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THE DIRECTIONAL RADIATIVE CHARACTERISTICS
OF CONICAL CAVITIES AND THEIR RELATION
TO LUNAR PHENOMENA

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Introduction

The directional radiative properties of surfaces are almost always ne-
glected in the calculation of thermal radiative transfer. Yet it has long

been recognized that few, if any, real surfaces follow the idealized scatter-

ing laws which are almost universally assumed; that is, either specular

(mirrorlike) or diffuse (cosine law) reflections, or some combination of these

as proposed in Ref. [1].

One surface which reflects in an anomalous manner is that of the moon.

If the moon's characteristics were those of an ideal diffuse reflector, the

brightness of the surface should decrease in proportion to the cosine of the

angle between the viewer and the normal to a point on the lunar surface. This,

of course, implies that the moon should appear brightest at its center, and quite

dark near the limb. Anyone who has viewed the full moon knows that this is not

the case.

Astronomers have puzzled for many years in an effort to determine the

structure of the moon's surface by comparison of the reflective characteristics

of various substances to the lunar characteristics, but without notable success.

(For example, see Refs. [2 to 7]).

Radar reflection studies of the lunar surface, which began shortly after

the close of World War II, shed some light on the structure of the moon's face.

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Daniels [8] for example, states that "a general picture of the lunar surface deduced from radar data is that of a surface pockmarked by telescopically invisible meteor pits having a wide range of sizes down to a limit that has not yet been determined."

Recent closeup photograhs of the surface of the moon taken by the Ranger lunar probes appear to bear out this view.

Radar reflectivity measurements are made at such wavelengths that the results may not accurately portray results which occur in the visible range of the spectrum [2]. In the visible range, much smaller detail (on the order of 1 mm to a few cm) on the surface can affect the scattering, and seems to do so quite strongly. Orlova [6] presents data on the lunar surface reflectivity that shows strong reflection in the direction of incident solar radiation.

Because it is known from both analytical and experimental studies (see, for example, Refs. [9] and [10]) that cavities of various geometries can reflect strongly in the direction of incident radiation, an analytical study of the directional absorptivity characteristics of conical cavities was undertaken. It was felt that conical cavities would provide a reasonable thermal model of the meteor craters or other cavities to be found on the surface of the moon in addition to the academic interest of such results. It might then be possible to model completely the reflectivity characteristics of the lunar surface, including the dependence on angle of incident solar radiation.

In the present paper, a beam of parallel radiation is taken as striking a right circular conical cavity at a given angle of incidence to the cone axis. The cone is assumed to have a diffusely reflecting surface and has a given cone angle. A straightforward Monte Carlo analysis of this case is used to determine the directional reflectivity of the cone. Parameters varied are the cone angle, surface

absorptivity, and angle of incidence of the solar radiation. Comparison is then made to the lunar characteristics.

Analysis

The geometry analyzed is a right circular conical cavity having diffuse, gray walls. Parallel radiation assumed to consist of discrete bundles of energy is impinging on the conical cavity. Polarization effects are neglected and the base radius is taken as unity.

Method of Solution

The Monte Carlo method is used to find the apparent absorptivity and the angular distribution of reflected energy from the conical cavity. This method consists of following discrete bundles of incident energy through their probable paths in the region of the cone, taking into account the diffuse reflections within the cone and the absorption of energy bundles at the gray internal surface. Those bundles not absorbed within the cone are tallied in the angular increment $(\Delta \gamma_i', \Delta \zeta_j')$ on a fictitious hemisphere that subtends the mouth of the cone from which they leave (see Fig. 1).

The apparent absorptivity of the cone for a given set of parameters - surface absorptivity, cone angle, and incident angle of radiation - is then calculated as the fraction of the total incident bundles absorbed within the cone. The directional reflectivity $\rho_{\zeta, \zeta', \gamma'}$ is calculated from the number of bundles that leave the cone per unit solid angle. This is equivalent to the reflected energy in a unit solid angle around (γ', ζ') divided by the total energy incident on the cone.

The major difficulty in this type of analysis is finding the optimum geometrical relations between various imaginary triangles within the cone in order to describe the paths of the bundles in terms of the parameters of the problem.

This is mainly an exercise in analytical geometry, but in this case leads to equations for the bundle paths which are transcendental in form. Because the Monte Carlo technique depends on many repetitive calculations, this could lead to difficulties for complex problems. A variety of methods exist for circumventing this pitfall, but die to the simplicity of the present problem, the transcendental equations were solved by a Newton-Raphson technique modified to include third-order terms. This is relatively time-consuming, but because the program ran rapidly overall it was adequate for this problem.

A complete derivation of the equations and a flow chart for the solution are given in Ref. [11]. The amount of computer time required is primarily a function of the cone angle, θ, because of the increased number of internal reflections for small cone angles. For $\theta = 1^{\circ}$, the running time for one value each of absorptivity and incident angle and for 50,000 particle histories was about nine minutes. The same program, but for $\theta = 179.8^{\circ}$, ran for 3.5 minutes. These runs gave the entire distribution of directional reflectivity over the hemisphere and the apparent absorptivity. Generally, running time was less than 5 minutes for each set of parameters and 50,000 particle histories.

The size of the angular increments affected the number of cases needed to get meaningful results. Various combinations of $\Delta\gamma'_i$ and $\Delta\zeta'_j$ were tried and evaluated.

The "best" combination is, of course, a compromise: smaller increments yield more values for plotting and show the variation of reflectivity with angle more clearly, but require a prohibitively high number of particle histories to obtain statistically meaningful data for each increment. On the other hand, choice of larger increments shortens the computer running time by reducing the number of histories necessary to obtain sound statistics; at the same time, reflectivity variations as a function of angle are obscured, (see Ref.[11]). For

the directional reflectivity calculations, $\Delta\gamma' = 15^{\circ}$ and $\Delta\zeta' = 6^{\circ}$ proved a staisfactory compromise.

Discussion of Results

Conical cavity results

Figure 2(a) shows the variation of apparent absorptivity of a 60° cone for various values of surface absorptivity and incident angle. All results are shown in the plane normal to the cone base and parallel to the incident radiation. Figure 2(b) shows how the apparent absorptivity varies as the cone angle is changed. As the cone angle becomes large, the results approach those for a flat plate; that is, the apparent absorptivity nears the surface absorptivity. For small cone angles, the multiple internal reflections cause an increase in apparent absorptivity.

Figure 3 shows the effect of different surface absorptivities on the directional reflectivity of a specific conical cavity. The results shown are in the plane containing the cone axis and the direction of incident radiation. The curves are not similar in the mathematical sense, because proportionately greater attenuation occurs at those angles where the reflected radiation has undergone multiple reflections. In Ref. [9], it is shown that the directional absorptivity for energy incident at a given angle is equal to the directional emissivity at the same angle for any isothermal gray cavity. The results of Figs. 3 and 4 can thus be used for determining the apparent emissivities of cones.

The reflectivity is demonstrated in the four parts of Fig. 4. Of interest here is the way in which the radiation is reflected strongly in the direction of the incident radiation. As the cone angle is increased this effect becomes less noticeable, and the results approach those for a flat plate.

In Fig. 5, the calculated standard deviation around the Monte Carlo points

is demonstrated for a particular case to indicate the accuracy of the results.

A more complete set of results for conical cavities is given in Ref. [11].

Relation of the results to lunar phenomena

Because the behavior of the conical cavities studied here corresponds to that of the lunar surface, that is, strong reflections are present in the direction of incident radiation, it is of interest to compare directly the directional results of these two surfaces.

Bennett [12] follows a similar approach, modeling the reflective characteristics of the lunar surface by an approximate solution of the reflective characteristics of an idealized surface. His model consists of a series of spheroidal cavities with a diffuse plane surface between them. The expression given for the normalized brightness is

$$\left[\frac{\rho_{\zeta,\zeta'}}{\rho_{normal,\,\zeta'}}\right] = 0.55 \cos \zeta + 0.45 \, V \tag{1}$$

where ζ is the angle of incidence, ζ' the angle of reflection, and V is the fraction of projected area which is illuminated within the spheroidal cavity. The two constants (0.55 and 0.45) were evaluated from the data for the lunar surface viewed normally and Eq. (1) of course fits the data for this case quite well. However, for other viewing angles, the function gives a poor fit, and does not give maximum reflectivity at full moon.

Figure 6 presents Bennett's observed and computed results, and a favorable comparison is made with the results for a conical cavity for cone angle 30° and surface absorptivity of 0.500.

Orlova [4,6] has published experimental measurements of the directional reflectivity of the lunar surface. However, she normalized all results by the maximum value at zero angle of incidence, thereby allowing only a relative comparison of the results for different angles of incidence. Orlova's results for three

angles of incidence are presented in Fig. 7 as the solid lines, while the dotted lines are again for a conical cavity of cone angle 30° with a surface absorptivity of 0.5. Comparison could perhaps be made better by varying the cone angle and absorptivity to get optimum agreement if the lunar surface needed more accurate thermal modeling. In addition, a specular component of reflection on the conical surface might improve the relation.

It needs to be made clear that the aurthors do not hypothesize a lunar surface made up of cavities in the shape of right circular cones with diffusely reflecting surfaces. Known polarization effects of the moon's reflected radiation imply much about the microscopic composition of the surface which sheds doubt on a diffuse reflection model even for the microstructure [5] and the cone angles found to correlate here are substantially less than those expected of the only near-conical cavities known to exist; that is, meteor craters.

Halajian [13] discusses the experimental data available in relation to the lunar surface at some length, and concludes that a highly porous cohesive rock froth best correlates with all available data on the surface characteristics. The results herein tend to support the photometric evidence backing this view.

Conclusions

The directional reflectivity of a right circular cone with 30° cone angle and a surface absorptivity of 0.500 compares well with the experimental photometric results for the lunar surface.

From this it can be inferred that the lunar surface could have many cavities with steep walls, whose structure is larger than the wavelengths of visible light, but smaller than is visible to present earth-based or lunar-probe observations.

Results for conical cavities with cone angles near those for observed lunar craters do not correlate with observed lunar photometric results, implying that

these craters, even if of considerably smaller size than those observed to date, contribute little to the reflectivity characteristics of the moon.

Of analytical interest, it was found that the Monte Carlo technique worked well in this type of calculation, and is a useful tool for carrying out more complex problems of radiative interchange.

<div align="center">References</div>

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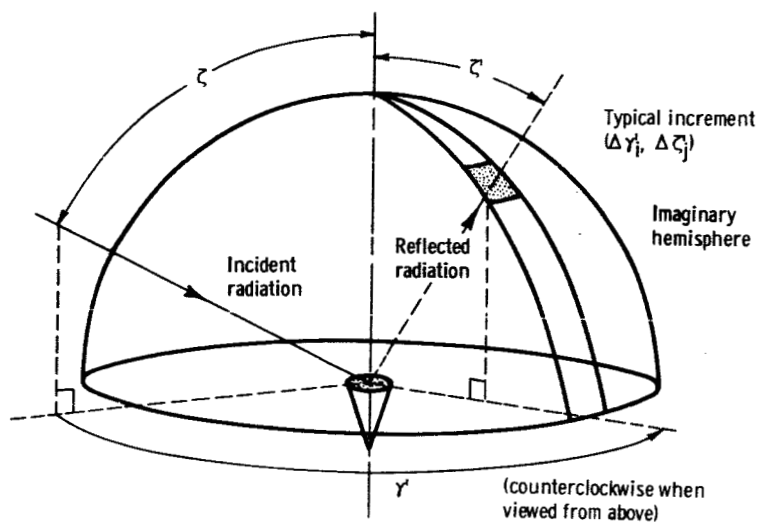
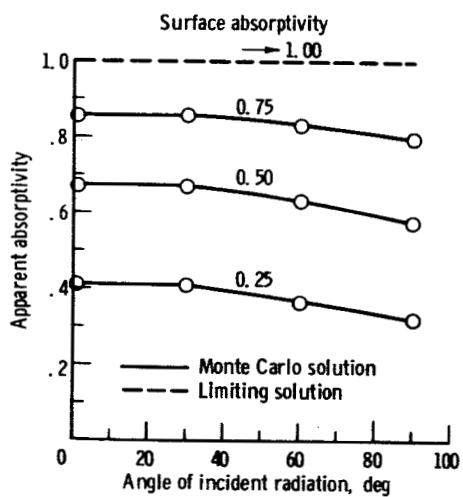Fig. 1 Coordinates for energy reflected from conical cavity.

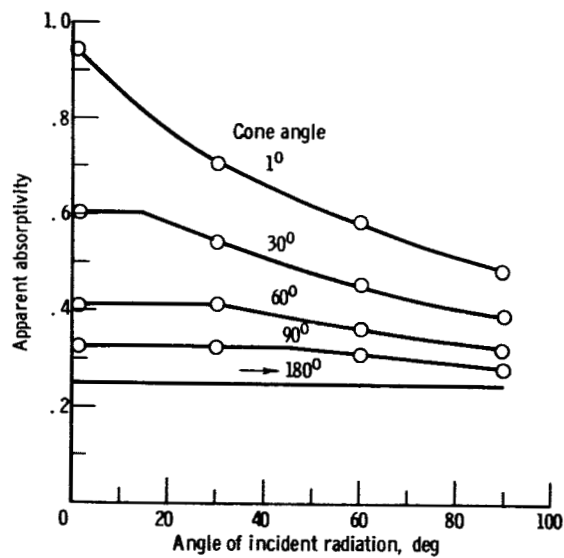Fig 2(a). Apparent absorptivity of conical cavity.
Cone angle, 60⁰.

Fig. 2(b) Apparent absorptivity of right circular cones.
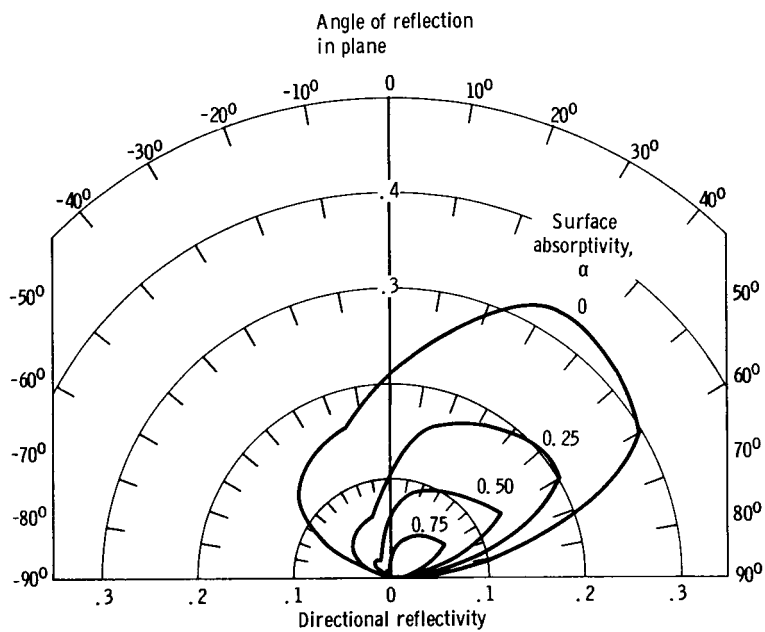Cone surface absorptivity, 0. 25.

Angle of reflection
in plane

Surface
absorptivity,
α

Directional reflectivity

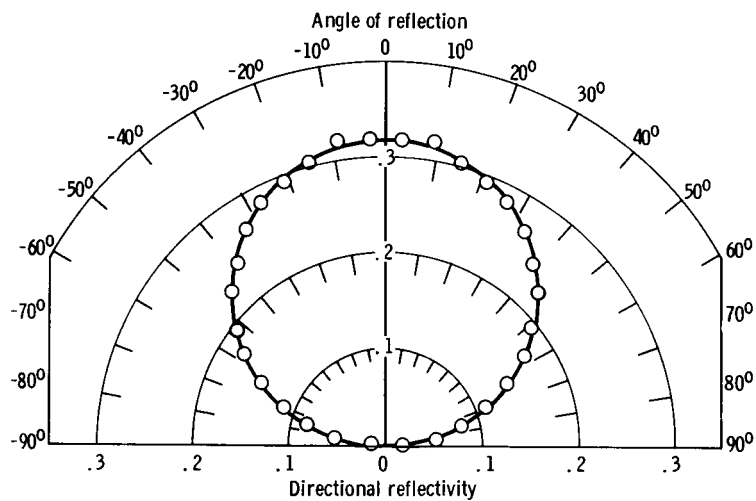Fig. 3 Directional reflectivity of conical cavity. Incident angle, 60°; cone
angle, 30°.

Angle of reflection in plane parallel
to incident radiation

(a) Cone angle, 1^{0}.

Directional reflectivity
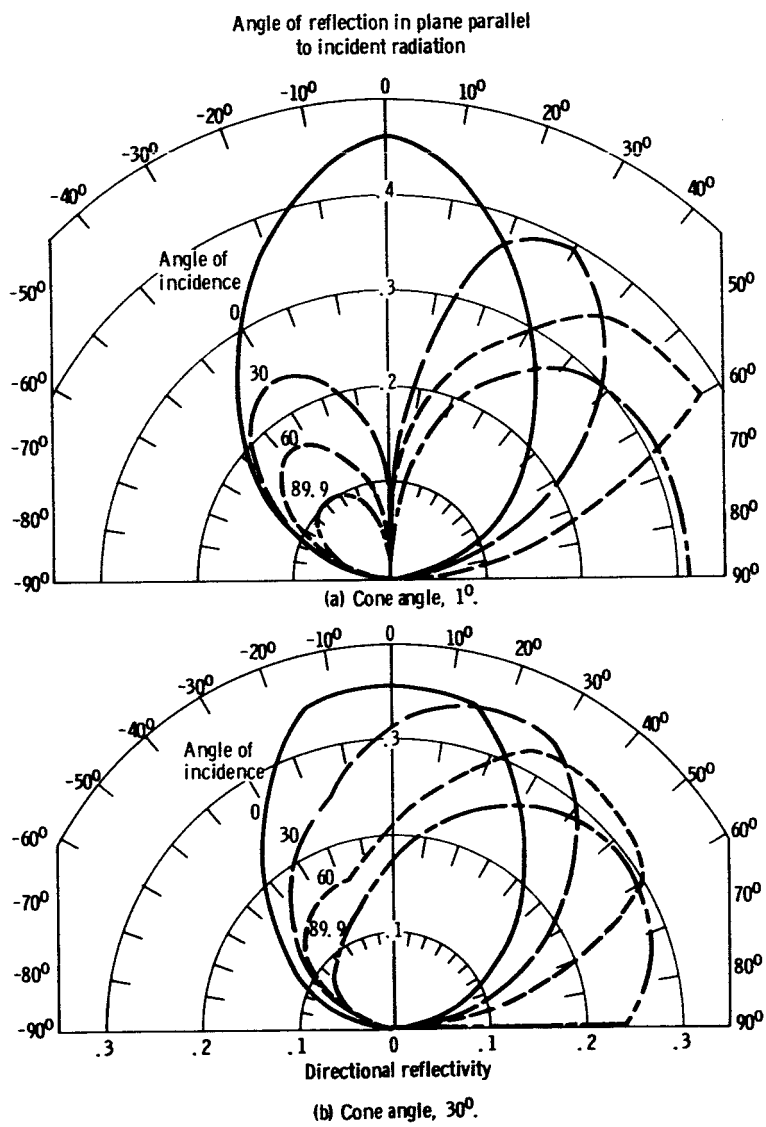
(b) Cone angle, 30^{0}.

Fig. 4 Directional reflectivity of conical cavity. Cone absorptivity, 0.

Angle of reflection in plane parallel
to incident radiation
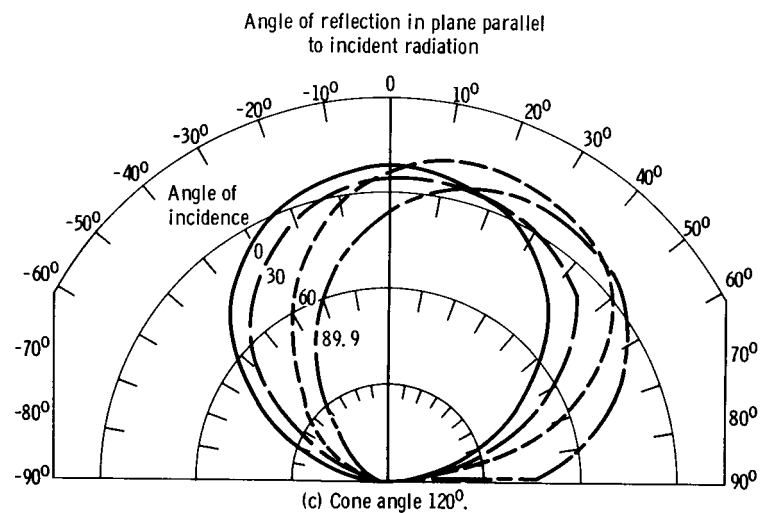
(c) Cone angle 120°.

——— Analytical flat-plate solution
O   Monte Carlo results

Angle of reflection

Directional reflectivity

(d) Cone angle 179.8°. Comparison of Monte Carlo and analytical solutions.

Fig. 4 Concluded.

Angle of reflection in plane parallel
to incident radiation
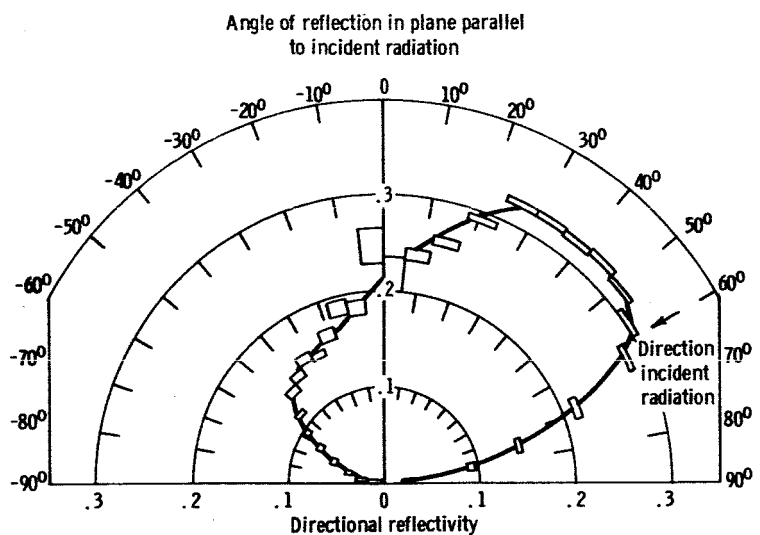
Fig. 5 Directional reflectivity of conical cavity showing expected standard deviation. Cone absorptivity, 0; cone angle, 30°.
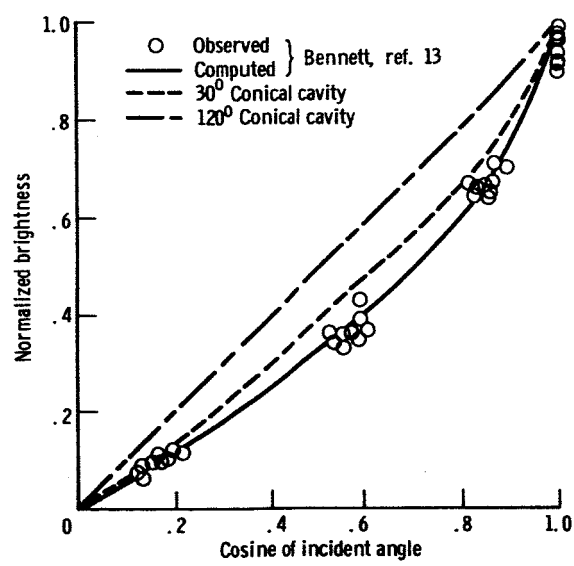

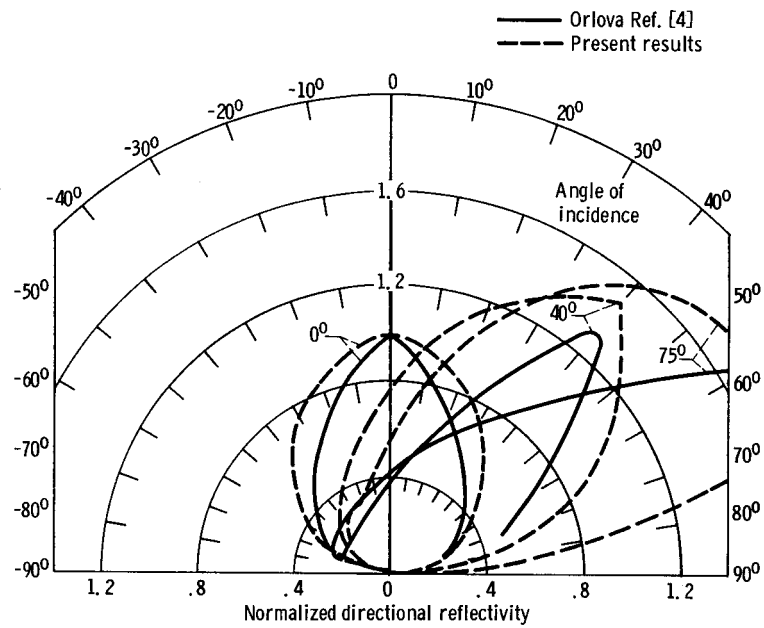Fig. 6 Comparison of observed and calculated lunar normalized brightness at 0° viewing angle.

Fig. 7 Normalized directional reflectivity of lunar surface (after Orlova Ref. [4]) compared to conical cavity results.