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Plane Wave Scattering from a
Rough Surface with Correlated Large and
Small Scale Orders of Roughness

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ABSTRACT

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A rough surface is assumed to be composed of a slowly
varying large-scale roughness, and a rapidly varying small-
scale roughness, and that these are correlated. The exponential
form of both the auto-correlations of these surfaces and their
cross-correlation were used. This paper shows that the return
from such a composite surface for a plane wave incidence can
have either one of the following terms contribute a major portion
of the total return power, namely the large scale roughness
term, small scale roughness term and the cross-correlation term.
It is therefore concluded that contribution due to each term must
be evaluated before neglecting its effect on the total return
power. These results are equally applicable to acoustic and
radar cases.

Introduction

A number of rough surface models have been proposed in the analysis of electromagnetic wave scattering from a rough surface. The simplest model is that of a surface, $\zeta(x,y)$ where roughness, i.e., deviation from the mean surface height, can be expressed by a single statistical distribution. A better model of the rough surfaces often met in practical applications is to consider these surfaces as the superposition of several types of roughness. For example, terrain may consist of such large scale features as rocks and mountains on which are superposed small scale roughnesses due to rock, vegetation, etc. As another example, the typical sea surface is comprised of "swell", "sea", and "ripple". For the case of radar return from the moon and planets several papers [1,2] have suggested that small scale roughness, rather than large scale, may play an important part in the backscatter signal.

Beckmann [3] has considered such a composite rough surface model where the surface $\zeta(x,y)$ is the superposition of n types of roughnesses

$$\zeta(x,y) = \zeta_1(x,y) = \zeta_2(x,y) + \ldots + \zeta_n(x,y)$$

where ζ_1, $\zeta_2 \ldots \zeta_n$ are independent stationary random functions each with its own statistical distribution and correlation function. However, natural evidence about the surface of the earth, and perhaps the surfaces of the planets, indicates that small scale roughnesses as caused by such factors as wind or water erosion, landslides or rockfalls, and artificial handling

are definitely dependent upon the local large scale roughness.

In light of such natural evidence this paper will investigate scattering from a composite rough surface where the large and the small scale roughness are correlated. Although this paper deals with a surface composed of two dependent random processes, the procedures used can be readily extended to any number of dependent random processes that might constitute the surface.

General Theory

The general Kirchoff solution for a perfectly conductive rough surface whose mean value coincides with the x-y plane and whose radii of curvature are always greater than the incident wavelength is, according to Beckmann (4) :

$$E\,(\Theta_1;\Theta_2;\Theta_3) = \frac{E_N F}{A} \int_{-X}^{X} \int_{-Y}^{Y} \exp\,\left[\,i\,\vec{v}\cdot\vec{r}\,\right]\,dx\,dy$$

The scattering geometry is as shown in Figure 1. Furthermore, $A = 4XY$, the area of the scattering surface; $\vec{k}_1$ and $\vec{k}_2$ are the propagation vectors of the incident and scattered waves. $\vec{v} = \vec{k}_1 - \vec{k}_2$, with rectangular components:

$$v_x = \frac{2\pi}{\lambda}\,(\sin\Theta_1 - \sin\Theta_2\,\cos\Theta_3)$$

$$v_y = \frac{-2\pi}{\lambda}\,(\sin\Theta_2\,\sin\Theta_3)$$

$$v_z = \frac{-2\pi}{\lambda}\,(\cos\Theta_1 + \cos\Theta_2)$$

$\vec{r} = x\vec{i} + y\vec{j} + (x,y)\,\vec{k}$, the radius vector to any point in the rough surface.

$$F = \frac{1 + \cos \Theta_1 \cos \Theta_2 - \sin \Theta_1 \sin \Theta_2 \cos \Theta_3}{\cos \Theta_1 + \cos \Theta_2}$$

$E_N = \dfrac{ike^{ikR_O}A}{2\pi R_O}$, the amplitude of the field reflected by a

smooth plane surface of area A at distance R_O for normal incidence

of a unit plane wave, $E_1 = e^{i(\vec{k}_1 \cdot \vec{r})}$.

In the derivation of (1) edge effect terms are neglected

for it is assumed $A \gg \lambda^2$; depolarization effects are ignored by

considering only the scalar value of $\vec{E}_2$ as the question of direc-

tion of $\vec{E}_2$ is more difficult to solve. Experimental measurements

indicate that the scattered field is largely determined by sur-

face roughness rather than by surface conductivity, so the assump-

tion of perfect conductivity for the rough surface is not unduly

restrictive.

The surface roughness $\zeta(x,y)$ will be considered as the sum

of a large scale roughness ζ_L and a small scale roughness ζ_S

(see Figure 2); $\zeta(x,y) = \zeta_L(x,y) + \zeta_S(x,y)$ (2)

Each roughness component is assumed to have zero mean value.

Now the r.m.s. scattered power, $\langle E_2 E_2^* \rangle$ is given by

$$\langle E_2 E_2^* \rangle = \frac{E_N^2 F^2}{A} \int_{-X}^{X} \int_{-X}^{X} \int_{-Y}^{Y} \int_{-Y}^{Y} \exp\left[iv_x(x_1 - x_2) + iv_y(y_1 - y_2)\right]$$

$$\left\langle e^{iv_z(\zeta_1 - \zeta_2)} \right\rangle dx_1 dx_2 dy_1 dy_2 \qquad (3)$$

where $\zeta_1 = \zeta(x_1, y_1)$ $\zeta_2 = \zeta(x_2, y_2)$

By introducing a separation parameter, τ, defined as

$$\tau = \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 \right]^{1/2} \tag{4}$$

equation (3) becomes:

$$\left\langle E_2 E_2^* \right\rangle = \frac{2\tau F^2 E_N^2}{\lambda} \int_0^\infty J_0(v_{xy}\tau) \left\langle e^{iv_z(\zeta_1 - \zeta_2)} \right\rangle \tau d\tau \tag{5}$$

where $J_0(v_{xy}\tau)$ is the Bessel function of the first kind and order zero with argument $v_{xy}\tau = \tau(V_x^2 + V_y^2)^{1/2}$. Since only the regions near $\tau = 0$ contribute to the integral, the range of integration in (5) has, for convenience, been made infinite.

Now $\left\langle e^{iv_z(\zeta_1 - \zeta_2)} \right\rangle = \left\langle e^{iv_z\left[(Z_1 + Z_3) - (Z_2 + Z_4)\right]} \right\rangle =$ (6)

$$\int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty W_4(Z_1, Z_2, Z_3, Z_4) e^{iv_z\left[(Z_1 + Z_3) - (Z_2 + Z_4)\right]} dz_1 dz_2 dz_3 dz_4$$

with

$$Z_1 = \zeta_S(x_1, y_1) \qquad Z_2 = \zeta_S(x_2, y_2)$$

$$Z_3 = \zeta_L(x_1, y_1) \qquad Z_4 = \zeta_L(x_2, y_2)$$

$W_4(Z_1, Z_2, Z_3, Z_4)$ is the four dimensional probability density function for the variables Z_1, Z_2, Z_3, Z_4.

If $Z_1, Z_2, Z_3,$ and Z_4 are normally distributed and statistically dependent random variables, W_4 takes the form:

$$W_4(Z_1, Z_2, Z_3, Z_4) = (2\pi)^{-2} |M|^{\frac{1}{2}} \exp\left(-\frac{1}{2|M|} \sum_{i=1}^4 \sum_{j=1}^4 M_{ij} Z_i Z_j\right) \tag{7}$$

In the above expression $|M|$ is the determinant of the matrix $\left[M \right]$ that has as its i-jth element, $d_{ij} = \left\langle Z_i Z_j \right\rangle$, the second moment

of the random variables Z_i and Z_j, and M_{ij} is the cofactor of d_{ij} in the determinant $|M|$.

Evaluation of the d_{ij} gives

$$[M] = \begin{bmatrix} \sigma_S^2 & \sigma_S^2 \rho_S(\tau) & \sigma_S \sigma_L \rho_{SL}(0) & \sigma_S \sigma_L \rho_{SL}(\tau) \\ \sigma_S^2 \rho_S(\tau) & \sigma_S^2 & \sigma_S \sigma_L \rho_{SL}(\tau) & \sigma_S \sigma_L \rho_{SL}(0) \\ \sigma_S \sigma_L \rho_{SL}(0) & \sigma_S \sigma_L \rho_{SL}(\tau) & \sigma_L^2 & \sigma_L^2 \rho_L(\tau) \\ \sigma_S \sigma_L \rho_{SL}(\tau) & \sigma_S \sigma_L \rho_{SL}(0) & \sigma_L^2 \rho_L(\tau) & \sigma_L^2 \end{bmatrix} \tag{8}$$

where σ_S and σ_L are the standard deviations of ζ_S and ζ_L, $\rho_S(\tau)$ and $\rho_L(\tau)$ are the normalized auto-correlation functions of ζ_S and ζ_L, and $\rho_{SL}(\tau)$ is the normalized cross-correlation function of ζ_S and ζ_L. In the above evaluation it has been assumed that the surface is statistically isotropic, i.e., that $\langle \zeta_S(x_1,y_1) \, \zeta_L(x_2,y_2) \rangle = \langle \zeta_L(x_1,y_1) \, \zeta_S(x_2,y_2) \rangle$

Now eq. (6) may be written as:

$$\left\langle e^{iv_z (\zeta_1 - \zeta_2)} \right\rangle = H\left(v_z, -v_z, v_z, -v_z\right) \tag{9}$$

where $H(v_1, v_2, v_3, v_4)$ is the characteristic function of the joint probability density $W_4(Z_1, Z_2, Z_3, Z_4)$.

For W_4 of the form given by eq. (7), Middleton (5) gives the characteristic function as

$$H(v_1, v_2, v_3, v_4) = \exp\left[-\frac{1}{2} \sum_{j=1}^{4} \sum_{k=1}^{4} v_i v_k d_{jk} \right] \tag{10}$$

so that using eq. (8):

$$\left\langle e^{iv_z(\zeta_1 - \zeta_2)} \right\rangle = \left[\exp\left[-v_z^2 \left\{ (\sigma_S^2 + \sigma_L^2 + 2\sigma_S\sigma_L\rho_{SL}(0)) \right] \right. \right.$$

$$\left. - (\sigma_S^2\rho_S(\tau) + \sigma_L^2\rho_L(\tau) + 2\sigma_S\sigma_L\rho_{SL}(\tau) \right\} \right] \qquad (11)$$

Setting,

$$\sigma^2 = \sigma_S^2 + \sigma_L^2 + 2\sigma_S\sigma_L\rho_{SL}(0) \qquad (12)$$

$$g = v_z^2\sigma^2$$

$$G(\tau) = -v_z^2 \left\{ \sigma_S^2\rho_S(\tau) + \sigma_L^2\rho_L(\tau) + 2\sigma_S\sigma_L\rho_{SL}(\tau) \right\}$$

and applying equation (11) to equation (5),

$$\left\langle E_2 E_2^* \right\rangle = \frac{2\pi F^2 E_N}{A} \int_0^\infty J_0(V_{xy}\tau) \, \exp\left[G-g \right] \tau d\tau \qquad (13)$$

Evaluation of the Scattered Power For Slight Surface Roughness

For slight surface roughness the following assumptions are made:

$$\sigma_L \gg \sigma_S \qquad (14)$$

$$\frac{\sigma}{\lambda} = \frac{\sigma L}{\lambda} \leq 0.035$$

so that

$$g_{max} = \frac{4\pi\sigma^2}{\lambda^2} \leq 0.2 \qquad (15)$$

$$G_{max} = G_{max}(0) = g_{max} \leq 0.2$$

In order to avoid a divergent integral it is necessary to rearrange equation (13) by noting that

$$\left\langle E_2 E_2^* \right\rangle = \left\langle E_2 \right\rangle \left\langle E_2^* \right\rangle + D\{E_2\} \tag{16}$$

where

$$D\{E_2\} = \left\langle \left| E_2 - \left\langle E_2 \right\rangle \right|^2 \right\rangle$$

or $D\{E_2\}$ is the variance of the complex variable E_2. This variance $D\{E_2\}$ may be written in terms of an integral over the scattering surface as

$$D\{E_2\} = \frac{2\pi F^2 E_N^{\,2}}{A} \int_0^\infty J_0 (V_{xy}\tau) \left[\left\langle e^{iv_z(\zeta_1 - \zeta_2)} \right\rangle - \left\langle e^{iv_z\zeta_1} \right\rangle \left\langle e^{iv_z\zeta_2} \right\rangle \right] \tau d\tau$$

$$\tag{17}$$

Now,

$$\left\langle e^{iv_z\zeta_1} \right\rangle = \left\langle e^{iv_z\zeta_2} \right\rangle = \left\langle e^{iv_z\zeta} \right\rangle \tag{18}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w_2(w_1, w_2) \, e^{iv_z(w_1 + w_2)} \, dw_1 \, dw_2 = H(v_z, v_z)$$

where

$$w_1 = \zeta_S(x,y), \quad w_2 = \zeta_L(x,y),$$

and

$$H(v_1, v_2) = \exp\left(-\frac{1}{2} \left[\sigma_S^{\,2} v_1^{\,2} + 2\sigma_S\sigma_L\rho_{SL}(0) + \sigma_L^{\,2} v_z^{\,2} \right] \right)$$

Thus be using equation (12),

$$\left\langle e^{iv_z\zeta} \right\rangle = \exp\left(-\frac{1}{2}g\right) = \exp\left(-\frac{1}{2}v_z^2\sigma^2\right) \tag{19}$$

Substituting (19) into (17), and using (13) and (16),

$$D\{E_2\} = \frac{2\pi F^2 E_N^2}{A} \int_0^\infty J_0(v_{xy}\tau)\left[\exp(G-g) - \exp(-g)\right]\tau d\tau \tag{20}$$

$$= \frac{2\pi F^2 E_N^2 e^{-g}}{A} \int_0^\infty J_0(v_{xy}\tau)\left[\left(\sum_{m=0}^\infty \frac{G}{m!} - 1\right)\right]\tau d\tau$$

$$= \frac{2\pi F^2 E_N^2 e^{-g}}{A} \sum_{m=1}^\infty \frac{1}{m!} \int_0^\infty J_0(v_{xy}\tau)\, G^m\, \tau d\tau$$

The last expression was obtained by the interchange of integration and summation.

Using equation (16) the r.m.s. scattered power will be:

$$\left\langle E_2 E_2^* \right\rangle = E_N^2 e^{-g} F^2\left[\rho_0^2 + \frac{2\pi}{A}\sum_{m=1}^\infty \frac{1}{m!}\int_0^\infty J_0(v_{xy}\tau)\, G^m \tau d\tau\right] \tag{21}$$

Since

$$\left\langle E_2 \right\rangle = \frac{FE_N}{A} \int_{-X}^X \int_{-Y}^Y e^{i(v_x x + v_y z)}\left\langle e^{iv_z\zeta}\right\rangle d_x d_y = \rho_0 E_N F e^{-\frac{g}{2}}$$

with

$$\rho_0 = \frac{\sin v_x X \sin v_y Y}{v_x v_y XY}$$

The first term, $E_N^2 e^{-g} F^2 \rho_o^2$, of equation (21) corresponds to reflection in the "specular" direction, i.e., where $\Theta_1 = \Theta_2$, and $\Theta_3 = 0$. This first term has a value only in the immediate neighborhood of the "specular" direction and is the predominant term for scattering in this direction when the surface is slightly rough.

The second or infinite series term of (21) corresponds to the diffuse scattering from the rough surface. Since the surface has slight overall roughness, $G(\tau)$ is small and only the first term of the series is required to evaluate $E_2 E_2^*$:

$$\left\langle E_2 E_2^* \right\rangle = E_N^2 e^{-g} F^2 \left[\rho_o 2 + \frac{2\pi}{A} \int_0^\infty J_o (v_{xy}) G(\tau) \tau d\tau \right] \tag{22}$$

Studies of most terrain data indicate that exponential correlation functions adequately describe the large and small scale roughnesses, i.e.,

$$\rho_S(\tau) = e^{-|\tau|/\beta_S} \qquad \rho_L(\tau) = e^{-|\tau|/\beta_S} \tag{23}$$

where β_S and β_L are the correlation distances of the large and small scale roughnesses.

The assumption will be made that the cross-correlation function is also exponential with a correlation distance of $\sqrt{\beta_S \beta_L}$,

$$\rho_{SL}(\tau) = K e^{-|\tau| \sqrt{\beta_S \beta_L}} \qquad |K| \le 1 \tag{24}$$

Evaluating $G(\tau)$ in terms of equation (23) and (24), and applying the identity,

$$\int_0^{\infty} J_0(v_{xy}\tau)\, e^{-m\tau/\beta}\, \tau d\tau = \frac{\beta^2}{m_2(1 + v_{xy}^2\, \beta^2/m^2)^{3/2}}$$

equation (22) becomes,

$$\left\langle E_2 E_2^* \right\rangle = E_N^2 e^{-g} F^2 \left[\rho_0^2 + \frac{2\pi}{A} v_z^2 \frac{\sigma_S^2 \beta_S^2}{(1 + v_{xy}^2 \beta_L^2)^{3/2}} \right.$$

$$\left. + \frac{2K\sigma_S\sigma_L\beta_S\beta_L}{(1 + v_{xy}^2\beta_S\beta_L)^{3/2}} + \frac{\sigma_L^2\beta_L^2}{(1 + v_{xy}^2\beta_L^2)^{3/2}} \right] \tag{25}$$

Thus, the diffusive scattering term from a slightly rough surface has been decomposed into three terms that respectively correspond to the effect of small scale roughness alone, the effect of large scale roughness alone, and the interaction between large and small scale roughness.

Evaluation of the Scattered Power For Moderate Surface Roughness

For moderate surface roughness it will be assumed that

$$\sigma_L \gg \sigma_S$$

$$\frac{\sigma}{\lambda} = \frac{\sigma_L}{\lambda} \leq 0.1 \tag{26}$$

so that

$$g_{max} = G_{max} \; (\tau) \leq 1.0 \qquad (27)$$

Using the auto-correlation and cross-correlation functions defined by (23) and (24), and approximating $\langle E_2 E_2^* \rangle$ by the first two terms of the series, equation (21) gives

$$\langle E_2 E_2^* \rangle = E_N^{\,2} F^2 e^{-g} \left[\rho_o^{\,2} + \frac{2\pi}{A} v_z^{\,2} \left\{ \frac{\sigma_S^{\,2} \beta_S^{\,2}}{(1 + v_{xy}^{\,2} \beta_S^{\,2})^{3/2}} + \right. \right.$$

$$\left. + \frac{2K \sigma_S \sigma_L \beta_S \beta_L}{(1 + v_{xy}^{\,2} \beta_S \beta_L)^{3/2}} + \frac{\sigma_L^{\,2} \beta_L^{\,2}}{(1 + v_{xy}^{\,2} \beta_L^{\,2})^{3/2}} \right\} + \frac{\pi}{A} v_z^{\,4} \left\{ \frac{\sigma_S \sigma^4 \beta_S^{\,2}}{4(1 + v_{xy}^{\,2} \frac{\beta_S^{\,2}}{4})^{3/2}} + \right.$$

$$+ \frac{\sigma_L^{\,4} \beta_L^{\,2}}{4(1 + v_{xy}^{\,2} \beta_L^{\,2}/4)^{3/2}} + \frac{K \sigma_S^{\,2} \sigma_L^{\,2} \beta_S \beta_L}{(1 + v_{xy}^{\,2} \beta_L \beta_L/4)^{3/2}} + \frac{2\sigma_S^{\,2} \sigma_L^{\,2} \beta_1^{\,2}}{(1 + v_{xy}^{\,2} \beta_1^{\,2})^{3/2}}$$

$$\left. \left. + \frac{4K \sigma_S^{\,3} \sigma_L^{\,2} \beta_2^{\,2}}{(1 + v_{xy}^{\,2} \beta_2^{\,2})^{3/2}} + \frac{4K \sigma_S \sigma_L^{\,3} \beta_3^{\,2})}{(1 + v_{xy}^{\,2} \beta_3^{\,2})^{3/2}} \right\} \right] \qquad (28)$$

where

$$\hat{\beta}_1 - \frac{\beta_S \beta_L}{\beta_L \beta_S} \qquad \beta_2 = \frac{\beta_S \sqrt{\beta_S \beta_L}}{\beta_S + \sqrt{\beta_S \beta_L}} \qquad \beta_3 = \frac{\beta_L \sqrt{\beta_S \beta_L}}{\beta_L + \sqrt{\beta_S \beta_L}}$$

More terms of the series that appears in (21) could be used to calculate $\langle E_2 E_2^* \rangle$ for surfaces of greater roughness, but the number of terms to be evaluated soon becomes prohibitive for the series does not converge rapidly for large values of $G(\tau)$.

Evaluation of the Scattered Power for Extreme Surface Roughness

This last case finds the widest application since it best describes the roughness of most terrains.

Here it is assumed that

$$g_{max} = G_{max} (\tau) \gg 1. \tag{29}$$

Rather than evaluate the infinite series in (21), it is better in this case to use a saddle-point integration of (13).

Since σ and σ_L are large, exp $(G-g)$, with G and g defined as in (12), will have a significant value only for values of τ near zero. Thus, the exponential auto-correlation and cross-correlation functions of (23) and (24) are replaced by the approximations,

$$\rho_S (\tau) = 1 - \frac{\tau}{\beta_S} \qquad \rho_L (\tau) = 1 - \frac{\tau}{L} \tag{30}$$

$$\rho_{SL} (\tau) = K (1 - \frac{\tau}{\sqrt{\beta_S \beta_L}})$$

So that

$$\exp (G-g) = \exp \left[-\tau \, \Sigma \right] \tag{31}$$

with

$$\Sigma = v_z^{\,2} \left(\frac{\sigma_S^{\,2}}{\beta_S} + \frac{\sigma_L^{\,2}}{\beta_L} + \frac{2K\sigma_S\sigma_L}{\sqrt{\beta_S\beta_L}} \right) \tag{32}$$

After completion of the integration of (13),

$$\left\langle E_2 E_2^* \right\rangle = \frac{2\pi F^2 E N^2 \Sigma}{A(\Sigma^{2} + v_{xy}^{\,2})} \, 3/2 \tag{33}$$

The scattering solution of (33) is essentially of the same form as given by Beckmann (3) for a composite rough surface that is the superposition of independent normal distributed random processes with exponential correlation functions. The statistical dependence of the large and small scale roughness produces just the same effect as though the rough surface consisted of three independent processes with standard deviations $\sigma_S, \sigma_L, \sqrt{2K\sigma_S\sigma_L}$, and correlation distances β_S, β_L, and $\sqrt{\beta_S\beta_L}$.

Conclusion

Inspection of the results of (25), (28) and (33) points out

the error of neglecting small scale roughness $\frac{\sigma_S}{\lambda}$ and its r.m.s.

slope $\frac{\sigma_S}{\beta_S}$ in any calculation of power reflected from a rough surface.

Also, these results clearly show the effect of statistical dependences

between the roughness components of a composite rough surface.

In each of the three cases described in this paper, the correlation

between large and small scale roughness produces terms in the

scattered power solution that vanish when the two roughness components

are independent.

The importance of the small scale roughness and the statistical

dependence between large and small scale roughness is easily

illustrated for an extremely rough surface. For such a terrain assume

σ_S - 10λ, σ_L - σ - 1000λ and σ_L/β_L - 0.02. Then for various

values of small scale roughness, the relative value of the three terms

that comprise the factor Σ appearing in equation (33) are given in the

following table.

RMS SMALL SCALE SLOPE	TERMS OF Σ		
	$\dfrac{\sigma_S^2}{\beta_S}$	$\dfrac{\sigma_L^2}{\beta_L}$	$\dfrac{2K\sigma_S\sigma_L}{\sqrt{\beta_L\beta_S}}$
0.1	1λ	20λ	$9K\lambda$
1.0	10λ	20λ	$28K\lambda$
5.0	50λ	20λ	$64K\lambda$

Furthermore, the above results of equations (25), (28), and (33)
used in previously reported work (6,7) on polarized radar return and
its dependence upon surface roughness parameters, yield useful
results in the analysis of radar return from a rough surface.

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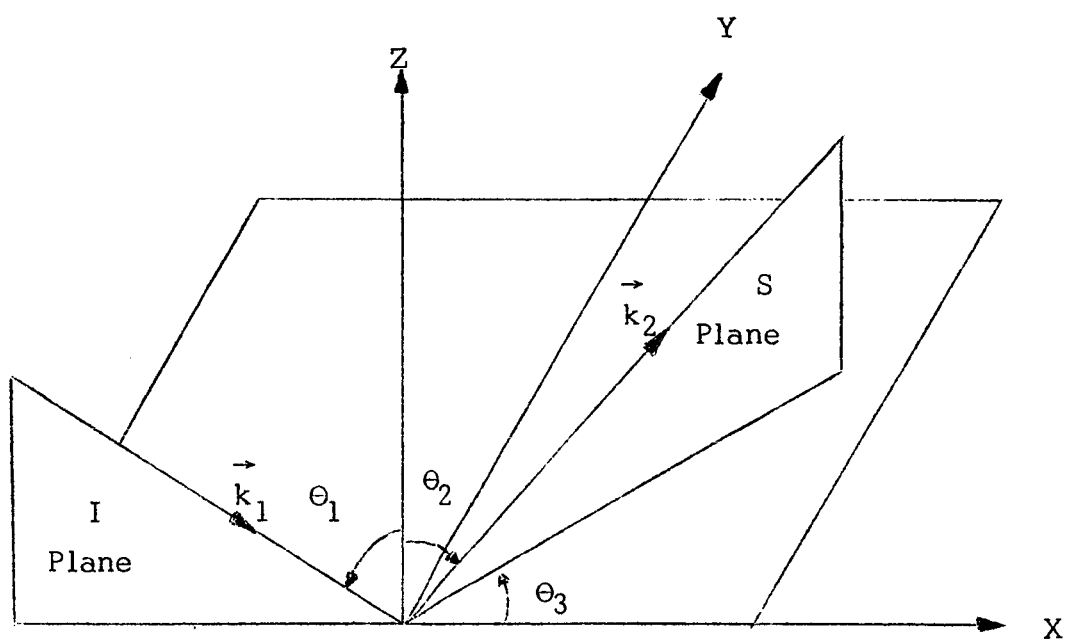
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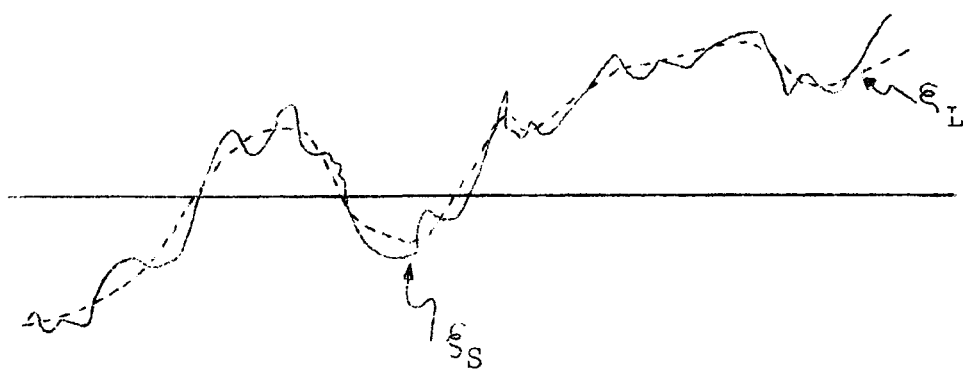
Fig. 1

Fig. 2

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