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AN INVESTIGATION OF SCHRÖTER'S RULE

by

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INTRODUCTION

When one encounters a discussion of the nature of lunar craters and the processes which resulted in their formation, one inevitably finds "Schröter's Rule" mentioned as a factor to be considered in evaluating the merits of the impact theory versus other proposed theories of formation.

The generally accepted statement of Schröter's Rule is (Kopal(1), Gilvarry(2)) that the volume of material in the rim of a lunar crater is roughly equal to the volume of the depression formed below the surrounding ground level. Some authors (Baldwin(3), Khabakov(4)) however prefer a statement of the rule which implies a strict one-to-one ratio between the rim volume and the crater volume.

It is important to realize at this point that Schröter's statement was made as a result of direct observation of lunar craters (Schröter(5)); in discussions concerning the relative merits of the various processes of crater formation it is introduced as empirical evidence in support of the meteoric impact origin of lunar craters.

The reasoning behind the use of a statement of Schröter's Rule in any form as a substantiation of the impact theory of crater formation is obvious. If the only mechanism causing a crater is one which displaces the original terrain by digging out a pit and piling up the original contents of the pit around its edge, then there can not be more material piled around the pit than was originally dug out from the ground. Implicit in this argument is the assumption that the mechanism which operates on the terrain to produce the depression does not add appreciable material to the system by virtue of its own volume. That is, the volume of the impacting body must be small compared to the volume of lunar surface which it displaces to form the crater.

RIM - CRATER DISTINCTION

In order to determine the relationship between the volume of an impact crater and the volume of material in the rim around the crater it is first necessary to establish a criterion for determining which portions of a lunar formation constitue the crater and which constitute the rim. In this study "crater volume" was taken as any depression below the undisturbed ground level, and "rim volume" as any elevation above the original level in the region of the crater.

The problem of distinguishing between rim and crater then resolves to one of finding the original surface level of the area in which a given crater has been formed. This is accomplished by determining the undisturbed surface level as near the crater as possible and then making the assumption that the region in which the crater was formed was originally at the same level as the surrounding regions, or constituted a level survace connecting the adjacent regions.

This means of determining the original surface level can be used with available lunar surface contour maps; the original level is taken as the first contour encountered, when proceeding radially from the center of the crater, which does not exhibit polar symmetry about the center of the crater and therefore probably existed prior to the time of the crater's formation.

Having thus established a criterion for distinguishing crater volume from rim volume as well as defining the outer limits of the rim extension from the center of the crater, it is possible to test Schröter's Rule by using observational data in the form of lunar contour maps.

CALCULATION OF VOLUMES

Data for the evaluation of Schröter's Rule were taken from multiple digital cross-sections of selected lunar craters prepared from Aeronautical Chart and Information Service lunar contour maps at Boston University[6]. The geometry of a typical cross-section as adapted from the Catalogue of Lunar Craters Cross Sections I is illustrated below:

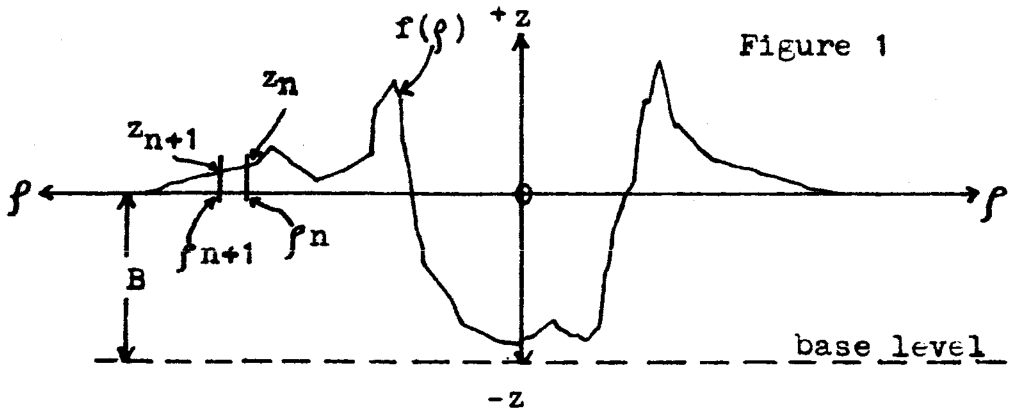


Figure 1

The line O is the chosen polar axis of the crater; B is the height of the original ground level above an arbitrary zero reference level; z is the height of the post-impact surface above (or below) B; ρ is the radius.

If z_n and z_{n+1} are two adjacent points in the digital cross-section and are connected by a straight line, $f(\rho)$, the moment about O of the element of cross-section between z_n and z_{n+1} is:

$$M = \int_{\rho_n}^{\rho_{n+1}} \rho f(\rho)\, d\rho \quad . \qquad (1)$$

Since $f(\rho) = a\rho + b$,

$$M = (a/3)(\rho_{n+1}^3 - \rho_n^3) + (b/2)(\rho_{n+1}^2 - \rho_n^2) \, , \qquad (2)$$

where

$$a = (z_{n+1} - z_n)/(\rho_{n+1} - \rho_n) \qquad (3)$$

is the slope of the line segment, and

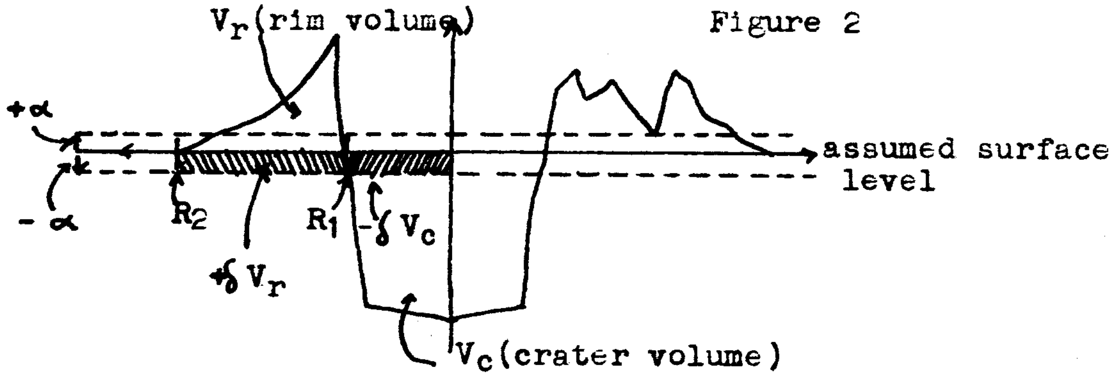
$$b = (z_n \rho_{n+1} - z_{n+1}\rho_n)/(\rho_{n+1} - \rho_n) \qquad (4)$$

is the projected intercept of the line segment on the polar axis.

By the theorem of Pappus, the moment of an element of cross-sectional area multiplied by an angle of rotation θ (in radians) about the axis O represents a volume element of the solid if it is assumed to possess polar symmetry throughout that angle of rotation. Since some error is introduced by this assumption into the volume calculations, the resultant volumes and their ratios are only meant to be approximations. In most cases the symmetry angle is only 45° so that the approximations should be better than order-of-magnitude.

The probable error in the relative height of a given contour line for the data used in this study was estimated by the ACIS to be 100 meters. This error introduces an uncertainty into the determination of the original surface level of the impact area, and thus also into the final rim volume and crater volume as shown in Fig. 2 below:

Figure 2



The probable error, δV_r, in the rim volume resulting from the uncertainty, α, in original surface level is

$$\delta V_r = \theta\,(R_2^2 - R_1^2)\alpha, \qquad (5)$$

and the probable error, δV_c, in the crater volume is

$$\delta V_c = \theta R_1^2 \alpha . \qquad (6)$$

R_1 is the average radius of the crater(depression); R_2 is the average outer radius of the rim; α is the uncertainty in height (100 meters in this case) of the original surface level.

If a Gaussian distribution is assumed for the probable error α the probable error in the ratio of rim-to-crater volume is (see for example, Baird[7]) , where R is the ratio,

$$R = \left[(\partial R/\partial V_r)^2 \delta V_r^2 + (\partial R/\partial V_c)^2 \delta V_c^2 + 2(\partial R/\partial V_r)(\partial R/\partial V_c)\delta V_r \delta V_c\right]^{\frac{1}{2}} . \qquad (7)$$

Since $R = V_r/V_c$,

$$R = (1/V_c)(\delta V_r^2 + R^2 \delta V_c^2 - 2R\delta V_r \delta V_c)^{\frac{1}{2}} . \qquad (8)$$

The mixed term is retained in equation (7) because δV_r and δV_c are not independent perturbations; Fig. 2 shows that when the error α is such that δV_r is positive, then δV_c is negative.

A Fortran II program (Appendix A, Program 1) was written to perform the volume calculations, find the ratios of rim-to-crater volume, and determine the uncertainty in these ratios. Another program (Appendix A, Program 2) was written to put the data cards used in a previous study (which produced NASA Report No. 16) into convenient form for input to Program 1.

RESULTS

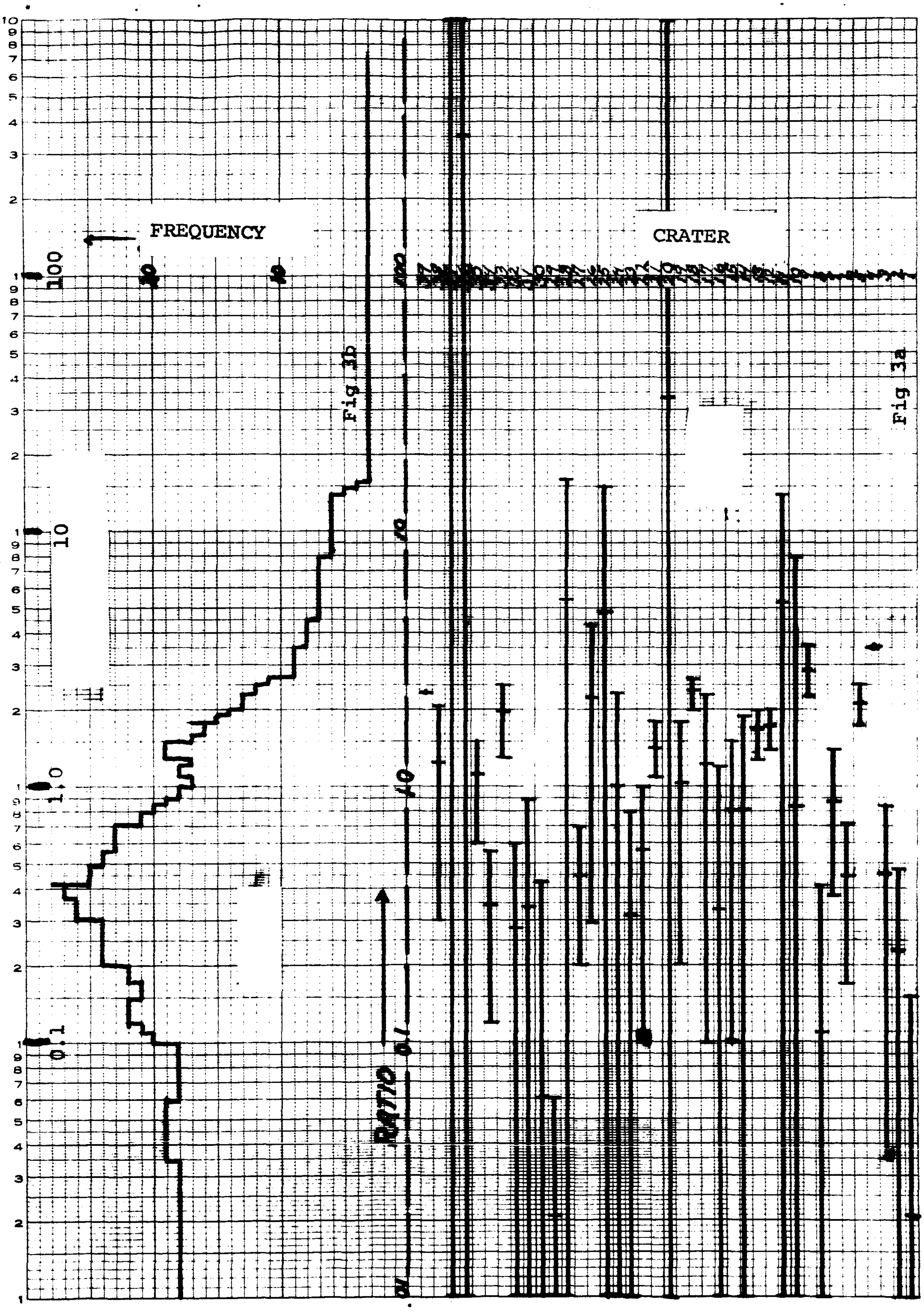
Values of the rim volume, the crater volume, the ratio of rim volume to crater volume, and the probable error in the ratio have been obtained for thirty-nine lunar craters from the output of Program 1; these results appear in Table 1, page 8.

A glance at the values of Ratio and Delta Ratio in the Table, which represent the rim-to-crater volume ratios and the probable error in those ratios respectively, indicates that a large proportion of the ratios are smaller than their corresponding errors, while most of the remaining values of Ratio have errors comparable to their magnitudes. Since the indicated errors have arisen only as a result of the uncertainty in the original lunar surface level, the implication is that, given a probable error of at least 100 meters in surface height, it is not possible to distinguish rim volume from crater volume with sufficient accuracy to make a valid statement concerning their ratio. Furthermore, since the same probable error of 100 meters applies to every determination of surface altitude used in this study, the probable errors in the ratios determined here represent minimum probable errors since only the effect of the error in the outermost contour height has been considered. Also, the error introduced by assuming polar symmetry for a given crater through the angle Θ has not been estimated.

In Figure 3-a, the values obtained for the ratio of

TABLE 1

CRATER		RIM VOL (KM3)	CRATER VOL (KM3)	RATIO	DELTA RATIO
ABULFEDA	1	1.166E 02	5.582E 03	2.089E-02	1.340E-01
AGRIPPA	2	5.196E 02	2.247E 03	2.313E-01	2.519E-01
ALFRAGANUS	3	1.514E 02	3.386E 02	4.471E-01	4.127E-01
ARCHIMEDES	4	4.229E 03	1.179E 03	3.588E 00	5.133E-02
AUTOLYCUS	5	1.736E 03	8.073E 02	2.150E 00	3.536E-01
BULLIALDUS	6	1.489E 03	3.296E 03	4.519E-01	2.785E-01
CAPELLA	7	8.564E 02	9.618E 02	8.904E-01	5.179E-01
CONON	8	5.891E 01	5.203E 02	1.132E-01	3.039E-01
COPERNICUS	9	1.389E 04	4.763E 03	2.916E 00	6.066E-01
DAMOISEAU	10	3.881E 02	4.596E 02	8.443E-01	1.031E 00
DOPELMAYER	11	1.575E 03	2.987E 02	5.273E 00	6.702E 00
ERATOSTHENES	12	2.950E 03	1.710E 03	1.725E 00	3.081E-01
EULER	13	3.280E 02	1.933E 02	1.696E 00	3.398E-01
GASSENDI	14	1.517E 03	1.862E 03	8.149E-01	1.197E 00
GASSENDI A	15	2.479E 02	3.087E 02	8.032E-01	7.503E-01
GASENDI B	16	8.112E 01	2.465E 02	3.291E-01	8.950E-01
GAUDIBERT	17	2.410E 02	1.977E 02	1.219E 00	1.101E 00
GAY LUSSAC	18	3.055E 02	1.286E 02	2.376E 00	3.040E-01
GUTENBERG	19	1.912E 03	1.833E 03	1.043E 00	8.393E-01
HANSTEEN	20	6.888E 02	2.034E 00	3.387E 02	6.758E 04
KANT	21	9.214E 02	6.482E 02	1.421E 00	3.547E-01
LANSBERG	22	5.994E 02	1.057E 03	5.668E-01	4.504E-01
MANILIUS	23	3.383E 02	1.048E 03	3.227E-01	4.496E-01
MERSENIUS P	24	2.862E 02	2.832E 02	1.011E 00	1.327E 00
MOSTING	25	1.237E 03	2.538E 02	4.873E 00	1.057E 01
PITATIUS	26	1.785E 03	7.887E 02	2.263E 00	2.040E 00
PLINIUS	27	8.953E 02	1.998E 03	4.482E-01	2.511E-01
REINER	28	1.432E 03	2.641E 02	5.422E 00	1.099E 01
REINHOLD	29	3.413E 02	1.598E 04	2.136E-02	3.687E-02
ROSS	30	2.297E 01	3.752E 02	6.123E-02	3.643E-01
SABINE	31	1.185E 02	3.387E 02	3.499E-01	6.341E-01
SIRSALIS	32	4.440E 02	1.566E 03	2.836E-01	3.300E-01
TARUNTIUS	33	4.142E 02	2.091E 02	1.981E 00	6.040E-01
THEOPHILUS	34	3.497E 03	1.018E 04	3.437E-01	2.298E-01
TIMOCHARIS	35	6.233E 02	5.523E 02	1.129E 00	4.917E-01
TRIESNECKER	36	1.581E 03	4.424E 00	3.574E 02	4.254E 04
UKERT	37	1.471E 03	2.195E-01	6.702E 03	1.003E 07
VITELLO	38	5.279E 02	4.211E 02	1.254E 00	9.045E-01
ZOLLNER	39	1.797E 03	7.691E 02	2.336E 00	4.262E-04


FREQUENCY
CRATER
Fig 3b
Fig 3a
RATIO
100
10
1.0
0.1

rim-to-crater volume have been plotted on a log scale along with the corresponding probable errors for each crater. Figure 3-b is a frequency histogram constructed from Figure 3-a; it represents the average possible frequency of occurence for ratios of rim volume to crater volume from the calculated values of Ratio and their probable errors, Delta Ratio. The high frequency of occurence at very low ratios arises from the large percentage of probable errors which exceed 100% of the stated value for Ratio. There is no indication that the ratio of 1.0 is preferred for rim-to-crater volumes; indeed, the histogram indicates a probable ratio of less than 1.0 with the highest probability near 0.4.

CONCLUSIONS

The huge uncertainties in volume ratios generated by the seemingly small probable altitude error of 100 meters indicate the following:

1. Any statements concerning the ratio of rim volume to crater volume as determined by less accurate means than those employed in obtaining the data used in this study are likely to be invalid;

2. Schröter's Rule does not hold for the group of craters tested in this study.

This is not to say that Schröter's Rule is not true for lunar craters in general, however, any statement such as Schroter's Rule applied to lunar craters in general cannot be supported unless much more accurate means of determining relative surface heights are developed.

ACKNOWLEDGMENT

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REFERENCES

1. Kopal, Zdenek, Physics and Astronomy of the Moon, Academic Press, 1962.

2. Gilvarry, J. J., "Effects of Loss of Lunar Mass by Meteoric Impact" in Icarus, 3 (2), 121-129, Nov. 2, 1964.

3. Baldwin, R. B., The Measure of the Moon, University of Chicago Press, 1963.

4. Khabakov, A. V., "Characteristic Features of the Relief of the Moon" in The Moon: A Russian View, A. V. Markov, Edit., University of Chicago Press, 1960.

5. Schröter, J. H., Selenotopographische Fragmente, 2 vols. Göttingen: Lilienthal & Helmst, 1791-1802.

6. Hawkins, G. S., W. H. Zack, and S. M. Saslow, Catalogue of Lunar Craters Cross Sections I: Craters With Peaks, Report No. 16, Sept., 1965, NASA G246-62

7. Baird, D. C., An Introduction to Measurement Theory and Experiment Design, Prentice-Hall, 1962.

APPENDIX A

PROGRAM 1 CALCULATION OF LUNAR CRATER VOLUMES

```
      READ 1000,N
 1000 FORMAT(I4)
      PRINT 1010
 1010 FORMAT(28H CRATER  RIM VOL  CRATER VOL4X46HRATIO   DELTA RATIO  DE
     1PTH   PTP DIAM  RC DIAM3X18HDELTA HC  DELTA RC/)
      PUNCH 1011
 1011 FORMAT(13X55HCRATER   RIM VOL   CRATER VOL      RATIO   DELTA RAT
     1IO)
      DO 100 JM=1,N
      DIMENSION INIT(2),Y(2,500),RAD(2,500),YS(500),HC(500)
      DIMENSION CRADC(2),RRADC(2),RMAX(2)
      READ 2004,XLAT,XLONG,PR,NPR
 2004 FORMAT(F6.2,2X,F6.2,2X,F4.0,2X,I3)
      PDIA=0.0
      ADIAR=0.0
      ADIAC=0.0
      DELTA=0.0
      RAREA=0.0
      CAREA=0.0
      DELY=0.0
      ALT=0.0
      EC=0.0
      ER=0.0
      CVOL=0.0
      RVOL=0.0
      THETA=2.0*3.14159/PR
      DO 160 JP=1,NPR,2
      READ 2002,INIT(1),HCS1,HWM1,HCB1,INIT(2),HCS2,HWM2,HCB2
 2002 FORMAT(2(I3,1X,3(F7.3,1X)))
      ALT=HWM1+HWM2-HCB1-HCB2+ALT
      ISPES=INIT(1)+INIT(2)+1
      FISPE=ISPES
      BPRI=HCS1-HCS2
      SLOPE=ABSF(BPRI/(FISPE-1.0))
      READ 2003,(HC(J),J=1,ISPES)
 2003 FORMAT(10F7.3)
      DO 2000 M=1,ISPES
      FM=M
      IF(BPRI)11,13,13
   11 R=FM-1.0
      B=HCS1
      GO TO 12
   13 R=ISPES-M
      B=HCS2
   12 BASE=R*SLOPE
      YS(M)=HC(M)-(BASE+B)
 2000 CONTINUE
      RY=0.0
      CY=0.0
      R1=0.0
```

```
      R2=0.0
      DO 155 L=1,2
      YMOMC=0.0
      YMOMR=0.0
      INITL=INIT(L)+1
      DO 125 KN=1,INITL
      RAD(L,KN)=INIT(L)+1-KN
      GO TO (5,6),L
    5 Y(1,KN)=YS(KN)
      GO TO 125
    6 MM=ISPES+1-KN
      Y(2,KN)=YS(MM)
  125 CONTINUE
      YMAX=Y(L,1)
      DO 222 IN=2,INITL
      IF(Y(L,IN))222,222,3
    3 IF(Y(L,IN)-YMAX)222,222,4
    4 YMAX=Y(L,IN)
      RMAX(L)=RAD(L,IN)
  222 CONTINUE
      RC=0.0
      RR=0.0
      DO 150 K=2,INITL
      KM1=K-1
      S1=Y(L,KM1)-Y(L,K)
      BVAL=(Y(L,KM1)*RAD(L,K)-Y(L,K)*RAD(L,KM1))/(RAD(L,K)-RAD(L,KM1))
      IF(Y(L,K)*Y(L,KM1))35,36,37
   35 RADI=-BVAL/S1
      RADO=RAD(L,KM1)
      XMOM=S1/3.*(RADO**3.-RADI**3.)+BVAL/2.*(RADO**2.-RADI**2.)
      YMOM=(S1**2)/3.*(RADO**3-RADI**3)+S1*BVAL*(RADO**2-RADI**2)+(BVAL*
     1*2)*(RADO-RADI)
      VOL=ABSF(XMOM)*THETA
      AREA=S1/2.*(RADO**2.-RADI**2.)+BVAL*(RADO-RADI)
      RADO=RADI
      RADI=RAD(L,K)
      IF(Y(L,KM1))45,45,46
   45 CVOL=CVOL+VOL
      CAREA=CAREA+AREA
      YMOMC=YMOMC+YMOM
      RC=RC+1.0
      GO TO 60
   46 RVOL=RVOL+VOL
      RAREA=RAREA+AREA
      YMOMR=YMOMR+YMOM
      RR=RR+1.0
      GO TO 60
   36 IF(Y(L,K)-Y(L,KM1))37,39,37
   39 GO TO 150
   37 RADO=RAD(L,KM1)
      RADI=RAD(L,K)
   60 XMOM=S1/3.*(RADO**3.-RADI**3.)+BVAL/2.*(RADO**2.-RADI**2.)
      YMOM=(S1**2)/3.*(RADO**3-RADI**3)+S1*BVAL*(RADO**2-RADI**2)+(BVAL*
```

```
     1*2)*(RADO-RADI)
      VOL=ABSF(XMOM)*THETA
      AREA=S1/2.*(RADO**2.-RADI**2.)+BVAL*(RADO-RADI)
      IF(Y(L,K))62,62,63
   62 CVOL=CVOL+VOL
      CAREA=CAREA+AREA
      YMOMC=YMOMC+YMOM
      RC=RC+1.0
      GO TO 150
   63 RVOL=RVOL+VOL
      RAREA=RAREA+AREA
      YMOMR=YMOMR+YMOM
      RR=RR+1.0
  150 CONTINUE
      IF(RAREA)51,51,50
   50 RRADC(L)=RVOL/(THETA*RAREA)
      RY=YMOMR/RAREA
   51 IF(CAREA)53,53,52
   52 CRADC(L)=CVOL/(THETA*CAREA)
      CY=YMOMC/CAREA
   53 DELY=DELY+RY-CY
      R1=RR+R1
      R2=RC+RR+R2
  155 CONTINUE
      DIAP=RMAX(1)+RMAX(2)
      PDIA=PDIA+DIAP
      RDIA=RRADC(1)+RRADC(2)
      CDIA=CRADC(1)+CRADC(2)
      ADIAR=ADIAR+RDIA
      ADIAC=ADIAC+CDIA
      DELTA=DELTA+(RDIA-CDIA)/2.0
      R1=R1/2.0
      R2=R2/2.0
      PUNCH 8000,R1,R2
 8000 FORMAT(5X,2(1PE9.3))
      EC=2.0*THETA*(R1**2)*.1+EC
      ER=2.0*THETA*((R2**2)-(R1**2))*.1+ER
      PUNCH 9000, ER,EC
 9000 FORMAT(2(1PE9.3))
  160 CONTINUE
      PDIA=PDIA*2.0/PR
      ADIAR=ADIAR*2.0/PR
      ADIAC=ADIAC*2.0/PR
      DELTA=DELTA*2.0/PR
      DELY=DELY*2./PR
      PROP=RVOL/CVOL
      DELR=(1.0/CVOL)*SQRTF((ER**2)+(PROP**2)*(EC**2)-2.0*ER*EC*PROP)
      ALT=ALT/PR
      PUNCH 1013,JM,RVOL,CVOL,PROP,DELR
 1013 FORMAT(14XI4,3X1PE9.3,3X1PE9.3,4X1PE9.3,3X1PE9.3)
      PRINT 1003,JM,RVOL,CVOL,PROP,DELR,ALT,PDIA,ADIAR,DELY,DELTA
 1003 FORMAT(1X,I4,3X,1P4(E9.3,2X),1P5(E8.2,2X))
  100 CONTINUE
      END
```

```
   PROGRAM 2    RESEQUENCE OF CRATER PROFILE DATA
      DIMENSION  XY(1000),M(2),HCS(2),HCM(2),HWM(2),HCB(2)
      DO 25 IN=1,NL
      READ 99,IA
   99 FORMAT(1X,15H               ,I3)
      I=0
      K=0
      PRINT 100,XLAT,XLONG,PR,NPR
   20 DO 7 J=1,2
      READ 100,XLAT,XLONG,PR,NPR
  100 FORMAT(2(F6.2,2X),F4.0,2X,I3)
      PUNCH 99,IA
      PUNCH 100,XLAT,XLONG,PR,NPR
      PRINT 99,IA
    7 READ 101,M(J),HCS(J),HCM(J),HWM(J),HCB(J)
  101 FORMAT(I3,4(2X,F7.3))
    6 READ 102,HC
  102 FORMAT(F7.3)
      IF(HC)11,15,14
   14 I=I+1
      XY(I)=HC
      K=K+1
      GO TO 6
   11 IF(HC+1.0)16,15,15
   16 M(1)=K
      K=-1
      GO TO 6
   15 M(2)=K
      PUNCH 2002,(M(J),HCS(J),HWM(J),HCB(J),J=1,2)
      PRINT 2002,(M(J),HCS(J),HWM(J),HCB(J),J=1,2)
 2002 FORMAT(2(I3,1X,3(F7.3,1X)))
      PUNCH 3000,(XY(L),L=1,I)
 3000 FORMAT(10F7.3)
      PRINT 3000,(XY(L),L=1,I)
      I=0
      K=0
      IF(HC)20,25,20
   25 CONTINUE
      END
```