NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report No. 32-887

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NOTE

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ABSTRACT

A requirement has existed at the Jet Propulsion Laboratory (JPL) for an instrument capable of measuring the insertion loss of transmission-line components beyond the normal precision capability of commercially available equipment. These measurements are necessary for the calibration of low-noise microwave receiving systems used in planetary/lunar astronomy and space communications programs. The instrumentation which was developed has a short-term jitter of 0.0004 db peak-to-peak and long-term drift of typically 0.0015 db/hr. Accuracy of the measurements is limited at the higher insertion-loss values by a 0.1% instrumentation nonlinearity. The precision at very low insertion-loss levels is shown to be better than 10^{-4} db. Details of the precision measurement techniques are presented for WR 430 Waveguide components.

I. INTRODUCTION

Accurate measurements of the parameters of passive microwave components are required in the assembly and evaluation of low-noise microwave receiving systems (Ref. 1). These measurements are used in the calibrations that are needed in the planetary and lunar radar experiments, radio astronomy, and space communications systems that form part of the program of Supporting Research and Advanced Development for the Deep Space Instrumentation Facility (DSIF) of JPL.

Thus there arose a need for an instrument that could measure the insertion loss of low-loss coaxial and waveguide components in the Laboratory as well as at the Goldstone Tracking Station, and which could do this beyond the normal precision capability of commercially available equipment. At that time the most precise, commercially available test set was the Weinschel Dual Channel Insertion Loss Test Set (Ref. 2), which has a precision of 0.02 db per 10 db (or 0.01 db, whichever is greater). This test set is essentially a laboratory system and its precision did not meet the JPL requirements. Measurements of the required precision had, however, been achieved (Ref. 3) but at the cost of extreme care and extraordinary temperature stabilization. Two suitable systems were developed at JPL. One uses an ac ratio-transformer technique (Ref. 4) and, in common with commercial insertion-loss test sets, requires modulation of the signal source. This Report describes the other system, which is a dc-potentiometer method. The dc-potentiometer test set does not require modulation of the signal source, eliminates ac ground-loop problems, incorporates increased temperature stability, can be operated from ac mains or batteries, and is portable.

II. EQUIPMENT DESCRIPTION

A dual-channel insertion-loss test set (Fig. 1) has been constructed almost entirely from commercially available equipment and components. The signal level is sampled with power meters at two points, and the dc outputs are compared on a null indicator. Amplitude instabilities in the signal generator do not affect the null, except for nonlinearity effects in the power meters. A precision divider is used to obtain a null both before and after inserting the microwave component to be measured at Point A. The difference in the divider ratio (converted to db) is a measure of the insertion loss of the component.

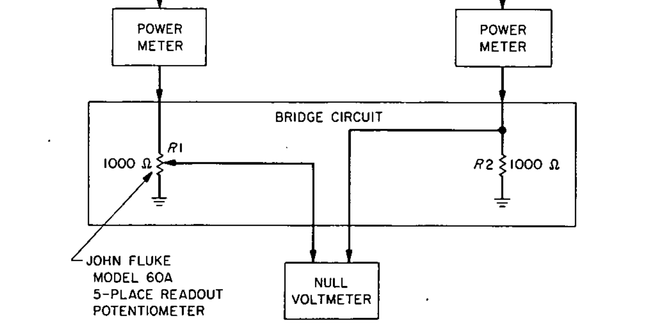

Fig. 1. Block diagram of microwave insertion-loss test set

The technology of precision dc dividers is more than adequate for the accuracy required. A photograph of the test set is shown in Fig. 2 in a configuration utilizing WR 112 Waveguide components. The waveguide is matched with screw tuners so that the voltage standing wave ratio (VSWR) in either direction from Point A is less than 1.01.

The internal circuitry of the power meter has an output which is proportional to the square root of the applied power. In order to have a linear output with respect to power, a squaring circuit used on the output of the power meters gives straight-line segments approximating square-law nonlinearity. The output linearity of the power meters was greatly improved by utilizing the output across the feedback resistor,[1] thereby bypassing the squaring-circuit output. A detail schematic of the test set is shown in Fig. 3.

Switch S_{1A}, in the power meter circuitry, enables the power meters to be operated in the normal manner when not used to measure insertion loss. Switch S_1 is a two-pole, four-position switch in the comparator circuitry. Positions 1 and 2 are used with the zero adjustment for Power Meters 2 and 1, respectively, to set the bias with RF power removed. This bias sets the quiescent current of the transistors in the feedback paths to the power meters. Position 4 is used to balance the -18-v power-meter supplies with R_1, and to define the null position between the power meters when the -18-v supplies are slightly out of balance, just prior to the final readout-potentiometer null adjustment. A steady-state unbalance between the -18-v

[1]Suggested by Mr. F. Praman of Hewlett–Packard, Palo Alto, Calif. These are Resistors R160–R166 shown in Fig. 5-3 of HP Instruction Manual for the HP 431B power meter.

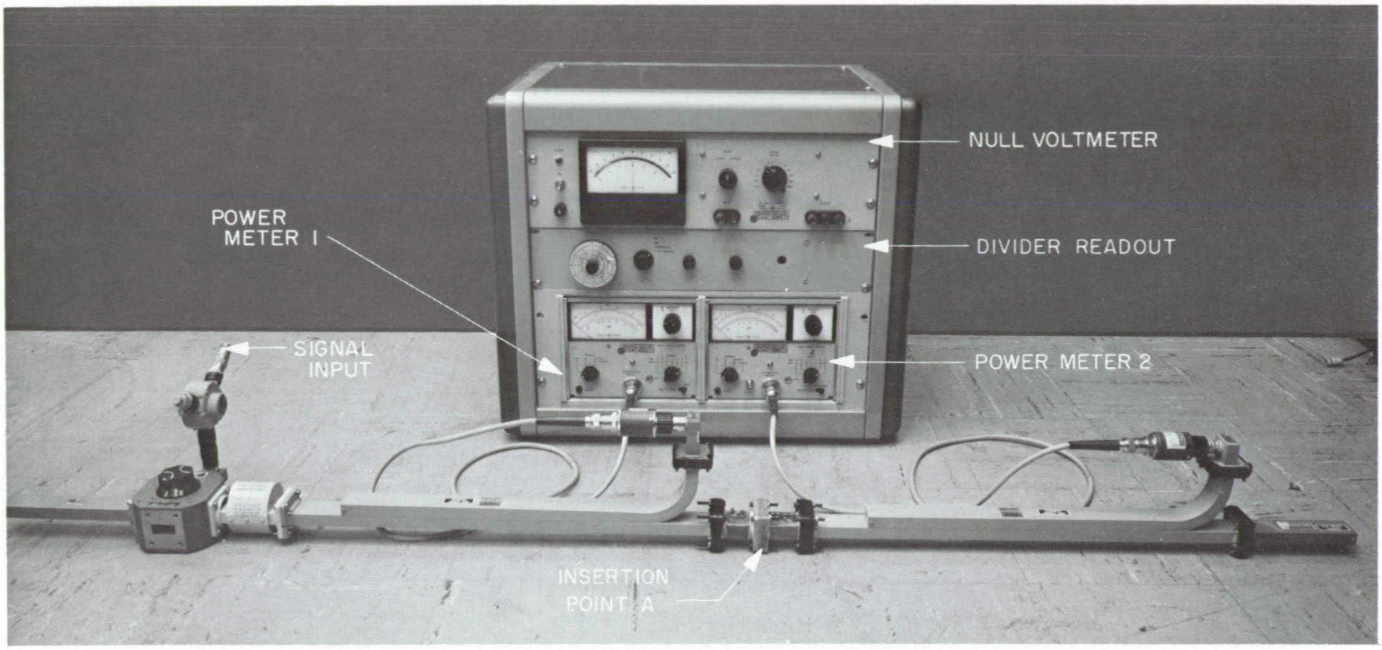

Fig. 2. Photograph of insertion-loss test set with an 8448-Mc waveguide configuration

supplies is a source of measurement nonlinearity. Incorrect changes in the −18-v balance between readings reduces the insertion-loss measurement resolution. Position 3 is the final readout position. The 5000-ohm helipots in the bridge circuit can be set so that a full-scale reading on the power meter is obtained with RF power applied (after Step 1 in the measurement procedure). This is done only as an initial setting, and is convenient for occasionally checking the RF power level.

A later version of the test set which uses a common power supply eliminated the need for the equalizing adjustment of the power supplies.

Modulation of the signal source is not required with the Hewlett–Packard HP 431B Power Meter, thus eliminating possible errors from klystron signal-source double moding and modulator instability. A stability test was made of the test set by recording the output of the null voltmeter; a typical data sample is shown in Fig. 4. Short-term jitter is about 0.0004 db peak-to-peak, and long-term drift is typically 0.0015 db/hr.

The operating instructions for the test set are as follows:

A. Initial Setup

1. With Switch S_{1A} in the normal power-meter position (Transistor Q107 connected to the power-meter range switch), perform the normal power-meter zeroing and nulling.

2. Set the signal-generator power level (typically 1.5 mw).

B. Measurement Procedure

1. Switch S_{1A} to the external range resistor.

2. Remove the RF power.

   a. Switch S_1, Position 4: adjust R_1 for a zero (±0.5 mv) on the null voltmeter.

   b. Switch S_1, Position 1: adjust zero for Power Meter 2 for 45 mv (±2 mv) on the null voltmeter.

   c. Switch S_1, Position 2: adjust zero for Power Meter 1 for a null (±2 mv) on the null voltmeter.

3. Apply RF power and Switch S_1 to Position 3.

4. Adjust the precision potentiometer for a null and record the reading ratio.

These steps are repeated several times with the waveguide parts connected together first without the unknown and then with the unknown inserted at Point A in Fig. 2. The ratios are converted with tables to db and then differences are taken and averaged.

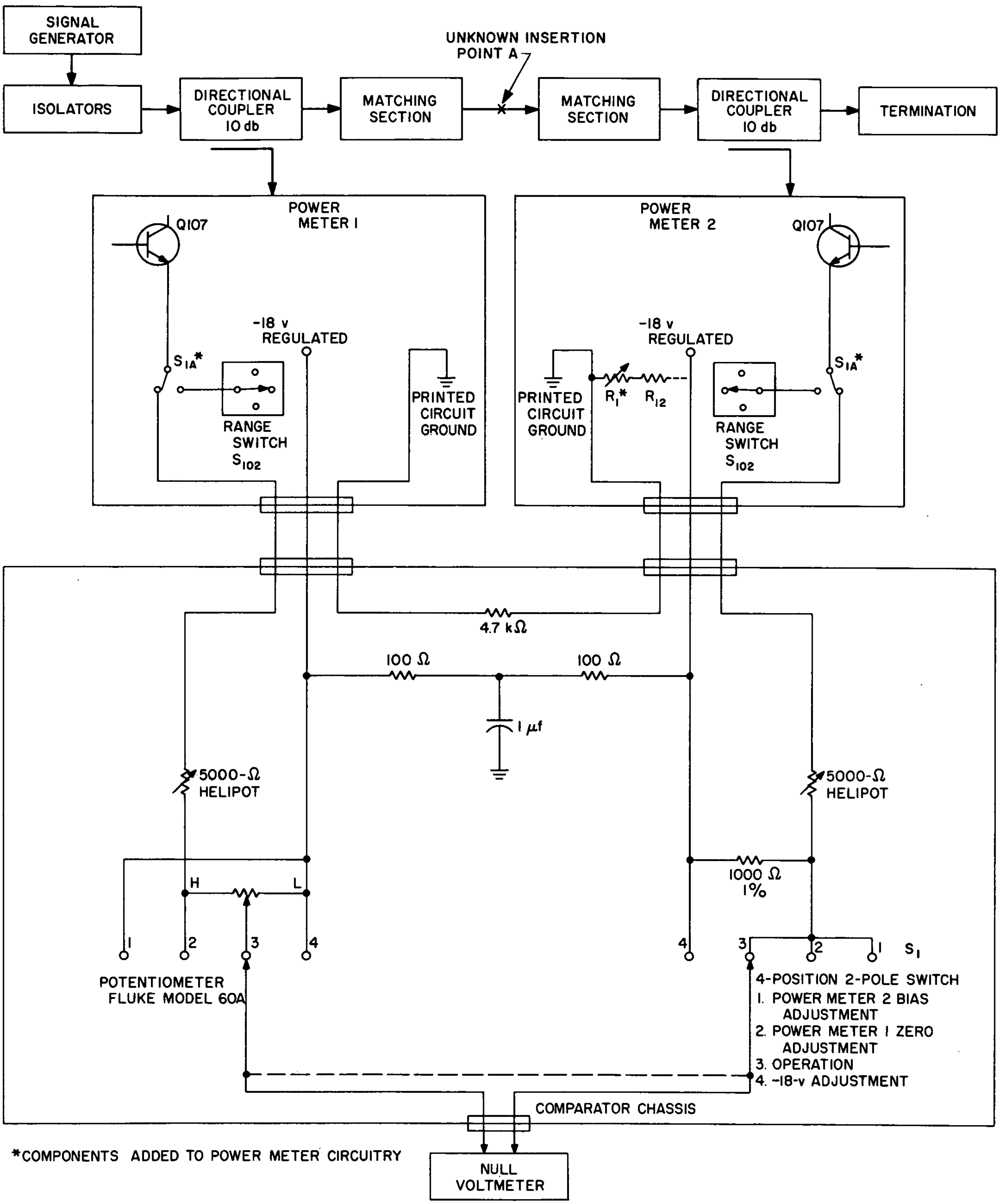

Fig. 3. Detail schematic of the test-set configuration

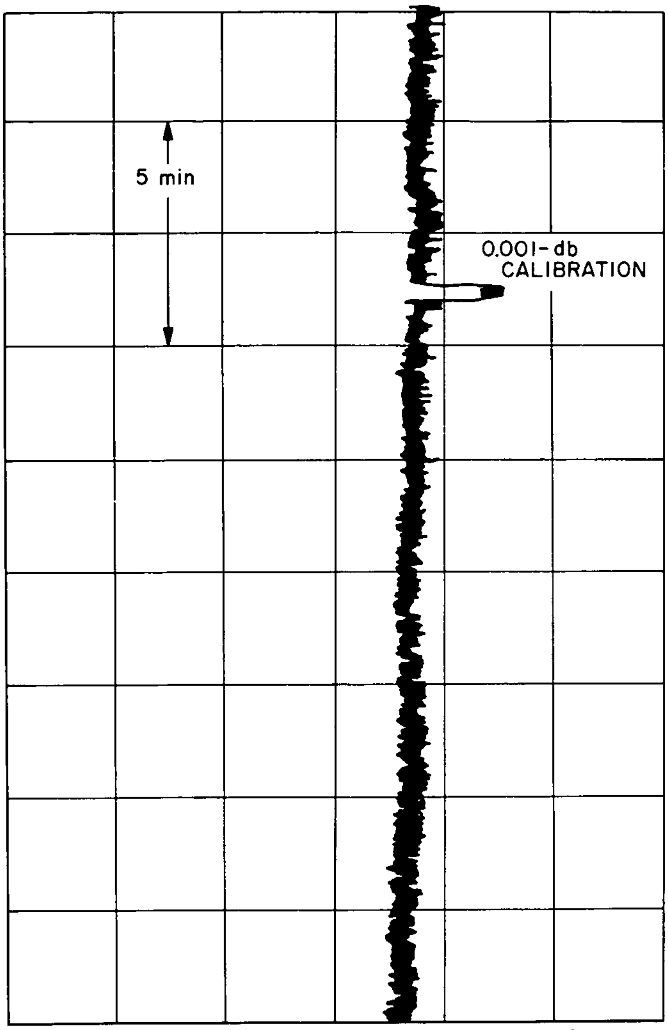

Fig. 4. Stability recording of microwave insertion-loss instrumentation

C. *Performance*

As an example of typical test-set performance, a right-angle WR 112 Waveguide section was measured eight consecutive times.

1. Average insertion loss was 0.0282 db.

2. Maximum difference from this average was 0.0017 db.

3. Average difference of all measurements from 0.0282 db was 0.0008 db, which includes the nonrepeatability of connecting and disconnecting waveguide flanges.

Many insertion-loss measurements of other components have been made with comparable results (Ref. 5).

The measurement of linearity of an insertion-loss test set is very difficult. One method used with limited success was to measure attenuators separately and in various combinations. The measured insertion loss of the combinations should be that obtained by summing the individually measured insertion losses. This method is time consuming and is especially vulnerable to mismatch errors. The most successful technique used a WR 112 Waveguide Hewlett–Packard Model H-382A Rotary Vane Attenuator, Serial No. 1103, which was modified[2] for high angular resolution and calibrated (Test No. 805434) by the National Bureau of Standards (NBS). The calibration frequency was 8448 Mc. Table 1 shows the difference between the NBS calibrations and the insertion-loss test set for three separate Hewlett–Packard Model 478A Thermistor Mounts over an attenuation range from 1.0 to 20 db. The indicated nominal attenuation is the amount of attenuation change produced by rotating the vane through the required angle. The variable attenuator is not removed from the microwave test set during these measurements. The RF power applied to the thermistors was 1.5 mw and the dc bias was 45 mv for these tests. Figure 5 shows a plot of the attenuation differences. It was observed that a change of 5 mv in the bias resulted in approximately 0.02 db change in the readout of 20 db. It is more important that the bias not change with high attenuations than with low attenuations. The NBS "estimated accuracy" is shown in dotted lines. The difference in db as compared with the precision H-band rotary-vane attenuator is less than 0.10% for nominal values of attenuation in db to 16 db for these three particular thermistor mounts.

[2]Measurement Specialties Laboratory, Inc., Model H 200, Ser. 101.

Table 1. Difference between measured and calibrated insertion loss for H-band rotary-vane attenuator

Calibrated attenuator setting, db	Calibration (National Bureau of Standards), db, minus measured insertion loss		
	Thermistor mount Serial 8506	Thermistor mount Serial 7731	Therimstor mount Serial 8525
1.0	−0.0003	+0.0010	+0.0013
3.0	+0.00032	+0.0033	+0.0051
6.0	+0.0055	+0.0040	+0.006
10.0	+0.002	+0.004	+0.007
16.0	−0.019	−0.008	+0.002
20.0	−0.045	−0.031	−0.017

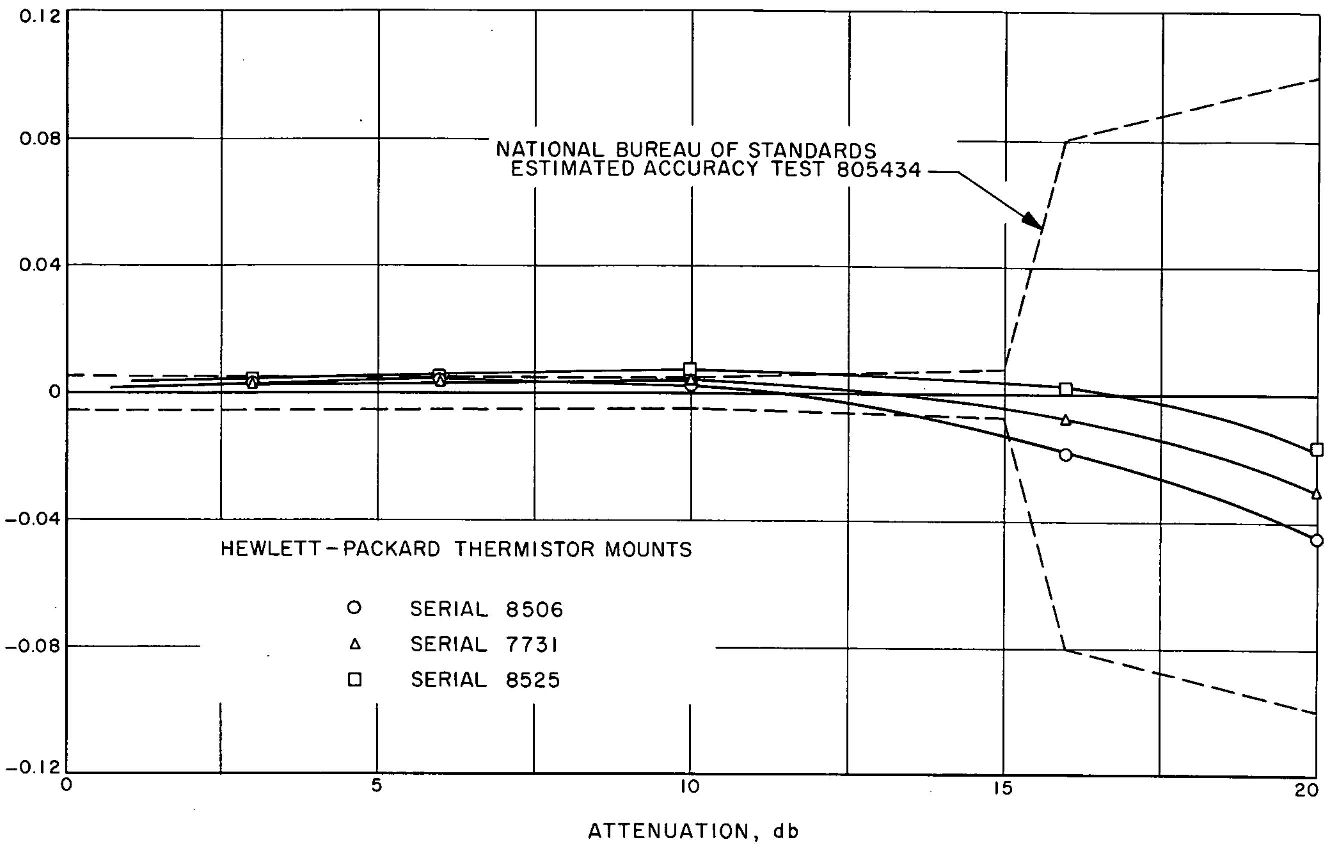

Fig. 5. Measured attenuation difference versus attenuation for three thermistor mounts

III. PRECISION MEASUREMENT TECHNIQUES

A. First-Order Analysis

For precision measurements (accuracy on the order of 10^{-4}), statistical methods are required. The least-squares method (Ref. 6) can be used to advantage. A plot of the measured insertion-loss data y can be made as a function of measurement number m. For relation to drift error, this method must assume a uniform time spacing of measurements. Experimentally, this can be realized to a good approximation. A best-fit straight line can be determined from the modified data points (Fig. 6). The modified data points are obtained from the original data points y by adding a constant $\bar{L}/2$ to each power-level measurement with the unknown disconnected, and subtracting $\bar{L}/2$ from each power-level measurement with the unknown connected. The standard deviation σ of the modified data points from the best-fit straight line $a + bx$ is

$$(\sigma)^2 = \frac{1}{m}\sum_{o}(y + \bar{L}/2 - a - bx)^2 + \frac{1}{m}\sum_{e}(y - \bar{L}/2 - a - bx)^2 \qquad (1)$$

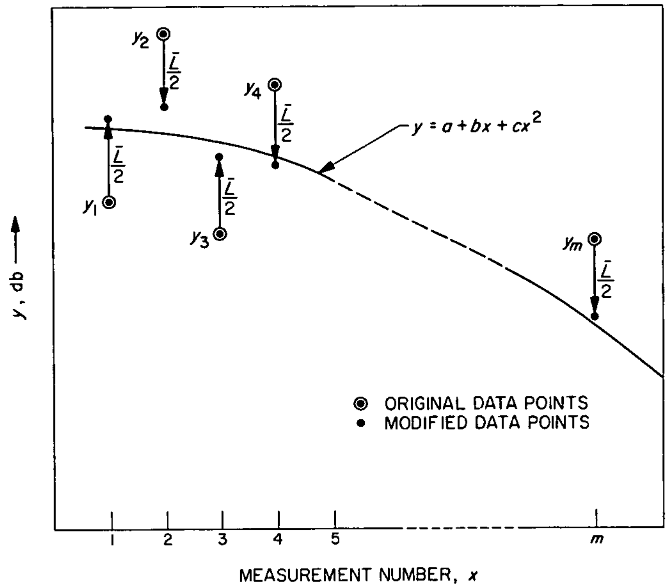

Fig. 6. First-order insertion-loss data reduction

where

y = Insertion-loss test-set reading, db

x = Measurement number

$\bar{L}$ = Average measured insertion loss, db

a = Constant of best-fit straight line, db

b = Slope of best-fit straight line, db/measurement number

m = Total number of measurements

o = Odd measurements (unknown disconnected)

e = Even measurements (unknown connected)

In order to determine the values of a, b, and $\bar{L}$ for which the standard deviation is a minimum, Eq. (1) is differentiated with respect to a, b, and $\bar{L}$, and is equated to zero (Ref. 6, p. 240).

$$\left.\begin{aligned}
&\sum_o (y + \bar{L}/2 - a - bx) + \sum_e (y - \bar{L}/2 - a - bx) = 0\\
&\sum_o x\,(y + \bar{L}/2 - a - bx) + \sum_e x\,(y - \bar{L}/2 - a - bx) = 0\\
&\sum_o (y + \bar{L}/2 - a - bx) - \sum_e (y - \bar{L}/2 - a - bx) = 0
\end{aligned}\right\} \tag{2}$$

Rearranging,

$$\left.\begin{aligned}
&-\frac{\bar{L}}{2} + am + b\sum x = \sum y\\
&-\frac{\bar{L}}{2}\left(\sum_o x - \sum_e x\right) + a\sum x + b\sum x^2 = \sum xy\\
&-\frac{\bar{L}}{2} + a + b\left(\sum_o x - \sum_e x\right) = \sum_o y - \sum_e y
\end{aligned}\right\} \tag{3}$$

Solving for $\bar{L}$,

$$\bar{L} = 2\,\frac{\begin{vmatrix} \sum y & m & \sum x\\ \sum xy & x & \sum x^2\\ \left(\sum_o y - \sum_e y\right) & 1 & \left(\sum_o x - \sum_e x\right)\end{vmatrix}}{\begin{vmatrix} -1 & m & \sum x\\ -\left(\sum_o x - \sum x\right) & \sum x & \sum x^2\\ -m & 1 & \left(\sum x - \sum_e x\right)\end{vmatrix}} \tag{4}$$

A simplification can be made if the middle measurement number is chosen as the origin so that the summation of all odd powers of x is zero (Ref. 6, p. 252). This is discussed in more detail in Section IV. Then,

$$\left.\begin{aligned}
\bar{L} &= \frac{\sum_e y}{(m-1)/2} - \frac{\sum_o y}{(m+1)/2}\\
a &= \frac{\sum_e y}{m-1} + \frac{\sum_o y}{m+1}\\
b &= \frac{\sum xy}{\sum x^2}
\end{aligned}\right\} \tag{5}$$

The probable errors of the odd and even data points are (Ref. 6, p. 167)

$$\left.\begin{aligned}
PE_{yo} &= 0.6745\left(\frac{\sum_o [y + (\bar{L}/2) - a - bx]^2}{(m+1)/2}\right)^{1/2}\\
PE_{ye} &= 0.6745\left(\frac{\sum [y - (\bar{L}/2) - a - bx]^2}{(m-1)/2}\right)^{1/2}
\end{aligned}\right\} \tag{6}$$

By expanding $\bar{L}$ in terms of the coefficients of the data points and summing (Ref. 6, p. 229), the probable error $PE_{\bar{L}}$ of the mean insertion loss $\bar{L}$ is found to be

$$PE_{\bar{L}} = \left(\frac{(PE_{yo})^2}{(m-1)/2} + \frac{(PE_{ye})^2}{(m-1)/2}\right)^{1/2} \tag{7}$$

B. Second-Order Analysis

In many applications of insertion-loss measurement, the maximum precision is required. For example, the accurate calibration of cooled microwave terminations is of fundamental importance to the evaluation of low-noise receiving systems. The precision measurement of the insertion loss of the transmission-line components is a basic requirement in the calibration of the termination. In view of the wide application and great importance of high-precision insertion-loss measurements, a second-order statistical analysis of the data reduction method has been carried out. The derivation is similar to the first-order analysis and is detailed in Appendix A.

The second-order analysis fits a parabola $a' + b'x + c'x^2$ to the modified data points (Fig. 6). This accounts for both drifts that occur at a constant rate and drift rates that change at a constant rate during the measurement period. The constant b' is the drift rate, or drift per measurement number in db, and is equal to b in the first-order analysis. The constant c' is the rate of change of the drift rate. The analysis minimizes the standard deviation σ' of the modified data points from the best fit parabola and yields

$$\overline{L} = \frac{2}{\Delta}\{\sum y[\underline{\overline{X}}\sum x^2 - \sum x^4] - \sum x^2 y[m\underline{\overline{X}} - \sum x^2] + \underline{\overline{Y}}[m\sum x^4 - (\sum x^2)^2]\} \quad (8)$$

$$a' = \frac{1}{\Delta}\{-\sum y[(\underline{\overline{X}})^2 + m\sum x^4] + \sum x^2 y[-\underline{\overline{X}} + m\sum x^2] - \underline{\overline{Y}}[-\sum x^4 + \underline{\overline{X}}\sum x^2]\} \quad (9)$$

$$b' = \frac{\sum xy}{\sum x^2} \quad (10)$$

$$c' = \frac{1}{\Delta}\{\sum y[-\underline{\overline{X}} + m\sum x^2] - \sum x^2 y[m^2 - 1] + \underline{\overline{Y}}[-\sum x^2 + m\underline{\overline{X}}]\} \quad (11)$$

where

$$\Delta = (-1)[\underline{\overline{X}}\sum x^2 - \sum x^4] + X[m\underline{\overline{X}} - \sum x^2] - m[m\sum x^4 - (\sum x^2)^2]$$

$$\underline{\overline{X}} = \sum_o x^2 - \sum_e x^2$$

$$\underline{\overline{Y}} = \sum_o y - \sum_e y$$

The probable errors of the data points are

$$\left.\begin{aligned} PE_{yo} &= 0.6745\left(\frac{\sum_o [y + (\overline{L}/2) - a' - b'x - c'x^2]^2}{(m+1)/2}\right)^{1/2} \\ PE_{ye} &= 0.6745\left(\frac{\sum_e [y - (\overline{L}/2) - a' - b'x - c'x^2]^2}{(m-1)/2}\right)^{1/2} \end{aligned}\right\} \quad (12)$$

and the probable error of the mean insertion loss is

$$PE_{\overline{L}} = ([PE_{yo}]^2 \sum_o \alpha^2 + [PE_{ye}]^2 \sum_e \alpha^2)^{1/2} \quad (13)$$

where α, the coefficients of the data points, are given by

$$\begin{aligned} \sum_o \alpha^2 = \frac{4}{\Delta^2}\Big\{ & [\underline{\overline{X}}\sum x^2 + (m-1)\sum x^4 - (\sum x^2)^2]^2 \left(\frac{m+1}{2}\right) \\ & - 2[\underline{\overline{X}}\sum x^2 + (m-1)\sum x^4 - (\sum x^2)^2][m\underline{\overline{X}} - \sum x^2] \\ & \times \sum_o x^2 + [m\underline{\overline{X}} - \sum x^2]^2 \sum_o x^4 \Big\} \end{aligned} \quad (14)$$

and

$$\begin{aligned} \sum_e \alpha^2 = \frac{4}{\Delta^2}\Big\{ & [\underline{\overline{X}}\sum x^2 - (m+1)\sum x^4 + (\sum x^2)^2]^2 \left(\frac{m-1}{2}\right) \\ & - 2[\underline{\overline{X}}\sum x^2 - (m+1)\sum x^4 + (\sum x^2)^2] \\ & \times [m\underline{\overline{X}} - \sum x^2]\sum_e x^2 + [m\underline{\overline{X}} - \sum x^2]^2 \sum_e x^4 \Big\} \end{aligned} \quad (15)$$

Equations (5)–(13) have been programmed for an IBM 1620 Computer (Appendix B), and are used with the insertion-loss measurement data reduction. If the probable errors resulting from the use of Eqs. (6) or (12) indicate significant difference between odd and even point values, a fault is indicated in one of the transmission-line connections. The probable errors given by (6) and (7)—and (12) and (13)—are due only to the random measurement errors and do not indicate bias or nonlinearity errors.

Measurement data for a precision insertion-loss evaluation may be reduced both by the first-order and the second-order analyses. This technique yields two values for the mean insertion loss, each with an associated probable error. The value of L from the data reduction technique which gives the smaller probable error is chosen.

IV. EXPERIMENTAL RESULTS

This section describes part of one application of the dc-potentiometer test set to the high-precision measurement of microwave insertion loss. This task was the calibration of a liquid-nitrogen-cooled waveguide termination at 2295 Mc. The calibration required the precision measurement of the insertion loss of various WR 430 Waveguide transmission-line sections as well as the evaluation of a silicone grease film and a mylar window between waveguide flanges.

The WR 430 Waveguide microwave calibration heads for the insertion-loss test set are shown in Fig. 7. The

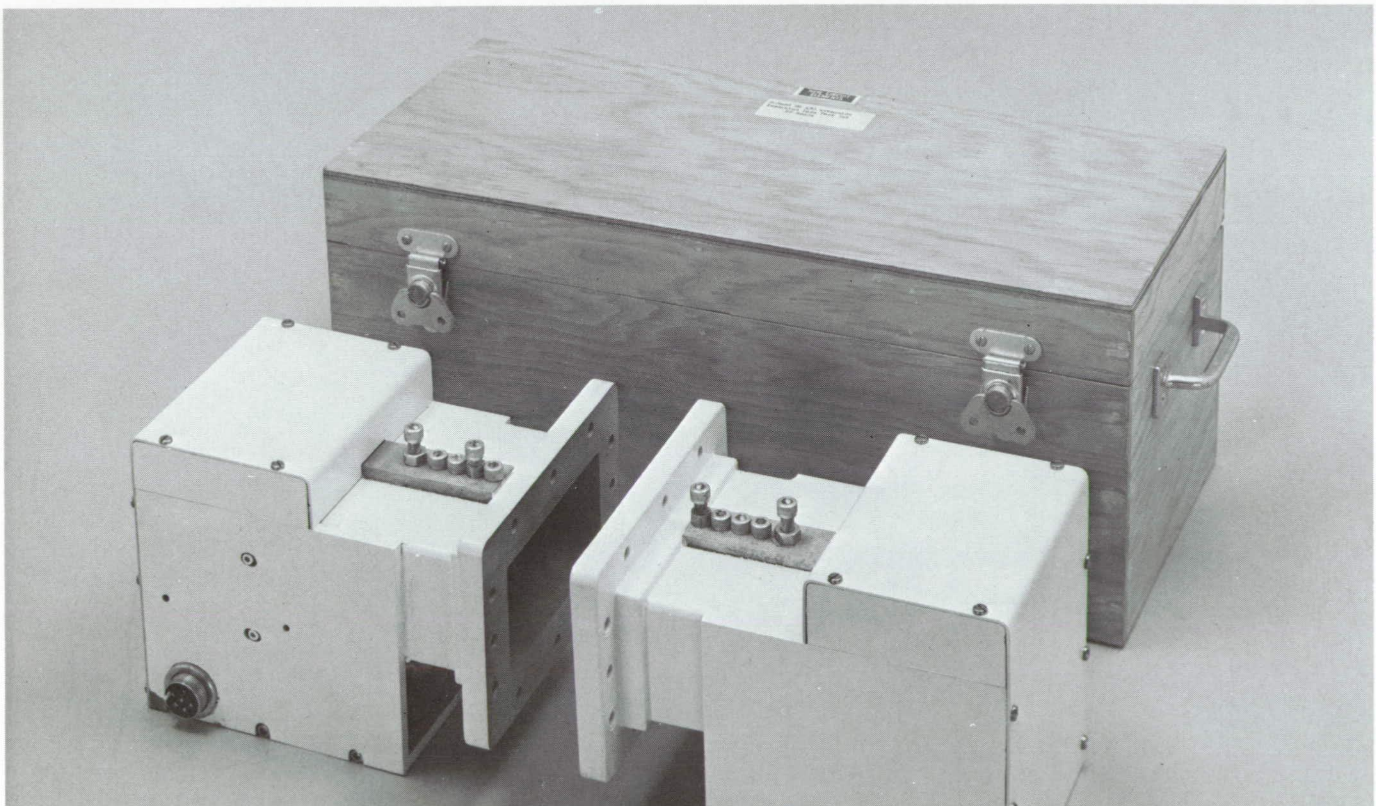

Fig. 7. Photograph of the WR 430 Waveguide microwave calibration heads of insertion-loss test set

special heavy flanges are pinned for proper waveguide alignment. The VSWR was evaluated with a precision waveguide sliding termination and reduced to less than 1.005. Each transmission-line test section (Part Nos. 226 and 239) shown in Fig. 8 was measured, as well as a section of copper waveguide (Part No. 240) and various combinations of No. 239 and No. 240. The insertion loss of Nos. 226 and 239 is fundamental to the calibration of the cooled termination. Part No. 240 was examined in order that the silicone grease film and the mylar window could be evaluated in conjunction with No. 239.

Part No. 226 is a stainless steel WR 430 Waveguide section 4.0 in. long. It has a wall thickness of 0.025 in., and copper plating and gold flash on the inside of 0.000120 in. and 0.000010 in. respectively. It is filled with a half-wavelength piece of polystyrene foam. The VSWR of this section is less than 1.005. The polystyrene window is required to prevent moisture condensation which will form on thin membrane windows. Part No. 239 is a 4.0-in. length of brass waveguide section with an iridite plate. Part No. 240 is a 4.0-in. length of copper waveguide section.

The following insertion-loss measurements were made:

Part Nos. 226, 239, and 240 alone.

Part Nos. 239 and 240 in combination.

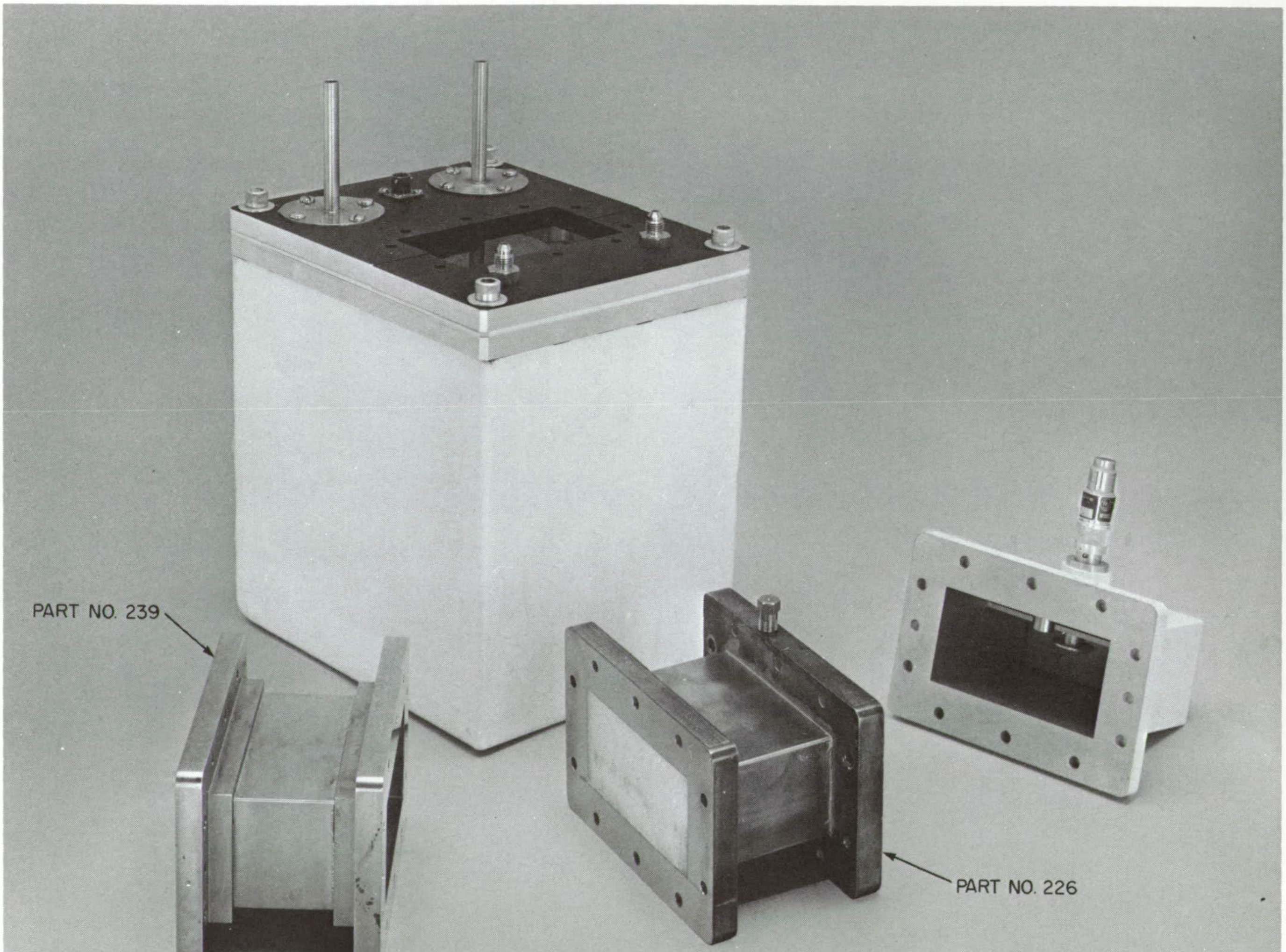

Fig. 8. Photograph of WR 430 Waveguide liquid-nitrogen-cooled termination assembly

Part Nos. 239 and 240 with a 0.001-in.-thick mylar window between them.

Part Nos. 239 and 240 with a thin silicone grease film on their mating flanges.

Five sets of eleven measurements were made on each unknown. After every set the match of each calibration head was checked and corrected if the VSWR became 1.005 or worse. The flanges of the calibration heads and of the unknown were lapped after every set as well. All of the waveguide flanges were pinned except the stainless steel Part No. 226. This section was aligned with the bolt holes using a drill rod before bolting together.

Table 2. Insertion loss of WR 430 Waveguide Part No. 239, Set No. 2 (first-order analysis) at 2295 Mc

Ratio R	y, db	Difference from straight-line fit, db $\times 10^{-3}$	Insertion loss L, db
0.99665	0.029146	−0.34339	0.00244
0.99637	0.031586	−0.12339	0.00244
0.99665	0.029146	0.18900	0.00218
0.99640	0.031325	0.14748	0.00279
0.99672	0.028536	0.11137	0.00227
0.99646	0.030802	0.15686	0.00279
0.99678	0.028013	0.12091	0.00200
0.99655	0.030017	−0.09520	0.00244
0.99683	0.027577	0.21763	0.00192
0.99661	0.029494	−0.08575	0.00296
0.99695	0.026532	−0.29551	
First-order constants:		a = 0.02940 db	
		b = −0.00027 db/measurement	
Mean insertion loss:		$\bar{L}$ = 0.00249 db	
Probable errors:		PE_{yo} = 0.00029 db	
		PE_{ye} = 0.00008 db	
		$PE_{\bar{L}}$ = 0.00007 db	

Table 2 shows a typical data set taken on Part No. 239. This table is also the format of the computer output for the straight-line (first-order) analysis. The first column lists the readings taken. The y values in the second column are those ratios converted to db by

$$y = 20 \log_{10} R \qquad (16)$$

The third column shows the difference of each y value from the straight-line fit and the last column is the insertion loss in db for each pair of readings. The probable errors from Eqs. (6) and (7) are also shown. The same data are shown in Table 3 with the second-order analysis and are plotted in Figs. 9 and 10. The translations in the

Table 3. Insertion loss of WR 430 Waveguide Part No. 239, Set No. 2 (second-order analysis) at 2295 Mc

Ratio R	y, db	Difference from second-order curve, db $\times 10^{-3}$	Insertion loss L, db
0.99665	0.029146	−0.09634	0.00244
0.99637	0.031586	0.02484	0.00244
0.99665	0.029146	0.13959	0.00218
0.99640	0.031325	0.07337	0.00279
0.99672	0.028536	−0.08627	0.00227
0.99646	0.030802	0.00864	0.00279
0.99678	0.028013	−0.07672	0.00200
0.99655	0.030017	−0.16931	0.00244
0.99683	0.027577	0.16822	0.00192
0.99661	0.029464	0.06249	0.00296
0.99695	0.026532	−0.04847	
Second-order constants:		a' = 0.02958 db	
		b' = 0.00027 db/measurement	
		c' = −0.00002 db/(measurement)^2	
Mean insertion loss:		$\bar{L}$ = 0.00242 db	
Probable errors:		PE_{yo} = 0.00007 db	
		PE_{ye} = 0.00006 db	
		$PE_{\bar{L}}$ = 0.00004 db	

x-axes used for simplification of the analysis consist of subtracting the middle measurement number from each original measurement number. In order to expand the scale, the constants a and a' are subtracted from the first- and second-order curves respectively and also from the data points modified by $\bar{L}/2$. The curves pass through the origin due to the scale translations. The experimental points in Figs. 9 and 10 may not have the same appearance because different $\bar{L}/2$ values are added to and subtracted from the odd and even data points.

The averages for the five sets of independent measurements for Part No. 239 are shown in Table 4 using the

Table 4. Mean insertion loss of WR 430 Waveguide Part No. 239 at 2295 Mc

Measurement set No.	Mean insertion loss L, db	Probable errors		
		PE_{yo}, db	PE_{ye}, db	PE_{L}, db
1	0.00232	0.00024	0.00018	0.00013
2	0.00249	0.00010	0.00007	0.00006
3	0.00222	0.00014	0.00012	0.00009
4	0.00229	0.00015	0.00006	0.00007
5	0.00242	0.00007	0.00006	0.00004

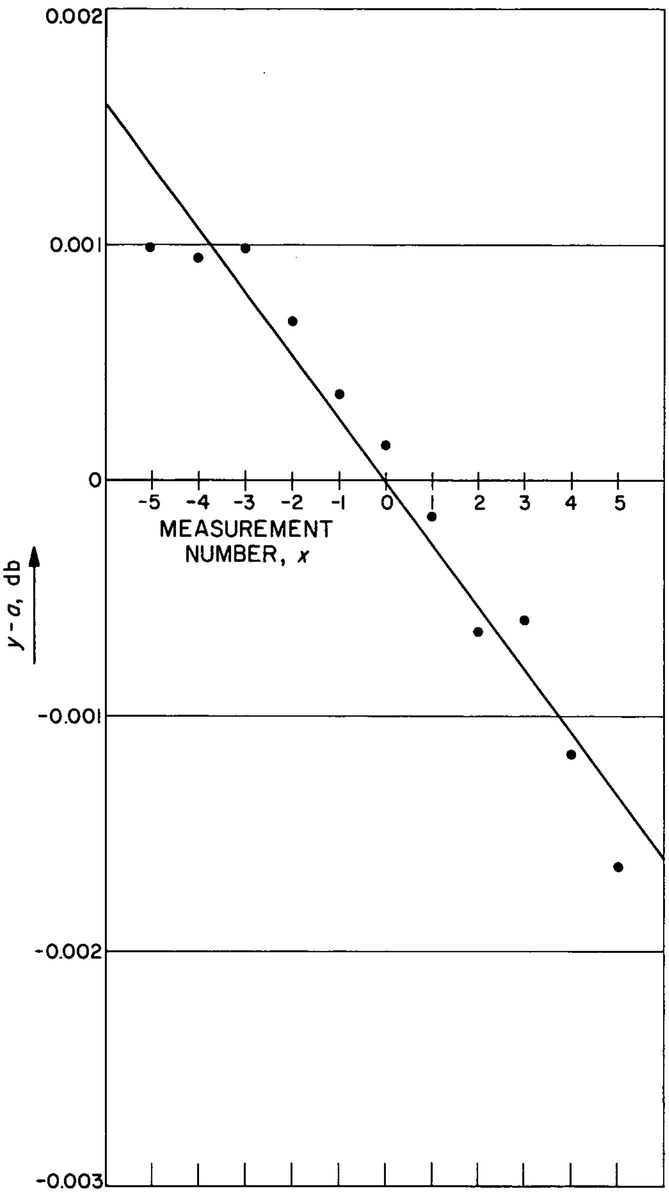

Fig. 9. Experimental insertion-loss data for Part No. 239, Set No. 2, with best least-squares straight line

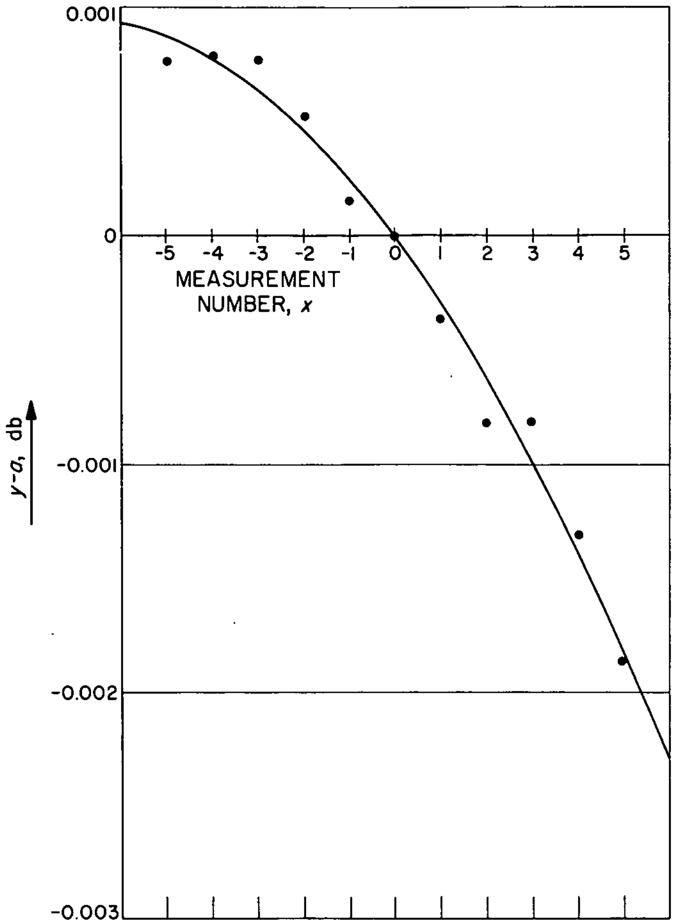

Fig. 10. Experimental insertion-loss data for Part No. 239, Set No. 2, with best second-order least-squares curve

curve with the lowest probable error for $\bar{L}$. The mean insertion loss for each set $\bar{L}_1$ to $\bar{L}_5$ is shown as well as probable errors of the odd and even y values, and, finally, the probable error of the mean insertion loss of each set. The weighted mean of the insertion loss is then found with the probable error of each set used as a weighting factor, and the grand mean $\bar{\bar{L}}$ is shown in Table 5. The average measured insertion loss of each set is shown as a horizontal bar in Fig. 11. The associated probable errors are shown as vertical lines. The grand weighted mean $\bar{\bar{L}}$ and its probable error are also shown.

Table 5. Summary of 2295-Mc insertion-loss measurements of WR 430 Waveguide components

Part No.	Description	Insertion loss $\bar{\bar{L}}$, db	Probable error of $\bar{\bar{L}}$, db $\times 10^{-4}$
226	Stainless steel	0.00690	0.33
239	Brass	0.00239	0.28
240	Copper	0.00118	0.31
239, 240	Sum	0.00357	0.42
239, 240	Combination	0.00358	0.38
239, 240	With mylar window	0.00413	0.27
239, 240	With silicone grease	0.00355	0.29

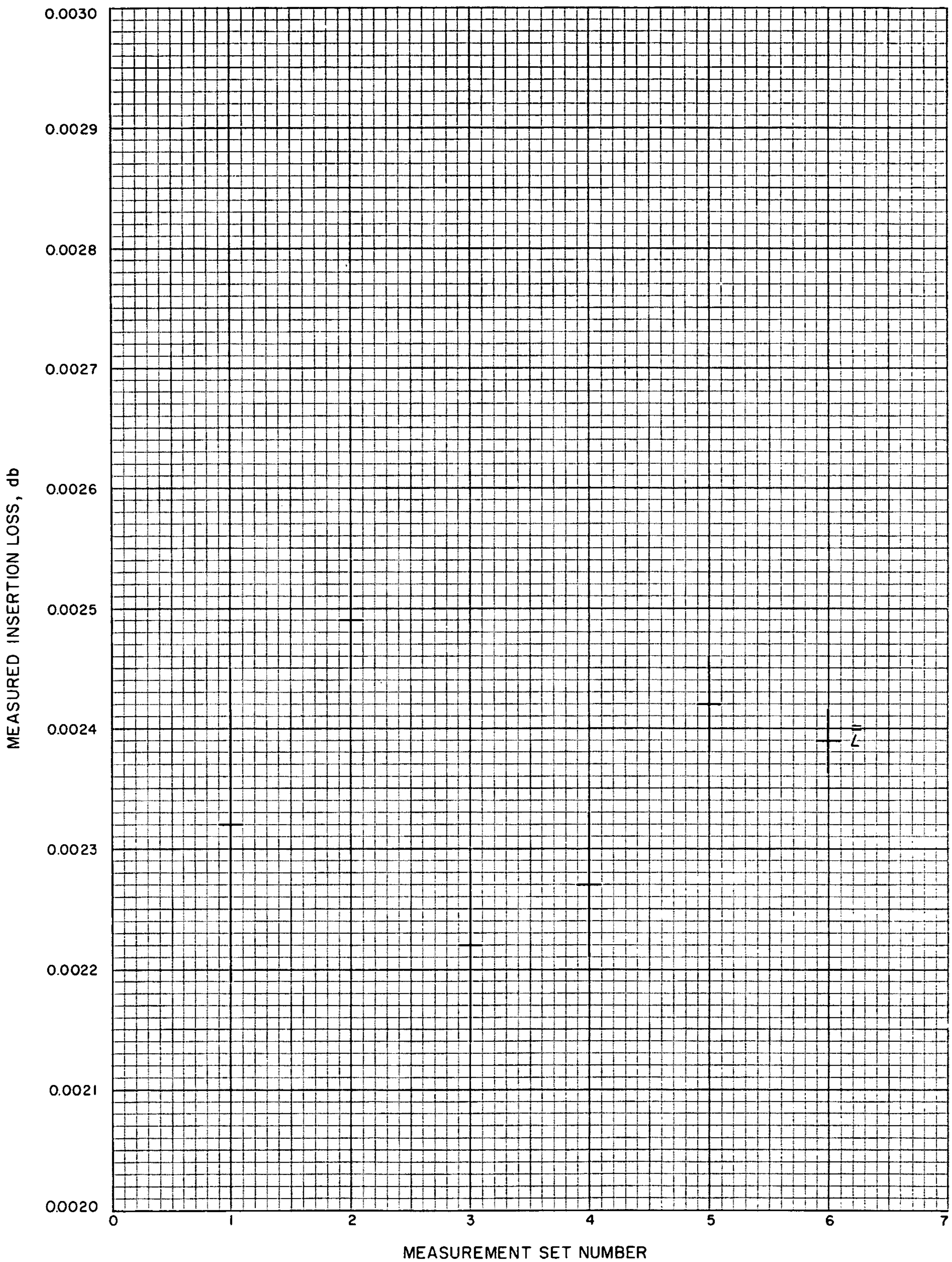

Fig. 11. Graph of insertion-loss measurements of Part No. 239 at 2295 Mc

A nonweighted mean $\bar{\bar{L}}'$ of $\bar{L}_1$–$\bar{L}_5$ was also calculated with its associated probable error. This mean differed at most by a few parts in 10^5 from the weighted mean. If the probable error of $\bar{\bar{L}}'$ were appreciably greater than the probable error of $\bar{\bar{L}}$, bias errors would be indicated for some of the individual sets. Measurement time is approximately 3 hr for the five sets of readings required for a grand mean.

The insertion loss of the brass waveguide Part No. 239 is shown to be 0.00239 db with a probable error of 0.28×10^{-4} db; the insertion loss of the copper Part No. 240 is 0.00118 db with a probable error of 0.31×10^{-4} db. The sum of these two insertion losses is 0.00357 db, with their probable errors combined as 0.42×10^{-4} db. This sum, 0.00357 db, must be compared with the measured value of the insertion loss of Nos. 239 and 240 in combination, which is 0.00358 db—a difference of 10^{-5} db. The probable errors of the sums are slightly higher than the probable errors of the combinations, as expected.

The theoretical insertion loss for No. 240 (copper waveguide 4 in. in length) is 0.00091 db (Ref. 7). This theoretical value does not account for surface roughness, metal impurities, or nonhomogeneity. A correction estimate for surface roughness increases the insertion loss by 15% (Ref. 7) to 0.00105 db. This agrees with our measurement to within 0.00013 db.

Part No. 240 was then used in conjunction with No. 239 to evaluate a 0.001-in.-thick mylar window. This material is presently under consideration as an additional window. Table 5 shows that the excess insertion loss due to the mylar window is 0.0055 db with a probable error of less than 10^{-4} db.

A thin silicone grease film is sometimes required between the flanges of waveguide sections of dissimilar metal. This film helps prevent corrosion which changes the insertion loss. The effect of a silicone grease film between the flanges of Nos. 239 and 240 was investigated. The results are presented in Table 5 and these show that (a) the effect of the film on the insertion loss is negligible, and (b) the insertion loss appears to drop when the grease is added. The lower insertion loss in (b) may be apparent, and explained by the spread of the probable errors. On the other hand, this effect may be real and the insertion loss lower owing to the high dielectric constant of the grease filling the low spots of the flange face and thus decreasing the reactance.

V. CONCLUSION

The average of the probable errors of the measured values of $\bar{\bar{L}}$ from Table 5 is 0.31×10^{4} db. It may be concluded, therefore, that if statistical methods are used with a sufficient number of observations, and if the measurement procedure is carried out as described with the indicated precautions, then insertion loss can be measured with a probable error of less than 10^{-4} db. This does not include the bias and nonrandom errors and waveguide tolerance errors. The estimated overall accuracy of the test set, expressed in terms of the insertion loss measured, is tabulated in Table 6.

The upper operating frequency is presently limited to 40 Gc by the thermistor mount characteristics of the Hewlett–Packard 431B Power Meter, although the test set has operated at frequencies as high as 90 Gc with transitions, resulting in slight performance degradation.

Table 6. Estimated accuracy of microwave insertion-loss test set

Insertion-loss measurement range, db	Accuracy, db
0–10	±0.0001 ±0.1% of measurement value in db
10–15	±0.5% of measurement value in db
15–20	±1.0% of measurement value in db

NOMENCLATURE

a, a', b, b', c'	First- and second-order analysis coefficients
e	Even measurements (unknown connected)
$\bar{L}$	Mean insertion loss from one set of measurements
$\bar{\bar{L}}$	Mean of mean insertion losses from several sets of measurements
m	Total number of data points in single measurement set
o	Odd measurements (unknown disconnected)
PE_a	Probable error of a
PE_b	Probable error of b
PE_c	Probable error of c
$PE_{\bar{L}}$	Probable error of $\bar{L}$
PE_{ye}	Probable error of even data points
PE_{yo}	Probable error of odd data points
S_1, S_{1A}	Switches
x	Measurement number
$\underline{\bar{X}}$	$\sum x^2 - \sum_o x^2$
y	Data point
$\underline{\bar{Y}}$	$\sum y - \sum_o y$
α	Coefficients of data points
Δ	Determinant on x and m
σ	Standard deviation

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APPENDIX A

Derivation of the Second-Order Equations

The measured data points are modified in the same manner as in the first-order analysis. A constant $\bar{L}/2$ is added to each power-level measurement with the unknown disconnected, and subtracted from each power-level measurement with the unknown connected. The standard deviation σ' of the modified data points from the best-fit parabola $a' + b'x + c'x^2$ is given by

$$(\sigma')^2 = \frac{1}{m}\sum_{o}(y + \bar{L}/2 - a' - b'x - c'x^2)^2 + \frac{1}{m}\sum_{e}(y - \bar{L}/2 - a' - b'x - c'x^2)^2 \qquad \text{(A-1)}$$

where

a' = constant of second-order curve, db

b' = slope of second-order curve, db/measurement number

c' = rate of change of the slope of the second-order curve, db/(measurement number)^2

Differentiating Eq. (A-1) with respect to a', b', c', and L, and equating to zero gives

$$\left.\begin{aligned}
&\sum y + \bar{L}/2 - a'm - b'\sum x - c'\sum x^2 = 0\\
&\sum xy + \bar{L}/2\left(\sum_{o} x - \sum_{e} x\right) - a'\sum x - b'\sum x^2 - c'\sum x^3 = 0\\
&\sum x^2 y + \bar{L}/2\left(\sum_{o} x^2 - \sum_{e} x^2\right) - a'\sum x^2 - b'\sum x^3 - c'\sum x^4 = 0\\
&\left(\sum_{o} x - \sum_{e} x\right) + m(\bar{L}/2) - a' - b'\left(\sum_{o} x - \sum_{e} x\right) - c'\left(\sum_{o} x^2 - \sum_{e} x^2\right) = 0
\end{aligned}\right\} \qquad \text{(A-2)}$$

If the middle measurement number is chosen as the origin, then the summation of all odd powers of x is zero. Thus

$$\sum_{o} x = \sum_{e} x = \sum x = \sum x^3 = 0 \qquad \text{(A-3)}$$

and writing

$$\sum_{o} x^2 - \sum_{e} x^2 = \underline{X} \qquad \text{(A-4)}$$

$$\sum_{o} y - \sum_{e} y = \underline{Y} \qquad \text{(A-5)}$$

Equations (A-2) then become

$$\left.\begin{aligned}
\sum y &= -\bar{L}/2 + a'm + c'\sum x^2\\
\sum xy &= b\sum x^2\\
\sum x^2 y &= -\underline{X}(\bar{L}/2) + a'\sum x^2 + c\sum x^4\\
\underline{Y} &= -m(\bar{L}/2) + a' + c\underline{X}
\end{aligned}\right\} \qquad \text{(A-6)}$$

Solving for $\bar{L}$, a', b', and c':

$$\bar{L} = 2\,\frac{\begin{vmatrix} \sum y & m & 0 & \sum x^2 \\ \sum xy & 0 & \sum x^2 & 0 \\ \sum x^2 y & \sum x^2 & 0 & \sum x^4 \\ \underline{Y} & 1 & 0 & \underline{X} \end{vmatrix}}{\begin{vmatrix} -1 & m & 0 & \sum x^2 \\ 0 & 0 & \sum x^2 & 0 \\ \underline{X} & \sum x^2 & 0 & \sum x^4 \\ -m & 1 & 0 & \underline{X} \end{vmatrix}} \tag{A-7}$$

$$\bar{L} = \frac{2}{\Delta}\{\sum y\,[\underline{X}\sum x^2 - \sum x^4] - \sum x^2 y\,[m\underline{X} - \sum x^2]$$

$$+ \underline{Y}\,[m\sum x^4 - (\sum x^2)^2]\} \tag{A-8}$$

where

$$\Delta = (-1)\,[\underline{X}\sum x^2 - \sum x^4] + \underline{X}\,[mX - \sum x^2]$$
$$- m\,[m\sum x^4 - (\sum x^2)^2]$$

$$a' = \frac{\begin{vmatrix} -1 & \sum y & 0 & \sum x^2 \\ 0 & \sum xy & \sum x^2 & 0 \\ \underline{X} & \sum x^2 y & 0 & \sum x^4 \\ -m & \underline{Y} & 0 & \underline{X} \end{vmatrix}}{\begin{vmatrix} -1 & m & 0 & \sum x^2 \\ 0 & 0 & \sum x^2 & 0 \\ \underline{X} & \sum x^2 & 0 & \sum x^4 \\ -m & 1 & 0 & \underline{X} \end{vmatrix}} \tag{A-9}$$

$$a' = \frac{1}{\Delta}\{-\sum y\,[(\underline{X})^2 + m\sum x^4] + \sum x^2 y\,[-\underline{X} + m\sum x^2]$$
$$- \underline{Y}\,[-\sum x^4 + \underline{X}\sum x^2]\} \tag{A-10}$$

$$b' = \frac{1}{\Delta}\begin{vmatrix} -1 & m & \sum y & \sum x^2 \\ 0 & 0 & \sum xy & 0 \\ \underline{X} & \sum x^2 & \sum x^2 y & \sum x^4 \\ -m & 1 & \underline{Y} & \underline{X} \end{vmatrix} \tag{A-11}$$

$$b' = \frac{\sum xy}{\sum x^2} \tag{A-12}$$

$$c' = \frac{1}{\Delta}\begin{vmatrix} -1 & m & 0 & \sum y \\ 0 & 0 & \sum x^2 & \sum xy \\ \underline{X} & \sum x^2 & 0 & \sum x^2 y \\ -m & 1 & 0 & \underline{Y} \end{vmatrix} \tag{A-13}$$

$$c' = \frac{1}{\Delta}\{\sum y\,[-\underline{X} + m\sum x^2] - \sum x^2 y\,[m^2 - 1] + \underline{Y}\,[-\sum x^2 + mX]\} \tag{A-14}$$

The probable errors of the data points are

$$\left.\begin{aligned} PE_{yo} &= 0.6745\left(\frac{\sum_o [y + (\overline{L}/2) - a' - b'x - c'x^2]^2}{(m+1)/2}\right)^{1/2} \\ PE_{ye} &= 0.6745\left(\frac{\sum_e [y - (\overline{L}/2) - a' - b'x - c'x^2]^2}{(m-1)/2}\right)^{1/2} \end{aligned}\right\} \tag{A-15}$$

To derive a probable error of the mean insertion loss L, (A-8) is written in the form

$$\overline{L} = \frac{2}{\Delta}\{\sum_o \alpha y + \sum_e \alpha y\} \tag{A-16}$$

This is expanded in terms of α, the coefficients of the data points, as

$$\overline{L} = \frac{2}{\Delta}\{\alpha_1 y_1 + \alpha_3 y_3 + \cdots\} + \frac{2}{\Delta}\{\alpha_2 y_2 + \alpha_4 y_4 + \cdots\} \tag{A-17}$$

Substituting (A-8) into (A-16) and (A-17) and summing over the data points gives expressions for the odd and even coefficients:

$$\sum_o \alpha^2 = \frac{4}{\Delta^2}\Big\{[\underline{X}\sum x^2 + (m-1)\sum x^4 - (\sum x^2)^2]^2\left(\frac{m+1}{2}\right) - 2\,[\underline{X}\sum x^2 + (m-1)\sum x^4 - (\sum x^2)^2]\,[m\underline{X} - \sum x^2]\sum_o x^2 + [m\underline{X} - \sum x^2]^2\sum_o x^4\Big\} \tag{A-18}$$

$$\sum_e \alpha^2 = \frac{4}{\Delta^2}\Big\{[\underline{X}\sum x^2 - (m+1)\sum x^4 + (\sum x^2)^2]^2\left(\frac{m-1}{2}\right) - 2\,[\underline{X}\sum x^2 - (m+1)\sum x^4 + (\sum x^2)^2]\,[m\underline{X} - \sum x^2]\sum_e x^2 + [m\underline{X} - \sum x^2]^2\sum_e x^4\Big\} \tag{A-19}$$

The probable error of the mean insertion loss is then

$$PE_{\overline{L}} = ([PE_{yo}]^2\sum_o \alpha^2 + [PE_{ye}]^2\sum_e \alpha^2)^{1/2} \tag{A-20}$$

APPENDIX B

Computer Programs

These two programs, one for the first-order analysis, the other for the second-order analysis, are written in Fortran for an IBM 1620 Computer. Symbol m is the total number of observed points R, and each program can handle a maximum of $m = 50$. The values of R and M are the inputs required. The output of each program prints the observed points R; the ratios y converted to db; the individual insertion-loss measurement L; the straight line and parabola constants a, b, and a', b', c'; the probable errors of the odd and even data points, PE_{yo}, PE_{ye}; the mean insertion losses $\overline{L}$; and the probable errors of the mean insertion loss, $PE_{\overline{L}}$.

I. FIRST-ORDER ANALYSIS

```
 1 FORMAT(1H1,48HINSERTION LOSS DATA REDUCTION,1ST ORDER ANALYSIS)
 2 FORMAT(1H0,6X1HR,12X1HY,15X1HE,14X2HL1)
 3 FORMAT(1H ,F9.6,5XE11.5,5XE11.5,5XE11.5)
 5 FORMAT(I5)
 6 FORMAT(F10.0)
 7 FORMAT(1H0,4X2HA=E11.5,5X2HB=E11.5)
 8 FORMAT(1H ,4X,5HPEYO=E11.5,3X5HPEYE=E11.5)
 9 FORMAT(1H ,4X2HL=E11.5,6X4HPEL=E11.5)
12 FORMAT(1H ,F9.6,2(5X,E11.5))
   DIMENSION R(50),Y(50),X(50),E(50),EL1(50)
25 READ 5,M
   EM=M
   N=M-1
   PRINT 1
   PRINT 2
   SUME=0.0
   SUMO=0.0
   SUMXY=0.0
   SUMXQ=0.0
   SEOQ=0.0
   SEEQ=0.0
   DO 30 I=1,M
   READ 6,R(I)
30 Y(I)=-20.0*0.43429*LOGF(R(I))
   DO 32 I=1,N,2
32 EL1(I)=Y(I+1)-Y(I)
   DO 33 I=2,M,2
33 EL1(I)=Y(I)-Y(I+1)
   DO 35 I=2,M,2
35 SUME=SUME+Y(I)
   ELE=(2./(EM-1.))*SUME
   DO 40 I=1,M,2
40 SUMO=SUMO+Y(I)
   ELO=(2./(EM+1.))*SUMO
   EL=ELE-ELO
   A=(ELE+ELO)/2.
   XMIN=(-(EM-1.0)/2.0)-(1.0)
   DO 45 I=1,M
   X(I)=XMIN+1.
   XMIN=X(I)
   XY=X(I)*Y(I)
   XQ=X(I)**2
   SUMXY=SUMXY+XY
45 SUMXQ=SUMXQ+XQ
   B=SUMXY/SUMXQ
   DO 50 I=1,M,2
   E(I)=Y(I)+EL/2.-A-B*X(I)
   EOQ=E(I)**2
50 SEOQ=SEOQ+EOQ
   DO 55 I=2,M,2
   E(I)=Y(I)-A-B*X(I)-EL/2.
   EEQ=E(I)**2
55 SEEQ=SEEQ+EEQ
   PEYO=.6745*SQRTF(SEOQ/((EM+1.)/2.))
   PEYE=.6745*SQRTF(SEEQ/((EM-1.)/2.))
   F1=(2./(EM+1.))*PEYO**2
   F2=(2.0/(EM-1.0))*PEYE**2
   PEL=SQRTF(F1+F2)
   DO 60 I=1,N
60 PRINT 3,R(I),Y(I),E(I),EL1(I)
   PRINT 12,R(M),Y(M),E(M)
   PRINT 7,A,B
   PRINT 8,PEYO,PEYE
   PRINT 9,EL,PEL
   GO TO 25
   END
```

II. SECOND-ORDER ANALYSIS

```
 1 FORMAT(1H1,48HINSERTION LOSS DATA REDUCTION,2ND ORDER ANALYSIS)
 2 FORMAT(1H0,6X1HR,9X1HY,13X1HE,13X2HLI)
 3 FORMAT(1H ,4(4X,E11.5))
 4 FORMAT(I5)
 5 FORMAT(F10.0)
 6 FORMAT(1H0,2HA=E11.5,3X2HB=E11.5,3X2HC=E11.5)
 7 FORMAT(1H ,5HPEYO=E11.5,5X5HPEYE=E11.5)
 8 FORMAT(1H ,2HL=E11.5,6X4HPEL=E11.5)
 9 FORMAT(1H ,3(3X,E11.5))
   DIMENSION R(50),Y(50), X(50), E(50), ELI(50)
25 READ4,M
   PRINT1
   PRINT2
   EM=M
   N=M-1
   DO35 I=1,M
   READ 5,R(I)
35 Y(I)=-20.0*0.43429*LOGF(R(I))
   Q=0.0
   F=0.0
   G=0.0
   H=0.0
   P=0.0
   S=0.0
   U=0.0
   V=0.0
   W=0.0
   EO=0.0
   EE=0.0
   DO40 I=1,N,2
   ELI(I)=Y(I+1)-Y(I)
40 CONTINUE
   DO42 I=2,N,2
   ELI(I)=Y(I)-Y(I+1)
42 CONTINUE
   DO44I=1,M,2
   Q=( +Y(I)
44 CONTINUE
   DO46 I=2,N,2
   F=F+Y(I)
46 CONTINUE
   IC=(-M+1)/2-1
   DO48I=1,M
   X(I)=IC+I
48 CONTINUE
   DO50 I=1,M,2
   G=G+(X(I))**2
50 CONTINUE
   DO52 I=2,N,2
   H=H+(X(I))**2
52 CONTINUE
   DO54 I=1,M
   P=P+(X(I))**4
54 CONTINUE
   DELTA=-(G-H)*(G+H)+P+(G-H)*(EM*(G-H)-(G+H))-EM*(EM*P-(G+H)**2)
   DO56 I=1,M
   S=S+Y(I)*(X(I))**2
56 CONTINUE
  0T=(2./DELTA)*((Q+F)*((G-H)*(G+H)-P)-S*(EM*(G-H)-(G+H))
  1+(Q-F)*(EM*P-(G+H)**2))
   DO58 I=1,M
   U=U+X(I)*Y(I)
58 CONTINUE
  0A=(1./DELTA)*((Q+F)*((G-H)**2-EM*P)+S*(H-G+EM*(G+H))
  1-(Q-F)*(-P+(G-H)*(G+H)))
   B=U/(G+H)
  0C=(1./DELTA)*((Q+F)*(H-G+EM*(G+H))-S*(EM**2-1.)
  1+(Q-F)*(-G-H+EM*(G-H)))
   DO6C I=1,M,2
   E(I)=Y(I)+T/2.-A-B*X(I)-C*(X(I))**2
   EO=E(I)**2 +EO
60 CONTINUE
   DO62 I=2,N,2
   E(I)=Y(I)-T/2.-A-B*X(I)-C*(X(I))**2
   EE=E(I)**2 +EE
62 CONTINUE
   PEYO=.6745*SQRTF(EO/((EM+1.)/2.))
   PEYE=.6745*SQRTF(EE/((EM-1.)/2.))
   DO64 I=1,M,2
   V=V+(X(I))**4
64 CONTINUE
   DO66 I=2,N,2
   W=W+(X(I))**4
66 CONTINUE
  0AA=((EM+1.)/2.)*(((G-H)*(G+H)+(EM-1.)*P-(G+H)**2)**2)
  1-(2.)*G*(EM*(G-H)-(G+H))*((G+H)*(G-H)+(EM-1.)*P-(G+H)**2)
  2+V*((EM*(G-H)-(G+H))**2)
  0BB=((EM-1.)/2.)*(((G-H)*(G+H)-(EM-1.)*P+(G+H)**2)**2)
  1-(2.)*H*(EM*(G-H)-(G+H))*((G+H)*(G-H)-(EM-1.)*P-(G+H)**2)
  2+W*((EM*(G-H)-(G+H))**2)
   PEL=2.*SQRTF(AA*((PEYO/DELTA)**2)+BB*((PEYE/DELTA)**2))
   DO68 I=1,N
   PRINT 3,R(I),Y(I),E(I),ELI(I)
68 CONTINUE
   PRINT 9,R(M),Y(M),E(M)
   PRINT 6,A,B,C
   PRINT 7,PEYO,PEYE
   PRINT 8,T,PEL
   GO TO 25
   END
```