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FORTRAN IV COMPUTER PROGRAM FOR THE EVALUATION OF NATURAL FREQUENCIES AND UNSTABLE VALUES OF THE THRUST FREQUENCY FOR A CIRCULAR CYLINDRICAL SHELL SUBJECTED TO A GIMBALED PERIODICALLY VARYING END THRUST

Research & Analysis Section Tech. Memo. #100

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1.0 INTRODUCTION

This report presents a FORTRAN IV Computer Program that operationally is compatible with the FORTRAN processors of the IBM 7094 and the UNIVAC 1107 digital computers. Additional details regarding the individual processors, actual machine compilation and object program execution, and so forth, are available in separate programming and operations reference manuals.

Specifically, this report presents a FORTRAN IV Computer Program that evaluates the natural frequencies of a thin cylindrical shell with simply-supported edge conditions [Eq. (47)] (all equation numbers in square brackets in this report are those appearing in ref. (1)) and then evaluates unstable values of the thrust frequency for given [Eq. (56)] initial conditions.

The Section 2.0 STABILITY CONSIDERATIONS-is presented in this report to facilitate the reader's understanding of the equations that are solved by the computer program (main program and subroutines) which is discussed at some length in the Section 3.0-FORTRAN IV COMPUTER PROGRAM. The reader may refer to APPENDIX A. - GENERAL INFORMATION ON THE SOLUTION OF THE STABILITY EQUATIONS - for mathematical details on the method of iteration used in the FORTRAN IV Computer Program.

2.0 STABILITY CONSIDERATIONS

2.1 Development of Equations

Since this report is to be regarded as a companion report to the report, "Response of A Circular Cylindrical Shell to a Gimbaled Periodically Varying End Thrust", Technical Memo. #38, Research and Analysis Section, Northrop Space Laboratories, Huntsville, Alabama, November 20, 1964, all equations presented in this report will have the numbers of those same equations in Tech. Memo. #38, shown in square brackets.

As presented in Tech. Memo. #38, the Matheau equation is

$$\frac{d^2\alpha}{d\tau_1} + (a - 2q \cos 2\tau_1)\alpha = 0 \qquad (1), \quad [16]$$

where

$$a = \frac{8\pi r L T_o K}{I\Omega^2} \quad , \quad q = \frac{\gamma a}{2} \qquad (2), \quad [15]$$

with ranges

$$0<a<1 \quad , \qquad 0<q\leq 0.05 \qquad (3), \quad [17]$$

Without detailed derivation (see p. 40 of ref. 2) it is shown that the

$$\sin^2 \frac{\beta\pi}{2} = \Delta(0) \sin^2 \frac{\pi a^{\frac{1}{2}}}{2} \qquad (4), \quad [21]$$

where for small values of q

$$\Delta(0) = 1 + \frac{\pi q^2}{4a^{\frac{1}{2}}(1-a)} \cot \frac{\pi a^{\frac{1}{2}}}{2} \qquad (5), \quad [22]$$

which, after substituting $q = \gamma a/2$ and simplifying becomes

$$\Delta(0) = 1 + \frac{\pi a^{3/2} \gamma^2}{16(1-a)} \cot \frac{\pi a^{\frac{1}{2}}}{2} \qquad (6) \quad \text{[not listed]}$$

The expression for β is written

$$\beta = \frac{2}{\pi} \left[\sin^{-1} \left\{ [\Delta(0)]^{\frac{1}{2}} \sin \frac{\pi a^{\frac{1}{2}}}{2} \right\} \right] \qquad (7) \quad \text{[not listed]}$$

which shows that β is a function of Ω by virtue of equation (6) and equation (2) in this report. The transcendental stability equations will now be presented.

2.2 Presentation of Stability Equations

2.2.1 Subroutine UNST1 solves Equation [82]

$$\overbrace{[(m-2)^2 + \epsilon_m \beta]}^{\textcircled{i}} \frac{\Omega}{\omega} = \omega_{0k}$$

where

$$\epsilon_m = \begin{cases} -\frac{1}{2} & \text{if } m=1 \\ \frac{1}{2} & \text{if } m=2 \\ \frac{1}{2} & \text{if } m=3 \end{cases}$$

$k = 1, 2, \ldots N$

$m = 1, 2, 3$

m	ⓘ	k	ω_{0k}
1	$1 - \frac{\beta}{2}$	1	ω_{01}
2	$\frac{\beta}{2}$	2	ω_{02}
3	$1 + \frac{\beta}{2}$	3	ω_{03}
		.	.
		.	.
		.	.
		N	ω_{0N}

2.2.2 Subroutine UNST2 solves Equation [83]

$$\frac{\Omega}{\omega} = \omega_{j0}^{i}$$

where

$i = 1, 2$

$j = 1, 2, \ldots, M$

i	j	ω_{j0}^{i}
1	1	ω_{10}^{1}
2	1	ω_{10}^{2}
1	2	ω_{20}^{1}
2	2	ω_{20}^{2}
1	3	ω_{30}^{1}
2	3	ω_{30}^{2}
.	.	.
.	.	.
.	.	.
.	.	.
1	M	ω_{M0}^{1}
2	M	ω_{M0}^{2}

2.2.3 Subroutine UNST3 solves Equation [84]

$$\overbrace{\left|(m-2)^2 \cdot (n-2)^2 + (\varepsilon_m-\varepsilon_n)\beta\right|}^{②} \frac{\Omega}{\omega} = \omega^{i}_{j0}$$

where

$i = 1, 2$

$j = 1, 2, \ldots M$

$m = 1, 2, 3$

$n = 1, 2, 3$ $\quad m \neq n$

$$\varepsilon_m = \begin{cases} -\frac{1}{2} & \text{if } m=1 \\ \frac{1}{2} & \text{if } m=2 \\ \frac{1}{2} & \text{if } m=3 \end{cases} \qquad \varepsilon_n = \begin{cases} -\frac{1}{2} & \text{if } n=1 \\ \frac{1}{2} & \text{if } n=2 \\ \frac{1}{2} & \text{if } n=3 \end{cases}$$

i	j	ω^{i}_{j0}
1	1	ω^{1}_{10}
2	1	ω^{2}_{10}
1	2	ω^{1}_{20}
2	2	ω^{2}_{20}
1	3	ω^{1}_{30}
2	3	ω^{2}_{30}
.	.	.
.	.	.
.	.	.
.	.	.
1	M	ω^{1}_{M0}
2	M	ω^{2}_{M0}

m	n	②
1	1	not admissible
2	1	$-1 + \beta$
3	1	β
1	2	$1 - \beta$
2	2	not admissible
3	2	1
1	3	$-\beta$
2	3	-1
3	3	not admissible

2.2.4 Subroutine UNST4 solves Equation [85]

$$\overbrace{\left|(m-2)^2 + (n-2)^2 + (\varepsilon_m+\varepsilon_n)\beta\right|}^{③} \frac{\Omega}{\omega} = \omega_{j0}^{i}$$

where

$i = 1, 2$

$j = 1, 2, \ldots M$

$m = 1, 2, 3$

$n = 1, 2, 3 \quad n \neq m$

$$\varepsilon_m = \begin{cases} -\frac{1}{2} & \text{if } m=1 \\ \frac{1}{2} & \text{if } m=2 \\ \frac{1}{2} & \text{if } m=3 \end{cases} \qquad \varepsilon_n = \begin{cases} -\frac{1}{2} & \text{if } n=1 \\ \frac{1}{2} & \text{if } n=2 \\ \frac{1}{2} & \text{if } n=3 \end{cases}$$

i	j	ω_{j0}^{i}
1	1	ω_{10}^{1}
2	1	ω_{10}^{2}
1	2	ω_{20}^{1}
2	2	ω_{20}^{2}
1	3	ω_{30}^{1}
2	3	ω_{30}^{2}
.	.	.
.	.	.
.	.	.
1	M	ω_{M0}^{1}
2	M	ω_{M0}^{2}

m	n	③
1	1	not admissible
2	1	1
3	1	2
1	2	1
2	2	not admissible
3	2	$1 + \beta$
1	3	2
2	3	$1 + \beta$
3	3	not admissible

2.2.5 Subroutine UNST5 solves Equation [86]

$$\underbrace{2[(m-2)^2 + \epsilon_m \beta]}_{④} \frac{\Omega}{\omega} = \omega_{j0}^{i}$$

where

$i = 1, 2$

$j = 1, 2, \ldots M$

$m = 1, 2, 3$

$$\epsilon_m = \begin{cases} -\frac{1}{2} & \text{if } m=1 \\ \frac{1}{2} & \text{if } m=2 \\ \frac{1}{2} & \text{if } m=3 \end{cases}$$

i	j	ω_{j0}^{i}
1	1	ω_{10}^{1}
2	1	ω_{10}^{2}
1	2	ω_{20}^{1}
2	2	ω_{20}^{2}
1	3	ω_{30}^{1}
2	3	ω_{30}^{2}
•	•	•
•	•	•
•	•	•
•	•	•
1	M	ω_{M0}^{1}
2	M	ω_{M0}^{2}

m	④
1	$2 - \beta$
2	β
3	$2 + \beta$

2.2.6 Subroutine UNST6 solves Equation [87]

$$\underbrace{[(m-2)^2 + \epsilon_m \beta]\frac{\Omega}{\omega}}_{⑤} = \omega_{jk}^{i}$$

where

$i = 1, 2, 3$

$j = 1, 2, \ldots M$

$k = 1, 2, \ldots N$

$m = 1, 2, 3$

$$\epsilon_m = \begin{cases} -\frac{1}{2} & \text{if } m=1 \\ \frac{1}{2} & \text{if } m=2 \\ \frac{1}{2} & \text{if } m=3 \end{cases}$$

i	j	ω_{jk}^{i}	k	m	⑤
1	1	ω_{1k}^{1}	$k = 1, 2, \ldots N$	1	$1 - \frac{1}{2}\beta$
2	1	ω_{1k}^{2}		2	$\frac{1}{2}\beta$
3	1	ω_{1k}^{3}		3	$1 + \frac{1}{2}\beta$
1	2	ω_{2k}^{1}			
2	2	ω_{2k}^{2}			
3	2	ω_{2k}^{3}			
1	3	ω_{3k}^{1}			
2	3	ω_{3k}^{2}			
3	3	ω_{3k}^{3}			
.	.	.			
.	.	.			
.	.	.			
1	M	ω_{Mk}^{1}			
2	M	ω_{Mk}^{2}			
3	M	ω_{Mk}^{3}	$k = 1, 2, \ldots N$		

2.2.7 Subroutine UNST7 solves Equation [88]

$$\overbrace{\left|(m-2)^2 - 1 + \varepsilon_m \beta\right|}^{⑥} \frac{\Omega}{\omega} = \omega_{jk}^{i}$$

where

$i = 1, 2, 3$

$j = 1, 2, \ldots M$

$k = 1, 2, \ldots N$

$m = 1, 2, 3$

$$\varepsilon_m = \begin{cases} -\frac{1}{2} & \text{if } m=1 \\ \frac{1}{2} & \text{if } m=2 \\ \frac{1}{2} & \text{if } m=3 \end{cases}$$

i	j	ω_{jk}^{i}	k	m	⑥
1	1	ω_{1k}^{1}	$k = 1, 2, \ldots N$	1	$-\frac{1}{2}\beta$
2	1	ω_{1k}^{2}	.	2	$-1 + \frac{1}{2}\beta$
3	1	ω_{1k}^{3}	.	3	$\frac{1}{2}\beta$
1	2	ω_{2k}^{1}	.		
2	2	ω_{2k}^{2}	.		
3	2	ω_{2k}^{3}	.		
1	3	ω_{3k}^{1}	.		
2	3	ω_{3k}^{2}	.		
3	3	ω_{3k}^{3}	.		
.	.	.	.		
.	.	.	.		
.	.	.	.		
1	M	ω_{Mk}^{1}	.		
2	M	ω_{Mk}^{2}	.		
3	M	ω_{Mk}^{3}	$k = 1, 2, \ldots N$		

2.2.8 Subroutine UNST8 solves Equation [89]

$$\overbrace{\left|(m-2)^2 + 1 + \epsilon_m \beta\right|}^{⑦} \frac{\Omega}{\omega} = \omega^{i}_{jk}$$

where

$i = 1, 2, 3$

$j = 1, 2, \ldots M$

$k = 1, 2, \ldots N$

$m = 1, 2, 3$

$$\epsilon_m = \begin{cases} -\frac{1}{2} & \text{if } m=1 \\ \frac{1}{2} & \text{if } m=2 \\ \frac{1}{2} & \text{if } m=3 \end{cases}$$

i	j	ω^{i}_{jk}	k	m	⑦
1	1	ω^{1}_{1k}	$k = 1, 2, \ldots, N$	1	$2 - \frac{1}{2}\beta$
2	1	ω^{2}_{1k}	•	2	$1 + \frac{1}{2}\beta$
3	1	ω^{3}_{1k}	•	3	$2 + \frac{1}{2}\beta$
1	2	ω^{1}_{2k}	•		
2	2	ω^{2}_{2k}	•		
3	2	ω^{3}_{2k}	•		
1	3	ω^{1}_{3k}	•		
2	3	ω^{2}_{3k}	•		
3	3	ω^{3}_{3k}	•		
•	•	•	•		
•	•	•	•		
•	•	•	•		
1	M	ω^{1}_{Mk}	•		
2	M	ω^{2}_{MK}	•		
3	M	ω^{3}_{MK}	$k = 1, 2, \ldots, N$		

2.2.9 Subroutine UNST9 solves Equation [90]

$$\overbrace{\left|(m-2)^2 \cdot (n-2)^2 + (\epsilon_m - \epsilon_n)\beta\right|}^{⑧} \frac{\Omega}{\omega} = \omega_{jk}^{i}$$

where

$i = 1,2,3$

$j = 1, 2, \ldots, M$

$k = 1,2, \ldots, N$

$m = 1,2,3$

$n = 1,2,3 \quad m \neq n$

$$\epsilon_m = \begin{cases} -\frac{1}{2} & \text{if } m=1 \\ \frac{1}{2} & \text{if } m=2 \\ \frac{1}{2} & \text{if } m=3 \end{cases} \qquad \epsilon_n = \begin{cases} -\frac{1}{2} & \text{if } n=1 \\ \frac{1}{2} & \text{if } n=2 \\ \frac{1}{2} & \text{if } n=3 \end{cases}$$

i	j	ω_{jk}^{i}	k	m	n	⑧
1	1	ω_{1k}^{1}	$k = 1,2,\ldots,N$	1	1	not admissible
2	1	ω_{1k}^{2}	•	2	1	$-1 + \beta$
3	1	ω_{1k}^{3}	•	3	1	β
1	2	ω_{2k}^{1}	•	1	2	$1 - \beta$
2	2	ω_{2k}^{2}	•	2	2	not admissible
3	2	ω_{2k}^{3}	•	3	2	1
1	3	ω_{3k}^{1}	•	1	3	$-\beta$
2	3	ω_{3k}^{2}	•	2	3	$-\beta$
3	3	ω_{3k}^{3}	•	3	3	not admissible
•	•	•	•			
•	•	•	•			
•	•	•	•			
1	M	ω_{Mk}^{1}	•			
2	M	ω_{Mk}^{2}	•			
3	M	ω_{Mk}^{3}	$k = 1,2,\ldots,N$			

2.2.10 Subroutine UNST10 solves Equation [91]

$$\underbrace{\left|(m-2)^2 + (n-2)^2 + (\epsilon_m+\epsilon_n)\beta\right|\frac{\Omega}{\omega} = \omega^{i}_{jk}}_{⑨}$$

where

$i = 1,2,3$
$j = 1,2,\ldots M$
$k = 1,2,\ldots N$
$m = 1,2,3$
$n = 1,2,3$ $\quad m \neq n$

$$\epsilon_m = \begin{cases} -\frac{1}{2} & \text{if } m=1 \\ \frac{1}{2} & \text{if } m=2 \\ \frac{1}{2} & \text{if } m=3 \end{cases} \qquad \epsilon_n = \begin{cases} -\frac{1}{2} & \text{if } n=1 \\ \frac{1}{2} & \text{if } n=2 \\ \frac{1}{2} & \text{if } n=3 \end{cases}$$

i	j	ω^{i}_{jk}	k	m	n	⑨
1	1	ω^{1}_{1k}	$k = 1,2,\ldots,N$	1	1	not admissible
2	1	ω^{2}_{1k}	.	2	12	1
3	1	ω^{3}_{1k}	.	3	1	2
1	2	ω^{1}_{2k}	.	1	2	1
2	2	ω^{2}_{2k}	.	2	2	not admissible
3	2	ω^{3}_{2k}	.	3	2	$1+\beta$
1	3	ω^{1}_{3k}	.	1	3	2
2	3	ω^{2}_{3k}	.	2	3	$1+\beta$
3	3	ω^{3}_{3k}	.	3	3	not admissible
.	.	.	.			
.	.	.	.			
.	.	.	.			
1	M	ω^{1}_{Mk}	.			
2	M	ω^{2}_{Mk}	.			
3	M	ω^{3}_{Mk}	$k = 1,2,\ldots,N$			

2.2.11 Subroutine UNST11 solves Equation [92]

$$\overbrace{2[(m-2)^2 + \varepsilon_m \beta]}^{⑩} \frac{\Omega}{\omega} = \omega_{jk}^{i}$$

where

$i = 1,2,3$

$j = 1,2,\ldots,M$

$k = 1,2,\ldots,N$

$m = 1,2,3$

and

$$\varepsilon_m = \begin{cases} -\frac{1}{2} & \text{if } m=1 \\ \frac{1}{2} & \text{if } m=2 \\ \frac{1}{2} & \text{if } m=3 \end{cases}$$

i	j	ω_{jk}^{i}	k	m	⑩
1	1	ω_{1k}^{1}	$k = 1,2,\ldots,N$	1	$2 - \beta$
2	1	ω_{1k}^{2}	•	2	β
3	1	ω_{1k}^{3}	•	3	$2 + \beta$
1	2	ω_{2k}^{1}	•		
2	2	ω_{2k}^{2}	•		
3	2	ω_{2k}^{3}	•		
1	3	ω_{3k}^{1}	•		
2	3	ω_{3k}^{2}	•		
3	3	ω_{3k}^{3}	•		
•	•	•	•		
•	•	•	•		
•	•	•	•		
1	M	ω_{Mk}^{1}	•		
2	M	ω_{Mk}^{2}	•		
3	M	ω_{Mk}^{3}	$k = 1,2,\ldots,N$		

2.3 Limiting Cases

2.3.1 Limiting case of β for $\Delta(0) = 1$.

Referring to equation (7) in this report, the expression for β is presented again

$$\beta = \frac{2}{\pi}\left\{\sin^{-1}\left[[\Delta(0)]^{\frac{1}{2}}\sin\frac{\pi a^{\frac{1}{2}}}{2}\right]\right\} \tag{7}$$

For the limiting case of $\Delta(0) = 1$, the equation becomes

$$\beta = a^{\frac{1}{2}} = \frac{2}{\Omega}\left(\frac{2\pi r H T_e K}{I}\right)^{\frac{1}{2}} \tag{8}$$

2.3.2 Limiting Cases of the Parameter a

Referring to equation (6) in this report, the expression for $\Delta(0)$ is presented again

$$\Delta(0) = 1 + \frac{\pi a^{3/2}\gamma^2}{16(1-a)}\cot\frac{\pi a^{\frac{1}{2}}}{2} \tag{6}$$

It can be shown, by the application of L'Hospital's rule on limits, that the following limits exist

$$\lim_{a \to 0}\left(\frac{\pi a^{3/2}\gamma^2}{16(1-a)}\cot\frac{\pi a^{\frac{1}{2}}}{2}\right) = 0 \tag{9}$$

$$\lim_{a \to 1}\left(\frac{\pi a^{5/2}\gamma^2}{16(1-a)}\cot\frac{\pi a^{\frac{1}{2}}}{2}\right) = \frac{\pi^2\gamma^2}{64} \tag{10}$$

As shown by the results of equations (9) and (10) the following value and/or expression for $\Delta(0)$ exists

$$\Delta(0) \to 1 \quad \text{as} \quad a \to 0$$

$$\Delta(0) \to 1 + \frac{\pi^2 \gamma^2}{16} \quad \text{as} \quad a \to 1$$

A small table of values of $\Delta(0)$ for selected values of γ is presented below

$\Delta(0)$ as $a \to 0$	$\Delta(0)$ as $a \to 1$	γ	γ^2
1	1.0000154	0.01	1.00×10^{-4}
1	1.0003850	0.05	2.50×10^{-3}
1	1.0015400	0.10	1.00×10^{-2}
1	1.0096300	0.25	6.25×10^{-2}
1	1.0385000	0.50	25.0×10^{-2}

3.0 FORTRAN IV COMPUTER PROGRAM

3.1 General

The following FORTRAN IV Computer Program has been developed to calculate the natural frequencies and the unstable values of thrust frequency from data consisting of the parameters, H, r, t, ρ, E, ν, ω, m, n, T_o, γ, Ω, $\Delta\Omega$ and K. The manner in which these data words are read into the computer is presented under FORTRAN IV COMPUTER PROGRAM INPUT AND OUTPUT INFORMATION of this report.

The computer program consists of the following parts:

3.1.1 Main Program - FORTRAN IV Computer Program for the Evaluation of Natural Frequencies (OMG) and Unstable Values of the Thrust Frequency (COMG) for a Simply-Supported, Circular Cylindrical Shell.

3.1.2 Subroutine NAFREQ - Subroutine for the Evaluation of Natural Frequencies (OMG). Subroutine calculates 10-values of OMG(0, K, I), 20-values of OMG(J, 0, I) and 300-values of OMG(J, K, I).

3.1.3 Subroutine UNST1 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation [82]. Consideration of three (3) cases, involving 30 values.

3.1.4 Subroutine UNST2 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation [83]. Consideration of one (1) case, involving 20 values.

3.1.5 Subroutine UNST3 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation [84]. Consideration of nine (9) cases (3-inadmissible), involving 120 values.

3.1.6 Subroutine UNST4 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation [85]. Consideration of nine (9) cases (3 inadmissible), involving 120 values.

3.1.7 Subroutine UNST5 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation [86]. Consideration of three (3) cases, involving 60 values.

3.1.8 Subroutine UNST6 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation [87]. Consideration of three (3) cases, involving 900 values.

3.1.9 Subroutine UNST7 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation [88]. Consideration of three (3) cases, involving 900 values.

3.1.10 Subroutine UNST8 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation [89]. Consideration of three (3) cases, involving 900 values.

3.1.11 Subroutine UNST9 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation [90]. Consideration of nine (9) cases (3 inadmissible), involving 1800 values.

3.1.12 Subroutine UNST10 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation [91]. Consideration of nine (9) cases (3 inadmissible), involving 1800 values.

3.1.13 Subroutine UNST11 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation [92]. Consideration of three (3) cases, involving 900 values.

3.2 FORTRAN IV Legend for Input Data

The FORTRAN IV designation for input data is as follows:

Input Parameter	FORTRAN IV Name
H, half length	H
r, radius	R
t, thickness	T
ρ, mass density	RHO
E, modulus	E
ν, Poisson's ratio	V
ω, test frequency	W
m, axial variation term	M
n, radial variation term	N
T_o, constant thrust	TO
γ, ratio, variable to constant thrust	GAM
Ω, thrust frequency	COMG
K, control constant	CK

Full details on the manner in which these data are read into the computer is presented under FORTRAN IV Computer Program Input and Output Information.

3.3 FORTRAN IV Legend for Computed Variables

Due to the restrictions on variable names in the FORTRAN IV language, many of the computed variables were reassigned. The following table gives a complete list of computed variables and their corresponding FORTRAN IV designation.

ω_{jk}^{i}, natural frequency	OMG(J,K,I)
Ω_{jk}^{i}, thrust frequency	COMG(J,K,I)
a_{jk}^{i}	A(J,K,I)
b_{jk}^{i}	B(J,K,I)
$[\Delta(0)]_{jk}^{i}$	DELO(J,K,I)
c_{jk}^{i}	C(J,K,I)
d_{jk}^{i}	D(J,K,I)
f_{jk}^{i}	F(J,K,I)
β_{jk}^{i}	BETA(J,K,I)
$(\beta/2)_{jk}^{i}$	BETA12(J,K,I)
$f_1(1)$, $f_1(2)$	F1(L)
X_1, X_2	X(L)
d_1	D1
τ_1	C1
τ_2	C2

Range on subscripts: $j = 0, 1, \ldots, M$
$k = 0, 1, \ldots, N$
$i = 1, 2, 3$

3.4 FORTRAN IV Computer Program Input and Output Information

3.4.1 Input

The input data which must be punched on cards are the terms H, R, T, RHO, E, V, W, M, N, TO, GAM, CMG, CK. These data are punched on two cards.

The first data card will contain H in spaces 1 through 10, R in spaces 11 through 20, T in spaces 21 through 30, RHO in spaces 31 through 40, E in spaces 41 through 50, V in spaces 51 through 60, and W in spaces 61 through 70.

The second data card will contain M in spaces 1 through 5, N in spaces 6 through 10, TO in spaces 11 through 20, GAM in spaces 21 through 30, COMG in spaces 31 through 40, DCOMG in spaces 41 through 50 and CK in spaces 51 through 60.

The READ STATEMENTS for the first and second data card will be as follows:

```
2    READ(5,102) H, R, T, RHO, E, V, W
102  FORMAT(7F10.5)

3    READ(5, 103) M, N, TO, GAM, COMG, CK
103  FORMAT (215, 4E10.5)
```

A definition of input data is given under FORTRAN IV Legend for Input Data.

3.4.2 Output

The values of natural frequencies ω_{jk}^{i} [OMG(J,K,I)] are printed in floating point numbers except the values of J and K which are integer values.

The unstable values of the thrust frequency Ω_{jk}^{i} [COMG(J,K,I)] are printed in floating point numbers except the values of J and K which are integer values. A descriptive title, properly identifying the equation number and the particular case, is printed before each section of the program printout.

3.5 FORTRAN IV Source Program

3.5.1 General Statement

The program which defines the operations which the computer is to do and which is written by the programmer in the FORTRAN IV language is called the FORTRAN IV Source Program. With this in mind, and with reference to the FORTRAN IV Legend for Computed Variables presented earlier in this report, the reader should have little difficulty in understanding the following Program Listing.

3.5.2 Program Listing

```
C
C     MAIN PROGRAM
      COMMON L,S,U,V,W,PI,M,N,OMG(20,20,3),R,H,TO,CK,RI,GAM
C     NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C     FORTRAN IV COMPUTER PROGRAM FOR THE EVALUATION OF NATURAL
C     FREQUENCIES (OMG) AND UNSTABLE VALUES OF THE THRUST FREQUENCY
C     (COMG) FOR A SIMPLY-SUPPORTED, CIRCULAR CYLINDRICAL SHELL.
      REAL L
1     WRITE (6,100)
100   FORMAT(1H1,20X,31HLEGEND OF TERMS USED IN PROGRAM,//,5X,30HH ----
     1HALF LENGTH OF CYLINDER,/,5X,25HR ---- RADIUS OF CYLINDER,/,5X,33H
     2T       THICKNESS OF CYLINDER WALL,/,5X,40HRHO -- MASS DENSITY OF C
     3YLINDER MATERIAL,/,5X,38HE ---- YOUNGS MODULUS FOR THE MATERIAL,/,
     45X,21HV ---- POISSONS RATIO,/,5X,26HW ---- REFERENCE FREQUENCY,/,5
     55X,38HM ---- NUMBER OF AXIAL VARIATION TERMS,/,5X,39HN ---- NUMBER
     6OF RADIAL VARIATION TERMS,/,5X,30HJ,K -- SUBSCRIPTS OF THE ARRAY,/
     7,5X,34HOMG(J,K,1) --- NATURAL FREQUENCY 1,/,5X,34HOMG(J,K,2) --- N
     8ATURAL FREQUENCY 2,/,5X,34HOMG(J,K,3) --- NATURAL FREQUENCY 3,/,5X
     9,28HTO --- CONSTANT THRUST FORCE,/,5X,38HGAM -- VARIABLE THRUST/CO
     1NSTANT THRUST,/,5X,48HCOMG -- CAPITAL OMEGA, INSTABLE SEARCH FREQU
     2ENCY,/,5X,39HDCOMG -- DELTA CAPITAL OMEGA, INCREMENT,/,5X,20HCK--C
     3ONTROL CONSTANT)
2     READ (5,102) H,R,T,RHO,E,V,W
102   FORMAT (7F10.5)
3     READ (5,103) M,N,TO,GAM,COMG,DCOMG,CK
103   FORMAT (2I5,5E10.5)
4     WRITE (6,104) H,R,T,RHO,W,E,V,M,N,TO,GAM,COMG,DCOMG,CK
104   FORMAT(1H1,10X,62HUNSTABLE VALUES OF THE THRUST FREQUENCY(COMG) FO
     1R THE CYLINDER,//,3X,14HHALF LENGTH = ,3PE15.5,10X,9HRADIUS = ,3PE
     215.5,10X,12HTHICKNESS = ,1PE15.5,//,3X,15HMASS DENSITY = ,E15.8,9X
     3,22HREFERENCE FREQUENCY = ,E15.5,//,3X,17HYOUNGS MODULUS = ,1PE15.
     45,7X,17HPOISSONS RATIO = ,E15.5,//,3X,34HNUMBER OF AXIAL VARIATION
     5 TERMS = ,I5,10X,35HNUMBER OF RADIAL VARIATION TERMS = ,I5,//,3X,2
     62HCONSTANT THRUST, TO = ,1PE15.5,3X,6HGAM = ,E15.5,3X,21HCAPITAL O
     7MEGA,COMG = ,E15.5,//,3X,28HDELTA CAPITAL OMEGA,DCOMG = ,E15.5,3X,
     824HCONTROL CONSTANT K,CK = ,E15.5,//)
      PI=3.141592654
      L=H/R
      S=T/H
      U=RHO*H*H*W*W/E
      CALL NAFREQ (L,S,U,V,W,PI,M,N,OMG)
5     WRITE (6,105)
105   FORMAT (20X,41HTABLE OF NATURAL FREQUENCIES, RAD PER SEC,//,4X,2HJ
     1,,4X,2HK,,8X,9HOMEGA (1),11X,9HOMEGA (2),10X,9HOMEGA (3),//)
106   FORMAT (2I5,3E20.8//)
      DO 6 J=1,M
      DO 6 K=1,N
      J1=J-1
      K1=K-1
6     WRITE (6,106) J1,K1,(OMG(J,K,I),I=1,3)
      RM=2.*H*2.*PI*R*T*RHO
      RI=RM*(.5*R*R+.333*H*H)*386.
      CALL UNST1 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST2 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST3 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST4 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST5 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST6 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST7 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST8 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST9 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
```

```
      CALL UNST10(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST11(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      GO TO 2
      END
```

```
C     FOR NAFREQ
C     NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C     SUBROUTINE  FOR THE EVALUATION OF NATURAL FREQUENCIES(OMG).
C     SUBROUTINE CALCULATES 10 VALUES OF OMG(J,K,I), 20 VALUES OF
C     OMG(J,0,I), AND 300 VALUES OF OMG(J,K,I)
      SUBROUTINE NAFREQ (L,S,U,V,W,PI,M,N,OMG)
      DIMENSION A(3,3),ROOTR(3),ROOTI(3),OMG(20,20,3)
      REAL L
      U1=U*(1.-V*V)
      DO 1 J=1,M
      DO 1 K=1,N
      DO 1 I=1,3
1     OMG(J,K,I)=0.0
      DO 2 J=1,M
      DO 2 K=1,N
      J1=J-1
      K1=K-1
      FJ=J1
      FK=K1
      IF(J-1)3,3,4
3     IF(K-1)2,2,5
5     OMG(J,K,1)=L*FK*W/SQRT(2.*U*(1.+V))
      GO TO 2
4     IF(K-1)6,6,7
6     A(1,1)=.25*(PI*FJ)**2/U1
      A(1,2)=.5*V*L*FJ*PI/U1
      A(2,1)=A(1,2)
      A(2,2)=(L*L+S*S*(FJ*PI)**4/192.)/U1
      CALL HESSEN(A,2)
      CALL GREIG(A,2,ROOTR,ROOTI,0)
      DO 8 I=1,2
      IF(ROOTI(I))9,10,9
10    IF(ROOTR(I))9,11,11
9     WRITE (6,100)J,K,I,ROOTR(I),ROOTI(I)
100   FORMAT(23HOROOT IS NOT ACCEPTABLE 3I5,2E15.8)
      GO TO 8
11    OMG(J,K,I)=W*SQRT(ABS(ROOTR(I)))
8     CONTINUE
      GO TO 2
7     A(1,1)=.25*((PI*FJ)**2+2.*(1.-V)*(L*FK)**2)/U1
      A(1,2)=-.25*FJ*FK*PI*L*(1.+V)/U1
      A(1,3)=.5*V*L*PI*FJ/U1
      A(2,1)=A(1,2)
      A(2,2)=.125*((1.-V)*(FJ*PI)**2+8.*(L*FK)**2)/U1
      A(2,3) = -1.*L*L*FK/U1
      A(3,1)=A(1,3)
      A(3,2)=A(2,3)
      A(3,3)=(12.*L*L+S*S*((.5*FJ*PI)**2+L*L*FK*FK)**2)/(12.*U1)
      CALL HESSEN(A,3)
      CALL GREIG(A,3,ROOTR,ROOTI,0)
      DO 12 I=1,3
      IF(ROOTI(I))13,14,13
14    IF(ROOTR(I))13,15,15
13    WRITE (6,101)J,K,I,ROOTR(I),ROOTI(I)
101   FORMAT(23HOROOT IS NOT ACCEPTABLE,3I5,2E15.8)
      GO TO 12
15    OMG(J,K,I)=W*SQRT(ABS(ROOTR(I)))
12    CONTINUE
2     CONTINUE
      RETURN
```

END

```
C FOR HESSEN                                                        HESS0002
C     SUBROUTINE TO PUT MATRIX IN UPPER HESSENBERG FORM.            HESS0003
      SUBROUTINE HESSEN(A,M)                                        HESS0004
      DIMENSION A(3,3), B(2)                                        HESS0005
      DOUBLE PRECISION SUM                                          HESS0006
      IF (M - 2) 30,30,32                                           HESS0007
   32 DO 40 LC = 3,M                                                HESS0008
      N = M - LC + 3                                                HESS0009
      N1 = N - 1                                                    HESS0010
      N2 = N - 2                                                    HESS0011
      NI = N1                                                       HESS0012
      DIV = ABS(A(N,N-1))                                           HESS0013
      DO 2 J = 1,N2                                                 HESS0014
      IF(ABS(A(N,J))- DIV) 2,2,1                                    HESS0015
    1 NI = J                                                        HESS0016
      DIV = ABS(A(N,J))                                             HESS0017
    2 CONTINUE                                                      HESS0018
      IF(DIV) 3,40,3                                                HESS0019
    3 IF(NI - N1) 4, 7,4                                            HESS0020
    4 DO 5 J = 1,N                                                  HESS0021
      DIV = A(J,N1)                                                 HESS0022
      A(J,N1) = A(J,NI)                                             HESS0023
    5 A(J,NI) = DIV                                                 HESS0024
      DO 6 J = 1,M                                                  HESS0025
      DIV = A(NI,J)                                                 HESS0026
      A(N1,J) = A(N1,J)                                             HESS0027
    6 A(N1,J) = DIV                                                 HESS0028
    7 DO 26 K = 1, N1                                               HESS0029
   26 B(K) = A(N,K)/A(N,N-1)                                        HESS0030
      DO 45 J = 1,M                                                 HESS0031
      SUM = 0.0                                                     HESS0032
      IF (J - N1) 46,43,43                                          HESS0033
   46 IF(A(N,J))  41,43,41                                          HESS0034
   41 A(N,J) = 0.0                                                  HESS0035
      DO 42 K = 1,N1                                                HESS0036
      A(K,J) = A(K,J) - A(K,N1)*B(J)                                HESS0037
   42 SUM = SUM + A(K,J)*B(K)                                       HESS0038
      GO TO 45                                                      HESS0039
   43 DO 44 K = 1,N1                                                HESS0040
   44 SUM = SUM + A(K,J)*B(K)                                       HESS0041
   45 A(N1,J) = SUM                                                 HESS0042
   40 CONTINUE                                                      HESS0043
   30 RETURN                                                        HESS0044
      END
```

```
C     FOR GRELG
C        PROGRAM TO CALL GR TRANSFORMATION, MAXIMUM ITER IS 50.
      SUBROUTINE GRELG(A,M,ROOTR,ROOTI,IPRNT)
      DIMENSION A(3,3),ROOTR(3),ROOTI(3)
      N = M
      IF(IPRNT)  80,81,80
   80 WRITE (6,104)
   81 ZERO = 0.0
      JJ=1
  177 XMN=0.0
      XLZ=0.0
      AA = 0.0
      B = 0.0
      C = 0.0
      DD = 0.0
      R=0.0
      SIG=0.0
      ITER = 0
   17 IF(N-2)     15,14,12
   13 IF(IPRNT)  82,83,82
   82 WRITE (6,105)A(1,1)
   83 ROOTR(1) = A(1,1)
      ROOTI(1) = 0.0
    1 RETURN
   14 JJ=-1
   12 X = (A(N-1,N-1) - A(N,N))**2
      S = 4.0*A(N,N-1)*A(N-1,N)
      ITER = ITER + 1
      IF(ABS(S/X).GT. 1.0E-8) GO TO 15
   16 IF(ABS(A(N-1,N-1))-ABS(A(N,N)))  32,32,31
   31 E = A(N-1,N-1)
      G = A(N,N)
      GO TO 33
   32 G = A(N-1,N-1)
      E = A(N,N)
   33 F = 0.
      H = 0.
      GO TO 24
   15 S = X + S
      X = A(N-1,N-1) + A(N,N)
      IF(S)     18,19,19
   19 SQ=SQRT(S)
      F=0.0
      H=0.0
      IF (X) 21,21,22
   21 E=(X-SQ)/2.0
      G=(X+SQ)/2.0
      GO TO 24
   22 G=(X-SQ)/2.0
      E=(X+SQ)/2.0
      GO TO 24
   18 F = SQRT(-S)/2.0
      E=X/2.0
      G=E
      H=-F
   24 IF(JJ)  28,70,70
   70 D = 1.0E-10*(ABS(G) + F)
      IF(ABS(A(N-1,N-2)) .GT. D)  GO TO 26
   28 IF(IPRNT)  84,85,84
   84 WRITE (6,105)E,F, ITER
```

```
      WRITE (6,105)C,H                                                  GRCN0061
85    ROOTR(N) = E                                                      GRCN0062
      ROOTI(N) = F                                                      GRCN0063
      ROOTR(N-1) = G                                                    GRCN0064
      ROOTI(N-1) = H                                                    GRCN0065
      N=N-2                                                             GRCN0066
      IF(JJ)  1,177,177                                                 GRCN0067
   26 IF(ABS(A(N,N-1)) .GT. 1.0E-10*ABS(A(N,N)))  GO TO 50              GRCN0068
29    IF(IPRNT)  86,87,86                                               GRCN0069
86    WRITE (6,105)A(N,N), ZERO, IIER                                   GRCN0070
87    ROOTR(N) = A(N,N)                                                 GRCN0071
      ROOTI(N) = 0.0                                                    GRCN0072
      N=N-1                                                             GRCN0073
      GO TO 177                                                         GRCN0074
50    IF(ABS(ABS(XNN/A(N,N-1))-1.0)-1.0E-6)  63,63,62                   GRCN0075
62    IF(ABS(ABS(XN2/A(N-1,N-2))-1.0)-1.0E-6)  63,63,700                GRCN0076
63    VG=ABS(A(N,N-1))-ABS(A(N-1,N-2))                                  GRCN0077
64    IF(VG)   67,67,66                                                 GRCN0078
66    IF(IPRNT)  88,85,88                                               GRCN0079
88    WRITE (6,107)A(N-1,N-2)                                           GRCN0080
      GO TO 84                                                          GRCN0081
67    IF(IPRNT)  89,87,89                                               GRCN0082
89    WRITE (6,107)A(N,N-1)                                             GRCN0083
      GO TO 86                                                          GRCN0084
 700  IF(IIER .GT. 50) GO TO 63                                         GRCN0085
 701  Z1=          ((E-AA)**2+(F-B)**2)/(E*E+F*F)                       GRCN0086
      Z2=       ((G-C)**2+(H-DD)**2)/(G*G+H*H)                       GRCN0087
      IF(Z1-0.25)  51,51,52                                             GRCN0088
51    IF(Z2-0.25)   53,53,54                                            GRCN0089
53    R=E*G-F*H                                                         GRCN0090
      SIG=E+G                                                           GRCN0091
      GO TO 60                                                          GRCN0092
54    R=E*E                                                             GRCN0093
      SIG=E+E                                                           GRCN0094
      GO TO 60                                                          GRCN0095
52    IF(Z2-0.25) 55,55,601                                             GRCN0096
55    R=G*G                                                             GRCN0097
      SIG=G+G                                                           GRCN0098
      GO TO 60                                                          GRCN0099
 601  R = 0.0                                                           GRCN0100
      SIG = 0.0                                                         GRCN0101
60    XNN=A(N,N-1)                                                      GRCN0102
      XN2=A(N-1,N-2)                                                    GRCN0103
      CALL GRT(A,N,R,SIG,D)                                             GRCN0104
      AA=E                                                              GRCN0105
      B=F                                                               GRCN0106
      C=G                                                               GRCN0107
      DD=H                                                              GRCN0108
      GO TO 12                                                          GRCN0109
104   FORMAT(////1X, 9HREAL PART  6X  14HIMAGINARY PART, 26X            GRCN0110
     1  13HTAKEN AS ZERO 6X 4HITER //)                                  GRCN0111
105   FORMAT(1X,E15.8,3X,E15.8, 42X I3)                                 GRCN0112
107   FORMAT(56X  E13.8)                                                GRCN0113
      END                                                               GRCN0114
```

```
C FOR GRT
      SUBROUTINE GRT(A,N,R,SIG,D)
      DIMENSION A(3,3), PSI(3), G(3)
      N1 = N - 1
      IA = N - 2
      IP = IA
      IF(N-3) 101,10,60
60    DO 12 J = 3,N1
      J1 = N - J
      IF(ABS(A(J1+1,J1))-D)     10,10,11
11    IF(ABS(A(J1+1,J1)*A(J1+2,J1+1)*(ABS(A(J1+1,J1+1)+A(J1+2,J1+2)      GRT 0011
     1-SIG)+ABS(A(J1+3,J1+2)))/(A(J1+1,J1+1)*(A(J1+1,J1+1)-SIG)+A(      GRT 0012
     21+1,J1+2)*A(J1+2,J1+1)+R))-D)      10,10,12
12    IP=J1
10    DO  14  J=1,IP
      J1=IP-J+1
      IF(ABS(A(J1+1,J1))-D)     13,13,14
14    IG=J1
13    DO  100  I=IP,N1
      IF(I-IP)    16,15,16
15    G(1)=A(IP,IP)*(A(IP,IP)-SIG)+A(IP,IP+1)*A(IP+1,IP)+R                GRT 0021
      G(2)=A(IP+1,IP)*(A(IP,IP)+A(IP+1,IP+1)-SIG)                         GRT 0022
      G(3)=A(IP+1,IP)*A(IP+2,IP+1)
      A(IP+2,IP)=0.0
      GO TO 19
16    G(1)=A(I,I-1)
      G(2)=A(I+1,I-1)
      IF(I-IA)    17,17,18
17    G(3)=A(I+2,I-1)
      GO TO 19
18    G(3)=0.0
19    XK = SIGN(SQRT(G(1)**2 + G(2)**2 + G(3)**2), G(1))                  GRT 0032
22    IF(XK)    23,24,23
23    AL=G(1)/XK+1.0
      PSI(1)=G(2)/(G(1)+XK)
      PSI(2)=G(3)/(G(1)+XK)
      GO TO 25
24    AL=2.0
      PSI(1)=0.0
      PSI(2)=0.0
25    IF(I-IG)    26,27,26
26    IF(I-IP)    29,28,29
28    A(I,I-1)=-A(I,I-1)
      GO TO 27
29    A(I,I-1)=-XK
27    DO 30  J=I,N
      IF(I-IA)  31,31,32
31    C=PSI(2)*A(I+2,J)
      GO TO 33
32    C=0.0
33    E=AL*(A(I,J)+PSI(1)*A(I+1,J)+C)
      A(I,J)=A(I,J)-E
      A(I+1,J)=A(I+1,J)-PSI(1)*E
      IF(I-IA)    34,34,30
34    A(I+2,J)=A(I+2,J)-PSI(2)*E
30    CONTINUE
      IF(I-IA)    35,35,36
35    L=I+2
      GO TO  37
36    L=N
```

```
37    DO  40  J=IQ,L
      IF(I-IA)  38,38,39
38    C=PSI(2)*A(J,I+2)
      GO TO  41
39    C=0.0
41    E=AL*(A(J,I)+PSI(1)*A(J,I+1)+C)
      A(J,I)=A(J,I)-E
      A(J,I+1)=A(J,I+1)-PSI(1)*E
      IF(I-IA)  42,42,40
42    A(J,I+2)=A(J,I+2)-PSI(2)*E
40    CONTINUE
      IF(I-N+3)  43,43,100
43    E=AL*PSI(2)*A(I+3,I+2)
      A(I+3,I)=-E
      A(I+3,I+1)=-PSI(1)*E
      A(I+3,I+2)=A(I+3,I+2)-PSI(2)*E
100   CONTINUE
101   RETURN
      END
```

```
C1 FOR UNST1
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (82).
C       CONSIDERATION OF THREE (3) CASES, INVOLVING 30 VALUES.
        SUBROUTINE UNST1(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
       1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),BETA12(20,20
       2,3),F1(4),X(4)
        DO 1 J=1,M
        DO 1 K=1,N
        DO 1 I=1,3
        COMG(J,K,I)=0.0
1       CONTINUE
101     FORMAT (20X,60HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
       1R EQUATION (82), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
       2//)
102     FORMAT (2I5, 1E20.8//)
103     FORMAT (20X,33HFIRST CASE, LOWER CASE M EQUAL 1.,//)
104     FORMAT (20X,34HSECOND CASE, LOWER CASE M EQUAL 2.,//)
105     FORMAT (20X,33HTHIRD CASE, LOWER CASE M EQUAL 3.,//)
        DO 2 J=1,1
        DO 2 K=1,N
        DO 2 I=1,1
        START=1.2
        START1=1.
55      X(1)=START*(OMG(J,K,I))
        DO 3 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)30,31,30
30      C2=2.**28
        IF(B(J,K,I)-C2)300,301,301
31      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 32
300     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1))))))/TAN(B(J,K,I))
32      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)33,33,34
34      START=START-.1
        IF(START)325,325,35
325     COMG(J,K,I)=.33333333
        GO TO 2001
33      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        BETA12(J,K,I)=.5*(BETA(J,K,I))
        F1(L)=(OMG(J,K,I))/(1.-BETA12(J,K,I))
3       X(L+1)=F1(L)
4       X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)36,37,36
36      C2=2.**28
        IF(B(J,K,I)-C2)360,401,401
37      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 38
360     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1))))))/TAN(B(J,K,I))
38      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
```

```
      IF(ABS(D(J,K,I))-1.)39,39,6
39    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(1.-BETA12(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,395
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)5,5,6
6     X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)4,4,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,1)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
501   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
5     COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
2     CONTINUE
      WRITE(6,101)
      WRITE(6,103)
      DO 7 J=1,1
      DO 7 K=1,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,1),I=1,1)
7     CONTINUE
      DO 8 J=1,1
      DO 8 K=1,N
      DO 8 I=1,1
      START=1.2
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**20
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
```

```
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START-.1
      IF(START)425,425,45
425   COMG(J,K,I)=.33333333
      GO TO 2002
43    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(BETA12(J,K,I))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(BETA12(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)10,10,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
502   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
```

```
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
6     CONTINUE
      WRITE(6,101)
      WRITE(6,104)
      DO 13 J=1,1
      DO 13 K=1,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,1)
13    CONTINUE
      DO 14 J=1,1
      DO 14 K=1,N
      DO 14 I=1,1
      START=.1
      START1=1.
55    X(1)=START*(OMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
50    C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
51    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
500   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
52    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
54    START=START+.1
      IF(START-1.0)55,55,525
525   COMG(J,K,I)=.33333333
      GO TO 2003
53    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(1.+BETA12(J,K,I))
15    X(L+1)=F1(L)
16    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
56    C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
57    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
560   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
58    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,1))-1.)59,59,18
59    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(1.+BETA12(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
590   IF(ABS(X(3))-100.)592,593,593
592   C1=.001
```

```
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
      IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)16,16,203
203   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
503   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.66666666
2003  CONTINUE
14    CONTINUE
      WRITE(6,101)
      WRITE(6,105)
      DO 19 J=1,1
      DO 19 K=1,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,1)
19    CONTINUE
      RETURN
      END
```

```
C
C  FOR UNST2
C      NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C      SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (83).
C      CONSIDERATION OF ONE (1) CASE, INVOLVING 20 VALUES.
       SUBROUTINE UNST2(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
       DIMENSION OMG(20,20,3),COMG(20,20,3)
101    FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
      1R EQUATION (83), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
      210X,10HCOMEGA (2),//)
102    FORMAT (2I5, 2E20.8//)
       DO 1 J=1,M
       DO 1 K=1,N
       DO 1 I=1,3
       COMG(J,K,I)=0.0
1      CONTINUE
       DO 2 J=2,M
       DO 2 K=1,1
       DO 2 I=1,2
       COMG(J,K,I)=OMG(J,K,I)
2      CONTINUE
       WRITE (6,101)
       DO 3 J=2,M
       DO 3 K=1,1
       J1=J-1
       K1=K-1
       WRITE (6,102) J1,K1,(COMG(J,K,1),I=1,2)
3      CONTINUE
       RETURN
       END
```

```
$I FOR UNST3
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (84).
C       CONSIDERATION OF 9 CASES (3 INADMISSIBLE), INVOLVING 120 VALUES.
        SUBROUTINE UNST3(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
       1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),BETA12(20,20
       2,3),F1(4),X(4)
101     FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
       1R EQUATION (84), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
       210X,10HCOMEGA (2),//)
102     FORMAT (2I5, 2E20.8//)
103     FORMAT (10X,78HFIRST CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQUA
       1L 1, (CASE NOT ADMISSIBLE).,///)
104     FORMAT (10X,56HSECOND CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
       1AL 1,//)
105     FORMAT (10X,55HTHIRD CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
       1L 1,//)
106     FORMAT (10X,56HFOURTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQU
       1AL 2,//)
107     FORMAT (10X,78HFIFTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQUA
       1L 2, (CASE NOT ADMISSIBLE).,///)
108     FORMAT (10X,55HSIXTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
       1L 2,//)
109     FORMAT (10X,57HSEVENTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQ
       1UAL 3,//)
110     FORMAT (10X,56HEIGHTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
       1AL 3,//)
111     FORMAT (10X,78HNINTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
       1L 3, (CASE NOT ADMISSIBLE).,///)
        DO 1 J=1,M
        DO 1 K=1,N
        DO 1 I=1,3
        COMG(J,K,I)=0.0
1       CONTINUE
        WRITE (6,101)
        WRITE (6,103)
        DO 2 J=2,M
        DO 2 K=1,1
        DO 2 I=1,2
        START=.1
        START1=1.
35      X(1)=START*(OMG(J,K,I))
        DO 3 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)30,31,30
30      C2=2.**28
        IF(B(J,K,I)-C2)300,301,301
31      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 32
300     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
32      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)33,33,34
34      START=START+.1
        IF(START-1.0)35,35,325
325     COMG(J,K,I)=.33333333
        GO TO 2001
```

```
33      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(L)=(OMG(J,K,I))/(ABS(-1.+BETA(J,K,I)))
3       X(L+1)=F1(L)
4       X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)36,37,36
36      C2=2.**28
        IF(B(J,K,I)-C2)360,401,401
37      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 38
360     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
38      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)39,39,6
39      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(3)=(OMG(J,K,I))/(ABS(-1.+BETA(J,K,I)))
        D1=X(3)-F1(3)
        IF(ABS(X(3))-1000.)390,394,394
390     IF(ABS(X(3))-100.)392,393,393
392     C1=.001
        GO TO 395
393     C1=.01
        GO TO 395
394     C1=.1
395     IF(ABS(D1)-C1)5,5,6
6       X(1)=X(2)
        F1(1)=F1(2)
        X(2)=X(3)
        F1(2)=F1(3)
        START1=START1+1.
        IF(START1-100.)4,4,201
201     A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        IF(A(J,K,I)-1.0)330,501,501
330     B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
        C(J,K,I)=SQRT(B(J,K,I))
        C3=.01
        IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331     COMG(J,K,I)=.88888888
        GO TO 2001
332     COMG(J,K,I)=.11111111
        GO TO 2001
301     COMG(J,K,I)=.77777777
        GO TO 2001
401     COMG(J,K,I)=.99999999
        GO TO 2001
5       COMG(J,K,I)=ABS(X(3))
        IF(A(J,K,I)-1.)2001,501,501
501     COMG(J,K,I)=.66666666
2001    CONTINUE
2       CONTINUE
        WRITE(6,101)
        WRITE(6,104)
        DO 7 J=2,M
        DO 7 K=1,1
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
```

```
7       CONTINUE
        DO 8 J=2,M
        DO 8 K=1,1
        DO 8 I=1,2
        START=.6
        START1=1.
45      X(1)=START*(OMG(J,K,I))
        DO 9 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)40,41,40
40      C2=2.**28
        IF(B(J,K,I)-C2)400,302,302
41      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 42
400     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
42      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)43,43,44
44      START=START-.1
        IF(START)425,425,45
425     COMG(J,K,I)=.33333333
        GO TO 2002
43      F(J,K,I)=PI-ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(L)=(OMG(J,K,I))/(ABS(BETA(J,K,I)))
9       X(L+1)=F1(L)
10      X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)46,47,46
46      C2=2.**28
        IF(B(J,K,I)-C2)460,402,402
47      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 48
460     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
48      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)49,49,12
49      F(J,K,I)=PI-ASIN(D(J,K,1))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(3)=(OMG(J,K,I))/(ABS(BETA(J,K,I)))
        D1=X(3)-F1(3)
        IF(ABS(X(3))-1000.)490,494,494
490     IF(ABS(X(3))-100.)492,493,493
492     C1=.001
        GO TO 495
493     C1=.01
        GO TO 495
494     C1=.1
495     IF(ABS(D1)-C1)11,11,12
12      X(1)=X(2)
        F1(1)=F1(2)
        X(2)=X(3)
        F1(2)=F1(3)
        START1=START1+1.
        IF(START1-1000.)10,10,202
202     A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        IF(A(J,K,I)-1.0)430,502,502
```

```
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
302   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
8     CONTINUE
      WRITE(6,101)
      WRITE(6,105)
      DO 13 J=2,M
      DO 13 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=1,1
      DO 14 I=1,2
      START=.1
      START1=1.
55    X(1)=START*(COMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
50    C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
51    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
500   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
52    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
54    START=START+.1
      IF(START-1.0)55,55,525
525   COMG(J,K,I)=.33333333
      GO TO 2003
53    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(1.-BETA(J,K,I)))
15    X(L+1)=F1(L)
16    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
56    C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
57    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
560   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
```

```
     1)))))/TAN(B(J,K,I))
58    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,18
59    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(1.-BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
590   IF(ABS(X(3))-100.)592,593,593
592   C1=.001
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)16,16,203
203   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
303   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.06666666
2003  CONTINUE
14    CONTINUE
      WRITE(6,101)
      WRITE(6,106)
      DO 19 J=2,M
      DO 19 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
19    CONTINUE
      WRITE (6,101)
      WRITE (6,107)
      WRITE (6,101)
      WRITE (6,108)
      DO 20 J=2,M
      DO 20 K=1,1
      DO 20 I=1,2
      COMG(J,K,I)=OMG(J,K,I)
20    CONTINUE
      DO 21 J=2,M
      DO 21 K=1,1
      J1=J-1
```

```
      K1=K-1
      WRITE (6,102) J1,K1,(OMG(J,K,I),I=1,2)
21    CONTINUE
      DO 22 J=2,M
      DO 22 K=1,1
      DO 22 I=1,2
      START=.6
      START1=1.
65    X(1)=START*(OMG(J,K,I))
      DO 23 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)60,61,60
60    C2=2.**28
      IF(A(J,K,I)-C2)600,304,304
61    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 62
600   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
62    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)63,63,64
64    START=START-.1
      IF(START)625,625,65
625   OMG(J,K,I)=.33333333
      GO TO 2004
63    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(-1.*(BETA(J,K,I))))
23    X(L+1)=F1(L)
24    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)66,67,66
66    C2=2.**28
      IF(A(J,K,I)-C2)660,404,404
67    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 68
660   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
68    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)69,69,26
69    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(-1.*(BETA(J,K,I))))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)690,694,694
690   IF(ABS(X(3))-100.)692,693,693
692   C1=.001
      GO TO 695
693   C1=.01
      GO TO 695
694   C1=.1
695   IF(ABS(D1)-C1)25,25,26
26    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)24,24,204
```

```
204     A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        IF(A(J,K,I)-1.0)630,504,504
630     B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
        C(J,K,I)=SQRT(B(J,K,I))
        C3=.01
        IF(ABS(C(J,K,I)-X(3))-C3)631,631,632
631     COMG(J,K,I)=.88888888
        GO TO 2004
632     COMG(J,K,I)=.11111111
        GO TO 2004
304     COMG(J,K,I)=.77777777
        GO TO 2004
404     COMG(J,K,I)=.99999999
        GO TO 2004
25      COMG(J,K,I)=ABS(X(3))
        IF(A(J,K,I)-1.)2004,504,504
504     COMG(J,K,I)=.66666666
2004    CONTINUE
22      CONTINUE
        WRITE(6,101)
        WRITE(6,109)
        DO 27 J=2,M
        DO 27 K=1,1
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
27      CONTINUE
        DO 28 J=2,M
        DO 28 K=1,1
        DO 28 I=1,2
        COMG(J,K,I)=OMG(J,K,I)
28      CONTINUE
        WRITE (6,101)
        WRITE (6,110)
        DO 29 J=2,M
        DO 29 K=1,1
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
29      CONTINUE
        WRITE (6,101)
        WRITE (6,111)
        RETURN
        END
```

```
      SUBROUTINE UNST4(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
C     FOR UNST4
C     NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C     SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (85).
C     CONSIDERATION OF 9 CASES (3 INADMISSIBLE), INVOLVING 120 VALUES.
      DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
     1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),BETA12(20,20
     2,3),F1(4),X(4)
101   FORMAT (20X,60HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
     1R EQUATION (85), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
     210X,10HCOMEGA (2),//)
102   FORMAT (2I5, 2E20.8//)
103   FORMAT (10X,73HFIRST CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQUA
     1L 1, (CASE NOT ADMISSIBLE).,///)
104   FORMAT (10X,56HSECOND CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
     1AL 1,//)
105   FORMAT (10X,55HTHIRD CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
     1L 1,//)
106   FORMAT (10X,56HFOURTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQU
     1AL 2,//)
107   FORMAT (10X,73HFIFTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQUA
     1L 2, (CASE NOT ADMISSIBLE).,///)
108   FORMAT (10X,55HSIXTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
     1L 2,//)
109   FORMAT (10X,57HSEVENTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQ
     1UAL 3,//)
110   FORMAT (10X,56HEIGHTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
     1AL 3,//)
111   FORMAT (10X,73HNINTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
     1L 3, (CASE NOT ADMISSIBLE).,///)
      DO 1 J=1,M
      DO 1 K=1,N
      DO 1 I=1,3
      COMG(J,K,I)=0.0
1     CONTINUE
      DO 2 J=2,M
      DO 2 K=1,1
      DO 2 I=1,2
      COMG(J,K,I)=OMG(J,K,I)
2     CONTINUE
      WRITE (6,101)
      WRITE (6,105)
      WRITE (6,101)
      WRITE (6,104)
      DO 3 J=2,M
      DO 3 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
3     CONTINUE
      DO 4 J=2,M
      DO 4 K=1,1
      DO 4 I=1,2
      COMG(J,K,I)=.5*(OMG(J,K,I))
4     CONTINUE
      WRITE (6,101)
      WRITE(6,105)
      DO 5 J=2,M
      DO 5 K=1,1
      J1=J-1
```

```
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
5     CONTINUE
      DO 6 J=2,M
      DO 6 K=1,1
      DO 6 I=1,2
      COMG(J,K,I)=OMG(J,K,I)
6     CONTINUE
      WRITE (6,101)
      WRITE (6,106)
      DO 7 J=2,M
      DO 7 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
7     CONTINUE
      WRITE (6,101)
      WRITE (6,107)
      DO 8 J=2,M
      DO 8 K=1,1
      DO 8 I=1,2
      START=.1
      START1=1.
35    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)30,31,30
30    C2=2.**28
      IF(B(J,K,I)-C2)300,301,301
31    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 32
300   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
32    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)33,33,34
34    START=START+.1
      IF(START-1.0)35,35,325
325   COMG(J,K,I)=.33333333
      GO TO 2001
33    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)36,37,36
36    C2=2.**28
      IF(B(J,K,I)-C2)360,401,401
37    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 38
360   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
38    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)39,39,12
39    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
```

```
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)10,10,201
201   A(J,K,I)=(8.*PI*R*H*T0*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*T0*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,1)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
501   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.06666666
2001  CONTINUE
6     CONTINUE
      WRITE(6,101)
      WRITE(6,108)
      DO 13 J=2,M
      DO 13 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=1,1
      DO 14 I=1,2
       COMG(J,K,I)=.5*(COMG(J,K,I))
14    CONTINUE
      WRITE(6,101)
      WRITE(6,109)
      DO 15 J=2,M
      DO 15 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
15    CONTINUE
      DO 16 J=2,M
      DO 16 K=1,1
      DO 16 I=1,2
      START2=.1
      START1=1.
45    X(1)=START*(COMG(J,K,I))
```

```
      DO 17 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
      C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START+.1
      IF(START-1.0)45,45,425
425   COMG(J,K,I)=.33333333
      GO TO 2002
45    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
17    X(L+1)=F1(L)
18    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,20
49    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,495
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)19,19,20
20    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)18,18,202
202   A(J,K,I)=(8.*PI*R*H*TO*(K)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*(K)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
```

```
      GO TO 2002
302   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
19    COMG(J,K,I)=ABS(X(J))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.06666666
2002  CONTINUE
16    CONTINUE
      WRITE (6,101)
      WRITE (6,110)
      DO 21 J=2,M
      DO 21 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
21    CONTINUE
      WRITE (6,101)
      WRITE (6,111)
      RETURN
      END
```

```
*
*1 FOR UNSTS
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (86).
C       CONSIDERATION OF THREE (3) CASES, INVOLVING 60 VALUES.
        SUBROUTINE UNSTS (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
       1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),F1(4),X(4)
101     FORMAT (20X,60HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
       1R EQUATION (86), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
       210X,10HCOMEGA (2),//)
102     FORMAT (2I5, 2E20.8//)
103     FORMAT (20X,33HFIRST CASE, LOWER CASE M EQUAL 1.,//)
104     FORMAT (20X,34HSECOND CASE, LOWER CASE M EQUAL 2.,//)
105     FORMAT (20X,33HTHIRD CASE, LOWER CASE M EQUAL 3.,//)
        DO 1 J=1,M
        DO 1 K=1,N
        DO 1 I=1,3
        COMG(J,K,I)=0.0
1       CONTINUE
        DO 2 J=2,M
        DO 2 K=1,1
        DO 2 I=1,2
        START=.1
        START1=1.
35      X(1)=START*(OMG(J,K,I))
        DO 3 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)30,31,30
30      C2=2.**28
        IF(B(J,K,I)-C2)300,301,301
31      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 32
300     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
32      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)33,33,34
34      START=START+.1
        IF(START-1.0)35,35,325
325     COMG(J,K,I)=.33333333
        GO TO 2001
33      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(L)=(OMG(J,K,I))/(2.-BETA(J,K,I))
3       X(L+1)=F1(L)
4       X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)36,37,36
36      C2=2.**28
        IF(B(J,K,I)-C2)360,401,401
37      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 38
360     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
38      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)39,39,6
39      F(J,K,I)=ASIN(D(J,K,I))
```

```
      F1(3)=(OMG(J,K,I))/(2.-ETA(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)5,5,6
6     X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)4,4,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
301   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
5     COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
2     CONTINUE
      WRITE(6,101)
      WRITE (6,103)
      DO 7 J=2,M
      DO 7 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
7     CONTINUE
      DO 8 J=2,M
      DO 8 K=1,1
      DO 8 I=1,2
      START=.6
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
```

```
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START-.1
      IF(START)425,425,45
425   COMG(J,K,I)=.33333333
      GO TO 2002
45    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(BETA(J,K,I))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(A(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(BETA(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,495
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)10,10,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
302   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
8     CONTINUE
      WRITE (6,101)
      WRITE (6,104)
```

```
      DO 13 J=2,M
      DO 13 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=1,1
      DO 14 I=1,2
      START=.1
      START1=1.
55    X(1)=START*(OMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
50    C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
51    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
500   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
52    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
54    START=START+.1
      IF(START-1.0)55,55,525
525   COMG(J,K,I)=.33333333
      GO TO 2003
53    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(2.+BETA(J,K,I))
15    X(L+1)=F1(L)
16    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
56    C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
57    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
560   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
58    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,18
59    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(2.+BETA(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
590   IF(ABS(X(3))-100.)592,593,593
592   C1=.001
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
```

```
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)16,16,2003
2003  A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*TO*(K)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
503   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.06666666
2003  CONTINUE
14    CONTINUE
      WRITE (6,101)
      WRITE (6,105)
      DO 19 J=2,M
      DO 19 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
19    CONTINUE
      RETURN
      END
```

```
C1 FOR UNSTO
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (87).
C       CONSIDERATION OF THREE (3) CASES, INVOLVING 900 VALUES.
        SUBROUTINE UNSTO (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
       1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),BETA12(20,20
       2,3),F1(4),X(4)
101     FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
       1R EQUATION (87), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),,
       210X,10HCOMEGA (2),10X,10HCOMEGA (3),//)
102     FORMAT (2I5, 3E20.8//)
103     FORMAT (20X,33HFIRST CASE, LOWER CASE M EQUAL 1.,//)
104     FORMAT (20X,34HSECOND CASE, LOWER CASE M EQUAL 2.,//)
105     FORMAT (20X,33HTHIRD CASE, LOWER CASE M EQUAL 3.,//)
        DO 1 J=1,M
        DO 1 K=1,N
        DO 1 I=1,3
        COMG(J,K,I)=0.0
1       CONTINUE
        DO 2 J=2,M
        DO 2 K=2,N
        DO 2 I=1,3
        START=.1
        START1=1.
35      X(1)=START*(OMG(J,K,I))
        DO 3 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)30,31,30
30      C2=2.**28
        IF(B(J,K,I)-C2)300,301,301
31      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 32
300     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
32      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)33,33,34
34      START=START+.1
        IF(START-1.0)35,35,325
325     COMG(J,K,I)=.33333333
        GO TO 2001
33      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        BETA12(J,K,I)=.5*(BETA(J,K,I))
        F1(L)=(OMG(J,K,I))/(1.-BETA12(J,K,I))
3       X(L+1)=F1(L)
4       X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)36,37,36
36      C2=2.**28
        IF(B(J,K,I)-C2)360,401,401
37      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 38
360     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
38      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
```

```
      IF(ABS(D(J,K,I))-1.)39,39,6
39    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(1.-BETA12(J,K,I))
      D1=X(J)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)5,5,6
6     X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)4,4,201
201   A(J,K,I)=(8.*PI*R*H*TO*(K)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*(K)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
301   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
5     COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
2     CONTINUE
      WRITE(6,101)
      WRITE(6,103)
      DO 7 J=2,M
      DO 7 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
7     CONTINUE
      DO 8 J=2,M
      DO 8 K=2,N
      DO 8 I=1,3
      START=1.
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*(K)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(D(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
```

```
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START-.1
      IF(START)425,425,45
425   COMG(J,K,I)=.33333333
      GO TO 2002
43    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(BETA12(J,K,I))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(BETA12(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)10,10,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*(K)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
502   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
```

```
      IF(A(J,K,1)-1.)2002,502,502
502   COMG(J,K,1)=.66666666
2002  CONTINUE
6     CONTINUE
      WRITE(6,101)
      WRITE(6,104)
      DO 13 J=2,M
      DO 13 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=2,N
      DO 14 I=1,3
      START=.1
      START1=1.
55    X(1)=START*(COMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
50    C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
51    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
500   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
52    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
54    START=START+.1
      IF(START-1.0)55,55,525
525   COMG(J,K,I)=.33333333
      GO TO 2003
53    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(1.+BETA12(J,K,I))
15    X(L+1)=F1(L)
16    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
56    C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
57    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
560   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
58    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,18
59    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(1.+BETA12(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
590   IF(ABS(X(3))-100.)592,593,593
592   C1=.001
```

```
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)16,16,203
203   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
503   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.06666666
2003  CONTINUE
14    CONTINUE
      WRITE(6,101)
      WRITE(6,105)
      DO 19 J=2,M
      DO 19 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
19    CONTINUE
      RETURN
      END
```

```
MAIN FOR NAFREQ
      COMMON L,S,U,V,W,PI,M,N,OMG(20,20,3),R,H,TO,CK,DI,GAM
C     NASA CONTRACT NASA-11255 (LONGITUDINAL VIBRATION RESEARCH)
C     FORTRAN IV COMPUTER PROGRAM FOR THE EVALUATION OF NATURAL
C     FREQUENCIES (OMG) AND UNSTABLE VALUES OF THE THRUST FREQUENCY
C     (COMG) FOR A SIMPLY-SUPPORTED, CIRCULAR CYLINDRICAL SHELL.
      REAL L
1     WRITE (6,100)
100   FORMAT(1H1,20X,31HLEGEND OF TERMS USED IN PROGRAM,//,5X,30HH ----
     1HALF LENGTH OF CYLINDER,/,5X,25HR ---- RADIUS OF CYLINDER,/,5X,33H
     2T       THICKNESS OF CYLINDER WALL,/,5X,40HRHO -- MASS DENSITY OF C
     3YLINDER MATERIAL,/,5X,38HE ---- YOUNGS MODULUS FOR THE MATERIAL,/,
     45X,21HV ---- POISSONS RATIO,/,5X,26HW ---- REFERENCE FREQUENCY,/,5
     5X,38HM ---- NUMBER OF AXIAL VARIATION TERMS,/,5X,39HN ---- NUMBER
     6OF RADIAL VARIATION TERMS,/,5X,30HJ,K -- SUBSCRIPTS OF THE ARRAY,/
     7,5X,34HOMG(J,K,1) --- NATURAL FREQUENCY 1,/,5X,34HOMG(J,K,2) --- N
     8ATURAL FREQUENCY 2,/,5X,34HOMG(J,K,3) --- NATURAL FREQUENCY 3,/,5X
     9,28HTO --- CONSTANT THRUST FORCE,/,5X,38HGAM -- VARIABLE THRUST/CO
     1NSTANT THRUST,/,5X,48HCOMG -- CAPITAL OMEGA, INSTABLE SEARCH FREQU
     2ENCY,/,5X,39HDCOMG -- DELTA CAPITAL OMEGA, INCREMENT,/,5X,20HCK--C
     3ONTROL CONSTANT)
2     READ (5,102) H,R,T,RHO,E,V,W
102   FORMAT (7F10.5)
3     READ (5,103) M,N,TO,GAM,COMG,DCOMG,CK
103   FORMAT (2I5,5E10.5)
4     WRITE (6,104) H,R,T,RHO,W,E,V,M,N,TO,GAM,COMG,DCOMG,CK
104   FORMAT(1H1,10X,62HUNSTABLE VALUES OF THE THRUST FREQUENCY(COMG) FO
     1R THE CYLINDER,//,3X,14HHALF LENGTH = ,3PE15.5,10X,9HRADIUS = ,3PE
     215.5,10X,12HTHICKNESS = ,1PE15.5,//,3X,15HMASS DENSITY = ,E15.8,9X
     3,22HREFERENCE FREQUENCY = ,E15.5,//,3X,17HYOUNGS MODULUS = ,1PE15.
     45,7X,17HPOISSONS RATIO = ,E15.5,//,3X,34HNUMBER OF AXIAL VARIATION
     5 TERMS = ,I5,10X,35HNUMBER OF RADIAL VARIATION TERMS = ,I5,//,3X,2
     62HCONSTANT THRUST, TO = ,1PE15.5,3X,6HGAM = ,E15.5,3X,21HCAPITAL O
     7MEGA,COMG = ,E15.5,//,3X,28HDELTA CAPITAL OMEGA,DCOMG = ,E15.5,3X,
     824HCONTROL CONSTANT K,CK = ,E15.5,//)
      PI=3.141592654
      L=H/R
      S=T/H
      U=RHO*H*H*W*W/E
      CALL NAFREQ (L,S,U,V,W,PI,M,N,OMG)
5     WRITE (6,105)
105   FORMAT (20X,41HTABLE OF NATURAL FREQUENCIES, RAD PER SEC,//,4X,2HJ
     1,,4X,2HK,,8X,9HOMEGA (1),11X,9HOMEGA (2),10X,9HOMEGA (3),//)
106   FORMAT (2I5,3E20.8//)
      DO 6 J=1,M
      DO 6 K=1,N
      J1=J-1
      K1=K-1
6     WRITE (6,106) J1,K1,(OMG(J,K,I),I=1,3)
      RM=2.*H*2.*PI*R*T*RHO
      RI=RM*(.5*R*R+.333*H*H)*386.
      CALL CHAIN(2)
      GO TO 2
      END
```

```
*
$1 FOR UNST7
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (88).
C       CONSIDERATION OF THREE (3) CASES, INVOLVING 900 VALUES.
        SUBROUTINE UNST7 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
       1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),BETA12(20,20
       2,3),F1(4),X(4)
101    FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
       1R EQUATION (88), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HOMEGA (1),
       210X,10HOMEGA (2),10X,10HOMEGA (3),//)
102    FORMAT (2I5, 3E20.8//)
103    FORMAT (20X,33HFIRST CASE, LOWER CASE M EQUAL 1.,//)
104    FORMAT (20X,34HSECOND CASE, LOWER CASE M EQUAL 2.,//)
105    FORMAT (20X,33HTHIRD CASE, LOWER CASE M EQUAL 3.,//)
       DO 1 J=1,M
       DO 1 K=1,N
       DO 1 I=1,3
       COMG(J,K,I)=0.0
1      CONTINUE
       DO 2 J=2,M
       DO 2 K=2,N
       DO 2 I=1,3
       START=1.
       START1=1.
35     X(1)=START*(OMG(J,K,I))
       DO 3 L=1,2
       A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
       B(J,K,I)=.5*PI*SQRT(A(J,K,I))
       IF(A(J,K,I)-1.)30,31,30
30     C2=2.**28
       IF(B(J,K,I)-C2)300,301,301
31     DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
       GO TO 32
300    DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,1
       1)))))/TAN(B(J,K,1))
32     C(J,K,1)=SQRT(ABS(DELO(J,K,I)))
       D(J,K,I)=(C(J,K,1))*(SIN(B(J,K,I)))
       IF(ABS(D(J,K,I))-1.)33,33,34
34     START=START-.1
       IF(START)325,325,35
325    COMG(J,K,I)=.33333333
       GO TO 2001
33     F(J,K,I)=PI-ASIN(D(J,K,I))
       BETA(J,K,I)=(2./PI)*(F(J,K,1))
       BETA12(J,K,1)=.5*(BETA(J,K,I))
       F1(L)=(OMG(J,K,I))/(ABS(-1.*BETA12(J,K,I)))
3      X(L+1)=F1(L)
4      X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
       A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
       B(J,K,I)=.5*PI*SQRT(A(J,K,I))
       IF(A(J,K,I)-1.)36,37,36
36     C2=2.**28
       IF(B(J,K,I)-C2)360,401,401
37     DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
       GO TO 38
360    DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,1
       1)))))/TAN(B(J,K,1))
38     C(J,K,1)=SQRT(ABS(DELO(J,K,I)))
       D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
```

```
      IF(ABS(D(J,K,1))-1.)39,39,6
39    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(-1.*BETA12(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)5,5,6
6     X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)4,4,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,1)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
501   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
5     COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
2     CONTINUE
      WRITE(6,101)
      WRITE(6,103)
      DO 7 J=2,M
      DO 7 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
7     CONTINUE
      DO 8 J=2,M
      DO 8 K=2,N
      DO 8 I=1,3
      START=.1
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
```

```
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START+.1
      IF(START-1.0)45,45,425
425   OMG(J,K,I)=.33333333
      GO TO 2002
43    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(-1.+BETA12(J,K,I)))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)400,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(-1.+BETA12(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)10,10,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      CS=.01
      IF(ABS(C(J,K,I)-X(3))-C )431,431,432
431   OMG(J,K,I)=.38888888
      GO TO 2002
432   OMG(J,K,I)=.11111111
      GO TO 2002
302   OMG(J,K,I)=.77777777
      GO TO 2002
402   OMG(J,K,I)=.99999999
      GO TO 2002
11    OMG(J,K,I)=ABS(X(3))
```

```
      IF(A(J,K,I)-1.)2002,502,502
  502 COMG(J,K,I)=.66666666
 2002 CONTINUE
    8 CONTINUE
      WRITE (6,101)
      WRITE(6,104)
      DO 13 J=2,M
      DO 13 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
   13 CONTINUE
      DO 14 J=2,M
      DO 14 K=2,N
      DO 14 I=1,3
      START=1.
      START1=1.
   55 X(1)=START*(OMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
   50 C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
   51 DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
  500 DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,1
     1)))))/TAN(B(J,K,I))
   52 C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
   54 START=START-.1
      IF(START)525,525,55
  525 COMG(J,K,I)=.33333333
      GO TO 2003
   53 F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(BETA12(J,K,I)))
   15 X(L+1)=F1(L)
   16 X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
   56 C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
   57 DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
  560 DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,1
     1)))))/TAN(B(J,K,I))
   58 C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,18
   59 F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(BETA12(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
  590 IF(ABS(X(3))-100.)592,593,593
  592 C1=.001
```

```
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)16,16,203
203   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
503   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.66666666
2003  CONTINUE
14    CONTINUE
      WRITE(6,101)
      WRITE(6,105)
      DO 19 J=2,M
      DO 19 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
19    CONTINUE
      RETURN
      END
```

```
*
&I FOR UNST
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (89).
C       CONSIDERATION OF THREE (3) CASES, INVOLVING 900 VALUES.
        SUBROUTINE UNSTB (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
       1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),BETA12(20,20
       2,3),F1(4),X(4)
101     FORMAT (20X,60HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
       1R EQUATION (89), RAD PER SEC.,///,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
       210X,10HCOMEGA (2),10X,10HCOMEGA (3),//)
102     FORMAT (215, 3E20.8//)
103     FORMAT (20X,33HFIRST CASE, LOWER CASE M EQUAL 1.,//)
104     FORMAT (20X,34HSECOND CASE, LOWER CASE M EQUAL 2.,//)
105     FORMAT (20X,33HTHIRD CASE, LOWER CASE M EQUAL 3.,//)
        DO 1 J=1,M
        DO 1 K=1,N
        DO 1 I=1,3
        COMG(J,K,I)=0.0
1       CONTINUE
        DO 2 J=2,M
        DO 2 K=2,N
        DO 2 I=1,3
        START=.1
        START1=1.
35      X(1)=START*(OMG(J,K,I))
        DO 3 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)30,31,30
30      C2=2.**28
        IF(B(J,K,I)-C2)300,301,301
31      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 32
300     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
32      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)33,33,34
34      START=START+.1
        IF(START-1.0)35,35,325
325     COMG(J,K,I)=.33333333
        GO TO 2001
33      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        BETA12(J,K,I)=.5*(BETA(J,K,I))
        F1(L)=(OMG(J,K,I))/(ABS(2.-BETA12(J,K,I)))
3       X(L+1)=F1(L)
4       X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)36,37,36
36      C2=2.**28
        IF(B(J,K,I)-C2)300,401,401
37      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 38
300     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
38      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
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```
      IF(ABS(D(J,K,I))-1.)39,39,6
39    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(2.-BETA12(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,395
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)5,5,6
6     X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)4,4,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,1)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
301   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
5     COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
2     CONTINUE
      WRITE (6,101)
      WRITE (6,103)
      DO 7 J=2,M
      DO 7 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
7     CONTINUE
      DO 8 J=2,M
      DO 8 K=2,N
      DO 8 I=1,3
      START=.1
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
```

```
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
      1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START+.1
      IF(START-1.0)45,45,425
425   COMG(J,K,I)=.33333333
      GO TO 2002
43    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(1.+BETA12(J,K,I)))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
      1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(1.+BETA12(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)10,10,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,1)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
502   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
```

```
      IF(A(J,K,I)-1.)2002,502,502
  502 COMG(J,K,I)=.66666666
 2002 CONTINUE
    6 CONTINUE
      WRITE(6,101)
      WRITE(6,104)
      DO 13 J=2,M
      DO 13 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
   13 CONTINUE
      DO 14 J=2,M
      DO 14 K=2,N
      DO 14 I=1,3
      START=.1
      START1=1.
      START2=1.0
   55 X(1)=START*(OMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
   50 C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
   51 DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
  500 DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
   52 C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
   54 START=START+.1
      IF(START-1.0)55,55,525
  525 COMG(J,K,I)=.33333333
      GO TO 2003
   53 F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(2.+BETA12(J,K,I)))
   15 X(L+1)=F1(L)
   16 X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
   56 C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
   57 DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
  560 DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
   58 C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,16
   59 F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(J)=(OMG(J,K,I))/(ABS(2.+BETA12(J,K,I)))
      J1=X(J)-F1(J)
      IF(ABS(X(J))-1000.)590,594,594
  590 IF(ABS(X(J))-100.)592,593,593
```

```
592   C1=.001
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,16
16    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)16,16,203
203   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
303   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.66666666
2003  CONTINUE
14    CONTINUE
      WRITE (6,101)
      WRITE (6,105)
      DO 19 J=2,M
      DO 19 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
19    CONTINUE
      RETURN
      END
```

```
C     FOR UNSTA
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (90).
C       CONSIDERATION OF 9 CASES(3 INADMISSIBLE), INVOLVING 1800 VALUES.
        SUBROUTINE UNST9 (PI,R,H,TO,CK,KI,GAM,OMG,M,N)
        DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
       1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),F1(4),X(4)
  101   FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
       1R EQUATION (90), RAD PER SEC.,///,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
       210X,10HCOMEGA (2),10X,10HCOMEGA (3),//)
  102   FORMAT (2I5, 3E20.8//)
  103   FORMAT (10X,78HFIRST CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQUA
       1L 1, (CASE NOT ADMISSIBLE).,///)
  104   FORMAT (10X,56HSECOND CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
       1AL 1,//)
  105   FORMAT (10X,55HTHIRD CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
       1L 1,//)
  106   FORMAT (10X,56HFOURTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQU
       1AL 2,//)
  107   FORMAT (10X,78HFIFTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQUA
       1L 2, (CASE NOT ADMISSIBLE).,///)
  108   FORMAT (10X,55HSIXTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
       1L 2,//)
  109   FORMAT (10X,57HSEVENTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQ
       1UAL 3,//)
  110   FORMAT (10X,56HEIGHTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
       1AL 3,//)
  111   FORMAT (10X,78HNINTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
       1L 3, (CASE NOT ADMISSIBLE).,///)
        DO 1 J=1,M
        DO 1 K=1,N
        DO 1 I=1,3
        COMG(J,K,I)=0.0
    1   CONTINUE
        WRITE (6,101)
        WRITE (6,103)
        DO 2 J=2,M
        DO 2 K=2,N
        DO 2 I=1,3
        START=.1
        START1=1.
   35   X(1)=START*(OMG(J,K,I))
        DO 3 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(KI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)30,31,30
   30   C2=2.**26
        IF(B(J,K,I)-C2)300,301,301
   31   DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 32
  300   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1))))))/TAN(B(J,K,I))
   32   C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)33,33,34
   34   START=START+.1
        IF(START-1.0)35,35,325
  325   COMG(J,K,I)=.33333333
        GO TO 2001
   33   F(J,K,I)=ASIN(D(J,K,I))
```

```
      BETA(J,K,I)=(2./PI)*(F(J,K,1))
      F1(L)=(OMG(J,K,I))/(ABS(-1.+BETA(J,K,I)))
3     X(L+1)=F1(L)
4     X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)36,37,36
36    C2=2.**28
      IF(B(J,K,I)-C2)360,401,401
37    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 38
360   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
38    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)39,39,6
39    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(-1.+BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)5,5,6
6     X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)4,4,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
301   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
5     COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
2     CONTINUE
      WRITE (6,101)
      WRITE (6,104)
      DO 7 J=2,M
      DO 7 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
7     CONTINUE
```

```
      DO 8 J=2,M
      DO 8 K=2,N
      DO 8 I=1,3
      START=.6
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START-.1
      IF(START)425,425,45
425   OMG(J,K,I)=.33333333
      GO TO 2002
43    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(BETA(J,K,I)))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)10,10,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
```

```
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
502   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
8     CONTINUE
      WRITE(6,101)
      WRITE(6,105)
      DO 13 J=2,M
      DO 13 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=2,N
      DO 14 I=1,3
      START=.1
      START1=1.
55    X(1)=START*(OMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TQ*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
50    C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
51    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
500   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
52    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
54    START=START+.1
      IF(START-1.0)55,55,525
525   COMG(J,K,I)=.33333333
      GO TO 2003
53    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(1.-BETA(J,K,I)))
15    X(L+1)=F1(L)
16    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TQ*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
56    C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
57    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
560   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
```

```
      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,18
59    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(1.-BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
590   IF(ABS(X(3))-100.)592,593,593
592   C1=.001
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)16,16,203
203   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
303   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.66666666
2003  CONTINUE
14    CONTINUE
      WRITE(6,101)
      WRITE(6,106)
      DO 19 J=2,M
      DO 19 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
19    CONTINUE
      WRITE (6,101)
      WRITE (6,107)
      DO 20 J=2,M
      DO 20 K=2,N
      DO 20 I=1,3
      COMG(J,K,I)=OMG(J,K,I)
20    CONTINUE
      WRITE (6,101)
      WRITE (6,108)
      DO 21 J=2,M
      DO 21 K=2,N
      J1=J-1
      K1=K-1
```

```
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
21    CONTINUE
      DO 22 J=2,M
      DO 22 K=2,N
      DO 22 I=1,3
      START=.6
      START1=1.
65    X(1)=START*(OMG(J,K,I))
      DO 23 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)60,61,60
60    C2=2.**28
      IF(B(J,K,I)-C2)600,304,304
61    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 62
600   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
62    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)63,63,64
64    START=START-.1
      IF(START)625,625,65
625   COMG(J,K,I)=.33333333
      GO TO 2004
63    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(-1.*BETA(J,K,I)))
23    X(L+1)=F1(L)
24    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)66,67,66
66    C2=2.**28
      IF(B(J,K,I)-C2)660,404,404
67    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 68
660   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
68    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)69,69,26
69    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(-1.*BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)690,694,694
690   IF(ABS(X(3))-100.)692,693,695
692   C1=.001
      GO TO 695
693   C1=.01
      GO TO 695
694   C1=.1
695   IF(ABS(D1)-C1)25,25,26
26    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)24,24,204
204   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
```

```
      IF(A(J,K,I)-1.0)630,504,504
630   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)631,631,632
631   COMG(J,K,I)=.88888888
      GO TO 2004
632   COMG(J,K,I)=.11111111
      GO TO 2004
304   COMG(J,K,I)=.77777777
      GO TO 2004
404   COMG(J,K,I)=.99999999
      GO TO 2004
25    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2004,504,504
504   COMG(J,K,I)=.66666666
2004  CONTINUE
22    CONTINUE
      WRITE (6,101)
      WRITE(6,109)
      DO 27 J=2,M
      DO 27 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
27    CONTINUE
      DO 28 J=2,M
      DO 28 K=2,N
      DO 28 I=1,3
      COMG(J,K,I)=COMG(J,K,I)
28    CONTINUE
      WRITE (6,101)
      WRITE (6,110)
      DO 29 J=2,M
      DO 29 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
29    CONTINUE
      WRITE (6,101)
      WRITE (6,111)
      RETURN
      END
```

```
C
C     FOR UNSTIO
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (91).
C       CONSIDERATION OF 9 CASES (3 INADMISSIBLE), INVOLVING 1800 VALUES.
        SUBROUTINE UNSTIO(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
       1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),F1(4),X(4)
101     FORMAT (20X,60HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
       1R EQUATION (91), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
       210X,10HCOMEGA (2),10X,10HCOMEGA (3),//)
102     FORMAT (2I5, 3E20.8//)
103     FORMAT (10X,78HFIRST CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQUA
       1L 1, (CASE NOT ADMISSIBLE).,///)
104     FORMAT (10X,56HSECOND CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
       1AL 1,//)
105     FORMAT (10X,55HTHIRD CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
       1L 1,//)
106     FORMAT (10X,56HFOURTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQU
       1AL 2,//)
107     FORMAT (10X,78HFIFTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQUA
       1L 2, (CASE NOT ADMISSIBLE).,///)
108     FORMAT (10X,55HSIXTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
       1L 2,//)
109     FORMAT (10X,57HSEVENTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQ
       1UAL 3,//)
110     FORMAT (10X,56HEIGHTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
       1AL 3,//)
111     FORMAT (10X,78HNINTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
       1L 3, (CASE NOT ADMISSIBLE).,///)
        DO 1 J=1,M
        DO 1 K=1,N
        DO 1 I=1,3
        COMG(J,K,I)=0.0
1       CONTINUE
        WRITE (6,101)
        WRITE (6,103)
        DO 2 J=2,M
        DO 2 K=2,N
        DO 2 I=1,3
        COMG(J,K,I)=OMG(J,K,I)
2       CONTINUE
        WRITE (6,101)
        WRITE (6,104)
        DO 3 J=2,M
        DO 3 K=2,N
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
3       CONTINUE
        DO 4 J=2,M
        DO 4 K=2,N
        DO 4 I=1,3
        COMG(J,K,I)=.5*(OMG(J,K,I))
4       CONTINUE
        WRITE (6,101)
        WRITE (6,105)
        DO 5 J=2,M
        DO 5 K=2,N
        J1=J-1
        K1=K-1
```

```
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
5     CONTINUE
      DO 6 J=2,M
      DO 6 K=2,N
      DO 6 I=1,3
      COMG(J,K,I)=OMG(J,K,I)
6     CONTINUE
      WRITE (6,101)
      WRITE (6,106)
      DO 7 J=2,M
      DO 7 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
7     CONTINUE
      WRITE (6,101)
      WRITE (6,107)
      DO 8 J=2,M
      DO 8 K=2,N
      DO 8 I=1,3
      START=.1
      START1=1.
35    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)30,31,30
30    C2=2.**28
      IF(B(J,K,I)-C2)300,301,301
31    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 32
300   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
32    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)33,33,34
34    START=START+.1
      IF(START-1.0)35,35,325
325   COMG(J,K,I)=.33333333
      GO TO 2001
33    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)36,37,36
36    C2=2.**28
      IF(B(J,K,I)-C2)360,401,401
37    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 38
360   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
38    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)39,39,12
39    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
      D1=X(3)-F1(3)
```

```
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,395
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)10,10,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
501   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
8     CONTINUE
      WRITE (6,101)
      WRITE (6,108)
      DO 13 J=2,M
      DO 13 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=2,N
      DO 14 I=1,3
      COMG(J,K,I)=.5*(OMG(J,K,I))
14    CONTINUE
      WRITE (6,104)
      WRITE (6,109)
      DO 15 J=2,M
      DO 15 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
15    CONTINUE
      DO 16 J=2,M
      DO 16 K=2,N
      DO 16 I=1,3
      START=.1
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 17 L=1,2
```

```
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START+.1
      IF(START-1.0)45,45,425
425   OMG(J,K,I)=.33333333
      GO TO 2002
45    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
17    X(L+1)=F1(L)
18    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,20
49    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,495
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)19,19,20
20    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)18,18,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   OMG(J,K,I)=.08888888
      GO TO 2002
432   OMG(J,K,I)=.11111111
      GO TO 2002
```

```
502   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
19    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
16    CONTINUE
      WRITE (6,101)
      WRITE (6,110)
      DO 21 J=2,M
      DO 21 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
21    CONTINUE
      WRITE (6,101)
      WRITE (6,111)
      RETURN
      END
```

```
*
*1 FOR UNST,1
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (92).
C       CONSIDERATION OF THREE (3) CASES, INVOLVING 900 VALUES.
        SUBROUTINE UNST11(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
       1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),F1(4),X(4)
101     FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
       1R EQUATION (92), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HOMEGA (1),
       210X,10HOMEGA (2),10X,10HOMEGA (3),//)
102     FORMAT (2I5, 3E20.8//)
103     FORMAT (20X,33HFIRST CASE, LOWER CASE M EQUAL 1.,//)
104     FORMAT (20X,34HSECOND CASE, LOWER CASE M EQUAL 2.,//)
105     FORMAT (20X,33HTHIRD CASE, LOWER CASE M EQUAL 3.,//)
        DO 1 J=1,M
        DO 1 K=1,N
        DO 1 I=1,3
        COMG(J,K,I)=0.0
1       CONTINUE
        DO 2 J=2,M
        DO 2 K=2,N
        DO 2 I=1,3
        START=.1
        START1=1.
35      X(1)=START*(OMG(J,K,I))
        DO 3 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)30,31,30
30      C2=2.**28
        IF(B(J,K,I)-C2)300,301,301
31      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 32
300     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
32      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)33,33,34
34      START=START+.1
        IF(START-1.0)35,35,325
325     COMG(J,K,I)=.33333333
        GO TO 2001
33      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(L)=(OMG(J,K,I))/(ABS(2.-BETA(J,K,I)))
3       X(L+1)=F1(L)
4       X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)36,37,36
36      C2=2.**28
        IF(B(J,K,I)-C2)360,401,401
37      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 38
360     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
38      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)39,39,6
39      F(J,K,I)=ASIN(D(J,K,I))
```

```
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(2.-BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)5,5,6
6     X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)4,4,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,1)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
301   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
5     COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
2     CONTINUE
      WRITE (6,101)
      WRITE (6,103)
      DO 7 J=2,M
      DO 7 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
7     CONTINUE
      DO 8 J=2,M
      DO 8 K=2,N
      DO 8 I=1,3
      START=.6
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1))))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
```

```
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START-.1
      IF(START)425,425,45
425   COMG(J,K,I)=.33333333
      GO TO 2002
45    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(BETA(J,K,I)))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)10,10,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
302   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
8     CONTINUE
      WRITE (6,101)
```

```
      WRITE (6,104)
      DO 13 J=2,M
      DO 13 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(OMG(J,K,I),I=1,3)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=2,N
      DO 14 I=1,3
      START=.1
      START1=1.
55    X(1)=START*(OMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
50    C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
51    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
500   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
52    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
54    START=START+.1
      IF(START-1.0)55,55,525
525   OMG(J,K,I)=.33333333
      GO TO 2003
53    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(2.+BETA(J,K,I)))
15    X(L+1)=F1(L)
16    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
56    C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
57    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
560   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
58    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,18
59    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(2.+BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
590   IF(ABS(X(3))-100.)592,593,595
592   C1=.001
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
```

```
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)16,16,203
  203 A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
  530 B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
  531 COMG(J,K,I)=.88888888
      GO TO 2003
  532 COMG(J,K,I)=.11111111
      GO TO 2003
  303 COMG(J,K,I)=.77777777
      GO TO 2003
  403 COMG(J,K,I)=.99999999
      GO TO 2003
   17 COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
  503 COMG(J,K,I)=.06666666
 2003 CONTINUE
   14 CONTINUE
      WRITE (6,101)
      WRITE (6,105)
      DO 19 J=2,M
      DO 19 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
   19 CONTINUE
      RETURN
      END
```

```
.
@I  MAP TRSN
        SEG  AFDEN-*(A,B,C,D)-E
A       SEG UNST1-UNST2-UNST3
B       SEG UNST4-UNST5-UNST6
C       SEG UNST7-UNST8
D       SEG UNST9-UNST10-UNST11
E       SEG NXPAF$-NXPAX$-EXP-ALOG-ATAN
```

```
* XGT PRSM
   480.          180.        1.0          2.591E-4        1.0E7        .3          100.0
   ''    ''        1.0E6      0.05         1.0             0.1          1.0
   480.          180.        1.0          2.591E-4        1.0E7        .333        100.0
   ''    ''        1.0E6      0.05         1.0             0.1          1.0
   990.          126.        .3125        2.591E-4        1.0E7        .333        100.0
   ''    ''        1.0E6      0.05         1.0             0.1          1.0
  2190.          198.        .5000        2.591E-4        1.0E7        .333        100.0
   ''    ''        1.0E6      0.05         1.0             0.1          1.0
* FIN
* FIN
```

```
*
@1  NAP PRSN
        CHN     1
        SEG     NAFSEG-NAFREQ-OREIG-GRT-HESSEN
        CHN     2
        SEG     DR1-UNST1-UNST2-UNST3
        CHN     3
        SEG     DR2-UNST4-UNST5-UNST6
        CHN     4
        SEG     DR3-UNST7-UNST8
        CHN     5
        SEG     DR4-UNST9-UNST10-UNST11
@1      FOR     DR1
        COMMON L,S,U,V,W,PI,M,N,OMG(20,20,3),R,H,TO,CK,RI,GAM
        CALL UNST1 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        CALL UNST2 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        CALL UNST3 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        CALL CHAIN(3)
        END
@1      FOR     DR2
        COMMON L,S,U,V,W,PI,M,N,OMG(20,20,3),R,H,TO,CK,RI,GAM
        CALL UNST4 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        CALL UNST5 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        CALL UNST6 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        CALL CHAIN(4)
        END
@1      FOR     DR3
        COMMON L,S,U,V,W,PI,M,N,OMG(20,20,3),R,H,TO,CK,RI,GAM
        CALL UNST7 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        CALL UNST8 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        CALL CHAIN(5)
        END
@1      FOR     DR4
        COMMON L,S,U,V,W,PI,M,N,OMG(20,20,3),R,H,TO,CK,RI,GAM
        CALL UNST9 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        CALL UNST10(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        CALL UNST11(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        CALL CHAIN(1)
        END
```

4.0 REFERENCES

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2. McLachlin, N. W., Theory and Application of Matheau Functions, The Clarendon Press, Oxford, England, 1951.

3. Kunz, K. S., Numerical Analysis, McGraw-Hill Book Company, Inc., New York, 1957.

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APPENDIX A

GENERAL INFORMATION ON SOLUTION OF THE STABILITY EQUATIONS

A-I. The Constraints of the Computer Program

To determine the roots of the instability equations a Fortran IV program using an accelerated variation of The Method of Iteration was used. The iteration formula was programmed so that if the procedure does not iterate to a value that satisfies the instability equations to a specified degree of accuracy for a given case then an easily spotted nonsense answer will be printed out for this case.

a. If after 10 initial guesses not one causes the argument of the arcsine, that appears as part of the function β in equation (7), to be less than one then

$$COMG(J,K,I) = .33333333$$

b. If the initial guess of Ω causes $B(J,K,I) = \pi a^{\frac{1}{2}}/2$, the argument of the trigonometric functions within β in equation (7), to be greater than 2^{28} then

$$COMG(J,K,I) = .77777777$$

c. If the iteration of Ω causes $B(J,K,I) = \pi a^{\frac{1}{2}}/2$ to be greater than 2^{28} then

$$COMG(J,K,I) = .99999999$$

d. If after the specified number of iterations (usually either 100 or 1000) the iterates are not equal to within a preset value of each other and

(1) The boundary condition of the Matheau [equation (1)], $0<a<1$ is not being met then

$$COMG(J,K,I) = .66666666$$

(2) The boundary condition is being met but $a \approx 1$ so that the iterate of Ω, X(3), is approximately equal to $C(J,K,I) = [\Delta(0)]^{\frac{1}{2}}$ [equation (6)] to within .01 then

$$COMG(J,K,I) = .88888888$$

(3) The boundary condition of $0<a<1$ is met and $X(3) \neq C(J,K,I)$ then

$$COMG(J,K,I) = .11111111$$

e. If iterates of Ω are equal to within the preset value of each other but the Matheau boundary condition is not being met again

$$COMG(J,K,I) = .66666666$$

A-II. The Iteration Procedure

a. The Method of Iteration

The iteration procedure used was an accelerated version of The Method of Iteration. In order that one might understand

this variant a brief review of The Method of Iteration is given. (For a more complete development see ref. 3) Given a function

$$f(x) = 0$$

One can always add a function $h(x)$

$$f(x) + h(x) = h(x)$$

to obtain

$$g(x) = h(x).$$

The simultaneous solutions of the resulting equations

$$y = g(x)$$

$$y = h(x)$$

are the real roots of

$$f(x) = 0 \quad \text{(ref. 1)}.$$

To find the roots of

$$f(x) = 0$$

by The Method of Iteration,

$$y = h(x)$$

must be rewritten in the form

$$x = H(y)$$

and for convergence the equations must satisfy the condition

$$\left| \frac{d[g(x)]}{dx} \right|_{x=p} < \left| \frac{d[h(x)]}{d} \right|_{x=p}$$

where p is the desired root of

$$f(x) = 0.$$

The iterates are formed by substituting an approximate value, x_o, of the root p into the equation

$$y = g(x)$$

to get

$$y_o = g(x_o).$$

A new approximate value of x is then derived from the equation

$$x = H(y),$$

$$x_1 = H(y_o).$$

This process is repeated until

$$\left| X_n - X_{n-1} \right| < C_1$$

and

$$\left| f(x_n) \right| < C_2$$

where C_1 and C_2 are a measure of the accuracy desired. If the slope of g(x) in the region of p is much smaller than h(x) then x_n will converge rapidly to the root.

Figure A-1 and Figure A-2 demonstrate the convergence of this iterative method graphically

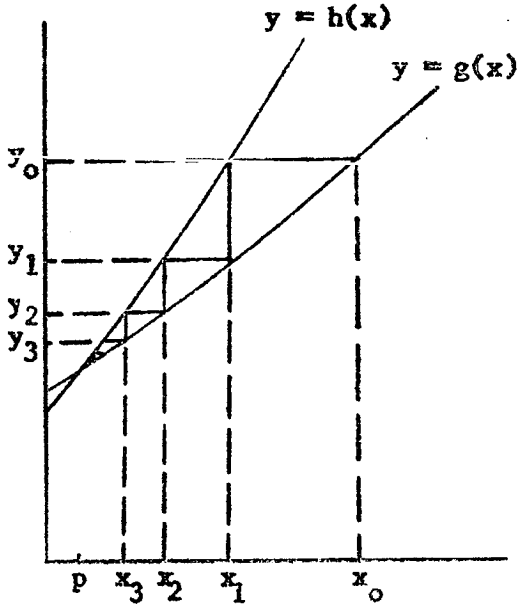

Figure A-1 - Slopes of g(x) and h(x) of Like Signs

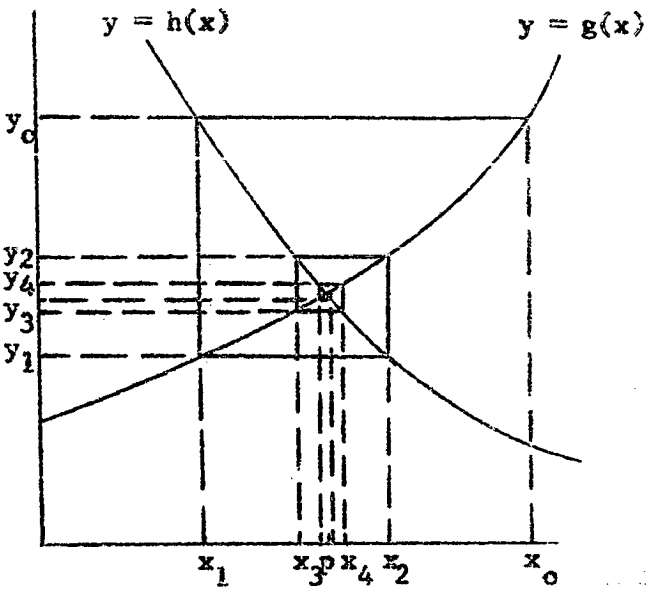

Figure A-2 - Slopes of g(x) and h(x) of Opposite Signs

In Figure A-1 the slopes of g(x) and h(x) are both of the same sign while in Figure A-2 they are of opposite sign. Points (x_o, y_o), (x_1, y_1),...(x_i, y_i)... lie on the curve $y = g(x)$ while points (x_1, y_o) (x_2, y_1) ... (x_i, y_{i-1}) ... lie on the curve $x = H(y)$ which is equivalent to the curve $y = h(x)$. When the slopes of the two functions have the same sign, the iteration follows a stepped path (Fig. A-1); when the slopes have opposite sign the iteration proceeds along a rectangular spiral (Fig. A-2).

Unfortunately, when one writes $f(x) = 0$ in the form

$$y = g(x)$$

$$y = h(x),$$

one often has difficulty making sure that the slope of $g(x)$ is much less than the slope of $h(x)$ at the point of intersection and also that $y = h(x)$ can be easily written in the form $x = H(y)$. However, if $h(x) = x$, then

$$f(x) = 0$$

becomes

$$f(x) + x = g(x) = x.$$

Now, using the equation $y = x$ one gets the simultaneous relation

$$y = g(x)$$

$$y = x$$

If the slope of $g(x)$ is small or zero, then these equations produce a rapidly converging iteration procedure. For a variant of this method involving derivatives see ref. 4.

b. An Accelerated Variation of the Method of Iteration

Another variant that will often accelerate the convergence of the iteration and does not use derivatives has an additional advantage of working even when

$$\left| \frac{dg(x)}{dx} \right|_{x=p} > \left| \frac{dh(x)}{dx} \right|_{x=p}$$

This method will be referred to as the Constructed Line Acceleration of the Method of Iteration. This variant also uses the two equations

$$y = g(x)$$

$$y = x$$

In this method, two points on

$$y = g(x)$$

are found by substituting the first approximate root, x_o, into

$$y = g(x)$$

yielding

$$y_o = g(x_o).$$

Substitution of y_o into

$$y = x$$

yields

$$y_o = x_1$$

The intersection of the line determined by the two points, (x_o, y_o) and (x_1, y_1) both on the curve $y = g(x)$, with the line

$$y = x$$

determines the abscissa of the third point. This process is repeated until the root is determined to within the desired degree of accuracy.

In Figure A-3 a graph of this method is presented. Notice that since

$$\left| \frac{d\,g(x)}{dx} \right|_{x=p} > \left| \frac{d\,h(x)}{dx} \right|_{x=p}$$

The standard form of The Method of Iteration would not work.

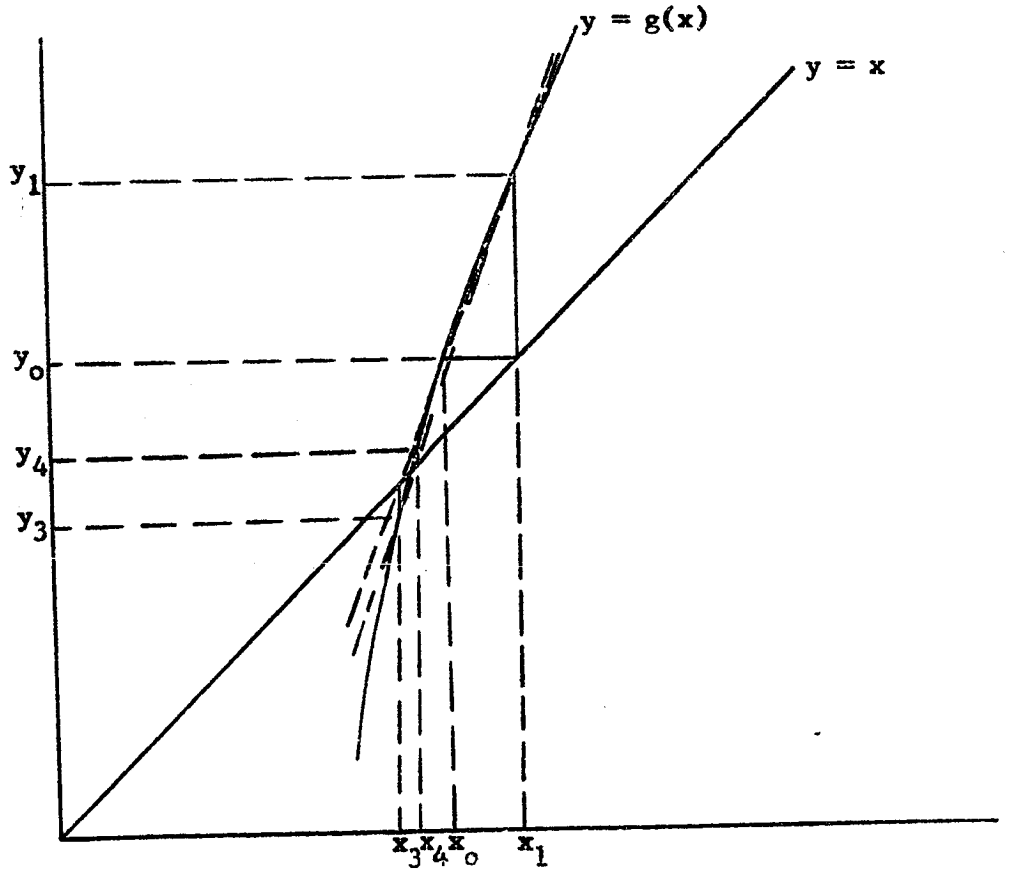

Figure A-3 - Constructed Line Acceleration of the Method of Iteration

The equation of the line through the two initial points on

$$y = g(x)$$

is

$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_0}{x_1 - x_0}$$

and at the intersection with the second of the two simultaneous equations from Figure A-3

$$y = x = x_2.$$

Combining

$$y = x_2$$

with the equation of the line through the two initial points one obtains an equation solvable for x_2, the abscissa of the point of intersection of these two lines. However, since

$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_0}{x_1 - x_0}$$

is an approximation of

$$y = g(x)$$

x_2 is an approximation of p, the root which is the abscissa of the point of intersection of

$$y = g(x)$$

and

$$y = x.$$

Thus substituting

$$y = x = x_2$$

in

$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_o}{x_1 - x_o}$$

one obtains

$$\frac{x_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_o}{x_1 - x_o}$$

or

$$x_2 = \frac{x_1 y_o - x_o y_1}{x_1 - x_o - y_1 + y_o}$$

or in general, the nth iterate is

$$x_n = \frac{x_{n-1}\, g(x_{n-2}) - x_{n-2}\, g(x_{n-1})}{x_{n-1} - x_{n-2} - g(x_{n-1}) + g(x_{n-2})}$$

Restrictions

A case where this iteration formula will not iterate is diagramed in **Figure A-4.**

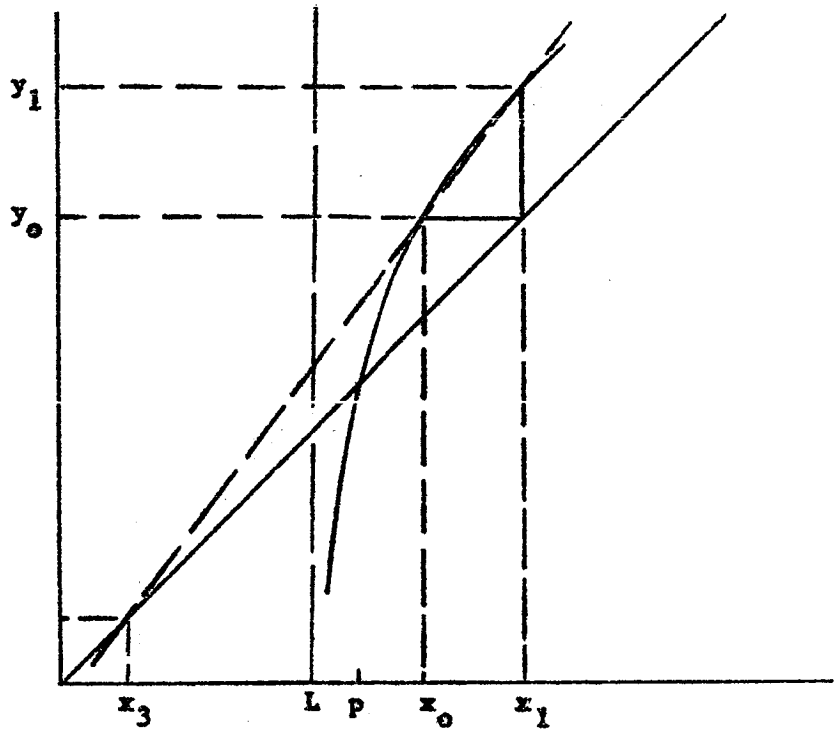

Figure A-4 - Example of Case Where The Constructed Line Acceleration Method Fails

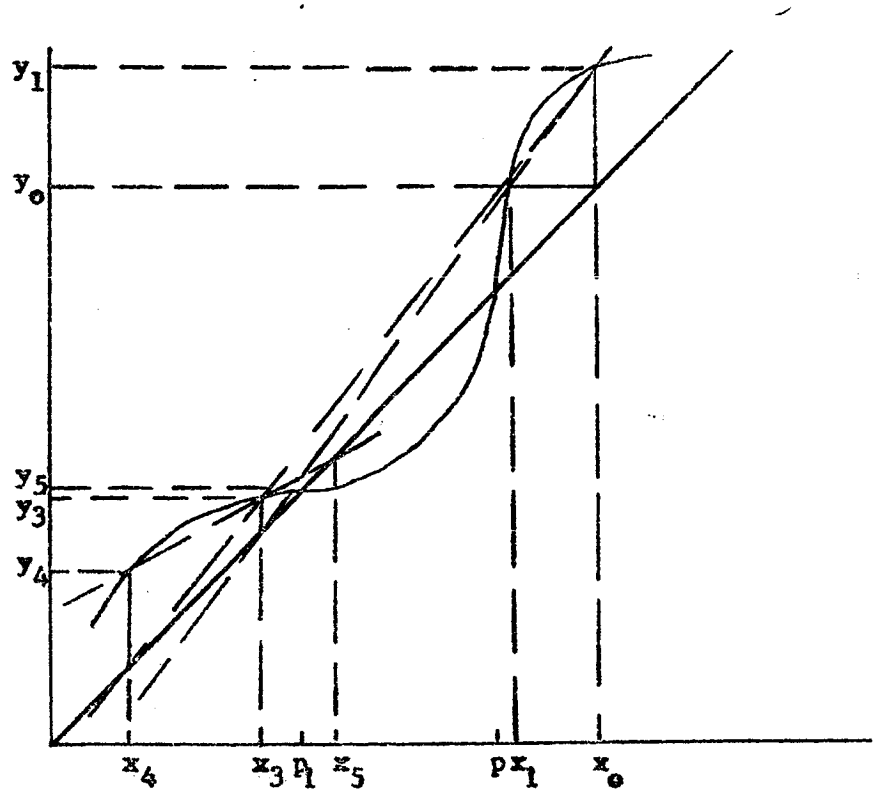

Figure A-5 - Another Case Where The Constructed Line Acceleration Method Fails

Since the function g(x) asymptotically approaches the line

$$x = L$$

there is no point of intersection between

$$y = g(x)$$

and

$$y = x_2.$$

But there is an intersection between

$$y = g(x)$$

and

$$x = y_2,$$

therefore, if the inverse of the function g(x) can be found, the iteration equation becomes

$$y_n = \frac{G(y_{n-1})\, y_{n-2} - G(y_{n-2})\, y_{n-1}}{G(y_{n-1}) - G(y_{n-2}) - y_{n-1} + y_{n-2}}$$

where

$$x_n = g(y_n) .$$

Another function that the constructed line acceleration method will not iterate properly is illustrated in Figure A-5.

In this case the iteration is proceeding to a root p_1 rather than the desired root p. However in the cases diagramed in Figures A-4 and A-5 and in the case where

$$\frac{y_1 - y_o}{x_1 - x_o} = 1$$

if x_o, the initial guess of p, the desired root, is near enough to p then the equation

$$x_n = \frac{x_{n-1}\, g(x_{n-2}) - x_{n-2}\, g(x_{n-1})}{x_{n-1} - x_{n-2} - g(x_{n-1}) + g(x_{n-2})}$$

will iterate x_n to p as long as g(x) is a single valued function and is defined on both sides of and at y = x.