TECHNICAL NOTE (R-69)

THE APPLICATION OF ONE-DIMENSIONAL ANALYSIS TO THRUST AUGMENTATION FOR THERMALLY PERFECT BUT CALORICALLY IMPERFECT GASES

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ABSTRACT

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One-dimensional analysis is applied to the problem of determining the flow parameters at the exit of a rocket thrust augmentor, given the chemical compositions of primary, secondary, and exit, and the flow parameters at the entrance (primary and secondary). The effect of burning auxiliary fuel (liquid) in the secondary is included.

With the exception of the static pressure ratios and mass flow ratios, the usual flow parameters and parameter ratios such as Mach number, specific heat ratio, total temperature and total pressure do not appear explicitly in the equations. These equations must generally be solved for each specific separate problem.

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LIST OF SYMBOLS

A	cross-sectional area
a	coefficient in polynomial for enthalpy
b	coefficient in polynominal for enthalpy at exit
C	constant in equation derived from momentum equation
c	mass fraction of element in molecule
H	molar enthalpy
h	enthalpy per unit mass
°K	degrees Kelvin
$\dot{m}$	mass flow rate
n	mass fraction of molecule
p	static pressure
R_o	universal gas constant
T	absolute temperature
u	velocity, parallel to axis of augmentor
W	molecular weight, effective or actual
$\dot{\mu}$	ratio of mass flow rate to primary mass flow rate
ρ	density
η	$(pA)/(p_p A_p)$

Subscripts

e	exit flow
f	fuel added to secondary
j	indicates a particular molecular form or a particular element
k	indicates a particular coefficient
o	indicates the first coefficient in the enthalpy series or total value as for total enthalpy (h_o)
p	primary flow
s	secondary flow
w	augmentor wall
x	component in direction of augmentor axis of symmetry

INTRODUCTION

One-dimensional analysis has been applied to perfect fluid flows for many years. In the case of ejector problems this application is particularly well known. Recently it has been applied to the problem of thrust augmentation (see Reference 1, for example).

In this paper, one-dimensional analysis is applied to rocket thrust augmentation for gases which follow the perfect gas law but for which the enthalpy is not a linear function of temperature but can be represented by a finite power series in temperature only, i. e., for which the specific heats are functions of temperature.

Specifically, the enthalpy is assumed to be represented by a fifth degree polynomial in the absolute temperature. Heats of formation are included as parts of the polynomial and therefore do not appear explicitly as such heats. Zeleznik and Gordon in Reference 2 give the polynomial coefficients for a large number of possible chemical forms for the two temperature ranges 300° - 1000° K and 1000° - 5000° K. The extension of the present procedure to a lower temperature range and for liquids within that range (as may be necessary when fuel is to be added to the secondary air flow) depends only on the determination of suitable coefficients for that range.

Because the enthalpy is not assumed to be a linear function of temperature, parameters and parameter ratios which are of importance for calorically perfect gases do not appear explicitly in the equations or in their solutions although several may be calculated if this is desired. Examples are total temperature, total pressure, specific heat ratios, and Mach numbers. (The combination γM^2 is recognizable in the momentum equation).

If this were desirable, the method could be extended to that for a plane-by-plane process in which the chemical kinetics of the flow was calculated.

STATEMENT OF THE PROBLEM

A schematic drawing to illustrate the problem is given in Figure 1. A rocket, instead of exhausting into the atmosphere, furnishes the primary flow (p) for a thrust augmentor. The secondary (s) is atmospheric air. Fuel (f) may or may not be added to the secondary. Complete mixing is assumed. The chemical compositions of primary, secondary, fuel added, and exit (e) are assumed known, along with the flow paramaters of primary and secondary. The problem is, given the component in the flow direction of the pressure force on the augmentor, to determine the flow parameters at the exit.

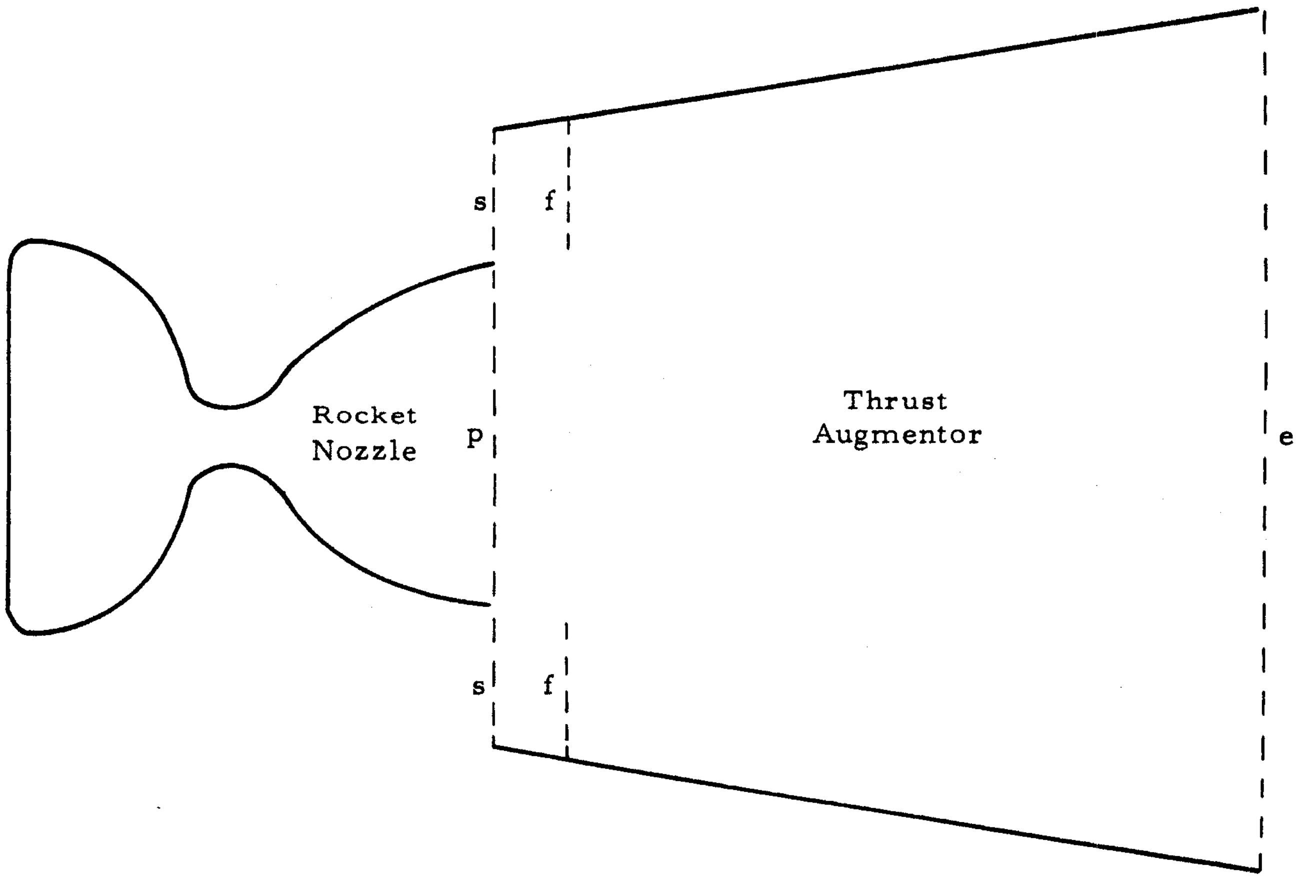

Figure 1

Schematic Diagram of Rocket Nozzle Plus Thrust Augmentor

THE EQUATIONS GOVERNING THE FLOW

The Perfect Gas Law

It is assumed that any gaseous component or any gaseous section of the flow acts as a thermally perfect gas, i.e., that

$$\frac{p}{\rho} = \frac{R_o}{W} T^* \tag{1}$$

where R_o is the universal gas constant and W is either actual or effective molecular weight. For any mixture of gases, W is then to be calculated from

$$W = \frac{\sum_j n_j}{\sum_j \frac{n_j}{W_j}} = \frac{1}{\sum_j \frac{n_j}{W_j}} \tag{2}$$

in which n_j is the mass fraction of component j and W_j is the molecular weight of that component. The effective molecular weights at relevant cross-sectional areas are calculable since the corresponding chemical compositions are assumed to be known.

Conservation of Mass

The mass flow rate leaving the augmentor must equal the sum of the mass flow rates entering, or

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*The c.g.s. system of units is considered to be used throughout this report. R_o, for example, might be expressed in ergs/mole°k.

$$\dot{m}_e = \dot{m}_p + \dot{m}_s + \dot{m}_f \tag{3}$$

where

$$\dot{m} = \rho u A \ . \tag{4}$$

If we let

$$\dot{\mu} = \frac{\dot{m}}{\dot{m}_p} \ , \tag{5}$$

then

$$\dot{\mu}_e = 1 + \dot{\mu}_s + \dot{\mu}_f \ . \tag{6}$$

Conservation of Elements

Also the rate at which each chemical element enters and leaves must be the same, i.e.,

$$\dot{m}_e \sum_j n_{ej}\, c_{jk} = \dot{m}_p \sum_j n_{pj}\, c_{jk} + \dot{m}_s \sum_j n_{sj}\, c_{jk} + \dot{m}_f \sum_j n_{fj}\, c_{jk} \ , \tag{7}$$

or

$$\dot{\mu}_e \sum_j n_{ej}\, c_{jk} = \sum_j n_{pj}\, c_{jk} + \dot{\mu}_s \sum_j n_{sj}\, c_{jk} + \dot{\mu}_f \sum_j n_{fj}\, c_{jk} \ , \tag{8}$$

where n_{ej}, for example, is the mass fraction of molecule j at the exit, and c_{jk} is the mass fraction of element k in this molecule. This equation must hold for each element k.

In reality, equation (7) or (8) would be used in selecting inputs for any computing program since the chemical composition at the exit is assumed to be known. It therefore provides a series of conditions under which the selected inputs are realistic.

The Momentum Equation

$$p_p A_p + p_s A_s - p_e A_e + \int p_w \, dA_x = \dot{m}_e u_e - \dot{m}_p u_p - \dot{m}_s u_s - \dot{m}_f u_f \,^* \quad (9)$$

where p_p, for example, is the pressure in the primary and $\int p_w \, dA_x$ is the component of the wall pressure force in the direction opposite the main flow, $\int p_w \, dA_x$ is therefore the thrust on the augmentor.

Conservation of Energy

$$\dot{m}_e\left[h_e\,(T) + \frac{1}{2}u_e^2\right] = \dot{m}_p\left[h_p + \frac{1}{2}u_p^2\right] +$$

$$\dot{m}_s\left[h_s + \frac{1}{2}u_s^2\right] + \dot{m}_f\left[h_f + \frac{1}{2}v_f^2\right] \quad (10)$$

or

$$\dot{\mu}_e\left[h_e\,(T) + \frac{1}{2}u_e^2\right] = h_p + \frac{1}{2}u_p^2 + \dot{\mu}_s\left[h_s + \frac{1}{2}u_s^2\right] + \dot{\mu}_f\left[h_f + \frac{1}{2}u_f^2\right] \quad (11)$$

in which h (T) is static enthalpy per unit mass. For any given chemical composition, it is assumed to be a function of temperature only.

The Static Enthalpy Equations

The molar enthalpy, H_j, for any component of the flow, is assumed to be approximated by

$$\frac{H_j}{R_o} = a_{oj} + \sum_{k=1}^{5} a_{jk} \frac{T^k}{k} \quad (12)$$

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*The fuel is assumed to be added as a liquid.

Seleznik and Gordon in Reference 2 tabulate values of the coefficients in this equation for a large number of elements and molecules dealt with in rocketry for two temperature ranges, 300° - 1000°K and 1000° - 5000°K. Their enthalpy base was an assigned value of zero at 298.15°K for their reference substances.*

Success in mathematically introducing cold rocket fuel into the secondary depends on obtaining suitable coefficients for this equation for a lower temperature range. It appears that this can be done.

The static enthalpy per unit mass of any flow is given by

$$h = \sum_j n_j h_j \tag{13}$$

where h_j is the static enthalpy per unit mass of molecular form j and is given by

$$h_j = \frac{H_j}{W_j} \tag{14}$$

so that

$$h = \sum_j \frac{n_j}{W_j} H_j \tag{15}$$

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*Heats of formation are automatically taken into account.

REDUCTION OF THE PROBLEM TO THAT OF SOLVING TWO SIMULTANEOUS EQUATIONS

Assuming that the wall pressure integral is known, the problem can be reduced to that of solving two simultaneous equations in two unknowns, velocity and temperature. Essentially the two equations are the energy and momentum equations.

Given the temperatures and velocities of primary, secondary, and fuel added, the right hand side of the energy equation is immediately calculable. h_p, for example, is given by

$$h_p = \sum_j n_{p_j} \frac{H_j}{W_j} = R_o \sum_j \frac{n_{p_j}}{W_j} \left[a_{o_j} + \sum_{k=1}^{5} a_{k_j} \frac{T^k}{k} \right] \qquad (16)$$

For the left-hand side of the equation, i.e. at the exit, the temperature is unknown. Here the enthalpy can be written

$$h_e = b_o + \sum_{k=1}^{5} b_k T^k \qquad (17)$$

where

$$b_o = \sum_j \frac{n_j}{W_j} a_{o_j}$$

and $\qquad (18)$

$$b_k = \sum_j \frac{n_j}{W_j} a_{k_j}$$

This is a fifth degree polynomial in the temperature with known coefficients. The energy equation then becomes a comparatively simple relation between temperature and velocity at the exit, namely,

$$\dot{\mu}_e \left[b_o + \sum_{k=1}^{5} b_k T_e{}^k + \frac{1}{2} u_e{}^2 \right] = \dot{\mu}_e h_{o_e} \tag{19}$$

where

h_{o_e} is total enthalpy per unit mass at the exit.

It is proposed to express the momentum equation in terms of the same two variables, using the remaining conditions where necessary. The mass flow rate equations, (4) and (5), can be written, using the perfect gas law, (1),

$$\dot{m} = \rho \; u \; A = \frac{p A W u}{R_o T} \tag{20}$$

and

$$\dot{\mu} = \eta \frac{W}{W_p} \frac{T_p}{T} \frac{u}{u_p} \tag{21}$$

where

$$\eta = \frac{p \; A}{p_p \; A_p} \tag{22}$$

The momentum equation, (9), can be written, using the above, as

$$\eta_e \left[1 + \frac{W_e \; u_e{}^2}{R_o \; T_e} \right] = 1 + \frac{W_p \; u_p{}^2}{R_o \; T_p} + \eta_s \left[1 + \frac{W_s \; u_s{}^2}{R_o \; T_s} \right] + \int \frac{p_w \; d A_x}{p_p \; A_p} + \frac{\dot{m}_f \; u_f}{p_p A_p} \tag{23}$$

(Here the ratio $\frac{W u^2}{R_o T}$ is equivalent to γM^2 for the calorically perfect gas. There appears to be no point here, however, in using such a conversion.) The right hand side of this equation, (23), contains only known functions, so one can write

$$\eta_e \left[1 + \frac{W_e \, u_e^2}{R_o \, T_e} \right] = C, \text{ say} \tag{24}$$

η_e can be eliminated using the known mass flow rate, $\dot{\mu}_e$, from the equation

$$\dot{\mu}_e = \eta_e \frac{W_e}{W_p} \frac{T_p}{T_e} \frac{u_e}{u_p} \tag{25}$$

so that Equation (24) can be written

$$\dot{\mu}_e \left[1 + \frac{W_e \, u_e^2}{R_o \, T_e} \right] = C \frac{W_e}{W_p} \frac{T_p}{T_e} \frac{u_e}{u_p} \tag{26}$$

On solving Equation (26) for u_e, one obtains

$$u_e = \frac{\frac{C}{\dot{\mu}_e} \frac{R_o}{W_p} \frac{T_p}{u_p} \pm \sqrt{\left(\frac{C}{\dot{\mu}_e} \frac{R_o}{W_p} \frac{T_p}{u_p} \right)^2 - 4 \frac{R_o \, T_e}{W_e}}}{2} \tag{27}$$

This is the desired form. The simultaneous solution of (27) and (19) by iteration yields both u_e and T_e. η_e can then be calculated from Equation (25) and $\frac{p_e}{p_p}$ can be obtained from (22). Thus the problem in principle is solved.

DISCUSSION OF RESULTS

Note that, as in the case of the corresponding equation for calorically perfect fluids, there are two roots of Equation (27) for each value of T_e. One of these must correspond to supersonic, the other to subsonic, exit flow.

This equation could easily be expressed in non-dimensional form by dividing by u_p, for example. However, in view of the fact that each separate problem will have its own specific solution, such a procedure could be questioned. It can be justified, at least when one is comparing solutions for calorically perfect and imperfect fluids.

Note also that total temperature and total pressure do not appear and, in fact, have no apparent significance. Much more significant is total enthalpy which is calculated as part of the procedure.

At least two separate but related studies of calculated results could be made. One would be the effect on the flow parameters of wall pressure distribution (thrust) at constant exit composition; and the other would be the effect on the flow parameters of varying exit composition at constant thrust. The latter might be useful in estimating the sensitivity of the flow parameters for a constant area duct to changes in chemical composition.

REFERENCES

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