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A SPACE-CHARGE-FLOW COMPUTER PROGRAM

by Carl D. Bogart and Edward A. Richley

Lewis Research Center
Cleveland, Ohio

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SUMMARY

A description of a computer program for the solution of two-dimensional and axisymmetric space-charge-flow problems is given. The program is written in FORTRAN IV for an IBM 7094 computer. Primary emphasis is placed on nomenclature, word definitions, and programing procedures. The program overlay chart and subroutine flow diagrams are presented, and example sets of input and output data for a variety of boundary conditions are given and fully explained. The program has been applied extensively for the analysis of various electrostatic-thrustor-design concepts, wherein positive charged-particle flow is conventional. It is equally applicable to various other problems, such as electron tube design in which the particles (electrons) have a negative charge. Output data listings include detailed information on potential distributions, particle trajectories, thrust, power and current densities, and other parameters of interest.

INTRODUCTION

Design requirements of electrostatic thrustors that are under investigation at the Lewis Research Center have given rise to the need for a numerical method of analysis of charged-particle trajectories subject to various boundary conditions. To fill this need, a computer program has been developed at Lewis that is capable of solving both two-dimensional and axisymmetric problems for charged-particle-flow conditions that range from zero charge density to space-charge limited. The program is presented herein.

Early versions of the program have been reported previously in references 1 and 2. Reference 1 deals with the two-dimensional space-charge-limited-flow problem and includes a program written in FORTRAN II for an IBM 7094 computer. In reference 2, a program for a special axisymmetric space-charge-limited-flow problem is given, and in a similar manner, other investigators have prepared computer programs for particular space-charge-flow problems (e.g., ref. 3). Subsequently, several additional features have been incorporated into the programs given in references 1 and 2, and they have now been merged into one overall program.

In references 1 and 2, emphasis was placed on the mathematical techniques that were employed to obtain numerical solutions of the various problems and the interpretation of results from the physical viewpoint. Very little attention was given to a description of program details from the computer programing viewpoint. Because of their length and complexity, this has led to difficulties regarding use of these earlier programs by independent investigators. In this report, the purpose is to describe the present computer program with emphasis on preparation of input data.

THE PROBLEM

In this section, the mathematical model will be established from a physical model, and the general method of solution will be described. While much of the information in this section has been reported previously (refs. 1, 2, and 4), a brief description of the problem and general method of solution is necessary for completeness.

The problem under consideration is either the two-dimensional or axisymmetric space-charge-flow problem for positively or negatively charged particles. For the investigator of various electrostatic-thrustor designs, numerical solutions are sought that will provide detailed information regarding parameters, such as potential distributions for various boundary conditions, ion trajectories, thrust, power, and current-density distributions. Many of these parameters are of interest in other fields of study, such as electron tube design.

Physical Model

The problem is formulated by first defining a region of interest of a prospective thrustor design such as shown in figure 1. The ideal theoretical performance of this thrustor as well as several other thrustor configurations is given in reference 5. In the design shown in figure 1, ions are formed on the downstream face of the ionizer, which together with the focus electrode is maintained at a high positive potential with respect to ground. The accel electrode is at a negative potential, and the resulting field causes acceleration and ejection of the ions that make up the exhaust beam. Along with the aforementioned parameters, the minimization of ion interception on the accel electrode is of particular importance to a prospective thrustor design.

Mathematical Model

For the purposes of the numerical analysis, the region of interest, depicted in fig-

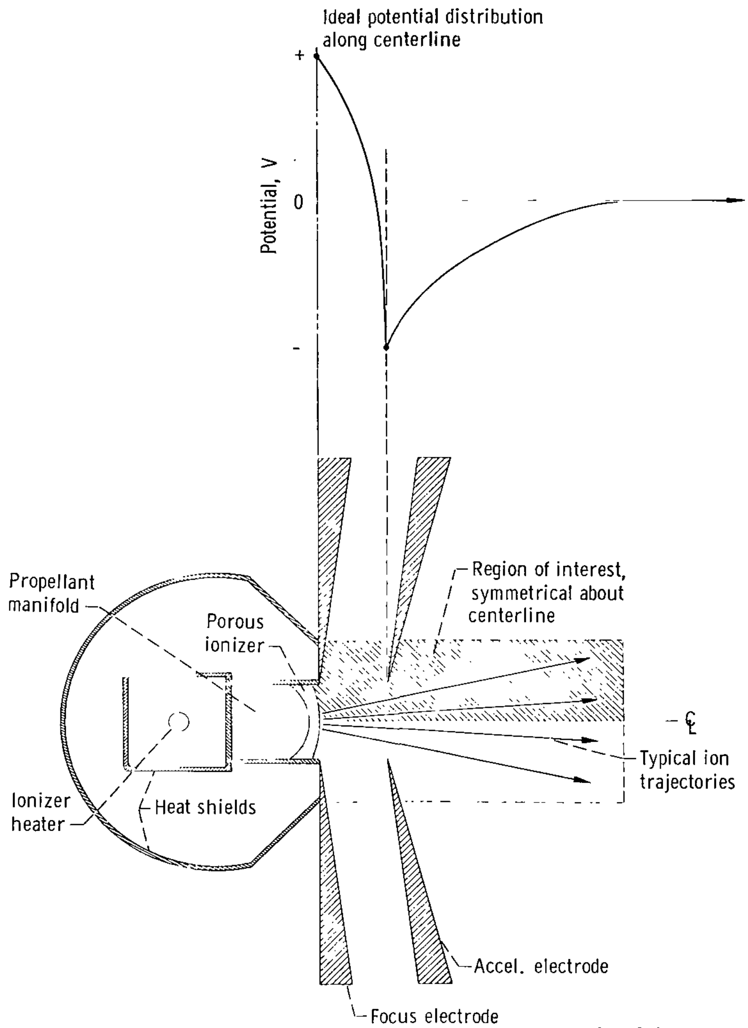


Figure 1. - Cross section of Lewis Research Center divergent-flow contact-ionization thrustor and ideal potential distribution.

ure 1, is laid out to scale, bounded, and overlaid with a uniform square mesh, as shown in figure 2. All mathematical symbols are defined in appendix A.

Boundary conditions may be given either in terms of a potential (such as along the ionizer and electrodes) or as a zero normal derivative of potential. This latter specification is applicable, for example, along the lower boundary of symmetry and is also a reasonable approximation along the upper boundary, particularly if the boundary is normal to the electrode surfaces and is sufficiently removed from the space-charge-flow region. Thus, the best location of some of the boundaries becomes a matter of judgment. For example, the right boundary is not physically well defined; however, for ion-thrustor operation, in which neutralizers are used, it can be approximated as a straight line of

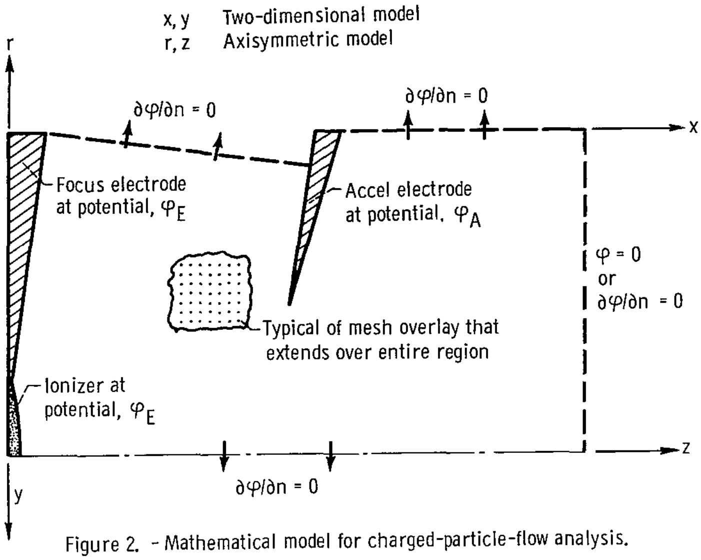
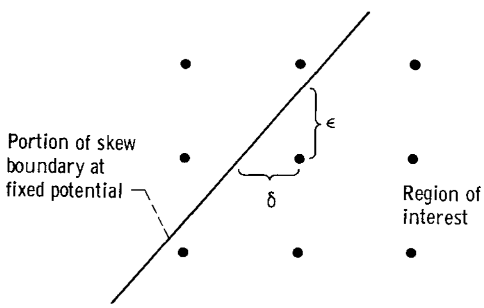


Figure 2. - Mathematical model for charged-particle-flow analysis.



(a)

zero potential. The effect of the location of this boundary on various solutions has been investigated and found to be negligible if the distance from the accel electrode is about the same as the distance between the ionizer and the accelerator (ref. 1).

Mesh points are numbered vertically starting at the upper left corner. The mesh may be made as fine or as coarse as desired within practical limits. The total number of mesh points employed is limited only by the storage capacity of the computer. Typical problems have used from 1500 to 3000 points.

Additional specifications required for preparation of input data are the calculated values of the various ϵ's and δ's (see sketch (a)), which are fractions of mesh-spacing distances measured from the mesh point under consideration to the boundary at a fixed potential. In these problem specifications, it is noteworthy that no restrictions are imposed with respect to the shape of the region.

The equation that must be solved for the bounded region is the Poisson equation, which can be written for the two-dimensional problem as

$$-\nabla^2\varphi(x, y) = \frac{1}{\epsilon_o}\rho(\varphi, x, y) \tag{1}$$

and for the axisymmetric problem as

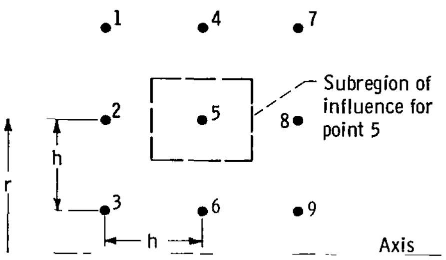
$$-\nabla^2\varphi(r, z) = \frac{1}{\epsilon_o}\rho(\varphi, r, z) \tag{2}$$

The functions φ and ρ are the continuous potential distribution and space-charge-density distribution functions, respectively. For the numerical, or discrete, problem, equations (1) and (2) are rewritten as

$$-\nabla^2 w(x, y) = f(w, x, y) \tag{3}$$

and

$$-\nabla^2 w(r, z) = f(w, r, z) \tag{4}$$



(b)

A five-point formula approximation of equation (3) or (4) is made for each mesh point of the bounded region. For example, the equation for a simple interior point, such as shown in sketch (b), is for the two-dimensional problem

$$w_5 - \frac{1}{4}(w_2 + w_4 + w_6 + w_8) = \frac{1}{4} f_5 h^2 \tag{5}$$

For the axisymmetric problem, the equation for point 5 is

$$r(4w_5 - w_2 - w_4 - w_6 - w_8) + \frac{h}{2}(w_6 - w_4) = r f_5 h^2 \tag{6}$$

Derivations of these equations as well as variations that arise for points near the various boundaries are given in references 1 and 2.

The equations, when written for each of N mesh points, form a set of N linear algebraic equations with N unknowns. These equations, arranged in proper order, may be written in matrix form as

$$A\underline{w} = \underline{k} \tag{7}$$

where A is a matrix consisting of the coefficients of the w's, and $\underline{w}$ and $\underline{k}$ are column vectors representing the potential distribution and space-charge-density distribution functions. The column vector $\underline{k}$ also contains the known boundary values. The method used for solving equation (7) is the Cyclic Chebyshev Semi-Iterative Method, and details regarding the mathematical techniques used are given in references 1 and 2.

GENERAL METHOD OF SOLUTION

The numerical solution of equation (7) in the absence of space charge presents no particular problem, and the Laplace potential distribution is readily obtained by straightforward iteration. With space charge present (due to the charged particle beam), the solution is more complex since values of the space-charge function that are required in equation (7) are not known a priori.

To begin the calculations that include space-charge effects, it is noted that

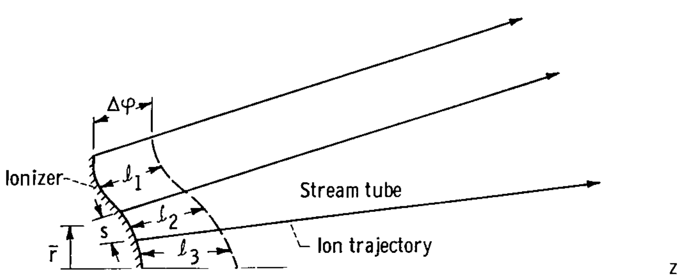
$$f(w, x, y) = \frac{1}{\epsilon_0} \frac{j(x, y)}{v(x, y)} \tag{8}$$

where j and v are the current-density and ion-speed functions. Similar equations apply in axisymmetric notation. Thus, if the functions j and v can be determined, values of f may be calculated for use in equation (7). The j and v functions are not known explicitly; however, they may be initially approximated from known physical laws, and additional iteration techniques may be employed. The initial j and v estimates are obtained with the aid of the Laplace potential distribution.

As discussed in reference 4, solutions may be obtained for either space-charge-limited-flow or less-than-space-charge-limited-flow problems.

Space-Charge-Limited Problems

The procedure to obtain values of j and v is as follows: The ionizer is divided into an arbitrary number of line segments of length s, as shown in sketch (c). Ion trajectories, which form the boundaries of stream tubes, are assumed to start from the ends of each line segment. As the trajectory moves to the right to each mesh column, its position is first estimated from the slope given by the velocity components at the previous mesh column. Its final position is determined by an iterative procedure that employs the equations of motion. The procedure and various trajectory possibilities are described in detail in reference 1. Velocities at any point in the region are determined from the law of conservation of energy.

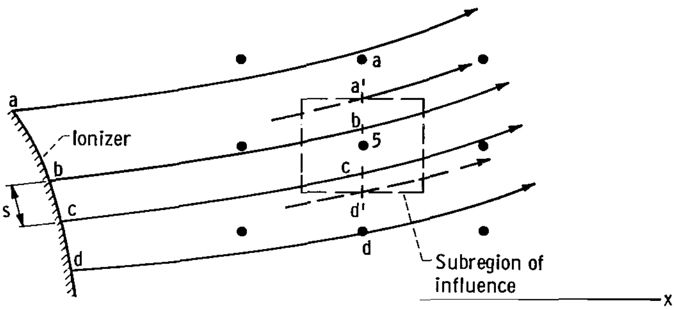


(c)

The current in each stream tube is found at the ionizer (see sketch (c)) by a parallel-plate approximation where the "plates" are assumed separated by a distance ℓ and a potential difference of $\Delta\varphi$. From Child's law, the space-charge-limited current density is given as

$$j_E = \frac{4}{9} \epsilon_o \sqrt{\frac{2q}{m}} \frac{\Delta\varphi^{3/2}}{\ell^2} \qquad (9)$$

In this calculation, either $\Delta\varphi$ or ℓ may be held constant for the various stream tubes. And as in the trajectory calculation, the Laplace potential distribution is used to obtain the initial $\Delta\varphi$ and ℓ values. Equation (9), if multiplied by the stream-tube cross-sectional area, gives the total current flowing in each stream tube. With the stream-tube current determined, the j(x, y) distribution is calculated as follows: Consider a typical mesh point in the region of interest as shown in sketch (d). To find the space-charge function f_5 chargeable to the subregion associated with point 5, it is necessary to sum the current contributions from each tube and divide by the subregion cross-sectional area (h × unit depth) and the magnitude of the average ion velocity in the x-direction in each stream tube. Thus, referring to sketch (d) gives



(d)

$$f_5 = \frac{1}{\epsilon_o h}\left[\frac{J_{a'b}}{\left(\bar{v}_x\right)_{a'b}} + \frac{J_{bc}}{\left(\bar{v}_x\right)_{bc}} + \frac{J_{cd'}}{\left(\bar{v}_x\right)_{cd'}}\right] \qquad (10)$$

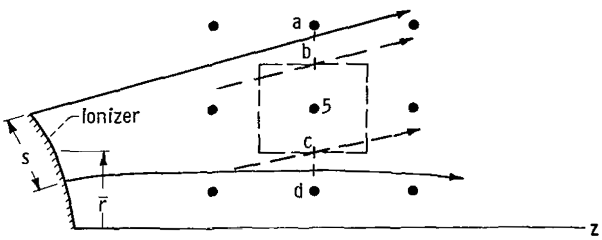
where

$$\left.\begin{aligned} J_{a'b} &= J_{ab}\left(\frac{a'b}{ab}\right) = \left(j_E s\right)_{ab}\left(\frac{a'b}{ab}\right) \\ J_{bc} &= \left(j_E s\right)_{bc} \\ J_{cd'} &= J_{cd}\left(\frac{cd'}{cd}\right) = \left(j_E s\right)_{cd}\left(\frac{cd'}{cd}\right) \end{aligned}\right\} \qquad (11)$$

and where $\left(\bar{v}_x\right)_{a'b}$, $\left(\bar{v}_x\right)_{bc}$, and $\left(\bar{v}_x\right)_{cd'}$ are the x-components of the average ion velocity and ab, a'b, bc, etc., are the stream-tube and/or line-segment designations, as shown in sketch (d).

The current density of the subregion is calculated by dividing by the subregion cross-sectional area taken normal to the x-direction; therefore, the x-components of the speeds must be used in equation (10).

It is also convenient in the axisymmetric problem to approximate the current density at the ionizer by using equation (9). The total current in each stream tube, however, must now be calculated as $J_t = j_E 2\pi \bar{r} s$, where $\bar{r}$ is the average radius to the ionizer line segment. For the space-charge-density calculation, area ratios must be used rather than the simple line-segment ratios employed in the two-dimensional problem. For example, consider the typical mesh point shown in sketch (e). Here,



(e)

$$f_5 = \frac{j_{bc}}{\epsilon_o \left(\bar{v}_z\right)_{bc}} = \frac{J_{bc}}{\epsilon_o \pi \left(r_b^2 - r_c^2\right)\left(\bar{v}_z\right)_{bc}} \tag{12}$$

where

$$J_{bc} = J_t \left(\frac{r_b^2 - r_c^2}{r_a^2 - r_d^2}\right) = j_E 2\pi \bar{r} s \left(\frac{r_b^2 - r_c^2}{r_a^2 - r_d^2}\right) \tag{13}$$

so that

$$f_5 = \frac{2 j_E \bar{r} s}{\epsilon_o \left(r_a^2 - r_d^2\right)\left(\bar{v}_z\right)_{bc}} \tag{14}$$

To recapitulate, space-charge-limited solutions to equation (7) are obtained by an iteration procedure in which the Laplace potential distribution is used to calculate initial ion trajectories and to obtain initial estimates of the current flowing in the stream tubes. The stream-tube current density at the ionizer is calculated from a Child's law relation for space-charge-limited-current flow. Values of the space-charge-density distribution are then determined and supplied in equation (7), which is solved to provide a new potential distribution. This completes one cycle. The new potential distribution then forms a basis for recalculation of trajectories and current densities, and the process continues until convergence is obtained. Details of the factors that affect the rate of convergence are given in reference 1. In general, from five to seven cycles are required with a total machine time of about 5 minutes.

Problems Less Than Space-Charge Limited

Previously, the current-density distribution of equation (8) was determined by first applying equation (9) to each stream tube and then using equation (10) or (14). Each subsequent iteration cycle of equation (7) produced a new potential distribution that is used to calculate new values of the emitter current density j_E.

On the other hand, j_E may be prescribed at some value, less than the space-charge-limited value, and held constant throughout the iterative procedure. This implies that the current in each stream tube remains constant. The current-density-distribution and velocity-distribution calculations required in equation (8) may then proceed in the same manner as described.

COMPUTER PROGRAM

The main program, LINKO, is divided into three major core loads, as shown in figure 3; each core load consists of several subroutines. Core load 1 is for data input and initialization. Core load 2 contains the subroutines used for calculation of the potential distribution. The charged-particle trajectory coordinates and space-charge-density distributions are calculated in core load 3.

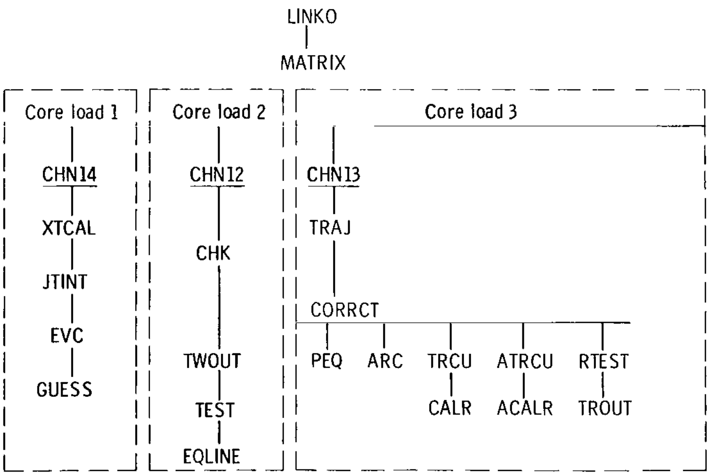


Figure 3. - Overlay chart of program.

Common statement symbols used in the program are defined in appendix B, and a complete listing of the program is given in appendix C. Brief descriptions of each of the core loads and subroutines along with schematic flow diagrams are presented in appendix D. An understanding of the input data preparation and output data listings is essential to use the program, and these topics will be discussed in detail in the sections that follow.

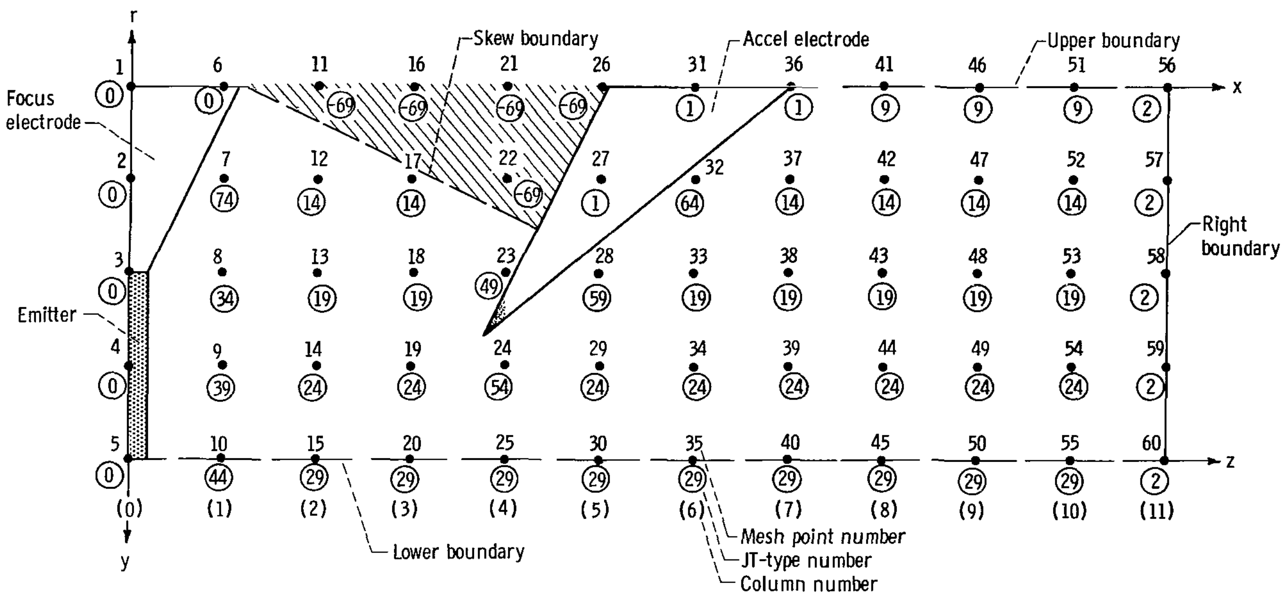


Figure 4. - Model for axisymmetric sample problem. Mesh spacing, 0.25 units; focuser and emitter potential, 1 kilovolt; accelerator potential, -1.0 kilovolt; right boundary, 0 kilovolt; normal derivative, zero along skew boundary and upper and lower boundaries.

Input Data

In this section the quantities required for preparation of input data are described. The explanation is based on the relatively simple model, shown in figure 4, that will be used as a sample problem for results to be presented in the next section.

In the explanation that follows, various subroutines appearing in core load 1 (fig. 3) that require input data are presented. Words are presented in the order in which they appear in each subroutine. Mesh point and mesh column numbering should always be ordered as shown in figure 4. The card column locations for the sample input data will be indicated in the input data listing.

It is important to note that, although the x, y or r, z coordinate orientation is as shown in figure 4, the program works only with x, y coordinates. Thus, unless otherwise noted in the following discussion, data input is always in terms of x, y coordinates.

Subroutine CHN14

NPIT. - If a first guess of the potential distribution is to be read in from subroutine GUESS, set NPIT = 0. If the potential distribution is available from subroutine BCDUMP (from prior computer run), set NPIT = 1.

NTP. - The absolute value of the maximum JT-type number plus 4 is given by NTP;

that is, NTP = $\left(|JT_{max}| + 4\right)$. The JT-type numbers are defined in the subsection subroutine XTCAL.

KAT. - This is a mesh column number and is used in connection with the test for current impingement on the first electrode for which an impingement test is desired (e. g., the accel electrode in fig. 4, p. 10). Of the mesh columns passing through the electrode, KAT is the minimum mesh column number where impingement can occur. If no impingement test is desired, set KAT = 0.

KATT. - Similar to KAT, KATT is the maximum mesh column number in the first electrode for which an impingement test is desired. If no impingement test is desired, set KATT = 0.

NT. - The total number of mesh points is given by NT.

NTJ. - Trajectories are equally spaced along the emitter (a spacing equivalent to 1 mesh width is usually adequate), and NTJ is equal to the number of spaces.

NTA. - Number of pairs of x, y coordinates used to specify the emitter surface is given by NTA. For the model shown in figure 4, only the beginning and the end pairs are needed since the emitter is a straight line.

KAN. - KAN is a mesh point number $\frac{1}{2}$ to $1\frac{1}{2}$ mesh widths away from the emitter surface centrally located with respect to the emitter surface. The space-charge-density function is checked for convergence at this point.

KBA. - KBA is a mesh point number such that if an equipotential line is taken through this mesh point, the equipotential line will be no closer than 1 mesh width from the emitter and no farther than 3 mesh widths. It may not be possible to satisfy these conditions simultaneously for some configurations in which case the selection of KBA should be based on the former condition. This equipotential line is used to calculate the emitter current density.

KAB. - KAB is a mesh column number. It is chosen to be the farthest mesh column to the right of the emitter required to establish a region in which the equipotential line will be located that is used to calculate the emitter current density.

KAT1. - Similar to KAT, KAT1 is the smallest mesh column number in the second electrode for the impingement test (if there is only one electrode as in fig. 4, or, if no impingement test is desired on the second electrode, set KAT1 = 0).

KAT2. - Similar to KATT, KAT2 is the largest mesh column number in the second electrode for the impingement test (if there is only one electrode, or if no impingement test is desired on the second electrode, set KAT2 = 0).

IAS. - For two-dimensional problems, set IAS = 0. For axisymmetric problems, set IAS = 1.

NPL. - The number of mesh points per mesh column is given by NPL.

KBB. - To calculate space-charge-limited current density, set KBB = 0. To specify the current density along the emitter set KBB = 1. At this point, five heading cards (card columns 1 to 72) are read into the program and printed out at the beginning of the data output listing.

JAS(J). - The Laplace potential distribution leads to an overestimation of the space-charge density and subsequently to a potential distribution that may be over-space-charge limited (i. e., there are potentials at mesh points near the emitter that are higher than the emitter potential). To check this condition, it is necessary only to examine a few points near the emitter. Thus, JAS(1) is the number of mesh points to be checked and JAS(2) to JAS[JAS(1) + 1] are the mesh point numbers. A maximum of 19 points can be checked with the present dimension of JAS.

AW. - Atomic weight (amu) of charged particles under consideration is given by AW, which is positive for positively charged particles and negative for negatively charged particles.

VA. - The emitter potential in volts is given by VA.

H. - The mesh size H is a fraction of chosen unit of length. Results will be in terms of whatever dimension is used for H.

VAT. - The potential of the maximum equipotential line to be printed out in volts is given by VAT.

VBT. - The potential of the minimum equipotential line to be printed out in volts is given by VBT.

SIZE. - This is the step size of equipotential lines to be printed out in volts. If SIZE = 0, no equipotentials are printed out.

RCU(J). - Read in only when the current density is specified (KBB = 1) along the emitter surface, that is, less-than-space-charge-limited problems. The current density at the emitter of the J^{th} segment of the emitter is RCU(J). The segments are bounded by charged-particle trajectories and are numbered consecutively, starting from 1, and increase with increasing y (decreasing r). Note for example, that if H is given in millimeters, the RCU(J)'s must be specified as amperes per millimeter squared.

At this point, the data from subroutine XTCAL is read in.

ATX(J). - This is the x-coordinate of the J^{th} pair of x, y coordinates given to specify the emitter surface.

ATY(J). - This is the y-coordinate of the J^{th} pair of x, y coordinates given to specify the emitter surface. By convention, ATY(J) $<$ ATY(J + 1) and usually these are coordinates of y-mesh lines except possibly for the first and/or last values. It should be emphasized that for axisymmetric problems it is the y-coordinate that is used (not the r-coordinate).

ER(J). - A set of y-coordinates of the electrode edges are required to calculate impingement on the electrodes. The largest y-coordinate of the first mesh column in the first electrode is ER(1) (corresponds to mesh column KAT). Similarly, ER(2) to ER(KATT - KAT + 1) are the largest y-coordinates for the remaining mesh columns in the first electrode. Likewise ER(KATT - KAT + 2) to ER(KAT2 + KATT + 2 - KAT - KAT1) are the y-coordinates for impingement on the second electrode, should there be a second electrode.

The data from subroutine JTINT is read in after the ER's.

XR. - This is the spectral radius of iteration matrix and it is the absolute value of the largest eigenvalue of the iteration matrix (see ref. 1). It is generated in subroutine EVC and is printed as part of the data output. Its value depends only on the physical configuration. Initially, a blank card is included to satisfy the read statement. For subsequent runs of the same configuration, XR is available from the data output.

Subroutine XTCAL

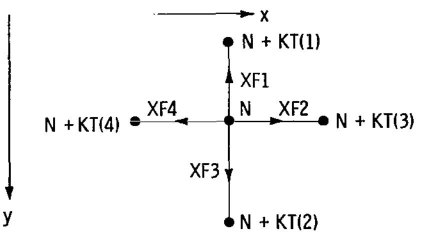
The finite-difference equation replacing Poisson's equation at the mesh point N has the form

$$U(N) = C_1(N)U(N + J_1) + C_2(N)U(N + J_2) + C_3(N)U(N + J_3) + C_4(N)U(N + J_4) + C_5(N)RH(N) \tag{15}$$

where the $U(N + J_i)$, $i = 1, \cdot \cdot \cdot, 4$, are potential values, the $C_i(N)$, $i = 1, \cdot \cdot \cdot, 5$, are coefficient weights, and RH(N) is the charge density at the N^{th} point. Subroutine XTCAL calculates the coefficients in equation (15), $C_1(N), \cdot \cdot \cdot, C_5(N)$, for the different mesh-point configurations. Generally, the coefficients depend on the distance from the mesh point to a boundary at a fixed potential and, in the axisymmetric case, on the radius as well. For either the two-dimensional or axisymmetric problem, the data input has the following format for each card:

KT(1), KT(2), KT(3), KT(4), XF1, XF2, XF3, XF4, R.

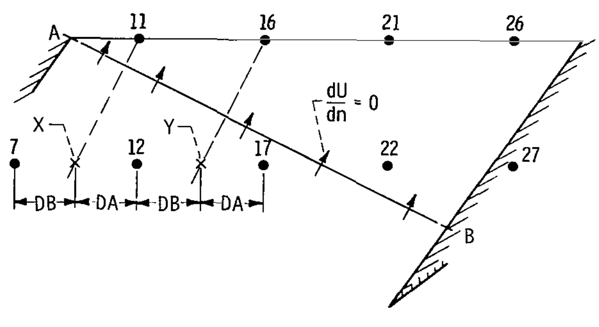
KT(1) to KT(4) are relative numbers that are added to the mesh point number. They are the numbers $J_1, \cdot \cdot \cdot, J_4$ in equation (15). XF1 to XF4 are the positive distances from the central mesh point N to either the boundary at a fixed potential or the nearest mesh point. The distance to the central mesh point from the axis of symmetry in the axisymmetric case is R (no entry required in two-dimensional problems). These quantities are further illustrated in sketch (f).



(f)

More specifically, KT(1) is used to select the upper vertical mesh point, and its value is usually equal to -1. For example, from examining figure 4 (p. 10), it is seen that for all mesh-point configurations, except for the ones along the upper boundary and the one associated with mesh point 24, KT(1) = -1. For mesh point 24, it is necessary to use an entry for KT(1) that will select a mesh point at a fixed potential (i. e., a mesh point located inside the accel electrode). Thus, KT(1) can have the entry 3 which picks out point 27, or 7, which picks out point 31. The XF1's are related to the KT(1)'s, and since they are zero for the upper boundary mesh configurations, the values used for these KT's are not important. It is convenient to use -1. A similar argument applies to other boundaries. The relative number used to select the lower vertical mesh point to be used in equation (15) is KT(2), and examination of figure 4 reveals that KT(2) = 1 except for mesh point 23. At mesh point 23, KT(2) can either be 4 or 8. The relative number picking out the horizontal point to the right is KT(3), and, in general, it is merely equal to the number of points per column. In every case in figure 4, KT(3) is 5 except for mesh point 23. For mesh point 23, KT(3) can be either 4 or 8, since as before, it is only necessary to pick out some point in the accel electrode so that the correct potential value will be used in equation (15). The relative number picking out the horizontal point to the left is KT(4), and, in general, it is equal to minus the number of points per column. In figure 4, KT(4) is seen to be -5 in every instance except for point 28 in which case it can either be -1 or 3. It should be mentioned that while the preceding remarks pertaining to mesh points 23 and 24 may make it appear as if these two points are rather special, an important feature illustrating the flexibility of the program has been demonstrated; that is, with this method of input data preparation, electrodes may be "infinitely thin," the only requirement being that a mesh point at the desired potential must exist somewhere in the array.

The distances XF1, . . ., XF4, are measured, respectively, in the upward vertical (-y) direction, the right horizontal (+x) direction, the downward vertical (+y) direction, and, the left horizontal (-x) direction (see sketch (f)). If any of the XF's equal zero, it is assumed in the program that the normal derivative equal to zero is specified in the corresponding direction, and the appropriate formula (refs. 1 and 2) is then used to calculate the coefficients. When it is desired to have a skew boundary that has the normal derivative equal to zero as the "boundary condition," the pertinent boundary points are described somewhat differently. Sketch (g) depicts a portion of the region from figure 4 that contains a skew boundary. Note in sketch (g) that a normal to line AB from point 11 intersects between points 7 and 12 at point X, which is at a distance DA from point 12 and a distance DB from point 7. Similarly, a normal to AB drawn from point 16 intersects at Y, which is at a distance DA from point 17 and DB from point 12. The normal derivative equal to zero



(g)

(dU/dn = 0) implies that an equipotential line passing through points 11 or 16 would be at right angles to the line AB and that if the potential at point 11, for example, were extrapolated along the line 11 X, the potential at point X would be at the same value. The potential at point 11 is not known but the potential at point X can be found by interpolation between values at points 7 and 12. Values of potential at points 7 and 12 are available because they are "interior points." Similarly, the potential at point 16 is the same as that at point Y. Thus, using linear interpolation gives the potential at point 11 as

$$U(11) = \left(\frac{DA}{DA + DB}\right)U(7) + \left(\frac{DB}{DA + DB}\right)U(12) \tag{17}$$

and at point 16 as

$$U(16) = \left(\frac{DA}{DA + DB}\right)U(12) + \left(\frac{DB}{DA + DB}\right)U(17) \tag{18}$$

Recall that the general form of equation (15) is $U(N) = C_1(N)U(N+J_1) + C_2(N)U(N+J_2) + \cdot\cdot\cdot + C_5(N)RH(N)$. The $C_i(N)$, in general, are calculated from the finite-difference equations (by using the XF distances), but it is a desirable feature to be able to enter certain $C_i(N)$ coefficients (e.g., those associated with eqs. (17) and (18)) directly into the computer. This feature is accomplished by setting $XF1 = -C_1(N)$, $XF2 = C_2(N)$, $XF3 = C_3(N)$, $XF4 = C_4(N)$, and $R = C_5(N)$. The KT's of this card can then be assigned relative numbers to associate the coefficients with the corresponding mesh points, that is, for these special cards, KT(1) is associated with XF1, KT(2) with XF2, etc. Referring to equations (17) and (18) shows that the entries for the mesh configuration of mesh points 11 and 16 would then have the form -4, 1, 0, 0, $-\left(\frac{DA}{DA + DB}\right)$, $\left(\frac{DB}{DA + DB}\right)$, 0, 0, 0. Note that mesh points 12 and 17 are considered as regular points since the distances XF1 to XF4 are only different from a mesh spacing when measured to a boundary where the potential is held constant or in the special cases just described.

It will become evident in proceeding from mesh point to mesh point that duplication of all nine entries on each card can occur. To eliminate this duplication, all similar mesh points are assigned a JT-type number of the form $\pm(5n + 4)$, $n = 1, 2, \cdot\cdot\cdot$, (see fig. 4, p. 10). In this manner, a large number of mesh points can be represented by one data card. The method used to get the JT-type numbers into the program as data is described in the subroutine JTINT section.

As is shown in figure 4, the assignment of actual JT-type numbers is arbitrary as far as correlation with mesh point numbers. However, in correlating XTCAL data cards, the approach is to associate the first data card for XTCAL with mesh points of type ±9, the next, type ±14, etc. No XTCAL data cards are required for mesh points held at a fixed potential, although these points are also assigned JT-type numbers. The numbers 0 to 6 are reserved for these points. All points associated with the focus electrode may be assigned the JT-type number 0; all accel electrode points, type number 1; and so on to a maximum of type number 6. These mesh points will then have the potential values as prescribed in subroutine GUESS.

The JT-type numbers are of the form $\pm(5n + 4)$ because there are exactly five coefficients (eq. (15)) necessary for each mesh-point configuration. The sign designation, plus or minus, is used to describe the central mesh points further. A positive type number is used to designate mesh points whose coefficients $C_i(N + J_i)$ are calculated from the finite-difference equations. A negative type number is used to indicate those points that are free to change in value but have coefficients calculated from linear interpolation. Examples of this type of point are shown in figure 4 (p. 10), that is, points 11, 16, 21, 22, and 26. As previously discussed, they arise because of the skew boundary that has the boundary condition of normal derivative equal to zero specified. It is not necessary to make consecutive duplicate data cards: only one card per different mesh-point configuration is required. Also, no card is required for mesh points held at a fixed potential.

Subroutine JTINT

Once a JT-type number is assigned to each mesh point, the computer generates the matrix equation

$$I\underline{U} = C\underline{U} + \underline{K} \tag{16}$$

where, if M denotes the number of mesh points, I is the $M \times M$ identity matrix, C is the $M \times M$ matrix having entries in each row $C_1(N)$, $C_2(N)$, $C_3(N)$, and $C_4(N)$; K is the $1 \times M$ column vector with entries $C_5(N)RH(N)$, and U is the $1 \times M$ column vector of potentials. If the C matrix contained $M \times M$ nonzero entries, it would be prohibitive to store C in core, but C contains only four entries in each row (see eq. (15)) so that the C matrix can be specified by at most 4M numbers.

To enter the JT-type numbers into the program as data, they must be punched on cards. While it is possible to write out the JT-type numbers, one for each mesh point, punch them on cards, and read them into the JT-array, this method is inefficient for large M. A more general method is used.

Description of the JTINT data cards. - The first card has only one entry, JA, which is the number of cards that contain data for the JT array. The JT cards all have the same format, that is, KA; KB(1) to KB(13). For the axisymmetric geometry, KA is always equal to 1; the two-dimensional variation of KA will be discussed later. The number of consecutive mesh points to be assigned the JT-type number in KB(2g) is KB(2g - 1), g = 1, · · ·, 6.

The convention for setting up the KB array for an axisymmetric problem is to start at mesh point number 1 and count the number of consecutive mesh points in the first row (not column) that have the same JT-type number. Looking at the first row in figure 4 shows that this number, KB(1), is 2 since points 1 and 6 are boundary points at a fixed potential. Thus, the entry of KB(1) is 2, and KB(2) has the entry that is the JT-type number assigned to mesh points that are held at the focuser potential. Then KB(3) is seen to be 4, and KB(4) is the JT-type number assigned to these mesh points; KB(5) is 2, and KB(6) is the JT-type number assigned to these mesh points; KB(7) is 3, and KB(8) is the JT-type number associated with points 41, 46, and 51; KB(9) is 1, and KB(10) is the JT-type number associated with point 56. This completes the specification of the points of row 1; however, note that KB(11) and KB(12) have not yet been assigned. The specifications for row 2 may begin in KB(11) and KB(12), or they may be left blank and row 2 specifications begun on the next card with KB(1) and KB(2). The procedure is continued until all mesh points have been covered. Recall that the number of cards generated in this process is JA.

For an axisymmetric problem (IAS = 1), it is also necessary to supply a final card giving the distance from the axis of symmetry of the model to the top of the region. This number is denoted in the program by BASE.

For two-dimensional problems, the process is similar except that it starts at mesh point 1 and goes down the first column in assigning values to KB(K), K = 1, · · ·, 12, then down the second column, etc. The reason for going down the columns in two-dimensional problems is that it is frequently possible to repeat the KB specifications of a card several times.

Rather than punching the same card over and over again, the number of repetitions desired may be specified by KA. Note that utilization of this feature is not possible for axisymmetric problems wherein the coefficients C_i of equation (15) vary with respect to radius. Thus, if a columnwise procedure were used, each mesh point would require individual specification. JT-type number assignments for a two-dimensional problem are shown in figure 5.

Subroutine GUESS

Subroutine GUESS is used to "initialize" the potential field. Data are read in only if NPIT = 0.

GEP(J). - As discussed in the data input section for subroutine XTCAL, boundary points that are held at a specified potential are assigned JT-type numbers from 0 to 6.

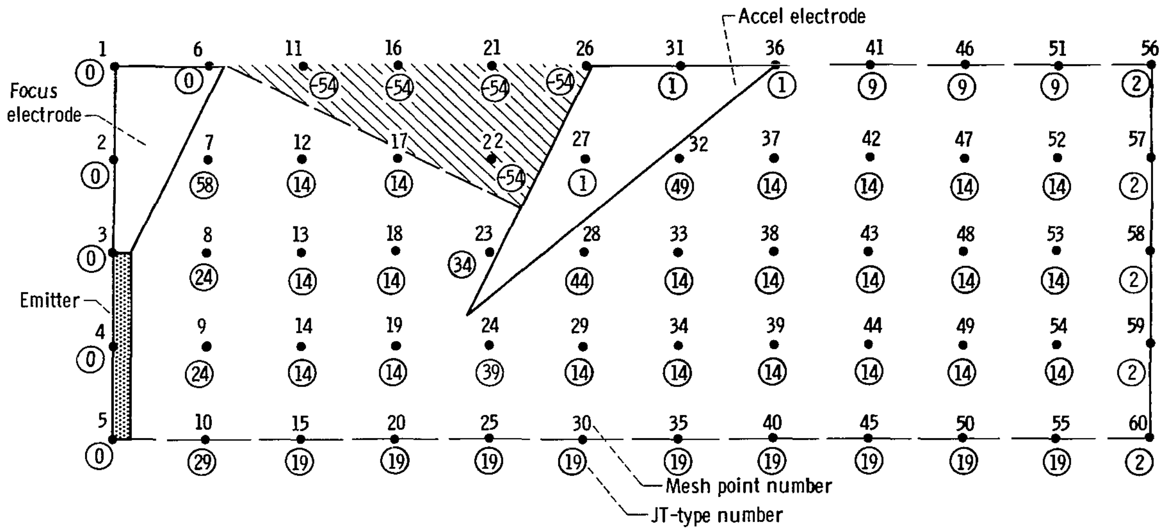


Figure 5. - Mesh points and JT-type numbers for two-dimensional sample problem.

Mesh points designated as type 0 are assigned the value of GEP(1), those of type 1 the value of GEP(2), etc. All other mesh points are initially given the value zero.

SAMPLE PROBLEMS

In the previous section, the input data words were presented and described. In this section a sample problem based on the axisymmetric model shown in figure 4 will be described; that is, numerical values will be assigned to the input data words, and results will be given. Machine listings of the input and output are presented in appendix E.

The mesh size shown in figure 4 is far too coarse to be used for meaningful analysis in the physical sense; however, for purposes of illustration of data preparation it is appropriate. As a result of the coarse mesh, the accelerator tip (shown as the shaded region in fig. 4, p. 10) extends into a mesh square and is not accounted for in impingement calculations. It will be assumed first that the model represents an electrostatic thrustor, which uses cesium as a propellant, and that the current density will be specified along the emitter. Voltages and distances will be as shown in figure 4. A set of input and output data for the same model treated as a space-charge-limited two-dimensional problem is also included in appendix E.

Input Data Preparation

The numerical values of the input quantities are given next. The data are presented in the order in which they are read in by the program (i. e., the order in which they appear in appendix E).

Subroutine CHN14. -

NPIT	0	A first guess of the potential distribution will be read in from subroutine GUESS.
NTP	78	$\left(\lvert JT_{max}\rvert + 4\right)$. The numerical value of NTP cannot be assigned until after the JT-type numbers have been determined (see fig. 4, p. 10).
KAT	4	The first column for impingement test
KATT	6	The last column through the accel electrode for which an impingement test is desired
NT	60	Total number of mesh points
NTJ	4	Number of spaces between trajectories
NTA	2	Because the emitter is a straight vertical line, only two pairs of coordinates are required to specify it.
KAN	9	Mesh point for convergence check
KBA	10	Mesh point locating equipotential for current density calculation
KAB	2	Mesh column locating region for KBA equipotential
KAT1	0	No second electrode for impingement
KAT2	0	No second electrode for impingement
IAS	1	Designates axisymmetric problem
NPL	5	Number of mesh points per column
KBB	1	Current density will be specified.

At this point, five heading cards are read.

JAS(1)	3	Number of mesh points to check for over space charge
JAS(2)	8	Mesh point to check for over space charge
JAS(3)	9	Mesh point to check for over space charge
JAS(4)	10	Mesh point to check for over space charge
JAS(5) to JAS(20)	0	Only three mesh points are checked for over space charge limited

Refer to appendix E and note that two cards are required to satisfy the READ statement for the JAS-array:

AW	132.91	amu
VA	1000.	V
H	0.25	arbitrary units
VAT	1000.	V
VBT	-1000.	V
SIZE	200.	V
RCU(1)	1.0 E-4	A/unit^2
RCU(2)	1.0 E-4	A/unit^2
RCU(3)	1.0 E-4	A/unit^2
RCU(4)	1.0 E-4	A/unit^2

As previously discussed in assigning JT-type numbers, the manner in which they are associated with the mesh points is arbitrary, the only requirement being that mesh points which are similar must be assigned the same type number. In table I the JT-type numbers and associated mesh points are indicated. The JT-type numbers and associated mesh points for which data are not necessary are given in the table at the left. For reference, the JT-type numbers may be punched on

JT-type number	Mesh points
0	1 to 6
1	27, 31, 36
2	56 to 60

TABLE I. - XTCAL DATA CARDS

Data									JT-type number	Reference mesh points (from fig. 4, p. 10)
KT(1)	(2)	(3)	(4)	XF1	XF2	XF3	XF4	R		
-1	1	5	-5	0.00	.25	.25	.25	1.00	9	41, 46, 51
-1	1	5	-5	.25	.25	.25	.25	.75	14	12, 17, 37, 42, 47, 52
-1	1	5	-5	.25	.25	.25	.25	.5	19	13, 18, 33, 38, 43, 48, 53
-1	1	5	-5	.25	.25	.25	.25	.25	24	14, 19, 29, 34, 39, 44, 49, 54
-1	1	5	-5	.25	.25	0.00	.25	0.00	29	15, 20, 25, 30, 35, 40, 45, 50, 55
-1	1	5	-5	.25	.25	.25	.20	.5	34	8
-1	1	5	-5	.25	.25	.25	.20	.25	39	9
-1	1	5	-5	.25	.25	0.00	.20	0.00	44	10
-1	4	4	-5	.25	.025	.050	.25	.5	49	23
3	1	5	-5	.125	.25	.25	.25	.25	54	24
-1	1	5	-1	.0834	.25	.25	.10	.5	59	28
-1	1	5	-5	.042	.25	.25	.05	.75	64	32
-4	1	5	-5	-.50	.50	0.00	0.00	0.00	-69	11, 16, 21, 22, 26
-1	1	5	-5	.165	.25	.25	.08	.75	74	7

the XTCAL data cards in card columns 73 to 80.

Subroutine CHN14 data (cont.). -

ATX(1), ATX(2)	.05, .05	x-emitter coordinates
ATY(1), ATY(2)	.50, 1.0	y-emitter coordinates
ER(1), . . ., ER(3)	.625, .400, .23	impingement coordinates

The data for subroutine JTINT are to be included at this point. Since the sample problem is being considered as an axisymmetric configuration, it will be recalled from the previous section that the ordering of the KB numbers will be from left to right, that is, along rows.

JA	7	This value cannot be assigned until the following JT cards (table II) have been established.

TABLE II. - JT DATA CARDS

KA	KB											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	2	0	4	-69	2	1	3	9	1	2		
1	1	0	1	74	2	14	1	-69	1	1	1	64
1	4	14	1	2								
1	1	0	1	34	2	19	1	49	1	59	5	19
1	1	2										
1	1	0	1	39	2	24	1	54	6	24	1	2
1	1	0	1	44	9	29	1	2				

BASE	1.0	BASE is read in only if IAS = 1, that is, for axisymmetric problems.

Subroutine CHN14 data (cont.). -

XR	----	No value for the spectral radius is available, but a blank card is necessary here.

Subroutine GUESS data. -

GEP(1)	1000.	V
GEP(2)	-1000.	V
GEP(3)	0.	V

The preceding values represent the potential values assigned, respectively, to the emitter and focuser, the accel electrode, and the straight boundary at the far right of the region. The values are associated with JT-type numbers 0, 1, and 2. After the Laplace potential distribution is determined, it is punched on cards as part of the output of the

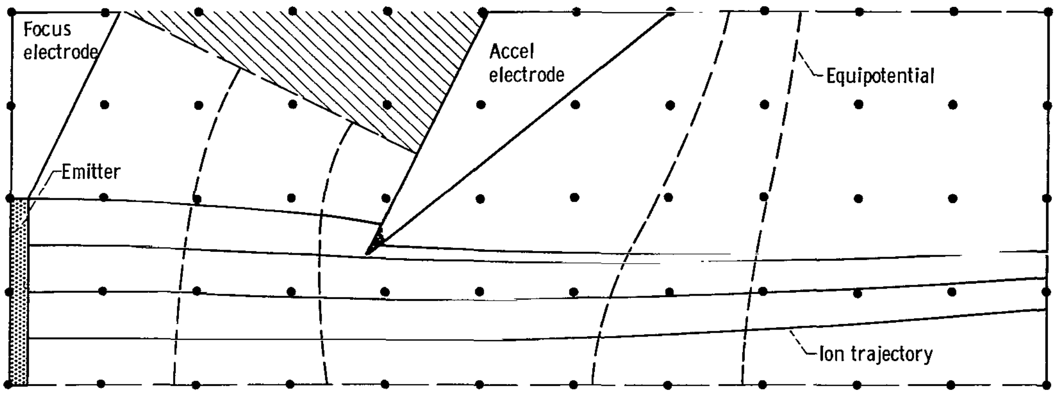


Figure 6. - Axisymmetric model. Focuser and emitter potential, 1 kilovolt; accelerator potential, -1 kilovolt; emitter current density specified at 12.7 milliamperes per square centimeter; percent impingement current at accelerator, 27.

program. If it is desired to run the problem again with the Laplace potential as the initial distribution, these cards should be included at this point, rather than data for subroutine GUESS, and NPIT set equal to 1.

Output Data Interpretation

A listing of the output data for the sample problem is given in appendix E, and results are plotted in figure 6. The listing begins with the five heading cards. The next portion of the data consists of the KT and XT from subroutine XTCAL. The XT are the coefficients of equation (15) that were calculated from the XF. Following the KT, XT printout is the number of iterations required to converge on the spectral radius XR and its value. The Laplace potential field convergence information and mesh point numbers along with their corresponding potential values are listed next, followed by the listing of the x, y coordinates of various Laplace equipotentials.

The quantities given next are self-explanatory. It should be noted that the trajectories listed at this point are calculated from the Laplace potential distribution. Convergence information is then given relative to the Poisson solution, where RHLOW, RHUP, and RH refer to the space-charge-density function at a predetermined "test" mesh point, KAN. The RH values at the various mesh points are the space-charge-density values multiplied by $H^2/4$, where H is the mesh spacing. Cycles 1, 2, 3, $\cdot\cdot\cdot$, n refer to successive iteration cycles, as described in reference 1. Finally, the converged Poisson solution is listed.

CONCLUDING REMARKS

The purpose of this report has been to describe in detail a computer program capable of solving a wide variety of space-charge-flow problems. The approach taken has been one in which emphasis has not been on the mathematical relations or physical interpretation, but rather to cover all aspects of input data preparation and output data interpretation. Toward this end, input words were fully defined and flow diagrams presented. Sample problems were then used to explain the program further.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 18, 1965.

APPENDIX A

MATHEMATICAL SYMBOLS

[The units used are the International System or SI.]

A	matrix of eq. (7)
f	space-charge-density distribution function for discrete case
h	mesh spacing
J	current
j	current density
$\underline{k}$	column vector of eq. (7)
ℓ	distance, defined in eq. (9) and sketch (c)
m	particle mass
q	unit charge
r	variable
$\bar{r}$	average radius to ionizer line segment
s	length of line segment
v	ion speed
w	potential distribution function for discrete case
$\underline{w}$	column vector of eq. (7)
x, y, z	variables
δ, ϵ	fraction of mesh spacing (see sketch (a))
ϵ_o	permittivity of free space
ρ	space-charge-density distribution function for continuous case
φ	potential distribution function for continuous case

Subscripts:

A	accelerator
E	ionizer
t	stream tube
x, z	direction

APPENDIX B

COMMON STATEMENT SYMBOLS

AREM	arc distance between trajectories at emitter surface
ATX(J)	problem specification (see data input discussion)
ATY(J)	problem specification (see data input discussion)
AX	x- coordinate of trajectories as they are calculated
AY(J)	y- coordinates of trajectories
BASE	see data input discussion
CU(J)	currents for stream tubes
CUD(J)	current densities at emitter surface
DC	distance used to sum tube currents
DCC	distance used to sum tube currents
DELY	mesh size in y-direction
DX	mesh size in x-direction
EPS	convergence criterion for potential field
ER(J)	see data input discussion
ETX(J)	beginning x-coordinates of trajectories at emitter surface
ETY(J)	beginning y-coordinates of trajectories at emitter surface
H	mesh size
IAS	see data input discussion
JAS(J)	see data input discussion
JD	intermediate storage
JOT	printout counter for trajectory coordinates
JT(J)	vector of type numbers
KAB	see data input discussion
KABB	see data input discussion
KAN	problem specification (see data input discussion)
KAT	program control word (see data input discussion)

KATT	program control word (see data input discussion)
KAT1	see data input discussion
KAT2	see data input discussion
KB(J)	used in subroutine JTINT as problem specification (see data input discussion) and then as cycle print control
KBA	see data input discussion
KBB	see data input discussion
KBF	indicates whether or not emitter surface extends to top boundary of region
KCH(J)	trajectory reflection counter
KRL	cycle counter
KT(J)	problem specification (see data input discussion)
LAST	indicates whether or not emitter surface extends to lower boundary of region
LB(J)	internal control parameters
LC(J)	internal control parameters
MO	indicates upper or lower bound test for RH
NAJ	number of tubes
NPIT	program control word (see data input discussion)
NRD	printout counter for trajectory coordinates
NRL	maximum number of cycles to converge to Poisson solution
NT	total number of mesh points
NTJ	number of trajectories
NURL	number of iterations on potential distribution
NXEP	mesh point number where maximum potential change is occurring
PTX(J)	x-coordinates of equipotential line used to calculate current density at emitter surface
PTY(J)	y-coordinates of equipotential line used to calculate current density at emitter surface
RCU(J)	see data input discussion
RH(J)	present space-charge-density function is stored in RH array except in subroutine EVC where an intermediate iterate on spectral radius is stored

RHDOWN	lower bound on RH
RHUP	upper bound on RH
RIN	width of region
RX	suppression factor for RH
SAU(J)	current at emitter surface
SEM	total emitter length
SIZE	problem specification (see data input discussion)
U(J)	present potential field is stored in U array except in subroutine EVC where an intermediate iterate on spectral radius is stored
UB(J)	potential field one cycle back is stored in UB array except in subroutine EVC where an intermediate iterate on spectral radius is stored
URH(J)	space-charge-density function one cycle back is stored in URH array
VA	emitter potential
VAT	problem specification (see data input discussion)
VBT	problem specification (see data input discussion)
VX(J)	x-velocity components of trajectories except in subroutine EQLINE where x-coordinates of equipotential lines are stored
VY(J)	y-velocity components of trajectories except in subroutine EQLINE where y-coordinates of equipotential lines are stored
XD	intermediate storage
XEP	storage for maximum potential change from iteration to iteration
XK	intermediate storage
XMP(J)	error factor
XQM	charge-to-mass ratio
XR	problem specification (see data input discussion)
XT(J)	coefficients in eq. (15)
YEP	permittivity of free space

APPENDIX C

FORTRAN LISTING

```
C     LINKO IS USED AS THE MAIN PROGRAM
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6  SIZE, U(3000), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
     7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
C     INITIALIZE COMMON AREA TO ZERO (19638 IS THE PRESENT LENGTH )
      DIMENSION AREM(1)
      DO 7 J=1,19638
 7    AREM(J)=0.0
C     TRANSFER TO DATA INPUT SUBROUTINE
 1       CALL CHN14
C     CHN12 CALCULATES THE POTENTIAL FIELD
 2       CALL CHN12
C     CHN13 CALCULATES TRAJECTORIES AND RHS
 3       CALL CHN13
C     SWITCH TO RESTART FOR NEXT DATA CASE
 6       IF(NURL) 1,2,2
         END
```

```
      SUBROUTINE MATRIX(N,B,X)
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6  SIZE, U(3000), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
     7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
         DIMENSION  B(300),X(100)
      M=3*N-2
         B(M+1)=0.0
      B(2)=B(2)/B(1)
      X(1)=X(1)/B(1)
         IF(N-1) 12,12,9
9     K=2
      DO 10 J=4,M,3
      B(J)=B(J)-B(J-1)*B(J-2)
      B(J+1)=B(J+1)/B(J)
      X(K)=(X(K)-B(J-1)*X(K-1))/B(J)
10    K=K+1
         K=K-1
      DO 11 J=1,M,3
      NB=M-J-1
      K=K-1
11    X(K)=X(K)-B(NB)*X(K+1)
12       RETURN
         END
```

```
      SUBROUTINE CHN14
C   CHN14 IS FOR DATA INPUT
     COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
    1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
    2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
    3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
    4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
    5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
    6  SIZE, U(3000), UB(3000), LRH(3000),VA, VAT, VBT, VX(51),
    7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
     READ(5,100) NPIT,NTP,KAT,KATT,NT,NTJ,NTA,KAN,KBA,KAB,KAT1,KAT2,IAS


      READ(5,100)NPL,KBB
     NTJ=NTJ-1
C  NPIT=NEGATIVE,  THE PROGRAM STOPS
C  NPIT=0,FIRST GUESS FOR POTENTIAL FIELD IS READ IN FROM GUESS
C  NPIT=+, POTENTIAL FIELD IS READ IN FROM BCREAD
C   NTP=MAXIMUM JT TYPE NUMBER +4
C  KAT=FIRST LINE IN FIRST GRID FOR IMPINGEMENT TEST
C  KATT=LAST LINE IN FIRST GRID FOR IMPINGEMENT TEST
C NT=TOTAL NUMBER OF POINTS
C NTJ=NUMBER OF TRAJECTORIES
C NTA=NUMBER OF INPUT COORDINATES FOR EMITTER
C KAN=TEST POINT FOR RHS
C KBA=TEST POINT FOR EQUIPOTENTIAL
C KAB=NUMBER OF LINES TO TRAVERSE TO OBTAIN EQUIPOTENTIAL LINE FOR CU
C CALCULATION
C  KAT1=FIRST LINE IN SECOND GRID FOR IMPINGEMENT TEST
C  KAT2=LAST LINE IN SECOND GRID FOR IMPINGEMENT TEST
C   IAS=0 FOR 2-D,  IAS=1 FOR 3-D
C   NPL=NUMBER OF POINTS PER LINE
C   KBB=0,- CURRENT DENSITY IS SPACE CHARGE LIMITED
C   KBB=+ CURRENT DENSITIES ARE SPECIFIED TUBEWISE BY RCU READ
C   NRL=MAX NUMBER OF CYCLES TO CONVERGE ON POISSON SOLUTION
      NRL=5
      KRL = NRL
C   KBF=UPPER SYMMETRY SWITCH
      KBF=1
C   RHUP= UPPER BOUND ON RH
      RHUP=0.0
C   RHDOWN= LOWER BOUND ON RH
      RHDOWN=0.0
C   KBH=READ  CONTROL FOR ER
      KBH=KATT-KAT+1
      IF(KAT2)16,16,17
17    KBH=KBH+KAT2-KAT1+1
C   PRINT CONTROL FOR LAPLACE TRAJ
16    JOT=200
C   PRINT CONTROL FOR POISSON TRAJ
      NRD=200
     DO 1 J=1,5
C   READ IN HEADING CARDS
      READ  (5,101)
1     WRITE (6,101)
```

```
      KABB=NTA-1
    LB(1)=0
    LB(2)=0
    LC(1)=NT/NPL-1
    LC(2)=NPL+1
    LC(3)=NPL
C JAS IS A VECTOR WHICH DETERMINES TEST POINTS TO BE CHECKED AGAINST
C EMITTER POTENTIAL
C   JAS(1)=NUMBER OF POINTS TO BE CHECKED AGAINST EMITTER POTENTIAL
C   JAS(2-(JAS(1)+1))=MESH POINT NUMBERS TO BE CHECKED
      READ  (5,100)(JAS(J),J=1,20)
C YEP=PERMITTIVEITY OF FREE SPACE
      YEP=8.854E-12
C   RX=RHS SUPRESSION FACTOR
      RX=.4
      EPS=.1
C   LAST=LOWER SYMMETRY TEST
      LAST=0
   READ (5,103) AW,VA,H
C  AW = ATOMIC WEIGHT OF ION
C VA=EMITTER POTENTIAL
C H=MESH SIZE
      XQM=9.649E7
C XQM=CHARGE TO MASS RATIO OF IONS
   XQM=XQM/AW
C   CC AND DCC ARE USED ONLY TO SUM THE CURRENT AT THE APPROPRIATE STEP
C   DC=DISTANCE FROM 0 TO RIGHT HAND SIDE OF ACCEL ELECTRODE
C   DCC EQUALS DISTANCE BETWEEN ACCEL AND DECEL ELECTRODES
      DC=FLOAT(KATT)*H
      DCC=FLOAT(KAT2-KATT)*H
      IF(KAT2.EQ.0) DCC=0.
   READ(5,103) VAT,VBT,SIZE
C   VAT = LARGEST POTENTIAL IN EQUIPOTENTIAL PRINTOUT
C   VBT = SMALLEST POTENTENTIAL IN EQUIPOTENTIAL PRINTOUT
C   SIZE = STEPSIZE FOR EQUIPOTENTIAL PRINTOUT
      XC=NPL-1
C   RIN = WIDTH OF REGION
      RIN=XC*H
      MO = 1
      IF(KBB) 35,35,36
 36   NN=NTJ+1
C   RCU=VALUES IF FIXED CURRENT DENSITY
      READ  (5,102)(RCU(J),J=1,NN)
35   CALL XTCAL(NTP)
C ATX=X-COORDINATES OF EMITTER
   READ (5,103) (ATX(J),J=1,NTA)
C ATY=Y-COORDINATES OF EMITTER
      READ  (5,103)(ATY(J),J=1,NTA)
C ER=TEST POINTS FOR IMPINGEMENT
      IF(KAT.EQ.0) GO TO 40
      READ  (5,103)(ER(J),J=1,KBH)
C   CHECK UPPER SYMMETRY
 40   IF(ATY(1)) 2,2,3
 2    KBF=-KBF
C   CHECK LOWER SYMMETRY
 3    IF(ABS(ATY(NTA)-RIN)-1.E-05*H) 30,30,37
```

```
 37       LAST=-1
 30       DO 4 J=1 ,NT
 4        JT(J) = 0
C   JTINT SETS UP JT-ARRAY
 34       CALL JTINT(NTP)
C  XR = SPECTRAL RADIUS
12     READ (5,103) XR
C CONDITIONAL EIGENVALUE CALCULATION
          IF((XR*(XR-1.)).LT.0. ) GO TO 19
13        CALL EVC
C CONDITIONAL EXIT
19     IF (NPIT)  25,14,27
C   GUESS INITIALIZES THE POTENTIAL FIELD
 14       CALL GUESS
          GO TO 18
27     CALL BCREAD(U(1),U(NT))
18      XM=LC(3)
        XN=LC(1)
C   XMP=ERROR FACTOR RELATING MESH SIZE
          XMP=.5*(XN*XM)**2/(XN*XN+XM*XM)
C   NURL=NUMBER OF ITERATIONS ON POISSON SOLUTION
          NURL=300
C  KB IS NOW PRINT CONTROL FOR POTENTIAL AND RHS
          KB(1)=1
          DO 15 J=2,13
 15       KB(J)=0
          DO 22 J=1,NT
          UB(J)=U(J)
          RH(J)=0.0
          IF(IABS(JT(J)).LE.8) JT(J)=0
          IF(JT(J))31,22,22
31     IF(LB(1)) 21,21,29
21     LB(1)=J
29     LB(2)=J
 22       CONTINUE
          WRITE(6,104)
          RETURN
25        CALL EXIT
100       FORMAT(14I5)
101       FORMAT(72H
     1                                                                        )
 102      FORMAT(7E10.5)
103       FORMAT(7F10.5)
 104      FORMAT(1H0,47X,17H LAPLACE SOLUTION )
          END
```

```
C     XTCAL CALCULATES COEFF FOR FINITE DIFFERENCE EQN
         SUBROUTINE XTCAL(N)
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6  SIZE, U(3000), UB(3000), LRH(3000),VA, VAT, VBT, VX(51),
     7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
C    XF1=DISTANCE UP (-Y DIRECTION)
C    XF1=0 IMPLIES NORMAL DERIVATIVE=0 IN UP DIRECTION
C       XF1 LESS THAN ZERO IMPLIES THAT THE XFS ARE LOADED INTO THE XT
C    SLOTS DIRECTLY AFTER THE SIGN IS CHANGED ON XF1
C    XF2=DISTANCE RIGHT (+X DIRECTION)
C    XF2=0 IMPLIES NOR DERIV=0 IN +X DIRECTION
C    XF3=DISTANCE DOWN (+Y DIRECTION)
C    XF3=0 IMPLIES NORMAL DERIVATIVE=0 IN DOWN DIRECTION
C    XF4=DISTANCE LEFT (-X DIRECTION)
C    XF4=0 IMPLIES NOR DERIV=0 IN -X DIRECTION
C    R=RADIUS FOR AXISYMMETRIC
C    XT(J)=COEFF UP
C    XT(J+1)=COEFF DOWN
C    XT(J+2)=COEFF RIGHT
C    XT(J+3)=COEFF LEFT
C    XT(J+4)=COEFF RHS
C    KT(J) IS RELATIVE NUMBER FOR XT(J)
C    KT(J+1) IS THE RELATIVE NUMBER FOR XT(J+1)
C    KT(J+2) IS THE RELATIVE NUMBER FOR XT(J+2)
C    KT(J+3) IS THE RELATIVE NUMBER FOR XT(J+3)
         WRITE(6,102)
         IF(IAS) 1,1,2
C    2-D CALCULATION
 1       DO 10 J=9,N,5
         ISW=0
         KEL=0
      K=J+3
         L=K+1
         JNUM=J
         READ(5,100)(KT(I),I=J,K),XF1,XF2,XF3,XF4, R

         IF(XF1) 14,15,15
C    STORE COEFF DIRECTLY
 14      XT(J)=-XF1
         XT(J+1)=XF2
         XT(J+2)=XF3
         XT(J+3)=XF4
         XT(J+4)=R
         JNUM=-JNUM
      GO TO 10
15    XF1=XF1/H
         XF2=XF2/H
         XF3=XF3/H
         XF4=XF4/H
C    CHECK FOR NOR DERIV=0 IN VERTICAL DIRECTION
```

```
      IF(XF1+XF3-1.) 16,16,17
16    IF(XF1) 18,18,19
18    XF1=1.
      ISW=1
      GO TO 17
19    XF3=1.
      ISW=-1
17    XF5=XF1+XF3
C   CHECK FOR NOR DERIV=C IN HORIZONTAL DIRECTION
      IF((XF2+XF4).GT.1.) GO TO 23
      IF(XF2.EQ.0.) GO TO 24
      XF4=1.
      KEL=-1
      GO TO 23
24    XF2=1.
      KEL=1
23    XF6=XF2+XF4
      XF7=XF1*XF3+XF2*XF4
      XT(J)=XF2*XF3*XF4/(XF5*XF7)
      XT(J+1)=XF1*XF2*XF4/(XF5*XF7)
      XT(J+2)=XF1*XF3*XF4/(XF6*XF7)
      XT(J+3)=XF1*XF2*XF3/(XF6*XF7)
      XT(J+4)=XF1*XF2*XF3*XF4*.5/XF7
      IF(ISW) 20,25,21
21    XT(J+1)=XT(J+1)+XT(J)
      XT(J)=0.
      GO TO 25
20    XT(J)=XT(J+1)+XT(J)
      XT(J+1)=0.
25    IF(KEL) 26,10,27
26    XT(J+2)=XT(J+2)+XT(J+3)
      XT(J+3)=0.
      GO TO 10
27    XT(J+3)=XT(J+3)+XT(J+2)
      XT(J+2)=0.
10    WRITE(6,103)(KT(I),I=J,K),(XT(I),I=J,L),JNUM

12    RETURN
C   AXISYM CALCULATION
2     DO 11 J=9,N,5
      K=J+3
      JNUM=J
      L=K+1
      KEL=0
      READ(5,100)(KT(I),I=J,K),XF1,XF2,XF3,XF4, R

      IF(XF1) 7,9,9
9     XF1=XF1/H
      XF2=XF2/H
      XF3=XF3/H
      XF4=XF4/H
      XF5=XF1+XF3
C   NORMAL DERIVATIVE CHECK
      IF((XF2+XF4).GT.1.) GO TO 28
      IF(XF2.EQ.0.) GO TO 29
      XF4=1.
```

```
      KEL=-1
      GO TO 28
29    XF2=1.
      KEL=1
28    XF6=XF2+XF4
      HR=.5*R
      HH=.5*H
      HE=.125*H
      IF(R) 3,3,13
C   NORMAL DERIVATIVE CHECK
13    IF(XF1+XF3-1.) 4,4,22
22    XF7=XF1*XF2*XF3*XF4
      XF8=HR*(XF2*XF4+XF1*XF3)/XF7+HE*(XF1-XF3)/(XF2*XF4)
      XF8=XF8*XF5*XF6
      XT(J)=(R/XF1+HH)*(.5*XF6/XF8)
      XT(J+1)=(R/XF3-HH)*(.5*XF6/XF8)
      XT(J+2)=(HR+HE*(XF1-XF3))*XF5/(XF2*XF8)
      XT(J+3)=(HR+HE*(XF1-XF3))*XF5/(XF4*XF8)
      XT(J+4)=.25*R*XF5*XF6/XF8
      GO TO 30
3     XT(J+1)=0.
      XL=XF1*XF1*.125
      XF8=(.25+XL/(XF2*XF4))*XF6
      XT(J)=   XF6/XF8*.25
      XT(J+2)=XL/(   XF2*XF8)
      XT(J+3)=XL/(   XF4*XF8)
      XT(J+4)=.5*XF6*XL/XF8
      GO TO 30
4     IF(XF1) 5,5,6
5     XF7=R-XF3*H*.25
      XF8=.5*XF6*(R/XF3-HH+XF3/(XF2*XF4)*XF7)
      XT(J)=C.
      XT(J+1)=.5*XF6*(R-XF3*HH)/(XF3*XF8)
      XT(J+2)=.5*XF3*XF7/(XF2*XF8)
      XT(J+3)=.5*XF3*XF7/(XF4*XF8)
      XT(J+4)=.25*XF3*XF6*XF7/XF8
      GO TO 30
6     XT(J+1)=0.
      XF7=R+XF1*H*.25
      XF8=.5*XF6*(R/XF1+HH+XF1*XF7/(XF2*XF4))
      XT(J)=.5*XF6*(R+HH*XF1)/(XF1*XF8)
      XT(J+2)=.5*XF1*XF7/(XF2*XF8)
      XT(J+3)=.5*XF1*XF7/(XF4*XF8)
      XT(J+4)=.25*XF1*XF6*XF7/XF8
30    IF(KEL) 32,11,31
31    XT(J+3)=XT(J+3)+XT(J+2)
      XT(J+2)=0.
      GO TO 11
32    XT(J+2)=XT(J+2)+XT(J+3)
      XT(J+3)=0.
      GO TO 11
7     XT(J)=-XF1
      XT(J+1)=XF2
      XT(J+2)=XF3
      XT(J+3)=XF4
      XT(J+4)=R
```

```
      JNUM=-JNUM
 11   WRITE(6,103)(KT(I),I=J,K),(XT(I),I=J,L),JNUM

      GO TO 12
100   FORMAT(4I5,5F10.0)
 102  FORMAT(111H    KT(JT)   KT(JT+1)   KT(JT+2)   KT(JT+3)
     1XT(JT)    XT(JT+1)   XT(JT+2)   XT(JT+3)   XT(JT+4)          JT    )
103   FORMAT(4I10,10X,5F10.7,I10)
      END
```

```
C    JTINT SETS UP THE JT ARRAY
           SUBROUTINE JTINT(NTP)
         COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
        1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
        2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
        3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
        4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
        5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
        6  SIZE, U(3000), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
        7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
           KB(13)=0
           IF(IAS.LT.1) GO TO 50
C    AXISYM JT-SETUP
           READ(5,1CC)JA
C    JA=NUMBER OF CARDS
C    JP=POINTS PER COLUMN
        JP=LC(3)
3       JG=NT+1
4       JK=1
           JD=JK
           DO 11 J=1,JA
           READ  (5,100)KA,(KB(K),K=1,12)

           KC=1
 7         JE=KB(KC)
           IF(JE) 11,11,8
 8         JF=KB(KC+1)
C    ERROR CHECK ON JT
           JFF=IABS(JF)
           IF(JFF.LT.9) GO TO 113
           IF(MOD(JFF-4,5).NE.0.OR.JFF.GE.NTP) GC TO 111
 113       DO 9 L=1,JE
           JT(JD)=JF
 9         JD=JD+JP
           KC=KC+2
           IF(JD-JG)  7,12,12
 12        JK=JK+1
           JD=JK
           GO TO 7
 11        CONTINUE
           READ(5,101)BASE
 31        RETURN
C    2-D JT SETUP
50      READ (5,100) JA
        JB=0
 60        DO 110J=1,JA
           READ  (5,10C)KA,(KB(K),K=1,12)

           KC = 1
           DO 10 K=1,KA
 70        JD = KB(KC)
           IF(JD) 10,10,80
 80        JE = KB(KC+1)
C    ERROR CHECK ON JT
           JFF=IABS(JE)
```

```
          IF(JFF.LT.9) GO TO 114
          IF(MOD(JFF-4,5).NE.0.OR.JFF.GE.NTP) GO TO 111
  114     DO 90L=1,JD
          JB = JB+1
C JT=POINT-TYPE VECTOR
  90      JT(JB) =  JE
          KC = KC+2
          GO TO 70
10        KC = 1
  110     CONTINUE
          GO TO 31
  111     WRITE(6,102)
          WRITE(6,100)KA,(KB(K),K=1,13)

          CALL EXIT
  100     FORMAT(14I5)
  101     FORMAT(F10.5)
  102     FORMAT(18H ERROR IN JT DATA.   )
          END
```

```
C       EVC CALCULATES THE SPECTRAL RADIUS OF THE ITERATION MATRIX
           SUBROUTINE EVC
        CCMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
      1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
      2 ,JOT, JT(3C00), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
      3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
      4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
      5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
      6  SIZE, U(3000), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
      7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
           DIMENSION AA(300),X(100)
           EQUIVALENCE (URH(1),X(1)),(URH(101),AA(1))
C    THE ABSOLUTE VALUE OF THE LARGEST EIGENVALUE OF THE MATRIX IS ITS
C          SPECTRAL RADIUS . THE TERMS EIGENVALUE AND SPECTRAL RADIUS
C          WILL BE USED INTERCHANGEABLY
C       INITIALIZE VECTOR ITERATES
        DO 3 J=1,NT
           RH(J)=0.0
  3        U(J)=C.0
           JS=-1
C       INITIALIZE EIGENVECTOR
        DO 8 JD=1,NT
        IF(JT(JD)-8) 6,6,7
  6        U(JD)=C.
           GO TO 8
  7        U(JD)=1.
  8        CONTINUE
C   NUMBER OF COLUMNS
        NLIN=LC(1)+1
        JM=LC(3)-1
C       BEGIN MINMAX PROCESS
  9        DO 22 KK=1,1C00
C       CALCULATE NEW VECTOR
           DO 29 ML=1,NLIN
C   FIRST MESH PCINT
        KZ=(ML-1)*LC(3)+1
C   LAST MESH PCINT
           KC=KZ+JM
           JV=1
           JU=1
           DO 38 K=KZ,KC
           JZ=JT(K)
           KU=KT(JZ)+K
           KD=KT(JZ+1)+K
           KR=KT(JZ+2)+K
           KL=KT(JZ+3)+K
           IF(JZ.GT.8) GO TO 37
           AA(JU)=0.
           AA(JU+1)=1.
           AA(JU+2)=0.
           SUM=0.
           GO TO 35
 37        SUM=U(KR)*XT(JZ+2)+U(KL)*XT(JZ+3)
           AA(JU)=-XT(JZ)
           AA(JU+1)=1.
```

```
         AA(JU+2)=-XT(JZ+1)
         IF((K.EQ.KZ).OR.((JT(K-1).GT.0).AND.(KT(JZ).EQ.-1))) GO TO 36
         AA(JU)=0.
 36      IF((K.EQ.KC).OR.((JT(K+1).GT.0).AND.(KT(JZ+1).EQ.1)))GO TO 35
         AA(JU+2)=0.
 35      X(JV)=SUM
         JV=JV+1
 38      JU=JU+3
       CALL MATRIX(LC(3),AA(2),X)
         JV=1
         DO 34 K=KZ,KC
         RH(K)=X(JV)
         IF(JT(K).LE.8) RH(K)=0.
 34      JV=JV+1
 29      CONTINUE
C    THE MATRIX MUST BE APPLIED TWICE AS ITS TWO-CYCLIC NATURE
C        REORDERS THE MESH POINTS AFTER ONE MULTIPLICATION.
C    THE SPECTRAL RADIUS SQUARED IS THE RESULT OF THE MINMAX
C        PROCEDURE AND THUS ITS SQUARE ROOT MUST BE TAKEN
C     SWITCH ALLOWING MATRIX TO BE APPLIED TWICE
         IF(JS) 13,15,15
13    DO 14 K=1,NT
C     STORE OLD VECTOR IN UB(J)
         UB(K)=U(K)
 14      U(K)=RH(K)
         GO TO 22
C     INITIALIZE FOR RATIO TEST
 15      XL=0.0
         XS=1.0
      DO 20 JD=1,NT
      IF(U(JD))20,20,16
16    X=RH(JD)/UB(JD)
         IF(XL-X) 17,18,18
C     XL IS MAXIMUM RATIO
 17      XL=X
      NNL=JD
 18      IF(XS-X) 20,20,19
C     XS IS MINIMUM RATIO
 19      XS=X
      NS=JD
 20      CONTINUE
         YL=RH(NNL)
       DO 21 JD=1,NT
C     SCALE SO THAT VECTOR DOESNT GET TOO  LARGE IN MAGNITUDE
21    U(JD)=RH(JD)/YL
C     CONVERGENCE TEST (1E-4 IS ARBITRARY)
         IF(XL-XS-1.0E-4) 24,24,22
 22      JS=-JS
 23      WRITE (6,102)XS,XL
24    XR=SQRT(.5*(XS+XL))
25    WRITE(6,101) XR,KK
C     ERROR CHECK
      IF((XR.GE.1.).OR.(XR.LE.0.))GO TO 28
 27      RETURN
 28      CALL EXIT
 100     FORMAT(14I5)
```

```
101     FORMAT(4HOXR=F11.8,2H  I5,39H ITERATIONS REQUIRED TO CONVERGE O
   *N XR    )
102     FORMAT(7HOXS XL  2F15.8)
        END
```

```
C    SUBROUTINE GUESS GENERATES A FIRST GUESS
           SUBROUTINE GUESS
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6  SIZE, U(3000), UB(3000), LRH(3000),VA, VAT, VBT, VX(51),
     7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
      DIMENSION GEP(7)
      EQUIVALENCE (UB(1),GEP(1))
C   GEP(1) IS ASSOCIATED WITH JT-TYPE 0,GEP(2) WITH TYPE1, ETC.
      READ(5,100)  (GEP(J),J=1,7)
         DO 1 J=1,NT
         U(J)=0.0
         IF(IABS(JT(J))-9) 2,1,1
 2       K=JT(J)+1
      U(J)=GEP(K)
         JT(J)=0
 1       CONTINUE
         RETURN
 100     FORMAT(7F10.5)
         END
```

```
      SUBROUTINE CHN12
C   CHN12 CALCULATES THE POTENTIAL FIELD
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1 DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3 KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4 NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5 RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6 SIZE, U(3000), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
     7 VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
      DIMENSION AA(300),X(100)
      EQUIVALENCE (URH(1),X(1)),(URH(101),AA(1))
      SX=1.0
      XM=.25*XR**2
      NLIN=LC(1)+1
      JA=LC(3)-1
1     XW=1.
      DO  23 NN=1,NURL
      XEP=0.
      ML=1
 2    DO 29 LL=ML,NLIN,2
      KZ=(LL-1)*LC(3)+1
      KC=KZ+JA
      JV=1
      JU=1
      DO 38 K=KZ,KC
      JZ=JT(K)
      KU=K+KT(JZ)
      KD=K+KT(JZ+1)
      KR=K+KT(JZ+2)
      KL=K+KT(JZ+3)
      IF(JZ.GT.0) GO TO 37
      AA(JU)=0.
      AA(JU+1)=1.
      AA(JU+2)=0.
      SUM=U(K)
      GO TO 35
 37   SUM=U(KR)*XT(JZ+2)+U(KL)*XT(JZ+3)+RH(K)*XT(JZ+4)
      AA(JU)=-XT(JZ)
      AA(JU+1)=1.
      AA(JU+2)=-XT(JZ+1)
      IF((K.EQ.KZ).OR.((JT(K-1).GT.0).AND.(KT(JZ).EQ.-1))) GO TO 36
      AA(JU)=0.
      SUM=SUM+U(KU)*XT(JZ)
 36   IF((K.EQ.KC).OR.((JT(K+1).GT.0).AND.(KT(JZ+1).EQ.1)))GO TO 35
      AA(JU+2)=0.
      SUM=SUM+U(KD)*XT(JZ+1)
 35   X(JV)=SUM
      JV=JV+1
 38   JU=JU+3
      CALL MATRIX(LC(3),AA(2),X)
      JV=1
      DO 34 K=KZ,KC
      DIF=X(JV)-U(K)
      U(K)=XW*DIF+U(K)
```

```
         DIF=ABS(DIF)
      IF(DIF.LE.XEP) GO TO 34
      XEP=DIF
      NXEP=K
 34      JV=JV+1
 29      CONTINUE
         IF(NN.GT.1) GO TO 11
      XW=1./(1.-2.*XM)
         GO TO 12
11    XW=1./(1.-XM*XW)
 12      IF(ML.GT.1) GO TO 3
         ML=2
         GO TO 2
 3    IF(LB(1)) 18,18,15
15    KG=LB(1)
C  CALCULATION OF NEGATIVE  JT-TYPE MESH POINTS
      KH=LB(2)
         DO 17 JD=KG,KH
         KE=-JT(JD)
         IF(KE) 17,17,16
C CONTRIBUTING POINTS ARE DETERMINED
 16      KU=KT(KE)+JD
         KD=KT(KE+1)+JD
         KL=KT(KE+2)+JD
         KR=KT(KE+3)+JD
         U(JD)=XT(KE+4)*RH(JD)+XT(KE)*U(KU)+XT(KE+1)*U(KD)+XT(KE+2)*U(KL
     1   )+XT(KE+3)*U(KR)
 17      CONTINUE
C  CONVERGENCE TEST
18    IF(XEP*XMP.LE.EPS) GO TO 24
 23      CONTINUE
 24      KNUM=JAS(1)
      IF(KNUM.LE.0) GO TO 27
C CHECK IF ANY POTENTIALS ARE ABOVE THOSE OF THE EMITTER
      DO 28 J=2,KNUM
         JN=JAS(J)
C  MULTIPLICATION BY XQM IS FOR SIGN CONVERSION ONLY
         IF((U(JN)-VA)*XQM) 28,28,25
28    CONTINUE
         GO TO 27
 25      SX=SX*RX
C KAN IS THE TEST POINT
         WRITE (6,101)JN,U(JN),SX
      DO 26 J=1,NT
         U(J)=UB(J)
 26      RH(J)=RH(J)*RX
      RHUP=RH(KAN)
         GO TO 1
27    CALL TEST(NN)
         RETURN
 101     FORMAT(38HOPOINT/VALUE/TOTAL SUPPRESSION OF RHS  I5,2E15.6)
         END
```

```
C PRINTOUT OF POTENTIAL FIELD
         SUBROUTINE TWOUT(KK)
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6  SIZE, U(3000), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
     7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
         NN=KK
         IF(MO) 2,10,2
2     WRITE(6,101) NN,XEP,NXEP
13    WRITE (6,102) NT
         WRITE(6,100)
         J=1
      DO 4 K=1,NT
      KT(J) =K
         XT(J)=U(K)
         J=J+1
         IF(J-9) 4,3,3
 3       WRITE (6,103)(KT(M),M=1,8),(XT(M),M=1,8)

         J=1
 4       CONTINUE
 5       IF(J-2) 8,6,6
 6       DO 7 K=J,8
         KT(K)=0
 7       XT(K)=0.0
         WRITE (6,103)(KT(M),M=1,8),(XT(M),M=1,8)

8     RETURN
 10      WRITE (6,104)
C POTENTIAL FIELD IS AVERAGED FOR FINAL VALUES
         JOT=NRD
C   SET SWITCH TO END PROBLEM
         NURL=-77
      DO 11  J=1,NT
 11      U(J)=.5*(U(J)+UB(J))
         GO TO 2
 100     FORMAT(1H0,10X,20H MESH POINT NUMBERS 45X,17H POTENTIAL VALUES)
 101     FORMAT(7H0AFTER I5,44H ITERATIONS ON U THE MAXIMUM CHANGE IN U
     *IS F10.5,32H VOLTS AND OCCURS AT MESH POINT I5)
102   FORMAT(1H0I5,9H U VALUES)
 103     FORMAT(1H 8I5,8F11.4)
 104     FORMAT(47H0 U-FIELD IS AVERAGE OF THIS AND PREVIOUS CYCLE   )
         END
```

```
C TEST CHECKS CYCLES AND PRINTOUTS
      SUBROUTINE TEST(KK)
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1 CC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3 KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4 NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5 RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6 SIZE, U(3000), UB(3000), LRH(3000),VA, VAT, VBT, VX(51),
     7 VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
      NN=KK
      IF(KRL-NRL) 2,1,2
1     CALL BCDUMP (U(1),U(NT))
      NURL=60
 2    IWRL=NRL-KRL+IABS(MO)
      IF(KB(IWRL)) 4,4,3
C CONDITIONAL PRINTOUT OF POTENTIAL FIELD AND EQUIPOTENTIALS
 3    CALL TWOUT(NN)
      CALL EQLINE
C SWITCH FOR LAST CYCLE
 4    IF(NURL) 6,5,5
C TEST ON CYCLES
 5    IF(KRL) 7,6,6
 6    RETURN
 7    WRITE (6,101)
C TRANSFER FOR NEW SET OF DATA
 8    NURL=-77
      RETURN
 101  FORMAT(11H1NEXT CASE.
      END
```

```
C EQLINE CALCULATES THE EQUIPOTENTIALS
         SUBROUTINE EQLINE
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6  SIZE, U(3000), UB(3000), LRH(3000),VA, VAT, VBT, VX(51),
     7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
      SUM=FLOAT(LC(1))*H
         XK=1.
C   NO EQUIPOTENTIALS IF SIZE = 0
      IF(SIZE) 1,27,1
1     POTEN=VAT
         WRITE(6,102)
      JC=LC(1)
         JD=LC(3)-1
      DX = H
 31      I=DX*100.0
C  SCALE MESH SIZE IF NECESSARY FOR PRINTING PURPOSES
         IF(I) 29,29,30
 29      DX=DX*10.0
         SUM=SUM*10.
         RIN=RIN*10.
         XK=XK*10.
         GO TO 31
 30      IF((RIN.LE.100.).AND.(SUM.LE.100.)) GO TO 41
         XK=XK/10.
         DX=DX/10.
         SUM=SUM/10.
         RIN=RIN/10.
         GO TO 30
 41      K=XK+.5
         IF(K.EQ.1) GO TO 2
         WRITE (6,101)H,DX,XK
 2    JE=LC(2)
         JED=JE+JD
         L=1
 3       AX=0.0
 33      DO 22 JJ=1,JC
         KS=1
 5       AAY=0.0
         DO 19 K=JE,JED
 6       IF(KS) 8,8,7
 7       M=1
       J=K-LC(3)
         GO TO 9
 8       J=K-1
C   CHECK TO SEE IF POTENTIAL IS BETWEEN U(J) AND U(K)
9     IF((U(K)-POTEN)*(U(J)-POTEN)) 10,10,18
 10      DIF=ABS(U(J)-U(K))
         IF(DIF) 13,13,11
 11      IF(M) 12,14,12
C   LINEAR INPERPOLATION IN HORIZONTAL DIRECTION
```

```
 12       VX(L)=ABS(U(J)-POTEN)/DIF*DX+AX
          VY(L)=AAY
          GO TO 15
 13       VX(L)=AX+DX
          VY(L)=AAY
          GO TO 15
 14       VX(L)=AX+DX
C   LINEAR INTERPOLATION IN VERTICAL DIRECTION
          VY(L)= ABS(U(J)-POTEN)/DIF*DX+AAY
 15       IF(L-6) 17,16,16
 16       WRITE (6,100)POTEN,(VX(I),VY(I),I=1,6)

          L=0
 17       L=L+1
 18       AAY=AAY+DX
 19       CONTINUE
          IF(KS) 21,21,20
 20       KS=0
          M=0
          JE=JE+1
          GO TO 5
 21       JE=JE+JD
          JED=JE+JD
          AX=AX+DX
 22       CONTINUE
          IF(L-2) 25,23,23
 23       DO 24 J=L,6
          VX(J)=0.0
 24       VY(J)=0.0
          WRITE (6,100)POTEN,(VX(I),VY(I),I=1,6)
C   STEP POTENTIAL DOWN

25      POTEN=POTEN-SIZE
       IF(POTEN-VBT) 27,2,2
27     RIN=RIN/XK
          RETURN
 100      FORMAT(17HOPOTENTIAL (X,Y) F8.1,2H  6(2H (F6.3,1H,F6.3,2H) ))
101    FORMAT(3H H=F11.6,5H  DX=F11.6,15H  SCALE FACTOR=F11.6)
 102      FORMAT(1H045X23H EQUIPOTENTIAL PRINTOUT     )
          END
```

```
      SUBROUTINE CHN13
C   CHN13 CALCULATES THE TRAJ AND RHS FOR 2-D CASES
       COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1  DC, DCC, DELY, DX, EPS, ER(4C), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6  SIZE, U(3000), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
     7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
      IF(NRL.EQ.KRL) NPIT=0
      NPIT=NPIT+1
      WRITE(6,100)NPIT
      SUMTWO=0.0
      SUMTRI=C.0
C   STORE LAST ESTIMATES
      DO 35 J=1 ,NT
      UB(J)=U(J)
 35   URH(J)=RH(J)
C   INITIALIZATION
      DO 1 J=1,51
C   SAU WILL HAVE EMITTER CURRENTS
      SAU(J)=0.0
C   ETX,ETY WILL HAVE COORD OF EMITTER SURFACE AT EQUAL ARC INCREMENTS
      ETX(J)=0.0
      ETY(J)=C.0
C   KCH=TRAJECTORY REFLECTION COUNTER
      KCH(J)=-1
C   AY(J) IS Y-COORD OF J-TH TRAJ
      AY(J)=0.0
C   VY(J) IS Y-COMPONENT OF VELOCITY OF J-TH TRAJ
      VY(J)=0.0
C   VX(J) IS X-COMPONENT OF VELOCITY OF J-TH TRAJ
      VX(J)=C.00001
C   CU(J) IS CURRENT IN J-TH TUBE
 1    CU(J)=0.0
      DO 11C0 J=1,99
C   PTX,PTY ARE COORD OF EQUIPOTENTIAL LINE USED TO CALCULATE
C     CURRENT DENSITY AT THE EMITTER
      PTY(J)=C.0
 1100 PTX(J)=0.0
      NJT=NTJ
C PEQ CALCULATES THE EQUIPOTENTIAL LINE IN ORDER THAT THE CURRENT
C DENSITIES CAN BE CALCULATED
 55   CALL PEQ
C ARC DIVIDES THE ARC LENGTH OF THE EMITTER INTO EQUAL INCREMENTS
C FOR THE CURRENT DENSITY CALCULATION
 54   CALL ARC
C KBA IS THE POINT NUMBER WHICH WILL BE TAKEN FOR THE EQUIPOTENTIAL
C LINE
      POTEN=U(KBA)
      DELU=ABS(VA-POTEN)
C XK=CONSTANT IN CHILDS LAW FORMULA*DELTA U**3/2
      XQ=ABS(XQM)
      XK=4.C/9.0*YEP*SQRT(2.0*XQ *DELU)*DELU
```

```
          NAJ=NTJ+1
        IF(JOT.LE.0) GO TO 2
          WRITE(6,105)
C   INITIALIZE
 2        AX=0.0
          NOT=JOT
        DX=H
        DELY=DX
C   JC=NUMBER OF LINES TO TRAVERSE
        JC=LC(1)
C   JE=FIRST MESH POINT IN SECOND LINE
        JE=LC(2)
C   JD=NUMBER OF MESH POINTS PER COLUMN
        JD=LC(3)
        SIGN=XCM/XQ
        RM=DX/YEP*SIGN
          IF(IAS.GT.0) RM=DX*RM
          XD=.5*XQM/DX
 6        DO 22 JN=1,JC
          JED=JE+JD-1
 7        AX=AX+DX
C TRCU CALCULATES THE CURRENT DENSITIES AND CALLS TRAJ WHICH
C CALCULATES THE TRAJECTORIES
          IF(IAS.EQ.0) GO TO 34
          CALL ATRCU(JE,JED)
          GO TO 10
 34       CALL TRCU(JE,JED)
C CORRCT CONDITIONALLY SHIFTS OR TERMINATES THE TRAJECTORIES
C AT THE GRIDS
10      CALL CORRCT(JN)
 11       IF((AX-3.0*DX-DC)*(AX-4.0*DX-DC)) 12,12,14
C SUMTWO IS THE CURRENT AFTER THE FIRST GRID
 12       NAJ=NAJ
          SUMTWO=0.5*(CU(1)+CU(NAJ))
          IF(IAS.GT.0) SUMTWO=2.*SUMTWO
          DO 13 K=3,NAJ
 13       SUMTWO=SUMTWO+CU(K-1)
          GO TO 17
 14       IF((AX-3.0*DX-DC-DCC)*(AX-4.0*DX-DC-DCC)) 15,15,17
C SUMTRI IS THE CURRENT AFTER THE SECOND GRID
 15       NAJ=NAJ
          SUMTRI=0.5*(CU(1)+CU(NAJ))
          IF(IAS.GT.0) SUMTRI=2.*SUMTRI
          DO 16 K=3,NAJ
 16       SUMTRI=SUMTRI+CU(K-1)
 17       IF(NOT) 19,19,18
 18       NOT=NOT-1
          NTJ=NTJ
       WRITE(6,101) AX,(K,KCH(K),AY(K),VX(K),VY(K),K=1,NTJ)

 19       DO 20 J=JE,JED
 20       RH(J)=0.0
          IF(IAS.EQ.0) GO TO 48
C  ACALR CALCULATES RHS FOR AXISYM
          CALL ACALR(JE,JED)
          GO TO 49
```

```
C   CALR CALCULATES THE RHS FOR 2-D
 48        CALL CALR(JE,JED)
 49        DO 21 K=JE,JED
 21        RH(K)=RH(K)*RM
 22        JE=JE+JD
           IW=NRL-KRL+1
           IF(KBB) 207,207,208
208     KK=NTJ-NJT
           NN=NJT+1
        DO 206  J=1,NN
           JJ=J+KK
 206       RCU(J)=RCU(JJ)
 207       NTJ=NJT
           JOT=IABS(KB(IW+1))
           IF(KB(IW)) 30,30,29
 29        SUM=0.0
           SU=0.0
C THRUST AND POWER CALCULATION
           CU=.5*CU
           NAJ=NAJ
           IF(IAS.EQ.0) CU(NAJ)=.5*CU(NAJ)
           DO 198 J=1,NAJ
        CU(J)=CU(J)*SIGN
        CUD(J)=CUD(J)*SIGN
        SAU(J)=SAU(J)*SIGN
           IF(J-1) 200,200,201
 200       V=VX(J)
           GO TO 199
 201       IF(J-NAJ) 202,203,203
 203       V=VX(J-1)
           GO TO 199
 202       V=.5*(VX(J)+VX(J-1))
 199       AY(J)=    CU(J)*V   /XQM
        VY(J)=.5*V*AY(J)
           SUM=SUM+AY(J)
198     SU=SU+VY(J)
           IF( IAS.GT.0)GO TO 41
           WRITE (6,109)(J,AY(J),J=1,NAJ)
           WRITE (6,110)SUM,SU
           GO TO 40
 41        WRITE(6,116)(J,AY(J),J=1,NAJ)
           WRITE(6,106) SUM,SU
 40        IF(IAS.GT.0) GO TO 47
           SAU(NAJ)=.5*SAU(NAJ)
           SAU(1)=.5*SAU(1)
C SUMONE IS THE INITIAL CURRENT
 47        SUMONE=0.0
           DO 8 J=1,NAJ
 8         SUMONE=SUMONE+SAU(J)
           WRITE (6,111)(J,CUD(J),J=1,NAJ)
           IF( IAS.GT.0) GO TO 39
           WRITE (6,108)(J,SAU(J),J=1,NAJ)
           WRITE (6,102)SUMONE
           ERI=SUMONE/ SEM
           WRITE (6,107)ERI
           ERI=ERI* SEM
```

```
          GO TO 38
 39       WRITE(6,115)(J,SAU(J),J=1,NAJ)
          WRITE(6,112)SUMONE
          ERI=SUMONE
 38    ERR=SUMTWO/SUMONE*SIGN
          IF(DC.LE.0.) GO TO 30
          ERA=ERR*ERI
          ERR=ERR*100.0
          IF(IAS.GT.0) GO TO 45
          WRITE (6,103)ERA,ERR
          IF(DCC.EQ.0.) GO TO 30
       ERR=SUMTRI/SUMONE*SIGN
          ERA=ERR*ERI
          ERR=ERR*100.0
          WRITE (6,104)ERA,ERR
          GO TO 30
 45       WRITE(6,113) ERA,ERR
          IF(DCC.EQ.0.) GO TO 30
       ERR=SUMTRI/SUMONE*SIGN
          ERA=ERR*ERI
          ERR=ERR*100.
          WRITE(6,114) ERA,ERR
 30       IF(NURL) 32,31,31
C RTEST CHECKS THE BOUNDS ON RHS
 31       CALL RTEST
 32       RETURN
100    FORMAT(8HCCYCLE  I2)
 101      FORMAT(1HOF8.4, 4XI2,13XI2,9XF9.4,2E13.5/(13XI2,13XI2,9XF9.4,
     *    2E13.5))
 102      FORMAT(23HOTOTAL INITIAL CURRENT= E12.6,19H AMPS/(UNIT H)      )
 103      FORMAT(36HOTRANSMITTED CURRENT AT ACCEL. GRID=  E12.6,24H AMPS/
     *(UNIT H) WHICH IS  F6.2,32H PERCENT OF THE INITIAL CURRENT.    )
 104      FORMAT(36HOTRANSMITTED CURRENT AT DECEL. GRID=  E12.6,24H AMPS/
     *(UNIT H) WHICH IS  F6.2,32H PERCENT OF THE INITIAL CURRENT.    )
 105      FORMAT(47HOX-COORD  TRAJ NUM  REFLECTION COUNTER  Y-COORD
     *    3X,23HX-VEL COMP   Y-VEL COMP )
 106      FORMAT(24HOTOTAL THRUST (NEWTONS)  E14.6//21H TOTAL POWER (WATT
     *S)  E14.6)
 107      FORMAT(33HOAVERAGE INITIAL CURRENT DENSITY= E12.6,17H AMPS/(UNI
     *T H)**2  )
 108      FORMAT(32HOINITIAL CURRENTS (AMPS/UNIT H)  //(7(1H I2,E14.6)))
 109      FORMAT(53HOTHRUST DISTRIBUTION BY STREAM TUBES (NEWTONS/UNIT H)
     *    //(7(1H I2,E14.6)))
 110      FORMAT( 31HOTOTAL THRUST (NEWTONS/UNIT H)  E14.6/27H TOTAL POWE
     *R (WATT/UNIT H)  E14.6)
 111      FORMAT(46HOINITIAL CURRENT DENSITIES (AMPS/(UNIT H)**2) //
     *    (7(1H I2,E14.6)))
 112      FORMAT(23HOTOTAL INITIAL CURRENT= E12.6, 5H AMPS  )
 113      FORMAT(37HOTRANSMITTED CURRENT AT ACCEL. GRID= E12.6,15H AMPS W
     *HICH IS F6.2,33H PERCENT OF THE INITIAL CURRENT.  )
 114      FORMAT(37HOTRANSMITTED CURRENT AT DECEL. GRID= E12.6,15H AMPS W
     *HICH IS F6.2,33H PERCENT OF THE INITIAL CURRENT.  )
 115      FORMAT(25HOINITIAL CURRENTS (AMPS) //(7(1H I2,E14.6)))
 116      FORMAT(47HOTHRUST DISTRIBUTION BY STREAM TUBES (NEWTONS)  //
     *    (7(1H I2,E14.6)))
          END
```

```
C TRAJ CALCULATES THE TRAJECTORY COORDINATES AND VELOCITIES
          SUBROUTINE TRAJ(M,KE,KED)
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6  SIZE, U(3000), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
     7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
          JE=KE
          JED=KED
          K=M
          AD=AY(K)/DX
          JX=AD
          XA=JX
          XA=AD-XA
          JP=JX+JE
      JS=4
          JQ=JP-JD
          UL=(1.0-XA)*U(JQ)+XA*U(JQ+1)
          UK=(1.0-XA)*U(JP)+XA*U(JP+1)
C CALCULATION OF LEFTHAND DERIVATIVE
          IF(XA-.5) 1,1,5
1         IF(JX) 2,2,3
2         YLA=2.0*XA*(U(JQ+1)-U(JQ))
          GO TO 4
3         YLA=(XA+.5)*U(JQ+1)-2.0*XA*U(JQ)+(XA-.5)*U(JQ-1)
4         DY=VY(K)/VX(K)
          JOX=JX
          GO TO 7
5         JX=JX+1
          JQ=JQ+1
          XA=XA-1.0
          IF(JX-JD+1) 3,6,6
6         YLA=2.0*XA*(U(JQ-1)-U(JQ))
          GO TO 4
C CALCULATION OF RIGHTHAND DERIVATIVE
7         XB=XA+DY
          IF(DY) 9,9,10
8         JX=JX-1
          XB=1.0+XB
9         IF(XB+.5) 8,12,12
11        XB=XB-1.0
          JX=JX+1
10        IF(XB-.5) 12,12,11
12        IF(JX) 14,17,16
13        JX=JX+2*(1-JD)
14        JX=-JX
          XB=-XB
15        JP=JE+JX
          YRA=(XB+.5)*U(JP+1)-2.0*XB*U(JP)+(XB-.5)*U(JP-1)
          IF(XB) 19,20,20
16        IF(JX-JD+1) 15,18,13
17        JP=JE
```

```
         YRA=2.0*XB*(U(JP+1)-U(JP))
         XB=ABS(XB)
         GO TO 20
 18      JP=JE+JX
         YRA=2.0*XB*(U(JP-1)-U(JP))
 19      JP=JP-1
         XB=1.0-ABS(XB)
 20      JQ=JP-JD
         USN=(1.0-XB)*U(JQ)+XB*U(JQ+1)
         UQN=(1.0-XB)*U(JP)+XB*U(JP+1)
         DUX=.5*(UK-UL-USN+UQN)
         VXB=VX(K)**2-2.*XQM*DUX
         VXX=SQRT(ABS(VXB))
         DT=2.0*DX/(VXX+VX(K))
         JS=JS-1
C YA=Y ACCELERATION
         YA=XD*(YLA+YRA)
C DY=DELTA Y INCREMENT
         DY=DT*(VY(K)-.5*YA*DT)/DX
         JX=JOX
         IF(JS) 21,21, 7
 21      IF(VXB) 28,29,29
28    WRITE(6,100)K,JE,JED,JP,JQ,AY(K),DUX,VX(K),VY(K),VXB,AX
 29      VX(K)=VXX
         VY(K)=VY(K)-YA*DT
         AY(K)=AY(K)+DY*DX
C REFLECTION OF TRAJECTORIES IF OUTSIDE BOUNDS
         IF(AY(K))22,23,23
 22      AY(K)=-AY(K)
      IF(IAS.LE.0) GO TO 25
      AY(K)=-1.
       CU(K)=0.
      CU(K+1)=0.
      GO TO 26
23    IF(AY(K)-RIN) 26,26,24
 24      AY(K)=RIN+RIN-AY(K)
 25      VY(K)=-VY(K)
 27      KCH(K)=KCH(K)+1
 26      CONTINUE
         RETURN
100   FORMAT(27HOTRAJ ATTEMPTS TO TURN BACK  //(5I5,6E15.6))
         END
```

```
C CORRCT CONDITIONALLY TERMINATES OR SHIFTS THE TRAJECTORIES
      SUBRCUTINE CCRRCT (JR)
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1 DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3 KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4 NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5 RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6 SIZE, U(3000), UB(3000), LRH(3000),VA, VAT, VBT, VX(51),
     7 VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
C    JN=MESH COLUMN NUMBER
      JN=JR
      IF(((JN-KAT)*(JN-KATT)).GT.0) GO TO 63
      J=JN-KAT+1
      GO TO 82
 63   IF(((JN-KAT1)*(JN-KAT2)).GT.0) GO TO 87
      J=JN-KAT1+KATT-KAT+2
 82   DO 64 K=1,NTJ
      IF(KCH(K).LT.0) GO TO 64
C    BEGIN IMPINGEMENT CHECK
      IF(AY(K).EQ.-1..OR.AY(K).GT.ER(J)) GO TO 64
      AA=AY(K)
      IF(K.EQ.1) CU(1)=0.
      IF(K.EQ.1.OR.(K.GT.1.AND.AY(K-1).EQ.-1.))GO TO 10
C    CORRECT CU(K) WHEN AY(K-1) HAS NCT TERMINATED
      AYD=AY(K-1)-AA
      AYDS=AY(K-1)-ER(J)
      CU(K)=AYDS/AYD*CU(K)
      AYDL=AYD-AYDS
      VY(K)=(VY(K)*AYDS+VY(K-1)*AYDL)/AYD
      VX(K)=(VX(K)*AYDS+VX(K-1)*AYDL)/AYD
      AY(K)=ER(J)
      IF(K.GE.NTJ) GO TO 13
      IF(AY(K-1).LE.ER(J).AND.AY(K+1).LE.ER(J)) GO TO 11
      IF(AY(K+1).GT.ER(J)) GO TO 12
      CU(K+1)=0.
      GO TO 14
C    CORRECT CU(K+1) WHEN AY(K+1) HAS NOT TERMINATED
 12   AYD=AY(K+1)-AA
      AYDS=AY(K+1)-ER(J)
      CU(K+1)=AYDS/AYD*CU(K+1)
      AYDL=AYD-AYDS
      AY(K)=ER(J)
      VY(K)=(AYDS*VY(K)+AYDL*VY(K+1))/AYD
      VX(K)=(AYDS*VX(K)+AYDL*VX(K+1))/AYD
      GO TO 14
 10   IF(K.LT.NTJ.AND.AY(K+1).LE.ER(J)) GO TO 11
      IF(K.EQ.NTJ) GO TO 13
      GO TO 12
 11   AY(K)=-1.
      CU(K+1)=0.
      GO TO 64
 13   IF(ER(J).GE.RIN) GO TO 11
C    CORRECT FOR LAST TUBE
      AYD=RIN-AA
```

```
      AYDS=RIN-ER(J)
      AYDL=AYD-AYDS
      AY(K)=ER(J)
      CU(K+1)=AYDS/AYD*CU(K+1)
      VY(K)=AYDS/AYD*VY(K)
      N=AX/DX+1.5
   N=N*LC(3)
   NN=LC(3)
      DIF=ABS(U(NN)-U(N))
      V=SQRT(2.*XQM*DIF)
      VX(K)=(AYDS*VX(K)+AYDL*V)/AYD
14    IF(ER(J).GE.RIN) GO TO 11
64    CONTINUE
87    RETURN
      END
```

```
      SUBROUTINE PEQ
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1 DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3 KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4 NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5 RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6 SIZE, U(3000), UB(3000), LRH(3000),VA, VAT, VBT, VX(51),
     7 VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
C PEQ CALCULATES THE EQUIPOTENTIAL LINE FOR THE CU CALCULATION
      POTEN=U(KBA)
      DX=H
      AL=.1*H
      NO=KABB+1
      AX=0.0
      L=0
      JE=LC(2)
      JD=LC(3)
20    DO 8 JJ=1,KAB
      AAY=0.0
      JED=JE+JD-1
      DO 7 K=JE,JED
      J=K-JD
      IF((U(K)-POTEN)*(U(J)-POTEN)) 1,1,7
 1    DIF=ABS(U(J)-U(K))
      L=L+1
C COORDINATES ARE TAKEN FOR EQUAL DELTA Y ONLY
      PTY(L)=AAY
      IF(DIF) 2,6,2
 2    DO 15 KJ=1,NO
      IF(ABS(AAY-ATY(KJ))-AL) 16,16,15
 15   CONTINUE
      GO TO 4
 16   IF(AX-ATX(KJ)) 3,3,4
 3    HK=AX+DX-ATX(KJ)
      AK=ATX(KJ)
      GO TO 5
 4    HK=DX
      AK=AX
 5    PTX(L)=ABS(U(J)-POTEN)/DIF*HK+AK
      GO TO 7
 6    PTX(L)=AX+DX
 7    AAY=AAY+DX
      AX=AX+DX
 8    JE=JE+JD
C   SORT Y-COORD IN INCREASING Y-ORDER
      DO 11 J=1,L
      LL=L-J+1
      T=0.0
      DO 10 I=1,LL
      IF(T-PTY(I)) 9,9,10
 9    T=PTY(I)
      NN=I
 10   CONTINUE
      PP=PTY(LL)
```

```
      PTY(LL)=T
         PTY(NN)=PP
      PP=PTX(LL)
      PTX(LL)=PTX(NN)
         PTX(NN)=PP
 11      CONTINUE
         KAP=0
C    CISCARD DUPLICATES
         J=2
 31      IF(PTY(J)-PTY(J-1)) 14,12,14
 12      NN=L-1-KAP
         KAP=KAP+1
         DO 13 JJ=J,NN
         PTX(JJ)=PTX(JJ+1)
 13      PTY(JJ)=PTY(JJ+1)
 14      J=J+1
         IF(J.LE.(L-KAP)) GO TO 31
         IF(JOT.LE.0) GO TO 30
         WRITE(6,100)POTEN
         NAL=L-KAP
         WRITE (6,101)(J,PTX(J),PTY(J),J=1,NAL  )
 30      RETURN
 100     FORMAT(57HOCURRENT DENSITIES ARE CALCULATED USING EQUIPOTENTIAL
     * OF F10.5,33H VOLTS WHICH HAS X-Y COORDINATES  )
 101     FORMAT(7(1H I2,2H (F5.3,1H,F5.3,2H) ))
         END
```

```
C ARC DIVIDES THE EMITTER INTO EQUAL ARC LENGTHS
      SUBROUTINE ARC
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1 DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3 KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4 NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5 RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6 SIZE, U(3000), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
     7 VY(51), XC, XEP, XK, XMP, XQM, XR, XT(1850),YEP
      DIMENSION LQ(100)
      EQUIVALENCE(LQ(1),RH(1))
C KABB=NUMBER OF INPUT COORDINATES FOR EMITTER LESS ONE
      NBH=NTJ+1
      BH=NBH
      SEM=0.0
C SUMMING TOTAL ARC LENGTH
      DO 1 J=1,KABB
      SA=SQRT((ATX(J+1)-ATX(J))**2+(ATY(J+1)-ATY(J))**2)
 1    SEM=SEM+SA
C   AREM=DELTA ARC LENGTH
      AREM=SEM/BH
      K=1
      REM=0.0
      J=1
      HYP=AREM
C CALCULATION OF COORDINATES
      ETX(K)=ATX(K)
      ETY(K)=ATY(K)
      K=K+1
 2    XDIF=ATX(J)-ATX(J+1)
      YDIF=ATY(J+1)-ATY(J)
      SA=SQRT(XDIF**2+YDIF**2)
      CSA=YDIF/SA
      SNA=XDIF/SA
 3    ETX(K)=ATX(J)-HYP*SNA
      ETY(K)=ATY(J)+HYP*CSA
      HYP=AREM+HYP
      K=K+1
      REM=REM+AREM
      IF(K-NBH) 4,4,6
 4    IF(SA-REM-AREM) 5,5,3
 5    J=J+1
      HYP=AREM-SA+REM
      REM=HYP-AREM
      GO TO 2
 6    CONTINUE
      ETX(K)=ATX(KABB+1)
      ETY(K)=ATY(KABB+1)
C   CHECK ON LOWER SYMMETRY
 12   IF(LAST) 17,18,18
 17   K=K+1
      NTJ=NTJ+1
      XDIF=ETX(K-1)-ETX(K-2)
      YDIF=ETY(K-1)-ETY(K-2)
```

```
      SA=ATAN(XDIF/YDIF)
      ETX(K)=ETX(K-1)+AREM*SIN(SA)
      ETY(K)=ETY(K-1)+AREM*COS(SA)
C CONDITIONAL PRINTOUT
 18   IF(JOT) 8,8,7
 7    WRITE (6,103)SEM,AREM
      NAL=KABB+1
      WRITE (6,100)(J,ATX(J),ATY(J),J=1,NAL )
      NAL=K-1
      IF(KBF) 13,14,14
 14   K=1
       GO TO 15
 13   K=2
 15   NUM=1
      DO 16 J=K,100
      LQ(J)=NUM
 16   NUM=NUM+1
      WRITE (6,102)          (LQ(J),ETX(J),ETY(J),J=K,NAL)
      DO 20 J=1,100
 20   RH(J)=0.
 8    RETURN
 100  FORMAT(24HOX-Y EMITTER COORDINATES //(7(1H I2,2H (F5.3,1H,F5.3,
     * 2H) )))
 102  FORMAT(28HOX-Y BEGIN TRAJ. COORDINATES //(7(1H I2,2H (F5.3,
     * 1H,F5.3,2H) )))
 103  FORMAT(41HOEMITTER ARC LENGTH , DELTA ARC LENGTH    2F10.5)
      END
```

```
C RTEST CHECKS ON THE UPPER AND LOWER BOUNDS
          SUBROUTINE RTEST
       COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
      1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
      2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
      3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
      4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
      5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
      6  SIZE, U(3000), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
      7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
C KAN=TEST POINT
          IF(KRL.EQ.NRL) WRITE(6,102)
          RT=RH(KAN)
          SX=1.0
C    NO CHECK IF CURRENT DENSITY IS SPECIFIED
          IF(KBB)25,24,25
 25       IF(KRL) 19,26,19
 26       IF(MO) 11,19,11
 24       IF(KRL) 22,23,22
 23       IF(MO) 11,22,11
C FIRST TWO CYCLES SET UPPER AND LOWER BOUNDS
 22       IF(KRL-NRL+1) 5,2,1
C RHUP=UPPER BOUND
 1        RHUP=RT
          GO  TO 19
 2        IF(RT-RHUP) 18,18,3
C RHDOWN=LOWER BOUND
 3        RHDOWN=RHUP
          MO=-MO
          RHUP=RT
 4        GO TO 19
C MO=+1 ,CHECK IS ON UPPER BOUND
C MO=-1 ,CHECK IS ON LOWER BOUND
C MO=0 ,NO CHECK
 5        IF(MO) 10,19, 8
 6        SX=SX*RX
       DO 7 J=1,NT
 7        RH(J)=RH(J)*RX
 8        IF(RHUP-RH(KAN)) 6,9,9
 9        IF(RH(KAN)-RHDOWN) 11,16,16
 10       IF((RT-RHDOWN)*(RT-RHUP)) 16,16,11
 11       POW=1.0/SX
       DO 12 J=1,NT
 12       RH(J)=.5*(RH(J)*POW+URH(J))
          KRL=1
          JJ=NRL+1
          KB(JJ)=77
          KB(JJ+1)=77
          WRITE (6,100)
          IF(KBB.GT.0) GO TO 15
          IF(MO) 14,13,13
 13       RHUP=RT
          GO TO 15
 14       RHDOWN=RT
 15       MO=0
```

```
 16      IF(MO) 18,19,17
 17      RHUP=RH(KAN)
         GO TO 19
 18      RHDOWN=RT
 19      XT(1)=.25*RHDOWN
         XT(2)=.25*RH(KAN)
         XT(3)=.25*RHUP
         WRITE(6,101)XT(1),KAN,XT(2),XT(3),KAN,U(KAN)
         IW=NRL-KRL+1
         IF(KB(IW)) 21,21,20
C TROUT PRINTS OUT THE RHS
 20      CALL TROUT
 21      MO=-MO
         KRL=KRL-1
         RETURN
 100     FORMAT(11HORF=AVERAGE    )
 101    FORMAT(8HORHDOWN=F11.7,4H RH(I4,2H)=F11.7,6H RHUP=F11.7,3H U(I4,
     *   2H)=F15.7)
 102     FORMAT(26HOSTART OF POISSON SOLUTION )
         END
```

```
C TRCUT PRINTS CLT THE RHS
        SUBROUTINE TROUT
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3C00), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6  SIZE, U(3C00), UB(3000), LRH(3000),VA, VAT, VBT, VX(51),
     7  VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
        IF(MO) 1,8,1
C NORMAL PRINT OUT
 1    WRITE (6,100) NT
        WRITE(6,104)
        J=1
      DO 3 K=1,NT
        XT(J)=.25*RH(K)
      KT(J)=K
        J=J+1
        IF(J-9) 3,2,2
 2      WRITE (6,101)(KT(M),M=1,8),(XT(M),M=1,8)

        J=1
 3      CONTINUE
       IF(J-2) 7,4,4
 4      DO 5 K=J,8
        KT(K)=0
 5      XT(K)=0.0
        WRITE (6,101)(KT(M),M=1,8),(XT(M),M=1,8)

 7      RETURN
C FINAL PRINT CUT IS AVERAGE
 8      WRITE (6,102)
        WRITE(6,103)
17    WRITE (6,1C0) NT
        WRITE(6,104)
        J=1
      CO 10 K=1,NT
        KT(J)=K
        XT(J)=.125*(RH(K)+URH(K))
        J=J+1
        IF(J-9) 10,9,9
 9      WRITE (6,101)(KT(M),M=1,8),(XT(M),M=1,8)

        J=1
 10     CONTINUE
      IF(J-2) 7,11,11
 11     DO 12 K=J,8
        KT(K)=C
12      XT(K)=C.0
        WRITE (6,101)(KT(M),M=1,8),(XT(M),M=1,8)

100   FORMAT(1HCI5,10H RH VALUES)
 101    FORMAT(1H 8I5,8F11.4)
 102    FORMAT(47HORHOLTPLT IS AVERAGE OF THIS AND PREVIOUS CYCLE   )
```

```
103   FORMAT(27HOCONVERGED POISSON SOLUTION  )
104   FORMAT(1H0,10X,20H MESH POINT NUMBERS ,49X,10H RH VALUES
      END
```

```
C   TRCU CALCULATES THE CURRENT AT THE EMITTER AND INITIALIZES
C      THE TRAJECTORIES
          SUBROUTINE TRCU(KE,KED)
       COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
      1  DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
      2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
      3  KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MC,
      4  NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
      5  RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
      6  SIZE, U(3000), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
      7  VY(51), XC, XEP, XK, XMP, XQM, XR, XT(1850),YEP
       DIMENSION US(2),UR(2)
       XQ=ABS(XQM)
          JE=KE
          JED=KED
          ALA=.2*H
          ACAN=1.0
          ISW=0
          K=1
 116      IS=0
          LL=0
          KK=K
C CHECK ON TERMINATED TRAJECTORIES
          IF(AY(K)+1.0) 16,16,18
C CHECK ON UNINIALIZED TRAJECTORIES
 18       IF(KCH(K)) 20,15,15
 20       IF(ISW) 22,22,1
 22       IF(AY(K)*ACAN) 1,1,19
 19       IS=1
          ISW=1
C CHECK IF TRAJECTORY CAN BE INIALIZED
 1        IF(AX-ETX(K+1)-ALA ) 21,21,2
 2        IF(AX-ETX(K+2)-ALA) 21,21,5
 5        N=-1
          ACAN=1.0
          KK=KK+1
          TA=(ETX(KK-1)-ETX(KK))/(ETY(KK-1)-ETY(KK))
          TB=(ETX(KK)-ETX(KK+1))/(ETY(KK)-ETY(KK+1))
          TB=-.5*(TA+TB)
          DE=ATAN(TB)
 33       XX=ETY(KK)-TB*ETX(KK)
          J=1
 3        N=N+1
 4        JJ=J+N
          TA=PTY(JJ)-PTY(JJ+1)
          TC=PTX(JJ)-PTX(JJ+1)
          YY=TC*PTY(JJ)-TA*PTX(JJ)
          CPX=(TC*XX-YY)/(TA-TB*TC)
          CPY=XX+CPX*TB
 7        IF(PTY(JJ+1)-CPY) 3,8,8
 8        LL=LL+1
 24       IF(AY(KK-1))9,9,10
C TRAJECTORIES INITIALIZED
 9        AY(KK-1)=ETY(KK)+TB*(AX-ETX(KK))
          ACAN=0.0
```

```
          XA=AY(KK-1)/DX
          JX=XA
          JP=JE+JX
          AD=JX
          XA=XA-AD
          UP=(1.C-XA)*U(JP)+XA*U(JP+1)
      V=SQRT(2.0*XQ*(ABS(VA-UP)))
          VX(KK-1)=V*COS(DE)
          VY(KK-1)=V*SIN(DE)
 10       US(LL)=CPX
          UR(LL)=CPY
          IF(K-NTJ) 12,11,12
C C  SPECIAL FOR LAST CURRENT DENSITY
 11       HT=AREM+AREM
          IF(LAST) 40,41,41
 40       HT=0.0
          GO TO 43
 41       KK=KK+1
          LL=2
 29       US(2)=    PTX(JJ+1)
          UR(2)=    PTY(JJ+1)
          PPX=.5*(ETX(KK-1)+ETX(KK))
          PX= .5*(US(1)+US(2))
          PPY=.5*(ETY(KK-1)+ETY(KK))
          PY= .5*(UR(1)+UR(2))
          GO TO 31
 12       IF(LL-2) 5,13,13
 13       HT=AREM
 23       PPX=.5*(ETX(KK-1)+ETX(KK))
          PX=.5*(US(1)+US(2))
          PPY=.5*(ETY(KK-1)+ETY(KK))
          PY=.5*(UR(1)+UR(2))
 31       DEX=SQRT((PX-PPX)**2+(PY-PPY)**2)
C CALCULATION OF CURRENT DENSITY
 17       CU(K+1)=HT/DEX**2*XK
          CUD(K+1)=CU(K+1)/HT
          IF(KBB)42,42,43
 43       CU(K+1)=RCU(K+1)*HT
          CUD(K+1)=RCU(K+1)
 42       SAU(K+1)=CU(K+1)
          KCH(K)=0
          IF(K-1) 14,14,26
C SPECIAL FOR FIRST CURRENT DENSITY
 14       DE=ETX(1)-ETX(2)
 102      DE=-DE/(ETY(1)-ETY(2))
 104      XX=ETY(1)-DE*ETX(1)
          J=0
 110      J=J+1
 109      TA=PTY(J)-PTY(J+1)
          TC=PTX(J)-PTX(J+1)
          YY=TC*PTY(J)-TA*PTX(J)
          CPX=(TC*XX-YY)/(TA-TC*DE)
          CPY=XX+CPX*DE
 107      IF(PTY(J+1)-CPY) 11C,1C8,108
 108      PX=.5*(ETX(1)+ETX(2))
          PPX=.5*(CPX+US(1))
```

```
      PY=.5*(ETY(1)+ETY(2))
      PPY=.5*(CPY+UR(1))
      DEX=SQRT((PX-PPX)**2+(PY-PPY)**2)
      HT=2.0*AREM
111   IF(KBF) 115,115,113
113   HT=.5*HT
      NTJ=NTJ+1
      NAJ=NAJ+1
      JR=NTJ
      DO 114KL=2,NTJ
      SAU(JR+1)=SAU(JR)
      CUD(JR+1)=CUD(JR)
      RCU(JR+1)=RCU(JR)
      CU(JR+1)=CU(JR)
      AY(JR)=AY(JR-1)
      VY(JR)=VY(JR-1)
      VX(JR)=VX(JR-1)
      KCH(JR)=KCH(JR-1)
114   JR=JR-1
      RCU(2)=RCU(1)
      RCU(1)=0.0
      CUD(1)=0.0
      CU(1)=C.0
      CU(2)=HT*XK/DEX**2
      CUD(2)=CU(2)/HT
      IF(KBB)44,44,45
45    CU(2)=HT*RCU(2)
      CUD(2)=RCU(2)
44    SAU(1)=CU(1)
      SAU(2)=CU(2)
      AY(1)=ETY(1)+DE*(AX-ETX(1))
      TB=ATAN(DE)
      XA=AY(1)/DX
      JX=XA
      JP=JE+JX
      AD=JX
      XA=XA-AD
      UP=(1.0-XA)*U(JP)+XA*U(JP+1)
    V=SQRT(2.C*XQ*ABS(VA-UP))
      VX(1)=V*COS(TB)
      VY(1)=V*SIN(TB)
      K=K+1
      J=51
      DO 118 I=1,50
      ETX(J)=ETX(J-1)
      ETY(J)=ETY(J-1)
118   J=J-1
      GO TO 26
115   CU(1)=HT*XK/DEX**2
      CUD(1)=CU(1)/HT
      IF(KBB)46,46,47
47    CU(1)=HT*RCU(1)
      CUD(1)=RCU(1)
46    SAU(1)=CU(1)
26    CONTINUE
21    IF(IS) 16,16,15
```

```
C TRAJ CALCULATES THE TRAJECTORIES AFTER THEY HAVE BEEN INIALIZED
 15       CALL TRAJ(K,JE,JED)
 16       K=K+1
          IF(NTJ-K) 117,116,116
 117      RETURN
          END
```

```
      SUBROUTINE CALR(KE,KED)
     COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
    1 CC, DCC, DELY, DX, EPS, ER(4C), ETX(51),ETY(51),H,IAS,JAS(20),JD
    2 ,JOT, JT(3C00), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
    3 KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
    4 NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
    5 RCU(51), RH(300C), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
    6 SIZE, U(3C00), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
    7 VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
      JE=KE
      JED=KED
      SWA=0.0
      DO 33 JJ=1,NAJ
      JO=0
      J=JJ
C IFCU(J)=0.,NO CONTRIBUTION
      IF(CU(J)) 33,33,1
C IF CU(1),SPECIAL
 1    IF(J -1) 2,2,5
 2    IF(KCH(J)) 4,4,3
 3    IF(VY(J)) 35,4,4
35   HT=2.0*(2.C*RIN-AY(J))
      NA=JE
      NB=JED
     WA=VX(J)
      WB=WA
      JO=1
      IF(SWA) 39,39,40
 39   SWA=1.C
     XH=AY(J)-2.0*RIN
      YL=-XH
      GO TO 23
 40   SWA=0.C
      XH=AY(J)
      YL=XH+HT
      GO TO 23
 4    HT=2.0*AY(J)
      XH=-AY(J)
      JX=AY(J)/DELY+1.0
      NA=JE
      NB=JX+JE
     WA=VX(J)
      WB=WA
      YL=AY(J)
      GO TO 23
C IF CU(NAJ),SPECIAL
 5    IF(J-NAJ) 9,6,33
 6    IF(KCH(J-1)) 8,8,7
 7    J=J-1
      IF(VY(JJ-1)) 8,22,22
8    HT=2.0*(RIN-AY(JJ-1))
      XH=AY(JJ-1)
      YL=XH+HT
      JX=AY(JJ-1)/DELY
      NA=JX+JE
```

```
          NB=JED
        WA=VX(JJ-1)
          WB=WA
          GO TO 23
 9        IF(KCH(J)) 11,34,34
C IF TRAJECTORIES HAVE REFLECTED,ADD IN MIRROR IMAGE
 34       IF(KCH(J-1)-KCH(J)) 10,11,15
 10       JO=1
          IF(VY(J)) 19,16,16
 11       HT=AY(J)-AY(J-1)
          IF(HT) 12,12,13
 12       HT=-HT
          XH=AY(J)
          JX=XH/DELY
          NB=AY(J-1)/DELY+1.0
        WA=VX(J)
        WB=VX(J-1)
          GO TO 14
 13       XH=AY(J-1)
          JX=XH/DELY
        WA=VX(J-1)
        WB=VX(J)
          NB=AY(J)/DELY+1.0
 14       NA=JX+JE
          NB=NB+JE
          YL=XH+HT
          GO TO 23
 15       JO=1
          IF(VY(J-1)) 19,19,16
 16       HT=AY(J)+AY(J-1)
          NA=JE
          IF(SWA) 17,17,18
 17       XH=-AY(J-1)
          YL=AY(J)
          JX=YL/DELY+1.0
          NB=JX+JE
        WA=VX(J-1)
        WB=VX(J)
          SWA=1.0
          GO TO 23
 18       SWA=0.0
          XH=-AY(J)
          YL=AY(J-1)
          JX=YL/DELY+1.0
          NB=JX+JE
        WA=VX(J)
        WB=VX(J-1)
          GO TO 23
 19     HT=RIN+RIN-AY(J)-AY(J-1)
          NB=JED
          IF(SWA) 20,20,21
 20       SWA=1.0
          XH=AY(J)
        YL=RIN+RIN-AY(J-1)
          JX=XH/DELY
          NA=JX+JE
```

```
      WA=VX(J)
      WB=VX(J-1)
      GO TO 23
 21   SWA=0.0
      XH=AY(J-1)
      YL=RIN+RIN-AY(J)
      JX=XH/DELY
      NA=JX+JE
      WA=VX(J-1)
      WB=VX(J)
      GO TO 23
 22   HT=2.0*(RIN+AY(J))
      JO=1
      NA=JE
      NB=JED
      WA=VX(J)
      WB=WA
      IF(SWA) 37,37,38
 37   SWA=1.0
      XH=-AY(J)
      YL=XH+HT
      GO TO 23
 38   SWA=0.0
      YL=AY(J)
      XH=YL-HT
 23   DO 32 K=NA,NB
      XA=K-JE
      IF(XA.LE.0.) JU=J-1
 25   XUD=(XA-.5)*DELY
 26   XX=XUD+DELY
      YU=AMAX1(XUD,XH)
      YD=AMIN1(XX,YL)
      IF(YD-YU) 32,32,27
 27   XA=.5*(YD+YU)
      W=WA+(XA-XH)*(WB-WA)/HT
      IF(K-JED) 29,28,28
 28   JU=NAJ-JJ
 29   IF(JU) 31,31,30
 30   IF(JO) 36,36,31
 36   W=.5*W
C CALCULATION OF RHS
 31   RH(K)=RH(K)+(YD-YU)*CU(JJ)/(HT*W)
 32   JU=0
      IF(SWA) 33,33,1
 33   CONTINUE
      RETURN
      END
```

```
C ATRCU CALCULATES THE TUBE CURRENTS ANC CALCULATES
C      THE TRAJECTORY COORD AND VELOCITIES IN SUB TRAJ
       SUBROUTINE ATRCU (KE,KED)
       COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1 DC, DCC, DELY, DX, EPS, ER(4C), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JOT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3 KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MO,
     4 NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5 RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6 SIZE, U(3000), UB(3000), URH(3000),VA, VAT, VBT, VX(51),
     7 VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
       DIMENSION US(2),UR(2)
       JE=KE
       JED=KED
       ALA=.2*H
       ACAN=1.0
       XQ=ABS(XQM)
       ISW=0
       K=1
 116   IS=C
       LL=C
       KK=K
C CHECK ON TERMINATED TRAJECTORIES
       IF(AY(K)+1.0) 16,16,18
C CHECK ON UNINIALIZED TRAJECTORIES
 18    IF(KCH(K)) 20,15,15
 20    IF(ISW) 22,22,1
 22    IF(AY(K)*ACAN) 1,1,19
 19    IS=1
       ISW=1
C CHECK IF TRAJECTORY CAN BE INIALIZED
 1     IF(AX-ETX(K+1)-ALA ) 21,21,2
 2     IF(AX-ETX(K+2)-ALA) 21,21,5
 5     N=-1
       ACAN=1.0
       KK=KK+1
       TA=(ETX(KK-1)-ETX(KK))/(ETY(KK-1)-ETY(KK))
       TB=(ETX(KK)-ETX(KK+1))/(ETY(KK)-ETY(KK+1))
       TB=-.5*(TA+TB)
       DE=ATAN(TB)
 33    XX=ETY(KK)-TB*ETX(KK)
       J=1
 3     N=N+1
 4     JJ=J+N
       TA=PTY(JJ)-PTY(JJ+1)
       TC=PTX(JJ)-PTX(JJ+1)
       YY=TC*PTY(JJ)-TA*PTX(JJ)
       CPX=(TC*XX-YY)/(TA-TB*TC)
       CPY=XX+CPX*TB
 7     IF(PTY(JJ+1)-CPY) 3,8,8
 8     LL=LL+1
 24    IF(AY(KK-1))9,9,10
C TRAJECTORIES INITIALIZED
 9     AY(KK-1)=ETY(KK)+TB*(AX-ETX(KK))
       ACAN=0.0
```

```
      XA=AY(KK-1)/DX
      JX=XA
      AD=JX
      JP=JE+JX
      XA=XA-AD
      UP=(1.0-XA)*U(JP)+XA*U(JP+1)
   V=SQRT(2.C*XQ*ABS(VA-UP))
      VX(KK-1)=V*COS(DE)
      VY(KK-1)=V*SIN(DE)
10    US(LL)=CPX
      UR(LL)=CPY
      IF(K-NTJ) 12,11,12
C C SPECIAL FOR LAST CURRENT DENSITY
11    HT=AREM+AREM
      IF(LAST) 40,41,41
40    RR=0.0
      GO TO 45
41    KK=KK+1
      LL=2
29    US(2)=   PTX(JJ+1)
      UR(2)=   PTY(JJ+1)
      PPX=.5*(ETX(KK-1)+ETX(KK))
      PX= .5*(US(1)+US(2))
      PPY=.5*(ETY(KK-1)+ETY(KK))
      PY= .5*(UR(1)+UR(2))
      GO TO 31
12    IF(LL-2) 5,13,13
13    HT=AREM
23    PPX=.5*(ETX(KK-1)+ETX(KK))
      PX=.5*(US(1)+US(2))
      PPY=.5*(ETY(KK-1)+ETY(KK))
      PY=.5*(UR(1)+UR(2))
31    DEX=SQRT((PX-PPX)**2+(PY-PPY)**2)
C CALCULATION OF CURRENT DENSITY
17    RR=BASE-PPY
      RR=6.2831853*RR
      CU(K+1)=AREM/DEX**2*XK*RR
      CUD(K+1)=CU(K+1)/(RR*AREM)
      IF(KBB)44,44,45
45    CU(K+1)=RCU(K+1)*RR*AREM
      CUD(K+1)=RCU(K+1)
44    SAU(K+1)=CU(K+1)
      KCH(K)=0
      IF(K-1) 14,14,26
C SPECIAL FOR FIRST CURRENT DENSITY
14    DE=ETX(1)-ETX(2)
102   DE=-DE/(ETY(1)-ETY(2))
104   XX=ETY(1)-DE*ETX(1)
      J=0
110   J=J+1
109   TA=PTY(J)-PTY(J+1)
      TC=PTX(J)-PTX(J+1)
      YY=TC*PTY(J)-TA*PTX(J)
      CPX=(TC*XX-YY)/(TA-TC*DE)
      CPY=XX+CPX*DE
107   IF(PTY(J+1)-CPY) 110,1C8,108
```

```
108     PX=.5*(ETX(1)+ETX(2))
        PPX=.5*(CPX+US(1))
        PY=.5*(ETY(1)+ETY(2))
        RR=BASE-PY
        RR=6.2831853*RR
        PPY=.5*(CPY+UR(1))
        DEX=SQRT((PX-PPX)**2+(PY-PPY)**2)
        NTJ=NTJ+1
        NAJ=NAJ+1
        JR=NTJ
        DO 114KL=2,NTJ
        SAU(JR+1)=SAU(JR)
        RCU(JR+1)=RCU(JR)
        CUD(JR+1)=CUD(JR)
        CU(JR+1)=CU(JR)
        AY(JR)=AY(JR-1)
        VY(JR)=VY(JR-1)
        VX(JR)=VX(JR-1)
        KCH(JR)=KCH(JR-1)
114     JR=JR-1
        RCU(2)=RCU(1)
        RCU(1)=0.0
        CUD(1)=0.0
        CU(1)=0.0
        CU(2)=AREM*XK/DEX**2*RR
        CUD(2)=CU(2)/(AREM*RR)
        IF(KBB)42,42,43
43      CU(2)=RCU(2)*RR*AREM
        CUD(2)=RCU(2)
42      SAU(1)=CU(1)
        SAU(2)=CU(2)
        AY(1)=ETY(1)+DE*(AX-ETX(1))
        TB=ATAN(DE)
        XA=AY(1)/DX
        JX=XA
        JP=JE+JX
        AD=JX
        XA=XA-AD
        UP=(1.0-XA)*U(JP)+XA*U(JP+1)
      V=SQRT(2.0*XQ*ABS(VA-UP))
        VX(1)=V*COS(TB)
        VY(1)=V*SIN(TB)
        K=K+1
        J=51
        DO 118 I=1,50
        ETX(J)=ETX(J-1)
        ETY(J)=ETY(J-1)
118     J=J-1
26      CONTINUE
21      IF(IS) 16,16,15
C ATRAJ CALCULATES THE TRAJECTORIES AFTER THEY HAVE BEEN INIALIZED
15      CALL TRAJ(K,JE,JED)
16      K=K+1
        IF(NTJ-K) 117,116,116
117     RETURN
        END
```

```
C    ACALR CALCULATES RHS FOR AXISYM CASES
      SUBROUTINE ACALR (KE,KED)
      COMMON AREM, ATX(51),ATY(51), AX, AY(51), BASE, CU(51), CUD(51),
     1 DC, DCC, DELY, DX, EPS, ER(40), ETX(51),ETY(51),H,IAS,JAS(20),JD
     2 ,JDT, JT(3000), KAB, KABB, KAN, KAT, KATT, KAT1, KAT2, KB(13),
     3 KBA, KBB, KBF, KCH(51), KRL, KT(1850), LAST, LB(2), LC(3), MD,
     4 NAJ, NPIT, NRD, NRL, NT, NTJ, NURL, NXEP, PTX(99), PTY(99),
     5 RCU(51), RH(3000), RHDOWN, RHUP, RIN, RX, SAU(51), SEM,
     6 SIZE, U(3000), UB(3000), LRH(3000),VA, VAT, VBT, VX(51),
     7 VY(51), XD, XEP, XK, XMP, XQM, XR, XT(1850),YEP
      DATA  PI/3.1415927/
          JE=KE
          JED=KED
          SWA=0.0
      DO 33 JJ=2,NAJ
          JD=0
          J=JJ
C IFCU(J)=0.,NO CONTRIBUTION
          IF(CU(J)) 33,33,5
C IF CU(NAJ),SPECIAL
 5        IF(J-NAJ) 9,6,33
6      HT=(RIN-AY(JJ-1))
          XH=AY(JJ-1)
          YL=XH+HT
          JX=AY(JJ-1)/DELY
          NA=JX+JE
          NB=JED
        WA=VX(JJ-1)
          WB=WA
          GO TO 23
 9        IF(KCH(J)) 11,34,34
C IF TRAJECTORIES HAVE REFLECTED,ADD IN MIRROR IMAGE
 34       IF(KCH(J-1)-KCH(J)) 19,11,19
 11       HT=AY(J)-AY(J-1)
          IF(HT) 12,12,13
 12       HT=-HT
          XH=AY(J)
          JX=XH/DELY
          NB=AY(J-1)/DELY+1.0
        WA=VX(J)
        WB=VX(J-1)
          GO TC 14
 13       XH=AY(J-1)
          JX=XH/DELY
        WA=VX(J-1)
        WB=VX(J)
          NB=AY(J)/DELY+1.0
 14       NA=JX+JE
          NB=NB+JE
          YL=XH+HT
          GO TO 23
19     HT=RIN+RIN-AY(J)-AY(J-1)
          JD=1
          R1=BASE-AY(J)
          R2=BASE-AY(J-1)
```

```
      NB=JED
      IF(SWA) 20,20,21
 20   SWA=1.0
      FA=R1**2/(R1**2+R2**2)
      XH=AY(J)
   YL=RIN
      JX=XH/DELY
      NA=JX+JE
    WA=VX(J)
    WB=VX(J-1)
      GO TO 23
 21   SWA=0.0
      XH=AY(J-1)
   YL=RIN
      FA=R2**2/(R1**2+R2**2)
      JX=XH/DELY
      NA=JX+JE
    WA=VX(J-1)
    WB=VX(J)
 23   DO 32 K=NA,NB
      XA=K-JE
      XUD=(XA-.5)*DELY
 41   XX=XUD+DELY
      IF(K-JED) 25,26,26
26 XX=RIN
   IF(BASE.EQ.RIN) GO TO 25
      XX=XX+.5*DX
      IF(XX.GT.BASE) XX=BASE
 25   YU=AMAX1(XUD,XH)
      YD=AMIN1(XX,YL)
      IF(YD-YU) 32,32,27
 27   XA=.5*(YD+YU)
      W=WA+(XA-XH)*(WB-WA)/HT
      YU=BASE-YU
      YD=BASE-YD
      AO=YU**2-YD**2
      AT=2.0*BASE*(YL-XH)+XH**2-YL**2
      XX=BASE-XX
      XUD=BASE-XUD
      AC=PI*(XUD**2-XX**2)
      IF(JO) 45,45,46
 46   AO=AO*FA
 45   RH(K)=RH(K)+CU(JJ)*AO/(AT*AC*W)
 32   CONTINUE
      IF(SWA) 33,33,9
 33   CONTINUE
      RETURN
      END
```

APPENDIX D

FLOW DIAGRAMS

Flow diagrams (figs. 7 to 28) and brief explanations of all the sections of the program and various subroutines, as shown in figure 3, are presented in this appendix. Some of the nomenclature used herein is further defined in COMMON STATEMENT SYMBOLS (appendix B) and/or in the section Input Data Preparation.

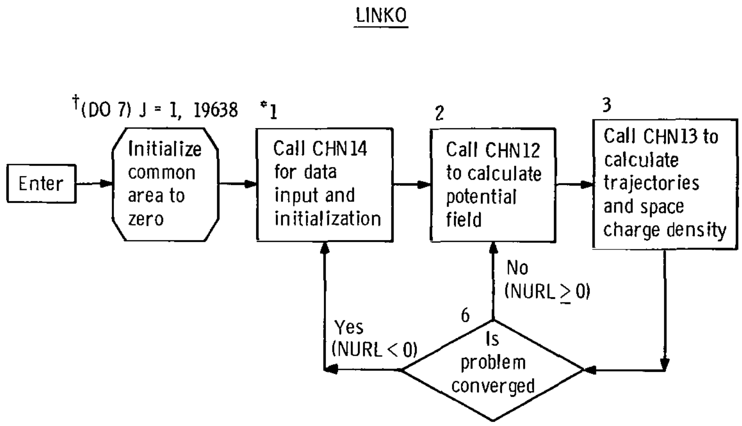


Figure 7. - LINKO is used as the Main Program to transfer control between the potential field calculation and computation of the ion trajectories and space-charge density. †( ) Notation to indicate repetitive operation (FORTRAN DO loop). *(These numbers correspond to FORTRAN statement numbers in appendix B.)

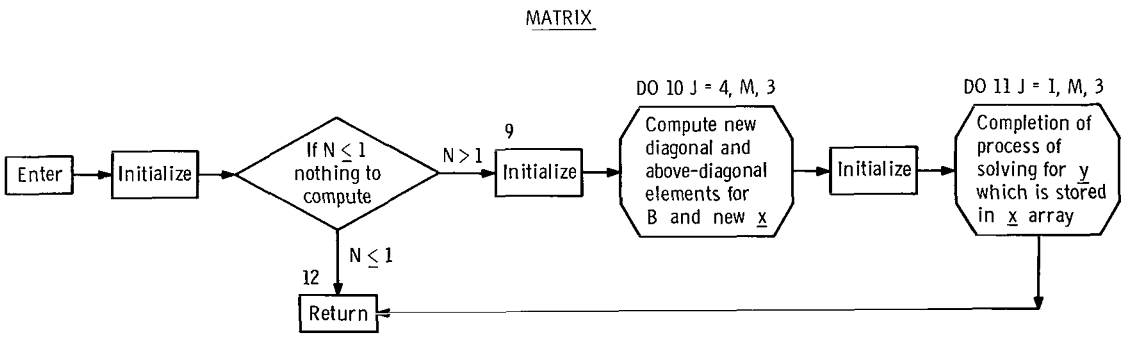


Figure 8. - Subroutine MATRIX solves by Gaussian elimination the matrix equation $B\underline{y} = \underline{x}$ for the vector $\underline{y}$, where B is a tri-diagonal matrix of order N stored in an N x 3 array and $\underline{x}$ is a given N x 1 column vector.

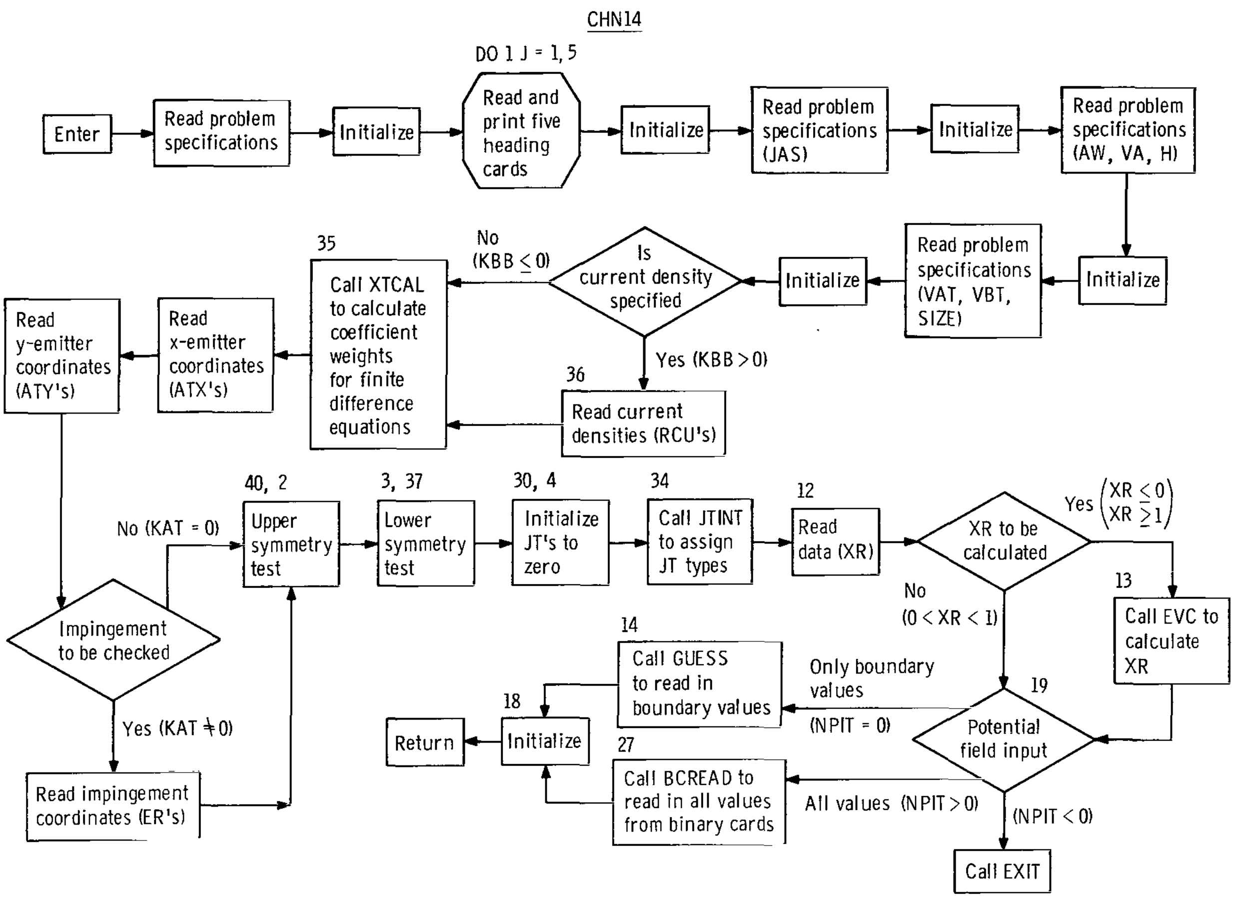


Figure 9. - Subroutine CHN14 is for data input and initialization.

XTCAL

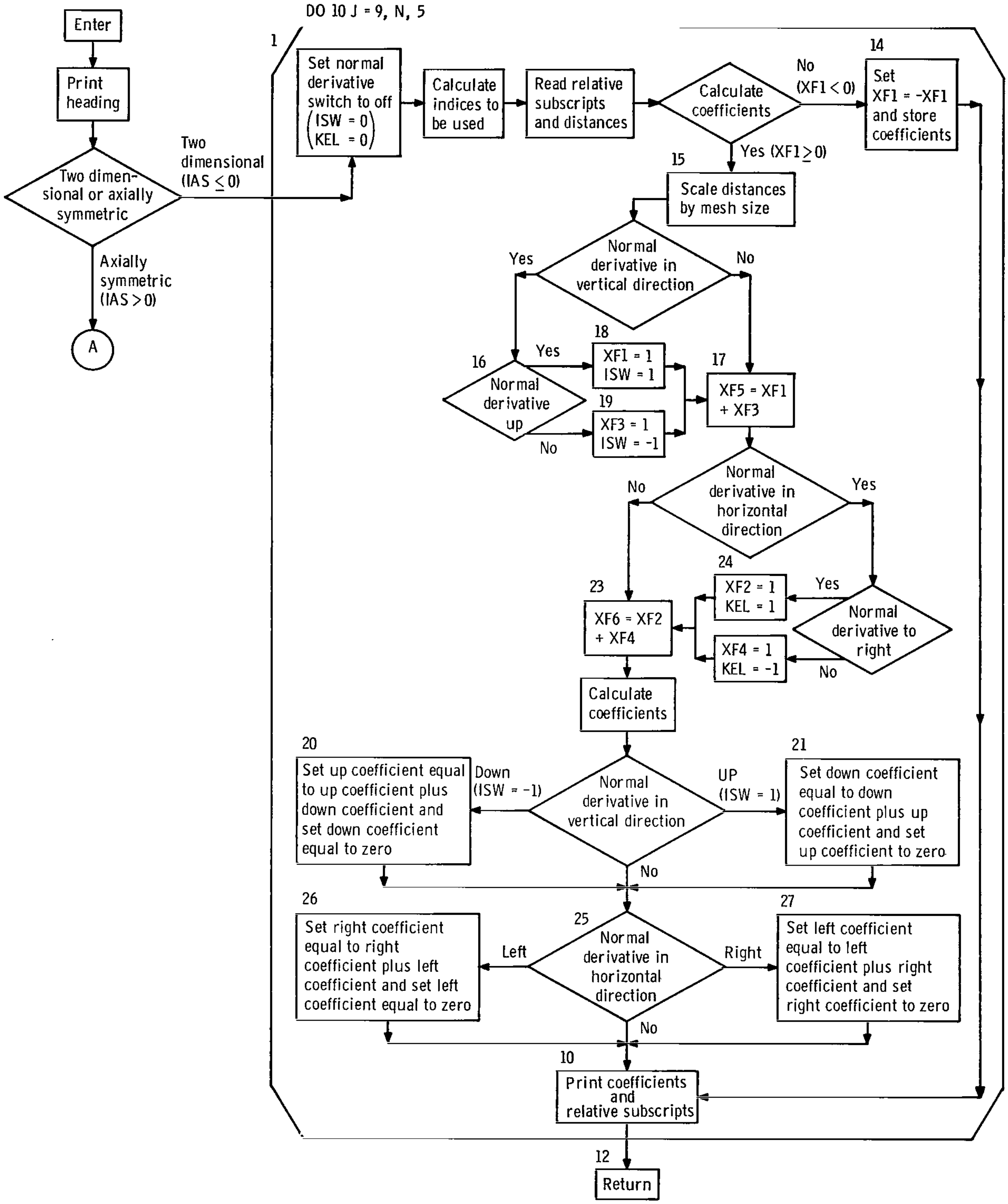


Figure 10. - Subroutine XTCAL calculates the coefficients in the finite-difference approximation to Poisson's equation.

XTCAL

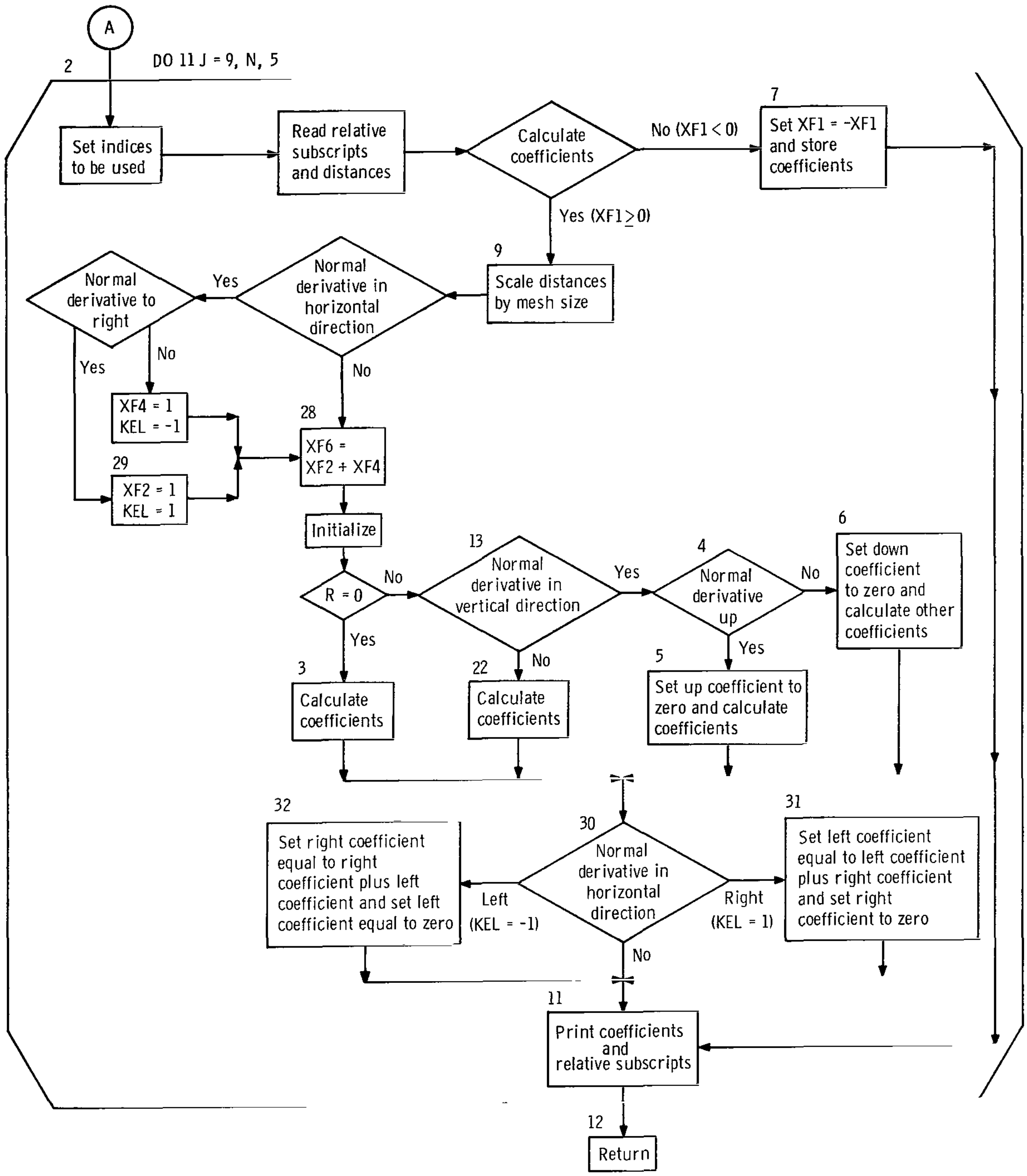


Figure 10. - Concluded.

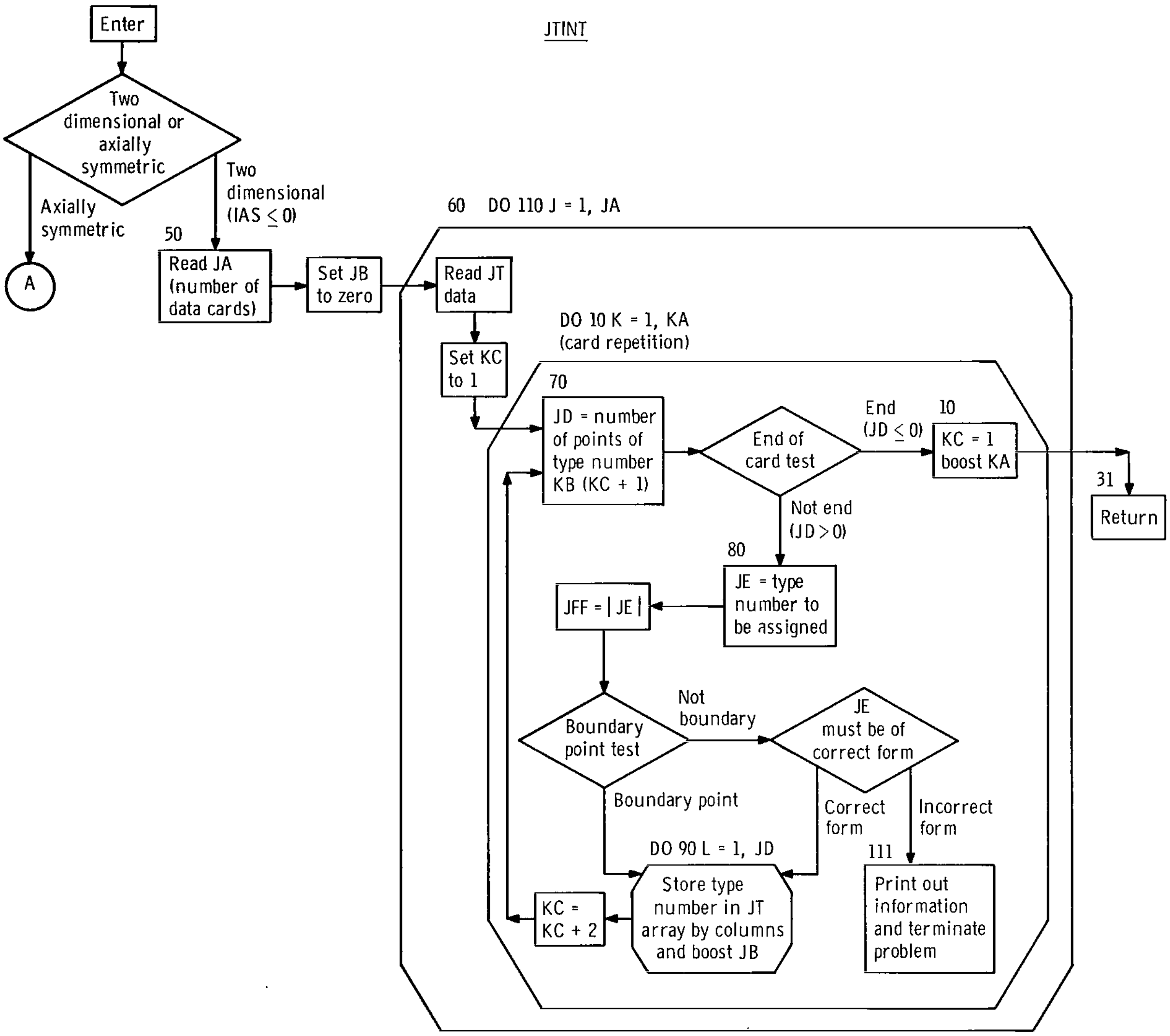


Figure 11. - Subroutine JTINT sets up the JT-type array.

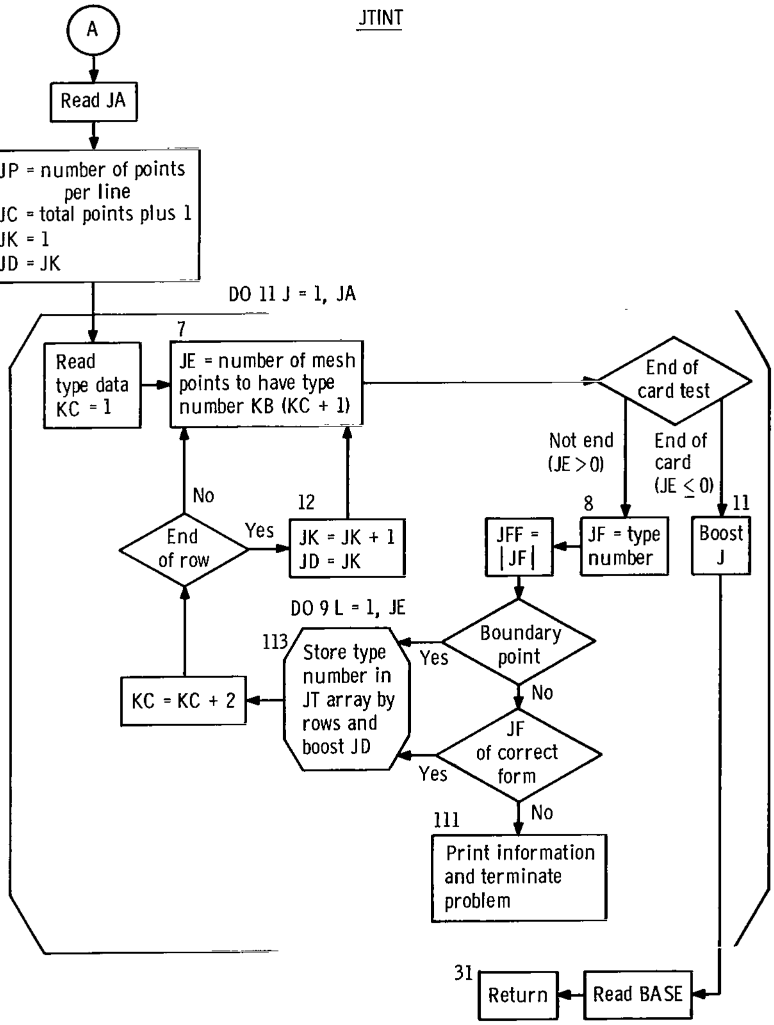


Figure 11. - Concluded.

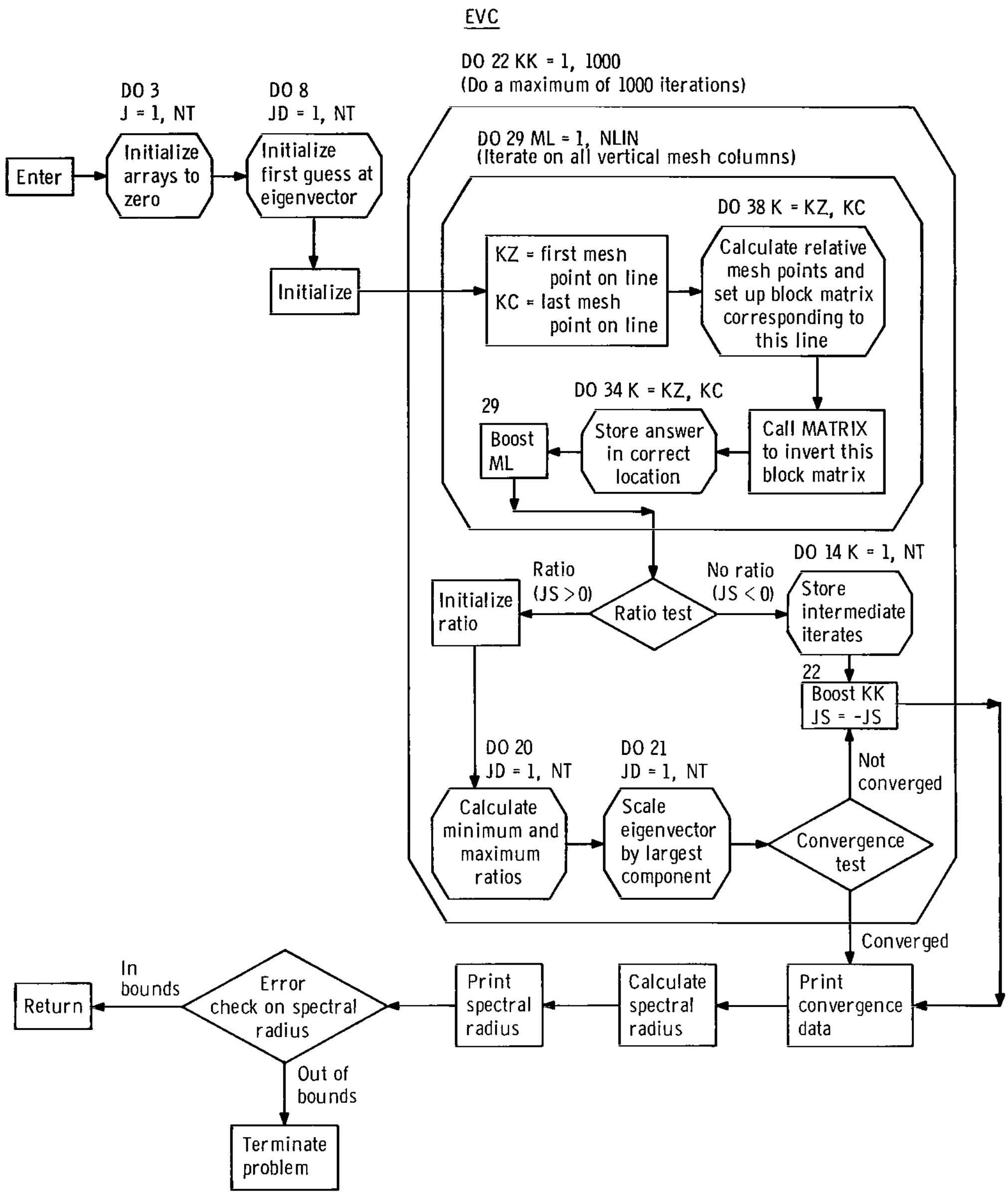


Figure 12. - Subroutine EVC calculates the spectral radius of the iteration matrix. The spectral radius is solved by a minimax process and as the matrix is two-cyclic, the ratio is calculated every other iteration.

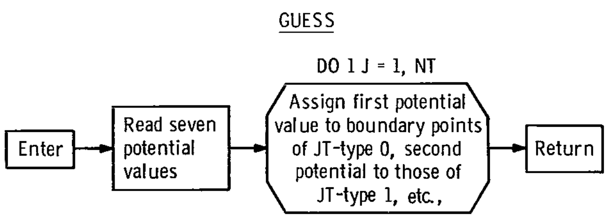


Figure 13. - Subroutine GUESS reads in potential values and assigns them to the proper boundaries.

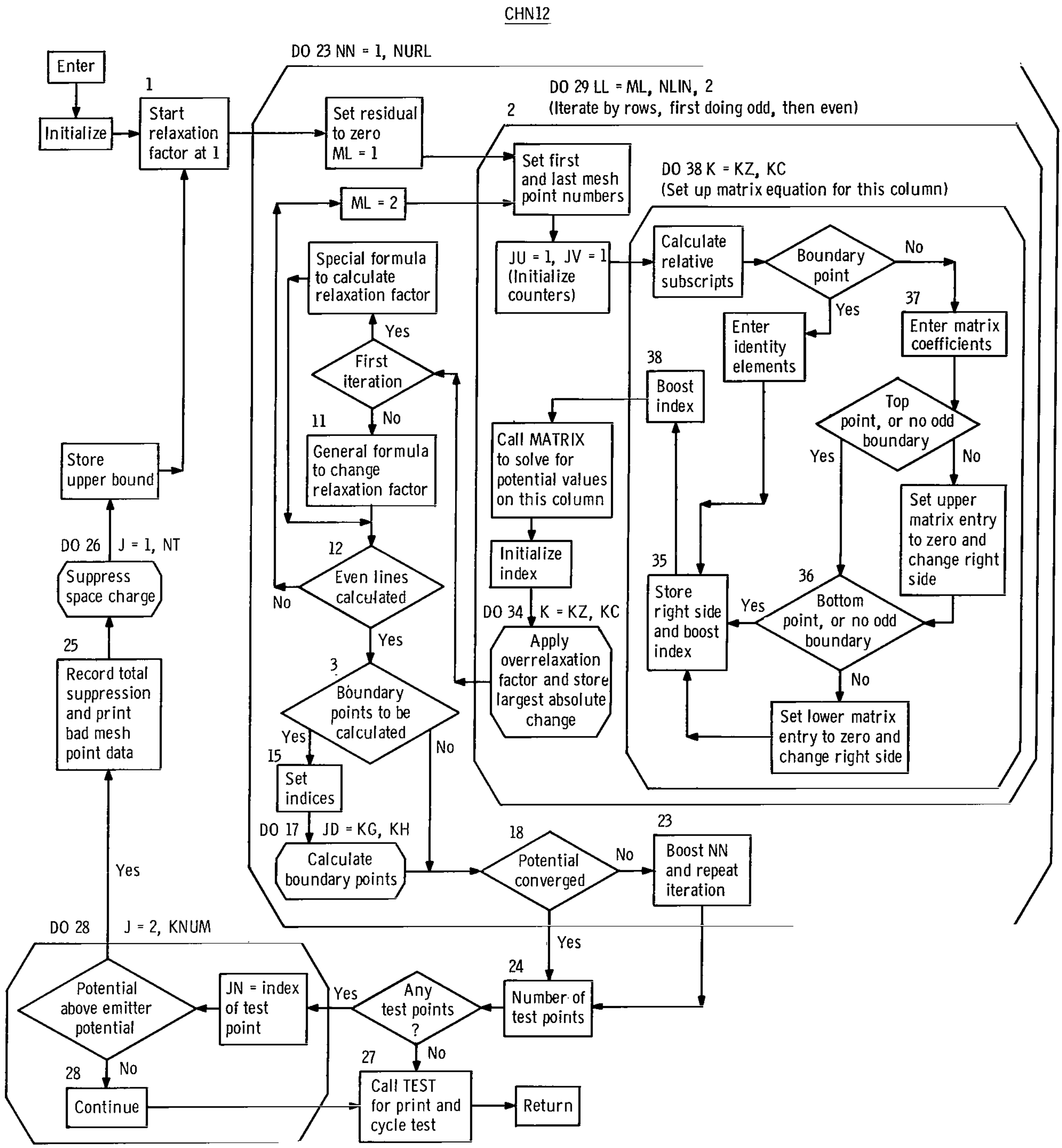


Figure 14. - Subroutine CHN12 calculates the potential field distribution.

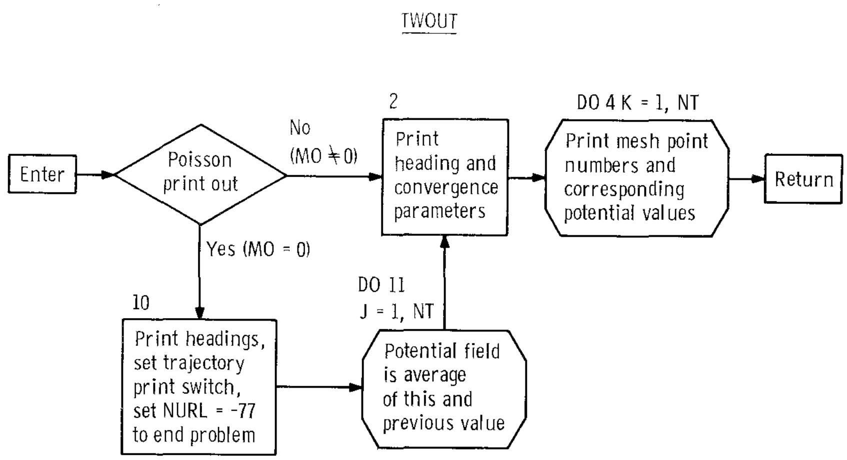


Figure 15. - Subroutine TWOUT is used to print out the potential field.

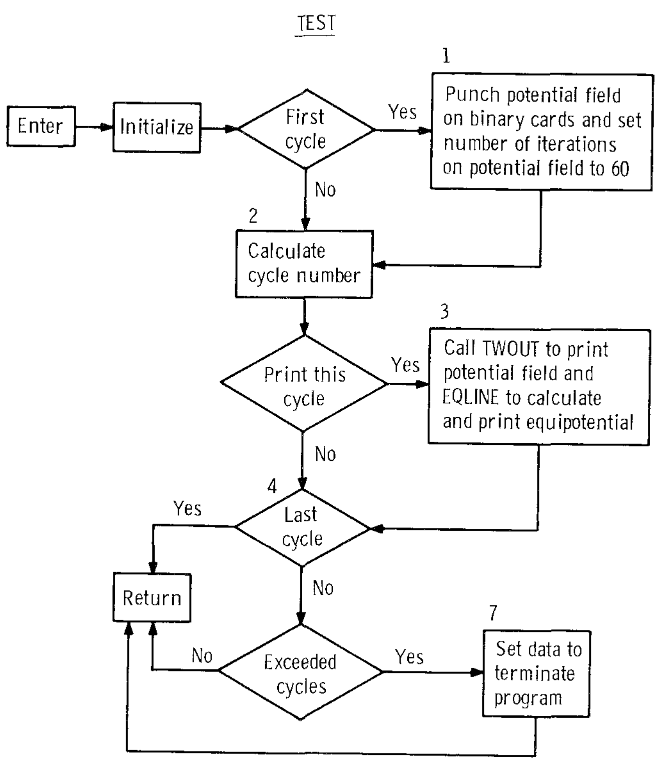


Figure 16. - Subroutine TEST controls printing of potential field and terminates program if maximum number of cycles to converge on RH is exceeded.

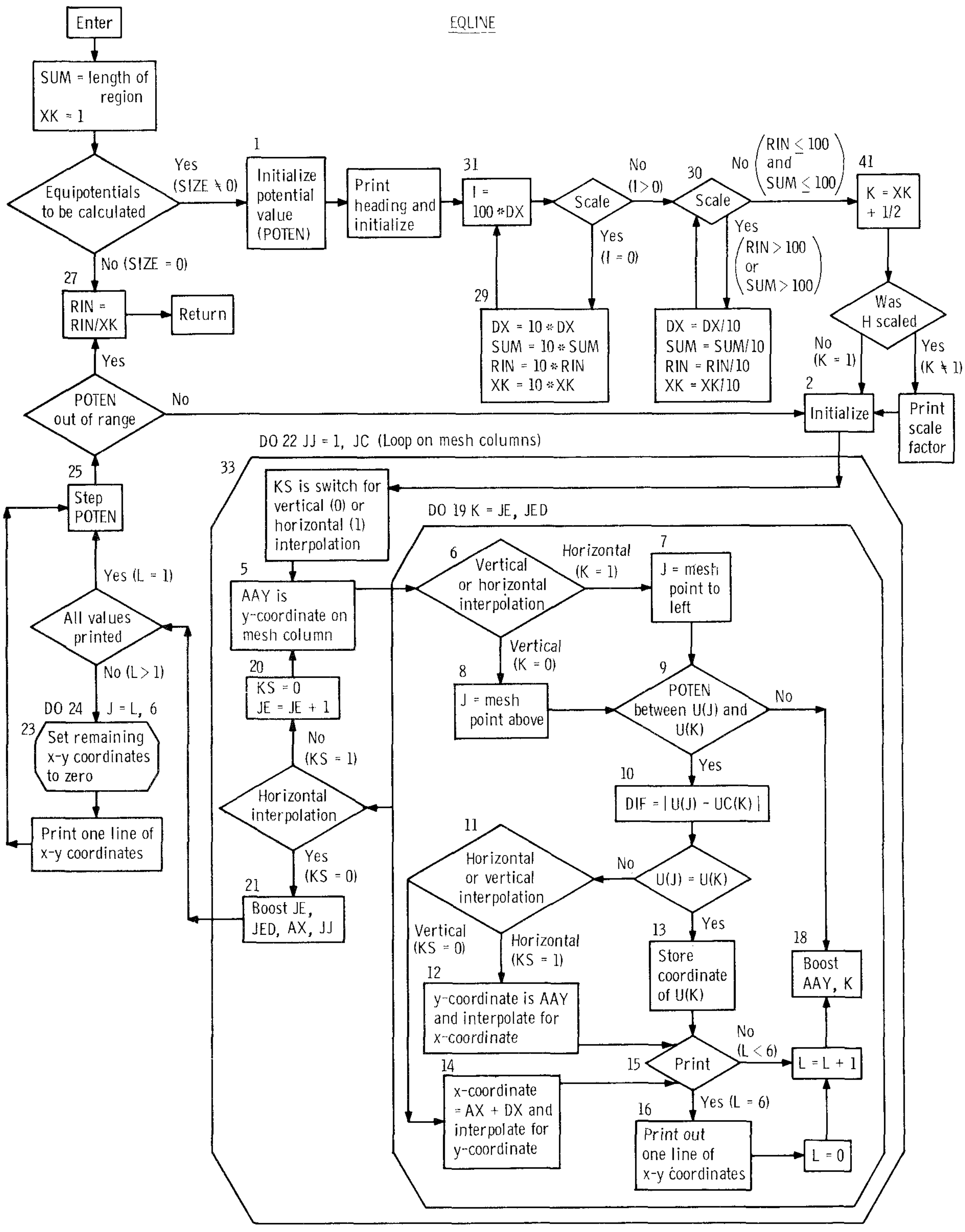


Figure 17. - Subroutine EQLINE calculates and prints equipotential lines.

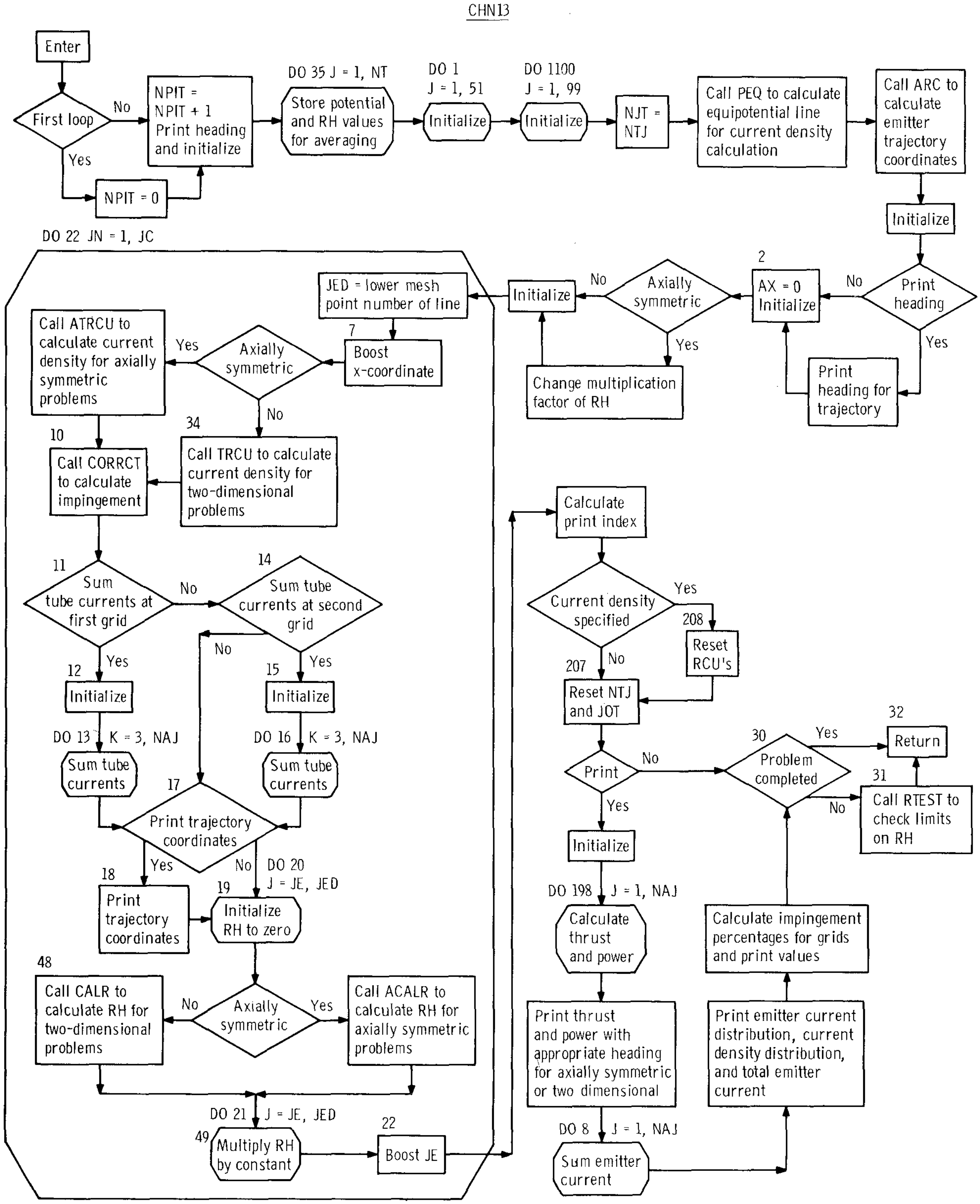


Figure 18. - Subroutine CHN13 coordinates the trajectory calculation and space-charge-density-function calculation.

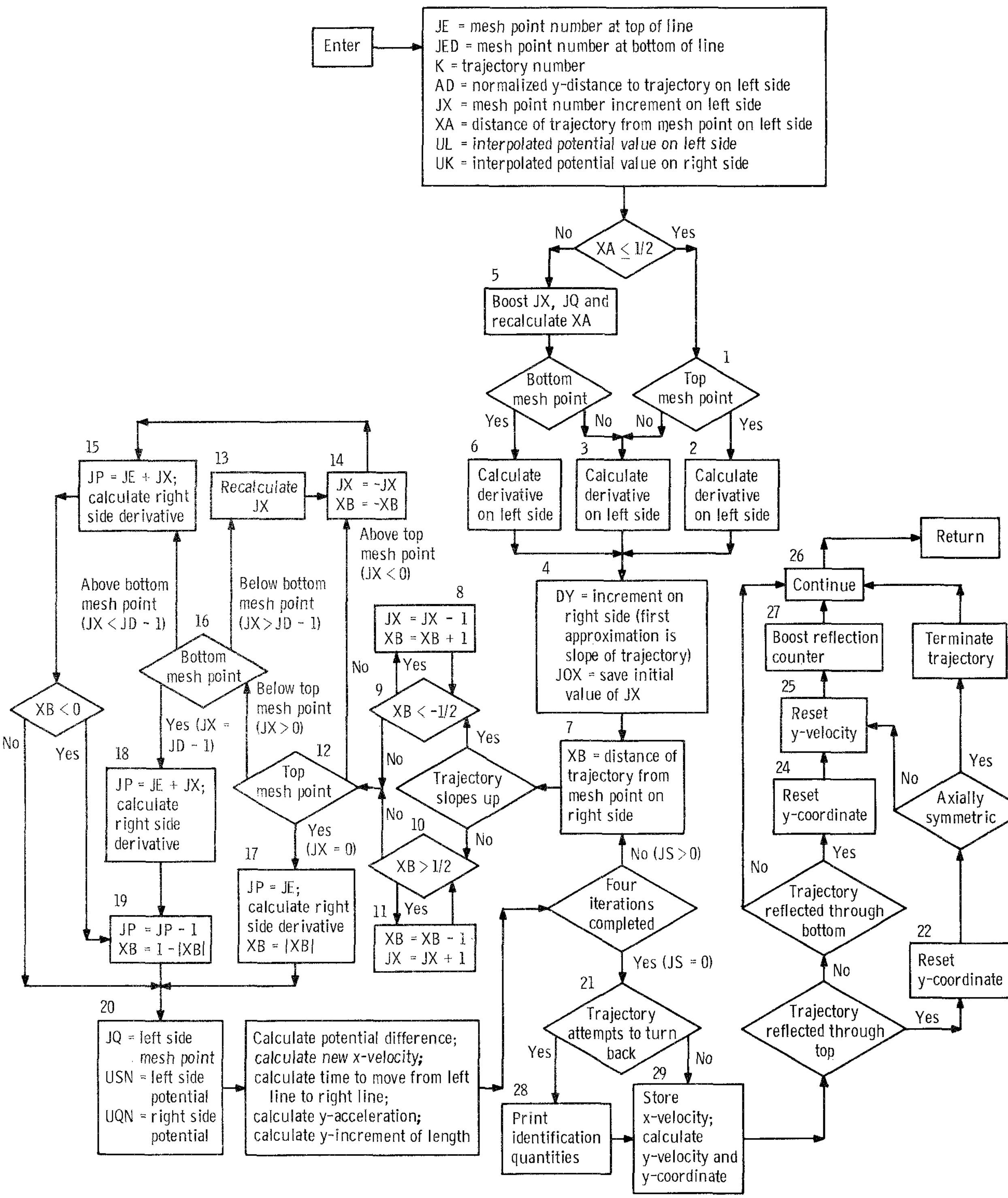


Figure 19. - Subroutine TRAJ calculates the trajectory velocities and coordinates.

CORRCT

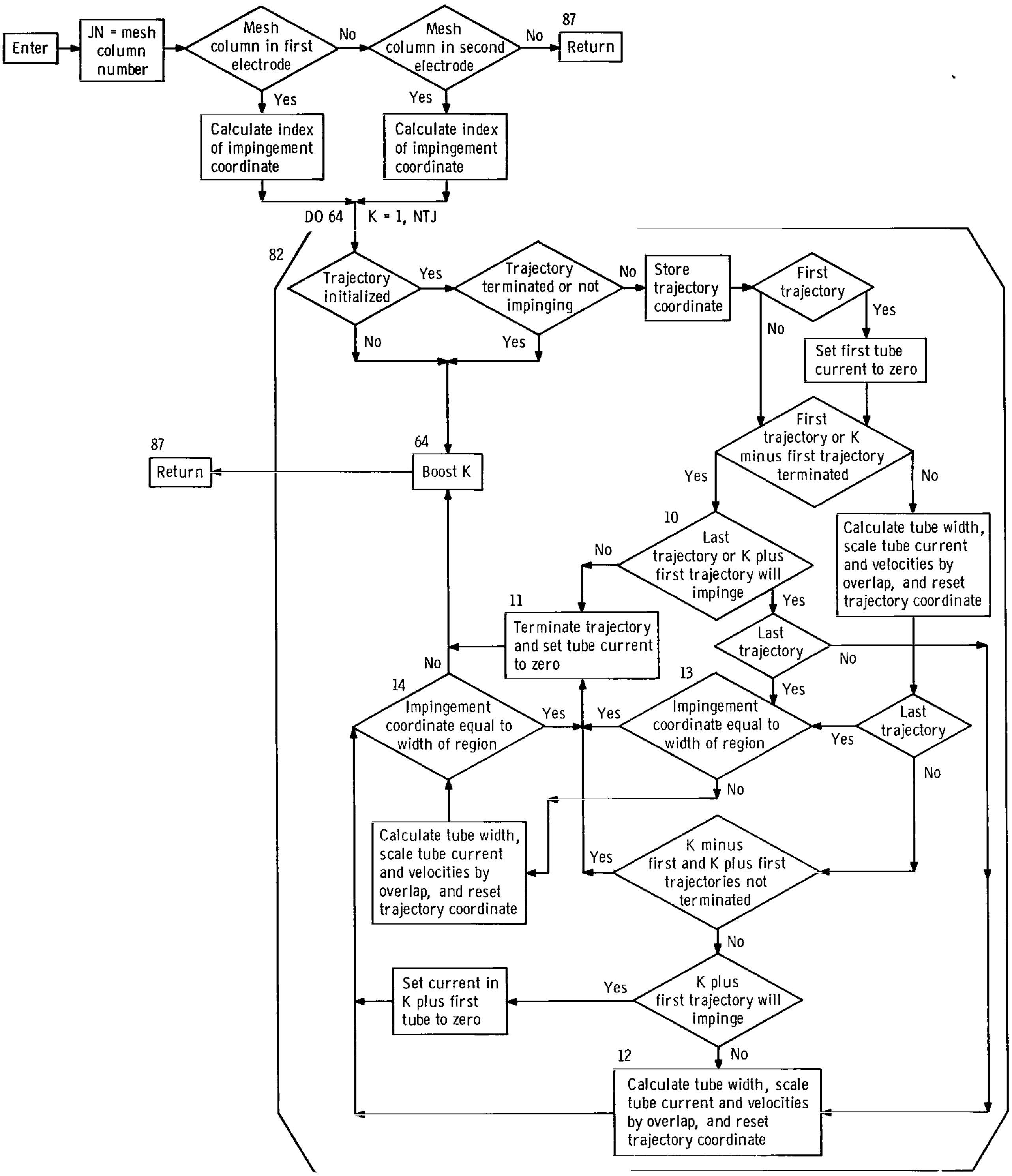


Figure 20. - Subroutine CORRCT computes the impingement on the designated electrodes.

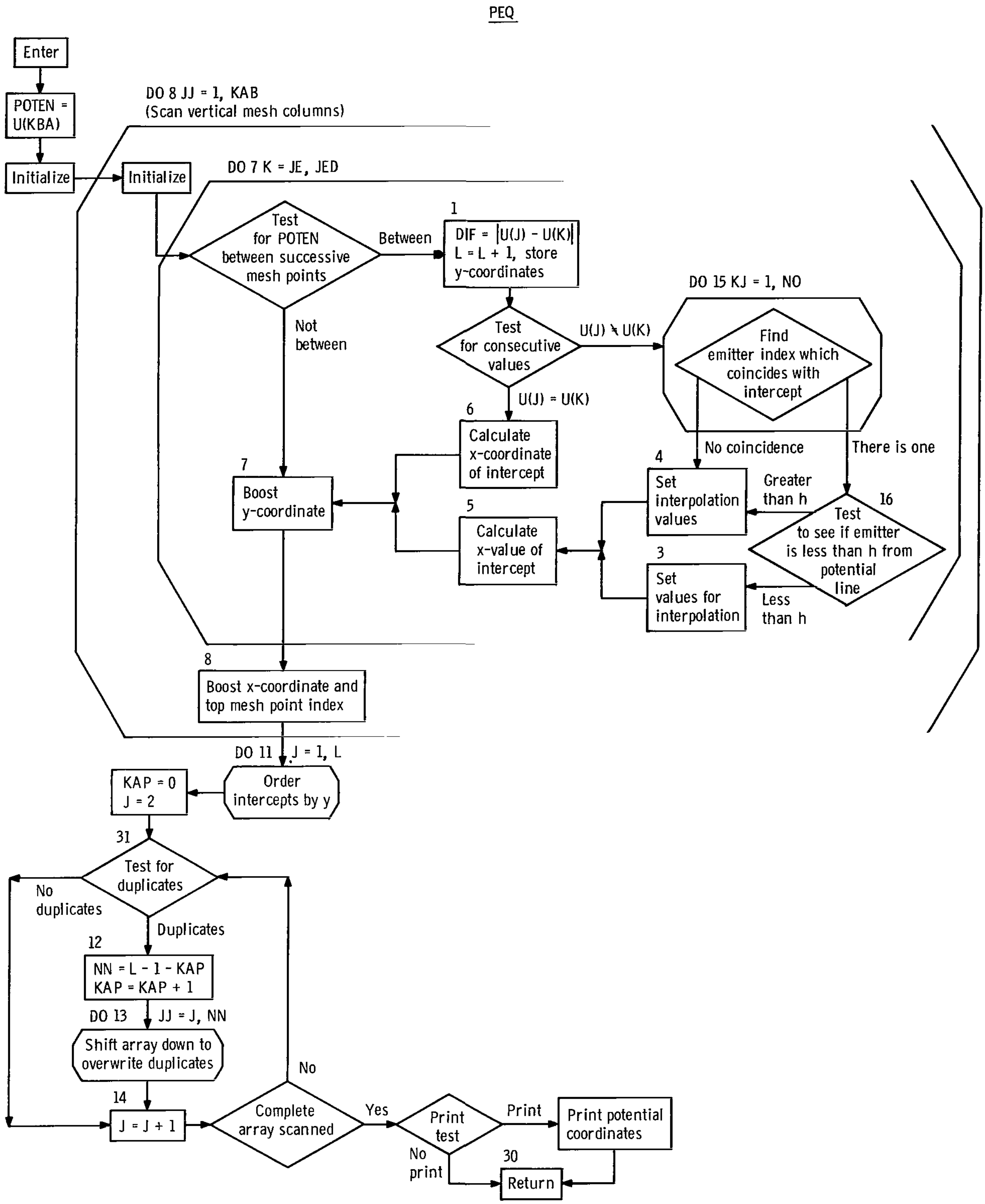


Figure 21. - Subroutine PEQ calculates the equipotential line that is used in conjunction with the emitter to calculate the emitter current density.

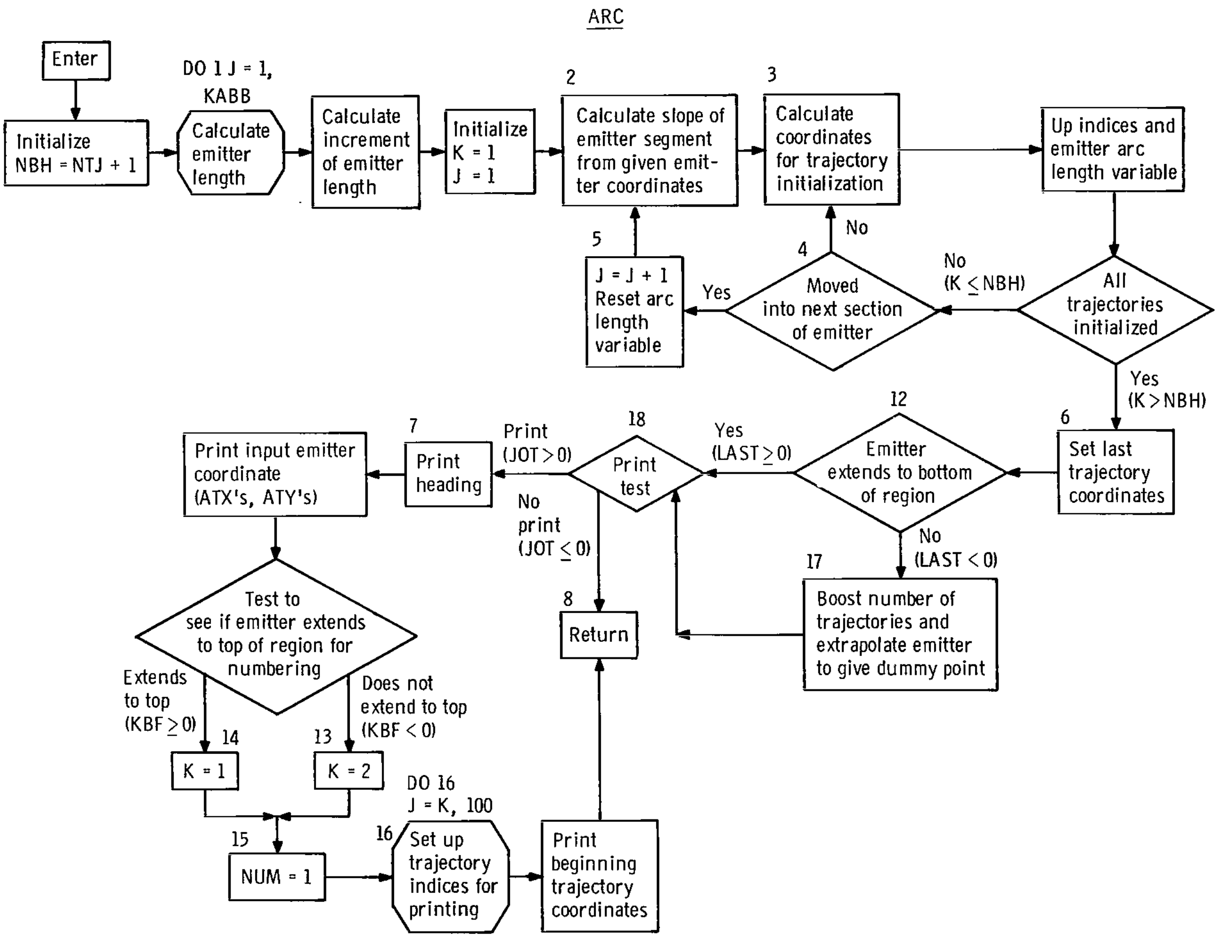


Figure 22. - Subroutine ARC divides the emitter segment into equal increments for intialization of trajectories and calculation of emitter current and current density.

RTEST

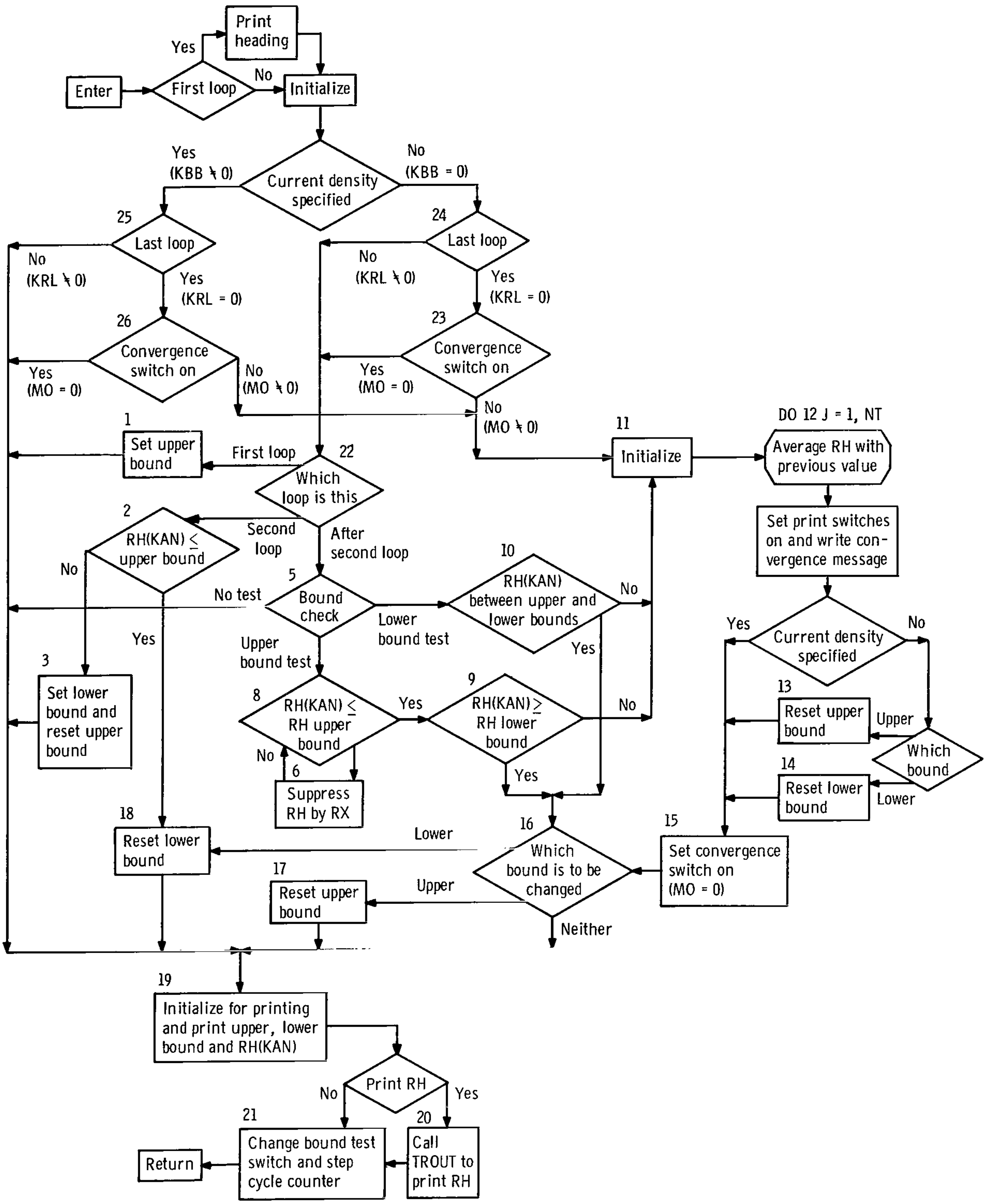


Figure 23. - Subroutine RTEST determines the convergence of the Poisson solution.

TROUT

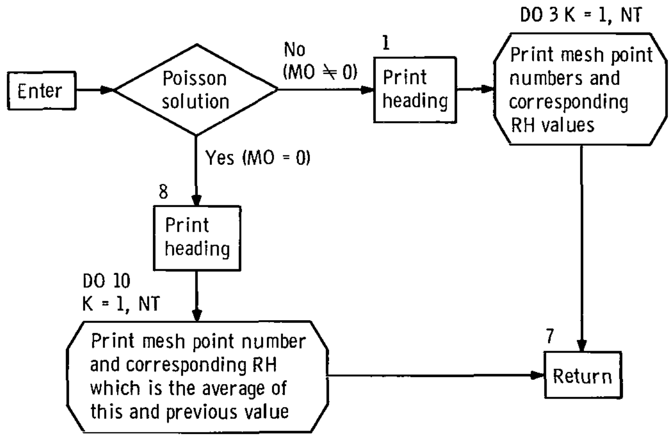


Figure 24. - Subroutine TROUT is used to print RH, the space-charge-density function multiplied by the square of the mesh size divided by 4.

TRCU

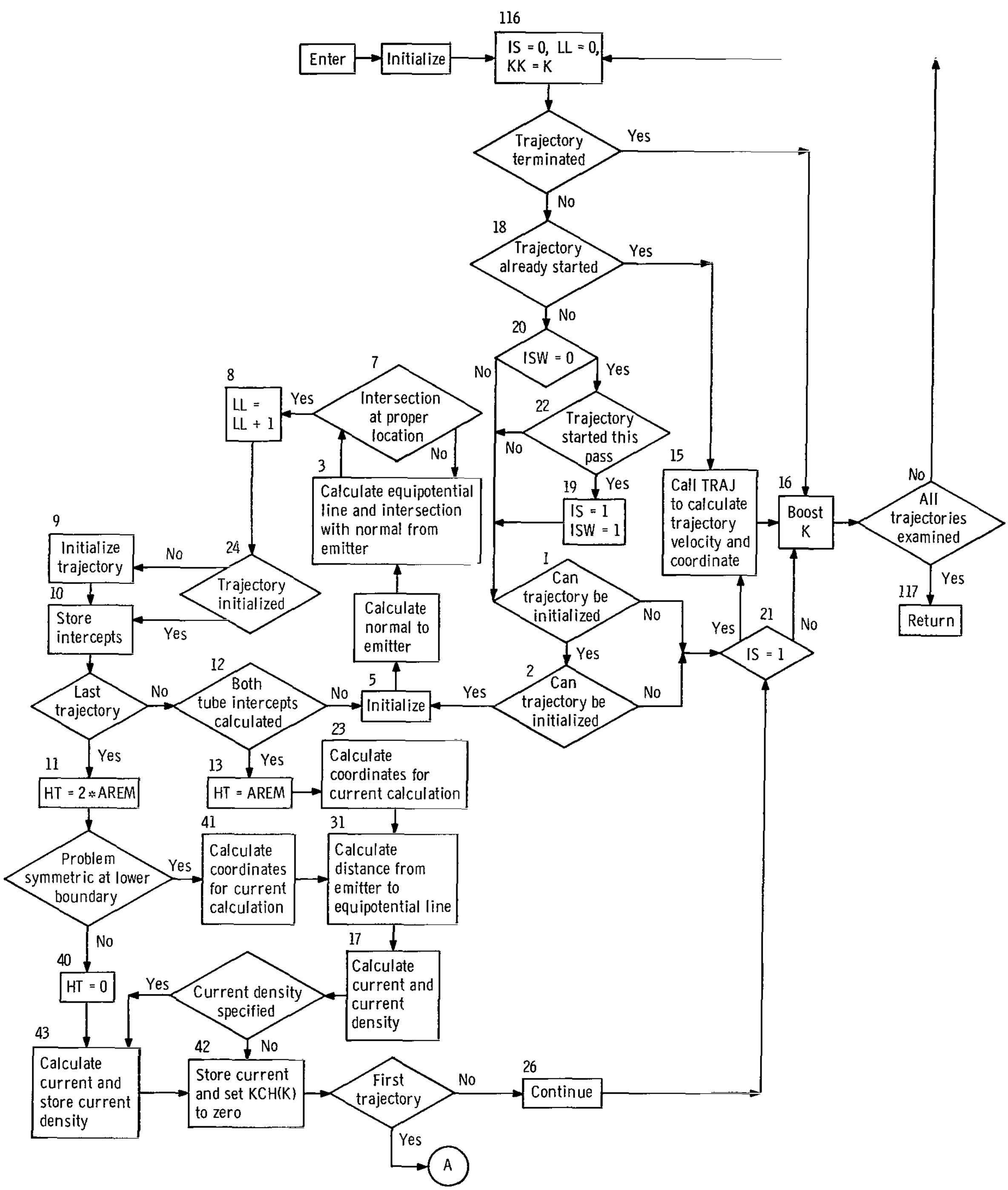


Figure 25. - Subroutine TRCU initializes the trajectory coordinates and calculates the current in each stream tube for two-dimensional problems.

Subroutine ATRCU initializes the trajectory coordinates and calculates the current in each stream tube for axially symmetric problems. The flow chart for subroutine ATRCU is similar to the one for TRCU. The differences are in the internal equations for calculating the current, as it is calculated for an annular section in ATRCU but only for a rectangular segment in TRCU, and in the calculation of the first stream tube, as it is not possible for it to be symmetric to the upper boundary for axially symmetric problems, while this possibility is accounted for in TRCU.

TRCU

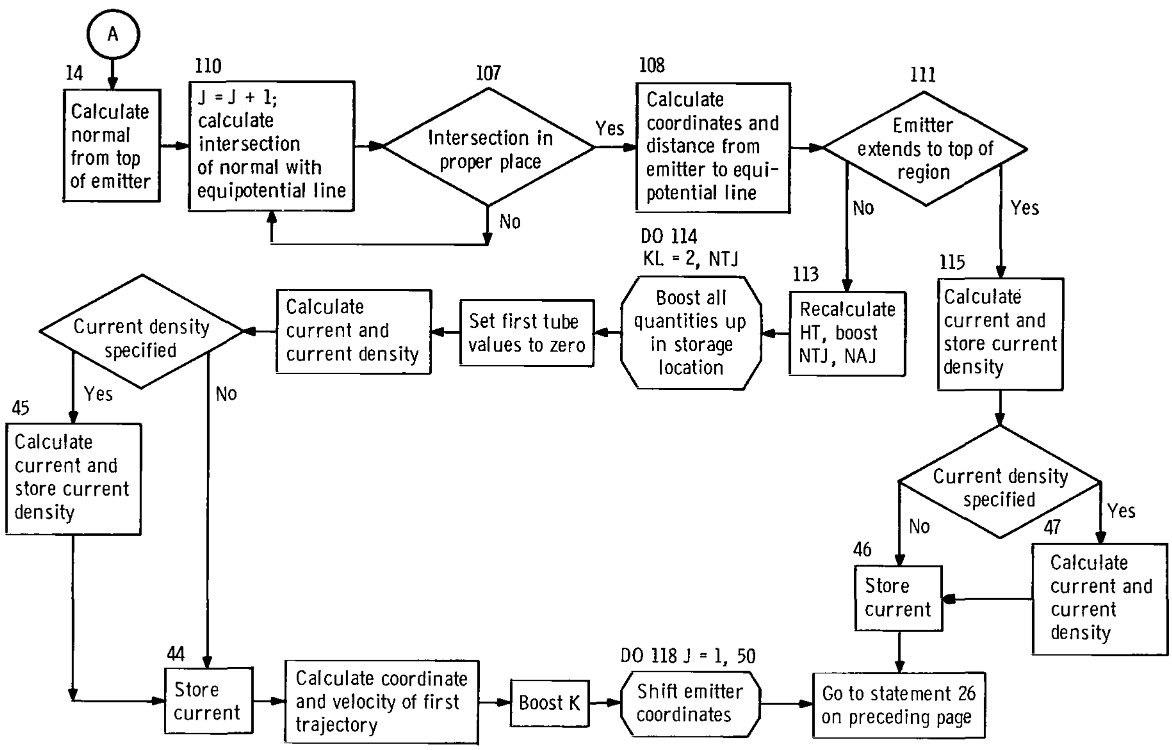


Figure 25. - Concluded.

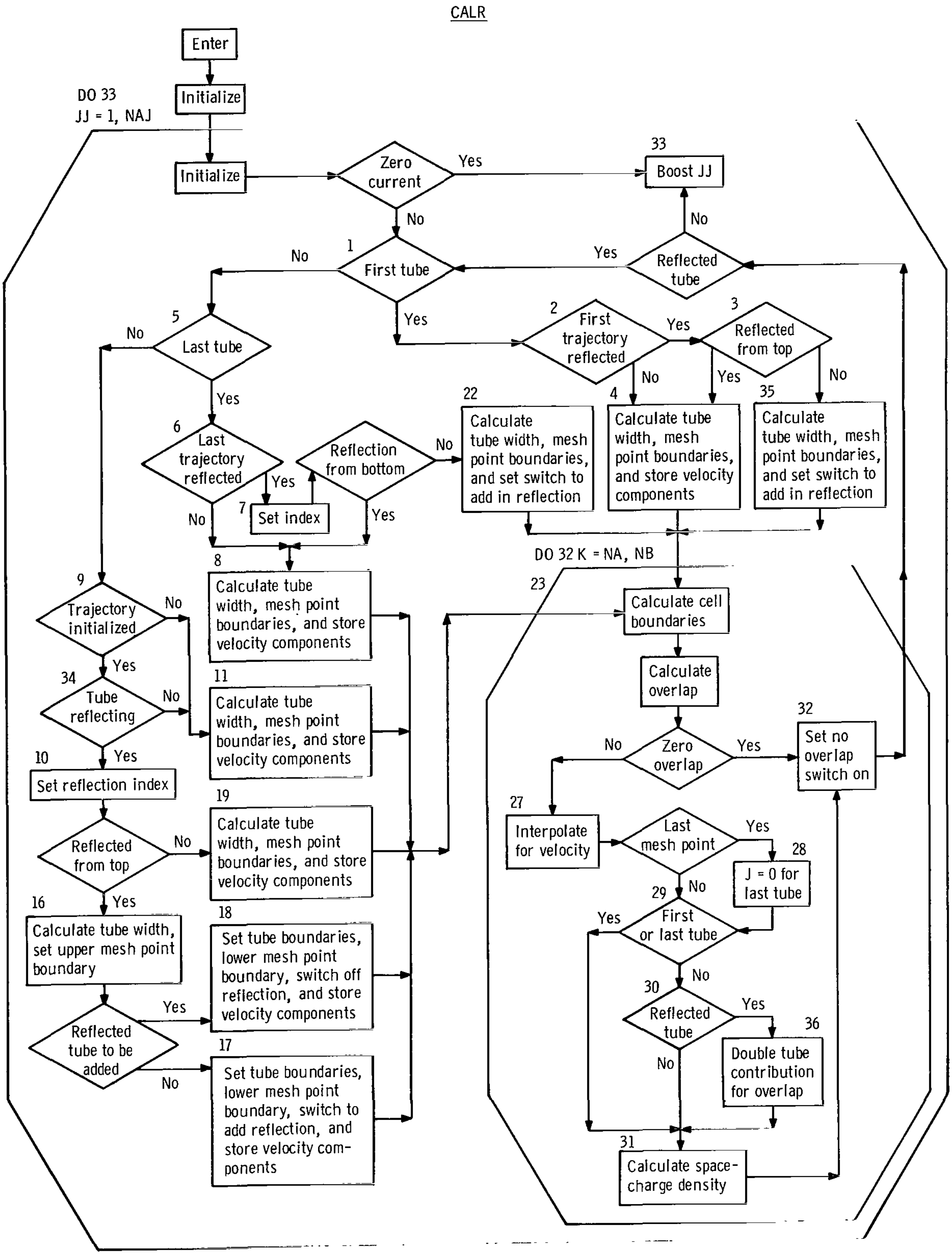


Figure 26. - Subroutine CALR calculates the space-charge-density function for two-dimensional geometries.

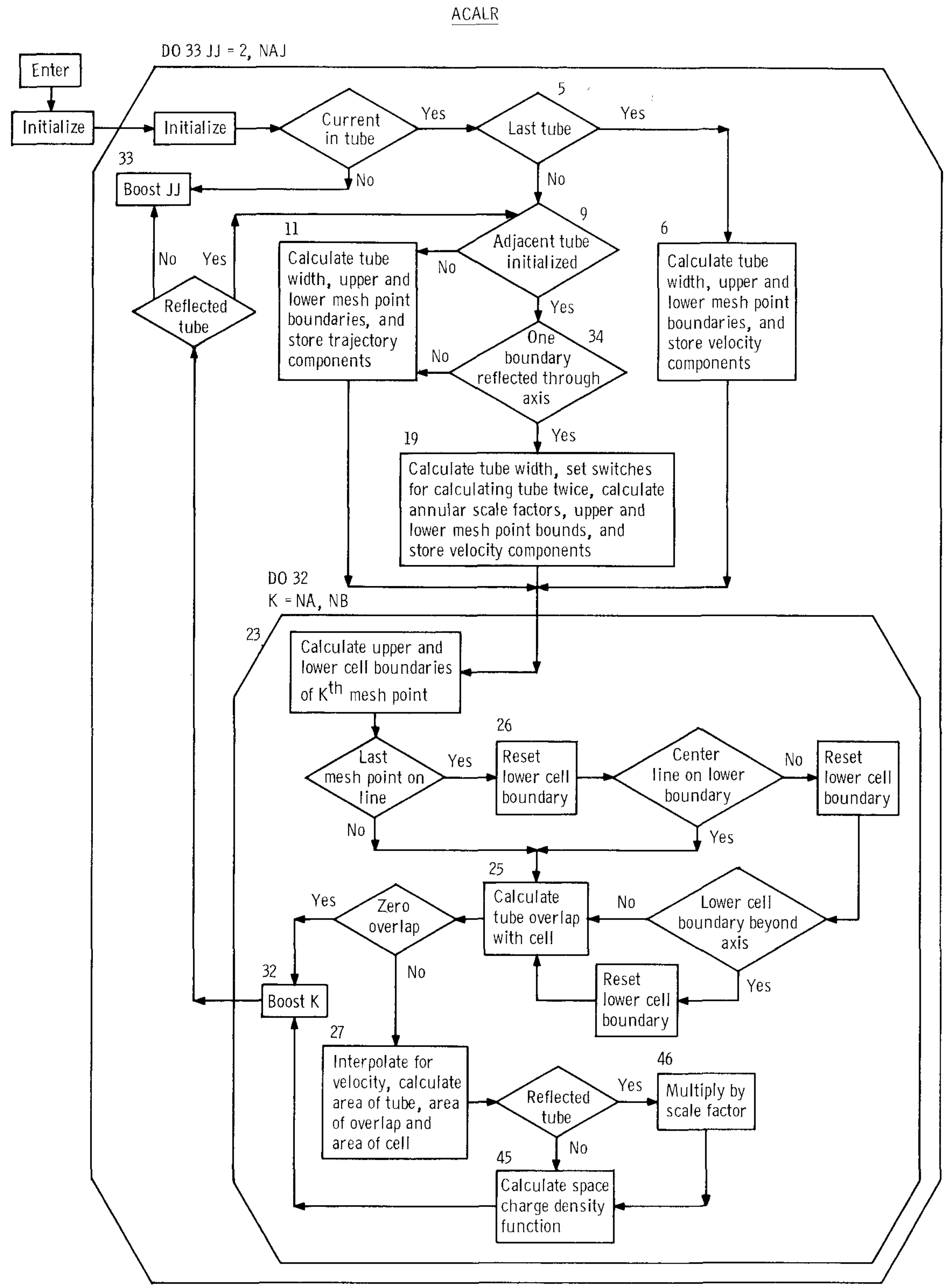


Figure 27. - Subroutine ACALR calculates the space-charge-density function for axisymmetric geometries.

APPENDIX E

SAMPLE PROBLEMS

Contained herein is a listing of the necessary input data cards as well as the computer calculated output data for an axisymmetric problem with current density specified. Results are plotted in figure 6 (p. 22). Also included is a two-dimensional problem that is solved as a space-charge-limited-flow problem.

Input Data Cards

```
                     CARD COLUMN NUMBERS
12345678901234567890123456789012345678901234567890123456789012345678901234567890

   0   78    4    6   60    4    2    9   10    2    0        1        CARD 1
   5    1                                                              CARD 2
1       SAMPLE PROBLEM TO DEMONSTRATE LEWIS RESEARCH CENTER            HEADING
       ION THRUSTOR PROGRAM                                            HEADING
   AXI-SYMMETRIC                                                       HEADING
   CURRENT DENSITY SPECIFIED                                           HEADING
                                                                       HEADING
   3    8    9   10                                                    JAS
                                                                       JAS
132.91    1000.     0.25                                              AW,VA,H
1000.     -1000.    200.                                              VATVBTSI
1.0  E-4 1.0  E-4 1.0  E-4 1.0  E-4                                    RCU
  -1    1    5   -5 0.        .25       .25       .25       1.        JT=9
  -1    1    5   -5 .25       .25       .25       .25       .75       JT=14
  -1    1    5   -5 .25       .25       .25       .25       .5        JT=19
  -1    1    5   -5 .25       .25       .25       .25       .25       JT=24
  -1    1    5   -5 .25       .25       0.        .25       0.        JT=29
  -1    1    5   -5 .25       .25       .25       .20       .5        JT=34
  -1    1    5   -5 .25       .25       .25       .20       .25       JT=39
  -1    1    5   -5 .25       .25       0.        .20       0.        JT=44
  -1    4    4   -5 .25       .025      .050      .25       .5        JT=49
   3    1    5   -5 .125      .25       .25       .25       .25       JT=54
  -1    1    5   -1 .0834     .25       .25       .10       .5        JT=59
  -1    1    5   -5 .042      .25       .25       .05       .75       JT=64
  -4    1    5   -5 -.50      .50       0.        0.        0.        JT=-69
  -1    1    5   -5 .165      .25       .25       .08       .75       JT=74
.05       .05                                                         ATX
.50       1.0                                                         ATY
.625      .40       .23                                               FR
   7                                                                  JA
   1    2    0    4  -69    2    1    3    9    1    2                 KA,KBS
   1    1    0    1   74    2   14    1  -69    1    1    1   64    0  KA,KBS
   1    4   14    1    2                                              KA=KBS
   1    1    0    1   34    2   19    1   49    1   59    5   19    0  KA,KBS
   1    1    2                                                        KA,KBS
   1    1    0    1   39    2   24    1   54    6   24    1    2    0  KA,KBS
   1    1    0    1   44    9   29    1    2                           KA,KBS
1.0                                                                   BASE
                                                                      XR
1000.     -1000.    0.                                                GUESS
```

Output Data Listing

SAMPLE PROBLEM TO DEMONSTRATE LEWIS RESEARCH CENTER
ION THRUSTER PROGRAM
AXI-SYMMETRIC
CURRENT DENSITY SPECIFIED

KT(JT)	KT(JT+1)	KT(JT+2)	KT(JT+3)	XT(JT)	XT(JT+1)	XT(JT+2)	XT(JT+3)	XT(JT+4)	JT
-1	1	5	-5	0.	0.4827586	0.2586207	0.2586207	0.2586207	9
-1	1	5	-5	0.2916667	0.2083333	0.2500000	0.2500000	0.2500000	14
-1	1	5	-5	0.3125000	0.1875000	0.2500000	0.2500000	0.2500000	19
-1	1	5	-5	0.3750000	0.1250000	0.2500000	0.2500000	0.2500000	24
-1	1	5	-5	0.6666667	0.	0.1666667	0.1666667	0.1666667	29
-1	1	5	-5	0.2777778	0.1666667	0.2469136	0.3086420	0.2222222	34
-1	1	5	-5	0.3333333	0.1111111	0.2469136	0.3086420	0.2222222	39
-1	1	5	-5	0.6153846	0.	0.1709402	0.2136752	0.1538462	44
-1	4	4	-5	0.0651042	0.2473958	0.6250000	0.0625000	0.0312500	49
3	1	5	-5	0.5797101	0.1159420	0.1521739	0.1521739	0.1739130	54
-1	1	5	-1	0.4603994	0.1063245	0.1237932	0.3094830	0.0945295	59
-1	1	5	-5	0.4939706	0.0672723	0.0731262	0.3656310	0.0471444	64
-4	1	5	-5	0.5000000	0.5000000	0.	0.	0.	-69
-1	1	5	-5	0.2225901	0.1102924	0.1617254	0.5053920	0.1098512	74

XR= 0.79946046   50 ITERATIONS REQUIRED TO CONVERGE ON XR

LAPLACE SOLUTION

AFTER 12 ITERATIONS ON U THE MAXIMUM CHANGE IN U IS 0.00851 VOLTS AND OCCURS AT MESH POINT 12

60 U VALUES

MESH POINT NUMBERS								POTENTIAL VALUES							
1	2	3	4	5	6	7	8	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000	895.9544	766.8270
9	10	11	12	13	14	15	16	714.3282	700.0223	705.8136	515.6727	365.5303	292.8042	273.5507	277.1696
17	18	19	20	21	22	23	24	38.6665	-168.9056	-228.1868	-229.9403	-252.4775	-543.6250	-918.3443	-837.2217
25	26	27	28	29	30	31	32	-740.4435	-771.8125	-1000.0000	-978.6510	-899.8868	-863.8362	-1000.0000	-984.5808
33	34	35	36	37	38	39	40	-913.5292	-862.4311	-843.0268	-1000.0000	-868.6906	-797.9154	-758.0294	-744.6000
41	42	43	44	45	46	47	48	-692.3663	-658.5857	-623.7481	-600.5139	-592.4560	-447.7889	-438.1004	-423.4583
49	50	51	52	53	54	55	56	-412.1742	-408.0782	-221.2972	-218.5140	-213.3298	-208.9571	-207.3180	0.
57	58	59	60	0	0	0	0	-0.	0.	-0.	0.	0.	0.	0.	0.

EQUIPOTENTIAL PRINTOUT

POTENTIAL (X,Y)	1000.0	(0.250, 0.)	(0. , 0.250)	(0. , 0.500)	(0. , 0.750)	(0. , 1.000)	(0.250, 0.)
POTENTIAL (X,Y)	1000.0	(0.250, 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)
POTENTIAL (X,Y)	800.0	(0.214, 0.500)	(0.175, 0.750)	(0.167, 1.000)	(0.250, 0.436)	(0.420, 0.)	(0.313, 0.250)
POTENTIAL (X,Y)	600.0	(0.445, 0.250)	(0.354, 0.500)	(0.318, 0.750)	(0.309, 1.000)	(0.500, 0.139)	(0.562, 0.)
POTENTIAL (X,Y)	400.0	(0.479, 0.500)	(0.436, 0.750)	(0.426, 1.000)	(0.500, 0.443)	(0.678, 0.)	(0.561, 0.250)
POTENTIAL (X,Y)	200.0	(0.665, 0.250)	(0.577, 0.500)	(0.545, 0.750)	(0.537, 1.000)	(0.750, 0.081)	(0.786, 0.)
POTENTIAL (X,Y)	0.	(0.671, 0.500)	(0.641, 0.750)	(0.636, 1.000)	(0.750, 0.297)	(0.881, 0.)	(0.767, 0.250)
POTENTIAL (X,Y)	0.	(2.750, 0.)	(2.750, 0.250)	(2.750, 0.500)	(2.750, 0.750)	(2.750, 1.000)	(2.750, 0.)
POTENTIAL (X,Y)	0.	(2.750, 0.250)	(2.750, 0.500)	(2.750, 0.750)	(0. , 0.)	(0. , 0.)	(0. , 0.)
POTENTIAL (X,Y)	-200.0	(0.736, 0.750)	(0.735, 1.000)	(0.750, 0.631)	(0.975, 0.)	(0.852, 0.250)	(0.760, 0.500)

POTENTIAL (X,Y)	-200.0	(2.524, 0.)	(2.521, 0.250)	(2.516, 0.500)	(2.511, 0.750)	(2.509, 1.000)	(0. , 0.)
POTENTIAL (X,Y)	-400.0	(0.938, 0.250)	(0.827, 0.500)	(0.821, 0.750)	(0.833, 1.000)	(1.000, 0.127)	(1.071, 0.)
POTENTIAL (X,Y)	-400.0	(2.303, 0.)	(2.293, 0.250)	(2.278, 0.500)	(2.265, 0.750)	(2.260, 1.000)	(0. , 0.)
POTENTIAL (X,Y)	-600.0	(0.894, 0.500)	(0.903, 0.750)	(0.931, 1.000)	(1.000, 0.288)	(1.167, 0.)	(1.031, 0.250)
POTENTIAL (X,Y)	-600.0	(1.988, 1.000)	(2.000, 0.766)	(2.094, 0.)	(2.066, 0.250)	(2.030, 0.500)	(2.001, 0.750)
POTENTIAL (X,Y)	-800.0	(0.961, 0.500)	(0.985, 0.750)	(1.000, 0.421)	(1.000, 0.846)	(1.140, 0.250)	(1.121, 1.000)
POTENTIAL (X,Y)	-800.0	(1.250, 0.031)	(1.281, 0.)	(1.745, 0.500)	(1.649, 0.750)	(1.609, 1.000)	(1.750, 0.493)
POTENTIAL (X,Y)	-800.0	(1.913, 0.)	(1.832, 0.250)	(0. , 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)
POTENTIAL (X,Y)	-1000.0	(1.250, 0.250)	(1.250, 0.250)	(1.250, 0.250)	(1.500, 0.)	(1.250, 0.250)	(1.500, 0.)
POTENTIAL (X,Y)	-1000.0	(1.750, 0.)	(1.750, 0.)	(1.750, 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)

CYCLE 1

CURRENT DENSITIES ARE CALCULATED USING EQUIPOTENTIAL OF 700.02231 VOLTS WHICH HAS X-Y COORDINATES
1 (0.379,0.250) 2 (0.292,0.500) 3 (0.258,0.750) 4 (0.250,1.000)

EMITTER ARC LENGTH , DELTA ARC LENGTH 0.50000 0.12500

X-Y EMITTER COORDINATES

1 (0.050,0.500) 2 (0.050,1.000)

X-Y BEGIN TRAJ. COORDINATES

1 (0.050,0.500) 2 (0.050,0.625) 3 (0.050,0.750) 4 (0.050,0.875)

X-COORD	TRAJ NUM	REFLECTION COUNTER	Y-COORD	X-VEL COMP	Y-VEL COMP
0.2500	1	0	0.5000	0.18400E 05	0.
	2	0	0.6250	0.19408E 05	0.
	3	0	0.7500	0.20366E 05	0.
	4	0	0.8750	0.20620E 05	0.
0.5000	1	0	0.5151	0.30366E 05	0.29413E 04
	2	0	0.6337	0.31218E 05	0.17690E 04
	3	0	0.7552	0.32045E 05	0.10844E 04
	4	0	0.8772	0.32262E 05	0.45616E 03
0.7500	1	0	0.5432	0.41180E 05	0.51183E 04
	2	0	0.6501	0.41707E 05	0.30070E 04
	3	0	0.7648	0.42218E 05	0.17881E 04
	4	0	0.8809	0.42239E 05	0.65682E 03
1.0000	1	0	0.6250	0.52109E 05	0.42150E 04
	2	0	0.6655	0.51951E 05	0.27732E 04
	3	0	0.7731	0.51534E 05	0.12980E 04
	4	0	0.8825	0.50922E 05	-0.38773E 02
1.2500	1	0	0.6425	0.52960E 05	0.31308E 04
	2	0	0.6762	0.52809E 05	0.17083E 04
	3	0	0.7769	0.52494E 05	0.29859E 03

	4	0	0.8802	0.52253E 05	-0.92167E 03
1.5000	1	0	0.6553	0.52289E 05	0.22822E 04
	2	0	0.6825	0.52183E 05	0.92128E 03
	3	0	0.7769	0.51998E 05	-0.28952E 03
	4	0	0.8749	0.51849E 05	-0.12980E 04
1.7500	1	0	0.6650	0.50761E 05	0.16981E 04
	2	0	0.6856	0.50667E 05	0.37581E 03
	3	0	0.7746	0.50528E 05	-0.68531E 03
	4	0	0.8680	0.50408E 05	-0.15365E 04
2.0000	1	0	0.6725	0.48375E 05	0.12880E 04
	2	0	0.6866	0.48294E 05	-0.12746E 02
	3	0	0.7704	0.48219E 05	-0.96813E 03
	4	0	0.8597	0.48123E 05	-0.17071E 04
2.2500	1	0	0.6788	0.45404E 05	0.10551E 04
	2	0	0.6859	0.45326E 05	-0.23786E 03
	3	0	0.7647	0.45299E 05	-0.11364E 04
	4	0	0.8503	0.45219E 05	-0.18144E 04
2.5000	1	0	0.6845	0.41996E 05	0.94331E 03
	2	0	0.6842	0.41913E 05	-0.34820E 03
	3	0	0.7580	0.41918E 05	-0.12212E 04
	4	0	0.8397	0.41846E 05	-0.18713E 04
2.7500	1	0	0.6903	0.38193E 05	0.90961E 03
	2	0	0.6819	0.38100E 05	-0.38200E 03
	3	0	0.7503	0.38129E 05	-0.12476E 04
	4	0	0.8280	0.38059E 05	-0.18897E 04

THRUST DISTRIBUTION BY STREAM TUBES (NEWTONS)

1 0. 2 0.792058E-06 3 0.128856E-05 4 0.772716E-06 5 0.257337E-06

TOTAL THRUST (NEWTONS) 0.311067E-05

TOTAL POWER (WATTS) 0.592785E-01

INITIAL CURRENT DENSITIES (AMPS/(UNIT H)**2)

1 0. 2 1.000000E-04 3 1.000000E-04 4 1.000000E-04 5 1.000000E-04

INITIAL CURRENTS (AMPS)

1 0. 2 0.343612E-04 3 0.245437E-04 4 0.147262E-04 5 0.490874E-05

TOTAL INITIAL CURRENT=0.785398E-04 AMPS

TRANSMITTED CURRENT AT ACCEL. GRID= 0.592525E-04 AMPS WHICH IS 75.44 PERCENT OF THE INITIAL CURRENT.

START OF POISSON SOLUTION

RHDOWN= 0. RH(9)= 8.7754C11 RHUP= 0. U(9)= 714.3281937

60 RH VALUES

MESH POINT NUMBERS RH VALUES

1 2 3 4 5 6 7 8 0. 0. 0. 0. 0. 0. 0. 4.0842

9	10	11	12	13	14	15	16	8.7754	8.5585	0.	0.	2.3490	5.8381	5.6656	0.
17	18	19	20	21	22	23	24	0.	1.4718	4.9237	4.6164	0.	0.	0.	4.6622
25	26	27	28	29	30	31	32	3.9701	0.	0.	0.	4.5936	3.7429	0.	0.
33	34	35	36	37	38	39	40	0.	4.6895	3.3977	0.	0.	0.	4.8723	3.1376
41	42	43	44	45	46	47	48	0.	0.	0.	5.1553	2.9123	0.	0.	0.
49	50	51	52	53	54	55	56	5.5372	2.7211	0.	0.	0.	6.0326	2.5649	0.
57	58	59	60	0	0	0	0	0.	0.	6.6805	2.4476	0.	0.	0.	0.

```
CYCLE   2

RHDOWN=  0.        RH(    9)=  9.0973064 RHUP=  0.        U(    9)=     734.9583206

CYCLE   3

RHDOWN=  0.        RH(    9)=  9.1062878 RHUP=  0.        U(    9)=     735.5056915

CYCLE   4

RHDOWN=  0.        RH(    9)=  9.1065317 RHUP=  0.        U(    9)=     735.5205154

CYCLE   5

RHDOWN=  0.        RH(    9)=  9.1065382 RHUP=  0.        U(    9)=     735.5209579

CYCLE   6

RH=AVERAGE

RHDOWN=  0.        RH(    9)=  9.1065348 RHUP=  0.        U(    9)=     735.5205307

CYCLE   7

THRUST DISTRIBUTION BY STREAM TUBES (NEWTONS)

 1  0.            2  0.687437E-06   3  0.128652E-05   4  0.771840E-06   5  0.257158E-06

TOTAL THRUST (NEWTONS)   0.300296E-05

TOTAL POWER (WATTS)   0.571508E-01

INITIAL CURRENT DENSITIES (AMPS/(UNIT H)**2)

 1  0.            2  1.000000E-04   3  1.000000E-04   4  1.000000E-04   5  1.000000E-04

INITIAL CURRENTS (AMPS)

 1  0.            2  0.343612E-04   3  0.245437E-04   4  0.147262E-04   5  0.490874E-05

TOTAL INITIAL CURRENT=0.785398E-04 AMPS

TRANSMITTED CURRENT AT ACCEL. GRID= 0.572759E-04 AMPS WHICH IS  72.93 PERCENT OF THE INITIAL CURRENT.

RHDOWN=  0.        RH(    9)=  9.1065351 RHUP=  0.        U(    9)=     735.5207520

RHOUTPUT IS AVERAGE OF THIS AND PREVIOUS CYCLE

CONVERGED POISSON SOLUTION

   60 RH VALUES
```

MESH POINT NUMBERS								RH VALUES							
1	2	3	4	5	6	7	8	0.	0.	0.	0.	0.	0.	0.	4.2070
9	10	11	12	13	14	15	16	9.1065	8.9058	0.	0.	2.4018	5.9097	5.7551	0.
17	18	19	20	21	22	23	24	0.	1.5300	4.8753	4.5924	0.	0.	0.	4.5164
25	26	27	28	29	30	31	32	3.8570	0.	0.	0.	4.4666	3.5396	0.	0.
33	34	35	36	37	38	39	40	0.	4.5697	3.1736	0.	0.	0.	4.7608	2.8615
41	42	43	44	45	46	47	48	0.	0.	0.	5.0488	2.5902	0.	0.	0.
49	50	51	52	53	54	55	56	5.4304	2.3564	0.	0.	0.	5.9130	2.1576	0.
57	58	59	60	0	0	0	0	0.	0.	6.5134	1.9906	0.	0.	0.	0.

U-FIELD IS AVERAGE OF THIS AND PREVIOUS CYCLE

AFTER 1 ITERATIONS ON U THE MAXIMUM CHANGE IN U IS 0.00007 VOLTS AND OCCURS AT MESH POINT 10

60 U VALUES

MESH POINT NUMBERS								POTENTIAL VALUES							
1	2	3	4	5	6	7	8	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000	898.4876	778.7139
9	10	11	12	13	14	15	16	735.5207	723.6093	710.8577	523.2277	381.3708	319.1874	303.1791	285.2541
17	18	19	20	21	22	23	24	47.2805	-156.2632	-206.0557	-204.3045	-244.7012	-536.6829	-917.1026	-826.3643
25	26	27	28	29	30	31	32	-723.1529	-768.3414	-1000.0000	-976.1431	-884.4876	-844.5840	-1000.0000	-983.8279
33	34	35	36	37	38	39	40	-906.4961	-842.9459	-820.5594	-1000.0000	-864.8654	-787.8471	-735.5510	-719.6834
41	42	43	44	45	46	47	48	-688.1255	-652.4277	-612.1475	-576.6889	-566.7832	-442.8870	-431.9096	-412.6915
49	50	51	52	53	54	55	56	-389.7871	-384.6211	-218.1396	-214.5997	-206.3915	-192.8332	-191.2206	0.
57	58	59	60	0	0	0	0	-0.	0.	-0.	0.	0.	0.	0.	0.

EQUIPOTENTIAL PRINTOUT

POTENTIAL (X,Y)	1000.0	(0.250, 0.)	(0. , 0.250)	(0. , 0.500)	(0. , 0.750)	(0. , 1.000)	(0.250, 0.)
POTENTIAL (X,Y)	1000.0	(0.250, 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)
POTENTIAL (X,Y)	800.0	(0.226, 0.500)	(0.189, 0.750)	(0.181, 1.000)	(0.250, 0.456)	(0.423, 0.)	(0.316, 0.250)
POTENTIAL (X,Y)	600.0	(0.449, 0.250)	(0.362, 0.500)	(0.331, 0.750)	(0.324, 1.000)	(0.500, 0.148)	(0.565, 0.)
POTENTIAL (X,Y)	400.0	(0.488, 0.500)	(0.451, 0.750)	(0.442, 1.000)	(0.500, 0.467)	(0.683, 0.)	(0.565, 0.250)
POTENTIAL (X,Y)	200.0	(0.670, 0.250)	(0.584, 0.500)	(0.557, 0.750)	(0.551, 1.000)	(0.750, 0.090)	(0.790, 0.)
POTENTIAL (X,Y)	0.	(0.677, 0.500)	(0.652, 0.750)	(0.649, 1.000)	(0.750, 0.308)	(0.885, 0.)	(0.770, 0.250)
POTENTIAL (X,Y)	0.	(2.750, 0.)	(2.750, 0.250)	(2.750, 0.500)	(2.750, 0.750)	(2.750, 1.000)	(2.750, 0.)
POTENTIAL (X,Y)	0.	(2.750, 0.250)	(2.750, 0.500)	(2.750, 0.750)	(0. , 0.)	(0. , 0.)	(0. , 0.)
POTENTIAL (X,Y)	-200.0	(0.747, 0.750)	(0.748, 1.000)	(0.750, 0.720)	(0.979, 0.)	(0.856, 0.250)	(0.764, 0.500)
POTENTIAL (X,Y)	-200.0	(2.491, 0.750)	(2.489, 1.000)	(2.500, 0.618)	(2.521, 0.)	(2.517, 0.250)	(2.508, 0.500)
POTENTIAL (X,Y)	-400.0	(0.941, 0.250)	(0.830, 0.500)	(0.828, 0.750)	(0.844, 1.000)	(1.000, 0.133)	(1.074, 0.)
POTENTIAL (X,Y)	-400.0	(2.236, 0.750)	(2.229, 1.000)	(2.250, 0.639)	(2.298, 0.)	(2.287, 0.250)	(2.265, 0.500)
POTENTIAL (X,Y)	-600.0	(0.896, 0.500)	(0.909, 0.750)	(0.941, 1.000)	(1.000, 0.292)	(1.170, 0.)	(1.034, 0.250)
POTENTIAL (X,Y)	-600.0	(1.963, 0.750)	(1.946, 1.000)	(2.000, 0.586)	(2.090, 0.)	(2.059, 0.250)	(2.015, 0.500)
POTENTIAL (X,Y)	-800.0	(0.962, 0.500)	(0.989, 0.750)	(1.000, 0.423)	(1.000, 0.814)	(1.142, 0.250)	(1.158, 1.000)

POTENTIAL (X,Y) -800.0 (1.250, 0.034) (1.284, 0.) (1.724, 0.500) (1.600, 0.750) (1.551, 1.000) (1.750, 0.461)

POTENTIAL (X,Y) -800.0 (1.910, 0.) (1.826, 0.250) (0. , 0.) (0. , 0.) (0. , 0.) (0. , 0.)

POTENTIAL (X,Y) -1000.0 (1.250, 0.250) (1.250, 0.250) (1.250, 0.250) (1.500, 0.) (1.250, 0.250) (1.500, 0.)

POTENTIAL (X,Y) -1000.0 (1.750, 0.) (1.750, 0.) (1.750, 0.) (0. , 0.) (0. , 0.) (0. , 0.)

CYCLE 8

CURRENT DENSITIES ARE CALCULATED USING EQUIPOTENTIAL OF 723.60932 VOLTS WHICH HAS X-Y COORDINATES
1 (0.489,0.) 2 (0.367,0.250) 3 (0.285,0.500) 4 (0.257,0.750) 5 (0.250,1.000)

EMITTER ARC LENGTH , DELTA ARC LENGTH 0.50000 0.12500

X-Y EMITTER COORDINATES

1 (0.050,0.500) 2 (0.050,1.000)

X-Y BEGIN TRAJ. COORDINATES

1 (0.050,0.500) 2 (0.050,0.625) 3 (0.050,0.750) 4 (0.050,0.875)

X-COORD	TRAJ NUM	REFLECTION COUNTER	Y-COORD	X-VEL COMP	Y-VEL COMP
0.2500	1	0	0.5000	0.17925E 05	0.
	2	0	0.6250	0.18779E 05	0.
	3	0	0.7500	0.19596E 05	0.
	4	0	0.8750	0.19816E 05	0.
0.5000	1	0	0.5142	0.29983E 05	0.27124E 04
	2	0	0.6327	0.30721E 05	0.15246E 04
	3	0	0.7546	0.31441E 05	0.93598E 03
	4	0	0.8769	0.31625E 05	0.39060E 03
0.7500	1	0	0.5405	0.40959E 05	0.47518E 04
	2	0	0.6470	0.41405E 05	0.25918E 04
	3	0	0.7630	0.41837E 05	0.15264E 04
	4	0	0.8800	0.41827E 05	0.52980E 03
1.0000	1	0	0.6250	0.52064E 05	0.35371E 04
	2	0	0.6599	0.51902E 05	0.22244E 04
	3	0	0.7696	0.51394E 05	0.92984E 03
	4	0	0.8808	0.50728E 05	-0.25556E 03
1.2500	1	0	0.6389	0.52874E 05	0.22951E 04
	2	0	0.6676	0.52713E 05	0.10078E 04
	3	0	0.7714	0.52281E 05	-0.18571E 03
	4	0	0.8772	0.52011E 05	-0.12283E 04
1.5000	1	0	0.6474	0.52133E 05	0.12675E 04
	2	0	0.6701	0.52011E 05	0.44516E 02
	3	0	0.7688	0.51720E 05	-0.89237E 03
	4	0	0.8702	0.51550E 05	-0.16657E 04
1.7500	1	0	0.6517	0.50551E 05	0.50196E 03
	2	0	0.6686	0.50437E 05	-0.67879E 03
	3	0	0.7631	0.50196E 05	-0.14056E 04
	4	0	0.8613	0.50059E 05	-0.19590E 04

2.0000	1	0	0.6527	0.48116E 05	-0.94276E 02
	2	0	0.6637	0.48011E 05	-0.12500E 04
	3	0	0.7549	0.47847E 05	-0.18080E 04
	4	0	C.8507	0.47738E 05	-0.21836E 04
2.2500	1	0	0.6511	0.45129E 05	-0.51220E 03
	2	0	0.6559	0.45023E 05	-0.16575E 04
	3	0	0.7444	0.44920E 05	-0.20944E 04
	4	0	0.8384	0.44834E 05	-0.23417E 04
2.500C	1	C	0.6473	0.41775E 05	-0.79243E 03
	2	0	0.6455	0.41662E 05	-0.19363E 04
	3	0	0.7318	0.41608E 05	-0.22865E 04
	4	0	0.8246	0.41543E 05	-0.24407E 04
2.7500	1	0	0.6420	0.38169E 05	-0.90586E 03
	2	0	0.6330	0.38040E 05	-0.20509E 04
	3	0	0.7172	0.38068E 05	-0.23635E 04
	4	0	C.8092	0.38033E 05	-0.24774E 04

THRUST DISTRIBUTION BY STREAM TUBES (NEWTONS)

1 0. 2 C.687437E-06 3 0.128652E-05 4 0.771840E-06 5 0.257158E-06

TOTAL THRUST (NEWTONS) 0.300296E-05

TOTAL POWER (WATTS) 0.5715C8E-01

INITIAL CURRENT DENSITIES (AMPS/(UNIT H)**2)

1 0. 2 1.0000C0E-04 3 1.000000E-04 4 1.000000E-04 5 1.000000E-04

INITIAL CURRENTS (AMPS)

1 0. 2 0.343612E-C4 3 0.245437E-04 4 0.147262E-04 5 0.490874E-05

TOTAL INITIAL CURRENT=0.785398E-04 AMPS

TRANSMITTED CURRENT AT ACCEL. GRID= C.572759E-04 AMPS WHICH IS 72.93 PERCENT OF THE INITIAL CURRENT.

01 UNIT05, EOF. FIL000 ZEC000

Input Data Cards

```
                                          CARD COLUMN NUMBERS
12345678901234567890123456789012345678901234567890123456789012345678901234567890


    0   63    4    6   60    4    2    9   10    2    0
    5    0
1        SAMPLE PROBLEM TO DEMONSTRATE LEWIS RESEARCH
      CENTER ION THRUSTOR PROGRAM
         TWO-DIMENSIONAL
         SPACE CHARGE LIMITED

    3    8    9   10

 132.91    1000.     .25
 1000.    -1000.     200.
   -1    1    5   -5 0.        .25       .25       .25                  9
   -1    1    5   -5 .25       .25       .25       .25                 14
   -1    1    5   -5 .25       .25       0.        .25                 19
   -1    1    5   -5 .25       .25       .25       .20                 24
   -1    1    5   -5 .25       .25       0.        .20                 29
   -1    4    4   -5 .25       .025      .050      .25                 34
    3    1    5   -5 .125      .25       .25       .25                 39
   -1    1    5   -1 .0834     .25       .25       .10                 44
   -1    1    5   -5 .042      .25       .25       .05                 49
   -4    1    0    0 -.50      .50       0.        0.        0.       -54
   -1    1    5   -5 .165      .25       .25       .08                 59
 .05       .05
 .50       1.0
 .625      .40       .23
    9
    1    5    0
    1    1    0    1   59    2   24    1   29
    2    1  -54    3   14    1   19
    1    2  -54    1   34    1   39    1   19
    1    1  -54    1    1    1   44    1   14    1   19
    1    1    1    1   49    2   14    1   19
    1    1    1    3   14    1   19
    3    1    9    3   14    1   19
    1    5    2

 1000.    -1000.     0.
```

Output Data Listing

SAMPLE PROBLEM TO DEMONSTRATE LEWIS RESEARCH
CENTER ION THRUSTOR PROGRAM
TWO-DIMENSIONAL
SPACE CHARGE LIMITED

KT(JT)	KT(JT+1)	KT(JT+2)	KT(JT+3)	XT(JT)	XT(JT+1)	XT(JT+2)	XT(JT+3)	XT(JT+4)	JT
-1	1	5	-5	0.	0.5000000	0.2500000	0.2500000	0.2500000	9
-1	1	5	-5	0.2500000	0.2500000	0.2500000	0.2500000	0.2500000	14
-1	1	5	-5	0.5000000	0.	0.2500000	0.2500000	0.2500000	19
-1	1	5	-5	0.2222222	0.2222222	0.2469136	0.3086420	0.2222222	24
-1	1	5	-5	0.4444444	0.	0.2469136	0.3086420	0.2222222	29
-1	4	4	-5	0.0555556	0.2777778	0.6060606	0.0606061	0.0333333	34
3	1	5	-5	0.4444444	0.2222222	0.1666667	0.1666667	0.1666667	39
-1	1	5	-1	0.4088604	0.1363958	0.1299268	0.3248169	0.0909487	44
-1	1	5	-5	0.4653067	0.0781715	0.0760870	0.3804348	0.0456522	49
-4	1	0	0	0.5000000	0.5000000	0.	0.	0.	-54
-1	1	5	-5	0.1967052	0.1298254	0.1632653	0.5102041	0.1077551	59

XR= 0.84812816   60 ITERATIONS REQUIRED TO CONVERGE ON XR

LAPLACE SOLUTION

AFTER 13 ITERATIONS ON U THE MAXIMUM CHANGE IN U IS 0.00658 VOLTS AND OCCURS AT MESH POINT 45

60 U VALUES

MESH POINT NUMBERS								POTENTIAL VALUES							
1	2	3	4	5	6	7	8	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000	874.8293	727.8214
9	10	11	12	13	14	15	16	679.3458	668.8804	662.2943	449.7594	298.9202	244.3187	236.1440	206.3770
17	18	19	20	21	22	23	24	-37.0053	-226.2191	-237.1353	-212.9401	-307.4717	-577.9377	-929.6562	-753.7015
25	26	27	28	29	30	31	32	-613.6343	-788.9688	-1000.0000	-953.7076	-800.2259	-734.1939	-1000.0000	-975.4640
33	34	35	36	37	38	39	40	-853.4273	-759.3019	-722.6900	-1000.0000	-828.1147	-725.2353	-660.8605	-637.9573
41	42	43	44	45	46	47	48	-659.8769	-611.7608	-558.5390	-520.9495	-507.4205	-415.9846	-400.5105	-376.2075
49	50	51	52	53	54	55	56	-356.9758	-349.8219	-203.0415	-198.0900	-188.8058	-180.9254	-177.9181	0.
57	58	59	60	0	0	0	0	-0.	0.	-0.	0.	0.	0.	0.	0.

EQUIPOTENTIAL PRINTOUT

POTENTIAL (X,Y)	1000.0	(0.250, 0.)	(0. , 0.250)	(0. , 0.500)	(0. , 0.750)	(0. , 1.000)	(0.250, 0.)
POTENTIAL (X,Y)	1000.0	(0.250, 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)
POTENTIAL (X,Y)	800.0	(0.184, 0.500)	(0.156, 0.750)	(0.151, 1.000)	(0.250, 0.377)	(0.398, 0.)	(0.294, 0.250)
POTENTIAL (X,Y)	600.0	(0.412, 0.250)	(0.325, 0.500)	(0.296, 0.750)	(0.290, 1.000)	(0.500, 0.073)	(0.534, 0.)
POTENTIAL (X,Y)	400.0	(0.441, 0.500)	(0.411, 0.750)	(0.405, 1.000)	(0.500, 0.332)	(0.644, 0.)	(0.526, 0.250)
POTENTIAL (X,Y)	200.0	(0.628, 0.250)	(0.547, 0.500)	(0.523, 0.750)	(0.520, 1.000)	(0.750, 0.007)	(0.753, 0.)
POTENTIAL (X,Y)	0.	(0.731, 0.250)	(0.642, 0.500)	(0.627, 0.750)	(0.631, 1.000)	(0.750, 0.212)	(0.850, 0.)
POTENTIAL (X,Y)	0.	(2.750, 0.)	(2.750, 0.250)	(2.750, 0.500)	(2.750, 0.750)	(2.750, 1.000)	(2.750, 0.)
POTENTIAL (X,Y)	0.	(2.750, 0.250)	(2.750, 0.500)	(2.750, 0.750)	(0. , 0.)	(0. , 0.)	(0. , 0.)
POTENTIAL (X,Y)	-200.0	(0.738, 0.500)	(0.731, 0.750)	(0.743, 1.000)	(0.750, 0.465)	(0.948, 0.)	(0.825, 0.250)
POTENTIAL (X,Y)	-200.0	(2.498, 0.250)	(2.485, 0.500)	(2.473, 0.750)	(2.468, 1.000)	(2.500, 0.154)	(2.504, 0.)

POTENTIAL (X,Y)	-400.0	(0.918, 0.250)	(0.812, 0.500)	(0.829, 0.750)	(0.867, 1.000)	(1.000, 0.086)	(1.048, 0.)
POTENTIAL (X,Y)	-400.0	(2.217, 0.500)	(2.184, 0.750)	(2.170, 1.000)	(2.250, 0.255)	(2.269, 0.)	(2.251, 0.250)
POTENTIAL (X,Y)	-600.0	(0.883, 0.500)	(0.926, 0.750)	(0.991, 1.000)	(1.000, 0.266)	(1.152, 0.)	(1.013, 0.250)
POTENTIAL (X,Y)	-600.0	(1.938, 0.500)	(1.859, 0.750)	(1.823, 1.000)	(2.000, 0.305)	(2.061, 0.)	(2.014, 0.250)
POTENTIAL (X,Y)	-800.0	(0.954, 0.500)	(1.000, 0.408)	(1.000, 0.684)	(1.132, 0.250)	(1.249, 0.750)	(1.250, 0.013)
POTENTIAL (X,Y)	-800.0	(1.250, 0.751)	(1.263, 0.)	(1.251, 0.750)	(1.500, 0.642)	(1.604, 0.500)	(1.750, 0.318)
POTENTIAL (X,Y)	-800.0	(1.897, 0.)	(1.782, 0.250)	(0. , 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)
POTENTIAL (X,Y)	-1000.0	(1.250, 0.250)	(1.250, 0.250)	(1.250, 0.250)	(1.500, 0.)	(1.250, 0.250)	(1.500, 0.)
POTENTIAL (X,Y)	-1000.0	(1.750, 0.)	(1.750, 0.)	(1.750, 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)

CYCLE 1

CURRENT DENSITIES ARE CALCULATED USING EQUIPOTENTIAL OF 668.88045 VOLTS WHICH HAS X-Y COORDINATES
1 (0.495,0.) 2 (0.371,0.250) 3 (0.284,0.500) 4 (0.256,0.750) 5 (0.250,1.000)

EMITTER ARC LENGTH , DELTA ARC LENGTH 0.50000 0.12500

X-Y EMITTER COORDINATES

1 (0.050,0.500) 2 (0.050,1.000)

X-Y BEGIN TRAJ. COORDINATES

1 (0.050,0.500) 2 (0.050,0.625) 3 (0.050,0.750) 4 (0.050,0.875)

X-COORD	TRAJ NUM	REFLECTION COUNTER	Y-COORD	X-VEL COMP	Y-VEL COMP
0.2500	1	0	0.5000	0.19879E 05	0.
	2	0	0.6250	0.20746E 05	0.
	3	0	0.7500	0.21577E 05	0.
	4	0	0.8750	0.21753E 05	0.
0.5000	1	0	0.5132	0.31909E 05	0.27388E 04
	2	0	0.6315	0.32522E 05	0.13883E 04
	3	0	0.7537	0.33124E 05	0.79870E 03
	4	0	0.8761	0.33214E 05	0.24523E 03
0.7500	1	0	0.5374	0.42122E 05	0.44160E 04
	2	0	0.6428	0.42254E 05	0.19884E 04
	3	0	0.7597	0.42367E 05	0.10171E 04
	4	0	0.8772	0.42171E 05	0.94421E 02
1.0000	1	0	0.6250	0.51713E 05	0.18312E 04
	2	0	0.6500	0.51457E 05	0.71830E 03
	3	0	0.7617	0.50377E 05	-0.26921E 03
	4	0	0.8742	0.49435E 05	-0.12048E 04
1.2500	1	0	0.6283	0.52208E 05	-0.46216E 03
	2	0	0.6481	0.51983E 05	-0.15119E 04
	3	0	0.7557	0.51080E 05	-0.21590E 04
	4	0	0.8644	0.50623E 05	-0.27280E 04

1.5000	1	0	0.6219	0.51217E 05	-0.21946E 04
	2	0	0.6367	0.51046E 05	-0.31806E 04
	3	0	0.7420	0.50486E 05	-0.34097E 04
	4	0	0.8489	0.50214E 05	-0.35370E 04
1.7500	1	0	0.6081	0.49568E 05	-0.33575E 04
	2	0	0.6181	0.49412E 05	-0.43143E 04
	3	0	0.7227	0.49019E 05	-0.42514E 04
	4	0	0.8296	0.48843E 05	-0.40724E 04
2.0000	1	0	0.5887	0.47225E 05	-0.41589E 04
	2	0	0.5937	0.47075E 05	-0.51043E 04
	3	0	0.6990	0.46836E 05	-0.48469E 04
	4	0	0.8073	0.46757E 05	-0.44556E 04
2.2500	1	0	0.5647	0.44426E 05	-0.46366E 04
	2	0	0.5644	0.44269E 05	-0.55812E 04
	3	0	0.6713	0.44143E 05	-0.52257E 04
	4	0	0.7821	0.44156E 05	-0.47086E 04
2.5000	1	0	0.5369	0.41291E 05	-0.48808E 04
	2	0	0.5310	0.41119E 05	-0.58266E 04
	3	0	0.6401	0.41070E 05	-0.54351E 04
	4	0	0.7541	0.41166E 05	-0.48533E 04
2.7500	1	0	0.5059	0.37839E 05	-0.49576E 04
	2	0	0.4938	0.37645E 05	-0.59041E 04
	3	0	0.6053	0.37660E 05	-0.55050E 04
	4	0	0.7233	0.37833E 05	-0.49026E 04

THRUST DISTRIBUTION BY STREAM TUBES (NEWTONS/UNIT H)

1 0. 2 0.101495E-05 3 0.407873E-05 4 0.443956E-05 5 0.458354E-05

TOTAL THRUST (NEWTONS/UNIT H) 0.141168E-04
TOTAL POWER (WATT/UNIT H) 0.266433E-00

INITIAL CURRENT DENSITIES (AMPS/(UNIT H)**2)

1 0. 2 0.553128E-03 3 0.629138E-03 4 0.683093E-03 5 0.703633E-03

INITIAL CURRENTS (AMPS/UNIT H)

1 0. 2 0.691410E-04 3 0.786423E-04 4 0.853866E-04 5 0.879541E-04

TOTAL INITIAL CURRENT=0.321124E-03 AMPS/(UNIT H)

AVERAGE INITIAL CURRENT DENSITY=0.642248E-03 AMPS/(UNIT H)**2

TRANSMITTED CURRENT AT ACCEL. GRID=0.271506E-03 AMPS/(UNIT H) WHICH IS 84.55 PERCENT OF THE INITIAL CURRENT.

START OF POISSON SOLUTION

RHDOWN= 0. RH(9)= 54.0542150 RHUP= 54.0542150 U(9)= 679.3458099

60 RH VALUES

MESH POINT NUMBERS								RH VALUES							
1	2	3	4	5	6	7	8	0.	0.	0.	0.	0.	0.	0.	24.0276
9	10	11	12	13	14	15	16	54.0542	57.0842	0.	0.	14.3230	35.7465	37.7167	0.

17	18	19	20	21	22	23	24	0.	9.6175	29.0589	29.9902	0.	0.	0.	25.7305
25	26	27	28	29	30	31	32	24.9610	0.	0.	0.	26.2311	22.6091	0.	0.
33	34	35	36	37	38	39	40	0.5670	27.1733	20.4508	0.	0.	3.5287	26.2379	18.6547
41	42	43	44	45	46	47	48	0.	0.	6.4331	25.8554	17.2311	0.	0.	10.2188
49	50	51	52	53	54	55	56	25.0918	16.1353	0.	0.	14.9869	23.9396	15.3354	0.
57	58	59	60	0	0	0	0	0.	21.0651	22.2965	14.8248	0.	0.	0.	0.

CYCLE 2

RHDOWN= 27.5502851 RH(9)= 27.5502851 RHUP= 54.0542150 U(9)= 853.3342285

CYCLE 3

RHDOWN= 27.5502851 RH(9)= 41.7501826 RHUP= 41.7501826 U(9)= 758.0848389

CYCLE 4

RHDOWN= 33.7710090 RH(9)= 33.7710090 RHUP= 41.7501826 U(9)= 808.6294403

CYCLE 5

RHDOWN= 33.7710090 RH(9)= 38.0834851 RHUP= 38.0834851 U(9)= 779.9215012

CYCLE 6

RH=AVERAGE

RHDOWN= 35.5938687 RH(9)= 36.8386769 RHUP= 38.0834851 U(9)= 795.5128098

CYCLE 7

THRUST DISTRIBUTION BY STREAM TUBES (NEWTONS/UNIT H)

1 0. 2 0. 3 0.207486E-05 4 0.225072E-05 5 0.228166E-05

TOTAL THRUST (NEWTONS/UNIT H) 0.660724E-05
TOTAL POWER (WATT/UNIT H) 0.120300E-00

INITIAL CURRENT DENSITIES (AMPS/(UNIT H)**2)

1 0. 2 0.348945E-03 3 0.355756E-03 4 0.358511E-03 5 0.357064E-03

INITIAL CURRENTS (AMPS/UNIT H)

1 0. 2 0.436181E-04 3 0.444695E-04 4 0.448138E-04 5 0.446330E-04

TOTAL INITIAL CURRENT=0.177534E-03 AMPS/(UNIT H)

AVERAGE INITIAL CURRENT DENSITY=0.355069E-03 AMPS/(UNIT H)**2

TRANSMITTED CURRENT AT ACCEL. GRID=0.131763E-03 AMPS/(UNIT H) WHICH IS 74.22 PERCENT OF THE INITIAL CURRENT.

RHDOWN= 35.5938687 RH(9)= 36.2508059 RHUP= 38.0834851 U(9)= 791.0378571

RHOUTPUT IS AVERAGE OF THIS AND PREVIOUS CYCLE

CONVERGED POISSON SOLUTION

60 RH VALUES

MESH POINT NUMBERS								RH VALUES							
1	2	3	4	5	6	7	8	0.	0.	0.	0.	0.	0.	0.	17.9619
9	10	11	12	13	14	15	16	36.5447	36.5534	0.	0.	10.3752	21.5913	21.4151	0.
17	18	19	20	21	22	23	24	0.	7.5518	16.2042	15.6423	0.	0.	0.	13.2213
25	26	27	28	29	30	31	32	12.0545	0.	0.	0.9608	12.8432	10.4053	0.	0.
33	34	35	36	37	38	39	40	2.7482	12.1621	8.9666	0.	0.	5.3169	10.9179	7.8054
41	42	43	44	45	46	47	48	0.	0.	8.4435	9.3437	6.8942	0.	0.	11.8661
49	50	51	52	53	54	55	56	7.6627	6.1796	0.	3.4263	12.0433	5.9281	5.6113	0.
57	58	59	60	0	0	0	0	8.3867	9.7986	5.1438	5.1438	0.	0.	0.	0.

U-FIELD IS AVERAGE OF THIS AND PREVIOUS CYCLE

AFTER 8 ITERATIONS ON U THE MAXIMUM CHANGE IN U IS 0.00473 VOLTS AND OCCURS AT MESH POINT 44

60 U VALUES

MESH POINT NUMBERS								POTENTIAL VALUES							
1	2	3	4	5	6	7	8	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000	892.0716	796.0714
9	10	11	12	13	14	15	16	790.0117	790.5113	696.5851	501.0987	395.5515	390.0618	397.9575	260.6426
17	18	19	20	21	22	23	24	20.1866	-146.5260	-109.6380	-64.4640	-257.1412	-534.4687	-922.4115	-682.4409
25	26	27	28	29	30	31	32	-499.1073	-767.2344	-1000.0000	-932.6057	-705.7480	-615.2989	-1000.0000	-967.7610
33	34	35	36	37	38	39	40	-792.8840	-644.0203	-592.2148	-1000.0000	-789.0769	-638.1402	-533.8799	-501.3821
41	42	43	44	45	46	47	48	-616.5369	-550.4070	-457.9888	-395.6505	-376.7771	-365.3327	-338.0239	-281.5294
49	50	51	52	53	54	55	56	-251.3279	-241.9987	-168.7470	-154.8273	-126.2424	-116.7851	-113.2813	0.
57	58	59	60	0	0	0	0	-0.	0.	-0.	0.	0.	0.	0.	0.

EQUIPOTENTIAL PRINTOUT

POTENTIAL (X,Y)	1000.0	(0.250, 0.)	(0. , 0.250)	(0. , 0.500)	(0. , 0.750)	(0. , 1.000)	(0.250, 0.)
POTENTIAL (X,Y)	1000.0	(0.250, 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)
POTENTIAL (X,Y)	800.0	(0.245, 0.500)	(0.238, 0.750)	(0.239, 1.000)	(0.250, 0.490)	(0.415, 0.)	(0.309, 0.250)
POTENTIAL (X,Y)	600.0	(0.437, 0.250)	(0.372, 0.500)	(0.369, 0.750)	(0.371, 1.000)	(0.500, 0.124)	(0.555, 0.)
POTENTIAL (X,Y)	400.0	(0.497, 0.500)	(0.494, 0.750)	(0.499, 1.000)	(0.500, 0.489)	(0.670, 0.)	(0.553, 0.250)
POTENTIAL (X,Y)	200.0	(0.657, 0.250)	(0.590, 0.500)	(0.595, 0.750)	(0.607, 1.000)	(0.750, 0.063)	(0.779, 0.)
POTENTIAL (X,Y)	0.	(0.682, 0.500)	(0.695, 0.750)	(0.715, 1.000)	(0.750, 0.280)	(0.876, 0.)	(0.759, 0.250)
POTENTIAL (X,Y)	0.	(2.750, 0.)	(2.750, 0.250)	(2.750, 0.500)	(2.750, 0.750)	(2.750, 1.000)	(2.750, 0.)
POTENTIAL (X,Y)	0.	(2.750, 0.250)	(2.750, 0.500)	(2.750, 0.750)	(0. , 0.)	(0. , 0.)	(0. , 0.)
POTENTIAL (X,Y)	-200.0	(0.972, 0.)	(0.849, 0.250)	(0.767, 0.500)	(0.789, 0.750)	(0.828, 1.000)	(2.460, 0.)
POTENTIAL (X,Y)	-200.0	(2.438, 0.250)	(2.381, 0.500)	(2.345, 0.750)	(2.332, 1.000)	(0. , 0.)	(0. , 0.)
POTENTIAL (X,Y)	-400.0	(0.939, 0.250)	(0.832, 0.500)	(0.877, 0.750)	(0.943, 1.000)	(1.000, 0.129)	(1.070, 0.)
POTENTIAL (X,Y)	-400.0	(1.992, 0.750)	(1.953, 1.000)	(2.000, 0.733)	(2.215, 0.)	(2.177, 0.250)	(2.082, 0.500)
POTENTIAL (X,Y)	-600.0	(0.896, 0.500)	(0.964, 0.750)	(1.000, 0.292)	(1.000, 0.862)	(1.168, 0.)	(1.035, 0.250)
POTENTIAL (X,Y)	-600.0	(1.217, 1.000)	(1.416, 1.000)	(1.500, 0.962)	(1.600, 0.750)	(1.750, 0.591)	(1.948, 0.250)
POTENTIAL (X,Y)	-600.0	(1.803, 0.500)	(2.000, 0.063)	(2.016, 0.)	(0. , 0.)	(0. , 0.)	(0. , 0.)

POTENTIAL (X,Y) -800.0 (C.961, C.500) (1.000, 0.421) (1.000, 0.628) (1.143, 0.250) (1.250, 0.035) (1.250, 0.646)

POTENTIAL (X,Y) -800.0 (1.285, 0.) (1.487, 0.500) (1.500, 0.490) (1.735, 0.250) (1.750, 0.237) (1.880, 0.)

POTENTIAL (X,Y) -1000.0 (1.25C, C.250) (1.250, 0.250) (1.250, 0.250) (1.500, 0.) (1.250, C.250) (1.500, 0.)

POTENTIAL (X,Y) -1000.0 (1.750, 0.) (1.750, 0.) (1.750, 0.) (0. , 0.) (0. , 0.) (0. , 0.)

CYCLE 8

CURRENT DENSITIES ARE CALCULATED USING EQUIPOTENTIAL OF 790.51134 VOLTS WHICH HAS X-Y COORDINATES
1 (0.423,0.) 2 (0.315,0.25C) 3 (0.253,0.500) 4 (0.249,0.750) 5 (0.250,1.000)

EMITTER ARC LENGTH , DELTA ARC LENGTH 0.50000 0.12500

X-Y EMITTER COORDINATES

1 (C.050,C.500) 2 (0.05C,1.000)

X-Y BEGIN TRAJ. COORDINATES

1 (C.050,0.50C) 2 (0.050,0.625) 3 (0.050,0.750) 4 (0.050,0.875)

X-COORD	TRAJ NUM	REFLECTION COUNTER	Y-COORD	X-VEL COMP	Y-VEL COMP
C.2500	1	0	C.5000	0.17207E 05	0.
	2	0	0.6250	0.17335E 05	0.
	3	0	0.7500	0.17461E 05	0.
	4	0	0.8750	0.17451E 05	0.
0.5000	1	0	0.5085	0.29625E 05	0.15987E 04
	2	0	0.6259	0.29692E 05	0.17752E 03
	3	0	0.7501	0.29759E 05	0.24140E 02
	4	0	0.8743	0.29663E 05	-0.12880E 03
0.7500	1	0	0.5235	0.40752E 05	0.26086E 04
	2	0	0.626C	0.40468E 05	-0.14826E 03
	3	0	0.7487	0.40141E 05	-0.41414E 03
	4	0	0.8714	0.39735E 05	-0.68234E 03
1.0000	1	0	-1.0000	0.52451E 05	0.27984E 04
	2	0	0.6250	0.51104E 05	-0.22442E 04
	3	0	0.7407	0.49490E 05	-0.24776E 04
	4	0	0.8621	0.48134E 05	-0.25935E 04
1.2500	1	0	-1.0000	0.52451E 05	0.27984E 04
	2	0	0.6062	0.51334E 05	-0.54675E 04
	3	0	0.7210	0.49816E 05	-0.53157E 04
	4	0	0.8431	0.49051E 05	-0.47907E 04
1.5000	1	0	-1.0000	0.52451E 05	0.27984E 04
	2	0	0.5726	0.49728E 05	-0.81195E 04
	3	0	0.6885	0.48698E 05	-0.75167E 04
	4	0	0.8150	0.48313E 05	-0.61550E 04
1.7500	1	0	-1.0000	0.52451E 05	0.27984E 04
	2	0	0.5255	0.47549E 05	-0.10188E 05
	3	0	0.6447	0.46797E 05	-0.92106E 04
	4	0	0.7799	0.46686E 05	-0.71860E 04

2.0000	1	0	-1.0000	0.52451E 05	0.27984E 04
	2	0	0.4659	0.44683E 05	-0.11796E 05
	3	0	0.5905	0.44238E 05	-0.10527E 05
	4	0	0.7382	0.44481E 05	-0.80020E 04
2.2500	1	0	-1.0000	0.52451E 05	0.27984E 04
	2	0	0.3942	0.41543E 05	-0.12941E 05
	3	0	0.5264	0.41373E 05	-0.11435E 05
	4	0	0.6903	0.41980E 05	-0.85679E 04
2.5000	1	0	-1.0000	0.52451E 05	0.27984E 04
	2	0	0.3110	0.38425E 05	-0.13661E 05
	3	0	0.4530	0.38523E 05	-0.12009E 05
	4	0	0.6368	0.39454E 05	-0.88704E 04
2.7500	1	0	-1.0000	0.52451E 05	0.27984E 04
	2	0	0.2177	0.35432E 05	-0.13905E 05
	3	0	0.3715	0.35863E 05	-0.12230E 05
	4	0	0.5786	0.37138E 05	-0.89573E 04

THRUST DISTRIBUTION BY STREAM TUBES (NEWTONS/UNIT H)

1 0. 2 0. 3 0.209802E-05 4 0.226902E-05 5 0.230177E-05

TOTAL THRUST (NEWTONS/UNIT H) 0.666880E-05
TOTAL POWER (WATT/UNIT H) 0.121546E-00

INITIAL CURRENT DENSITIES (AMPS/(UNIT H)**2)

1 0. 2 0.350769E-03 3 0.357920E-03 4 0.361040E-03 5 0.359966E-03

INITIAL CURRENTS (AMPS/UNIT H)

1 0. 2 0.438462E-04 3 0.447401E-04 4 0.451300E-04 5 0.449957E-04

TOTAL INITIAL CURRENT=0.178712E-03 AMPS/(UNIT H)

AVERAGE INITIAL CURRENT DENSITY=0.357424E-03 AMPS/(UNIT H)**2

TRANSMITTED CURRENT AT ACCEL. GRID=0.132853E-03 AMPS/(UNIT H) WHICH IS 74.34 PERCENT OF THE INITIAL CURRENT.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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