ADMINISTRATIVE REPORT

NUMBER 1

Covering the Period: 1 July - 31 December 1965

RESEARCH GRANT NGR 52 - 046 - 001

Lie Series Solutions of Partial Differential Equations

INSTITUTE FOR THEORETICAL PHYSICS

UNIVERSITY OF INNSBRUCK, (AUSTRIA)

Contractor and Principal Investigator: Univ.-Prof.Dr.F. CAP

I) Scientific Work Done in the Period of the Report

Our scientific program is concerned with the application of Lie series to the solution of differential equations - a powerful tool which has been introduced by GROEBNER, Professor at the University of Innsbruck. In particular, we are attempting to obtain Lie solutions for the 33 equations resulting from the separation of the HELMHOLTZ equation in 11 coordinate systems. It is to be hoped that certain advantages - as far as accuracy and velocity of numerical computation are concerned - will emerge from the use of Lie series compared to the conventional methods. In doing these investigations, our interest is focused on two special fields, at the present stage:

1) a separation of the HELMHOLTZ equation in elliptic cylinder coordinates led to far-going considerations of MATHIEU functions and their representation by means of Lie series. It is by following this promising branch that we wrote a code which enables us to compute the complete solution of the MATHIEU equation, the MATHIEU functions of first class (ce_n and se_n) and the fundamental system of the solutions.and for a calculation of the eigenvalues occurring in the MATHIEU equations. The representation of solutions of the MATHIEU equation by means of Lie series has been reported in Scientific Report no. 1. Furthermore, a code has been written for an expansion of MATHIEU functions into a FOURIER series which is going to be compared to the Lie expansion, on the computer.

2) considerations aiming to a general code for all 33 equations resulting from the separation of the HELMHOLTZ equation insofar as evidently these equations are special cases of the general second order differential equation for which a total solution in terms of

Lie series could be given (Scientific Report No. 2). Together with error estimates to be obtained in the next future these formulas seem to be a sound foundation upon which numerical calculations of great extent can be done. -

In the course of our investigations, we also took the use of LAPLACE transformation into consideration, encouraged by the good results one of us (SCHETT) achieved by applying it to the BESSEL equation (to be published); unfortunately, the use of this method does not seem to be of great advantage at the present stage of our investigation due to implications we found out only after a considerable amount of work.

II) Activities of the Collaborators

The team consists of:

Professor Dr. Ferdinand CAP

Dietmar FLORIANI

Alois SCHETT

Dr. Juergen WEIL

In the following we give a short survey as to how the work reported in Chapter I) has been done by the individual persons of the team:

Professor Dr. Ferdinand CAP: was responsible for the coordination and general supervision of the work, comprising general investigations of the applicability of Lie series to various problems and plenty of discussions with the collaborators, usually held once or twice a week. In particular, his is the idea of splitting known functions from the Lie solution, in this way proposing a promising alternative - as far as computer time is concerned - to the general formula.

Dietmar FLORIANI: after generally dealing with the separation of the HELMHOLTZ equation in the 11 coordinate systems, turned to the MATHIEU equation and wrote a code for its solutions as represented by Lie series for the λ-eigenvalues belonging to certain types of solutions as well as for a FOURIER expansion of the MATHIEU functions for the purpose of comparison. At present, he is busy computing the λ-eigenvalues of the MATHIEU equation. From 8 - 14 October he participated in the Congress of Austrian Physicists in Vienna and wrote a report on some of the lectures given there. He is - in collaboration with Prof. CAP - the author of Scientific Report No. 1.

Alois SCHETT: initially was concerned with investigations on the use of LAPLACE transformation for solving the MATHIEU and other equations of our field; then he turned to the solution of the general second order differential equation by means of Lie series and obtained two alternative formulas for the solution. He is - in collaboration with J. WEIL - the author of Scientific Report No. 2.-- Besides, he participated in the International Conference on Polarization Phenomena of Nucleons at Karlsruhe, the Conference on Stable Isotopes at Leipzig and the Symposium on Criticality Problems at Stockholm and wrote reports on them.

Dr. Juergen WEIL: initially being concerned with considerations on the use of LAPLACE transformation and attending a course on computing by our ZUSE, eventually turned his attention to the solution of the general second order differential equation; he is the co-author of Scientific Report No. 2. - Besides, he participated in the National Physicists Conferences of Sarajevo and Bologna and wrote reports on them.

III) Future Plans

According to Chapter I), our activities are proceeding on two lines, the MATHIEU function line and the general one. As to the MATHIEU line, we plan to write a (λ,q)-map, i.e., to continue the eigenvalue program mentioned above. Furthermore, we are concerned with preliminary considerations on the elaboration of a table of MATHIEU functions by means of Lie series. Besides, a comparison of the accuracy of Lie and FOURIER representations is being done, at present. The general line will turn to the problem of error estimating (see Scientific Report No. 2) and to convergence improving considerations in analogy to KNAPP; after having laid a solid foundation in this way, we will begin to compute numerically, i.e., to write a code for the general solution given in Scientific Report No. 2, whence it will be possible to start the evaluation of the individual 33 equations resulting from the separation of the HELMHOLTZ equation. - Besides, we intend to open a third line of progress aiming to physical problems to be solved by Lie series, at the same time being a proposal for a continuation of the contract:

1) The problem of the asymmetrical heavy top. [illegible]lutions to

2) Higher approximations to particle trajectories in accelerators.

3) A problem emerging from a lecture given by RUMYANTSEV in Athens Conference, Summer 1965 in which the problem of gravity gradient stabilization of artificial satellites was shown to lead to MATHIEU functions. Apparently, work on this field seems to be very promising.