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HYDROGEN MASER FREQUENCY COMPARISON WITH A CESIUM BEAM STANDARD

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

ABSTRACT

A direct measurement of the hyperfine energy-level splitting (splitting the energy band into sublevels) of the hydrogen atom has been made by comparing the frequency of an atomic hydrogen maser and a cesium-beam frequency standard. The two standards were compared for the period of one day. The measured value of the $F = 1$, $M_f = 0 \rightarrow F = 0$, $M_f = 0$ hyperfine transition corrected to zero field is 1420405751.781 ± 0.016 (A-1 Time Scale).

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INTRODUCTION

A direct comparison between a hydrogen maser and a cesium beam tube frequency standard has been made at Goddard Space Flight Center. This is one of the first direct measurements of the hyperfine splitting of the energy levels of the hydrogen atom compared to the hyperfine splitting of the cesium atom (References 1 and 2). The cesium beam standard used was a Hewlett-Packard 5060A and the hydrogen maser was a Varian H-10.

Figure 1 shows the system block diagram for the measurement of f_x, a frequency which is 31 Mc below the output frequency of the maser and phase locked to the maser. The 31 Mc difference between f_x and f_m is a property of the maser synthesizer system employed which is explained fully in Appendix A. The frequency f_{cs}, due to the zero-field hyperfine separation for the $_{55}Cs^{133}$ atom, is by definition exactly 9,192,631,770.00 cps (Reference 3) consistent with the A-1 (Atomic Time) time scale. The 5060A used in the measurement provides outputs at 1 and 5 Mc in the UT-2 (Universal Time) time scale. Therefore all calculations are made with UT-2 scale frequencies, and a correction to the A-1 scale is made after the calculations.

The frequencies presented in Figure 1 are related to f_x and f_{cs} by Equations 1 through 7.

$$f_1 = 5 \text{ Mc}, \tag{1}$$

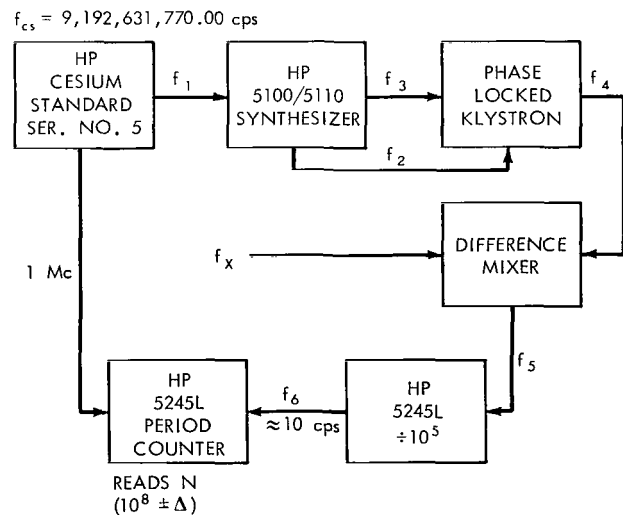$$f_2 = 30405772.98 \text{ cps}, \tag{2}$$

$$f_3 = 136 \text{ Mc}, \tag{3}$$

$f_{cs} = 9,192,631,770.00$ cps

Figure 1—System block diagram for measurement of f_x.

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$$f_4 = 10\, f_3 + f_2 \, , \tag{4}$$

$$f_5 = f_4 - f_x \, , \text{ and} \tag{5}$$

$$f_6 = \frac{f_5}{10^5} = \frac{1}{(10^5 + \Delta') \times 10^{-6}} \approx 10 \text{ cps} \, , \tag{6}$$

where Δ' is in microseconds, or

$$f_6 = \frac{10^3}{(10^8 + \Delta) \times 10^{-6}} \text{ cps} = \frac{10^8}{(10^8 + \Delta)} \text{ Mc}$$

where Δ is in nanoseconds. Therefore

$$f_x = f_4 - f_5$$

$$= 10\, f_3 + f_2 - 10^5\, f_6$$

$$= 1390.40577298 - \frac{10^8}{10^8 + \Delta} \text{ Mc} \, . \tag{7}$$

COMPARISON RUNS

Six comparison runs between the two standards were made, each lasting about 5 hours. The resolution of ±1 count on the counter produces negligible error. The maser cavity was tuned to better than 5 parts in 10^{12} prior to the measurements by the pressure shift technique (References 4 and 5). The data from the first run are presented in Table 1. The mean value of Δ as well as the standard deviation and the standard deviation of the mean are determined as shown in this table.

From Equation 7,

$$f_x = 1390.40577298 - \frac{10^8}{10^8 + \overline{\Delta} \pm \frac{\sigma}{\sqrt{n}}} \text{ Mc} \, .$$

With

$$10^5\, f_6 = \frac{10^8}{10^8 + \overline{\Delta} \pm \frac{\sigma}{\sqrt{n}}} = \frac{1}{1 + \left(\overline{\Delta} \pm \frac{\sigma}{\sqrt{n}}\right) 10^{-8}} \, ,$$

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$$10^5 \, f_6 \bigg|_{\overline{\Delta} + \frac{\sigma}{\sqrt{n}}} = \frac{1}{1 + 0.4117 \times 10^{-8}} \approx 999{,}999.9959 \text{ cps} \; ,$$

and

$$10^5 \, f_6 \bigg|_{\overline{\Delta} - \frac{\sigma}{\sqrt{n}}} = \frac{1}{1 + 0.1637 \times 10^{-8}} \approx 999{,}999.9983 \text{ cps} \; ,$$

therefore

$$10^5 \, f_6 = 999{,}999.9971 \pm 0.0012 \text{ cps} .$$

By adding the 31 Mc offset between f_m and f_x, it follows that

$$f_m = 1420405772.9829 \pm 0.0012 \text{ cps} .$$

Converting to A-1,

$$f_{m_{A-1}} = f_m \left(1 - 1.5 \times 10^{-8}\right)$$

$$= 1420405751.6769 \pm 0.0012 \text{ cps} .$$

This measured maser frequency must be corrected for wall shift, magnetic field, and second order doppler shift in order to determine the zero-field hyperfine splitting of hydrogen. These corrections are:

Δf wall shift* $= -0.043$ cps ± 0.0023,

Δf 2nd order doppler $= -0.062$ cps, and

Δf C field $= 0.00105$ cps. The sum of these is

$\Sigma \Delta f = -0.1040$ cps;

Table 1

Comparison Data Points.

Data Point	Deviation (Δ) from Nominal Period (nanoseconds)	Deviation Squared (Δ^2) (nanoseconds^2)
1	-0.13	0.0169
2	+0.75	0.5625
3	+0.54	0.2916
4	+0.12	0.0144
5	+0.04	0.0016
6	-0.64	0.4096
7	+0.09	0.0081
8	+0.18	0.0324
9	+0.46	0.2116
10	+0.04	0.0016
11	+0.47	0.2209
12	+1.28	1.6384
13	+0.54	0.2916

$$\Sigma \Delta = 3.74 \qquad \Sigma \Delta^2 = 3.7012$$

$$\overline{\Delta} = 0.2877 \qquad \overline{\Delta^2} = 0.2847$$

$$(\overline{\Delta})^2 = 0.08277$$

$$\sigma = \sqrt{\overline{\Delta^2} - (\overline{\Delta})^2} = \sqrt{0.2847 - 0.0828} = 0.45$$

$$= \text{Standard Deviation}$$

$$\frac{\sigma}{\sqrt{n}} = \frac{0.45}{\sqrt{13}} = 0.124 = \text{Standard Deviation of the Mean}$$

---

*Wall material is FEP Teflon. The value determined for 5-1/2" bulb at Bomac Division, Varian Associates.

therefore

$$F_{m_{A-1}} = f_{m_{A-1}} - \Sigma\Delta f$$

$$= 1420405751.7809 \pm 0.0012 \text{ cps }.$$

If $\pm 5 \times 10^{-12}$ is allowed for error in the hydrogen maser and $\pm 1 \times 10^{-11}$ is allowed for error in the cesium standard, and if the errors are combined as random errors, the measured frequency of hydrogen becomes 1420405751.781 $\pm$0.016 cps for the first of the six runs. The results of all the runs are tabulated in Table 2.

Table 2

Results of Comparison Runs One Through Six

| Run Number	1	2	3	4	5	6
Maser frequency 1420405751. + (cps)	0.781	0.781	0.781	0.780	0.783	0.782
RMS error of mean (cps)	±0.0012	0.0010	0.0011	0.0012	0.0017	0.0010

COMPARISON WITH PREVIOUS MEASUREMENTS

The value obtained for the zero-field hyperfine frequency of hydrogen agrees well with the result of Peters et al. (Reference 1) which was 1420405751.778 $\pm$0.016 cps and that obtained by Kantaschoff et al. (Reference 2) which was ...751.785 $\pm$0.016 cps. It also agrees well with the value measured by Crampton, et al. (Reference 6) which was ...751.800 $\pm$0.028 cps. It is slightly outside the combined statistical errors of the value found by Markowitz et al. (Reference 7) which is ...751.827 $\pm$0.02 cps when all corrections are applied.

It is significant to note that in the measurement described here a Hewlett-Packard cesium beam standard and the Varian H-10-1 hydrogen maser were compared. The comparison made by Peters et al. involved the Varian H-10-2 maser and a different Hewlett-Packard cesium standard.

During the time in which the hydrogen/cesium comparison was made, Maser 1 was compared with Maser 2. Figure 2 shows the fractional frequency stability measured during both comparisons. The variations present between the masers may be correlated with ambient temperature variations and are discussed further in a forthcoming paper on the hydrogen maser performance. It is interesting to note that the scale of the two plots differs by 1 order of magnitude.

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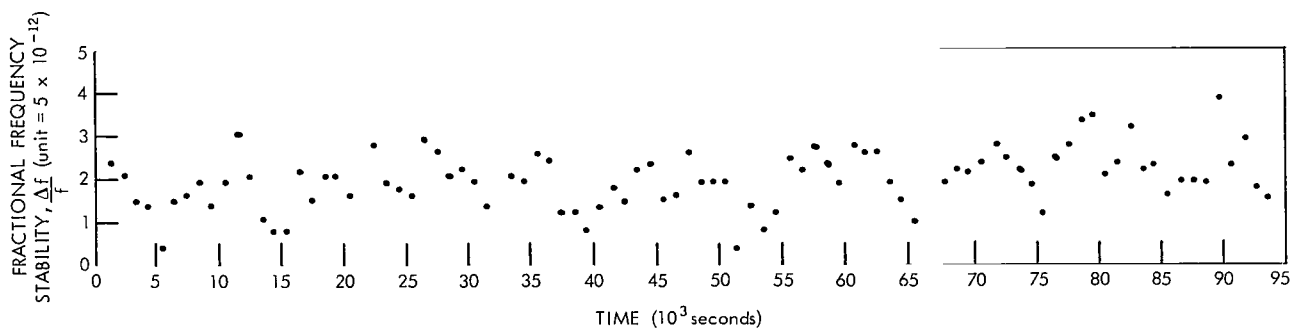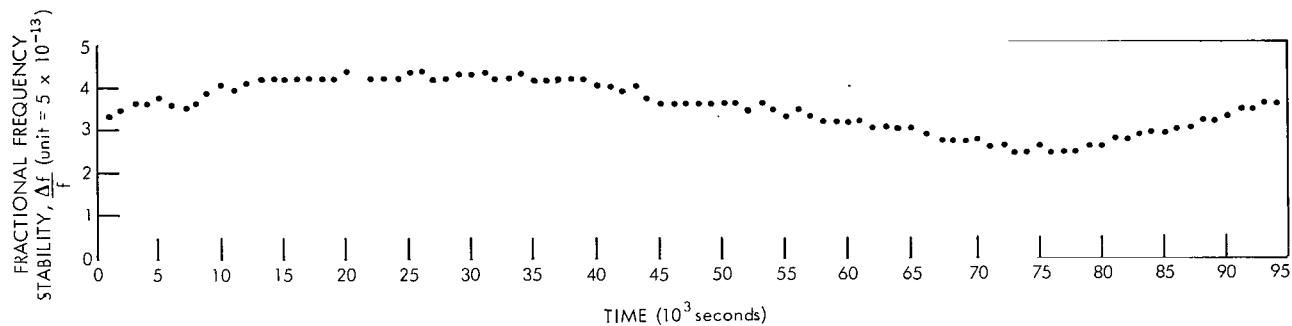
Figure 2—Comparison of the H-10-1 maser (maser 1): (a) With the 5060A cesium–beam frequency standard, serial number 5, (b) With the H-10-2 maser (maser 2).

CONCLUSIONS

The spread of the results of this comparison and those in References 1 and 2 is ±2.5 parts in 10^{12}. If this result is considered, along with the spread of data in Figure 2, it appears that the $±1 \times 10^{-11}$ error taken for the cesium standard is conservative. The $±5 \times 10^{-12}$ tuning error taken for the maser is also conservative, because the two masers differed by less than 5×10^{-13} after tuning.

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Appendix A

Hydrogen Maser Synthesizer and Measurement System

Introduction

In mid-November, 1964, two atomic hydrogen maser oscillators were delivered by Varian Associates to the Goddard Space Flight Center. The primary objective of the hydrogen maser program is the evaluation of the applicability of these ultra stable oscillators to NASA mission requirements. Before work could begin toward meeting this primary objective, it was necessary to satisfy two intermediate goals:

1. The development of an adequately sensitive technique for the measurement of the accuracy and stability of the masers, as well as the measurement of other precision atomic and crystal standards against the maser.

2. The development of a synthesis technique for deriving signals at useful frequencies characterized by the frequency accuracy and stability of the hydrogen maser.

The method in which these goals have been met is the subject of this appendix. In summary, the described measurement system can measure the frequency and frequency stability of the two oscillators under test with an error of only ±2 parts in 10^{-15} for a 1000 second averaging time. On the analog recorder employed, the full scale (8" paper) deflection corresponds to a frequency change $\Delta F/F$ of 2 parts in 10^{-13}.

The synthesizer system employed produces a 1 Mc signal on either the UT-2 or the A-1 time scale which exhibits the accuracy and fractional frequency stability $\Delta F/F_{1Mc}$ of the hydrogen maser. In addition, by using an auxiliary frequency synthesizer and an available phase locked klystron, signals at frequencies up to 4.5 Gc can be synthesized from the hydrogen maser.

Basic Synthesis Technique

The technique for synthesis of a precision 1 Mc signal, which is coherently related to the hydrogen maser signal, is shown in simplified form in Figure A1. The maser signal at 1420.40577298 Mc (UT-2) is amplified in a low-noise (4.0db) preamplifier and then heterodyned down to 31 Mc. The local oscillator signal, at 1389.40577298 Mc, and the phase detector reference are coherently synthesized from the 1 Mc VCXO. The output of the phase detector is then fed back through the low pass filter, phase locking the 1 Mc VCXO to the hydrogen maser. The noise bandwidth of the loop is approximately 1 cps. Thus, for averaging times longer than 1 second,
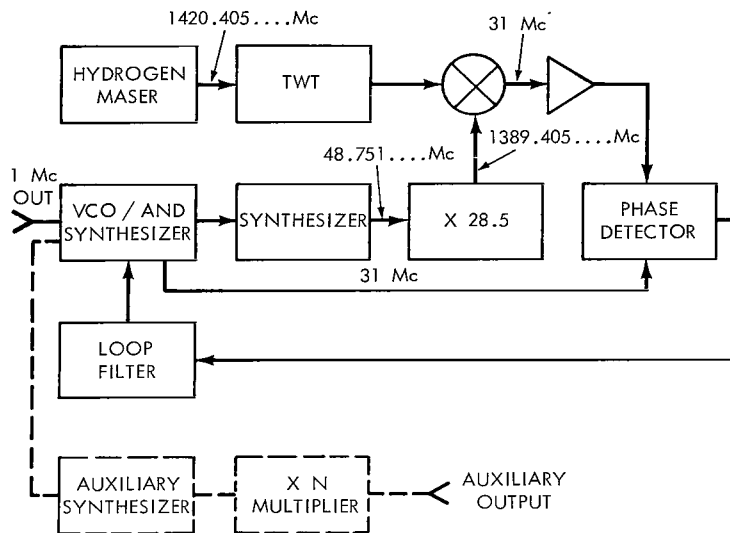
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Figure A1 —Simplified diagram of technique for synthesis of a
precision 1-megacycle signal.

the 1 Mc signal will exhibit the fractional frequency stability of the hydrogen maser; for averaging times shorter than 1 cps the 1 Mc signal will exhibit the inherent stability of the 1 Mc VCXO. One task which remains is the optimization of this loop bandwidth so that the best characteristics of each oscillator are retained in the output 1 Mc signal.

The synthesis of frequencies other than 1 Mc is shown by the dotted lines in Figure A1. In practice, up to three auxiliary synthesizers may be driven from the basic VCXO/synthesizer. Each auxiliary synthesizer will produce frequencies up to 50 Mc in 0.01 cps steps. This range may be extended into the microwave region through the use of appropriate multipliers or phase-locked oscillators.

Measurement Technique

It was determined that the maser frequency measurement system should have the following characteristics:

1. It should be sensitive to frequency changes as small as ± 1 part in 10^{-14} .

2. The sensitivity to frequency changes should be independent of any offset between the frequencies being compared.

3. The system should be capable of comparing frequencies over a wide range of averaging times.

4. The output of the system should be essentially reduced data on frequency changes.

5. The system should have the inherent capability of measuring other atomic and crystal frequency standards against the maser.

A measurement system which is relatively simple and easy to implement is shown in Figure A2. This system was the first one established at GSFC but fell short of the desired characteristics listed above. The output of the system was the actual beat note between the two masers, resulting in a laborious data reduction problem.

The system shown in Figure A3, however, possesses all of the desired characteristics. The nominal 1388 Mc and 1389 Mc signals, it should be remembered, are phase-locked to the two

hydrogen masers, thus they exhibit the stability of the masers. By heterodyning down to 1 Mc, the fractional frequency instability $\Delta F/F$ is increased by a factor of 1.4×10^3. In order to multiply the error further, the 1 Mc beat note is again multiplied to 360 Mc and heterodyned down to 10 Mc. The 350 Mc reference is obtained by multiplying the phase-locked 1 Mc signal which is provided by the synthesizer loop. This 1 Mc is more stable than the 1 Mc produced by the mixer by a factor of 1.4×10^3, and therefore it contributes essentially no error. The 10 Mc beat note is then divided by a factor of 10^6 in a conventional frequency counter, producing a 10 cps output. The period of this 10 cps signal is measured in a second conventional counter. The counter output is displayed on both an analog and a digital recorder. The counter error of ± 1 count corresponds to a frequency measurement error of $\pm 2 \times 10^{-15}$ for 1000 second averages, $\pm 2 \times 10^{-14}$ for 100 second averages, etc.

When it is desired to compare the hydrogen maser with other atomic or crystal standards, one of the maser synthesizer loops is opened, and the standard to be tested is substituted at 1 Mc or 5 Mc for the 1 Mc VCXO shown in Figure A1. This operation takes virtually no time, since nothing else in the system need be changed.
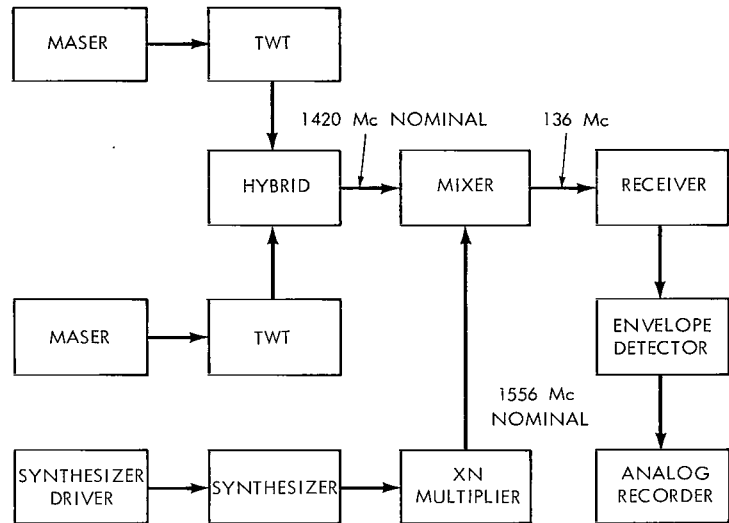

Figure A2—System block diagram of a beat-frequency technique for frequency measurement.
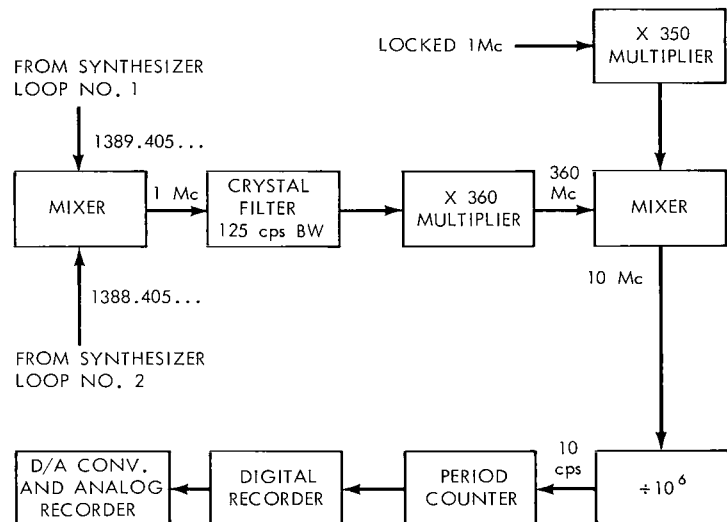

Figure A3—System block diagram of an improved beat-frequency technique for frequency measurement.

Figure A4 is a diagram of the complete synthesis and measurement scheme as it is presently implemented.

Typical Hydrogen Maser Performance Data

The top curve in Figure A5 is a plot of the frequency instability of the two hydrogen masers for a period of approximately 5 days. The lower curve is a plot of the ambient temperature
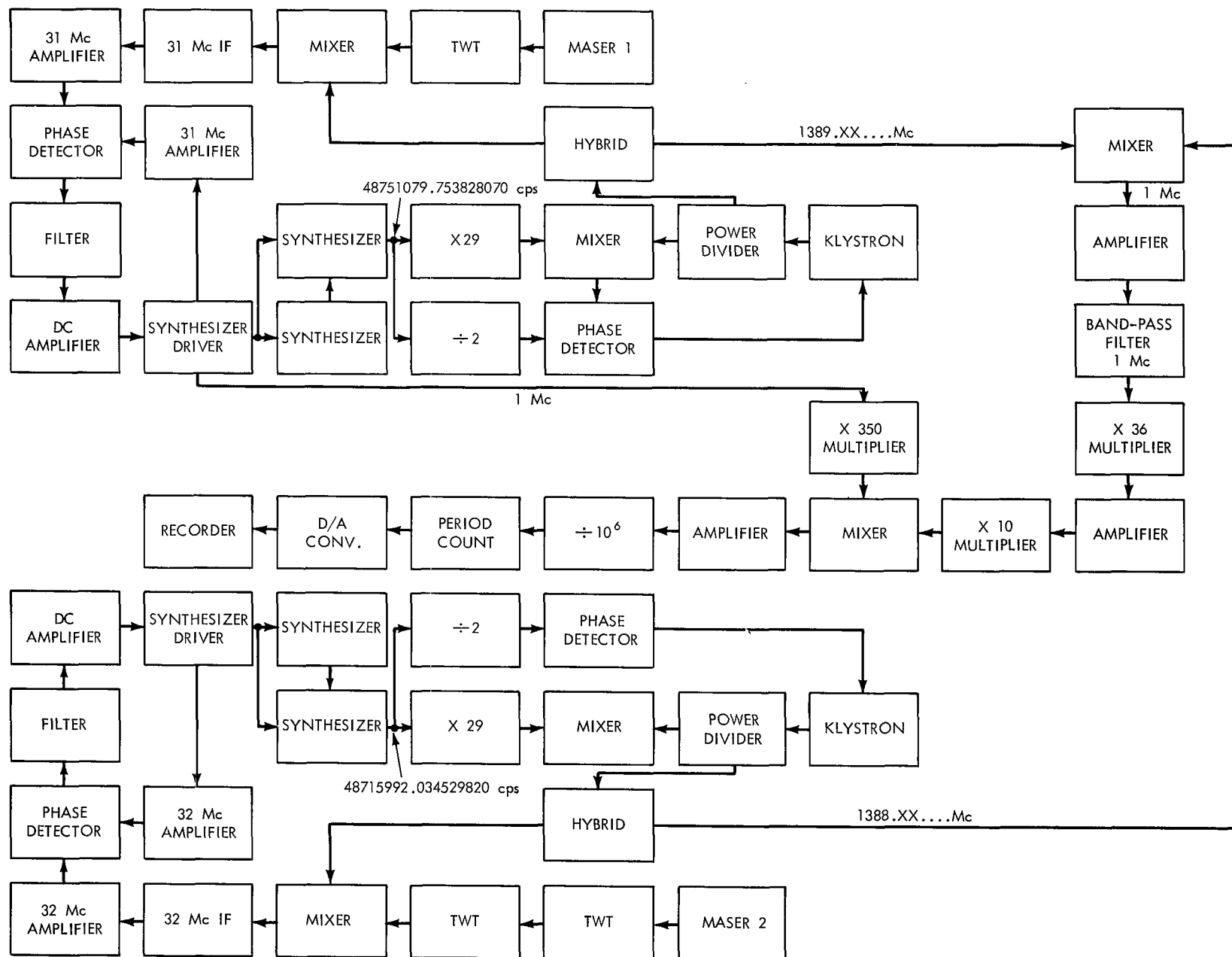
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Figure A4—System block diagram of the complete frequency synthesis and measurement technique.
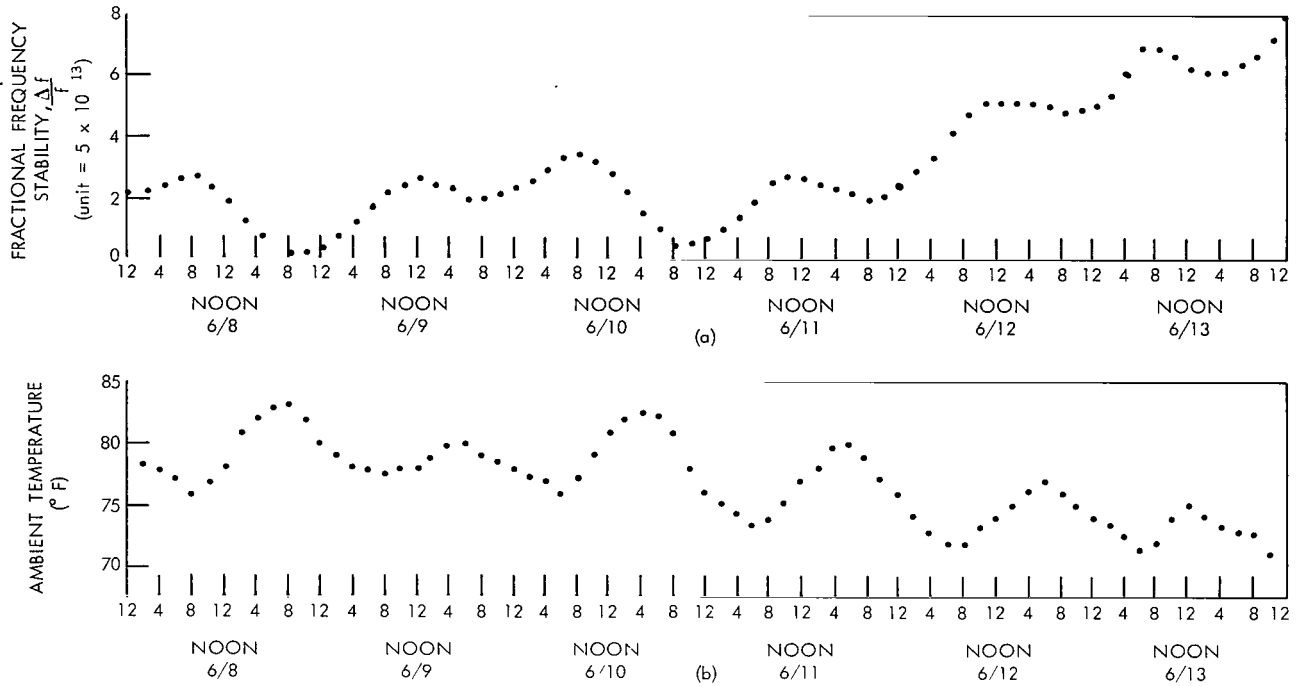
Figure A5—Comparison of the H-10-1 maser (maser 1) against the H-10-2 maser (maser 2) over an approximate 5-day period: (a) Fractional stability, (b) Ambient temperature.

in the laboratory for the same period. It is interesting to note that the daily fluctuations in maser frequency of approximately 2×10^{-12} peak-to-peak correlate very closely with ambient temperature. It is believed that these fluctuations are due to cavity pulling by the output coaxial circuitry in one of the masers. Therefore, additional thermal control at this point is being implemented which should eliminate the difficulty. It should be noted that systematic frequency drifts due to causes other than ambient variations have not been observed.

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