

DOUBLE COMPTON EFFECT WITH HIGH INTENSITY RADIATION*

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A semiclassical calculation of the double Compton effect of an electron subjected to a high-intensity, monochromatic, linearly-polarized electromagnetic wave (laser beam) is presented. The cross section for the simultaneous scattering of two photons of the laser beam and their conversion into one photon of twice the frequency (second harmonic generation) will be discussed in detail. The results may be summarized as follows: (1) there is no intensity-dependent frequency shift for the scattered photons; (2) the cross section is a very sensitive function of the laser beam frequency ω_c and the initial velocity of the electron; (3) for present day laser beam intensities and available electron velocities the total cross section turns out to be about 10^2 times the Thomson cross section ($6.65 \times 10^{-25} \text{ cm}^2$) under favorable conditions and is therefore readily amenable to observation; and (4) the calculations predict a marked dependence of the differential as well as the total cross section on the angle between the initial velocity of the electron and the direction of polarization of the laser beam.

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1) Introduction. Quantum mechanical theories of the interaction of high intensity radiation with matter have been studied by a variety of authors ¹⁾. Not surprisingly the interaction of a free electron with high intensity fields has drawn considerable attention. The Compton effect is the simplest of all radiation processes and therefore modifications of this effect due to the presence of high intensity radiation fields are of considerable theoretical interest. What has been found and generally agreed upon is that with increasing intensity I of the laser beam the cross section for Compton scattering will decrease more and more and at the same time other scattering channels will open up. These new channels are 2 photon, 3 photon, etc., scatterings by the electron; and since the scattered photon is connected to the initial photons (i.e., the 2 photons, 3 photons, etc., which are removed from the laser beam) via energy and momentum conservation, the frequency of the scattered photon will be $2\omega_0$, $3\omega_0$, etc., ω_0 being the laser beam frequency, provided that $\hbar\omega_0 \ll mc^2$ a relation which is certainly satisfied for visible light. Because of this fact this type of generalized Compton effect is called harmonic production. What is not agreed upon among various authors is the angular dependence of the differential cross section for harmonic production as well as its magnitude ²⁾. There is also some argument in the literature about an intensity dependent frequency shift. Brown and Kibble ³⁾ as well as Goldman ³⁾ have found such a shift whereas Fried and Eberly ³⁾ did not. Subsequently it has been shown by Fried ⁴⁾ that the so-called Volkov solutions for a Dirac electron subjected to a monochromatic classical electromagnetic field ⁵⁾ (used in the calculations of both Brown and Kibble and Goldman) do not have the correct asymptotic

conditions, i.e., they do not represent free electrons at $t = \pm \infty$ and it has been shown by the author that taking correct initial conditions, no intensity dependent frequency shift occurs⁶⁾.

In the following we will give a detailed account of a calculation of the cross section for the simultaneous scattering of two photons out of the laser beam (second harmonic production) in the experimentally interesting case $m v \gg \epsilon^{-1} \hbar \omega_0$, where v is the initial electron velocity. We will find that the cross section is a very sensitive function of the initial electron velocity, a fact that is not surprising since the effect is relativistic in origin. Previous calculations (Goldman and Brown and Kibble¹⁾) indicated a rather small cross section for the second harmonic generation quite in contradistinction to our findings (see Section 4). But since these authors did not use the correct wave functions for the electron, there is really no comparison possible. As a matter of fact, the wave functions we will be using, i.e., wave functions with the correct initial conditions differ widely from those used by earlier authors. In Section 2 the appropriate wave functions for an electron subject to an external electromagnetic plane wave field will be derived. In Section 3 we will proceed by calculating the relevant matrix elements for the first harmonic generation and the differential cross section for this process will be obtained. In Section 4 a detailed discussion of the cross section will be given. Certain approximations will be necessary in the course of the calculation and these will be clearly stated. These approximations, however, happen to be well realized for a wide range of experimental conditions.

2) The Wave Functions. A plane, linearly polarized (in the x direction) electromagnetic wave propagating into the z direction may be represented by the vector potential

$$\underline{A} = \varepsilon \frac{m c^2}{e} \underline{e}_x \cos k_c (z - ct), \quad (1)$$

Here $\underline{e}_x$ is a unit vector in the x direction of the observer's frame and ε is a dimensionless quantity given by

$$\varepsilon^2 = \frac{4\pi e^2 \hbar \bar{n}}{m^2 c^2 \omega_c}, \quad (2)$$

with $\bar{n}$ the number density of photons of the beam represented by eq. (1).

Since the intensity I of the laser beam (1) is obviously given by

$$I = \hbar \omega_c \bar{n} c, \quad (3)$$

ε can be related to the intensity I in a simple manner and hence measures the strength of the radiation field.

The Dirac equation of an electron subject to the radiation field

(1) reads:

$$i\hbar \frac{\partial \psi}{\partial t} = \left\{ \frac{c\hbar}{i} \underline{\alpha} \cdot \nabla + mc^2 \beta + E mc^2 \underline{\alpha}_r \cos k_r (z - ct) \right\} \psi, \quad (4)$$

with the usual Dirac matrices $\underline{\alpha}$ and β . We seek a solution of the form

$$\psi_r(\underline{h}, \underline{r}, t) = e^{i\underline{h} \cdot \underline{r} - i\Omega_r t} \varphi_r(\underline{h}, z - ct, t), \quad (5)$$

with

$$\Omega_r = c \left(h^2 + \left(\frac{mc}{\hbar} \right)^2 \right)^{1/2} \quad (6)$$

The conditions on the spinor φ are:

$$\lim_{E \rightarrow 0} \varphi_r(\underline{h}, z - ct, t) = u_r(\underline{h}), \quad (7)$$

$$\lim_{t \rightarrow +\infty} \varphi_r(\underline{h}, z - ct, t) = u_r(\underline{h}), \quad (8)$$

where $u_r(\underline{k})$ is just the free particle spinor with wave vector $\underline{k}$ and spin polarization r . The meaning of condition (7) is, of course, that without the electromagnetic field (1) we are dealing with a free electron and that of condition (8) is that the field (1) is switched on at $t = 0$. In other words, the solution (5) and its derivatives must be continuously smoothed on to the solution

$$\psi = e^{i \frac{\hbar \cdot r}{\hbar} - i \omega t} u_r(\underline{k}), \quad (9)$$

for $t < 0$. Putting

$$\psi_r(\underline{k}, z-ct, t) = \sum_{n=-\infty}^{+\infty} e^{i n k_0 (z-ct)}$$

$$\times F_n^{(r)}(\underline{k}, t), \quad (10)$$

we find that condition (8) is satisfied if

$$F_n^{(r)}(\underline{k}, 0) = \delta_{n0} u_r(\underline{k}) \quad (11)$$

and the Dirac equation (4) is satisfied provided that the spinors obey the following equations:

$$\left[\Omega - c \underline{h} \cdot \underline{\alpha} - \frac{mc^2}{\hbar} \beta + n c h_0 (1 - \alpha_z) \right] F_n^{(r)} + i \frac{d F_n^{(r)}}{dt} = \epsilon \frac{mc^2}{2\hbar} \alpha_x (F_{n+1}^{(r)} + F_{n-1}^{(r)}) \quad (12)$$

It is indeed easy to check that ψ as given by eqs. (9) and (10) is continuous at $t = 0$ and so are both its spacial and time derivatives.

Since the set of equations (12) is difficult to solve in general, we are introducing at this point the first of the approximations mentioned in the Introduction. The approximation consists in treating the parameter

ϵ as small and proceeding with a perturbation expansion in which the term on the right-hand side of eqs. (12) is considered small.¹¹⁾ This is actually no real restriction since even for the highest available laser beam intensities $I \approx 10^9$ watts cm^{-2} for red light ($\omega \approx 3 \cdot 10^{15}$ sec^{-1}), ϵ turns out to be only of the order of magnitude 10^{-4} . From the structure of the eqs. (12) we see then immediately that the spinors $F_n^{(r)}$ can be written in general:

$$F_n^{(r)}(\underline{h}, t) = \epsilon^{|n|} \sum_{m=0}^{\infty} \epsilon^m F_{n,m}^{(r)}(\underline{h}, t) \quad (13)$$

From eq. (13) and the condition (11) we have then in 0th order

$$F_{n,0}^{(r)0} = \int_{n_0} u_r(\underline{h}). \quad (14)$$

By simple iteration (treating ε as expansion parameter) we are now able to obtain the solution of the eqs. (12). We will show later (Section 3) that we only need the spinors $F_n^{(r)}$ to second order in ε . From eq. (13) we see then that we really only need $F_0^{(r)}$, $F_{\pm 1}^{(r)}$ and $F_{\pm 2}^{(r)}$. It is a matter of simple algebra to actually perform the iteration in ε up to second order. We will merely quote the result here. With the abbreviation

$$A_n = \Omega - c \underline{h} \cdot \underline{\alpha} - \frac{mc^2}{\hbar} \beta + n c h_0 (1 - \alpha_z), \quad (15)$$

we find for the various spinors $F_n^{(r)}$ correct to order ε^2 :

$$F_0^{(r)}(\underline{h}, t) = \left\{ 1 + \frac{\varepsilon^2}{4} \left(\frac{mc^2}{\hbar} \right)^2 e^{i t A_0} \int_0^t d\tau \right. \\ \times e^{-i \tau A_0} \alpha_x \left[A_{+1}^{-1} (1 - e^{i \tau A_{+1}}) + A_{-1}^{-1} (1 - e^{i \tau A_{-1}}) \right] \\ \left. \times \alpha_x \right\} u_r(\underline{h}), \quad (16)$$

$$F_{\pm 1}^{(r)}(\underline{h}, t) = \frac{\varepsilon}{2} \frac{m c^2}{\hbar} A_{\pm 1}^{-1} (1 - e^{i t A_{\pm 1}})$$

$$X \propto_x u_r(\underline{h}), \quad (17)$$

and finally:

$$F_{\pm 2}^{(r)}(\underline{h}, t) = \frac{\varepsilon^2}{4} \frac{(m c^2)^2}{i \hbar^2} e^{i t A_{\pm 2}} \int_0^t dt e^{-i \tau A_{\pm 2}}$$

$$X \propto_x A_{\pm 1}^{-1} (1 - e^{i \tau A_{\pm 1}}) \propto_x u_r(\underline{h}). \quad (18)$$

Eqs. (16), (17) and (18) in conjunction with eqs. (9) and (10) represent the solution of eq. (4) for $t > 0$. Eq. (9), of course, is the free particle solution valid for $t < 0$. Physically this signifies, as we have stressed before, that the field (1) is switched on at $t = 0$. At this point we like to compare our solution given above to the Volkov solution⁵⁾ used by other authors which, as we know, does not satisfy the initial condition (8).

In our notation up to first order in ε the Volkov solution with the field (1) is given by:

$$\begin{aligned} \psi_V = & \left\{ 1 + \varepsilon \frac{m c}{\hbar (h_z - c \Omega_1)} \left[\alpha_x (1 - \alpha_z) \cosh h_0 (z - ct) \right. \right. \\ & \left. \left. + \frac{h_x}{h_0} \sin h_0 (z - ct) \right] \right\} u_r(\underline{h}) e^{i \underline{h} \cdot \underline{r} - i \Omega_1 t}, \quad (19) \end{aligned}$$

an expression which bears little resemblance to the corresponding first-order equation (17).

3) Calculation of the Differential Cross Section for the Double Compton

Effect. The transition we wish to calculate is one in which a photon is created outside the laser beam. Since the laser beam is treated as a classical field, we must treat this process as a spontaneous emission of a photon by the electron, which in turn is moving in the laser beam.

But this means that we can use simple first-order radiation theory as amply described elsewhere⁷⁾. The matrix element for a spontaneous emission of a photon with wave vector $\underline{k}$ and polarization γ accompanied by a transition of the electron from the initial state $\psi_r(\underline{h}, \underline{r}, t)$ to the final state $\psi_r(\underline{h}', \underline{r}, t)$ is then given by

$$M(\underline{h}, \underline{r}; \underline{h}', \underline{r}'; \underline{k}, \gamma) = -\frac{ie}{\hbar} \int_0^t d\tau \int d^3r$$

$$\times \bar{\psi}_r(\underline{h}, \underline{r}, \tau) \underline{\alpha} \cdot \underline{A}_r(\underline{k}) \psi_r(\underline{h}', \underline{r}, \tau); \quad (20)$$

where the electron wave functions are given by eqs. (5) and (10) and

$$\underline{A}_r(\underline{k}) = \left(\frac{2\pi \hbar c}{V K} \right)^{1/2} \underline{e}_{\underline{k}}^{(\gamma)} e^{i \underline{k} \cdot \underline{r} - i \omega_k t}, \quad (21)$$

is the relevant part of the vector potential of the quantized radiation field and $\underline{\epsilon}_K^{(\gamma)}$ is a unit polarization vector. Inserting eqs. (9) and (10) into eq. (20) gives easily:

$$M(\underline{h} \nu; \underline{h}' \nu'; \underline{K} \gamma) = -\frac{ie}{\hbar} \left(\frac{2\pi \hbar c}{V K} \right)^{\frac{1}{2}} \sum_{n, n'} \delta(\underline{h} + (n-n')\underline{h}_0, \underline{\epsilon}_z - \underline{h}' - \underline{K}) \int_0^t d\tau \bar{F}_n^{(r)}(\underline{h}, \tau) \times \underline{\alpha} \cdot \underline{\epsilon}_K^{(\gamma)} F_{n'}^{(r)}(\underline{h}', \tau) \exp\{i\tau[\omega_L + (n-n')c h_0 - \omega_L' - c K]\} \quad (22)$$

The δ -function in eq. (22) is a Kronecker δ since we are using a large but finite quantization volume V for the photons and $\underline{\epsilon}_z$ is a unit vector in z direction (the direction of propagation of the laser beam). From expression (22) we see that the matrix element (20) is composed of various parts which signify physically the removal from the beam of 1, 2, 3, etc., photons of frequency $\omega_L = c h_0$ and their subsequent conversion into one photon outside the laser beam. Since we are particularly interested in the 2 photon effect, we put $n - n' = 2$ (n and n' range independently from $-\infty$ to $+\infty$) and find from eq. (13) that we have to know the spinors F_n only to order ϵ^2 for the leading term in an expansion in ϵ . The differential cross section for the two photon scattering is given in a well known manner from the square of the relevant amplitude by dividing with the flux $\bar{n} c$ of the laser beam.

If we also sum over the final electron states, we obtain after a little algebra for the differential cross section of the two photon-one photon scattering per frequency interval dK and per solid angle:

$$\begin{aligned} \frac{d\sigma_2(\underline{h}, r; \underline{K}, r)}{dK d\Omega} &= \varepsilon^2 \frac{e^4 K}{\pi m \omega_0 \hbar d\Omega'} \sum_r \lim_{t \rightarrow +\infty} \\ &\times \frac{d}{dt} \left| \int_0^t d\tau e^{i\tau(\Omega + 2\omega_0 - \Omega' - cK)} \sum_{n=0} F_{n,0}^{(r)}(\underline{h}, \tau) \right. \\ &\times \left. \underline{\alpha} \cdot \underline{\underline{K}} F_{n-2,0}^{(r)}(\underline{h}', \tau) \right|^2 \end{aligned} \quad (23)$$

Eq. (23) is correct to lowest order in ε . $\underline{h}'$ is given by momentum conservation:

$$\underline{h}' = \underline{h} + 2 \hbar_0 \underline{e}_z - \underline{K} \quad (24)$$

The spinors F_n are normalized according to ⁸⁾. Ω' is of course given by eq. (6) with $\underline{h}$ replaced by $\underline{h}'$. It is now our task to compute the matrix elements in spin space occurring in eq. (23) using the functions (16), (17) and (18) as defined by eq. (13) and derived in Section 2. The evaluation is facilitated by the following fact. In disentangling the matrix element (the sum over n in eq. (23)) we will find various terms, some time dependent and some time independent. Now, we only need to retain

the time independent terms. This is so because in the asymptotic limit $t = \infty$ we will get terms which are proportional to t and terms which are oscillatory as explained in detail in 7). The oscillatory terms we discard at once, their time average being zero, and of the terms which are time proportional we only retain the one corresponding to the time independent part of the matrix element because we have in general

$$\lim_{t \rightarrow \infty} \frac{d}{dt} \left| \int_0^t d\tau e^{i\omega\tau} \right|^2 = 2\pi \delta(\omega), \quad (25)$$

so that we obtain from the time independent terms of the sum over n in eq. (23):

$$\text{time independent part} \sim \delta(\Omega + 2\omega_0 - \Omega' - cK), \quad (26)$$

i.e., energy conservation, all other time proportional terms being multiplied with different δ -functions which cannot be satisfied simultaneously with eq. (24) for positive values of K . At this point we like to introduce two more simplifications. First, we assume $v \ll c$. If the initial velocity of the electron v is small compared to c , then according to eqs. (24) and (26) also, the final velocity v' is small compared to c . Second, we assume $m v \gg c^{-1} \hbar \omega_0$ or the electron

momentum is large compared to the momentum of a beam photon. This is of course no real restriction since it is always true down to very small electron energies indeed. Under these restrictions we can now put

$$\left. \begin{aligned} h &\approx h', \\ ck &\approx 2w_0. \end{aligned} \right\} (27)$$

We are now in a position to evaluate the matrix element of eq. (23). Inserting the expressions (16), (17) and (18) into eq. (23), keeping the approximation (27) in mind and retaining only the time independent part in lowest order in v/c , we will obtain certain expressions which we will write down presently. The calculation is facilitated by noting that for the matrix A_n defined in eq. (15) we have

$$e^{it} A_n = e^{it(\omega_0 + n\omega_0)} \left\{ \cos \omega_0 t - i \left(c^{-1} \underline{v} \cdot \underline{\alpha} + \alpha^{-1} \beta \right) \sin \omega_0 t \right\}, \quad (28)$$

and for its inverse:

$$A_n^{-1} = \frac{n}{|n|} (2\hbar\omega_0)^{-1} \left(1 - \frac{\underline{v}}{c} \cdot \underline{e}_z \right)^{-1} \times \left(c^{-1} \underline{v} \cdot \underline{\alpha} + \beta + \alpha \right), \quad (29)$$

with

$$\alpha = \left(1 + \frac{v^2}{c^2}\right)^{\frac{1}{2}} \quad (30)$$

Eqs. (28) and (29) have been derived in the approximation $h_0 \ll h$ as discussed above. We now list the matrix elements as obtained according to the prescriptions given above:

$$F_{1,0}^{(r)} \underline{\underline{\alpha}} \cdot \underline{\underline{e}}_{\underline{\underline{k}}}^{(r)} F_{-1,0}^{(r')} = \bar{u}_r(\underline{\underline{h}}) T_1 u_r(\underline{\underline{h}}'), \quad (31a)$$

$$F_{2,0}^{(r)} \underline{\underline{\alpha}} \cdot \underline{\underline{e}}_{\underline{\underline{k}}}^{(r)} F_{0,0}^{(r')} = \bar{u}_r(\underline{\underline{h}}) T_2 u_r(\underline{\underline{h}}'), \quad (31b)$$

$$F_{0,0}^{(r)} \underline{\underline{\alpha}} \cdot \underline{\underline{e}}_{\underline{\underline{k}}}^{(r)} F_{-2,0}^{(r')} = \bar{u}_r(\underline{\underline{h}}) T_3 u_r(\underline{\underline{h}}'), \quad (31c)$$

with

$$T_i = - \left(\frac{mc}{4\hbar h_0} \right)^2 \left(1 - c^{-1} \underline{\underline{v}} \cdot \underline{\underline{e}}_{\underline{\underline{z}}} \right)^{-2} \left\{ \alpha_x \left(c^{-1} \underline{\underline{v}} \cdot \underline{\underline{\alpha}} \right. \right. \\ \left. \left. + \beta + \alpha \right) \underline{\underline{\alpha}} \cdot \underline{\underline{e}}_{\underline{\underline{k}}}^{(r)} \left(c^{-1} \underline{\underline{v}} \cdot \underline{\underline{\alpha}} + \beta + \alpha \right) \alpha_x \right.$$

$$\begin{aligned}
& + \frac{1}{4} \alpha_x \left[1 - (\alpha^{-1} \underline{v} \cdot \underline{\alpha} + \alpha^{-1} \beta) \right] (\alpha^{-1} \underline{v} \cdot \underline{\alpha} + \beta + \alpha) \\
& \times \underline{\alpha} \cdot \underline{\Sigma}_K^{(r)} (\alpha^{-1} \underline{v} \cdot \underline{\alpha} + \beta + \alpha) \left[1 - (\alpha^{-1} \underline{v} \cdot \underline{\alpha} \right. \\
& \left. + \alpha^{-1} \beta) \right] \alpha_x \Big\}, \quad (32a)
\end{aligned}$$

$$\begin{aligned}
T_2 = & - \left(\frac{mc}{4\hbar h_0} \right)^2 (1 - \alpha^{-1} \underline{v} \cdot \underline{\Sigma}_z)^{-1} \alpha_x (\alpha^{-1} \underline{v} \cdot \underline{\alpha} + \beta + \alpha) \\
& \times \alpha_x (2 \alpha^{-1} \underline{v} \cdot \underline{\alpha} + 2 \alpha^{-1} \beta + \alpha^2 + \alpha^{-2}) \underline{\alpha} \cdot \underline{\Sigma}_K^{(r)}, \quad (32b)
\end{aligned}$$

$$T_3 = T_2^\dagger. \quad (32b)$$

Since v/c is small, we will take the Pauli limit for the matrix elements (31). It will turn out that the largest contributions are of the order $(v/c)^3$. The Pauli limit is described in detail in 9).

Here it suffices to say that a product of α matrices with an odd number of terms couples large and small components of the electron spinor u_r ; a product of an even number of α matrices couples only the large components and the connection between small and large components is in the Pauli limit:

$$u_S = (2c)^{-1} \underline{v} \cdot \underline{\sigma} u_L, \quad (33)$$

where u_S and u_L are two component spinors and $\underline{\sigma}$ consists of the usual Pauli matrices. We will not give the straightforward but somewhat tedious algebra, but merely quote the result.

With the definition

$$\left. \begin{aligned} \underline{a} &= a_x \underline{e}_x + a_y \underline{e}_y + a_z \underline{e}_z, \\ \underline{\hat{a}} &= a_x \underline{e}_x - a_y \underline{e}_y - a_z \underline{e}_z, \end{aligned} \right\} \quad (34)$$

we obtain to lowest order in v/c :

$$\Pi = \Pi_1 + \Pi_2 + \Pi_3 = - \left(\frac{mc}{4 \hbar h_0} \right)^2 \left(\frac{v}{c} \right)^3$$

$$X \left(9 \underline{x} + \underline{\hat{x}} + 6 \underline{x} \cdot \underline{\hat{x}} \underline{x} + 2 \underline{x} \cdot \underline{\hat{x}} \underline{\hat{x}} \right) \cdot \underline{e}_K^{(x)}. \quad (35)$$

Here $\underline{x}$ is a unit vector in the direction of the velocity of the electron and $\underline{\hat{x}}$ a unit vector generated from $\underline{x}$ by a rotation about the X axis by 180° according to the definition (34).

Collecting all results, observing eqs. (2), (3), (25), (27), (35) and the expression for the cross section (23), we obtain finally for the differential cross section for the conversion of 2 photons into one of twice the frequency

$$\frac{d\sigma(\underline{h}; \underline{K}, \gamma)}{d\Omega} = \frac{\pi}{16} r_0^2 \frac{I}{c} \left(\frac{emv^3}{\hbar^2 \omega_0^3} \right)^2 \times \left[(9\underline{x} + \underline{\hat{x}} + 6\underline{x} \cdot \underline{\hat{x}} \underline{x} + 2\underline{x} \underline{\hat{x}} \underline{\hat{x}}) \cdot \underline{e}_k^{(r)} \right]^2 \quad (36)$$

Here r_0 is the classical electron radius and I/c is the energy density of the laser beam. To obtain eq. (36) we also have averaged over the initial electron spin.

4) Discussion of the Cross Section. Let us list first the restrictions under which the expression for the cross section (36) is valid. It is clear from the derivation that we must have for one

$$v \ll c, \quad (37)$$

i.e., non-relativistic electrons. Next, we must have

$$mv \gg \hbar^{-1} \omega_0, \quad (38)$$

i.e., the electron momentum must be large compared to the photon momentum. This constitutes no experimental restriction. Even electrons with an energy of 10^{-2} eV satisfy requirement (38) if $\omega_0 \approx 3 \cdot 10^{15} \text{ sec}^{-1}$ (red light). A third requirement is obtained if we look at the wave functions developed in Section 2. There we stated that a perturbation expansion in the parameter ξ eq. (2) has been adopted. But a glance at the wave function eq. (17) and at the expression for the inverse of the matrix A eq. (29) shows that the restriction is not $\xi \ll 1$ but rather

$$\xi \frac{m v c}{\hbar \omega_0} \ll 1. \quad (39)$$

Because of the inequality (38), eq. (39) imposes a much greater restriction on ξ than the simple statement $\xi \ll 1$. Incidentally, the restriction (39) shows that we are working in the quantum limit; letting $\hbar$ go to zero condition (39) cannot be satisfied anymore. Indeed the cross section (36) contains $\hbar$ in an essential way. This situation is quite different from the single photon Thomson scattering in which case quantal and classical calculations give the same result.

Summing over the polarization in the final state of the scattered photon, we obtain from eq. (36) with the definition ($\underline{e}$ is a unit vector in the direction of propagation of the scattered photon)

$$\underline{S} = 9\underline{x} + \underline{\hat{x}} + 6\underline{x} \cdot \underline{\hat{x}} \underline{x} + 2\underline{x} \cdot \underline{\hat{x}} \underline{\hat{x}}, \quad (40)$$

the following result¹⁰⁾

$$\frac{dG_2(\underline{h}, \underline{K})}{d\Omega} = \frac{\pi}{16} r_0^2 \frac{I}{c} \left(\frac{emv^3}{\hbar^2 \omega_0^3} \right)^2 \times \left[S^2 - (\underline{S} \cdot \underline{e})^2 \right], \quad (41)$$

and for the total cross section

$$G_2(\underline{h}; \underline{K}) = G_0 \frac{\pi}{16} \frac{I}{c} \left(\frac{emv^3}{\hbar^2 \omega_0^3} \right)^2 S^2, \quad (42)$$

with $G_0 = 8\pi r_0^2/3$ the Thomson cross section. The vector $\underline{S}$ eq. (40) gives rise to an interesting effect. For let the direction of the velocity of the electron be in the x direction (in direction of the polarization of the laser beam). Then we have from (40) and (34)

$$S^2 = S_1^2 = 326, \quad (43)$$

and if the direction of the electron velocity is perpendicular to the direction of the polarization of the laser beam, we have

$$S^2 = S_2^2 = 16, \quad (44)$$

and from eq. (42) we see that the intensity of the scattered light differs by an order of magnitude depending on whether the electron velocity is parallel with, or perpendicular to, the direction of polarization of the laser beam ($S_1^2 / S_2^2 = 20.25$).

To conclude, let us turn to the magnitude of the cross section (42). The cross section is a very sensitive function of v/ω_0 . But we cannot arbitrarily increase v and decrease ω_0 to obtain a large cross section since the restrictions (37), (38), and in particular (39), have to be met. But as an example, let us take $\omega_0 = 2 \cdot 10^{15} \text{ sec}^{-1}$, $v = 10^9 \text{ cm sec}^{-1}$ and $I = 10^7 \text{ watts cm}^{-2}$. In this case $\epsilon m v c / \hbar \omega_0 \approx 0.06$ so that eq. (42) is applicable and the total cross section turns out to be $\sigma_1 \approx 50 \sigma_0$, i.e., 50 times the Thomson cross section for

the case in which the velocity of the electron and the polarization of the laser beam are perpendicular, and it is about 500 times the Thomson cross section in the opposite case.

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References and Footnotes

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- 2) Brown and Kibble (Ref. 1) differ from Fried and Eberly (Ref. 1).
- 3) See Ref. 1.
- 4) Z. Fried, private communication.
- 5) D. M. Volkov, Z. Physik 94, 250 (1935).
- 6) O. von Roos, Space Programs Summary No. 37-31, Vol. IV, Jet Propulsion Laboratory (unpublished).
- 7) O. von Roos, Phys. Rev., 135, A43 (1964).
- 8) S. S. Schweber, H. A. Bethe, F. de Hoffmann, Mesons and Fields, Row, Peterson and Co., 1955.
- 9) H. A. Bethe and E. E. Salpeter, Quantum Mechanics of One and Two Electron Atoms, Academic Press, New York, 1957.
- 10) For a general direction of polarization S^2 can be written

$$S^2 = 4 \left[4 + 13(\underline{x} \cdot \underline{f}_z)^2 + 16(\underline{x} \cdot \underline{f}_z)^4 + 48(\underline{x} \cdot \underline{f}_z)^6 \right]$$
 with the unit vector of polarization $\underline{f}_z$.
- 11) A more precise statement is given in Eq. (39).