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MEMBRANE STRESSES AND DEFLECTIONS OF THE CENTAUR AFT BULKHEAD (AC-6 AND ON)

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FOREWORD

The equations used in determining Centaur aft bulkhead stresses and deflections are derived and summarized in the main body of this report. These stresses and deflections, for various loading conditions, are graphically presented in Appendix A.

Numerical computation of the Appendix A data was accomplished on General Dynamics/Convair's (at that time Astronautics') IBM 650 computer by M. C. Fox, who also prepared the related figures.

This study was conducted under the provisions of Contract NAS3-3232.

SUMMARY

The Centaur aft bulkhead (AC-6 and on) is considered to be an isotropic shell of constant thickness. Scaled deflections and stress resultants are determined for the bulkhead under loading conditions based on uniform internal pressure and on liquid oxygen head pressure.

Figure A-1 (in Appendix A) is a plot of "hoop radius" (r_0) against aft bulkhead station. Figures A-2 through A-13 present scaled deflections and stress resultants versus r_0 for the six loading cases listed in Table A-1. Thus to find the value of a scaled deflection or stress resultant at any station on the aft bulkhead, first find the value of r_0 (from Figure A-1) corresponding to that station, then determine the value of the desired parameter from the appropriate figure.

By suitable superposition and interpolation, many different tanking situations may be investigated (see example, Subsection A1.2). Also, all values can be determined for liquids other than liquid oxygen by scaling of the plotted values, since all quantities vary linearly with specific weight (γ); an example of this type is given in Subsection A1.2.3.

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LIST OF SYMBOLS

Symbol	Description
γ	Specific weight of liquid (weight per unit volume)
ϕ	Angle between normal to shell and axis of revolution
θ	Angle between plane containing meridian and some datum plane
r_o	Radius of parallel circle ("hoop" radius)
r_1	Meridional radius of curvature
r_2	Radius of curvature in plane normal to meridian
N_ϕ	Meridional membrane stress resultant
N_θ	Hoop direction membrane stress resultant
p_r	Load per unit surface area, in direction normal to surface
p_ϕ	Load per unit surface area, in meridional direction
ϵ_ϕ	Strain in meridional direction
ϵ_θ	Strain in hoop direction
ν	Poisson's ratio
E	Young's modulus
t	Shell thickness
v, w	Meridional and normal deflection, respectively
μ, δ	Axial and radial deflection, respectively
ψ	Rotation of tangent to meridian
Ω	Polar angle
x, y, z	Cartesian coordinates (y in axial direction)
a, b	Semi-major and semi-minor axis, respectively, of an ellipse
y_1	Distance from equator of ellipse to free surface of internal liquid
h	Distance from point of observation to free surface of internal liquid

LIST OF SYMBOLS (CONTINUED)

Symbol	Description
n_x, n_y, n_z	Load factors in corresponding Cartesian directions
V	Volume (in ft^3) of aft bulkhead aft of Station (410.0 + y)

SECTION I

INTRODUCTION

1.1 SCOPE

This report presents aft bulkhead stresses and deflections derived by application of linear, small deflection membrane theory (see Reference 1, Chapter 14 and Reference 2, Chapter 2). The aft bulkhead is considered to be an isotropic shell of constant thickness; no account is taken of discontinuity bending stresses.

1.2 METHOD OF ANALYSIS

Scaled deflections (scaled by t, the thickness) and stress resultants are presented explicitly for the aft bulkhead geometry under two types of loading conditions:

a. Uniform internal pressure (see Figures A-2 and A-3).

b. Liquid oxygen ($\gamma = 69.0$ lb/ft^3) head pressure (see Figures A-4 through A-13).

Axial deflection (μ) at Station 410 is taken to be zero.

SECTION II

DEVELOPMENT OF STRESS AND DEFLECTION EQUATIONS

2.1 GENERAL SHELL EQUATIONS

General shell equations for linear stress resultants in a shell of revolution (Figure 2-1) under axisymmetric loading are developed below. Figure 2-2 illustrates loads and membrane stress resultants on a differential element of a shell.

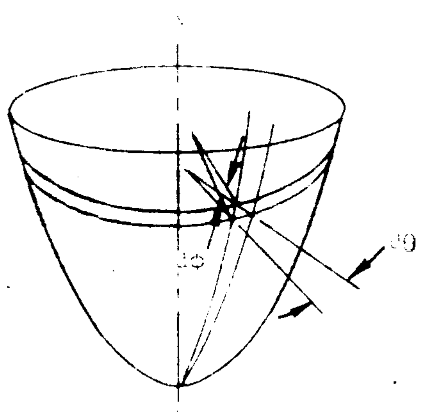

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Figure 2-1. Shell of Revolution

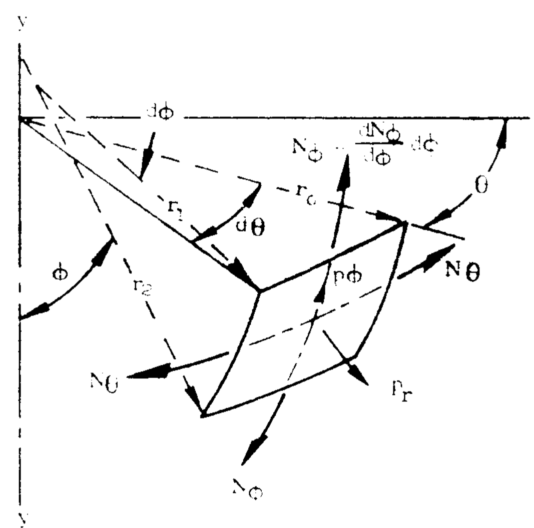

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Figure 2-2. Geometry and Stress Resultants

From Figure 2-2:

$$r_0 = r_2 \sin\phi \tag{2-1}$$

The equations below, extracted from Reference 2, Page 23, are the basic differential equations for the stress resultants:

$$\frac{d}{d\phi}(r_0 N_\phi) - r_1 N_\theta \cos\phi = -p_\phi r_0 r_1 \tag{2-2}$$

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = p_r \qquad (2\text{-}3)$$

From equations 2-2 and 2-3, we can get an explicit relation for N_ϕ:

$$N_\phi = \frac{1}{r_2 \sin^2\phi}\left[\int r_1\, r_2\, (p_r \cos\phi - p_\phi \sin\phi)\sin\phi\, d\phi + C_1\right] \qquad (2\text{-}4)$$

Equation 2-4 is simply a condition of over-all y-direction equilibrium for the part of the shell below the parallel circle ϕ = constant; the constant C_1 takes account of concentrated force. N_ϕ can be determined from equation 2-4 and then N_θ can be found from equation 2-2.

2.1.1 HOOKE'S LAW. Hooke's law expresses the linear stress-strain relation for an isotropic material. The equations below, which are taken from Reference 2, Page 87, are based on this relationship:

$$\epsilon_\phi = \frac{1}{Et}(N_\phi - \nu N_\theta) \qquad (2\text{-}5)$$

$$\epsilon_\theta = \frac{1}{Et}(N_\theta - \nu N_\phi) \qquad (2\text{-}6)$$

2.2 GENERAL AXISYMMETRIC SHELL DEFLECTIONS

General axisymmetric shell deflections are illustrated in Figures 2-3 and 2-4. The deflection equations that follow are taken from Reference 1, Page 446.

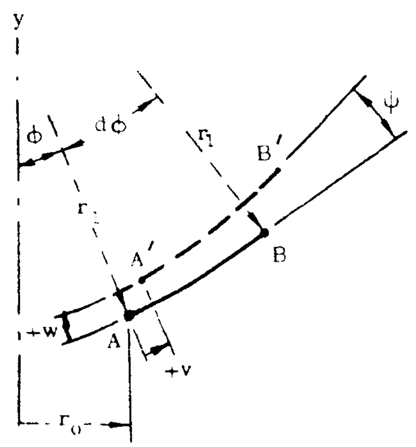

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Figure 2-3. Axisymmetric Shell Deflections

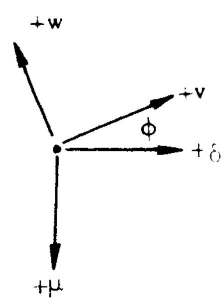

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Figure 2-4. Relation of Deflections

$$v = \sin\phi \left[\int \frac{f(\phi)}{\sin\phi}\, d\phi + C_2 \right] \tag{2-7}$$

where

$$f(\phi) = \frac{1}{Et} \left[N_\phi (r_1 + \nu r_2) - N_\theta (r_2 + \nu r_1) \right] \tag{2-8}$$

and

$$\epsilon_\theta = \frac{v}{r_2} \cot\phi - \frac{w}{r_2} \; ; \text{ or}$$

$$w = v \cot\phi - r_2 \epsilon_\theta \tag{2-9}$$

Also,

$$\psi = \frac{v}{r_1} + \frac{dw}{r_1\, d\phi} \tag{2-10}$$

From Figure 2-4:

$$\delta = -w \sin\phi + v \cos\phi,$$

or, using equation 2-9:

$$\delta = \epsilon_\theta r_o \tag{2-11}$$

Again, from Figure 2-4:

$$\mu = -w \cos\phi - v \sin\phi$$

or, using equation 2-9:

$$\mu = \epsilon_\theta r_o \cot\phi - \frac{v}{\sin\phi}$$

Substituting for v from equation 2-7 and for ϵ_θ from equation 2-11:

$$\mu = \delta \cot\phi - \int \frac{f(\phi)}{\sin\phi}\, d\phi + C_3 \tag{2-12}$$

where C_3 is determined from a boundary condition at a support.

Combining equations 2-9 and 2-10:

$$\psi = \frac{1}{r_1}\left[v + \frac{d}{d\phi}(v \cot\phi) - \frac{d}{d\phi}(r_2 \epsilon_\theta)\right]$$

Using equation 2-7, after differentiating and simplifying we obtain:

$$\psi = \frac{1}{r_1}\left[\frac{f(\phi)}{\tan\phi} - \frac{d}{d\phi}(r_2 \epsilon_\theta)\right] \qquad (2\text{-}13)$$

2.3 SUMMARY OF EQUATIONS FOR STRESS RESULTANTS AND DEFLECTIONS OF A SHELL OF REVOLUTION UNDER AXISYMMETRIC LOADING

A convenient set of equations for determining stress resultants N_ϕ and N_θ, and the scaled deflections δt, μt, and ψt are:

$$N_\phi = \frac{1}{r_2 \sin^2\phi}\left[\int r_1 r_2 (p_r \cos\phi - p_\phi \sin\phi)\sin\phi \, d\phi + C_1\right] \qquad (2\text{-}4)$$

where C_1 accounts for concentrated force

$$N_\theta = r_2\left(p_r - \frac{N_\phi}{r_1}\right) \qquad \text{from } (2\text{-}3)$$

$$\epsilon_\theta t = \frac{1}{E}(N_\theta - \nu N_\phi) \qquad \text{from } (2\text{-}5)$$

$$\delta t = (\epsilon_\theta t) r_o \qquad \text{from } (2\text{-}11)$$

$$\mu t = (\delta t) \cot\phi - \int \frac{f'(\phi)}{\sin\phi} d\phi + C_4 \qquad \text{from } (2\text{-}12)$$

where,

$$f'(\phi) = \frac{1}{E}\left[N_\phi (r_1 + \nu r_2) - N_\theta (r_2 + \nu r_1)\right] \qquad \text{from } (2\text{-}8)$$

and C_4 is determined from a boundary condition.

$$\psi t = \frac{1}{r_1}\left[\frac{f'(\phi)}{\tan\phi} - \frac{d}{d\phi}(r_2 \cdot \epsilon_\theta t)\right] \qquad \text{from } (2\text{-}13)$$

If the scaled deflections vt and wt are desired in addition to the scaled deflections δt and μt, inspection of Figure 2-4 gives:

$$v = \delta \cos\phi + \mu \sin\phi \quad \text{and} \quad w = -\delta \sin\phi + \mu \cos\phi$$

or,

$$vt = (\delta t)\cos\phi + (\mu t)\sin\phi \tag{2-14}$$

$$wt = -(\delta t)\sin\phi + (\mu t)\cos\phi \tag{2-15}$$

2.4 GEOMETRY OF AN ELLIPSE

Radii of curvature are found as functions of the Cartesian coordinates x and y; these coordinates in turn are related to the polar angle Ω (see Figure 2-5). Use of these coordinates is more convenient (for numerical computation and subsequent use of the data obtained from computation) than use of the surface coordinate φ.

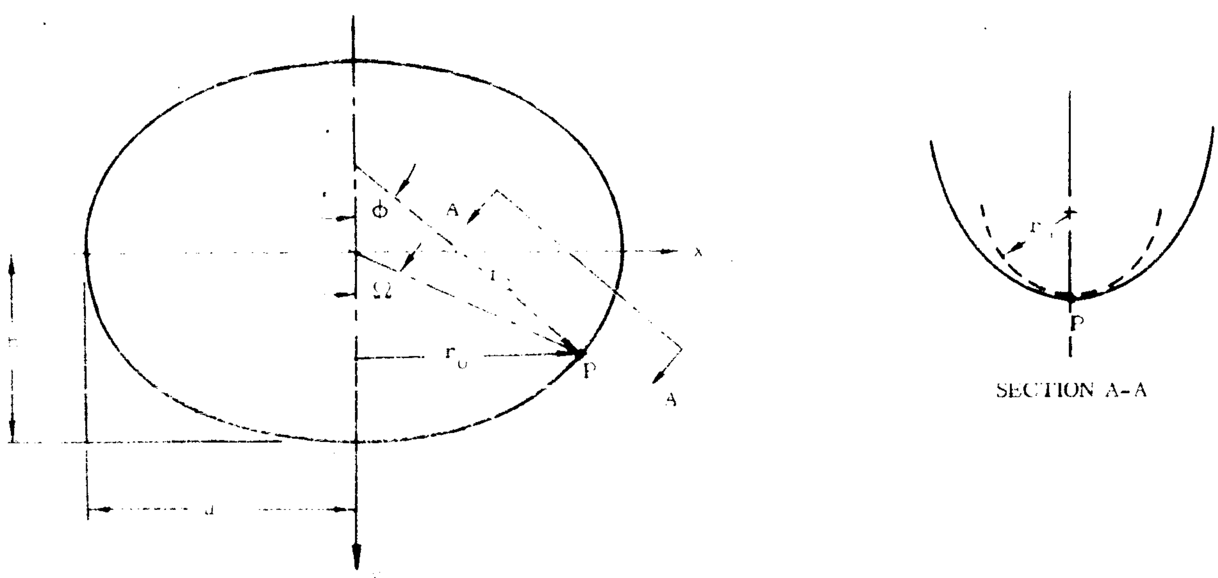

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Figure 2-5. Geometry of an Ellipse

The equation of an ellipse is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ or}$$

$$x = a\left(1 - \frac{y^2}{b^2}\right)^{1/2} = r_O \tag{2-16}$$

$$r_2 = \frac{(a^4 y^2 + b^4 x^2)^{1/2}}{b^2} \qquad (2\text{-}17)$$

(Reference 1, Page 440)

$$r_1 = r_2^3 \frac{b^2}{a^4} \qquad (2\text{-}18)$$

Also,

$$\sin\phi = \frac{x}{r_2} \text{ ; or} \qquad (2\text{-}19)$$

$$\phi = \sin^{-1}\frac{x}{r_2} \qquad (2\text{-}20)$$

Thus, starting with a value of y, the values of r_0 (=x), r_1, r_2, and ϕ can be determined.

It should be mentioned that the numerical work for determining aft bulkhead deflections (as well as the numerical work on bulkhead geometry in Reference 3) used the polar angle (Ω in Figure 2-5, but called θ in Reference 3) as the running parameter. The equations required if the polar angle is used are:

$$r_0 = x = y \tan\Omega$$

$$y = \frac{1}{\left(\frac{\tan^2\Omega}{a^2} + \frac{1}{b^2}\right)^{1/2}}$$

2.5 AFT BULKHEAD DEFLECTIONS UNDER ULLAGE OR LIQUID PRESSURE

The aft bulkhead cross-section is elliptic in shape as illustrated in Figure 2-6.

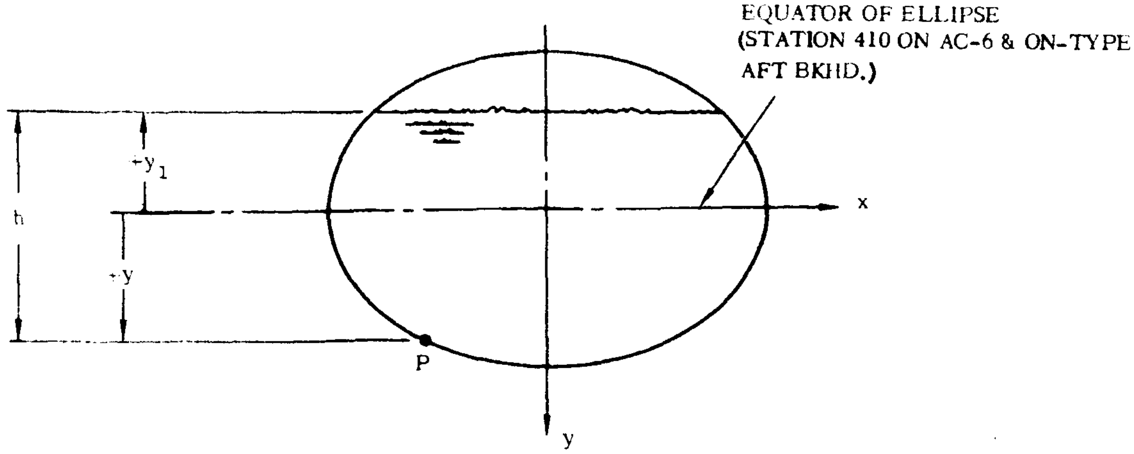

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Figure 2-6. Aft Bulkhead

For the aft bulkhead (material is 301 CRES, 3/4 hard):

$a = 43.5$ in.

$b = 60$ in.

$\nu = 0.30$ (Reference 1, Page 97)

$E = 27.0 \times 10^6$ psi (Reference 4, Memo 12.1; approx average room temperature E)

We take $\mu = 0$ at the equator.

2.5.1 UNIFORM INTERNAL PRESSURE. For uniform internal pressure (ullage pressure):

$p_r = 10$ psi (Case I, 10 psi chosen as a convenient "unit" pressure)

$p_\phi = 0$

Note that for this choice of p_r and p_ϕ, equation 2-4 simplifies to:

$$N_\phi = \frac{p_r\, r_2}{2} = 5\, r_2 \qquad (\text{For } p_r = 10 \text{ psi}) \tag{2-21}$$

2.5.2 LIQUID PRESSURE. For liquid pressure (from LO_2):

$p_r = \gamma h = \frac{69.0}{1728}(y + y_1)$ (Cases II through VI)

$p_\phi = 0$

Note that for this choice of p_r and p_ϕ, equation 2-4 simplifies to:

$$N_\phi = \frac{\gamma}{2}\left(\frac{V}{\pi\, r_0 \sin\phi} + h\, r_2\right) \tag{2-22}$$

$$= \frac{69.0}{2}\left(\frac{V}{\pi\, r_0 \sin\phi} + \frac{h\, r_2}{1728}\right)$$

where V = volume (in cubic feet) of aft bulkhead below Station $(410.0 + y)$.

In the programming of this type of loading, if $h < 0$, h and p_r are assumed to be zero.

2.5.3 APPLICATION OF EQUATIONS. For computation, we use the above p_r loads; the values of a, b, E, and υ (Page 2-7); the simplified relation (either equation 2-21 or 2-22) for N_ϕ; the geometrical relations (equations 2-16 through 2-20); and the equations summarized in Subsection 2.3. Using these quantities and relations, we determine the stress resultants and scaled deflections in the aft bulkhead under various loading conditions; these conditions are detailed in Table A-1 of Appendix A.

SECTION III

REFERENCES

1. Theory of Plates and Shells; S. Timoshenko and S. Woinowsky - Krieger; Second edition; Mc Graw-Hill Book Co., New York; 1959.

2. Stresses in Shells; W. Flugge; Springer-Verlag, Berlin; 1962.

3. Centaur Vehicle Fore and Aft Bulkhead Geometry; Contractor Report GD/C63-1249.

4. GD/C Structures Memoranda.

APPENDIX A

NUMERICAL DATA

A1.1 USE OF NUMERICAL DATA

Figure A-1 is a plot of r_0 against station for Centaur AC-6 and on aft bulkheads. Results of numerical work for the loading conditions listed in Table A-1 are shown in Figures A-2 through A-13; in these figures all quantities are plotted against r_0 (=x). Thus to find the value of any scaled deflection or stress resultant at a particular station on the aft bulkhead, find the value of r_0 corresponding to that station from Figure A-1, and then find the value of the scaled deflection or stress resultant from the appropriate figure. By suitable superposition and interpolation, many different tanking situations may be investigated (see example, Subsection A1.2). Note also that all quantities can be determined for liquids other than LO_2 by scaling of the plotted values, since all quantities are linear with γ (see Subsection A1.2.3).

A1.2 EXAMPLE OF USE OF NUMERICAL DATA

Assume we want δ and N_ϕ at Station 430.0 for LO_2 tanked to Station 400.0 and 17.5 psi ullage pressure.

Tank thickness: $t = 0.018$ in.

$r_0 = 53.1$ (At Station 430.0)

A1.2.1 δ CALCULATION. The deflection (δ) at Station 400.0 is determined as follows.

For LO_2 tanked to Station 388.0:

$$\delta t = -0.054 \times 10^{-3} \text{ in.}^2 \text{ (at } r_0 = 53.1 \text{ in.)}$$

For LO_2 tanked to Station 410.0:

$$\delta t = -0.056 \times 10^{-3} \text{ in.}^2 \text{ (at } r_0 = 53.1 \text{ in.)}$$

For LO_2 tanked to Station 400.0 (interpolating linearly):

$$\delta t = -0.056 \times 10^{-3} + \frac{(0.056 \times 10^{-3} - 0.054 \times 10^{-3})}{(410 - 388)} (410 - 400)$$

$$= -0.055 \times 10^{-3} \text{ in.}^2$$

For 10 psi ullage pressure:

$$\delta t = +0.067 \times 10^{-3} \text{ in.}^2 \text{ (at } r_o = 53.1 \text{ in.)}$$

For 17.5 psi ullage pressure:

$$\delta t = \frac{17.5}{10} (+0.067 \times 10^{-3}) = 0.117 \times 10^{-3} \text{ in.}^2$$

$$\text{Total } \delta t = 0.117 \times 10^{-3} - 0.055 \times 10^{-3} = +0.062 \times 10^{-3} \text{ in.}^2$$

$$\delta = \frac{\delta t}{t} = \frac{+0.062 \times 10^{-3}}{0.018} \text{; or}$$

$$\underline{\underline{\delta = 0.00344 \text{ in.}}}$$

A1.2.2 N_ϕ CALCULATION. The meridional membrane stress (N_ϕ) is determined as follows:

For LO_2 tanked to Station 388.0:

$$N_\phi = 72 \text{ lb/in. (at } r_o = 53.1 \text{ in.)}$$

For LO_2 tanked to Station 410.0:

$$N_\phi = 44 \text{ lb/in. (at } r_o = 53.1 \text{ in.)}$$

For LO_2 tanked to Station 400.0 (interpolating linearly):

$$N_\phi = 44 + \frac{(72-44)}{(410-388)} (410 - 400) = 57 \text{ lb/in.}$$

For 10 psi ullage pressure:

$$N_\phi = 327 \text{ lb/in. (at } r_o = 53.1 \text{ in.)}$$

For 17.5 psi ullage pressure:

$$N_\phi = \frac{17.5}{10} (327) = 572 \text{ lb/in.}$$

$$\text{Total } N_\phi = 57 + 572 \text{; or}$$

$$\underline{\underline{N_\phi = 629 \text{ lb/in.}}}$$

A1.2.3 N_ϕ CALCULATION FOR WATER. Assume now that we want to determine N_ϕ at Station 420.0 for water (γ = 62.4 lb/ft^3) tanked to Station 400.0 and 17.5 psi ullage pressure.

Tank thickness = t = 0.018 in.

$$r_0 = 53.1 \text{ in (At Station 420.0)}$$

All that must be done is to scale down the N_ϕ value (for LO_2 tanked to Station 400.0) by the ratio $\gamma_{H_2O}/\gamma_{LO_2}$ or,

for water tanked to Station 400.0:

$$N_\phi = \frac{62.4}{69.0}(57) = 52 \text{ lb/in.}$$

As before, for 17.5 psi ullage pressure:

$$N_\phi = 572 \text{ lb/in.}$$

Total N_ϕ = 52 + 572; or

$$\underline{N_\phi = 624 \text{ lb/in.}}$$

A1.3 SUMMARY OF NUMERICAL DATA (AC-6 AND ON-TYPE AFT BULKHEAD)

Liquid in aft bulkhead is assumed to have a specific weight γ of 69.0 lb/ft^3.

Aft bulkhead (and any liquid in it) is assumed to be at rest, i.e., n_y = 1.0 g, n_x = n_z = 0.

TABLE A-1. AFT BULKHEAD LOADING CONDITIONS

Case No.	Description of Loading	Figure No.	*Liquid Level (y_1)(in.)	Ullage Pressure (psi)
I	Uniform Internal Pressure	A-2, A-3	-43.5	10
II	LO_2 tanked to Station 432.0	A-4, A-5	-22.0	0
III	LO_2 tanked to Station 410.0	A-6, A-7	0	0
IV	LO_2 tanked to Station 388.0	A-8, A-9	+22.0	0
V	LO_2 tanked to Station 374.6	A-10, A-11	+35.4	0
VI	LO_2 tanked to Station 364.8	A-12, A-13	+45.2	0

Note:

* See Figure 2-6.

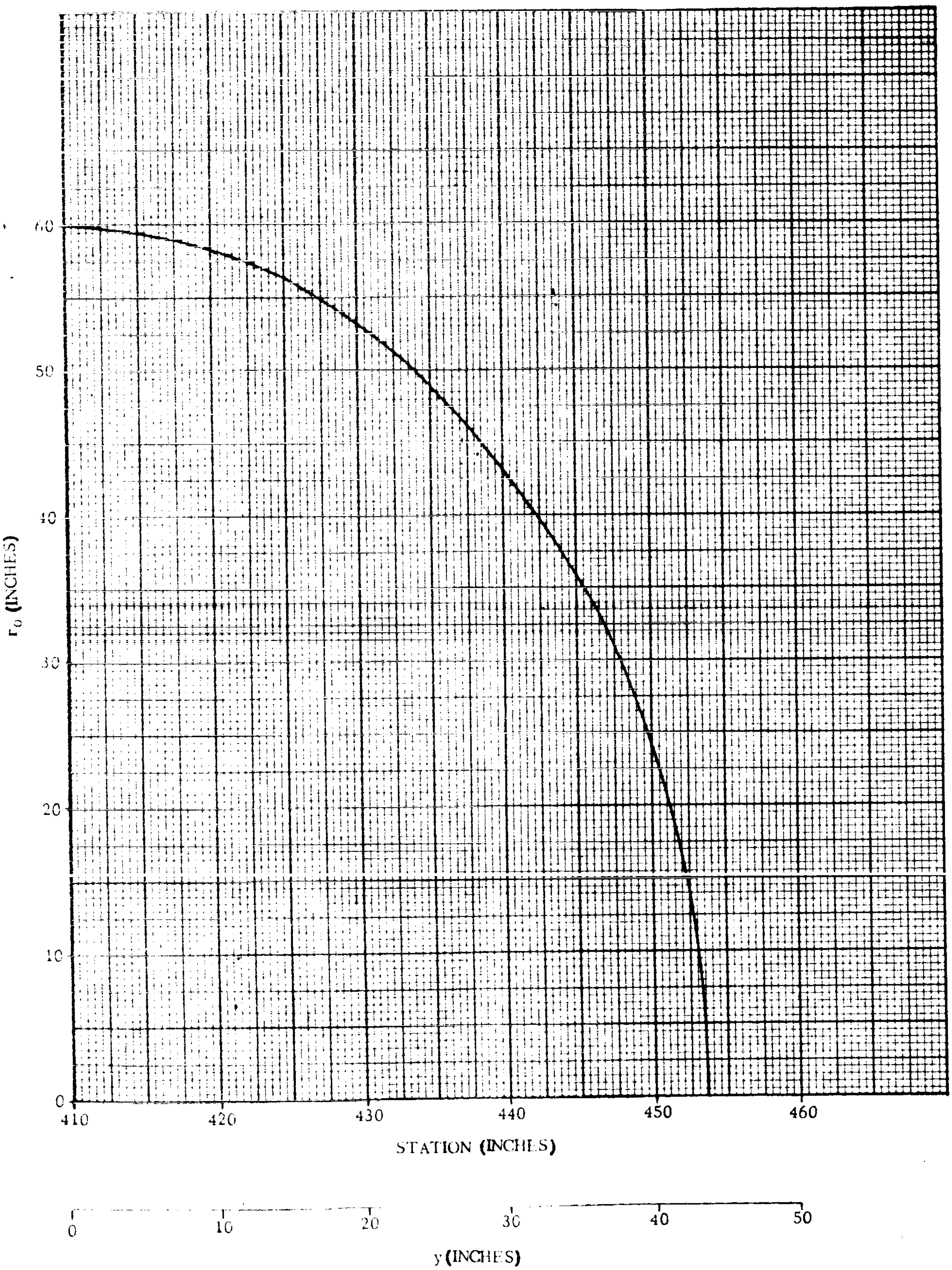

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Figure A-1. r_0 Versus Station - AC-6 and On - Type Aft Bulkhead

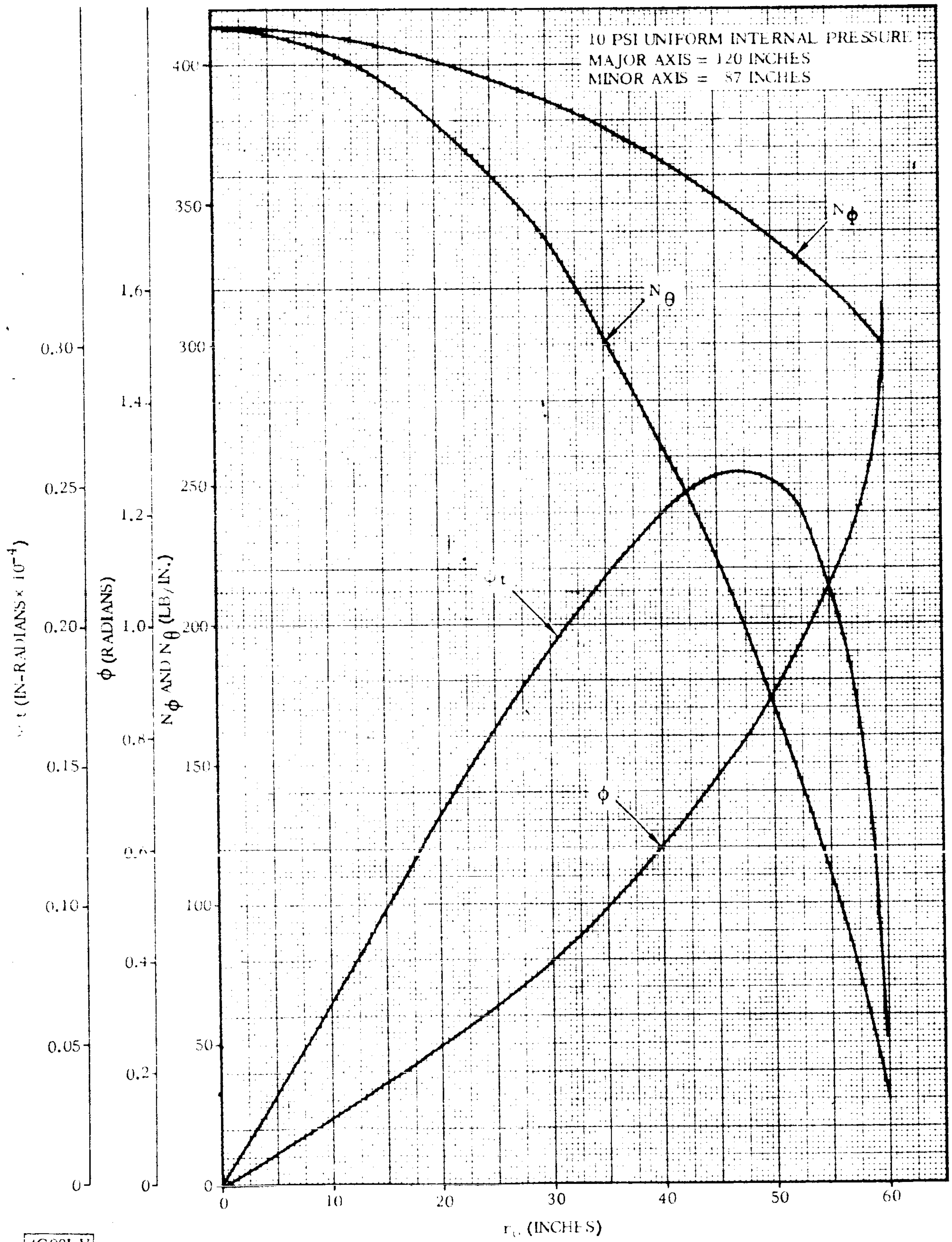

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Figure A-2. Bulkhead Parameters - Case I

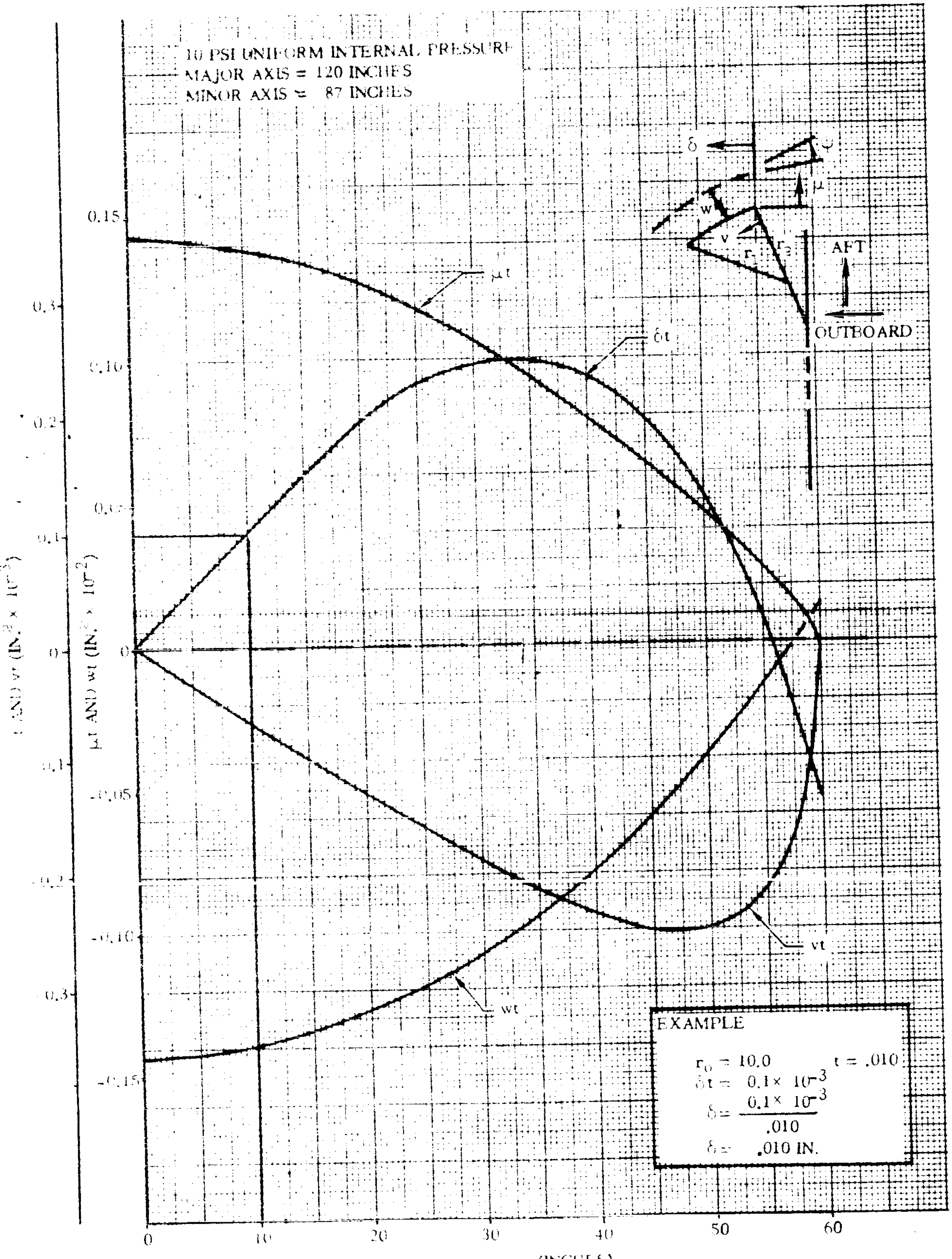

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Figure A-3. Bulkhead Parameters - Case I

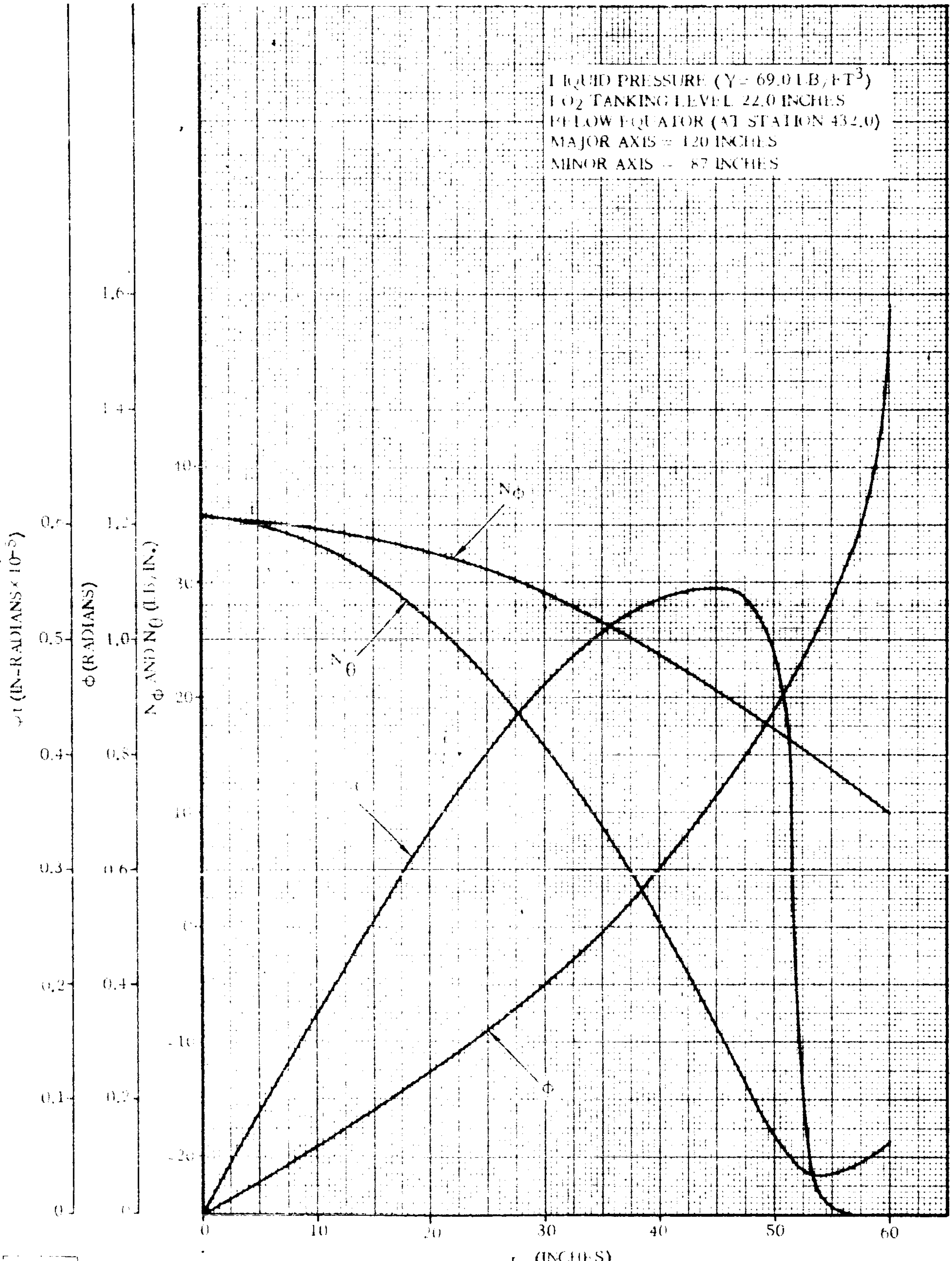

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Figure A-1. Bulkhead Parameters - Case II

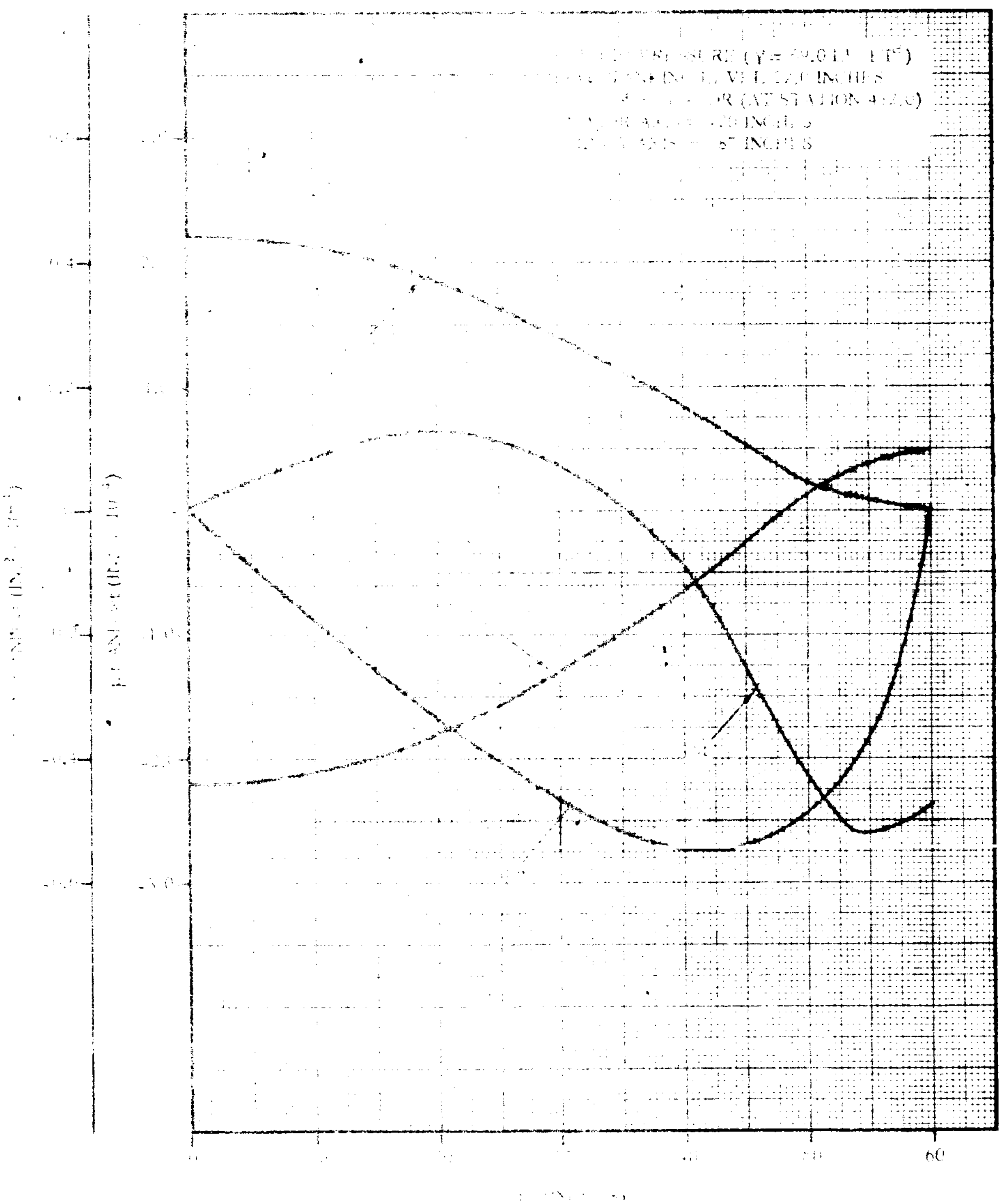
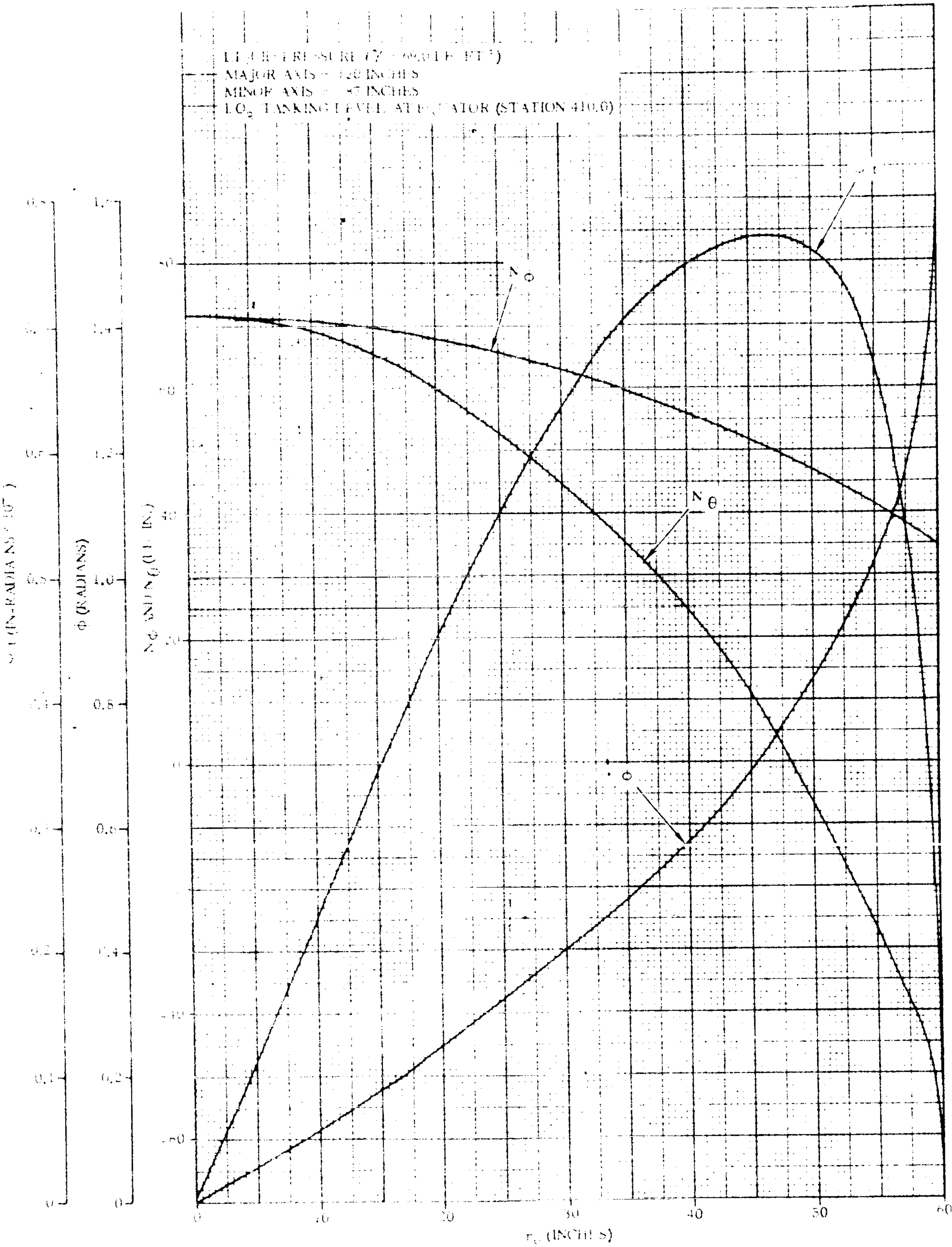


Figure A-


MAJOR AXIS = 120 INCHES
N_φ
N_θ
φ
φ (RADIANS)
r_o (INCHES)
0
10
20
30
40
50
60
0
0.2
0.4
0.6
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1.0
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1.4
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0.2
0.5

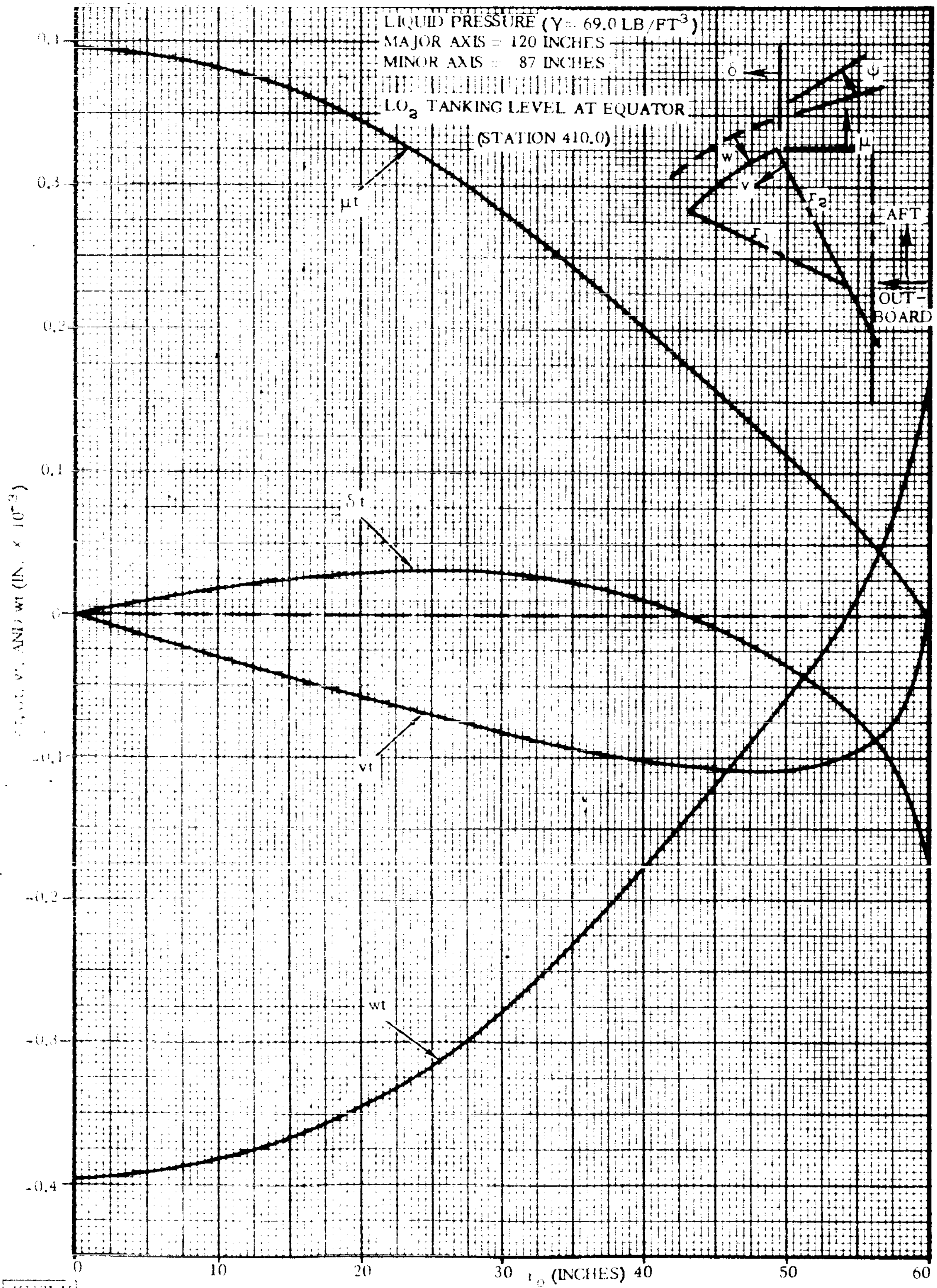

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Figure A-7. Bulkhead Parameters - Case III

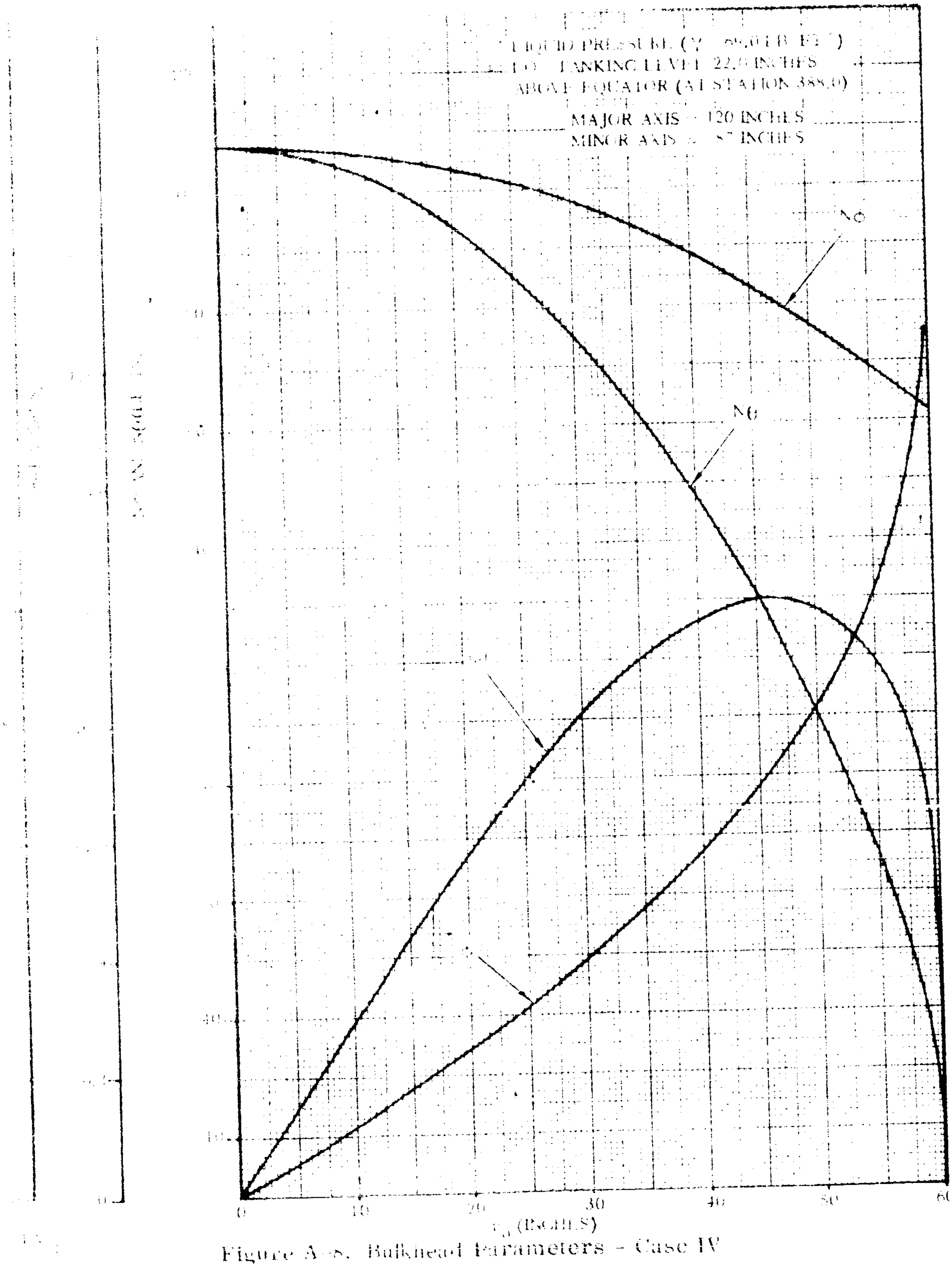

Figure A-8. Bulkhead Parameters - Case IV

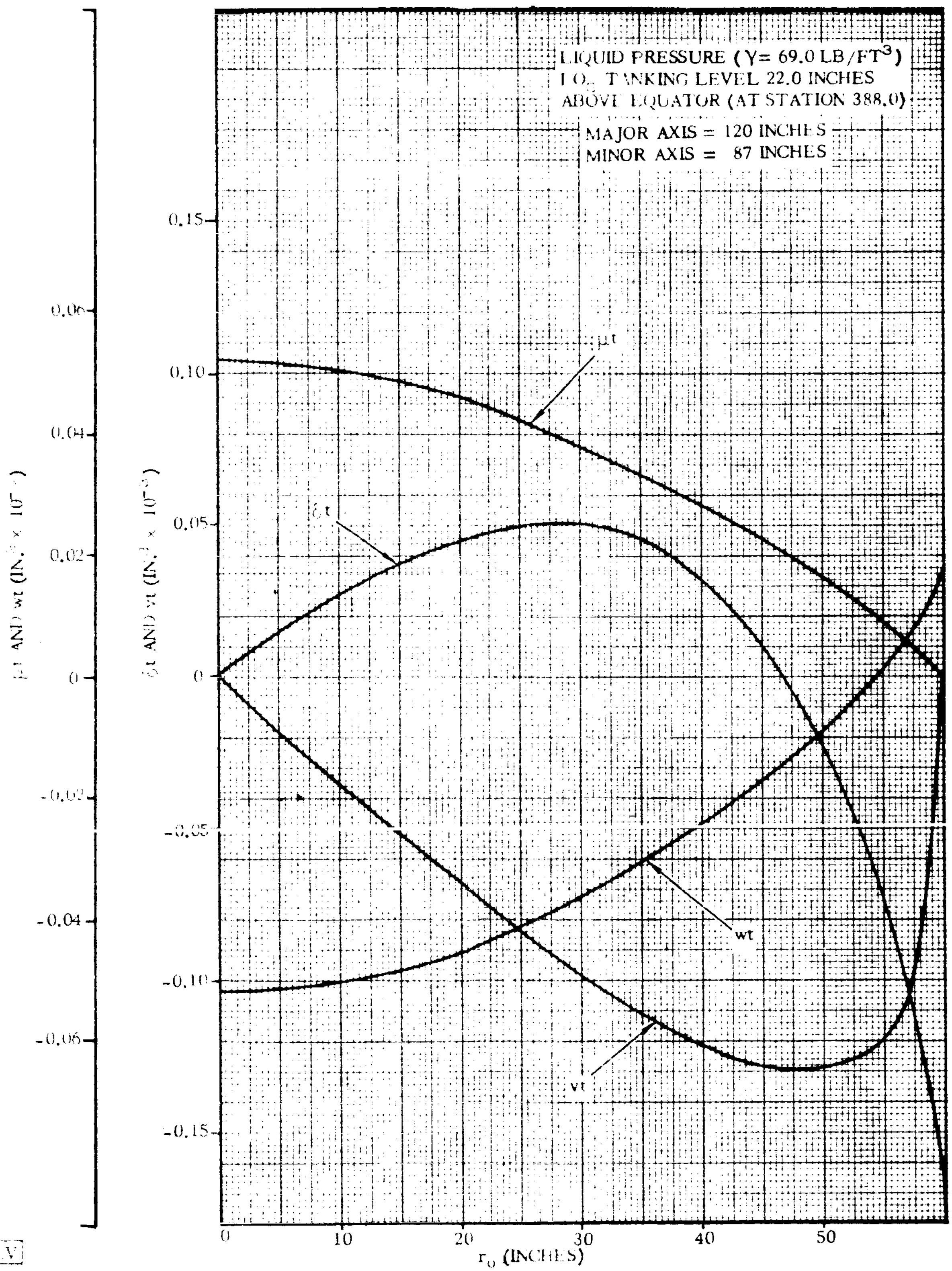

Figure A-9. Bulkhead Parameters - Case IV

LIQUID PRESSURE (γ = 69.0 LB/FT^3)
LO_2 TANKING LEVEL 35.4 INCHES ABOVE EQUATOR (AT STATION 374.6)

MAJOR AXIS = 120 INCHES
MINOR AXIS = 87 INCHES

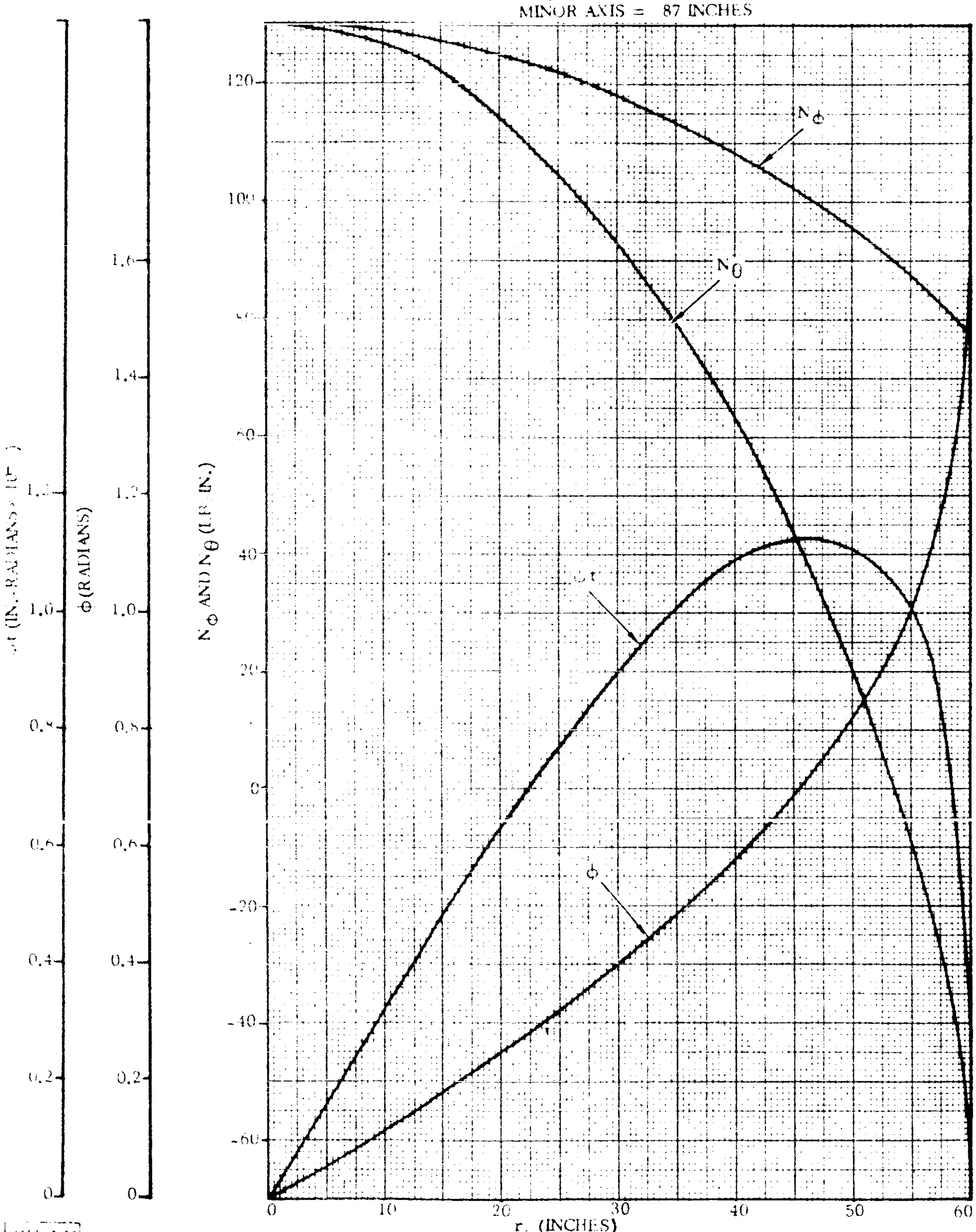

Figure A-10. Bulkhead Parameters - Case V

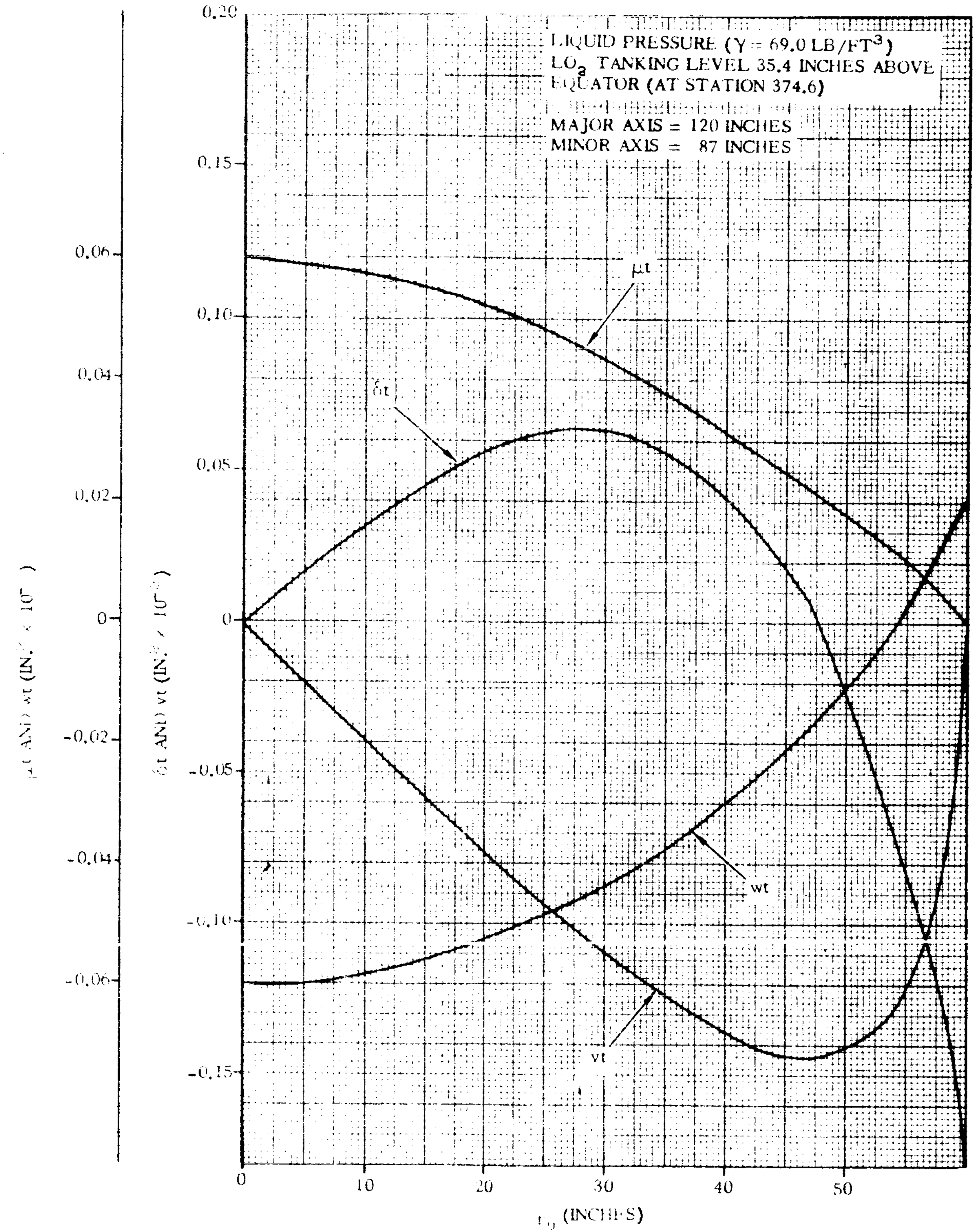

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Figure A-11. Bulkhead Parameters - Case V

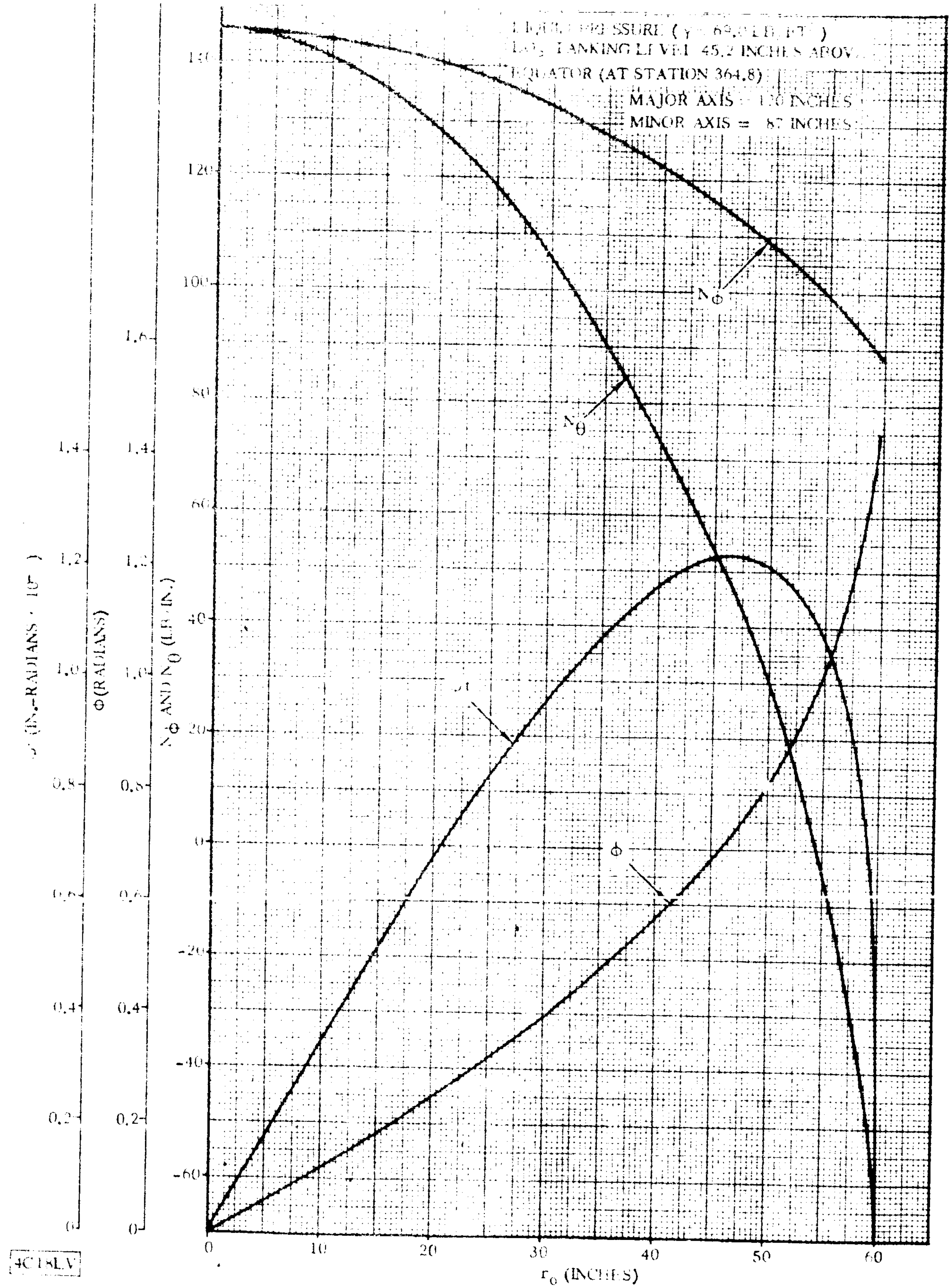

Figure A-12. Bulkhead Parameters - Case VI

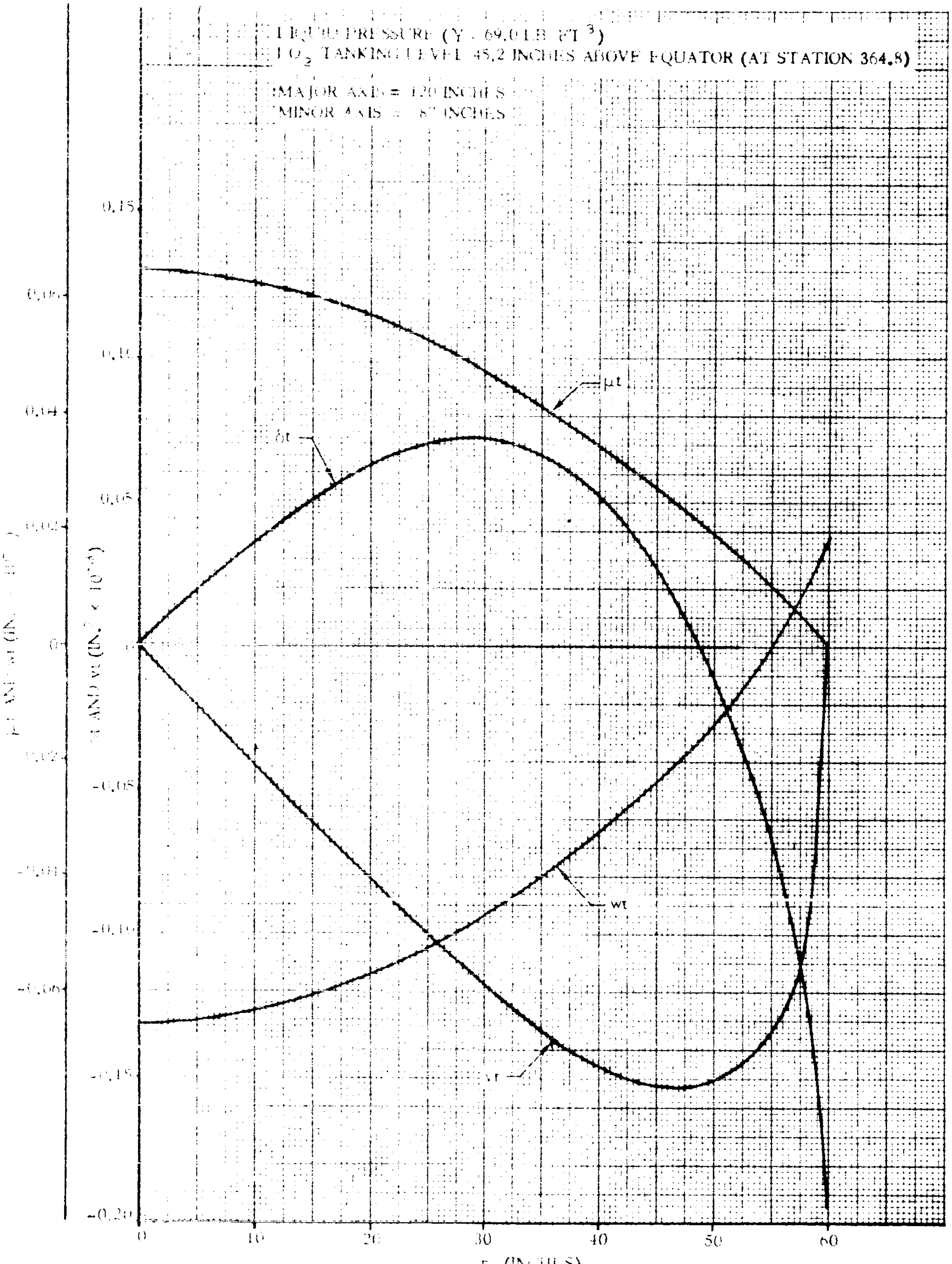

Figure A-13. Bulkhead Parameters - Case VI

GENERAL DYNAMICS

Convair Division