NASA TECHNICAL MEMORANDUM

NASA TM X- 52101

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CALCULATION OF RADIANT HEAT EXCHANGE
BY THE MONTE CARLO METHOD

by John R. Howell
Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION · WASHINGTON, D.C. · 1965

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ABSTRACT

Because of the complex nature of radiative heat-transfer calculations in many practical cases, especially where surfaces are involved, the use of digital computing machines becomes mandatory. It follows that methods that take advantage of the assets of these computers should be utilized. The Monte Carlo method is discussed as a means of solving radiative-transfer problems, and the literature dealing with its application is reviewed. An example problem is presented and discussed with regard to a Monte Carlo solution. Applications of the method to general problems in radiation are pointed out, and the areas where advantages over more conventional techniques exist are examined.

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SUMMARY

Because of the complex nature of radiative heat-transfer calculations in many practical cases, especially where surfaces are involved, the use of digital computing machines becomes mandatory. It follows that methods that take advantage of the assets of these computers should be utilized. The Monte Carlo method is discussed as a means of solving radiative-transfer problems, and the literature dealing with its application is reviewed. An example problem is presented and discussed with regard to a Monte Carlo solution. Applications of the method to general problems in radiation are pointed out, and the areas where advantages over more conventional techniques exist are examined.

INTRODUCTION

The transfer of energy by thermal radiation for all but extreme situations is considered to have a solid theoretical foundation. The chief difficulty in solving practical problems involving radiative transfer does not lie, therefore, in formulating the problem. Rather, the resulting formulation, especially when other modes of heat transfer must be considered, gives equations that are so complex in many cases that complete solution by analytical techniques is impossible, and solution by standard numerical methods impractical.

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This difficulty arises because of the nonlinearity of the energy balance equations. Here combined energy transfer is occurring by radiation, where fourth-power temperatures are present, and by conduction and/or convection, involving first-power temperatures. Further, the radiative terms in these equations take the form of multiple integrals, while conduction terms contain second derivatives. Further still, the radiative surface properties which appear are often functions of wavelength, direction, and temperature. If gases are involved, these variables plus pressure can strongly affect the local gas-radiation properties. The complete energy balance on each element of the system then takes the form of a nonlinear integro-differential equation.

Because of this, three approaches to radiant-interchange calculations have been used. First, radiation is assumed negligible, thus admitting defeat. Secondly, the problem is simplified to the point where solution is possible by the means at hand. This is done by making as many assumptions, reasonable if possible, as are necessary, and philosophically accepting the resulting loss of accuracy if not validity. Surfaces that are black, gray, diffuse, or specular and gases that are opaque, transparent, gray, or isothermal are assumptions that fall into this category. Few problems in radiative transfer are solved without explicitly or implicitly making one or more of these assumptions. Finally, as a third choice, more advanced techniques can be developed in the hope of accurately handling the mathematics of the more complex situations.

One step along this third path is the Monte Carlo technique. Monte Carlo is in essence a method for statistical sampling of events. This sampling is done by analysis of a model of the real physical problem being considered. The method was developed to a high degree by those working on the first atomic bomb.

In solving problems in thermal radiation, the method has some very great advantages over conventional methods. Of course, there are also a few disadvantages.

The Monte Carlo method, its application to thermal radiation problems, and its advantages and disadvantages for these problems will now be discussed at some length.

THE MONTE CARLO METHOD

Monte Carlo was developed as a way of treating problems in which the happenings at a given location are known, at least in the form of statistical distributions, but in which the equations that describe the interactions between locations are extremely difficult to solve. One example is the local neutron flux produced by the diffusion of neutrons in the core, reflector, and shielding of a nuclear reactor. Because the neutrons arising from the fission process undergo different sequences of absorption, fission, and scattering, a solution for the neutron flux at all points in a heterogeneous system can be difficult. However, the frequency of events occurring along the path of an individual neutron are fairly well understood. This leads to the idea of following sample neutrons, and determining the events along their paths by picking events at random from the appropriately weighted set of possibilities at each point. By letting each neutron sample represent a group of real neutrons, and by following enough samples, the flux at each point can be determined.

Of course, such a large number of simple calculations must be performed that a digital computer becomes a necessity. However, the fact that Monte Carlo depends on a large number of simple repetitive calculations and decisions

means that it is ideally suited for the computer.

APPLICATION TO THERMAL RADIATION

Applying the Monte Carlo method to thermal radiative transfer problems involves setting up a physical model which characterizes radiative processes. In neutron diffusion, sample neutrons obeying known scattering, absorption, and fission laws meet this need. In radiation, the corollary sample particle is the photon. However, if the photon itself is chosen as a particle to sample, the problem arises that the wavelength of the photon depends on its energy. It is more convenient to choose a common energy for all samples to be followed. Using this technique, each sample then becomes a bundle of photons with the same wavelength. Each sample can have a different number of photons according to its wavelength. However, the total energy carried by every bundle is the same.

With these bundles of energy as our samples, it becomes relatively straightforward to simulate radiative processes.

For an example, examine the energy transfer between element dA_1 at temperature T_1 and surface A_2, an infinite plane, at temperature $T_2 = 0$ (see Fig. 1). So that some previous statements about directional and spectral properties will gain substance, let element dA_1 have emissivity

$$\epsilon_1 = \epsilon_1(\lambda,\beta) \tag{1}$$

and let area 2 have emissivity

$$\epsilon_2 = \epsilon_2(\lambda,\beta) \tag{2}$$

and assume only that the emissivity of both surfaces is independent of circumferential angle θ (Fig. 1). This is the case for real surfaces prepared by sandblasting, plating, or etching.

For such a surface, the total emitted energy per unit time is

$$Q_{e,1} = \epsilon_{T,1} \sigma T_1^4 \, dA_1 \tag{3}$$

where $\epsilon_{T,1}$ is the total hemispherical emissivity given in this case by

$$\epsilon_T = 2\pi \int_0^\infty \int_0^{\pi/2} \frac{\epsilon_1 i_{\lambda,1} \sin \beta \cos \beta \, d\beta \, d\lambda}{\sigma T_1^4} \tag{4}$$

and $i_{\lambda,1}$ is the Planck spectral distribution of blackbody radiant intensity.

If it is assumed that $Q_{e,1}$, the total energy emitted per unit time by dA_1, is composed of N of the energy bundles described above, then the energy of each bundle, c, is simply

$$c = \frac{Q_{e,1}}{N} \tag{5}$$

To determine the energy transferred from element dA_1 to surface A_2, we now follow N bundles of energy through their emission from dA_1, and determine the number S_2 absorbed at A_2. If the energy reflected from A_2 back to dA_1 is neglected, the energy transferred per unit time from dA_1 to A_2 will be

$$Q_{1-2} = cS_2 \tag{6}$$

The next question is how is each individual bundle path determined and how is a wavelength assigned to each bundle? However this is done, the directions and wavelengths of the N bundles must conform to the constraints given by the emissivity of the surface and the laws governing radiative processes. For example, if we assign wavelengths to N bundles, the spectral distribution of emitted energy generated by the Monte Carlo process (comprised of the $cN_\lambda \, d\lambda$ for discrete intervals $d\lambda$) must closely approximate the spectrum of the actual emitted energy (plotted as $\pi \epsilon_\lambda i_\lambda \, d\lambda$ versus λ). To assure this, a number

of methods are available for choosing the energy-bundle properties (Refs. [1 and 2]). Leave the radiation problem momentarily for a physical interpretation of two common methods of choosing sample properties.

Choosing individual events. - Of the two methods to be outlined for randomly selecting events in a manner that obeys the physical constraints of the problem, the first is probably the most intuitively satisfying. It consists of choosing events directly from the curve of known probability of an event.

Consider the probability distribution of Fig. 2(a), given by the relation

$$p(\xi) = \frac{\xi^2}{1000} \tag{7a}$$

in the interval $0 < \xi \leq 10$ and $p(\xi) = 0$ elsewhere. Normalizing this relation by the area under the curve of Fig. 2(a), gives

$$P(\xi) = \frac{p(\xi)}{\int_0^{10} p(\xi)d\xi} = \frac{3\xi^2}{1000} \tag{7b}$$

Such a normalized probability curve is called a probability density function.

To choose values of ξ in such a manner that Eq. (7b) is satisfied by the distribution of chosen values, proceed as follows:

Two numbers, R_a and R_b, are chosen at random from a large set of numbers evenly distributed in the range (0-1). On the digital computer, such R values are selected randomly by means of either a random number generating subroutine or from stored sequences of random numbers. Choosing R values at random ensures that each event in a history will be independent of preceding events.

The two random numbers are then used to select a point $(P(\xi),\xi)$ on Fig. 2(b) by setting

$$P(\xi) = R_a; \quad \xi = R_b(\xi_{max} - \xi_{min}) = 10R_b$$

This point is then compared to the value of $P(\xi)$ at ξ computed from Eq. (7b).

If the randomly selected value lies above the computed value of $P(\xi)$, then

the randomly selected value of ξ is rejected and two new random numbers are

selected. Otherwise, the value ξ that has been found is used. Referring again

to Fig. 2(b), it is seen that such a procedure assures that the correct fraction

of ξ values selected for use will lie in each increment $d\xi$ after enough

selections are made.

The difficulty with such an event choosing procedure is that in some cases

a large portion of the values of ξ may be rejected. A more efficient method of

choosing ξ is therefore desirable. The method to be outlined is more efficient

for many of the distributions which occur in radiative-transfer Monte Carlo

calculations.

This method is, in short, to integrate the probability density function

$P(\xi)$ using the relation

$$R = \int_{-\infty}^{\xi} P(\xi')d\xi' \qquad (8)$$

where R can only take on values in the range (0-1) because of the properties

of $P(\xi)$. Eq. (8) is known as the cumulative distribution function. The

function R is then taken to be a random number, and values of ξ are obtained

by choosing R at random and solving Eq. (8) for the corresponding value of ξ.

To show that the probability density of ξ chosen in this way corresponds to

the required $P(\xi)$, we can again examine the probability density function of

Fig. 2(b).

Inserting the example $P(\xi)$ of Eq. (7b) into Eq. (8) gives

$$R = \int_0^\xi P(\xi')d\xi' = \frac{\xi^3}{1000} \tag{9}$$

Equation (9) is shown plotted in Fig. 3. Divide the range of ξ into a number of equal increments $\Delta\xi$. Suppose M values of R are now chosen in the range 0-1, and these M values are picked at equal intervals along R. There will be M values of ξ which correspond to these M values of R. The fraction of the M values of ξ which occurs per given increment $\Delta\xi$ is then

$$\left(\frac{M_{\Delta\xi}}{M}\right)\Big/\Delta\xi = \frac{\Delta R}{\Delta\xi} \tag{10}$$

But $\Delta R/\Delta\xi$ is of course an approximation to $(dR/d\xi)$ if a large enough value is chosen for M and small increments $\Delta\xi$ are examined. But $dR/d\xi$ can be seen from Eq. (7) to be simply $P(\xi)$ and it has been shown, therefore, that by choosing values of ξ in this manner the required probability distribution is indeed satisfied.

A similar procedure for use when the probability distribution is a non-separable function of more than one variable (for example, a reflectivity, which for real surfaces may have the angles of reflection (β, θ) depending on one another) is demonstrated in Ref. [1].

Selection of events for the example problem. - To return to the problem at hand, the wavelength of emission for the emitted bundle must be chosen.

It is assumed here that the surface properties are product functions of the two variables angle and wavelength, that is

$$\epsilon(\lambda,\beta) = \epsilon(\lambda)\epsilon(\beta)$$

This assumption is valid for many actual surfaces, since the wavelength variation of emissivity, for example, rarely depends on angle of emission. It follows, therefore, that dependence on either variable may be found by integrating out the other variable. Then the normalized probability of emission occurring in the interval $d\lambda$ is

$$P(\lambda) = \int_0^{\pi/2} P(\lambda,\beta)d\beta = \frac{2\pi\int_0^{\pi/2}\epsilon_1(\lambda,\beta)i_\lambda \sin\beta \cos\beta \, d\beta \, d\lambda}{\epsilon_T\sigma T_1^4} \qquad (11)$$

Substituting into Eq. (7) gives

$$R_\lambda = \frac{2\pi\int_0^\lambda\int_0^{\pi/2}\epsilon_1(\lambda',\beta)i_{\lambda'} \sin\beta \cos\beta \, d\beta \, d\lambda'}{\epsilon_T\sigma T_1^4} \qquad (12)$$

If the number of bundles N is very large, and this equation were solved for λ each time an R_λ were chosen, computing time could become too large for practical calculations. To circumvent this problem equations like Eq. (12) can be numerically integrated once over the range of λ, and a curve can be fitted to the result. A polynomial curve is often adequate, as is the case in this problem, giving

$$\lambda = A + BR_\lambda + CR_\lambda^2 + \ldots \qquad (13)$$

This equation rather than Eq. (12) is used in the problem-solving program.

Following a similar procedure for the cone angle of emission β gives the relation

$$R_\beta = \int_0^\beta\int_0^\infty P(\beta',\lambda)d\lambda \, d\beta' = \frac{2\pi\int_0^\beta\int_0^\infty\epsilon_1(\lambda,\beta')i_\lambda \sin\beta' \cos\beta' \, d\lambda \, d\beta'}{\epsilon_T\sigma T_1^4}$$

$$(14)$$

which can again be curve-fit to give

$$\beta = A' + B'R_\beta + C'R_\beta^2 + \ldots \qquad (15)$$

For a gray, diffuse surface, Eq. (12) reduces to

$$R_{\lambda,gray} = \frac{\pi \int_0^\lambda i_{\lambda'}\, d\lambda'}{\sigma T_1^4} = F_{0-\lambda} \tag{16}$$

where $F_{0-\lambda}$ is the well known fraction of blackbody emission in the wavelength interval $(0 - \lambda)$. Equation (14) for this case reduces to

$$R_{\beta,gray} = 2 \int_0^{\beta_1} \sin\beta' \cos\beta'\, d\beta' = \sin^2\beta_1 \tag{17}$$

or,

$$\sin\beta_1 = \sqrt{R_\beta} \tag{18}$$

The point to be made here is that computational difficulty in obtaining λ from either Eq. (13) or (16) is not greatly different, nor is it much different for obtaining β_1 from either Eq. (15) or (18). The difference is mainly in the auxiliary numerical integrations of Eqs. (12) and (14), which are performed once to get the curve fits for the nongray-nondiffuse solution. As far as the problem-solving program is concerned, the more difficult case may just as well be solved. Thus increasing problem complexity leads to only gradual increases in the complexity of the Monte Carlo program and similar gradual increases in computer time.

For emission of an individual energy bundle from surface dA_1, then, a wavelength λ can be chosen from Eq. (13), and a cone angle of emission β_1 can be chosen from Eq. (15). There remains only specification of the circumferential angle θ_1. Because of the assumption made earlier that emission did not depend on θ_1, it is easily shown by the formalism outlined, and is also fairly obvious from intuition, that θ_1 can be determined by

$$\theta_1 = 2\pi R_\theta \tag{19}$$

Because the position of plane A_2 with respect to dA_1 is known it is a simple matter to determine whether a given energy bundle will strike A_2 after leaving dA_1 in direction (β_1, θ_1). (It will hit A_2 if $\cos \theta_1 > 0$, as shown in Fig. 1.) If it misses, another bundle must be emitted from dA_1. If the bundle strikes A_2, it must be determined whether it is absorbed or reflected. To do this, geometry is used to find the angle of incidence β_2 of the bundle onto A_2.

$$\cos \beta_2 = \sin \beta_1 \cos \theta_1 \tag{20}$$

Knowing the absorptivity of A_2 from Kirchoff's Law

$$\alpha_2(\lambda,\beta) = \epsilon_2(\lambda,\beta) \tag{21}$$

and having determined the wavelength λ of the incident bundle from Eq. (13) and the incident angle β_2 from Eq. (15), the probability of absorption of the bundle at A_2 can be determined. The probability of absorption is simply the absorptivity of A_2 evaluated at β_2 and λ because the directional spectral absorptivity $\alpha_2(\lambda,\beta)$ is the fraction of energy incident on A_2 (in a given wavelength interval) from a given solid angle that is absorbed by the surface. The absorptivity is therefore the probability-density function for the absorption of incident energy. It is now easy to determine whether a given incident energy bundle is absorbed by using the first of the two event-choosing methods just outlined; that is, by comparing the surface absorptivity $\lambda_2(\lambda,\beta)$, which corresponds to $P(\xi)$ (the probability of absorption), with a random number R_α. If

$$R_\alpha \leq \alpha_2(\lambda,\beta) \tag{22}$$

the bundle of energy is absorbed, and a counter S_2 is increased by one. Otherwise, the bundle is assumed to be reflected, and is henceforth neglected. This

neglect is reasonable if the absorptivity of A_2 is large, or if the directional reflectivity is such that few bundles are reflected back in their original direction. If not, angles of reflection must be chosen from known directional reflectivities, and the bundle followed further along its path until absorption or loss from the system. For the purposes of this example, little is to be gained by following the bundle after reflection from surface A_2 because the derivation of the necessary relations is similar to that already presented. The bundles are therefore neglected.

A new bundle is now chosen and its history followed. This procedure is continued until all N bundles have been emitted from dA_1. The energy absorbed at A_2 is then calculated from Eq. (6).

This completes derivation of the equations needed for solution of the example problem. In putting together a flow chart (Fig. 4), some shortcuts can be used. For example, the angle θ can be computed first. If the bundle is not going to strike A_2 on the basis of the calculated θ, there is no point in computing λ and β for that bundle. Alternately, because θ values are isotropically distributed, it can be noted that exactly half the bundles must strike A_2. Therefore the calculated θ values can be constrained to the range $-\frac{\pi}{2} < \theta < \pi/2$. If N bundles are emitted in this range, then the calculated heat transfer will be

$$Q_{1-2} = \frac{cS_2}{2} \qquad (23)$$

The solution of this problem by Monte Carlo is now complete. An astute observer will note that this example could be solved without much difficulty by standard methods. A more astute observer might note further

that extension to only slightly more difficult problems would cause serious consequences for the standard treatments (for example, consider introducing a third surface with directional properties into the problem and accounting for all interactions). On the other hand, the author has found few radiation problems that will not yield to a Monte Carlo approach.

Some problems that have been solved by Monte Carlo are now discussed.

ADVANTAGES AND APPLICATIONS OF MONTE CARLO

Several means are available in the literature for solving radiative transport problems. These methods have been called "standard" or "conventional" methods herein, and include the techniques developed by Poljak [3], Hottel [4, 5], Oppenheim [6], and Gebhart [7] as well as formulation in terms of integral equations. Each of these has advantages for certain types of problems, and all will outshine the Monte Carlo approach in speed and accuracy over some limited range of radiation calculations that is outlined roughly by the complexity of the problem.

The chief usefulness of Monte Carlo to the thermal radiation analyst lies in this fact: Monte Carlo program complexity increases roughly in proportion to problem complexity for radiative interchange problems. This is an important advantage because conventional methods increase roughly with the square of complexity of the problem, due to the matrix form into which they fall. However, because Monte Carlo is somewhat more difficult to apply to the simplest problems, it is most effective in problems where complex geometries and variable properties must be considered. In complex geometries, Monte Carlo has the advantage that simple relations will specify the path of a given energy bundle, whereas most other methods involve explicit or implicit integrations over

surface areas. Such integrations become difficult when a variety of skewed
or curved surfaces are present. One application of Monte Carlo to such a
problem was in Reference [8], where the directional reflectivity and absorp-
tivity of conical cavities with diffuse internal reflections were calculated.
Here a straightforward Monte Carlo technique gave an easily programmed solu-
tion to a problem of not inconsiderable difficulty for conventional approaches.

Monte Carlo was first applied in thermal radiation to problems involving
gases. This had been suggested in Ref. [9], and the idea was applied in a
series of papers (Refs. [10 to 12]) which treat radiation through gases in
various geometries. One paper by Howell and Perlmutter Ref.[12] gives solu-
tions for the temperature distribution and heat transfer through a nongray gas
with temperature dependent properties, contained between infinite parallel black
plates. The cases of plates with much different temperatures, and of plates with
the same temperature and with a parabolic distribution of energy sources in the
gas are solved.

Fleck [13] about the same time presented a lengthy discussion of combined
conduction and radiation in absorbing-emitting media using a Monte Carlo approach,
including transient effects.

Howell, Strite, and Renkel [14, 15] used Monte Carlo in the more practical
problem of predicting the combined radiative and convective energy transfer in
the rocket nozzle of a gaseous-core nuclear propulsion system. They allow
(in Ref. [15]) the absorption coefficient of the propellant to vary with any two
variables (from wavelength, static pressure, and static temperature) evaluated
locally, and account for axial and (in Ref. [16]) radial temperature gradients.
In addition, the ability of a layer of optically thick gas injected along the
nozzle wall to attenuate the extreme predicted radiative fluxes was demonstrated
in Ref. [16].

Such a system, involving combined heat-transfer modes in a flowing

system with variable properties and an arbitrary axisymmetric bounding

surface would at best be a laborious test of the more conventional approaches

to radiative transfer. Yet the Monte Carlo program used was able to predict

temperature profiles in the propellant, converged to within 0.1 percent at

all points, along with axial heat-flux distributions at the nozzle wall. All

this was done in less than 10 minutes of machine time.

DISADVANTAGES OF MONTE CARLO TECHNIQUE

Monte Carlo calculations give results which fluctuate around the real

answer because the method is a repetitive experiment on a mathematical model

used in place of the actual physical situation. The uncertainty can usually

be found by applying the standard statistical tests, and can be reduced in the

same manner as experimental error; that is, by averaging over more tests

(bundle histories).

No rigorous test exists to guarantee the convergence of Monte Carlo

results to valid solutions. This has not as yet proven to be a difficulty in

thermal radiation problems. It would often be immediately obvious that con-

vergence to invalid solutions was occurring because of the limiting solutions

and physical constraints which are known for most radiation problems.

An example of the scatter of results and convergence criteria for a

radiation problem is shown in Fig. 5, taken from Ref. [12]. This figure

shows typical scatter in Monte Carlo calculations of the temperature dis-

tribution in a real gas between diffuse black infinite parallel plates at

different temperatures. The absorption coefficient κ of the gas is a

function of wavelength and local gas temperature. Because the limiting

diffusion and transparent solutions are known in this case and bracket the Monte Carlo results, the likelihood that convergence to an invalid solution is occurring is small. Also, the solutions follow what is known to be physically necessary; for example, the gas temperature increases monotonically from the colder to the hotter wall, and the gradient of emissive power in the gas varies in the expected manner with the property variations.

The results in Fig. 5 are compared in Ref. [12] to a number of approximate solutions involving various simplifying assumptions for the gas absorption coefficient, and agreement within the limits of the assumptions is good. The results presented in Fig. 5 required an iterative procedure because of the dependence of the gas absorption coefficient on temperature. Convergence to within 2 percent required three iterations, each involving 10,000 bundle histories.

Sample size. - Other difficulties can and do arise in sampling techniques, most of which have to do with obtaining an optimum sample size. For example, in the gas radiation problem of Fig. 5, the free path of an energy bundle between absorptions becomes quite short in optically thick gases. Any energy bundle undergoes a large number of absorptions and emissions in the gas before traversing the distance between parallel plates. This means that the computer time needed to obtain a sample of bundles transferred between the plates become prohibitively large. Conversely, for nearly transparent gases, relatively few absorptions occur in any given element of gas, and an insignificant energy sample is available to calculate the temperature of the gas element with accuracy. This accounts for the relatively greater scatter for the 0.5 centimeter plate-spacing results of Fig. 5.

Both these problems are common enough in transport processes that are mathematically related to radiative transport that special methods of "weighting" the free paths of bundles have been developed (e.g., see Ref. [2]) to obtain adequate samples, thereby saving computer time and gaining accuracy, albeit at the expense of added complexity.

In the gas-radiation problems available so far in the literature, it has been found that problems which cannot be accurately handled by either the trans-transparent or diffusion approximations fall into the range of mean free paths that allow efficient Monte Carlo programming without reverting to weighting techniques.

CLOSING REMARKS

Monte Carlo is discussed in the preceding section as a method suitable for use in the solution of complex problems in radiative transfer. A sample problem is outlined to demonstrate its application, and some of the advantages and disadvantages of the technique are discussed along with pertinent literature references.

From all this, certain conclusions emerge. First, Monte Carlo appears to have a definite advantage over other radiative-transfer calculation techniques when the difficulty of the problem being treated lies above some undefined level. Just where this level is cannot be established, probably being a function of the experience, competence, and prejudice of the individual working the problem. However, problems above this nebulous benchmark in complexity can be treated with greater flexibility, simplicity, and speed. Monte Carlo does lack a kind of generality common to other approaches in that each problem may require an individual technique, and a dash of ingenuity often helps. This places a greater burden on

the programmer's backlog of experience and intuition where standard methods may allow programming through "cookbook" application of their formalism.

Second, for thermal-radiation problems, the parameters and mathematical relations involved lie in ranges which allow straight-forward Monte Carlo programming without the need of the more exotic schemes occasionally necessary in other Monte Carlo transport studies.

Third, with all its advantages, the method suffers from certain problems. The worst of these are the statistical nature of the results, and the lack of guaranteed convergence. It should be noted that the latter fault is common to all methods when complex problems are treated.

Finally, it must be commented that the person using Monte Carlo techniques often develops a physical grasp of the problems encountered, because the model being analyzed is simple, and the mathematics describing it are therefore on an unsophisticated basis easily related to the physical model. This is in contrast to the rather poor physical interpretations and predictions which we can make when working with, say, a matrix of nonlinear second-order integro-differential equations.

LIST OF SYMBOLS

A surface area

c energy per sample bundle

$F_{0-\lambda}$ fraction of total blackbody emissive power that lies in the wavelength range $0 - \lambda$

i_λ Planck spectral distribution of blackbody radiant intensity (energy)/(unit projected surface area)(unit solid angle)(unit wavelength interval)(unit time)

$p(\xi)$ probability distribution

$P(\xi)$ probability density function, Eq. (8b)

Q energy per unit time

R number chosen at random from set of numbers evenly distributed in range

          (0 - 1)

S number of energy bundles absorbed at a surface per unit time

T absolute temperature

α absorptivity of a surface

β cone angle of emission (Fig. 1)

          or cone angle of incidence

ϵ emissivity of a surface

ϵ_T total hemispherical emissivity, Eq. (4)

κ absorption coefficient

λ wavelength

ξ variable

σ Stefan-Boltzmann radiation constant

θ circumferential angle of emission (Fig. 1)

Subscripts

e emitted

T total

1,2   refers to surface dA_1 or A_2, respectively

REFERENCES

1. Kahn, Herman: Applications of Monte Carlo. USAEC Rept. AECU-3259, the Rand
   Corp., Santa Monica, Calif., April 19, 1954.

2. Cashwell, E. D.; Everett, C. J.; and Rechard, O. W.: A Practical Manual
   on the Monte Carlo Method for Random Walk Problems. Rept. LA-2120,
   Los Alamos Scientific Lab., Jan. 1957.

3. Poljak, G., Tech. Physics USSR 1, 555, 1935.

4. Hottel, H. C.: Radiant Heat Transmission, in Heat Transmission by
   W. H. McAdams, Third Ed., Chapter 3, McGraw-Hill, New York, 1954.

5. Hottel, H. C. and Cohen, E. S.: Radiant Heat Exchange in a Gas-Filled
   Enclosure: Allowance for Nonuniformity of Gas Temperature. AIChE
   Jour. 4, no. 1, pp. 3-14, Mar. 1958.

6. Oppenheim, A. K.: Radiation Analysis by the Network Method. Trans.
   ASME 78, 725-735 (1956).

7. Gebhart, B.: Heat Transfer. McGraw-Hill, New York, 1961.

8. Polgar, L. G. and Howell, J. R.: Directional Thermal Radiative Properties
   of Conical Cavities. Proposed NASA Technical Note.

9. Bird, R. B.; Stewart, W. E.; and Lightfoot, E. N.: Notes on Transport
   Phenomena. John Wiley & Sons, New York, 1958, p. 259.

10. Howell, J. R. and Perlmutter, M.: Monte Carlo Solution of Thermal Transfer
    Through Radiant Media Between Gray Walls. ASME Jour. Heat Trans. 86,
    no. 1, Feb. 1964, pp. 116-122.

11. Perlmutter, M. and Howell, J. R.: Radiant Transfer Through a Gray Gas
    Between Concentric Cylinders Using Monte Carlo. ASME Jour. Heat Trans.
    86, no. 2, May 1964, pp. 169-179.

12. Howell, J. R. and Perlmutter, M.: Monte Carlo Solution of Radiant Heat
    Transfer in a Nongrey Nonisothermal Gas with Temperature Dependent
    Properties. AIChE Jour. 10, no. 4, July 1964, pp. 562-567.

13. Fleck, Joseph A., Jr.: The Calculation of Nonlinear Radiation Transfer
    by a Monte Carlo Method. Methods in Computational Physics, Vol. 1,
    Academic Press, 1963, pp. 43-65.

14. Howell, J. R.; Strite, M. K.; and Renkel, H.: Heat Transfer Analysis of

    Rocket Nozzles Using Very High-Temperature Propellants. AIAA Jour.

    $\underline{3}$, no. 4, April 1965, pp. 669-673.

15. Howell, J. R.; Strite, M. K.; and Renkel, H.: Analysis of Heat-Transfer Effects in

    Rocket Nozzles Operating with Very High-Temperature Hydrogen. NASA

    TR R-220, Feb. 1965.

16. Howell, J. R., and Strite, M. K.: Heat Transfer in Rocket Nozzles Using

    High Temperature Hydrogen Propellant with Real Property Variations.

    Paper presented at AIAA Propulsion Specialists Conference, Colorado
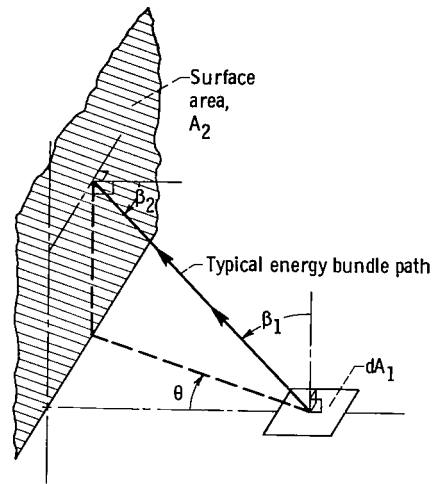
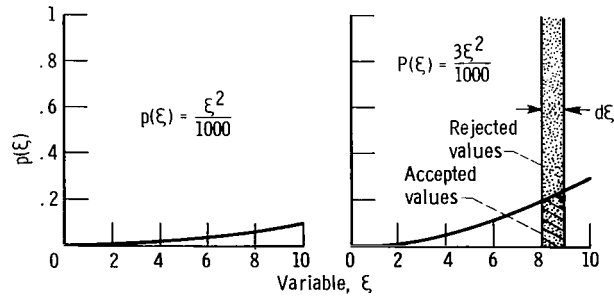   Springs, June 1965.

Fig. 1. – Geometry for example problem.

(a) Probability distribution of variable ξ.

(b) Probability density of variable ξ.

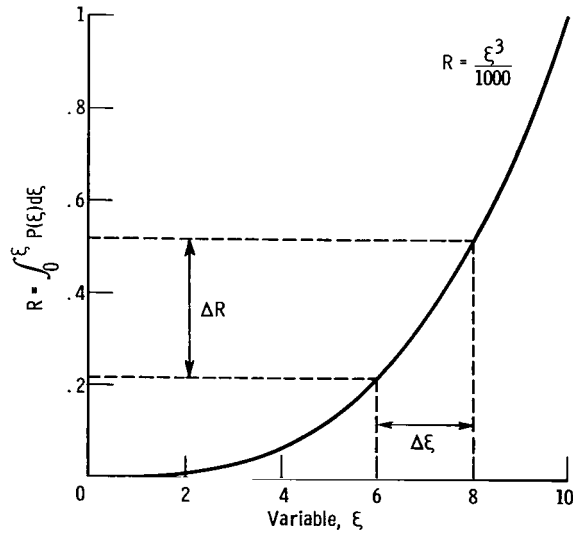Fig. 2. – Examples of probability distributions.

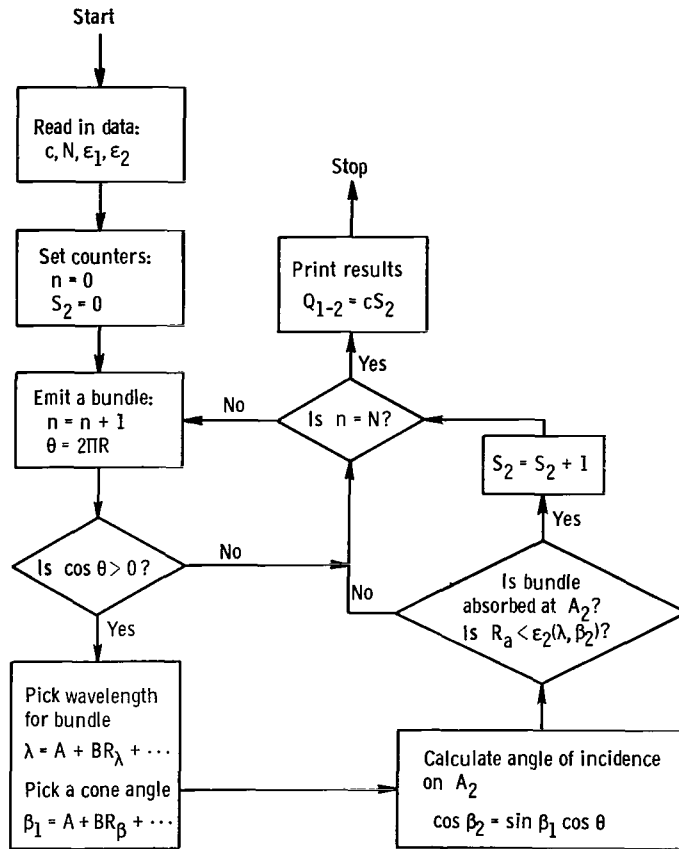Fig. 3. – Cumulative distribution of example.

Start

Read in data:
$c, N, \varepsilon_1, \varepsilon_2$

Set counters:
$n = 0$
$S_2 = 0$

Emit a bundle:
$n = n + 1$
$\theta = 2\pi R$

No

Stop

Print results
$Q_{1-2} = cS_2$

Yes

Is $n = N$?

No

$S_2 = S_2 + 1$

Yes

Is $\cos \theta > 0$?   No

Yes

No

Is bundle
absorbed at A_2?
Is $R_a < \varepsilon_2(\lambda, \beta_2)$?

Pick wavelength
for bundle
$\lambda = A + BR_\lambda + \cdots$

Pick a cone angle
$\beta_1 = A + BR_\beta + \cdots$

Calculate angle of incidence
on A_2

$\cos \beta_2 = \sin \beta_1 \cos \theta$

Fig. 4. – Computer flow chart for example program.

Solutions

$\left.\begin{array}{c} \bigcirc \\ \square \\ \triangle \end{array}\right\}$ Monte Carlo
Gas absorption coefficient, $\kappa = \kappa(\lambda, T)$

——— Least squares fit through Monte Carlo
results
— — — Limiting transparent solution
—·—·— Limiting diffusion solution

Plate
spacing,
cm

Limit → 0

0.5

3

20

Limit → ∞

1.0

.8

.6

.4

.2

$\dfrac{T_{g,x}^4 - T_1^4}{T_0^4 - T_1^4}$

0   .2   .4   .6   .8   1.0
Position between plates, x
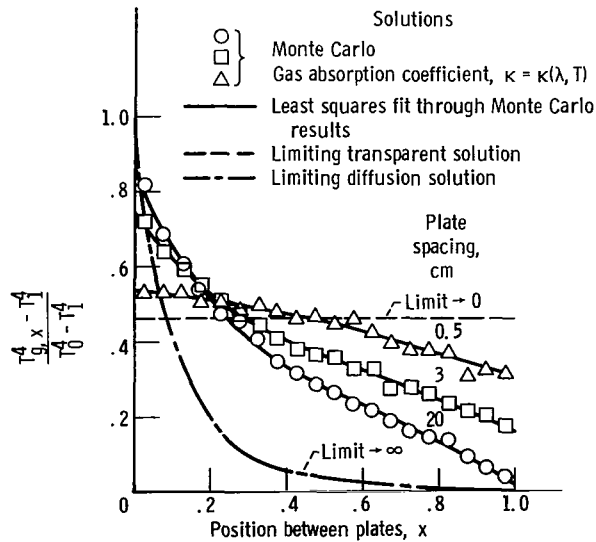
Fig. 5. – Emissive power distribution in hydrogen
between parallel plates; no gas heat source;
plate temperatures: $T_0 = 9500°$ K; $T_1 = 4500°$ K
(from Ref. 12).