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DYNAMICS OF T = 2 $\Sigma - \pi$ SYSTEM . I

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ABSTRACT 22181

A dynamical calculation using Balazs type N/D method is performed for the T = 2 $\Sigma - \pi$ system in $P_{3/2}$ state. Self consistent solutions for the position and residue of a resonance are obtained for wide range of the relevant Yukawa coupling constants and different sets of matching points. Some remarks are made regarding interpretation of the results.

Author

[redacted]

1. INTRODUCTION

Recently Pan and Ely[1,2] have presented evidence for the existence of a $T = 2\ \Sigma - \pi$ resonance of narrow width and mass $\sim$ 1415 Mev, in their experiments on K^- interaction with neutrons in carbon. If the existence of this resonance is confirmed in other experiments, it will have important implications for the unitary symmetry scheme of strongly interacting particles. The lowest representation of the group SU_3 in which this resonance can be placed in of dimension 27.

Some years back, several authors predicted the existence of such a resonance on basis of Chew-Low or strong coupling models[3]. It is however interesting to note that dynamical calculation of meson baryon resonances in the Octet model by Martin and Wali[4] did not indicate any resonance in the 27 dimensional representation of SU_3. This may however be due to the fact that the short range forces arising from the far away singularities were not adequately treated by them. Recently, Gyuk, Simmons and Tuan[5] have given some rough bootstrap arguments for the $T = 2\ \Sigma - \pi$ resonance and suggest that the most likely angular momentum state is $P_{3/2}$. In view of these indications, we have done a detailed dynamical analysis of this problem following the method of Balazs[6] in order to take account of the far away left hand singularities. In the present note we report the calculation of the $P_{3/2}$ system. Other angular momentum states are currently being investigated using the same method.

A rather welcome circumstance is that, unlike the case of $T = 0$ and $T = 1$ $\Sigma - \pi$ systems, this is essentially a single channel problem in the approximation of neglecting inelastic effects.

In Sec. 2 we present the details of the calculation. Contribution of the nearby Born cuts due to the exchange of Λ and Σ in the u-channel are exactly evaluated. Far away left hand singularities are replaced by Balazs poles. Self consistent solutions for position and residue of a $T = 2$ $\Sigma - \pi$ resonance (to be called Y_2^* in the following) are obtained by a computer. The results are discussed in Sec. 3. It is found that very good self consistent solutions do exist for a wide range of the Yukawa coupling constants. The question of dependence of the solutions on the choice of matching points is also examined and discussed.

2. DETAILS OF THE CALCULATION

As the kinematical details of meson baryon systems are fairly well known[7], we merely present the results briefly. Denoting the four momenta of incoming Σ^- and π^- by p_1, q_1 and those of the outgoing Σ^- and π^- by p_2, q_2 respectively, the invariants s, t, u are defined as usual[8]:

$$s = (p_1 + q_1)^2 = W^2$$

$$t = (p_1 - p_2)^2 = -2q^2\,(1 - \cos\theta) \tag{1}$$

$$u = (p_1 - q_2)^2 = 2(\Sigma^2 + \pi^2) - s + 2q^2\,(1 - \cos\theta)$$

where W q and θ denote respectively the total energy, magnitude of three momentum and scattering angle in the C M system of s channel.

As in reference 7, we choose the elastic scattering amplitude for $P_{3/2}$ Σ-π system to be

$$g(s) = W^2\, e^{i\delta_{1+}} \sin\delta_{1+} / q^3 \tag{2}$$

The important singularities of the partial wave amplitude (shown in Fig. 1) arise as follows:

(I) s channel : We have the usual right hand cut for[8]

$$(\Sigma + \pi)^2 = 91.6 \leq s \leq \infty \tag{3}$$

and a pole due to Y_2^* at

$$s = s_R = (m_{Y_2^*})^2 \tag{4}$$

(II) u and t channels: All the exchanged systems give rise to a cut from o to - ∞. In addition to this, exchange of Λ gives rise to the cut:

$$L_2 = 82.2 \leq s \leq L_1 = 85.0 \tag{5}$$

whereas exchange of Σ gives rise to the cut

$$L_4 = 71.4 \leq s \leq L_3 = 75.4 \tag{6}$$

Intermediate states with masses larger than $(\Sigma + \pi)^2$ (in particular, all Y*'s) give rise to left hand cuts below s = 57.3. Exchange of two pion system in t channel (in particular vector meson ρ) gives rise to a circular cut (not shown in Fig. 1). Contribution of this is known to be small compared to that of the exchanged baryons for the $P_{3/2}$ system and hence its contribution to nearby singularities will be neglected. Of course, contributions of all the exchanged systems to the far away cut (o to $-\infty$) are retained.

Except for some changes, we follow the general method of calculation using Balazs type poles described by Singh and Udgaonkar[9] and Pati[10]. The reader is referred to these works for details.

The amplitude g(s) is written as

$$g(s) = N(s)/D(s) \tag{7}$$

where, as usual D(s) has only the right hand unitarity cut and N(s) has only the left hand cuts. In the elastic approximation we have

$$D(s) = 1 - \frac{s-s_0}{\pi} \int_{(\Sigma+\pi)^2}^{\infty} \frac{q'^3\, N(s')\, ds'}{s'(s'-s)(s'-s_0)} \tag{8}$$

where s_0 is the arbitrary subtraction point which we choose for convenience in latter calculation to be the mid-point of the Λ-cut ($s_1 = 83.6$).

For the N(s) we write

$$N(s) = N_n(s) + N_f(s) \tag{9}$$

where

$$N_n(s) = \frac{1}{\pi} \int_{L_n} \frac{\{\mathrm{Im}\, g(s')\}\, D(s')\, ds'}{s' - s} \tag{10}$$

L_n refers to the nearby portion of the left hand cut. As discussed in references 9 and 10 $N_f(s)$ is written as

$$N_f(s) = \frac{R_3}{s - s_3} + \frac{R_4}{s - s_4} \tag{11}$$

where in our case the positions of the two poles are determined from Balazs curves to be

$$s_3 = -10 , \qquad s_4 = -500 \tag{12}$$

Residues R_3, R_4 are still unknown at this stage.

The Born terms due to Λ and Σ exchange are given by

$$g_Y (s) = \frac{g^2{}_{Y\Sigma\pi}}{32\pi q^4} \left[\{(W+\Sigma)^2 - \pi^2\} \quad \{(W+Y-2\Sigma\} Q_1 (x) \right.$$
$$\left. + \{(W-\Sigma)^2 - \pi^2\} \{W+2\Sigma-Y\} Q_2 (x) \right] \tag{13}$$

$$x = \frac{2(\Sigma^2+\pi^2) - s - Y^2}{2q^2} + 1 \tag{14}$$

where Y stands for Σ or Λ, $Q_1 (x)$, $Q_2 (x)$ for Legendre functions of second kind and $g_{Y\Sigma\Lambda}$ for the renormalized coupling constant.

Contribution of Y_2* in the s- channel is given by

$$g^{D}_{Y_2*}(s) = \frac{-\varkappa\,[(W+\Sigma)^2-\pi^2]}{[(W_R+\Sigma)^2-\pi^2]}\;\frac{1}{W-W_R} \tag{15}$$

where $\varkappa$ is the residue at $W=W_R=\sqrt{s_R}$. For a resonance it is related to the total width Γ by

$$\varkappa = \frac{W_R^{\,2}\Gamma}{2q_R^{\,3}} \tag{16}$$

where q_R is the c m momentum of $\Sigma\,\pi$ system at resonance.

Contribution due to exchange of Y_2* can be obtained by describing it by a Rarita Schwinger field[11] and is given by

$$g^{EX}_{Y_2*}(s) = \frac{2\,\varkappa\,C(s)}{\{(W_R+\Sigma)^2-\pi^2\}\,8\,q^4} \tag{17}$$

where

$$C(s) = \{(W+\Sigma)^2 - \pi^2\} \{\alpha(W)\ (W-2\Sigma-W_R) + \beta(W_R)(2\Sigma-W-W_R)\}\ Q_1\ (z)$$

$$+ \{(W-\Sigma)^2 - \pi^2\} \{\alpha(W)\ (W+2\Sigma + W_R) + \beta(W_R)(W_R-2\Sigma-W)\}\ Q_2\ (z) \tag{18}$$

$$\alpha(W) = \frac{2\ \Sigma^2 + 2\ \pi^2 + 2\ q_R^{\ 2} - W_R^{\ 2} - s}{2} \tag{19}$$

$$\beta(W_R) = [(W_R + \Sigma)^2 - \pi^2]^2/\underline{12}\ W_R^{\ 2} \tag{20}$$

$$z = \frac{2\ \Sigma^2 + 2\ \pi^2 - s - W_R^{\ 2}}{2\ q^2} + 1 \tag{21}$$

In order to obtain an idea of the relative contribution of various nearby singularities to $N_n(s)$ we temporarily made a linear approximation for the D(s) function, in the unphysical region (i.e., assuming resonance at $s_R \approx 103$)

$$D(s) = 1 - \frac{s - s_0}{s_R - s_0} \tag{22}$$

Using (22) and imaginary parts of the Born terms, we have checked explicitly that the contributions due to the exchange of Y_2^* (1415), Y_0^* (1405), Y_1^* (1385) to $N_n(s)$ for s in the physical region were much smaller than those due to the exchange of Σ and Λ for reasonable values of coupling constants[12].

Furthermore, as the cuts due to Λ and Σ exchange are quite short as compared to their mean distances from points on the right hand cut, we can replace them by poles at their mid-points (s_1, s_2) to a very good approximation. This was also explicitly checked[(13)].

Now we drop the assumption (22) for D(s) and write

$$N_n(s) = \frac{R_1}{s - s_1} + \frac{R_2\, D(s_2)}{s - s_2} \tag{23}$$

where

$$R_1 = -\frac{1}{\pi}\int_{L_2}^{L_1} \operatorname{Im} g(s')\, ds' \tag{24}$$

$$R_2 = -\frac{1}{\pi}\int_{L_4}^{L_3} \operatorname{Im} g(s')\, ds' \tag{25}$$

We note that by our normalization

$$(s_0 = s_1), \quad D(s_1) = 1 \quad .$$

Defining

$$F(s,\ s_0,\ s_i) = \frac{s-s_0}{\pi}\int_{(\Sigma+\pi)^2}^{\infty} \frac{q'^3\, ds'}{s'(s'-s)(s'-s_0)(s'-s_i)} \tag{26}$$

We obtain from (8), (9), (11) and (23)

$$D(s) = 1 - R_1 F(s, s_0\ s_1) - R_2 F(s, s_0, s_2)\ D\ (s_2)$$

$$- R_3 F(s, s_0, s_3) - R_4 F(s, s_0, s_4) \qquad (27)$$

$D(s_2)$ can be obtained from the last equation in terms of other parameters. On resubstituting we get $D(s)$ in terms of the two unknown parameters R_3, R_4 and the rest known constants or functions.

In order to determine R_3 and R_4 we match the values of $N(s)/D(s)$ obtained from (9), (11) (23) and (27) at two ≪ judiciously chosen ≫ points, called the matching points (s_{M_1}, s_{M_2}) with the value of the partial wave amplitude obtained from fixed energy dispersion relation[14]. This relation is to be used only where the partial wave expansion is expected to be valid.

The fixed energy dispersion relation for the invariant amplitudes[7] $A(s,t,u)$, $B(s,t,u)$ is given by

$$B(s,t,u) = \frac{R_\Lambda}{u-\Lambda^2} + \frac{R_\Sigma}{u-\Sigma^2} + \frac{R_{Y_2^*}}{s-s_R} + \frac{1}{\pi}\int du' \frac{\text{Im } B_u(u',s)\, du'}{u'-u}$$

$$+ \frac{1}{\pi}\int dt' \frac{\text{Im } B_t\,(t',s)\, dt'}{t'-t} \tag{28}$$

A similar expression holds for $A(s,t,u)$. The third term corresponds to contribution of Y_2^* in the s-channel. In consistency with our previous approximations we drop contributions from exchanges of vector mesons and higher mass states in t-channel. In u channel we retain contribution of Y_2^* exchange to the first integral although it is small. Hence we have

$$g(s) = g_\Lambda(s) + g_\Sigma(s) + g_{Y_2^*}{}^{D}(s) + g_{Y_2^*}{}^{EX}(s) \tag{29}$$

Now the matching equation is given by

$$\frac{N_n(s) + N_f(s)}{D(s)} = g(s) \tag{30}$$

which after some algebra reduces to an equation of the type

$$R_3 f_1 + R_4 f_2 + f_3 = 0 \tag{31}$$

where f_1, f_2, f_3 are complicated functions which need not be given.

We write Eq. (31) at two points, s_{M_1}, s_{M_2} and solve to obtain R_3 and R_4. D(s) and N(s) are now completely known.

The output position of resonance or bound state is given by

$$\mathrm{Re}\, D(s_{R\ out}) = 0 \tag{32}$$

and the output residue $\varkappa$ by

$$\varkappa_{out} = - \frac{1}{2W_{R\ out}} \left\{ \frac{N(s_{R\ out})}{\mathrm{Re}\, D'\ (s_{R\ out})} \right\} \tag{33}$$

The above calculation was carried out numerically on IBM 7094.

The following iterative procedure was adopted to find the self-consistent solutions. For a given set of values of

$$\frac{g_{\Sigma\Lambda\pi}^2}{4\pi} \ , \quad \frac{g_{\Sigma\Sigma\pi}^2}{4\pi}$$

and the two matching points s_{M_1}, s_{M_2}; $s_{R\ in}$ and $\varkappa_{in}$ were chosen. R_3 and R_4 were found by solving Eq. (31) at s_{M_1} and s_{M_2}. $s_{R\,out}$ and $\varkappa_{out}$ were then found from (32) and (33) and compared with SR_{in} and $\varkappa_{in}$. $s_{R\,out}$ and $\varkappa_{out}$ were chosen as input values for the next iteration. The whole procedure was repeated till $s_{R\,out}$ and $s_{R\,in}$, $\varkappa_{out}$ and $\varkappa_{in}$ came out to be equal within some preassigned accuracy. Various sets of starting values of $s_{R\,in}$ and $\varkappa_{in}$ were chosen. The calculation was repeated for various values of the coupling constants[15]

$$\frac{g_{\Sigma\Lambda\pi}^2}{4\pi} \ , \quad \frac{g_{\Sigma\Sigma\pi}^2}{4\pi}$$

as well as different sets of matching points. In this way, we have searched for solutions up to $s \approx 150$. The results are presented and discussed in the next section.

3. RESULTS AND DISCUSSION

As a result of the above procedure we found resonant solutions with very good input-output consistency ($|s_{Rout} - s_{Rin}| < 0.5$, $|\varkappa_{out} - \varkappa_{in}| < 0.5$) for rather wide range of values of the Yukawa coupling constants (both

$$\frac{g_{\Sigma\Lambda\pi}{}^2}{4\pi} , \quad \frac{g_{\Sigma\Sigma\pi}{}^2}{4\pi}$$

varying from 0 to 16) and different sets of matching points (MP). Table I shows the results for

$$\frac{g_{\Sigma\Lambda\pi}{}^2}{4\pi} = 11 \quad \text{and} \quad \frac{g_{\Sigma\Sigma\pi}{}^2}{4\pi} = 4 \ .$$

For the first set of MP ($s_{M_1} = 88$, $s_{M_2} = 65$) we see that for the above set of coupling constants a sharp low energy resonant solution does exist. The position and width are close to the experimental values[1,2]

$$s_R \approx 103 \qquad \Gamma < 50 \text{ Mev} \ .$$

As regards variation with coupling constants an interesting feature is found. Even when both the coupling constants

$$\left(\frac{g_{\Sigma\Lambda\pi}{}^2}{4\pi} , \ \frac{g_{\Sigma\Sigma\pi}{}^2}{4\pi} \right)$$

are set equal to zero,

many self consistent solutions are found, but they are spread out in the region between threshold and $s \approx 130$. As the coupling constants are increased, the solutions start becoming localized in the region near threshold, with slowly increasing value of $\varkappa$.

For the other sets of MP (78, 65; 67, 62; 88, 45) similar results are obtained. We find that the results are not sensitive to the variation of the matching points if they are chosen in such a way that one of them lies on either side of the nearby cuts (taken together). However, if one of them lies in between the nearby cuts or both of them lie on the same side of these cuts, the results are somewhat sensitive to the variations of the MP.

In view of this situation, we make the following remarks. One of the features of this problem is the crowding of singularities near the physical threshold as well as with respect to each other. This makes it difficult to find suitable MP. Ideally, the final results should be independent of the choice of these. But in practice certain amount of caution must be used. As discussed in reference 10, these points should be as far away from physical thresholds and the unknown left hand singularities as possible. Thus there is an extremely narrow region available for placing s_{M1}, s_{M_2}, and these points are necessarily close to the nearby cuts. Rather a posteriori, we can say that the « best set » of MP corresponds to having one of them on either side of the nearby cuts (taken together) since as mentioned above, for this situation the final results are not sensitive to the variation of each of these points. It is rather remarkable that this set of MP gives the position and residue of resonance quite close to the experimental values[1,2].

Thus we see that in the present procedure, we do obtain a self-consistent low lying resonance in T = 2, $P_{3/2}$ $\Sigma - \pi$ system[16]. Whether future experiments confirm the existence and quantum numbers of this resonance remains to be seen. It has been often remarked that the dynamical methods should not only predict existing resonances but also should rule out non-existing ones. If the resonance under discussion, for example, is not confirmed, one can seriously question the validity of the dynamical method used here[17], in cases where observed resonances have beey shown to be self-consistent.[9,10,18]

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6. L. A. P. Balazs, Phys. Rev. 126, 1220 (1962)

7. See, for example, S. C. Frautschi and J. D. Walecka, Phys. Rev., 120, 1486 (1960)

8. Σ, Λ, π etc. have been used to denote masses of these particles also. Values of s are throughout given in units of π^2.

9. V. Singh and B. M. Udgaonkar, Phys. Rev. 130, 1177 (1963)

10. J. C. Pati, Phys. Rev. 134 B, 387 (1964)

11. See, for example, reference 4 or E. Abers and C. Zemach, Phys. Rev. 131, 2305 (1963)

12. This was found to be approximately true even for the unphysical region not too far from the threshold.

13. Somewhat surprisingly, it was found that this approximation was good even for N(s) in the unphysical region as long as one is not too close to the cuts (at least for the matching points which we have chosen.)

14. It is suggested in reference 10 that in such an approximate

calculation, matching the N/D amplitude with g(s) at two different points may be better than matching the amplitudes and their first derivatives at a single point.

15. The coupling constants

$$\frac{g_{\Sigma\Lambda\pi}^2}{4\pi} \quad , \quad \frac{g_{\Sigma\Sigma\pi}^2}{4\pi}$$

are not known at present. Analysis of hyperfragment and hyperon-nucleon reactions (J. J. deSwart and C. K. Iddings, Phys. Rev. 130, 319 (1963)) is however, consistent with

$$\frac{g_{\Sigma\Lambda\pi}^2}{4\pi} \approx 11 \ , \quad \frac{g_{\Sigma\Sigma\pi}^2}{4\pi} \approx 4$$

These values are predicted by SU_3 symmetry scheme with the mixing parameter $\alpha \approx 0.25$.

16. However, recently, J. C. Pati and the present author (preprint) have shown that in the type of Balazs procedure where fixed energy dispersion relations are used to obtain residues of far

away poles, it is likely that one would get a self consistent bound state or resonant solution for any meson baryon system in $P_{3/2}$ state as long as the Born terms are not strongly repulsive. The result is mainly based upon the following considerations. (i) The Born terms and N_n are usually small (at least for small Yukawa coupling constants) as compared to the resonance or bound state term $(g_{Y_2^*}{}^{D}(s))$ in the present case, for example) and N_f respectively. (ii) The qualitative results of such calculations do not seem to have a marked dependence on the Yukawa coupling constants in a wide range. In our case, for reasonable values of coupling constants these terms are quite comparable. Still good self consistent solutions have been found. Thus the present calculation seems to give support to this result.

17. We have used fixed energy dispersion relations to obtain residues of the far away poles. It may also be interesting to use crossing symmetry for this purpose. See, for example, A. Tubis, T. Kanki and G. D. Doolen, Bull. Am. Phys. Soc., Vol. 10, 460, (1965).

18. G. L. Kane, Phys. Rev. 135 B, 843 (1964).

TABLE I

Some self consistent solutions for

$$\frac{g_{\Sigma\Lambda\pi}^2}{4\pi} = 11 \;, \quad \frac{g_{\Sigma\Sigma\pi}^2}{4\pi} = 4 \;.$$

Values of s and Γ are given in units of π^2 and Mev respectively. κ is dimensionless. Some solutions close to the given solutions with slightly different input-output self consistency have also been obtained.

s_{M_1}	s_{M_2}	s_{Rin}	s_{Rout}	κ_{in}	κ_{out}	Γ_{in}	Γ_{out}
88	65	98.5	98.1	10	10.1	17.6	16
78	65	115.9	115.7	12.0	12.0	155.6	153.7
67	62	128.8	128.6	15.0	14.9	380.7	374.3
88	45	99	98.6	10.4	10.4	20.3	18.4

FIGURE CAPTION

Singularities of the partial wave amplitude in the s-plane. Values of s are given in units of π^2.