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# APPLICATION OF THE INVERSE TECHNIQUE TO THE FLOW OVER A BLUNT BODY AT ANGLE OF ATTACK

*by William W. Joss*

Prepared under Contract No. NASr-119 *by*
CORNELL AERONAUTICAL LABORATORY, INC.
Buffalo, N. Y.

*for*

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## FOREWARD

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## ABSTRACT

A method for the numerical solution of the flow behind a nonaxisymmetric bow shock is described. This method has been programmed for an IBM 7044 digital computer for the case of an ideal gas, and the calculation time is approximately ten minutes. For a specified bow shock the program generates the subsonic flow, a portion of the supersonic flow, and the body shape which will support the specified shock. Sample results are shown for three different shock shapes.

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# INTRODUCTION

Aerodynamicists have for some time been interested in the problem of the flow around the nose of a blunt body moving at hypersonic speed. A number of methods have been devised for the solution of this problem. Most have been limited to the case of an axisymmetric body at zero angle of attack, and some have included nonequilibrium chemistry[1-8]. A few have considered, with certain restrictions, nonaxisymmetric bodies or bodies at angle of attack[9-12].

The methods of solution of the blunt-body problem can be divided into two broad classes; those defined as direct methods and those defined as inverse methods. In the direct method the shape of a particular body, its velocity and its angle of attack are specified. The solution then predicts the flow field over the nose of the body as well as the shape of the detached shock wave ahead of the body. In contrast, the inverse method begins with a specification of the shape of the shock wave. Then, for a given flow velocity and direction, the solution predicts the flow field behind the shock and the shape of the body supporting the shock.

This paper discusses an application of the inverse technique to the calculation of the flow field behind a nonaxisymmetric shock set at an arbitrary angle of attack. The solution is at the present time limited to an ideal gas. It has been the purpose of this work to generate a high-speed numerical program which could be used for the investigation of various aerodynamic problems associated with nonaxisymmetric flows, and for a test of the analytic relations that are obtained by making approximations to the complete equations. To accomplish this end the inverse method of calculation was chosen, since the single specification of the shape of the shock wave supplies the complete boundary condition for the problem, even when chemical reactions are included in the calculation.

One disadvantage of the inverse method is the fact that the body shape cannot be specified a priori, but must be calculated along with the flow field for any choice of bow shock shape. This characteristic of the method would require an iterative procedure if the flow around a specific body were to be studied. A second disadvantage is the known instability of the inverse method, in that any irregularity introduced into the calculation is quickly magnified in later steps. This aspect of the problem requires that the calculation procedure be very smooth and accurate, so that no perturbations are introduced by the numerical methods. For example, if a cylindrical coordinate system is used for the calculation, the cancellation of terms of order $1/r$ in the flow conservation equations introduces numerical perturbations at points near the axis ( $r$ small). Numerical smoothing routines during the calculation are then required to obtain a solution. However, the difficulty can be circumvented by using Cartesian coordinates, whereupon large and nearly equal subtractive terms do not appear. This is the method used in the present program, and no smoothing techniques have been required.

The results of several sample calculations are included in the present report. These calculations were performed to check the accuracy of the program and to determine the stability and calculation time. The accuracy is demonstrated by comparison with available results for an axisymmetric case. All of the test cases demonstrate that the calculations in the rectangular coordinate system are not affected by the instability. The calculation time is about 10 minutes on the IBM 7044.

## GENERAL METHOD

The general approach to the solution using the inverse technique will be briefly outlined here. A more detailed description of certain aspects of the method will follow.

An assumed bow shock shape is shown in Fig. 1. The solution is begun immediately behind the bow shock and is extended to the body via a numerical integration scheme. Initial values for the flow-field variables are first defined, through the oblique shock relations, at a large number of data points behind the shock. These initial data points are themselves defined by the intersections of planes of constant $y$ and $z$ on the surface of the shock wave. Representative traces are shown in Fig. 1. Each trace is identified with a subscript; the subscript $j$ is associated with a plane of constant $y$, the subscript $\ell$ with a plane of constant $z$. Therefore, each data point is associated with a particular $j\,\ell$ subscript.

It will be shown later that the flow conservation equations, originally containing partial derivatives with respect to $x$, $y$, and $z$, can be reduced to a set of equations containing only partial derivatives with respect to $x$ and total derivatives along the traces of constant $y$ and $z$. These total derivatives of the flow variables along the traces of constant $y$

and z can be evaluated by numerical differentiation, since a whole series of data points is located along each of these traces.

The procedure at any particular data point employs the reduced flow conservation equations to define partial derivatives of the flow variables with respect to x. The partial derivatives are computed using the numerically determined total derivatives evaluated along the two traces which intersect to form the data point.

Then, using the initial values of the flow variables at the particular data point, and the partial derivatives with respect to x which were determined as discussed above, an integration is performed along a ray, parallel to the x-axis, behind the initial data point. The solution for the values of the flow variables is advanced a distance $\Delta x$ behind the original data point. This procedure is performed simultaneously at all the initial data points on the shock and the solution is advanced the same distance, $\Delta x$, behind each point. In other words, a whole new set of data points is defined a distance $\Delta x$ behind the shock and forms a new surface, parallel to the shock, at that location. Since the values of the flow variables are known at every point on this new surface, the surface can be treated in exactly the same way as was the shock. That is, at every point new partial derivatives in the x-direction are computed utilizing total derivatives along traces in the displaced surface. These values of the x-derivatives are then used to integrate another step, $\Delta x$, into the flow field. The procedure is repeated until the integration rays, which are parallel to the x-axis, strike, or define, the surface of the body.

The solution is normally started with approximately twelve planes of constant z and 25 planes of constant y. The intersections of the traces of these planes on the surface of the shock therefore define 300 initial data points just behind the shock wave. Approximately 10 integration steps are normally taken before the solution reaches the surface of the body. Therefore, in a typical solution the properties of the flow field will be defined at approximately 3000 points.

## Definition of Initial Conditions

The initial conditions behind the bow shock depend on the freestream Mach number and the local angle $\beta_{jk}$ between the freestream velocity vector and the inward directed normal to the shock surface. This angle can be found through use of the relation

$$\cos \beta_{jk} = \vec{U}_o \; \vec{n} = u_o n_x + v_o n_y + w_o n_z = \sin \delta_{jk}$$

where $u_o$, $v_o$ and $w_o$ are the components of the unit freestream velocity vector and $n_x$, $n_y$ and $n_z$ are the components of the unit vector normal to the surface of the shock. $\delta_{jk}$ is the complement of $\beta_{jk}$ and is used in many of the latter equations.

If the shock is defined by an equation of the type

$$F(x, y, z) = 0$$

then the components of the unit vector normal to the shock are defined by

$$n_x = \frac{\partial F/\partial x}{\sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 + \left(\frac{\partial F}{\partial z}\right)^2}} \; , \; etc.$$

The freestream velocity vector lies in the x-y plane, which is the plane of symmetry for the description of the bow shock. Thus the component $w_o$ is defined to be zero, and the expression for $\sin \delta_{jk}$ becomes simply

$$\sin \delta_{jk} = u_o n_x + v_o n_y$$

where $u_o \equiv \cos \alpha$ and $v_o = \sin \alpha$, and $\alpha$ is the angle of attack; i.e., the angle between the freestream velocity and the x-axis. This expression for sin $\delta_{jk}$ is used in the oblique shock relations to define the initial values of the pressure and density as shown below. Initial values are denoted by the superscript $(\tilde{\ })$.

$$\tilde{p}_{jk} = \frac{1}{\gamma_o M_o^2} \left[ \frac{2\gamma_o \left(M_o \sin \delta_{jk}\right)^2 - (\gamma_o - 1)}{\gamma_o + 1} \right]$$

$$\tilde{\rho}_{jk} = \left[ \frac{(\gamma_o + 1)\left(M_o \sin \delta_{jk}\right)^2}{(\gamma_o - 1)\left(M_o \sin \delta_{jk}\right)^2 + 2} \right]$$

where $M_o$ is the value of the freestream Mach number and $\gamma_o$ is the freestream ratio of specific heats. Here, the pressure has been nondimensionalized by the freestream value of $\rho_o' U_o'^2$ and the density by $\rho_o'$. The primes $(\ )'$ indicate dimensional quantities and $U_o'$ is the magnitude of the total freestream velocity vector.

The relations for the initial velocity components, $\tilde{u}_{jk}$, $\tilde{v}_{jk}$ and $\tilde{w}_{jk}$ can conveniently be determined by resolving the initial velocity into components tangent and normal to the shock wave. These components can be further resolved into $u$, $v$ and $w$ components. The tangent components are unaltered when passing through the shock. The normal component suffers a normal-shock loss, so that the ratio of its magnitude behind the shock to its magnitude ahead of the shock is the reciprocal of the density ratio given above. The final expressions for the initial velocity components at a particular data point ( j, $k$) are

$$\tilde{u}_{jk} = \cos\alpha + \left[\left(\frac{1}{\tilde{\rho}} - 1\right) n_x\right]_{jk} \sin\delta_{jk}$$

$$\tilde{v}_{jk} = \sin\alpha + \left[\left(\frac{1}{\tilde{\rho}} - 1\right) n_y\right]_{jk} \sin\delta_{jk}$$

$$\tilde{w}_{jk} = \left[\left(\frac{1}{\tilde{\rho}} - 1\right) n_z\right]_{jk} \sin\delta_{jk}$$

In summary, the initial conditions, and in fact the entire problem, are characterized by the freestream value of the Mach number by $\gamma_0$ and by the slope of the shock at any point. In using the program, the only restrictions on the expression describing the particular shock shape chosen are that (1) the expression must be differentiable so that expressions for $n_x$, $n_y$, and $n_z$ may be found, and (2) the expression must yield a unique solution for the values of x, y, and z on the shock surface. Polynomial expressions for x in terms of y and z are well suited for use with the program.

Derivation of the Complete Differential Equations for Numerical Integrations

As discussed earlier, once the initial conditions are defined at a point directly behind the shock wave, a numerical integration is performed in the x-direction in order to generate the solution for the flow field. In order to perform this integration, one must know the partial derivatives with respect to x of the nondimensional flow variables, $p$, $\rho$, $u$, $v$, and $w$. This section will briefly describe the derivations of the x-derivative equations.

The nondimensionalized equations of motion for an ideal, inviscid gas written in a cartesian coordinate system are shown below:

$$
\left.
\begin{array}{ll}
\underline{\text{Continuity}} & \\
u\dfrac{\partial \rho}{\partial x} + \rho\dfrac{\partial u}{\partial x} + v\dfrac{\partial \rho}{\partial y} + \rho\dfrac{\partial v}{\partial y} + w\dfrac{\partial \rho}{\partial z} + \rho\dfrac{\partial w}{\partial z} = 0 & \text{(1a)} \\
\underline{\text{Momentum}} & \\
u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z} = -\dfrac{1}{\rho}\dfrac{\partial P}{\partial x} & \text{(1b)} \\
u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + w\dfrac{\partial v}{\partial z} = -\dfrac{1}{\rho}\dfrac{\partial P}{\partial y} & \text{(1c)} \\
u\dfrac{\partial w}{\partial x} + v\dfrac{\partial w}{\partial y} + w\dfrac{\partial w}{\partial z} = -\dfrac{1}{\rho}\dfrac{\partial P}{\partial z} & \text{(1d)} \\
\underline{\text{Entropy}} & \\
u\dfrac{\partial P}{\partial x} - \dfrac{\gamma P u}{\rho}\dfrac{\partial \rho}{\partial x} + v\dfrac{\partial P}{\partial y} - \dfrac{\gamma P v}{\rho}\dfrac{\partial \rho}{\partial y} & \\
\qquad + w\dfrac{\partial P}{\partial z} - \dfrac{\gamma P \omega}{\rho}\dfrac{\partial \rho}{\partial z} & \text{(1e)}
\end{array}
\right\} (1)
$$

The above equations, which contain partial derivatives with respect to x, y, and z, will be reduced to a set of equations involving only partial derivatives with respect to x and directional derivatives which will be numerically evaluated. The numerical derivatives are taken along the two traces in the surface which intersect to define the data point in question.

It remains to eliminate the terms which contain partial derivatives with respect to y and z and replace them with other terms which can be evaluated numerically. These other terms are derivatives along the trace defined by the intersection of the integration surface (initially the shock surface) with the plane of constant $z$, $\left(\frac{d}{ds}\Big|_z\right)$ or with the plane of constant $y$, $\left(\frac{d}{ds}\Big|_y\right)$. These directional derivatives are symbolized by a specific subscript for the coordinate that is held constant (y or z), and it is understood that the derivative is taken within the integration surface. If $\theta_y$ is defined as the angle between the x-axis and the trace defined by the plane of constant y, and $\theta_z$ as the angle between the x-axis and the trace defined by the plane of constant z, then, for example,

$$\frac{\partial \rho}{\partial y} = \frac{\frac{d\rho}{ds}\Big|_z - \left(\frac{\partial \rho}{\partial x}\right)\cos\theta_z}{\sin\theta_z} \tag{2}$$

$$\frac{\partial \rho}{\partial z} = \frac{\frac{d\rho}{ds}\Big|_y - \left(\frac{\partial \rho}{\partial x}\right)\cos\theta_y}{\sin\theta_y} \tag{3}$$

where $\theta_y$ and $\theta_z$ are geometry factors of the initial shock shape given by

$$\sin\theta_z = \frac{n_x}{\sqrt{n_x^2 + n_y^2}} \quad , \quad \sin\theta_y = \frac{n_x}{\sqrt{n_x^2 + n_y^2}}$$

Substitution of relations of the type of (2) and (3) into Eqs. (1) gives, after rearrangement and collection of terms within individual equations

$$\begin{aligned}
&B\left(\frac{\partial \rho}{\partial x}\right) + \rho\left(\frac{\partial u}{\partial x}\right) - \rho D\left(\frac{\partial v}{\partial x}\right) - \rho C\left(\frac{\partial w}{\partial x}\right) = E \\
&\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right) + B\left(\frac{\partial u}{\partial x}\right) = F \\
&-\frac{D}{\rho}\left(\frac{\partial P}{\partial x}\right) + B\left(\frac{\partial v}{\partial x}\right) = G \\
&-\frac{C}{\rho}\left(\frac{\partial P}{\partial x}\right) + B\left(\frac{\partial w}{\partial x}\right) = H \\
&B\left(\frac{\partial P}{\partial x}\right) - B\,\frac{\gamma P}{\rho}\left(\frac{\partial \rho}{\partial x}\right) = I
\end{aligned} \tag{4}$$

where

$$A = \left[ u - w \frac{\cos\theta_y}{\sin\theta_y} \right]$$

$$B = \left[ A - v \frac{\cos\theta_z}{\sin\theta_z} \right]$$

$$C = \cos\theta_y / \sin\theta_y$$

$$D = \cos\theta_z / \sin\theta_z$$

The terms on the right hand side are given by

$$E = -\frac{1}{\sin\theta_y}\left[ w \left.\frac{d\rho}{ds}\right|_y + \rho \left.\frac{dw}{ds}\right|_y \right] - \frac{1}{\sin\theta_z}\left[ v \left.\frac{d\rho}{ds}\right|_z + \rho \left.\frac{dv}{ds}\right|_z \right]$$

$$F = -\frac{w}{\sin\theta_y}\left[ \left.\frac{du}{ds}\right|_y \right] - \frac{v}{\sin\theta_z}\left[ \left.\frac{du}{ds}\right|_z \right]$$

$$G = -\frac{w}{\sin\theta_y}\left[ \left.\frac{dv}{ds}\right|_y \right] - \frac{1}{\sin\theta_z}\left[ \frac{1}{\rho} \left.\frac{dP}{ds}\right|_z + v \left.\frac{dv}{ds}\right|_z \right]$$

$$H = -\frac{1}{\sin\theta_y}\left[ \frac{1}{\rho} \left.\frac{dP}{ds}\right|_y + w \left.\frac{dw}{ds}\right|_y \right] - \frac{v}{\sin\theta_z}\left[ \left.\frac{dw}{ds}\right|_z \right]$$

$$I = -\frac{w}{\sin\theta_y}\left[ \left.\frac{dP}{ds}\right|_y - \frac{\gamma P}{\rho} \left.\frac{d\rho}{ds}\right|_y \right] - \frac{v}{\sin\theta_z}\left[ \left.\frac{dP}{ds}\right|_z - \frac{\gamma P}{\rho} \left.\frac{d\rho}{ds}\right|_z \right]$$

Equations (4) define the five unknown partial derivatives in terms of five simultaneous equations. It should be noted that the directional derivatives are all collected on the right hand side of each individual equation. The simultaneous solution of the above five equations yields the following expressions for the unknown partial derivatives

$$\frac{\partial P}{\partial x} = \frac{I}{B} + \frac{\gamma P}{\rho}\left(\frac{\partial \rho}{\partial x}\right)$$

$$\frac{\partial u}{\partial x} = \frac{1}{B}\left(F - \frac{1}{\rho}\frac{\partial P}{\partial x}\right)$$

$$\frac{\partial v}{\partial x} = \frac{1}{B}\left(G + \frac{D}{\rho}\frac{\partial P}{\partial x}\right)$$

$$\frac{\partial w}{\partial x} = \frac{1}{B}\left(H + \frac{C}{\rho}\frac{\partial P}{\partial x}\right)$$

where

$$\frac{\partial \rho}{\partial x} = \frac{\rho}{\frac{\gamma P}{\rho}\left[\frac{B^2}{\gamma P/\rho} - \left(1 + C^2 + D^2\right)\right]}\left[\frac{I}{B\rho}\left(1 + C^2 + D^2\right) + \left(HC + DG + \frac{EB}{\rho} - F\right)\right]$$

The overall procedure, then, is as follows. At each initial data point behind the shock wave, the initial values of $P$, $\rho$, $u$, $v$, and $w$ are computed as in the previous section. Then, at each data point, the directional derivatives of all five variables are obtained along the two traces which intersect to define the data point. The values of these derivatives are then used in the calculation of E, F, G, H, and I at each point. Then these parameters, along with the sin $\theta_y$, sin $\theta_z$, tan $\theta_y$, and tan $\theta_z$, which also vary from point to point, are used in the calculation of the partial derivatives with respect to x. Finally, the partial derivatives are numerically integrated forward in x until the solution reaches the body. In the present program, a 4th order Runge-Kutta method is used in the integration process.

The following section will discuss the method whereby the numerical derivatives discussed above are obtained.

## Numerical Derivative Evaluation

As discussed in the previous section, directional derivatives must be taken along the traces defined by the planes of constant y and z. These derivatives are required in order to compute the partial derivatives with respect to x at any particular data point. The sketch in Fig. 1 indicates the manner in which the data points are defined on the shock surface. The planes of constant y and z are separated by constant increments, $\Delta y$ and $\Delta z$. The increments in y are not necessarily equal to those in z .

When evaluating the numerical derivatives at any point, say the point defined $j = 3$, $k = 4$ in Fig. 1, the derivatives along the trace are first computed with respect to either y or z as appropriate and then converted to derivatives with respect to arc length. This is done in order to take advantage of the fact that the data points are equally spaced in terms of the coordinate distances whereas they are unequally spaced with respect to arc lengths. The program utilizes a least squares fitting routine. A fifth-order polynomial is fitted through the values of each of the variables at seven consecutive data points with the point in question normally being the central point. The values of y or z for the seven points being considered are temporarily changed by translating the origin so that the value of y or z at the central point is zero. For example, $\rho$ may be given on the integration surface by

$$\rho = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 + a_5 y^5 \ .$$

Then

$$\left.\frac{\partial \rho}{\partial y}\right|_{z,(x-\bar{x})} = a_1 + 2a_2 y + 3a_3 y^2 + 4a_4 y^3 + 5a_5 y^4$$

where $\tilde{x}$ is the initial value of x on the shock surface for a given value of y and z. Since the value of y is defined as zero at the point in question, the value of the derivative is given simply as

$$\left.\frac{\partial \rho}{\partial y}\right|_{z, x-\tilde{x}} = a_1$$

When the data points on the outer edges of the mesh are being considered, a slightly different procedure must be used. The values of the derivatives at the last three data points on a trace must be evaluated using a value of y or z not equal to zero. For example, since y is defined as zero at the central point, the value of y at the point defined by the uppermost plane of constant y is $3\,\Delta y$. Therefore, at that point, the formula for the derivative cannot be reduced to one term. All the terms must be considered, with a y of $3\,\Delta y$ being used in the calculation of the derivative. The above discussion also applies to the data points defined by the last three planes of constant z.

The coefficients used in the equation for the total derivatives are functions of the values of the variables at the seven data points. The formulas for the coefficients are

$$a_1 = \frac{1}{60\Delta \ell}\left[-F_{-3} + 9F_{-2} - 45\,F_{-1} + 45\,F_1 - 9F_2 + F_3\right]$$

$$a_2 = \frac{1}{264(\Delta \ell)^2}\left[-13\,F_{-3} + 67\,F_{-2} - 19\,F_{-1} - 70\,F_0 - 19\,F_1 + 67\,F_2 - 13\,F_3\right]$$

$$a_3 = \frac{1}{48(\Delta \ell)^3}\left[F_{-3} - 8\,F_{-2} + 13\,F_{-1} - 13\,F_1 + 8\,F_2 - F_3\right]$$

$$a_4 = \frac{1}{264(\Delta \ell)^4}\left[3F_{-3} - 7F_{-2} + F_{-1} + 6\,F_0 + F_1 - 7\,F_2 + 3\,F_3\right]$$

$$a_5 = \frac{1}{240(\Delta \ell)^5} \left[ - F_{-3} + 4F_{-2} - 5F_{-1} + 5F_1 - 4F_2 + F_3 \right]$$

where $\Delta \ell$ refers to either the increments in y or z as appropriate and F stands for one of the seven different values of the variables being differentiated. The subscripts on F define the data point from which each of the values of the variables are taken. The subscripts range from minus three through zero to plus three.

After the derivatives with respect to y or z have been computed as above, they are converted to derivatives with respect to arc length by applying the following formulas

$$\left.\frac{dF}{ds}\right|_y = \left.\frac{\partial F}{\partial z}\right|_{y,x-\tilde{x}} \frac{1}{(ds/dz)|_y}$$

$$\left.\frac{dF}{ds}\right|_z = \left.\frac{\partial F}{\partial y}\right|_{z,x-\tilde{x}} \frac{1}{(ds/dy)|_z}$$

The terms $\left.\frac{ds}{dz}\right|_y$ and $\left.\frac{ds}{dy}\right|_z$ are the values of the integrand in the formula for the arc length at any point on the trace. They are given by

$$\left.\frac{ds}{dz}\right|_y = \sqrt{1 + \left(\frac{dx}{dz}\right)_y^2} = \sqrt{1 + \left(\frac{n_z}{n_x}\right)^2}$$

$$\left.\frac{ds}{dy}\right|_z = \sqrt{1 + \left(\frac{dx}{dy}\right)_z^2} = \sqrt{1 + \left(\frac{n_y}{n_x}\right)^2}$$

In summary, the directional derivatives of the five variables $P, \rho, u, v,$ and $w$ are determined through a fifth order least squares fit through seven consecutive data points. The derivatives are first computed with respect to y or z and are then converted to derivatives with respect to arc length along the traces of constant y or z. These derivatives are then used in the reduced flow conservation equations to obtain the partial derivatives of the independent variables with respect to x.

## Definition of the Body

As discussed previously, the inverse method of solution is begun with a known shock shape and is then used to compute the shape of a compatible body. To obtain a definition of the position of the body, a relation which describes the conservation of mass along any integration ray is employed. At the point where the mass entering the shock wave is balanced by the mass loss, the body has been reached.

To consider this question in more detail, imagine a group of cylinders of vanishingly small cross-sectional area constructed around each individual data point on the shock, and extending parallel to the x-axis to the point where they strike the body. The mass flux entering such a cylinder per unit cross-sectional area (divided by the freestream mass flux per unit area) is given by

Flux entering per unit area of shock wave = $\vec{U}_0 \vec{n} = n_x \cos\alpha + n_y \sin\alpha$

To obtain the flux per unit area in a constant x plane, this relation must be divided by the cosine of the angle between the x-axis and the normal to the shock wave (i. e., must be divided by $n_x$). Thus

Flux entering per unit area $dy\,dz = \cos\alpha + \sin\alpha\left(\frac{n_y}{n_x}\right)$

The flux passing out through the walls of a cylinder of cross-sectional area $dy\,dz$ and length $dx$ is given as

Flux out = $\left[-E - (wC + vD)\frac{\partial\rho}{\partial x} - \rho C\frac{\partial w}{\partial x} - \rho D\frac{\partial v}{\partial x}\right] dx\,dy\,dz$

where the symbols are as defined in the section describing the derivation of the x-derivatives. This formula is simply a result of applying a differential continuity equation along the integration ray. The total mass flux out through the wall is integrated along with the values of the other variables in the problem and is subtracted from the initial mass flux passing in through the shock. This defines the value of the excess mass on any particular integration ray as a function of x. When the excess mass falls to zero along any ray, then that ray is said to have struck the body. In this way, every individual integration ray can be used to define an individual body point.

## RESULTS OF SAMPLE CALCULATIONS

The first test case was an axisymmetric shock wave at zero angle of attack. The shock shape chosen was the "elliptic-catenary" with an axis ratio of 1.0, i.e., an axisymmetric catenary shock. The freestream Mach number was 13.98. This allowed a comparison to be made with previous results generated by the program described in Ref. 3. The shock and body shape for this case are shown in Fig. 2. As is seen, the comparison between the location of the two bodies is excellent. A check of the values of the variables throughout the flow field also showed excellent agreement with the earlier results. Relatively small step sizes were chosen in this case, so that seventeen integration steps were taken before reaching the body. Later experience has shown that the integration step size can be increased considerably. The running time for this case was, however, still short, being approximately eleven minutes. It should be emphasized that no simplifications arise because the case is axisymmetric. For instance, none of the derivatives reduces to zero in the Cartesian coordinates as they would in cylindrical coordinates. Therefore, this case represents an excellent check of the entire program.

The next test case again utilized the elliptic-catenary shock but with an axis ratio, $b$ , of 2.0. That is, at a given x-location, the distance from the x-axis to the shock in the x-z plane was twice as great as the distance from the x-axis to the shock in the x-y plane. The results for this highly nonaxisymmetric case compared well with the results of an earlier (unpublished) case calculated using cylindrical coordinates. The cylindrical-coordinate case required extensive smoothing of the derivatives in order to run, whereas the case in Cartesian coordinates required no smoothing whatsoever.

The shapes of the shock and body are shown in the x-y and x-z planes in Fig. 3. Figure 4 is an isometric drawing of the body, which has an axis ratio considerably larger than that of the shock wave. It should be pointed out again that no smoothing has been applied to the solution, either during the calculation or in the drawing of the body shapes.

The next test case was calculated with an axisymmetric shock set at an angle of attack of 15°. The purpose of this case was to show that the introduction of the few terms which are zero when the angle of attack is zero do not cause any difficulty when computing the solution. Actually, once the initial conditions have been defined, it is unimportant to the program what the external conditions are. The runge-Kutta integration proceeds in exactly the same way for every case. The results of the calculation for this shock shape are shown in Fig. 5. A thinner shock layer is seen somewhat below the x-axis. The body curves back above the x-axis in order to support the shock.

The cases discussed above were test calculations to demonstrate the ability of the program to generate the subsonic flow behind arbitrary specified shock waves. The particular forms used in these test calculations were chosen for convenience, but it is emphasized that the program can be used directly with any smoothly varying shock wave that has a plane of symmetry. The accuracy of the results is attested to by the stable generation of the flow field and smooth supporting blunt bodies.

## SUMMARY AND CONCLUDING REMARKS

A method has been presented for the calculation of the flow over nonaxisymmetric bodies at angle of attack. The solution utilizes the inverse technique wherein the shock shape is specified and the body shape is computed. The shock shape is required to have a plane of symmetry, but is otherwise arbitrary. The solution has been coded for use on an IBM 7044, typical calculation times for a complete solution with 300 integration rays being of the order of ten minutes. To date, only an ideal gas has been considered.

Sample cases have been calculated which show that the method can be used to compute the flow behind highly nonaxisymmetric shocks and shocks at large angle of attack.

The present program can be utilized for the numerical investigation of questions concerning the subsonic flow region ahead of a blunt supersonic body. Of particular interest in this regard is the location of the stagnation point and the details of the stagnation flow region for blunt bodies at angle of attack. The trajectory of the maximum-entropy stream line in such cases is also of interest in connection with the question of whether or not this stream line wets the body surface.

Because of the short computing time required by the present program, it is felt that it would be practical to include the effects of finite rate chemistry in the calculation at a future date. In addition, the method may lend itself to an iteration procedure wherein the shock shape is progressively modified to obtain a desired body shape. The complete generality of the shock specification would make the program well suited for this application.

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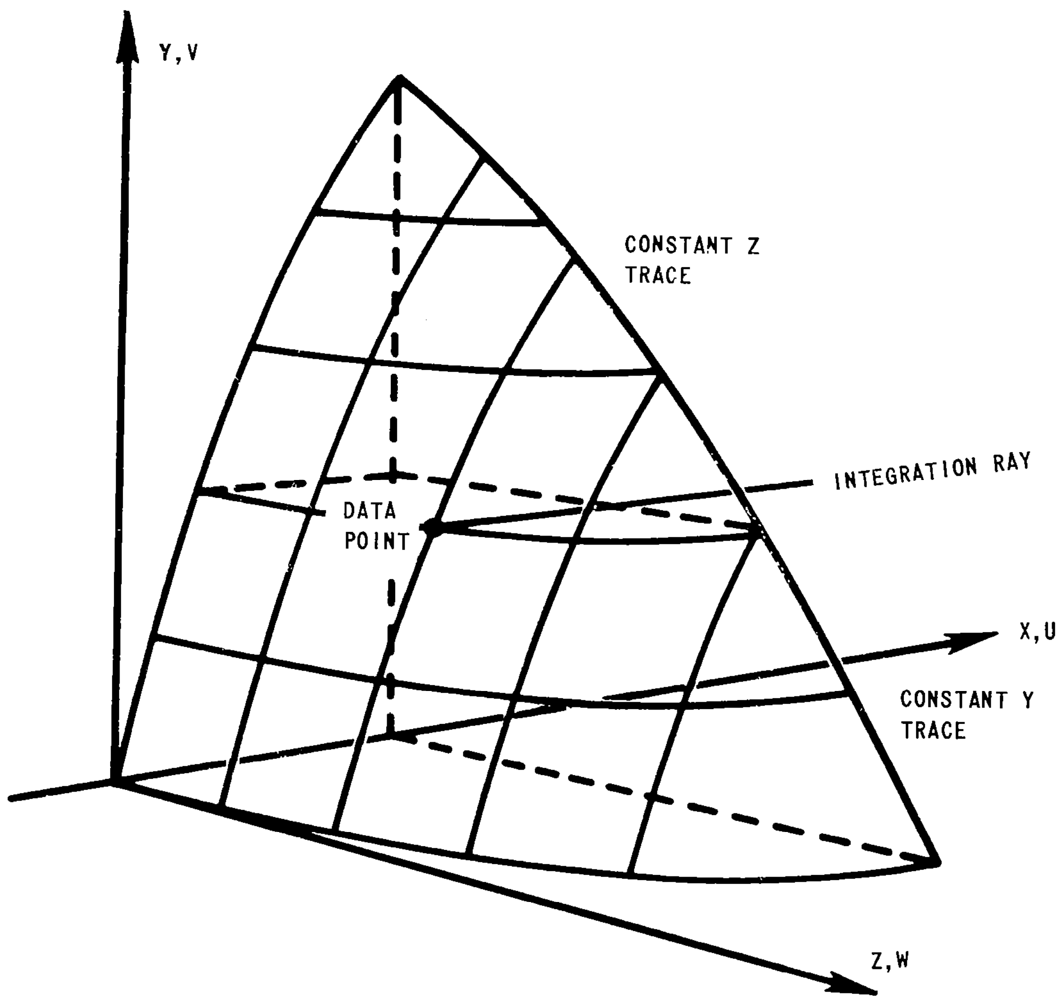


Figure 1 COORDINATE SYSTEM

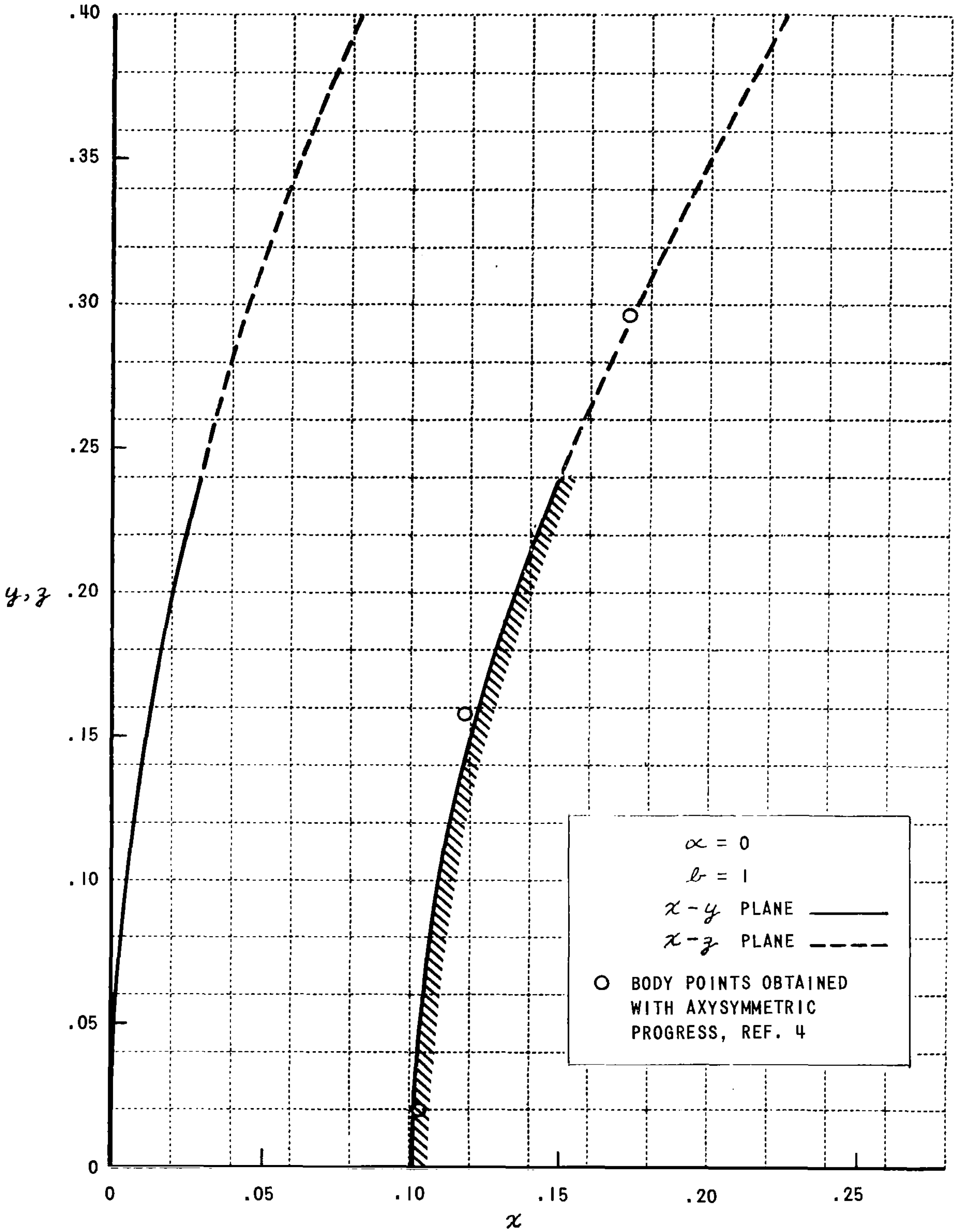


Figure 2 BODY GENERATED WITH AXYSYMMETRIC CATENARY SHOCK WAVE AND COMPARISON WITH PREVIOUS RESULTS

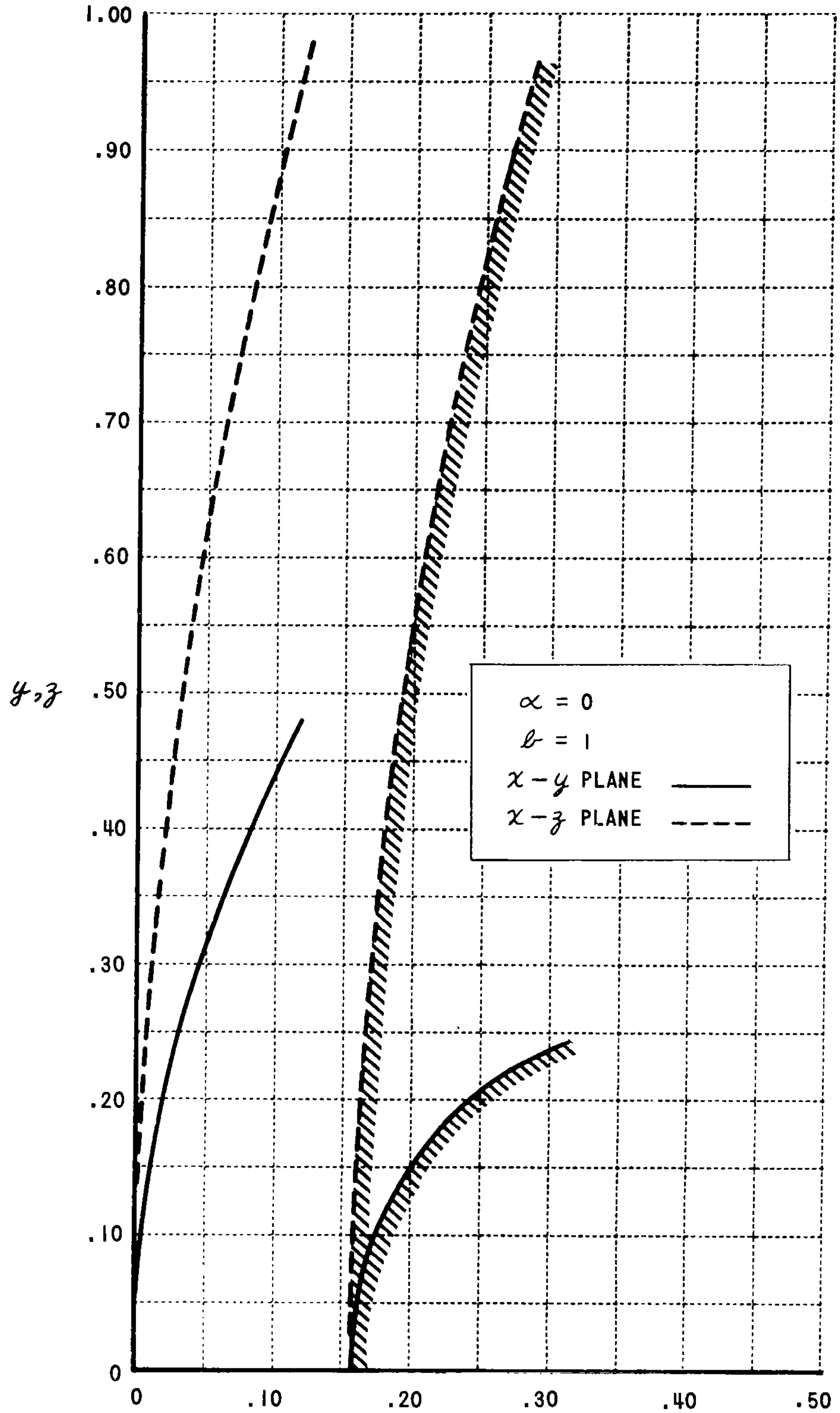


Figure 3 BODY GENERATED WITH ELLIPTIC-CATENARY SHOCK WAVE AT ZERO ANGLE OF ATTACK

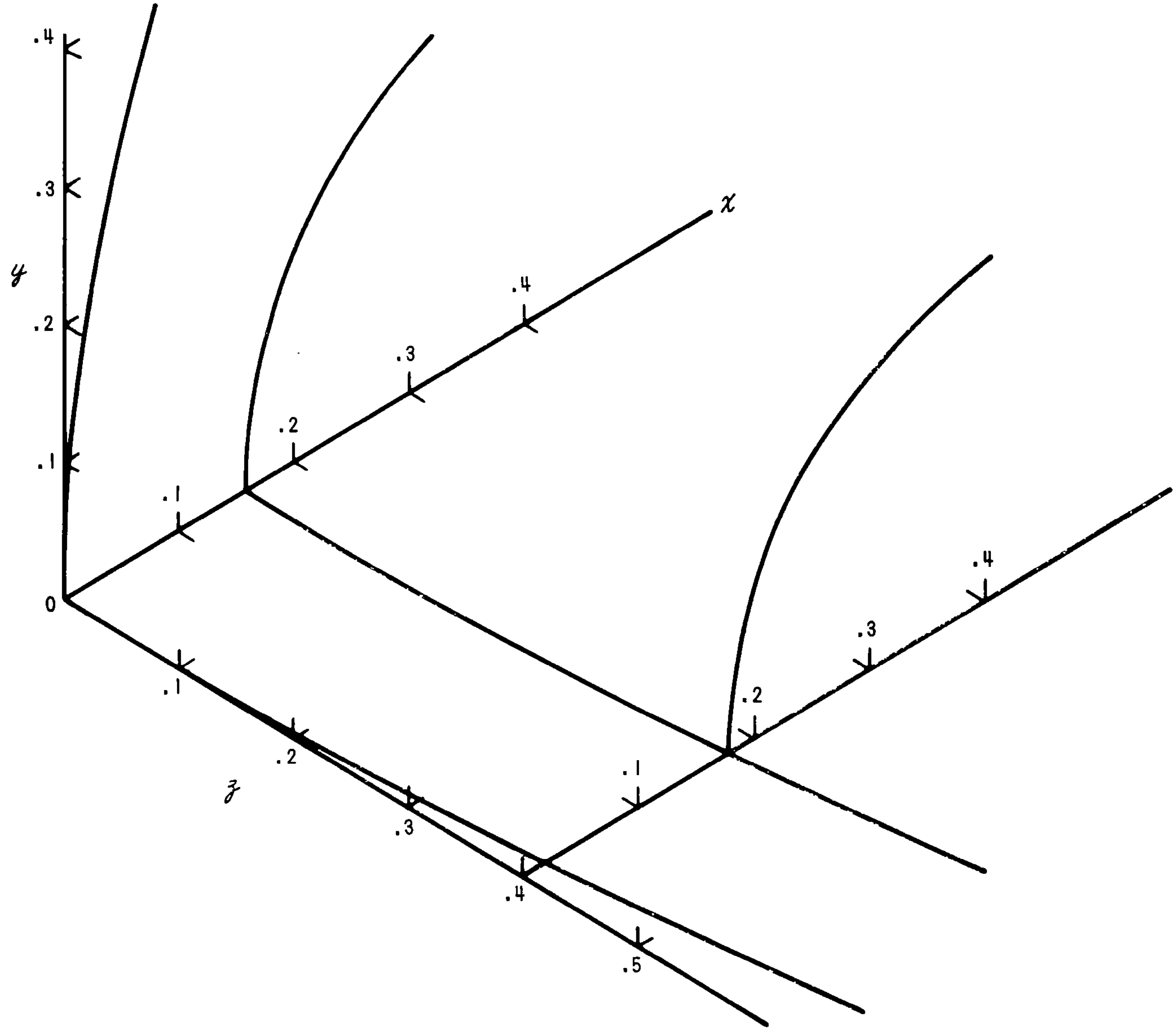


Figure 4 ISOMETRIC PLOT OF BODY GENERATED WITH ELLIPTIC-CATENARY SHOCK WAVE AT ZERO ANGLE OF ATTACK

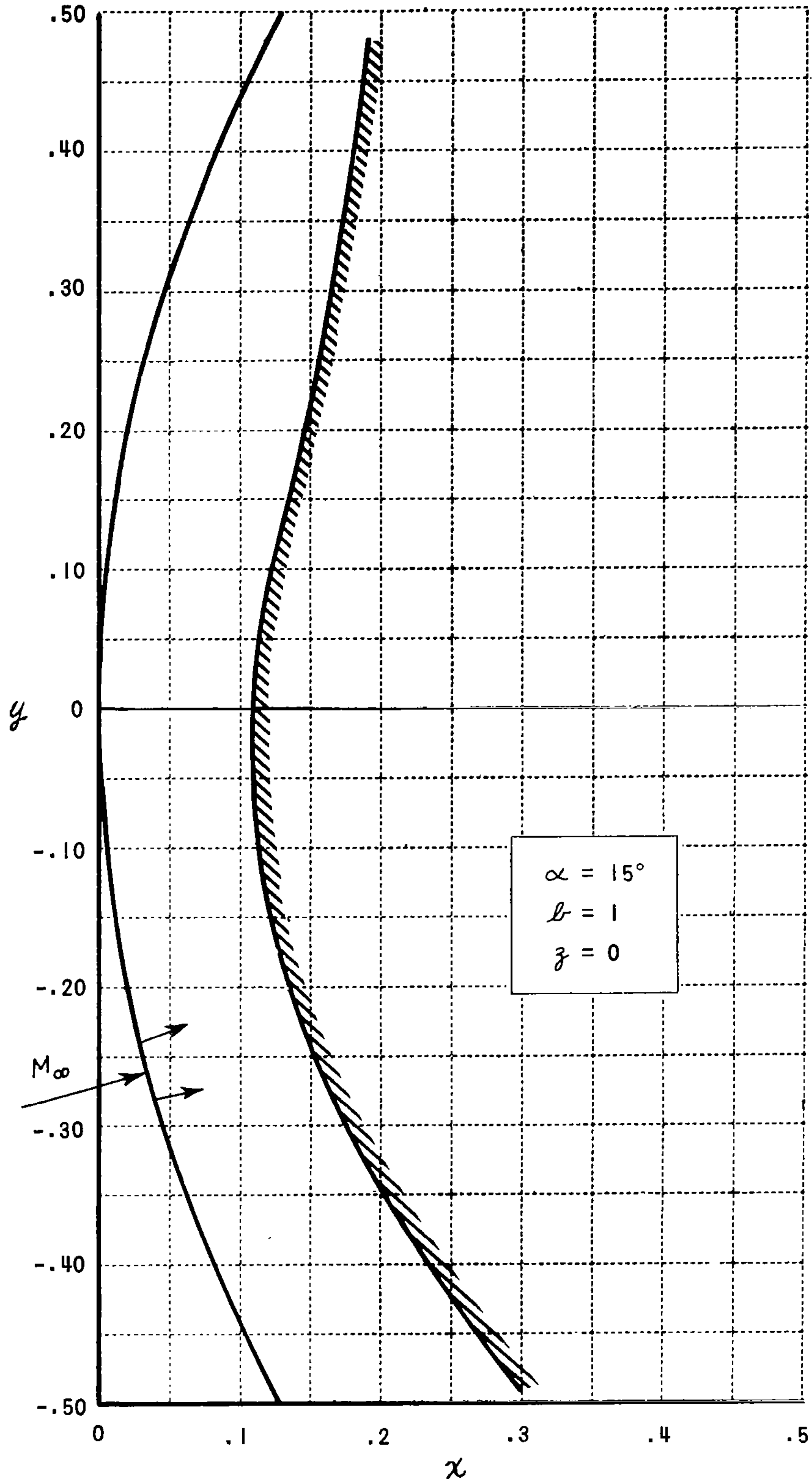


Figure 5 BODY GENERATED BY AXYSYMMETRIC CATENARY SHOCK WAVE PLACED AT 15 DEGREES ANGLE OF ATTACK



NASA-Langley, 1966 CR-445