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SEPARATED BASE FLOW HEAT TRANSFER ANALYSIS

by T. F. Greenwood

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RESEARCH LABORATORIES

BROWN ENGINEERING COMPANY, INC.

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ABSTRACT

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This report describes an analytical method for predicting the heat transfer to the base of a wedge or rearward facing step from a separated, turbulent, supersonic flow. Calculated values are compared with experimental data.

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LIST OF SYMBOLS

A	Area
a	Speed of sound
C	Crocco number
C'	A constant used with eddy viscosity
Cp	Specific heat at constant pressure
erf	Error function
Ge	Mass flowing between j and d streamline
h	Width of base of rearward facing blunt body or 2 times back step height
H	Enthalpy
I_1	Integral defined by Equation 18
I_2	Integral defined by Equation 19
I_3	Integral defined by Equation 22
k	Ratio of specific heats
M	Mach number
$\dot{q}$	Heat transfer rate
T	Absolute temperature
u	Velocity in the x or X direction
v	Velocity in the y or Y direction
x, y	Coordinates of the intrinsic coordinate system
X, Y	Coordinates of the reference coordinate system

LIST OF SYMBOLS (Continued)

δ	Boundary layer thickness
ϵ	Eddy viscosity
ζ	Dimensionless y-coordinate
η	Dimensionless coordinate
η_p	Position parameter
θ	Streamline angle
Λ	Stagnation temperature ratio
ξ	Transformed x-coordinate
ρ	Density
σ	Free-jet spreading parameter
ϕ	Dimensionless velocity
ψ	Dimensionless X-coordinate
Ω	Energy transport per unit width

<u>Subscripts</u>

1, 2, 3, 4	Refers to conditions of cross sections indicated in Figure 1
a	Refers to conditions of the flow in the isentropic stream adjacent to the dissipative region
b	Refers to conditions at the base region
d	Refers to the streamline whose kinetic energy is just sufficient to enter the receompression region (the discriminating streamline)
j	Refers to conditions along the jet boundary streamline
m	Refers to a coordinate shift in the mixing theory due to the momentum integral
o	Stagnation conditions

INTRODUCTION

The purpose of this report is to examine in some detail the existing methods and theories for calculating heat transfer in separated flow over a back step (modifying them if possible to give better results), and to compare these results with measured values. To do this, several theories were examined, modified, discussed, and compared with experimental data until the best correlation with experimental results was found. The heat transfer theories under consideration were those of Korst[3], Page and Dixon[2], and Chapman[15]. Appropriate experimental data was extremely difficult to find, so that the comparison of analytic values with experimental is not extensive. However, several sources were found (References 6, 7, 8, and 9), and these were used as the basis for judging the accuracy of the theoretical models.

DISCUSSION OF THEORETICAL MODEL

GENERAL DESCRIPTION OF THE FLOW MODEL

Korst's basic turbulent flow analysis can be applied to the problem of calculating the heat transfer to the base of a backward facing step or the base of a wedge or cylinder. Since the basic equations and assumptions of Korst's theory are given in detail in several other references (References 3, 5, and 10) only a brief review of his flow field analysis will be presented here.

The basic flow model is shown in Figure 1. The flow is divided into three sections: (a) a frictionless free stream, (b) a dissipative layer, and (c) a base region formed by the separation of the boundary layer due to a sudden recession of the guiding wall. Since this is a case of supersonic flow a Prandtl-Meyer expansion occurs in the free stream from 1 to 2. In region 2, constant pressure across the jet mixing region is assumed. At the end of the free jet, wall reattachment (symmetrical impingement with another stream) and recompression takes place. The flow is assumed to undergo an oblique shock from 3 to 4. The pressure in region 4 is impressed upon the viscous layer by the adjacent free stream.

The "j" streamline divides the amount of mass passing over the corner at 1 from that mass flow entrained by the viscous action of the free jet. The "d" streamline is called the discriminating streamline, and it is the streamline which has just sufficient kinetic energy at 3 to penetrate the pressure rise to 4. Streamlines above this one have higher kinetic energies and enter the recompression zone 4 with a finite velocity. Streamlines below have lower kinetic energies and are unable to penetrate the oblique shock separating 3 from 4. These streamlines are turned back to circulate in the base region. If there is no secondary flow, the conservation of mass in the base region requires that the j and d streamlines be identical.

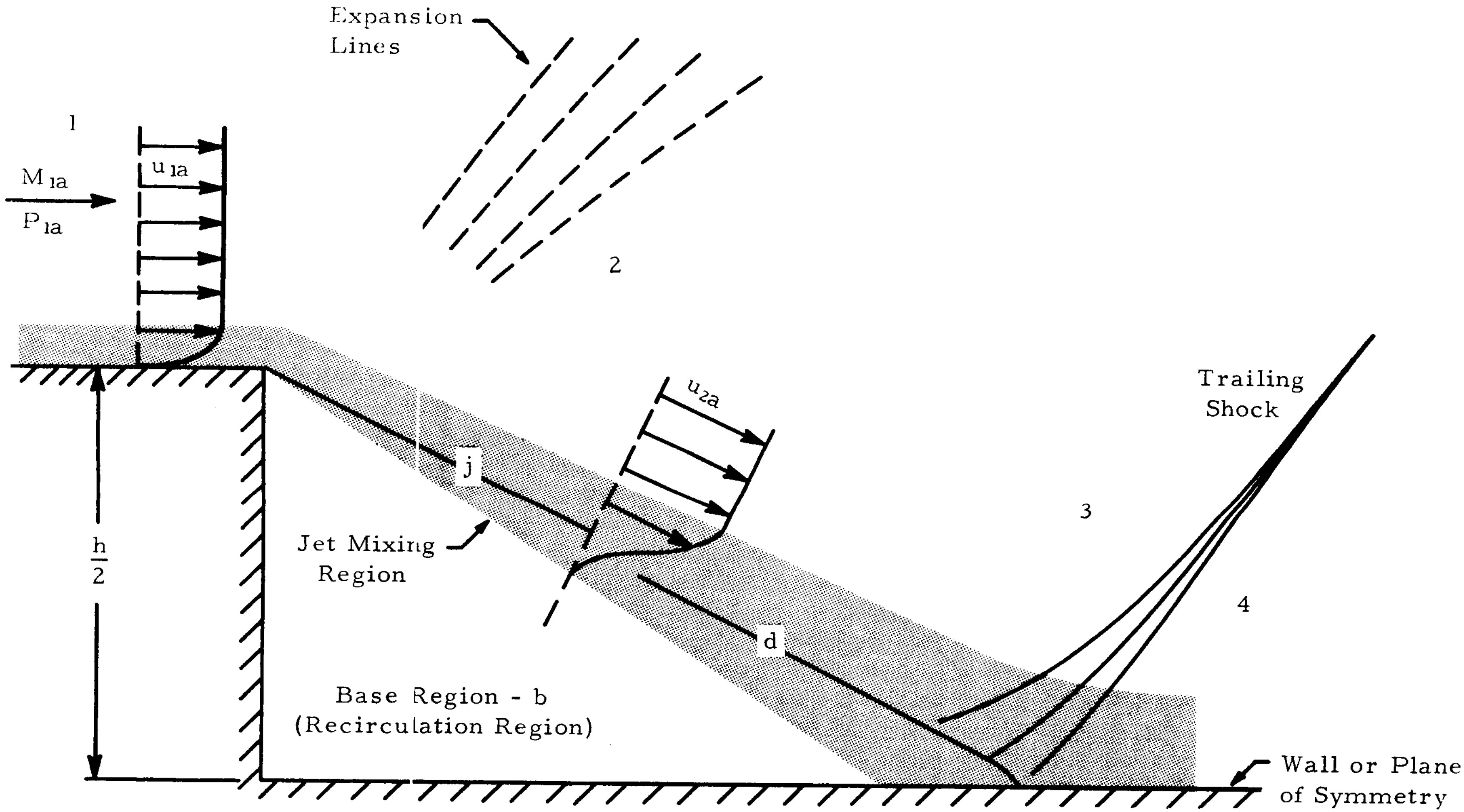


Figure 1. Korst's Basic Flow Model

JET MIXING ANALYSIS

For the general case of two-dimensional jet mixing at constant pressure, two coordinate systems are utilized (see Figure 2). The jet boundary of the "corresponding inviscid jet" (same terminology as used in References 3, 4, 5, and 10) forms the coordinate system (X, Y). A second set of coordinates (x, y) called "intrinsic coordinates" are defined for the viscous jet mixing region. The x axis is aligned with the center of the mixing region so that $y = 0$ at $u = \frac{1}{2} u_{2a}$. Consequently,

$$X = x$$

$$Y = y - y_m(x) \quad \text{with} \quad y_m(0) = 0 \quad .$$

The coordinate shift $y_m(x)$ can be determined by writing the momentum equation in the direction of the stream.

By making suitable assumptions (see Reference 3) the equation of motion for a plane-flow, constant-pressure mixing region can be linearized and expressed in the intrinsic coordinate system as

$$\frac{\partial u}{\partial x} = \frac{\varepsilon}{u_{2a}} \frac{\partial^2 u}{\partial y^2} \tag{1}$$

where ε is the apparent kinematic viscosity. By introducing the dimensionless variables

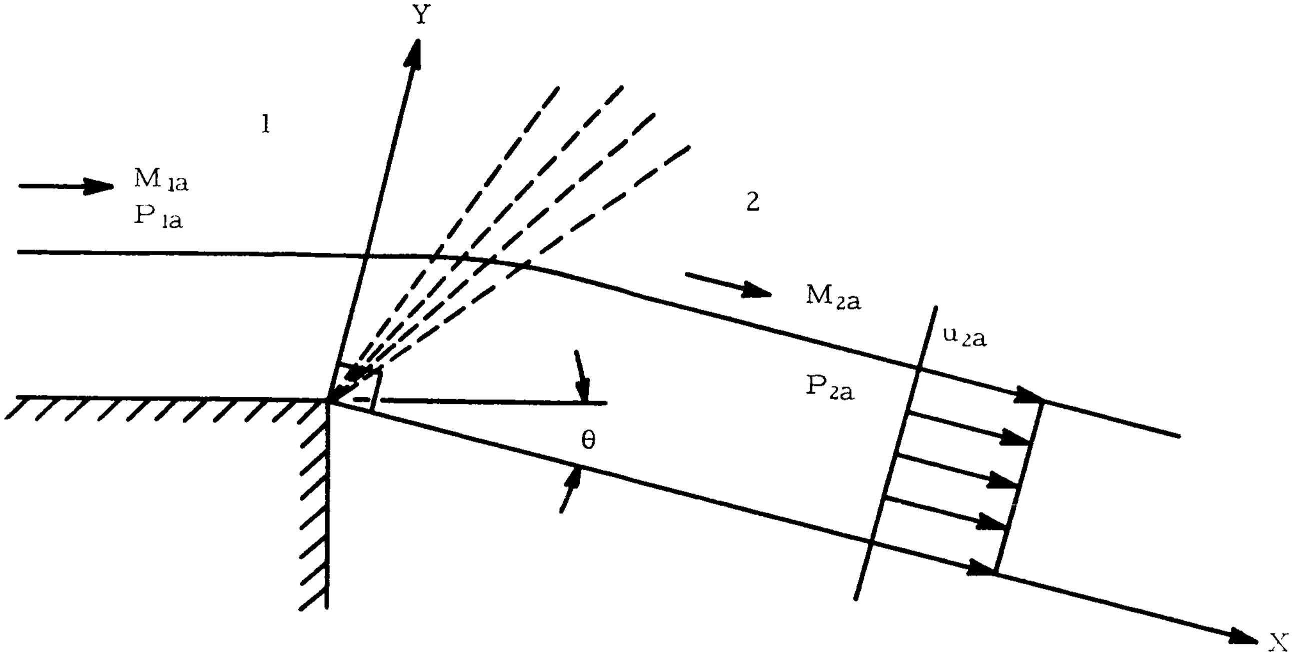
$$\phi \equiv \frac{u}{u_{2a}} , \quad \psi \equiv \frac{x}{\delta_2} , \quad \zeta \equiv \frac{y}{\delta_2}$$

where δ_2 is the thickness of the mixing region at section 2, the equation of motion becomes

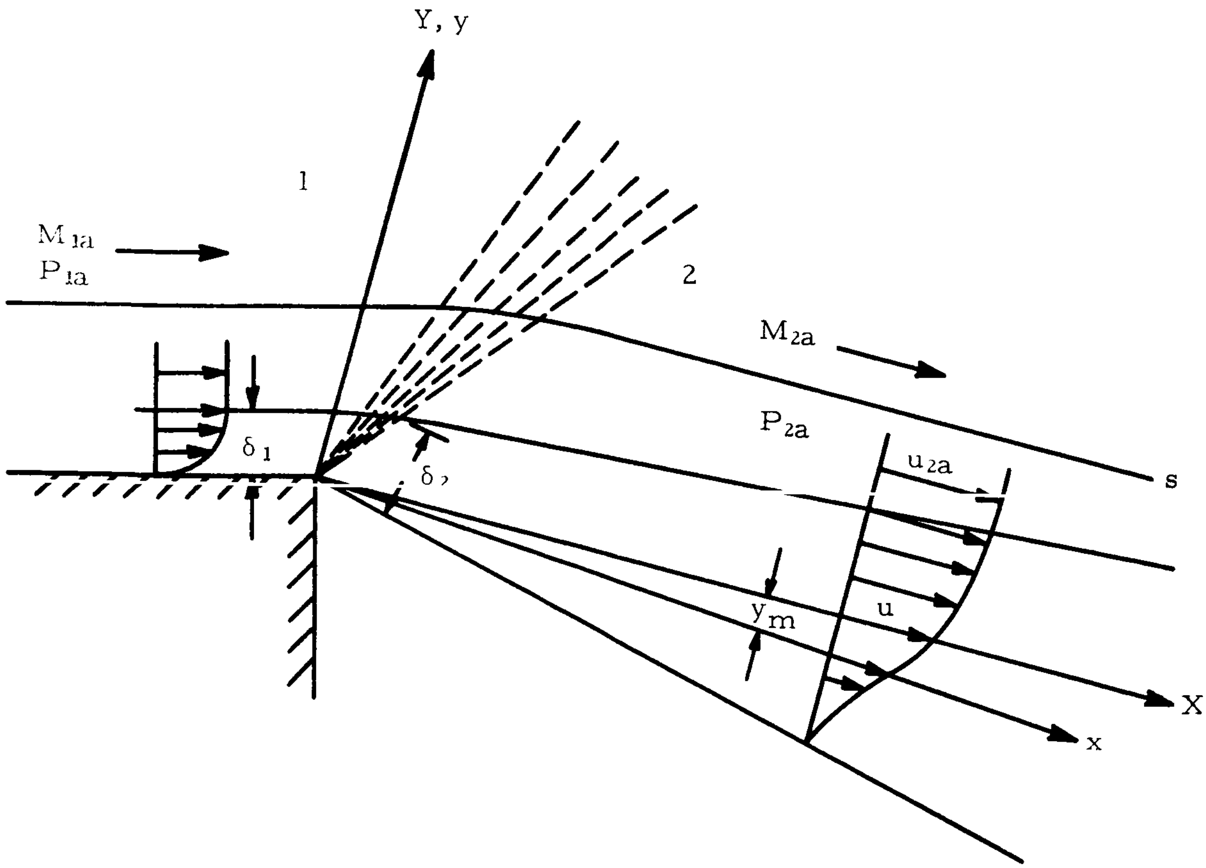
$$\frac{\partial \phi}{\partial \psi} = \frac{\varepsilon}{u_{2a} \, \delta_2} \frac{\partial^2 \phi}{\partial \zeta^2} \quad . \tag{2}$$

If we assume that ε is proportioned to $u_{2a}\, x$ (Reference 13), we can write

$$\varepsilon = C' \, u_{2a} \, x = C' \, u_{2a} \, \psi \, \delta_2 \quad . \tag{3}$$



a. Inviscid Jet



b. Viscous Jet

Figure 2. Corresponding Inviscid and Viscous Jets (s is a Streamline Outside the Viscous Mixing Region)

Equation 2 now becomes

$$\frac{\partial\phi}{\partial\psi} = C' \psi \frac{\partial^2\phi}{\partial\zeta^2} \quad . \tag{4}$$

A new variable ξ is defined by

$$C' \psi \equiv \frac{d\xi}{d\psi}$$

and

$$\xi \equiv \xi(\psi) = C' \int_0^\psi \psi \, d\psi = \frac{1}{2} C' \psi^2 \quad . \tag{5}$$

Equation 4 can now be simplified to

$$\frac{\partial\phi}{\partial\xi} = \frac{\partial^2\phi}{\partial\zeta^2} \quad . \tag{6}$$

By using the appropriate boundary and initial conditions (Reference 3) Equation 6 can be integrated to yield the velocity profile solution

$$\phi = \frac{1}{2} \left[1 + \operatorname{erf}(\eta - \eta_p)\right] + \frac{1}{\sqrt{\pi}} \int_{\eta-\eta_p}^{\eta} \left[\phi_2 \left(\frac{\eta - \beta}{\eta_p}\right)\right] e^{-\beta^2} \, d\beta$$

where $\eta_p = \dfrac{1}{2\sqrt{\xi}}$ and $\eta = \zeta \, \eta_p$.

For positions far downstream of the separation corner or for very thin oncoming boundary layers

$$\frac{x}{\delta_2} \to \infty, \text{ i.e., } \psi \to \infty \; .$$

This results in $\xi \to \infty$ and $\eta_p \to 0$. Since $\eta = \zeta \, \eta_p$ is now undetermined, it must be rewritten in the following way:

$$\eta = \zeta \, \eta_p = \zeta \frac{1}{2(\xi)^{\frac{1}{2}}} = \zeta \frac{1}{(2C'\psi)^{\frac{1}{2}}} = \frac{1}{(2C)^{\frac{1}{2}}} \frac{y}{x} = \sigma \frac{y}{x} \; ,$$

where

$$\sigma = \frac{1}{(2C')^{\frac{1}{2}}} .$$

σ is called the jet-spreading parameter and is determined experimentally. Several experimentally determined functions for σ are in general use today. The two values most used are:

$$\sigma = 12 + 2.758\ M_{2a} \qquad \text{(References 2 and 5)} \tag{7}$$

and

$$\left.\begin{array}{l} \sigma = 47.1\ C_{2a}^2 \text{ for } C_{2a}^2 > 0.23 \quad (8) \\ \sigma = 11 \text{ for } C_{2a}^2 < 0.23 \quad (9) \end{array}\right\} \text{(References 11 and 12)}$$

Since these equations are empirical they should be used with care. The velocity profile solution now becomes

$$\phi = \frac{1}{2}\,(1 + \text{erf}\ \eta) . \tag{10}$$

Although this profile is for fully developed flow only, it gives reasonable results for most practical cases. Almost all the theoretical work done in this area has used this simplification. Consequently this report will be restricted to the case where Equation 7 adequately represents the mixing profile.

TEMPERATURE PROFILE

For an apparent turbulent Prandtl Number of 1 the velocity profile can be related to the temperature profile by Crocco's relation. From Reference 3 the following quantity is obtained:

$$\Lambda \equiv \frac{T_o}{T_{oa}} = \frac{T_b}{T_{oa}} + \left(1 - \frac{T_b}{T_{oa}}\right)\phi \tag{11}$$

where T_b is the stagnation temperature of the base region and T_{oa} is the stagnation temperature of the inviscid free stream. By writing the energy equation in terms of the Crocco Number C, where C is defined as

$$C^2 \equiv \frac{H_o - H}{H_o} = \frac{u^2}{2\, Cp\, T_o} = \frac{M^2}{\frac{2}{k-1} + M^2} \quad ,$$

and since $\rho/\rho_{2a} = T_{2a}/T$ for a constant pressure mixing process, the energy equation can now be written as

$$\frac{\rho}{\rho_{2a}} = \frac{T_{2a}}{T} = \frac{1 - C_{2a}^2}{\Lambda - C_{2a}^2\, \phi^2} \quad . \tag{12}$$

y_m COORDINATE SHIFT

The coordinate shift, y_m, can be determined by writing a momentum balance below the s streamline between the corner and another downstream section.

$$\rho_{2a}\, u_{2a}^2\, Y_s = \int_{-\infty}^{+Y_s} \rho\, u^2\, dY$$

and in dimensionless coordinates,

$$\eta_m = \eta_s - (1 - C_{2a}^2) \int_{-\infty}^{\eta_s} \frac{\phi^2}{\Lambda - C_{2a}^2\, \phi^2}\, d\eta \tag{13}$$

and

$$\eta_m = y_m \frac{\sigma}{x} \quad .$$

The s streamline has been taken to be in the inviscid portion of the flow field so that η_m is large enough for $\phi_s \rightarrow 1$.

JET BOUNDARY STREAMLINE

A jet boundary streamline, j, can now be found which separates the mass originally flowing at the separation corner from that entrained from the dead air region. Writing the continuity equation between the corner and a downstream location for mass between streamlines s and j results in

$$\rho_{2a}\, u_{2a}\, Y_s = \int_{Y_j}^{Y_s} \rho\, u\, dY \quad . \tag{14}$$

Transforming coordinate systems and using Equations 9 and 10,

$$\int_{\eta_j}^{\eta_s} \frac{\phi}{\Lambda - C_{2a}^2\, \phi^2}\, d\eta = \int_{-\infty}^{\eta_s} \frac{\phi^2}{\Lambda - C_{2a}^2\, \phi^2}\, d\eta \quad . \tag{15}$$

η_j can be found for a given T_b/T_{oa} and C_{2a}^2, and from earlier definitions, remembering that

$$\phi_j = \frac{1}{2}\,(1 + \text{erf}\ \eta_j) \quad .$$

<u>MASS FLOW RATE</u>

The mass flowing per unit width between the j streamline and any other streamline d is given by

$$G_d = \int_{y_j}^{y_d} \rho\, u\, dy \quad . \tag{16}$$

Transforming coordinate systems and using Equations 9 and 10,

$$\frac{G_d\, \sigma}{x\,(1 - C_{2a}^2)\, \rho_{2a}\, u_{2a}} = \int_{\eta_j}^{\eta_d} \frac{\phi}{\Lambda - C_{2a}^2\, \phi^2}\, d\eta \quad . \tag{17}$$

When the d streamline is the reattaching streamline, G_d is the mass added to the dead air region. The integrals in Equations 13 and 14 are of two types:

$$\int_{\eta_j}^{\eta_d} \frac{\phi}{\Lambda - C_{2a}^2 \phi^2} d\eta = \int_{-\infty}^{\eta_d} \frac{\phi}{\Lambda - C_{2a}^2 \phi^2} d\eta - \int_{-\infty}^{\eta_j} \frac{\phi}{\Lambda - C_{2a}^2 \phi^2} d\eta$$

$$= I_1 \left(\eta_d, C_{2a}^2, \frac{T_o}{T_{oa}}\right) - I_1 \left(\eta_j, C_{2a}^2, \frac{T_b}{T_{oa}}\right)$$

and

$$\int_{-\infty}^{\eta_s} \frac{\phi^2}{\Lambda - C_{2a}^2 \phi^2} d\eta = I_2 \left(C_{2a}^2, \eta_s, \frac{T_b}{T_{oa}}\right)$$

where

$$I_1 \equiv \int_{-\infty}^{\eta} \frac{\phi}{\Lambda - C_{2a}^2 \phi^2} d\eta \tag{18}$$

and

$$I_2 \equiv \int_{-\infty}^{\eta} \frac{\phi^2}{\Lambda - C_{2a} \phi^2} d\eta \quad . \tag{19}$$

Equations 15 and 17 can be expressed as

$$I_{1s} - I_{1j} = I_{2s} \tag{20}$$

$$\frac{G_d \sigma}{x \rho_{2a} u_{2a} (1 - C_{2a}^2)} = I_{1d} - I_{1j} \tag{21}$$

where the subscript indicates the upper limit of the integral. I_1 and I_2 have been calculated on a digital computer for the isoenergetic case ($\Lambda = 1$) and are available in Reference 14. For changing values of T_b/T_{oa}, these integrals are given graphically in Reference 4.

ENERGY TRANSFER

Energy is transferred across the j streamline in the mixing region from the free stream to the dead air region by shear work, heat conduction, and heat convection. Mass flowing between streamlines j and d can also carry energy into the dead air region. The total amount of energy per unit width crossing section x = 0 is

$$\int_0^\infty \rho\, u\, Cp\, T_{oa}\, dy$$

and the total amount of energy per unit width crossing section x = x is

$$\int_{y_j}^\infty \rho\, u\, Cp\, T_o\, dy \quad .$$

Therefore, the net rate of energy transferred per unit width by shear work and heat transfer across the j streamline between x = 0 and x = x is the difference between the two previous equations or

$$\int_{y_j}^\infty \rho\, u\, Cp\, (T_{oa} - T_o)\, dy = \Omega_c \quad .$$

By using previously defined and/or derived expressions, Ω_c can be re-written in the following way: multiply by δ_2/δ_2 and change the variable of integration, remembering that $\zeta = y/\delta_2$

$$\Omega_c = Cp \int_{y_j}^\infty \left(\frac{\delta_2}{\delta_2}\right) \rho\, u\, (T_{oa} - T_o)\, dy = \delta_2\, Cp \int_{\zeta_j}^\infty (T_{oa} - T_o)\, \rho\, u\, d\zeta \ ;$$

multiply by $\left(\frac{\eta_p}{\eta_p}\right)\left(\frac{u_{2a}}{u_{2a}}\right)$ and again change the variable of integration, remembering that $\eta = \zeta\, \eta_p$ and $\phi = \frac{u}{u_{2a}}$

$$\Omega_c = \frac{\delta_2}{\eta_p} Cp \int_{\zeta_j}^{\infty} (T_{oa} - T_o)\,\rho\,\frac{u_{2a}}{u_{2a}}\,u\,\eta_p\,d\zeta = \frac{\delta_2}{\eta_p} Cp\,u_{2a} \int_{\eta_j}^{\infty} (T_{oa} - T_o)\,\rho\,\phi\,d\eta$$

$$\Omega_c = \left(\frac{\rho_{2a}}{\rho_{2a}}\right) \frac{\delta_2}{\eta_p} Cp\,u_{2a}\,T_{oa} \int_{\eta_j}^{\infty} \left(1 - \frac{T_o}{T_{oa}}\right) \rho\,\phi\,d\eta$$

$\frac{T_o}{T_{oa}} \equiv \Lambda$ and for constant pressure mixing, $\frac{\rho}{\rho_{2a}} = \frac{T_{2a}}{T}$

$$\Omega_c = \frac{\delta_2}{\eta_p} Cp\,u_{2a}\,T_{oa}\,\rho_{2a} \int_{\eta_j}^{\infty} (1 - \Lambda)\,\frac{\rho}{\rho_{2a}}\,\phi\,d\eta = \frac{\delta_2}{\eta_p} Cp\,u_{2a}\,T_{oa}\,\rho_{2a} \int_{\eta_j}^{\infty} (1 - \Lambda)\,\frac{T_{2a}}{T}\,\phi\,d\eta$$

$\frac{T_{2a}}{T} = \frac{1 - C_{2a}^2}{\Lambda - C_{2a}^2\,\phi^2}$ (Reference 3) and as defined earlier, $\eta = \zeta\,\eta_p$ and $\zeta = \frac{y}{\delta_2}$

$$\Omega_c = \frac{y}{\zeta}\,\frac{\zeta}{\eta}\,Cp\,u_{2a}\,T_{oa}\,\rho_{2a} \int_{\eta_j}^{\infty} \frac{(1 - \Lambda)\,(1 - C_{2a}^2)}{(\Lambda - C_{2a}^2\,\phi^2)}\,\phi\,d\eta =$$

$$\frac{y}{\eta}\,Cp\,u_{2a}\,T_{oa}\,\rho_{2a}\,(1 - C_{2a}^2) \int_{\eta_j}^{\infty} \frac{(1 - \Lambda)\,\phi}{\Lambda - C_{2a}^2\,\phi^2}$$

$$E \equiv \int_{\eta_j}^{\infty} \frac{(1 - \Lambda)\,\phi}{\Lambda - C_{2a}^2\,\phi^2}\,d\eta \quad \text{and} \quad \eta = \sigma\,\frac{y}{x}$$

and finally,

$$\Omega_c = \frac{x}{\sigma}\,\rho_{2a}\,u_{2a}\,Cp\,T_{oa}\,(1 - C_{2a}^2)\,E \quad . \tag{22}$$

E can also be written as

$$E \equiv (I_1 - I_3)_{\infty} - (I_1 - I_3)_{\eta j} \tag{23}$$

or by using Equation 12,

$$E = I_{3j} - \frac{T_b}{T_{oa}} I_{1j}$$

where

$$I_3 = I_3\left(\eta,\ C_{2a}^2,\ \frac{T_b}{T_{oa}}\right) \equiv \int_{-\infty}^{\eta} \frac{\Lambda\phi}{\Lambda - C_{2a}^2\,\phi^2}\,d\eta\ . \tag{24}$$

E and I_3 are graphically represented in Reference 4.

The rate of energy per unit width carried into the base region by mass flowing between the j and d streamlines is

$$\Omega_d = \int_{y_j}^{y_d} \rho\, u\, Cp\, T_o\, dy\ .$$

By the same procedure as before Ω_d can be put into the following form:

$$\Omega_d = \frac{x}{\sigma}\,\rho_{2a}\, u_{2a}\, Cp\, T_{oa}\,(1 - C_{2a}^2)\,(I_{3d} - I_{3j})\ . \tag{25}$$

The total rate of energy added per unit width to the enclosed base region from the outside flow is

$$\Omega = \Omega_c + \Omega_d = \frac{x}{\sigma}\,\rho_{2a}\, u_{2a}\, Cp\, T_{oa}\,(1 - C_{2a}^2)\left(I_{3j} - \frac{T_b}{T_{oa}} I_{1j}\right)$$

$$+ \frac{x}{\sigma}\,\rho_{2a}\, u_{2a}\, Cp\, T_{oa}\,(1 - C_{2a}^2)\,(I_{3d} - I_3)$$

$$\Omega = \frac{x}{\sigma}\,\rho_{2a}\, u_{2a}\, Cp\, T_{oa}\,(1 - C_{2a}^2)\left[I_{3d} - \frac{T_b}{T_{oa}} I_{1j}\right]\ . \tag{26}$$

For the case considered, i.e., flow over a back step, there is no secondary flow. Consequently, the j and d streamlines are identical, $G_d = 0$, and the rate of energy carried into the base region by mass flowing between the j and d streamlines is 0. The rate of energy added per unit width to the base region now becomes only Ω_c or

$$\Omega_c = \Omega = \frac{x}{\sigma} \rho_{2a} u_{2a} Cp T_{oa} (1 - C_{2a}^2) E \quad . \tag{22}$$

This is Korst's basic method for calculating heat transfer to the base region. It does not attempt to account for the resistance of the recirculating flow.

REFLECTED IMAGE APPROXIMATION

In order to calculate the heat transfer to the base of the step, the relationship between bulk temperature, T_b, and wall temperature, T_w, must be known. At the present time an exact expression for this relation is not available. However, reasonable results have been obtained by utilizing a reflected image approximation[1,2]. This method treats the reversed flow as a reflected image of the free turbulent jet (Figure 3). By using this approximation the normal boundary condition of zero velocity at the wall is violated, but the added mathematical complexity involved in using this boundary condition would prohibit a closed solution to the problem. The mass flow through the reflected image equals the actual reversed mass flow. Therefore,

$$\frac{T_w}{T_b} = \frac{T_b}{T_{oj}} \quad . \tag{27}$$

Also,

$$\frac{T_w}{T_{oa}} = \frac{T_b}{T_{oa}} \frac{T_w}{T_b} = \frac{T_b}{T_{oa}} \frac{T_b}{T_{oj}} = \frac{T_b}{T_{oa}} \frac{T_b}{T_{oa}} \frac{T_{oa}}{T_{oj}} \tag{28}$$

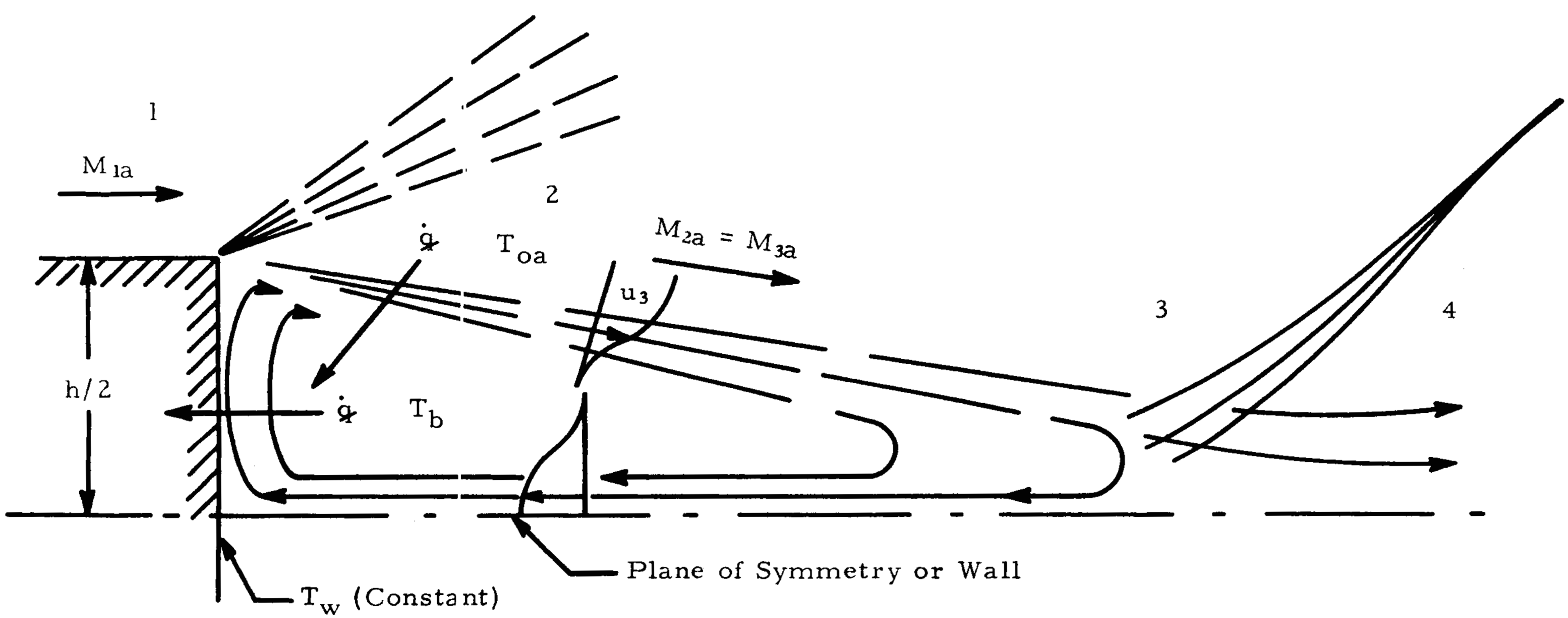


Figure 3. Heat Transfer Model

and from Equation 11,

$$\frac{T_{oj}}{T_{oa}} = \frac{T_b}{T_{oa}} + \left(1 - \frac{T_b}{T_{oa}}\right) \phi_j \quad .$$

Substituting this equation into Equation 19 yields

$$\frac{T_w}{T_{oa}} = \frac{\left(\frac{T_b}{T_{oa}}\right)^2}{\frac{T_b}{T_{oa}} + \phi_j - \frac{T_b}{T_{oa}} \phi_j} = \frac{\left(\frac{T_b}{T_{oa}}\right)^2}{\frac{T_b}{T_{oa}} (1 - \phi_j) + \phi_j} \quad . \tag{29}$$

HEAT TRANSFER CALCULATION

In calculating the heat transfer to the base of the step, two resistances (see Figure 3) must be recognized; that of the free jet mixing region and the recirculating flow. The flow which actually comes into contact with the base wall is the recirculating flow, but it is in contact for only a short portion of its total path. The total flow path of the recirculated flow includes its reversal at the point of recompression, its flow back along the axis of symmetry, and its flow along the base wall.

The bulk temperature between the centerline and the free stream is assumed to be that of the free stream, and the bulk temperature between the wall and the free stream is calculated by the reflected image approximation. The average bulk temperature that is used in making the heat transfer calculations is found by averaging the two previous values with respect to their path lengths using the following relationships:

$$\left(\frac{T_b}{T_{oa}}\right)_{cal} = \frac{A \left(\frac{T_b}{T_{oa}}\right)_{refl} + B \left(\frac{T_{oa}}{T_{oa}}\right)}{A + B} \tag{30}$$

where

$$A = \frac{h}{2} \quad , \tag{31}$$

$$B + A = \frac{h/2}{\sin\theta} \tag{32}$$

and θ is the streamline angle. From Equations 30, 31, and 32,

$$\left(\frac{T_b}{T_{oa}}\right)_{refl} = \frac{\left(\frac{T_b}{T_{oa}}\right)_{cal} - 1 + \sin\theta}{\sin\theta} \quad . \tag{33}$$

Equation 29 can now be written as

$$\frac{T_w}{T_{oa}} = \frac{\left(\frac{T_b}{T_{oa}}\right)^2_{refl}}{\phi_j + (1 - \phi_j)\left(\frac{T_b}{T_{oa}}\right)_{refl}} \quad . \tag{34}$$

Since we are concerned about steady state heat transfer, the heat transfer rate to the wall must equal the heat transfer rate through the mixing region. The heat transfer rate across the mixing region has already been calculated by Korst's energy integral method and is given by Equation 22. Reference 4 provides graphs and tables necessary for evaluating Equation 22. Therefore, there is now a way to calculate the heat transfer to the base of a step given various combinations of the parameters of the flow field.

COMPARISON OF CALCULATED VALUES WITH MEASURED VALUES

Experimental results for the physical model described by this analysis were found in reports by the Royal Aircraft Establishment[6], the University of Minnesota[1], and the General Applied Science Laboratories, Inc.[8, 9]. These results were compared with calculated values which were obtained by using both the Korst method and the technique suggested by Page and Dixon. The analysis by Chapman was developed primarily for laminar flow, and since this report is concerned with turbulent jet mixing the Chapman analysis could not be used. In all cases the Korst method was found to give values that were substantially larger than the measured values, and for this reason, these results are not discussed further. The technique developed by Page and Dixon was found to give more reasonable results when compared to experimental data and consequently this is the technique described and used in this report. (The method suggested by Page and Dixon is basically Korst's method, except that it includes a technique that attempts to account for the resistance of the recirculating flow.) Also, better results were obtained when the jet spreading parameter (σ) suggested by References 11 and 12, and given by Equation 8, was used instead of Equation 7 originally suggested by Korst and used by Page and Dixon.

The data from the Royal Aircraft Establishment is shown in Figure 4. The actual flow model used in these tests was a wedge (15° semi-apex angle), so the free stream Mach number of M_{1a} = 2.9 is that of the stream after accounting for the oblique shock and Prandtl Meyer expansion caused by the wedge angle. Both methods (Equations 7 and 8) for calculating the jet mixing coefficient (σ) were used and the results indicate (as mentioned earlier) that Equation 8 will give results closer to experimental values.

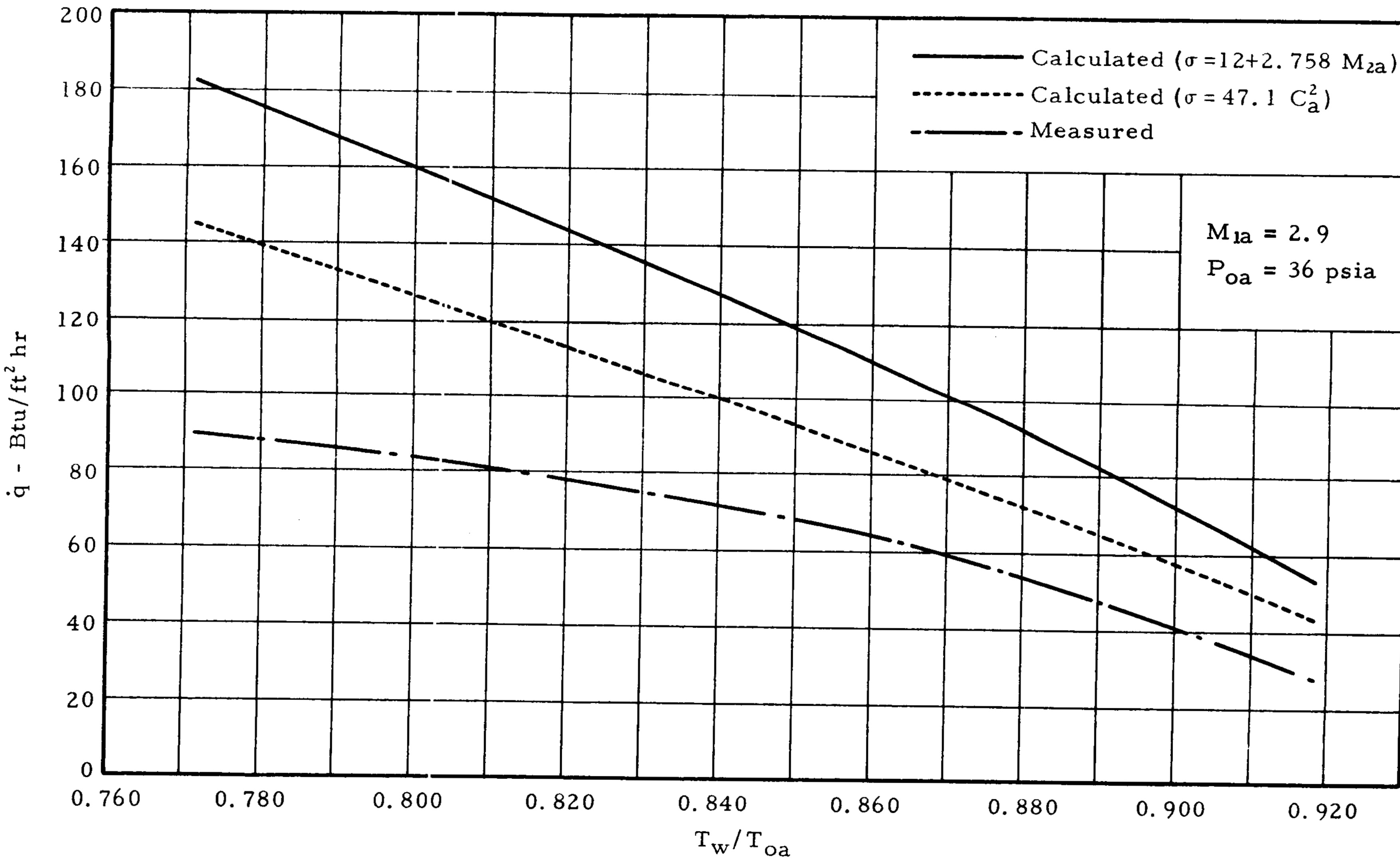


Figure 4. Heat Transfer Data from Royal Aircraft Establishment

Figure 5 presents data from the University of Minnesota. The calculated values are in all cases larger than those measured, but both show the same trend. As before, the jet mixing coefficient calculated by Equation 8 will give results closer to experimental values than those calculated by Equation 7.

The results from the GASL data are presented in Figure 6. However, the stagnation pressure (P_{oa}) of this data was not the same for all values of T_w/T_{oa}. This variation in P_{oa} makes the data difficult to compare, since the heat transfer rate is a function of P_{oa}. Also, since the model GASL used in obtaining their results was a cylinder 3.6 inches in diameter, the flow field is axisymmetric and not a plane two-dimensional flow. However, even with these problems the calculated values show the same trend as the measured and give an order of magnitude estimate of the heat transfer.

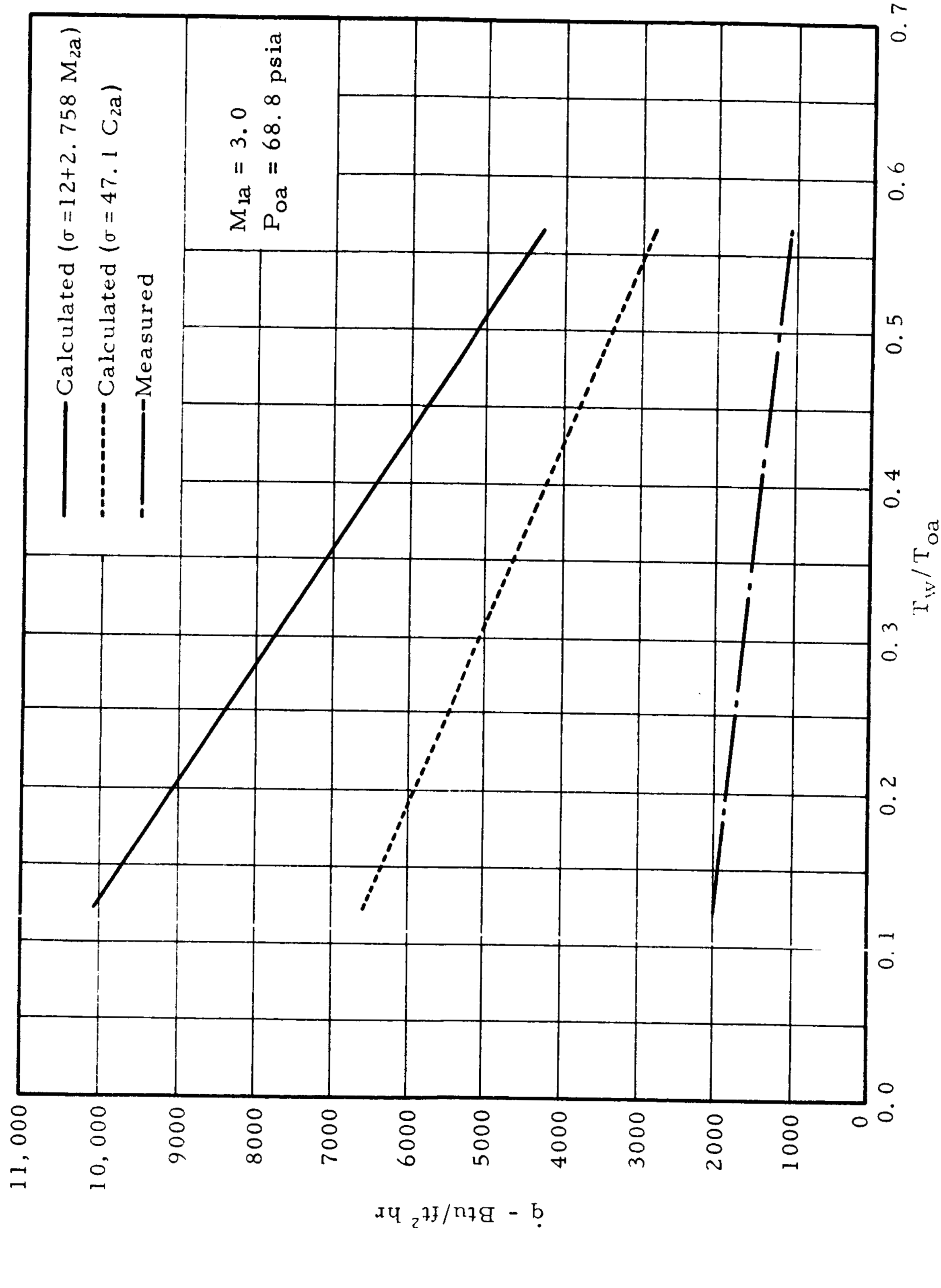


Figure 5. Heat Transfer Data from University of Minnesota

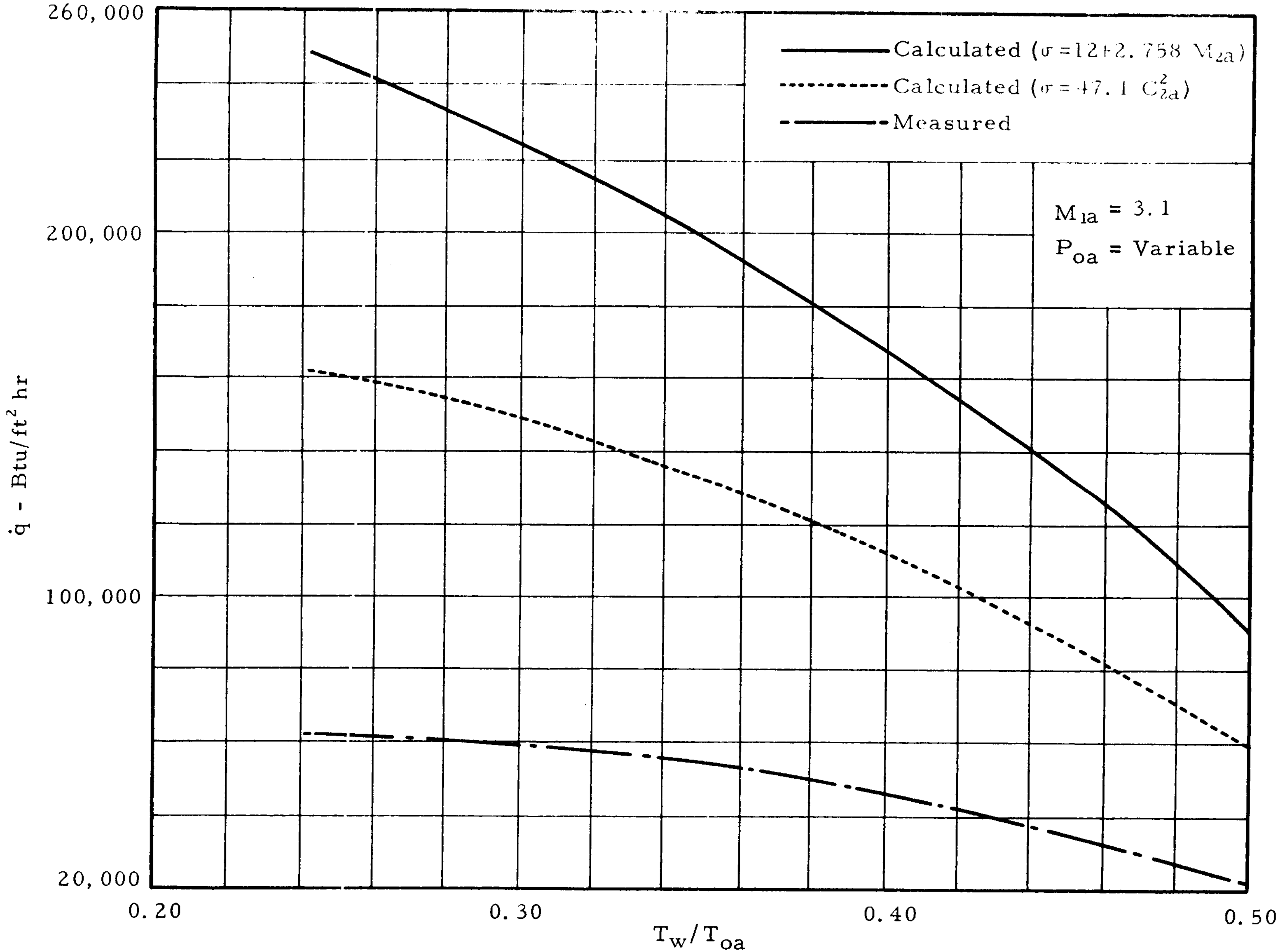


Figure 6. Heat Transfer Data from GASL

CONCLUSIONS

The heat transfer method suggested by Page and Dixon and modified by using the jet spreading parameter given by Equation 8 instead of Equation 7 will give better values for the base or back step heat transfer rate in separated, supersonic flow than any other method investigated. In every instance where measured data was compared with calculated values, the theory predicted values greater than measured values. Since this technique is relatively simple and straightforward it can be useful for providing good estimates. However, due to the oversimplification of the flow field and a basic lack of understanding of the physical phenomenon involved, this theory will not provide accuracy any better than about 50%. The error should always be conservative.

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13. ABSTRACT

This report describes an analytical method for predicting the heat transfer to the base of a wedge or rearward facing step from a separated, turbulent, supersonic flow. Calculated values are compared with experimental data.

14. KEY WORDS
separated flow
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