Experiment, is the meteorological balloon. An operational balloon system (Reference 1) with the capability of providing synoptic global weather information suitable for fast and accurate weather prediction would require thousands of balloons. Such a system would be required to determine the location of each balloon and provide a communications link over which meteorological data could be transmitted and yet not exceed the payload packaging requirements that would be imposed on such balloons. A brief summary of the packaging requirements includes: (1) low density to prevent damage to aircraft in case of a collision, (2) lightweight (less than five pounds) to meet the carrying capacity of the balloon, (3) low average power consumption consistent with balloon battery/solar cell systems (4) reliability sufficient to assure six months operation (5) ability to withstand the temperature extremes of the balloon environment, (6) low cost to insure the economic practicality of the system. The basic premise for the conception of any balloon system is then the degree of simplicity inherent in the balloon's electronic package.

2. OPERATIONAL OPLE CONSIDERATIONS

2.1 Omega System Description

The Omega System (Reference 2) was developed at the Naval Electronics Laboratory with assistance from several other organizations including the Harvard Cruft Laboratory and the Naval Research Laboratory. Evolution of the Omega system followed an extensive investigation of very-low-frequency propagation characteristics throughout the last decade. One result of these investigations has been to show that the 10 kc region of the VLF spectrum has a very low attenuation rate and exhibits exceptional phase stability. These characteristics permit world wide propagation of radio waves and allow phase measurements with an rms variation of less than five microseconds. Within this
frequency range, the radiated energy is propagated as a guided wave in the space between the earth and the reflecting ionosphere with an attenuation rate of nearly that due to inverse spreading loss. Near the transmitter the ground wave predominates and interference between the ground wave and the single-mode guided wave transmission causes phase shifts of considerable magnitude. Beyond a few hundred miles, the single-mode propagation dominates and the signal can be used for position measurements up to a distance of at least 5000 miles from the transmitter. Frequencies between 10 and 14kc were chosen for use by Omega because of the high excitation of the first mode and the low higher mode interference effects at sunrise and sunset.

The optimization of the Omega frequencies with respect to the above medium characteristics has been verified by experimental results. The experimental phase of the Omega program is essentially completed and an overall operational design of considerable flexibility has been established and is rapidly being implemented. An Omega Project Office under the Chief of Navy Material, has been established to direct the construction of the entire Omega network. Three complete operational stations providing coverage over the north-western quadrant of the earth are under construction and will be used for fleet evaluation tests early in 1966. An additional five operational stations are expected to complete the Omega network by 1969.

The operational Omega system will use eight VLF transmitting stations radiating 10 kilowatts of power each with an average separation between stations of about 5000 nautical miles. It is expected that all eight transmissions will be receivable at nearly every point on earth and that at least five of the eight will produce usable signals with only a short monopole receiving antenna. The Omega receiver measures the relative phase of the signals from at least two pairs of stations — that is, three transmitters. Two lines of position (isophase contours) are generated by the phase difference between each of the two transmitter pairs and the position of the receiver is established by the intersection of the two isophase hyperbolic contours. The very long base lines between stations results in position lines that diverge only slightly and that cross each other at nearly right angles. This geometric excellence, along with the high degree of phase stability and low attenuation rates of VLF radio signals, results in a reliable system with high absolute accuracy that varies little with geographical position.

The uncertainty in an Omega line of position can be summarized as one standard deviation of about three-tenths of a mile over a daytime propagation path and about twice that at night. By the time the Omega network becomes operational, it is expected that the rms fix error, for all causes combined, will be about one mile in the daytime and two miles at night (Reference 3). In recent
tests performed by the Naval Research Laboratory, the rendezvous or station keeping accuracies attained were around 200 yards (Reference 4). Thus, a fixed station can provide very accurate relative position measurements (and velocity measurements through continual tracking) of balloons in a large vicinity.

The Omega system presently being implemented provides for considerable flexibility and future expansion. The transmitted signal spectrum is shown in Figure 1 while the transmitting station time multiplexing scheme is shown in Figure 2. The primary transmission frequencies are 10.2 kc, 11.33 kc, and 13.6 kc each of which is phase modulated by a single tone of 11.33 cps, 45.33 cps and 226.66 cps tones, respectively. In addition, eight other frequencies are shown which are all sub-harmonics of 408 kc. Each of these eight frequencies is assigned to one of the eight transmitting stations in accordance with the time multiplexing diagram in Figure 2. These eight frequencies permit transmitter station identification but they are not essential to the position location function. By means of very low phase deviation, these frequencies also provide for inter-station communications for control and synchronization purposes.

Figure 1. Omega System Signal Spectrum

Ambiguity resolution is performed by successive measurements of the received phase of the carrier difference and modulation frequencies. The carrier frequencies and their modulating tones have been selected to permit construction of the difference frequencies listed in Table 1 along with the resulting ambiguity resolution steps. No dead reckoning, lane counting or log keeping is
necessary and the transmitted sequence will permit completely automatic operation on an "as required" basis.

<table>
<thead>
<tr>
<th>STATION</th>
<th>10.2</th>
<th>13.6</th>
<th>11.33</th>
<th>10.462 kcs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10.736</td>
<td>10.2</td>
<td>13.6</td>
<td>10.736 kcs</td>
</tr>
<tr>
<td></td>
<td>11.027</td>
<td>10.2</td>
<td>13.6</td>
<td>11.027 kcs</td>
</tr>
<tr>
<td></td>
<td>11.657 kcs</td>
<td>10.2</td>
<td>13.6</td>
<td>11.657 kcs</td>
</tr>
<tr>
<td></td>
<td>12.000 kcs</td>
<td>10.2</td>
<td>13.6</td>
<td>12.000</td>
</tr>
<tr>
<td></td>
<td>12.363 kcs</td>
<td>10.2</td>
<td>13.6</td>
<td>11.33</td>
</tr>
<tr>
<td></td>
<td>11.33</td>
<td>12.750 kcs</td>
<td>12.750</td>
<td>10.2</td>
</tr>
<tr>
<td></td>
<td>13.6</td>
<td>11.33</td>
<td>13.161 kcs</td>
<td>10.2</td>
</tr>
</tbody>
</table>

Figure 2. Omega Transmitted Signal Format

2.2 OPLE Functional Description

When the Omega system becomes operational, any vehicle will be able to locate its position with only appropriate VLF receiving equipment, suitable processing and display equipment, and a navigator equipped with necessary charts, instructions and procedural information. If all of the above mentioned items, except for the VLF receiving equipment, could be at a convenient central processing location, and if an adequate communication circuit could be provided between the VLF receiver on the vessel and the central processing equipments, then the position of the vessel could be ascertained at the central processing location and the size of the equipment aboard the vessel could be reduced accordingly.

In line with these thoughts, it seems reasonable to consider the possibility of electrically connecting the platform VLF receiver to the central processing location through a synchronous satellite repeater.
Table 1

Omega Ambiguity Resolution Steps

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Period</th>
<th>( \lambda /2 ) km</th>
<th>( \lambda /2 ) N.Miles</th>
<th>( \lambda /2 ) S.Miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.2 kcs</td>
<td>98 ( \mu s )</td>
<td>14.7</td>
<td>7.94</td>
<td>9.12</td>
</tr>
<tr>
<td>3.4 kcs</td>
<td>306 ( \mu s )</td>
<td>44.1</td>
<td>23.8</td>
<td>27.4</td>
</tr>
<tr>
<td>1.13 kcs</td>
<td>880 ( \mu s )</td>
<td>132.1</td>
<td>71.5</td>
<td>82.0</td>
</tr>
<tr>
<td>226.66 cs</td>
<td>4.48 ms</td>
<td>672</td>
<td>363</td>
<td>418</td>
</tr>
<tr>
<td>45.33 cs</td>
<td>22.06 ms</td>
<td>3309</td>
<td>1786</td>
<td>2057</td>
</tr>
<tr>
<td>11.33 cs</td>
<td>88.24 ms</td>
<td>13,236</td>
<td>7143</td>
<td>8229</td>
</tr>
</tbody>
</table>

In this way, both the platform and the satellite would essentially become transponders although different frequency conversion and stabilization schemes would be used for each. A frequency drift cancellation technique would be used to eliminate the need for a highly stable frequency source on the platform. In this technique the stability of the received interrogation carrier as generated by the ground station would be used to determine the platform transmitter stability, the only error introduced by the platform being proportional to the offset frequency between the received and transmitted carriers. The platform would contain a suitable VLF receiver, meteorological sensors, multiplexing equipment and a VHF transmitter. The VHF link for each platform would be of relatively narrow bandwidth since the VLF Omega Spectrum is narrow band by necessity and the meteorological sensors would be low in data rate.

2.3 Operational System Description

An operational system, as shown in Figure 3, would consist of, (1) an OPLE Control Center, (2) a Command and Data Acquisition Station, (3) a synchronous satellite, and (4) the OPLE platforms working in conjunction with the Omega network. The OPLE Control Center will originate all the control signals
that determine the sequencing of platform addresses, and the times of interrogation. The OPLE Control Center will receive the Omega transmissions to derive timing and to determine the state of the Omega net. Satellite availability times will come from the Satellite Control Center and the OPLE Control Center will then determine which platforms are to be interrogated and the times of interrogation. This information will be sent to the Satellite Command and Data Acquisition (CDA) Station and to the interrogation transmitter.

When a platform receives its address from the interrogation transmitter, it turns on a VHF transmitter, a VLF receiver, and starts a low precision timer. The incoming Omega signals are up-converted as received and relayed to the satellite along with an up-converter reference followed by data from the meteorological sensors in time sequence. The satellite again translates this signal in frequency and retransmits it to the CDA station which has previously been instructed by the satellite control center to accept the platform transmission. The CDA Station will relay this signal to a data processing center where signal-tracking and phase-measuring equipment will locate the position of the platform in much the same way as would a shipboard navigator using the conventional Omega system. The meteorological data will be processed to baseband and made available to the users.
Under the concept proposed above, the required platform-to-satellite contact time would be about three minutes and the required bandwidth would be about 2 kilocycles. Since the satellite transponder would have a much greater bandwidth capability than two kilocycles (say hundreds of kilocycles for example) it would be possible to frequency multiplex the platform transmissions to increase the number of platforms that could be serviced. That is, perhaps 50 different transmission frequencies could be divided up according to geographical areas. The fifty simultaneous interrogations, each of three minutes duration, would allow 1000 platform interrogations per hour or a total of 2000 platforms, each interrogated once every two hours or 4000 platforms, each interrogated every four hours. With three equatorial synchronous satellites, 6000 to 12,000 balloons could be serviced with world-wide coverage.

2.4 Global Coverage Considerations

The circle of illumination on the earth by an equatorial synchronous satellite has a radius of about 81 degrees of longitude at the equator. This circle is centered on the equator and extends to within about 9° of each pole. With two synchronous satellites in equatorial orbits spaced on opposite sides of the earth, the total coverage would include all but a segment circling the earth of about 17° of width as illustrated in Figure 4a. With three synchronous satellites in equatorial orbits equally spaced around the earth, complete coverage is provided.

![Figure 4a. Coverage by Two Equatorial Synchronous Satellites](image)

![Figure 4b. Polar Blind Region for Three Equatorial Synchronous Satellites](image)
from about 73° north latitude to 73° south latitude all around the earth. This coverage would extend to the arctic and antarctic circles which are located at approximately 66° latitude with a platform-to-satellite angle of about 7 degrees. The spherical triangular areas located at the poles which are not covered are illustrated by Figure 4b.

The extremely poor conductivity in the arctic permafrost and polar icecap regions will result in a different velocity of propagation at VLF as compared with other areas of the earth. The extent of this difference is difficult to determine at frequencies as low as 10 kc where the depth of penetration of the ratio wave is so great. However, the difference in phase delay is dependent upon the conductivity so that in principle, it is possible to derive correction coefficients for the polar regions in the same way as will be done for the land masses. The Omega project has been concerned with this potential problem and has established monitoring stations at College, Alaska (about 65° north latitude) and in Northern Norway (about 70° north latitude). Results to date indicate that phase disturbances are small in magnitude at least at these latitudes and should be correctable so that the main concern is to provide adequate signal strength to overcome the high attenuation in the polar regions. The location of the Omega transmitting sites have been selected with this limitation in mind. However, the possibility exists that difficulties may be experienced in the polar regions.

In the event that the Omega system is capable of providing adequate coverage over the polar regions, it is constructive to further discuss the possibility of global coverage with synchronous satellites. In particular, consider one synchronous satellite in an inclined plane along with two synchronous satellites in the equatorial plane. This system of satellites would provide coverage of each pole for one continuous period of time each day. The length of this time period depends on the angle of inclination of the orbital plane to the equatorial plane. A plot of the pole visibility time T in hours is given in Figure 5 for antenna elevation angles of 0, 5 and 10 degrees.

The combination of three synchronous satellites properly phased, where one has an orbital plane inclined by 30 degrees, would allow full earth coverage over each 24 hour period. The inclined plane allows the satellite to look over the poles and view the entire area not seen by the other two satellites. Each polar area would be entirely covered for a minimum of 4 hours per day. It seems that virtually any realistic operational requirement could be met by three phased synchronous satellites, each with properly selected inclination angles.

3. OPLE EXPERIMENT DESCRIPTION

The OPLE Experiment is designed to demonstrate the feasibility and to explore the potential operational system concepts discussed above. The experiment
Figure 5. Synchronous Satellite Pole Visibility Time vs Orbital Inclination

This figure illustrates the relationship between the pole visibility time and the satellite's orbital inclination. The graph shows how the visibility time changes with different antenna elevation angles, labeled as $a = 0^\circ$, $a = 5^\circ$, and $a = 10^\circ$.

The synchronous satellite pole visibility time vs orbital inclination consists of the following basic elements. (1) The synchronous ATS-C equipped with a VHF transponder/antenna system similar to the ATS-B system; (2) Twelve experimental platforms will be fabricated and deployed, (as described later) to fully demonstrate the operational capabilities of the OPLE System; (3) An OPLE Control Center will be established at the Goddard Space Flight Center to determine the position of the platforms by measuring the phase of the recovered Omega signals, initiate interrogation commands, and supervise the overall
operation of the experiment; (4) The CDA Station for the ATS-C which will be at Rosman, N.C.

3.1 Platform Description

The OPLE platform functional diagram is shown in Figure 6. A common antenna structure will be used for both the VLF and VHF receivers. At VHF it will have a 3 db beamwidth of 100 degrees and be circularly polarized with a maximum axial ratio of 3 db. The VLF antenna will be omnidirectional and will present sufficient aperture to allow the minimum VLF atmospheric noise to exceed the VLF receiver noise. The antenna coupler will provide the proper isolation and impedance match between the VHF transceiver, the VLF receiver and the antenna. The VHF receiver will contain an interrogation detector and may contain either a phase-locked-loop or a non-oven controlled oscillator with a stability of 1 part in $10^6$ to provide a stable reference for the VLF receiver converter. The address decoder will process the detected interrogation command to initiate action of the multiplexer which controls the platforms timing,
switching and data modulation functions. The VLF receiver converter accepts signals from the multiplexer and upconverts the signals to the proper VHF frequency. The VHF transmitter provides the necessary VHF power amplification for transmission to the synchronous satellite. The platform transmitter/antenna system will provide an effective radiated power of five watts.

Figure 7 shows a preliminary block diagram of the platform electronic package. The VHF frequencies given in Figure 7 are presented as an example and it is anticipated that the OPLE experimental frequencies will be within 5 mc of those shown. The VHF receiver will receive an interrogation signal carrier, which will always be available during those times when the OPLE experiment is utilizing the VHF satellite transponder. The VHF receiver will normally be on at all times and therefore will be designed to consume as little power as is technically possible. The Omega spectrum as received occupies 3.6 kc of bandwidth, however, to conserve satellite transponder bandwidth and to reduce the platform transmitter power the transmission bandwidth will be compressed to 2 kc. One possible method of accomplishing this is shown in Figure 7 with a resulting Omega spectrum as shown in Figure 8.

The operational sequence of the platform would conceptually be as follows. Upon receipt of the proper platform address the platform equipment is turned on and proceeds through the following sequence.

(a) Transmit acquisition tones

(b) Transmit received Omega signals

(c) Transmit meteorological sensor data preceeded by the platform address.

(d) Return to standby power.

The scheme shown uses a drift cancelling technique to provide a relatively stable output frequency from the platform. Two separate frequency sources are shown in Figure 7. One, \( f_2 \), is a multiple of the basic Omega tones and is used to derive the Omega calibration tones. This frequency and the corresponding source are common to all platforms. The second oscillator, \( f_1 + naf \) determines the platform channel frequency, where \( n \) is the channel number and \( af \) is the channel spacing. To obtain the maximum use of the available satellite transponder bandwidth (nominally 100 kc wide on ATS/C) each platform will be assigned a channel approximately 2.5 kc wide within the transponder bandwidth (See Figure 9). The naf term associated with oscillator \( f_1 \) determines which channel is assigned to a particular platform.
Figure 7. OPLE Platform Block Diagram
Some degree of protection against false interrogation is desirable since an unrequested response from a platform can be disturbing to the system and response to alien transmissions is most undesirable. For this reason, each interrogation sequence will include the transmission of a particular platform address followed by a bit-by-bit inverse of the same address. The platform
will perform bit-by-bit comparison in real time of the second received address with the first received address. In this way, single bit error rejection can be insured with a storage register of only half the length of the full interrogation sequence.

A thirteen bit platform address is sufficient for 6000 platforms in an operational system. That is, a thirteen bit address would permit 8192 unique platform addresses. Assuming that the single-bit error probability is of some value, say Pe for each platform, the probability that a platform other than the one addressed will respond can be approximated by the value of $1/13 \cdot Pe^2 \cdot (1-Pe)^2$. Using a single-bit error probability of one per thousand, the above value becomes insignificant. On the other hand, the probability that the correct platform will successfully recognize its address can be found from the value of $(1-Pe)^{26}$ which for the same Pe is approximately 0.974 or about 97 times out of one hundred. This value could be increased by using more than two consecutive address transmissions with a "majority vote" logic on the platform or by use of a more elaborate coding scheme such as an (n, k) group code. However, this additional circuitry may not be warranted since the platform control center will have immediate knowledge of a failure to respond and can re-initiate the interrogation.

3.2 Satellite Transponder Description

The satellite transponder will be nearly identical to the VHF transponder flown on ATS-B. Figure 10 is a functional block diagram and Table 2 lists the transponder's major characteristics. This transponder utilizes a despun phased-array antenna to allow the antenna beam to always remain positioned on the earth thereby compensating for the stabilizing spin of the satellite. The antenna gain will be between 8.5 and 10 db but for the purposes of link calculations the 8.5 db value was used. The antenna is composed of eight elements each of which has its individual receiver, phase shifter and transmitter all coupled together with a common intermediate frequency conversion section. The satellite transponder will receive the platform transmission, convert its frequency from the 148 Mc band to the 136 Mc band, and amplify it for transmission to the CDA Station. One primary requirement of the transponder receiver is that its equivalent front-end noise spectral density be less than the received spectral density by at least 10 to 1. This will assure that the signal-to-noise ratio established at the platform will not be materially degraded by the satellite, and will allow the ground equipment to take full advantage of the bandwidth reduction improvement. Table 3 shows that the output power from the transponder of approximately one watt per 2 kc channel will be sufficient to assure reliable signal processing. This transponder will also be used to relay interrogation commands from the ground station to the platforms and the right-hand column of Table 3 indicates the wide margins available from the transponder in this mode of operation.
Transponder Characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitter power output (per whip)</td>
<td>7dbw (5 watts)</td>
</tr>
<tr>
<td>Total power output</td>
<td>16.0dbw (40 watts)</td>
</tr>
<tr>
<td>Transmitter antenna gain</td>
<td>8.5db (minimum)</td>
</tr>
<tr>
<td>Transmitter losses (diplexers, cables, etc.)</td>
<td>1.5db</td>
</tr>
<tr>
<td>Effective radiated power</td>
<td>23.5dbw (230 watts)</td>
</tr>
<tr>
<td>Receiver noise figure</td>
<td>3.5db</td>
</tr>
<tr>
<td>Receiver bandwidth</td>
<td>100 kc</td>
</tr>
<tr>
<td>Receiving antenna gain</td>
<td>8.5db (minimum)</td>
</tr>
<tr>
<td>Weight</td>
<td>21.0 pounds</td>
</tr>
<tr>
<td>Power required</td>
<td>90 watts</td>
</tr>
</tbody>
</table>

3.3 Platform-to-Satellite Transmission Link

The limiting portion of the overall platform-to-processing communication link will be the platform-to-satellite path even though the satellite-to-platform
<table>
<thead>
<tr>
<th>Platform-Satellite-Ground Station</th>
<th>Ground Station-Satellite-Platform</th>
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<tbody>
<tr>
<td></td>
<td><strong>2 kc Bandwidth</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Location and Data Channel</strong></td>
</tr>
<tr>
<td></td>
<td><strong>0.5 kc Bandwidth</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Interrogation Channel</strong></td>
</tr>
<tr>
<td><strong>Up Link 148 Mc</strong></td>
<td><strong>Nominal</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Neg. Tol.</strong></td>
</tr>
<tr>
<td>Radiated Power</td>
<td>100 watts total 8.4 watts/channel</td>
</tr>
<tr>
<td>5 w minimum</td>
<td>Transmitter Power</td>
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<tr>
<td></td>
<td>(Programmable)</td>
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<tr>
<td>Platform Antenna Gain*</td>
<td>Ground Station Cable Losses</td>
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<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>Path Losses</td>
<td>Ground Station Antenna Gain</td>
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<td>168.6</td>
<td>28.0</td>
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<tr>
<td>Satellite Antenna Gain</td>
<td>Path Losses</td>
</tr>
<tr>
<td>8.5</td>
<td>168.6</td>
</tr>
<tr>
<td>Satellite Cable Losses</td>
<td>Satellite Antenna Gain</td>
</tr>
<tr>
<td>1.5</td>
<td>8.5</td>
</tr>
<tr>
<td>Satellite Receiver Noise Power‡</td>
<td>Satellite Cable Losses</td>
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<tr>
<td>-165.9 dbw</td>
<td>1.5</td>
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<tr>
<td></td>
<td>Satellite Receiver Noise Power‡</td>
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<td></td>
<td>-165.9 dbw</td>
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<tr>
<td>Received Signal-to Noise Ratio</td>
<td>11.3</td>
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<tr>
<td>7.8</td>
<td>21.2</td>
</tr>
<tr>
<td>Down Link 137 Mc</td>
<td>Received Signal-to Noise Ratio</td>
</tr>
<tr>
<td></td>
<td>40 watts total 1.0 watt/channel</td>
</tr>
<tr>
<td>1.0 watt/channel plus 3.9 dbw</td>
<td>Transmitter Power</td>
</tr>
<tr>
<td></td>
<td>3.9 dbw</td>
</tr>
<tr>
<td>Transmitter Power</td>
<td>Satellite Cable Losses</td>
</tr>
<tr>
<td>0.0 dbw</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Satellite Cable Losses</td>
</tr>
<tr>
<td>Satellite Cable Losses</td>
<td>1.5</td>
</tr>
<tr>
<td>Satellite Antenna Gain</td>
<td>Satellite Antenna Gain</td>
</tr>
<tr>
<td>8.5</td>
<td>8.5</td>
</tr>
<tr>
<td>Path Losses</td>
<td>Path Losses</td>
</tr>
<tr>
<td>167.7</td>
<td>167.7</td>
</tr>
<tr>
<td>Ground Station Antenna Gain</td>
<td>Platform Antenna Gain*</td>
</tr>
<tr>
<td>28.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Ground Station Cable Losses</td>
<td>Platform Cable Losses</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Ground Station Receiver Noise‡</td>
<td>Platform Receiver Noise‡</td>
</tr>
<tr>
<td>-165.9 dbw</td>
<td>-171.9 dbw</td>
</tr>
<tr>
<td>Received Signal-to-Noise Ratio</td>
<td>32.7</td>
</tr>
<tr>
<td>27.7</td>
<td>14.6</td>
</tr>
</tbody>
</table>

*Includes 3db polarization losses.

†Effective receiver temperature of 941°K.
path results in a lower received signal-to-noise ratio. The lower signal-to-noise ratio in the later path will not be a limiting factor since it is used only for interrogation and redundant coding will be used. The primary requirement of the platform-to-satellite link is that the added noise be insignificant compared with that already present at the VLF receiver output. The VLF receiver worst case output will consist of a number of discrete or quasi-discrete tones whose total power will be small compared with the total noise power across the 2 kc band. That is, the worst-case received signal-to-noise ratio will be below zero db. The Omega tones will then only be discernible and useful after extraction by narrowband tracking filters at the data receiving center. Figures 11 and 12 show anticipated signal-to-noise ratios versus receiver-transmitter distances. Figure 11 is an approximate lower bound whereas Figure 12 is an approximate upper bound on an experimentally determined range of values (references 5 and 6).

![Diagram](image)

**Figure 11. S/N Ratio vs Transmitter-Receiver Distance (D) for 10.2 kc**

A reasonably accurate model of the signal transmitted from the platform is a uniform density or flat-white-noise signal. The ultimate phase measurements will not be materially altered by the addition of noise with a similar density.
Assume 35db noise above 1μ v/m in 1cps BW

Received Signal Strengths taken from Al'pert Graphs

TRANSMITTER-RECEIVER DISTANCE IN THOUSANDS OF NAUTICAL MILES

Figure 12. S/N Ratio vs Transmitter-Receiver Distance for 10.2 kc (Reference 5)

distribution and with a total noise power of ten-to-one or less than that already present. Thus, a platform-to-satellite transmission link which can produce a 10db minimum received signal-to-noise ratio over a 2kc bandwidth will suffice. The amplitude linearity requirement of the communication link is not critical and a few percent of total harmonic and intermodulation distortion should be easily tolerated.

As with any time measuring scheme, uncontrolled phase distortion can produce large errors; however, with the system proposed here, serious problems should not exist. First of all, reference tones will be laid on with the received Omega signals at the platform, which will be carried along to the processing equipment. These tones will allow for conversion oscillator instabilities and doppler effects. In addition, up to five microseconds of group delay distortion is permissible by virtue of the finest resolution distance of one-half mile.

3.4 Control Center Description

The OPLE Control Center (OCC) will receive the satellite transponder's composite signal either from the DA Station via a land line or directly from a
receiver system directed at the satellite as shown in Figure 13. The OCC will also receive WWV transmissions to provide timing information and the Omega transmissions to determine the status and timing sequence of the Omega network. The OCC will process all platform transmissions and initiate all platform interrogations and commands. The received composite transponder signal will be applied to channel selectors and then to individual OPLE receivers which will determine the platforms position and detect the meteorological data. The OPLE receiver will without external rate aiding determine platform position at platforms with velocity up to 200 knots. The frequency synthesizer will be used to generate the Control Center's reference frequencies utilizing an atomic standard, the WWV receiver and the Omega monitoring receiver. An Omega synchronous selector will provide the OPLE receivers with the proper commutation pattern. A control console capable of being operated by one operator under nominal conditions will initiate all control center sequences and monitor the overall operating status of the OPLE experiment. The signal handler-computer will perform all the required automatic operations necessary for the performance of the OPLE experiment. The signal handler-computer will provide clocking information, process the OPLE receiver outputs by performing diurnal and coordinate corrections, decode the meteorological data to provide the sensor data, and provide a platform status signal to the control console. The control center will contain an address coder and interrogation transmitter. The interrogation transmitter will have a maximum power of 40 watts programmable in 3 db steps to 0.01 watts and a center frequency of 148 ±5 mc with a stability of 1 part in 10^{10}. The transmitter power in conjunction with simulated channel modulations will be used by the Control Center to control the operating level of the transponder. A service channel will also be established to provide a two way communications link between the platform deployment area and the Control Center. A VHF receiver with a nominal I.F. of 100 kc and a tuning range from 134 to 155 mc will be used in conjunction with a helical antenna with a nominal gain of 14 db to directly receive the satellite transmissions. The output of this receiver will go to both the platform processing circuits and a transponder monitoring unit which will determine the correct interrogation transmitter power and modulation level to maintain the optimum transponder operating level.

3.5 Platform Deployment

Twelve platform equipments will be fabricated and deployed for the ATS-C Experiment. Three platforms will be used to simulate balloon flights using aircraft. These air-borne platforms will provide information as to the optimum frequency of interrogations and reporting times required to minimize the platform power consumption without degrading the wind velocity determinations as measured by averaging the balloon tracks. These platforms will also be used
in a separate experiment especially designed to evaluate the performance of a differential or relative location network for tracking balloons in the neighborhood of a fixed monitoring location.

Three platforms will be placed on NASA/GSFC tracking ships to provide Omega location information over a wide area for long periods of time and to compare the Omega positional information with the ships navigation aids. Two platforms will be placed on ocean buoys if possible to provide a long-term free-drifting environment which will augment the limited airborne tests. Two platforms may be placed on aircraft in conjunction with communications experiments using the ATS-C transponder and to evaluate the performance of the system with high speed aircraft. The remaining four platforms will be placed at widely placed fixed sites to provide an experimental standard. These receivers will be used in a differential Omega system test which will enable the moving platforms to be located to within a few hundred feet and to enable wind velocity measurements to within a few knots (reference 4). At least one experiment will be performed with a van mounted receiver to evaluate performance on land. In order to closely control these activities, and to obtain real time information as to the operational status of each of the experimental platforms, a two-way voice service channel will be implemented through the transponder from the platforms to the control center.

4.0 FUTURE EXTENSIONS

One of the prime components of the proposed experiment, and of any future system, is the satellite antenna. At synchronous altitude, the angle subtended by the earth is 16.4 degrees. This would allow use of an antenna with approximately 20.1 db gain which would, assuming zero pointing error, provide a minimum of 17.1 db of gain over the entire illuminated area of the earth. An antenna pointing error of 25% (4.1 degrees) would necessitate using an antenna with a gain of 16 db gain, which would provide a minimum gain of 13.7 db over the illuminated area of the earth. Thus, the maximum allowable antenna gain is determined by the antenna pointing error, and since 4 degrees of error is well within the capability of present mechanical and electrical stabilization systems, a nominal 14 db gain antenna is easily feasible for an operational system. Table 4 shows the required nominal transmitter powers required for two hypothetical operational systems based on 14 db gain satellite antennas.

Once the feasibility of the OPLE concept is demonstrated by the experiment, it will be possible to design an operational data collection and location system capable of serving a wide variety of users on a global scale. One possible additional technique compatible with the OPLE System would be the use of an
Table 4
Operational Performance Predictions

<table>
<thead>
<tr>
<th></th>
<th>136-148 mc TRANSPONDER</th>
<th>400-450 mc TRANSPONDER</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2 kc CHANNEL BANDWIDTH</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SATELLITE RECEIVER NOISE POWER</strong></td>
<td>-165.4 dbw (1000&quot;k)</td>
<td>-168.4 dbw (500&quot;k)</td>
</tr>
<tr>
<td><strong>SATELLITE RECEIVER THRESHOLD LEVEL FOR MINIMAL OMEGA DISTORTION</strong></td>
<td>+10.0 db</td>
<td>+10.0 db</td>
</tr>
<tr>
<td><strong>SATELLITE CABLE AND DIPLEXER LOSSES</strong></td>
<td>1.5 db</td>
<td>1.5 db</td>
</tr>
<tr>
<td><strong>SATELLITE ANTENNA GAIN</strong></td>
<td>14.0 db</td>
<td>14.0 db</td>
</tr>
<tr>
<td><strong>MAXIMUM RANGE PATH LOSSES</strong></td>
<td>168.0 db</td>
<td>173.0 db</td>
</tr>
<tr>
<td><strong>PLATFORM ANTENNA GAIN</strong></td>
<td>0.0 db</td>
<td>0.0 db</td>
</tr>
<tr>
<td><strong>PLATFORM CABLE LOSSES</strong></td>
<td>0.5 db</td>
<td>0.5 db</td>
</tr>
<tr>
<td><strong>REQUIRED PLATFORM TRANSMITTER POWER</strong></td>
<td>+0.6 dbw (1.2 watts)</td>
<td>+2.6 dbw (1.8 watts)</td>
</tr>
</tbody>
</table>

automatic Omega receiver on the vehicles of users which have the space and power available. This receiver would reduce the Omega positional information to something on the order of 26 bits, thus greatly reducing the transmission power and/or transmission time required. A semi-automatic receiver currently being produced by the ITT Company for the U.S. Navy has 1 cubic foot of volume, weighs 45 pounds and requires 150 watts of primary power. This receiver contains cathode ray displays, electronic readout displays, battery pack and other auxiliary equipment not essential to automatic operation. Figures 14 and 15 are conceptual block diagrams of two Omega receivers in which microelectronic circuitry could be extensively used.

Using these concepts as a basis, commercial aircraft, including the Super Sonic Transport (see Appendix A), could be tied together in an air traffic control system capable of safely directing the anticipated air traffic for many decades to come. Also flight schedules and terminal traffic would be better controlled if timely knowledge was available as to exact arrival times. Steamship lines could also use this system for scheduling arrival times and port facilities to more economically operate and regulate their overall traffic.

Another possible application is as a recovery aid for the Apollo re-entry vehicle. With OPLE type equipment aboard the Apollo re-entry vehicle and the various units of the recovery force including the helicopters, it would be possible for a central control center to know the relative position of all the recovery
units and spacecraft to within 200 yards and the absolute position of any one of them to within 1 or 2 miles regardless of the landing site location.

Meteorologists would have a means of gaining atmospheric and surface data by means of balloons and buoys. In the case of tethered buoys, the Omega portion of the receiver could be off with only the data channel activated for power conservation and in the event of a broken tether, the Omega receiver could be activated thus giving the position of the wayward buoy to service vessels. Oceanographers desiring ocean current and sea state data could use the system in much the same way. Also Zoologists desiring to ascertain the migration route of various land and air-breathing sea animals could possibly use such a system (reference 7).

One control center operating the entire system would greatly increase the economy of the system as related to each individual user. Anyone desiring to use the system would inform the center of the intended region of operation, the platform frequency, and the number and time of the interrogations. The retrieved data and locations could then be sent to the user on a near real time
basis or be provided in a more leisurely manner depending on the users requirements. An unmanned platform could thus be tracked for scientific data and a manned conveyance would be tracked for safety control and scheduling purposes and in addition have a voice and communications channel to a central control center. It would then be possible to learn more concerning the global phenomena of the earth and for the higher speed and more complex transportation systems to be run in a safer and more efficient manner.
REFERENCES


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ACKNOWLEDGMENTS

The possibility of using the Omega Navigation System for the location of meteorological platforms was first brought to the authors' attention by Dr. John W. Townsend, Jr. of the Goddard Space Flight Center in January 1965. He referred the authors to Mr. H. S. Moore of the National Environmental Satellite Center for whom the Aerospace Corporation was performing a preliminary system definition of the GHOSTS and SCOMO concepts and in which the Omega System was considered the best means for balloon location. The final report (Reference 1) covering this work was made available to the authors in November 1965 after the work reported in this description of OPLE was completed.

The authors gratefully acknowledge the assistance and support of many governmental and private individuals and organizations who have been of great help during the formative stages of the OPLE concept.
Appendix A

Position and Velocity Determination on High-Speed Aircraft

I. Standard Omega Receiver Approach

The standard Omega receiver is directly applicable for use on high-speed aircraft. The only differences between operation on a stationary or slowly moving platform (less than 180 knots) are (1) the measured phase difference could vary at a noticeable rate, and (2) aircraft maneuvers could cause acceleration effects which affect the capability of the phase lock loops to lock on to an incoming signal.

The following is the form of the received signal from station A as seen by a receiver aboard a high speed aircraft (assuming no acceleration effects).

\[ S_A = \cos \left( \left( W_0 + W_{dA} \right) t + \phi_A \right) \]

The \( W_{dA} \) term is the doppler frequency caused by the aircraft motion relative to the transmitting station A; \( W_0 \) is the transmission frequency of the Omega station; and \( \phi_A \) is the phase angle determined by the initial location of the aircraft relative to the transmitting station.

The signal from station B will be of the same form and is given below:

\[ S_B = \cos \left( \left( W_0 + W_{dB} \right) t + \phi_B \right) \]

Putting \( S_A \) and \( S_B \) into a phase detector we obtain an output of the form:

\[ \phi \text{ difference} = (W_{dA} - W_{dB}) t + \phi_A - \phi_B \]

At any instant of time the above function defines isophase hyperbolic contours one of which includes the receiver position. The only difference between this and a slowly moving platform is that the \( W_{dA} - W_{dB} \) term for the slowly moving case is so small that a very long time is required before it causes a noticeable change in the \( \phi \) difference term.

The velocity vector could be determined by (1) measuring the position change over a known time interval (and calculating, \( \bar{V} \) average = \( \Delta \) position/\( \Delta \) time); by

27
determining the slope of the phase detector output, or by measuring doppler shifts and calculating the velocity component magnitudes with the following formula (Derived in Part II of the appendix, Eq. #4).

\[ |\vec{V}_k| = \text{VELOCITY COMPONENT TOWARD STATION } S_k \]

\[ |\vec{V}_k| = 3600 f_{dk} \lambda \]

where \( f_{dk} \) = doppler shift of frequency relative to station \( S_k \), \( \lambda \) = wavelength of ranging frequency.

Once the receiver coordinates and velocity components relative to two stations have been determined, the navigator can determine his velocity vector relative to the ground as follows:

\[ \vec{V} = x' \hat{i} + y' \hat{j} \]

where

\[ x' = \frac{V_1 \left( \frac{M_1}{y_1 - y_5} \right) - V_2 \left( \frac{M_2}{y_2 - y_5} \right)}{x_1 - x_5} - \frac{x_2 - x_5}{x_1 - x_5} - \frac{x_2 - x_5}{y_1 - y_5} - \frac{y_2 - y_5}{y_2 - y_5} \]

\[ y' = -\frac{x_2 - x_5}{y_2 - y_5} \left[ \frac{V_1 M_1}{y_1 - y_5} - \frac{V_2 M_2}{y_2 - y_5} \right] + \frac{V_2}{M_2} \left[ \frac{(y_2 - y_5)^2 + (x_2 - x_5)^2}{y_2 - y_5} \right] \]

\( x' \) = magnitude of aircraft velocity parallel to equator
\( y' \) = magnitude of aircraft velocity perpendicular to equator

The above formulae are derived as follows:
\( S_1 = \text{station 1} \)
\( S_2 = \text{station 2} \)

\( \vec{V}_1 = \text{velocity of aircraft toward } S_1 \)

Direction of \( \vec{V}_1 = \frac{x_1 - x_5}{M_1} i + \frac{y_1 - y_5}{M_1} j \)

\( \vec{V}_2 = \text{velocity of aircraft toward } S_2 \)

Direction of \( \vec{V}_2 = \frac{x_2 - x_5}{M_2} i + \frac{y_2 - y_5}{M_2} j \)

Where

\[ M_1 = \sqrt{(x_1 - x_5)^2 + (y_1 - y_5)^2} \]

\[ M_2 = \sqrt{(x_2 - x_5)^2 + (y_2 - y_5)^2} \]

\[ \vec{V}_1 = V_1 \left[ \frac{x_1 - x_5}{M_1} \, i + \frac{y_1 - y_5}{M_1} \, j \right] \]

Solving for equation of line \( L_1 \) (assuming origin located at \( X_5, Y_5 \)) where \( L_1 \) is \( \perp \) to \( \vec{V}_1 \) and passes through tip of \( \vec{V}_1 \).

\[ L_1: \ y = \frac{x_1 - x_5}{y_1 - y_5} x + b_1 \]

\[ V_1 \frac{y_1 - y_5}{M_1} = -\frac{x_1 - x_5}{y_1 - y_5} \, V_1 \frac{x_1 - x_5}{M_1} + b_1 \]

\[ b_1 = V_1 \left[ \frac{y_1 - y_5}{M_1} + \frac{(x_1 - x_5)^2}{M_1(y_1 - y_5)} \right] \]

\[ L_1: \ y = -\frac{x_1 - x_5}{y_1 - y_5} x + V_1 \left[ \frac{y_1 - y_5}{M_1} + \frac{(x_1 - x_5)^2}{M_1(y_1 - y_5)} \right] \]
equation of line $L_2$:

$$L_2: y = -\frac{x_2 - x_5}{y_2 - y_5} x + V_2 \left[ \frac{y_2 - y_5}{M_2} + \frac{(x_2 - x_5)^2}{M_2 (y_2 - y_5)} \right]$$

The vector from $(X_5, Y_5)$ to the intersection of the two lines $L_1$ and $L_2$ is the velocity vector of the aircraft with respect to the ground.

$$x' = \frac{V_1 M_1}{y_1 - y_5} - \frac{V_2 M_2}{y_2 - y_5} - \frac{x_1 - x_5}{x_1 - x_5} - \frac{x_2 - x_5}{x_1 - x_5}$$

$$y' = \frac{V_1 M_1}{y_1 - y_5} - \frac{V_2 M_2}{y_2 - y_5} - \frac{x_1 - x_5}{y_1 - y_5} - \frac{x_2 - x_5}{y_1 - y_5}$$

$$\bar{V} = x' \bar{i} + y' \bar{j}$$

The lane counting technique would be more applicable to high speed aircraft than the ambiguity resolution method using the sidetones. Flight time would be limited to several hours thereby decreasing the chance of losing a lane count that could occur on a ship which could require tracking for several days or an unmanned platform which could require tracking for several months. An aircraft would require almost continuous position and velocity information and less equipment would be required in the lane counting technique to achieve this.

For an aircraft velocity of 1800 knots perpendicular to the hyperbolic contours, a lane crossing would occur every sixteen (16) seconds (for an eight nautical mile wide lane). This rapid motion would require automatic operation with a minimum of functions for a navigator. At this velocity a temporary loss of signal could be critical. Therefore either other Omega tones would have to be utilized to check the lane count (by resolving the eight mile ambiguity) or external inputs would be necessary to periodically check and update the lane counter if necessary.
II. Direct computation of position and velocity vector from doppler frequencies:

The following scheme is based on the idea that the magnitude of the velocity of the receiver relative to each of four fixed transmitting stations uniquely determines both the position coordinates and the magnitude and direction of the velocity vector of the receiver.

The above diagram illustrates the problem. The aircraft is located at point A with coordinates \( (x, y) \) and the instantaneous velocity vector \( \vec{v} \). The unit direction vector from the aircraft (A) to each of the transmitting stations (s) is as follows:

\[
A - S_k : \frac{x - x_k}{M_k} \hat{i} + \frac{y - y_k}{M_k} \hat{j} \quad \text{(EQ. #1)}
\]

where

\[
k = 1, 2, 3, \text{ or } 4
\]

and

\[
M_k = \sqrt{(x_k - x)^2 + (y_k - y)^2}
\]

The magnitude of aircraft velocity toward a particular station is found by taking the dot product of the velocity vector and the unit direction vector.
Velocity of aircraft toward station \( S_k \):

\[
|\vec{V}|_k = m \left( \frac{x_k - x_5}{M_k} \right) + n \left( \frac{y_k - y_5}{M_k} \right)
\]  

(EQ. #2)

With four stations there are four equations of the above form.

The values of \( X_k \) and \( Y_k \) are known since the transmitter sites are fixed.

The \( |\vec{V}|_k \) quantities are calculated from the doppler frequencies as follows:

\[
PHASE \phi \quad \text{O} \quad \rightarrow \quad \pi \quad \text{IN RADIANS}
\]

\[
DISTANCE D \quad \text{O} \quad \frac{\lambda}{2} \quad \text{IN NAUTICAL MILES}
\]

For a moving platform

\[
D_k = k|\vec{V}|_k t
\]

therefore the \( \phi \) due to the velocity is:

\[
\phi_k = k \frac{2\pi|\vec{V}|_k t}{\lambda} \text{ radians}
\]

Since \( w = d\phi/dt \):

\[
w_{d, k} = k \frac{2\pi|\vec{V}|_k}{\lambda} \text{ radians/second}
\]

this angular frequency = \( 2\pi \times \text{doppler frequency} \)

\[
: f_{d, k} = k \frac{|\vec{V}|_k}{\lambda}
\]  

(EQ. #3)
for \( \lambda \) in nautical miles, \(|\vec{V}|_k\) in knots and \(f_{dk}\) in cycles per second; a conversion factor \(K\) of 1 hour per 3600 seconds is required. Solving for \(|\vec{V}|_k\)

\[
|\vec{V}|_k = 3600 \ f_{dk} \lambda \ \text{knots} \quad \text{(EQ. #4)}
\]

where \(f_{dk}\) is the frequency shift of the transmissions from station \(S_k\) as seen by the aircraft.

We are now left with four simultaneous equations and the four unknowns \(m, n, x5\) and \(y5\).

Solution of these equations will yield the instantaneous position coordinates and velocity vector.

Since we are concerned with the magnitude of the doppler frequency, which is directly proportional to the ranging frequency, it would be advantageous to use the highest frequencies to measure the doppler shift. For a velocity of 1800 knots, the following maximum doppler frequencies are obtained at 10.2 kc and 13.6 kc by using Equation #3.

\[
10.2 \ \text{kc} \rightarrow .03125 \ \text{cps maximum}
\]

\[
13.6 \ \text{kc} \rightarrow .04167 \ \text{cps maximum}
\]

A doppler frequency of 0.0003 cycles per second yields a value of approximately 17 knots, when a ranging frequency of 10.2 kc is used, as calculated by using Eq. #4. The frequency of the received signal containing this doppler frequency is 10200.0003 cps. This is a doppler frequency shift of 3 parts in \(10^8\) of the received signal.

Therefore, the ability to resolve velocity components to within 17 knots requires accuracies in the received signals (which are used to determine the doppler frequencies) of 3 parts in \(10^8\).
III. Acceleration Errors

In each of the previously mentioned cases acceleration effects due to changes in the velocity components of the aircraft relative to the transmitting stations will cause errors in position and velocity measurements. This is caused by the inability of second order loops to precisely track acceleration effects. In second order, type 1 loops an acceleration step input is tracked with a constant error. This would result in position and velocity errors.

There are two separate causes of acceleration for the case of a velocity vector with a constant magnitude.

1) Consider the aircraft velocity vector with a constant direction. For this case the relative velocity vector with respect to each transmitting station changes continuously. This effect causes an acceleration input to the measuring circuitry. The acceleration for this case is calculated as follows:

Rate of change of frequency due to maximum radial acceleration.
Graph A shows relationship between aircraft position with respect to Station \( S_1 \), and aircraft velocity as viewed by the station:

\[
|\vec{V}_1| = -|\vec{V}| \cos \phi
\]

\[
\frac{d|\vec{V}_1|}{dt} = |\vec{V}| \sin \phi \frac{d\phi}{dt}
\]

\[
\phi = \cot^{-1} \frac{D_2}{D_1}
\]

\[
D_2 = |\vec{V}| \cdot t
\]

\[
\phi = \cot^{-1} \frac{|\vec{V}| \cdot t}{D_1}
\]

\[
\frac{d\phi}{dt} = -\frac{1}{1 + \left(\frac{|\vec{V}| \cdot t}{D_1}\right)^2} \frac{|\vec{V}|}{D_1}
\]

\[
\sin \phi = \frac{D_1}{\sqrt{D_1^2 + D_2^2}} = \frac{1}{\sqrt{1 + \left(\frac{D_2}{D_1}\right)^2}}
\]

\[
\frac{d|\vec{V}_1|}{dt} = -|\vec{V}| \left[ \frac{1}{\sqrt{1 + \left(\frac{|\vec{V}| \cdot t}{D_1}\right)^2}} \right] \frac{1}{1 + \left(\frac{|\vec{V}| \cdot t}{D_1}\right)^2} \frac{|\vec{V}|}{D_1}
\]

\[
\frac{d|\vec{V}_1|}{dt} = \frac{|\vec{V}|^2}{D_1} \left[ \frac{1}{1 + \left(\frac{|\vec{V}| \cdot t}{D_1}\right)^2} \right]^{3/2}
\]

when

\[
t = \pm \infty, \quad \frac{d|\vec{V}_1|}{dt} = 0
\]
when

\[ t = 0, \quad \frac{d |\vec{V}|}{dt} = - \frac{|\vec{V}|^2}{D} \text{ max. acceleration.} \]

\[ \frac{df}{dt} = \frac{|\vec{V}|}{D} \frac{(n. \text{mi/hr})^2}{n \text{mi}} \times \frac{1 \text{ cycle}}{16 \text{ nmi}} \times \frac{1(\text{hr})^2}{(3600)^2 (\text{sec})^2} \]

\[ \text{max. rate of change of frequency:} \]

\[ \frac{df}{dt} = 0.482 \times 10^{-8} \frac{V^2}{D} \text{ cycles/sec}^2 \]

\[ v = \text{aircraft velocity (knots)} \]

\[ D = \text{distance between transmitting station and line that velocity of aircraft lies on (nautical miles).} \]

(2) Consider an aircraft velocity vector with a changing direction. This case is caused by aircraft maneuvers. The acceleration realized in this case can be several magnitude larger than that for case (1). The following analysis considers a 2 "g" or "panic" turn.

\[ \text{acceleration vector} \]

\[ \text{Aircraft} \]

\[ \text{transmitting station} \]

\[ a = \frac{v^2}{R} \]

\[ 2 \times 32.2 \text{ f/sec}^2 = 64.4 \text{ ft/sec}^2 \times \frac{(3600 \text{ sec})^2}{\text{hr}^2} \times \frac{1 \text{mi}}{5280 \text{ ft}} \times \frac{0.87 \text{ n.mi}}{1 \text{ mi}} \]

\[ a = 13.78 \times 10^5 \frac{\text{rmi}}{\text{hr}^2} \]
radius of turn \( R = \frac{v^2}{13.78 \times 10^5 \text{n.m.i.}} \)

for velocity \( v = 1800 \text{knots}; \ R = 2.35 \text{n.m.i.} \)

\[
\frac{df}{dt} = 0.0106 \frac{\text{n.mi}}{\text{sec}^2} \times \frac{1 \text{cycle}}{16 \text{n.m.}} = 6.63 \times 10^{-4} \text{cycle/sec}^2
\]

It might be necessary to provide inputs from equipment external to the Omega receiver to eliminate this effect.

Comparison of the two methods

The method of utilizing the Omega transmission that is described in Part 1 of the Appendix has several advantages over the second method. The first method requires the reception of signals only from three transmitting stations while the second method requires four signals. The standard receiver can be used when the aircraft is on the ground and also to achieve the high rendezvous accuracies while the doppler dependent system would be useless in both these cases.

One advantage of the doppler system is that it allows unique position determination without the ambiguities of the standard receiver. Therefore a loss of signal is important only during that time interval when the signal is not present and does not introduce a continuous error that has to be corrected. In addition only one ranging frequency is required therefore eliminating the need to process other frequencies.

Neither method has advantages over the other in eliminating errors due to accelerations since both require the phase lock tracking filters to recover the signals from noise before processing.

The doppler system eliminates errors introduced by the divergence of the hyperbolic contours since it does not make measurements on differences of received signals from different stations but makes measurements on each signal separately and therefore is essentially a circular system.

The preceding analysis is performed on a planar surface. The results prove the feasibility of the methods; however, they would have to be extended to a spherical surface to allow their use in converting the Omega signals to parameters in terms of longitude and latitude.