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GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

EQUATIONS FOR TWO-DIMENSIONAL ANALYSIS OF TOUCHDOWN DYNAMICS OF SPACECRAFT WITH HINGED LEGS INCLUDING ELASTIC, DAMPING, AND CRUSHING EFFECTS

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ABSTRACT

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An analytical approach is presented for determining the touchdown dynamics motion of spacecraft landing on the lunar surface. Spacecraft with hinged legs including elastic, damping, and crushing effects are considered. Before results of touchdown dynamics investigations can be intelligently correlated, comparison of approaches to the problem must be made. This report describes one such approach.

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DEFINITION OF SYMBOLS

(First and Second Legs, Respectively)

Symbol	Definition
a, d	distance from the foot to the leg-vehicle attachment point, measured normal to the vehicle's longitudinal axis
b, e	distance from the foot to the main member-vehicle attachment point, measured parallel to the vehicle's longitudinal axis
c, f	distance from the foot to the support member - vehicle attachment point, measured parallel to the vehicle's longitudinal axis
C_{m1}, C_{m2}	spring constant along the main member
C_{m10}, C_{m20}	crushing force along the main member
C_{s1}, C_{s2}	spring constant along the support member
C_{s10p}, C_{s20p}	crushing force designed to limit the compressive force along the support member
C_{s10n}, C_{s20n}	crushing force designed to limit the tension force along the support member
F_{c1}, F_{c2}	force along the main member, positive in compression
F_{s1}, F_{s2}	force along the support member, positive in compression
F_{n1}, F_{n2}	force normal to the surface, positive along the positive Y-axis
F_{t1}, F_{t2}	force tangential to the surface,positive along the positive X-axis
g	gravitational acceleration of the moon (or other body of interest)
k	vehicle's radius of gyration with respect to the center of gravity

DEFINITION OF SYMBOLS (Cont'd)

Symbol	Definition
L_1, L_3	distance from the center of gravity to the foot, measured normal to the longitudinal axis
L_2	original distance (before deflection) from the feet to the center of gravity, measured along the longitudinal axis
L_5, L_7	distance from the center of gravity to the leg attachment point, measured normal to the longitudinal axis
L_1', L_3'	distance from the center of gravity to the rocket stabilization motor
L_{m1}, L_{m2}	length of main member
L_{s1}, L_{s2}	length of support member
m	vehicle's mass
R_{m1}, R_{m2}	damping constant along the main member
R_{s1}, R_{s2}	damping constant along the support member
$\dot{S}_1$, $\dot{S}_2$	sliding velocity along the surface, positive along the positive X-axis
T_1, T_2	stabilization rocket motor thrust
t_{1b}, t_{2b}	stabilization rocket motor burning time
V_v	initial vertical velocity, positive downward
V_h	initial horizontal velocity, positive value gives a component along the positive X direction
W	weight of vehicle at lunar surface, mg
W_e	weight of vehicle on earth

DEFINITION OF SYMBOLS (Cont'd)

Symbol	Definition
X	center of gravity coordinate along the lunar surface
Y	center of gravity coordinate normal to the lunar surface
α_1, α_2	angle between the vertical and the line from the center of gravity to the foot
δ_1, δ_2	the stroke parallel to the vehicle's longitudinal axis
δ_{1to}	initial value of δ_1
δ_{m1}, δ_{m2}	deflection along the main member, positive deflection loads the member in compression
δ_{s1}, δ_{s2}	deflection along the support member, positive deflection loads the member in compression
δ_{1ff}, δ_{2ff}	the stroke parallel to the vehicle's longitudinal axis when the normal force becomes zero
δ_{m10}, δ_{m20}	elastic deflection along main member when crushing stops
δ_{s10p}, δ_{s20p}	positive elastic deflection along the support member when crushing stops, member in compression
δ_{s10n}, δ_{s20n}	negative elastic deflection (elongation) along the support member when crushing stops, member in tension
δ_{m1pp}, δ_{m2pp}	the "previous positive" or largest deflection in the main member which has previously been obtained in compression
δ_{s1pp}, δ_{s2pp}	the "previous positive" or largest deflection in the support member which has previously been obtained in compression
δ_{s1pn}, δ_{s2pn}	the "previous negative" or largest negative deflection (elongation) in the support member which has previously been obtained in tension
θ	slope of the lunar surface from the horizontal, positive counterclockwise

DEFINITION OF SYMBOLS (Cont'd)

Symbol	Definition
μ_1, μ_2	angle between the total force and the component of total force normal to surface, positive for positive tangential force
μ_{o1}, μ_{o2}	limiting value of μ, or angle of friction
tan μ_{o1}, tan μ_{o2}	coefficient of friction
φ	vehicle's attitude angle, positive counterclockwise and zero when the longitudinal axis is vertical.

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SUMMARY

Two-dimensional equations are presented for use in touchdown dynamics analysis of spacecraft with hinged legs including elastic, damping, and crushing effects. Purpose of the report is to document the equations.

SECTION I. INTRODUCTION

In the overall study of the design and performance of the Saturn V lunar logistics vehicle, analysis of touchdown dynamics of the landing stage is an important part. Such analysis influences the landing stage design as well as placing limits on the permissible vertical and horizontal velocities, vehicle attitude, and pitch rate at touchdown for a given design.

A touchdown dynamics program had previously been developed [1] and was used in the MSFC study of touchdown motion for the Lunar Logistics System [2]. These equations, however, did not account for elastic effects but assumed that the crushable material in the legs began to absorb energy upon impact without any prior elastic deflection. While it was recognized that the vehicle would have some elastic deflection, there was not sufficient appreciation for the magnitude of this deflection. After the MSFC Lunar Logistic System study was completed, the Space Technology Laboratories received a contract for a "Comparative Design Study of Modular Stage Concepts for Lunar Supply Operations" (contract NAS8-11022). The STL designs show that the elastic spring constant for each leg is so low that considerable elastic stroke is experienced before the legs start crushing. For a vertical landing on a level surface, the energy stored in elastic deflection is about 30 percent of the initial kinetic energy [3]. Thus, the vehicle would bounce off the surface with a velocity of about half the initial touchdown velocity. If the vehicles considered for lunar landing actually do have this large a percentage of energy stored in elastic deflection, then it is to be expected that the effect of this would significantly alter the touchdown dynamics motion.

The purpose of this report is to document the equations which have been developed for a new touchdown dynamics program for spacecraft with hinged legs including elastic, damping, and crushing effects in both the main and support leg members. The new program should prove to be a valuable extension to the touchdown dynamics analysis capability available for future analysis requirements. The author is indebted to Mr. John D. Capps, Computation Laboratory, who programmed the equations for the GE 225 digital computer.

SECTION II. MATHEMATICAL MODEL

The equations were developed corresponding to the two-dimensional model shown in Figure 1. This model is adequate for analysis of either three- or four-legged vehicles. Motion takes place in a plane such that two legs contact the surface simultaneously for the four-legged vehicle. For the three-legged vehicle, either the double leg or single leg can initially touchdown. Both the main (upper) and support (lower) member for each leg is assumed to undergo some elastic deflection with damping before the force is sufficient to begin crushing the inelastic crushable material. The main member crushes under compression only, but the support member is designed so that crushable material limits the load in both tension and compression. The force along the support member required to crush in tension may be different from that required to crush in compression.

The initial center of gravity coordinate normal to the lunar surface is

$$Y_o = (L_2 - \delta_{1to}) \cos (\varphi_o - \theta) + L_{1o} \sin (\varphi_o - \theta), \tag{1}$$

where δ_{1to} is the initial value assumed for the stroke of the first leg. This value is usually zero. This means the vehicle has just touched down on the surface. For some cases, it may be desirable to begin the solution with the first leg off the surface. In such cases, the value of δ_{1to} is a negative number.

The initial velocity components of the center of gravity normal and parallel to the surface are

$$\dot{Y}_o = - V_v \cos \theta - V_h \sin \theta \tag{2}$$

$$\dot{X}_o = V_h \cos \theta - V_v \sin \theta. \tag{3}$$

The stroke parallel to the vehicle's longitudinal axis for the first leg is

$$\delta_1 = L_2 - Y \sec (\varphi - \theta) + L_1 \tan (\varphi - \theta). \tag{4}$$

If $\delta_1 < 0$, the first leg is off the surface and all forces relating to the leg are zero. If $\delta_1 \geqq 0$, the leg is on the surface and the rate of stroking is obtained by

$$\dot{\delta}_1 = - \bar{A} + \dot{a} \tan (\varphi - \theta), \tag{5}$$

where

$$\bar{A} = \sec^2(\varphi - \theta) \left\{ \dot{Y} \cos (\varphi - \theta) + \left[Y \sin (\varphi - \theta) - L_1 \right] \dot{\varphi} \right\}. \tag{6}$$

Before the stroking rate can be obtained from Equation (5), the rate at which the dimension, a, is varying must be obtained. This is found to depend, among other things, upon the rate that the foot is sliding. The position of the first foot along the X-axis may be expressed by

$$S_1 = X - X_1, \tag{7}$$

where X is the coordinate of the center of gravity and X_1 is

$$X_1 = L_1 \cos (\varphi - \theta) - (L_2 - \delta_1) \sin (\varphi - \theta). \tag{8}$$

The sliding rate is thus

$$\dot{S}_1 = \dot{X} + L_1\dot{\varphi} \sin (\varphi - \theta) - \dot{a} \cos (\varphi - \theta) + (L_2 - \delta_1)\dot{\varphi} \cos (\varphi - \theta)$$
$$- \dot{\delta}_1 \sin (\varphi - \theta). \tag{9}$$

Now, since

$$Y = (L_2 - \delta_1) \cos (\varphi - \theta) + L_1 \sin (\varphi - \theta), \qquad (10)$$

then

$$\dot{S}_1 = \dot{X} + Y\dot{\varphi} - \dot{a} \cos (\varphi - \theta) - \dot{\delta}_1 \sin (\varphi - \theta). \qquad (11)$$

Substitution of Equation (5) into Equation (11) and solving for $\dot{a}$ yields

$$\dot{a} = \bar{A} \sin (\varphi - \theta) \cos (\varphi - \theta) + (\dot{X} + Y\dot{\varphi} - \dot{S}_1) \cos (\varphi - \theta). \qquad (11)$$

At any given time, everything is known to obtain $\dot{a}$ except the rate of sliding of the foot. The proper sliding rate is obtained by iteration. It is first assumed that the sliding rate, $\dot{S}_1$, is zero. Equations (11) and (5) are then solved for $\dot{a}$ and $\dot{\delta}_1$. The deflection rates in the main and support member are then obtained:

$$\dot{\delta}_{ml} = (b\dot{\delta}_1 - a\dot{a})/L_{ml} \qquad (12)$$

$$\dot{\delta}_{sl} = (c\dot{\delta}_1 - a\dot{a})L_{sl} \qquad (13)$$

where

$$b = b_o - \delta_1 \qquad (14)$$

$$c = c_o - \delta_1 \qquad (15)$$

$$L_{ml} = (a^2 + b^2)^{1/2} \qquad (16)$$

$$L_{sl} = (a^2 + c^2)^{1/2}. \qquad (17)$$

At any given time, the deflection of the main and support members are known from

$$\delta_{ml} = L_{ml_o} - L_{ml} \qquad (18)$$

$$\delta_{sl} = L_{sl_o} - L_{sl}. \qquad (19)$$

The force in the main member is then obtained from one of the following equations:

$$F_{c1} = C_{m1}\delta_{m1} + R_{m1}\dot{\delta}_{m1} \qquad (20)$$

$$\delta_{m1} \geqq 0$$

$$\delta_{m1} - \delta_{m10} < 0$$

$$\delta_{m1pp} - \delta_{m10} < 0$$

$$F_{c1} = C_{m10} \qquad (21)$$

$$\delta_{m1} \geqq 0$$

$$\delta_{m1} - \delta_{m10} \geqq 0$$

$$\delta_{m1} - \delta_{m1pp} > 0$$

$$F_{c1} = C_{m1}(\delta_{m1} - \delta_{m1pp} + \delta_{m10}) + R_{m1}\dot{\delta}_{1} \qquad (22)$$

$$\delta_{m1} \geqq 0$$

$$\delta_{m1} - \delta_{m1pp} \leqq 0$$

$$\delta_{m1} - \delta_{m1pp} + \delta_{m10} > 0$$

$$F_{c1} = 0 \qquad (23)$$

$$\delta_{m1} \geqq 0$$

$$\delta_{m1} - \delta_{m1pp} + \delta_{m10} \leqq 0$$

$$F_{c1} = C_{m1}\delta_{m1} + R_{m1}\dot{\delta}_{m1} \qquad (24)$$

$$\delta_{m1} < 0.$$

The force in the support member is obtained from one of the following equations:

$$F_{s1} = C_{s1}\delta_{s1} + R_{s1}\dot{\delta}_{s1} \tag{25}$$

$$\delta_{s1} \geqq 0$$

$$\delta_{s1} - \delta_{s10p} < 0$$

$$\delta_{s1pp} - \delta_{s10p} < 0$$

$$F_{s1} = C_{s10p} \tag{26}$$

$$\delta_{s1} \geqq 0$$

$$\delta_{s1} - \delta_{s10p} \geqq 0$$

$$\delta_{s1} - \delta_{s1pp} > 0$$

$$F_{s1} = C_{s1}(\delta_{s1} - \delta_{s1pp} + \delta_{s10p}) + R_{s1}\dot{\delta}_{s1} \tag{27}$$

$$\delta_{s1} \geqq 0$$

$$\delta_{s1} - \delta_{s1pp} \leqq 0$$

$$\delta_{s1} - \delta_{s1pp} + \delta_{s10p} > 0$$

$$F_{s1} = 0 \tag{28}$$

$$\delta_{s1} \geqq 0$$

$$\delta_{s1} - \delta_{s1pp} + \delta_{s10p} \leqq 0$$

$$F_{s1} = C_{s1}\delta_{s1} + R_{s1}\dot{\delta}_{s1} \tag{29}$$

$$\delta_{s1} < 0$$

$$\delta_{s1} - \delta_{s10n} > 0$$

$$\delta_{s1pn} - \delta_{s10n} > 0$$

$$F_{s1} = C_{s10n} \tag{30}$$

$$\delta_{s1} < 0$$

$$\delta_{s1} - \delta_{s10n} \leqq 0$$

$$\delta_{s1} - \delta_{s1pn} < 0$$

$$F_{s1} = C_{s1}(\delta_{s1} - \delta_{s1pn} + \delta_{s10n}) + R_{s1}\dot{\delta}_{s1} \tag{31}$$

$$\delta_{s1} < 0$$

$$\delta_{s1} - \delta_{s1pn} \geqq 0$$

$$\delta_{s1} - \delta_{s1pn} + \delta_{s10n} < 0$$

$$F_{s1} = 0 \tag{32}$$

$$\delta_{s1} < 0$$

$$\delta_{s1} - \delta_{s1pn} + \delta_{s10n} \geqq 0.$$

There are eight equations for F_{s1} corresponding to eight sets of conditions on the deflection. These equations can best be discussed with the aid of Figure 2. As the first leg contacts the surface, the leg will begin to shorten due to compression, or elongate due to tension. Assume, for example, that the leg begins to compress. Equation (25) will be used until $\delta_{s1} \geqq \delta_{s10p}$ after which Equation (26) will be used. If $F_{s1} > C_{s10p}$ while using Equation (25), F_{s1} is set equal to C_{s10p}. The leg continues to crush until $\delta_{s1} \leqq \delta_{s1pp}$, at which time crushing stops and the elastic deflection (which is equal to δ_{s10p} as δ_{s1} goes to zero) begins to reduce. Equation (27) is now used until $\delta_{s1} \leqq \delta_{s1pp} - \delta_{s10p}$. If F_{s1} becomes less than zero using Equation (27), the force is set to zero. The force remains zero as the leg elongates until the neutral deflection ($\delta_{s1} = 0$) is reached. Continued elongation will place the leg in tension and Equation (29) is used. Equations (29) through (32), when the leg is elongated ($\delta_{s1} < 0$), are used in the same way as Equations (25) through (28) when the leg is shortened. The additional restriction on Equation (25) that $\delta_{s1pp} < \delta_{s10p}$ is needed to keep Equation (25) from being used whenever $\delta_{s1pp} < 2\delta_{s10p}$ and δ_{s1} becomes less than δ_{s10p} again.

Once the forces F_{c1} and F_{s1} are obtained (having assumed $\dot{S}_1 = 0$), the normal and tangential forces acting on the foot are obtained:

$$F_{n1} = F_{c1} \cos (\eta_1 + \theta - \varphi) + F_{s1} \cos (\xi_1 + \theta - \varphi) \tag{33}$$

$$F_{t1} = F_{c1} \sin (\eta_1 + \theta - \varphi) + F_{s1} \sin (\xi_1 + \theta - \varphi) \tag{34}$$

where

$$\eta_1 = \tan^{-1}(a/b) \tag{35}$$

$$\xi_1 = \tan^{-1}(a/c). \tag{36}$$

Then,

$$\tan \mu_1 = F_{t1}/F_{n1}. \tag{37}$$

If $|\tan \mu_1| \leqq \tan \mu_{O1}$, then the assumption that $\dot{S}_1 = 0$ is correct and the foot is at rest. However, if $|\tan \mu_1| > \tan \mu_{O1}$, the foot cannot be at rest but must be moving at some rate along the surface. The proper sign for $\dot{S}_1$ is opposite the sign obtained for $\tan \mu_1$ since the tangential force acts to oppose the sliding motion. A value for $\dot{S}_1$ is then chosen (the magnitude of which is made proportional to $|\tan \mu_1| - \tan \mu_{O1}$) and new values for $\dot{a}$ and $\dot{\delta}_1$ obtained. The process is repeated and another $\tan \mu_1$ obtained. The proper $\dot{S}_1$ is obtained by iteration such that $|\tan \mu_1| = \tan \mu_{O1}$.

Once the proper forces have been obtained for the first leg, the forces in the second leg are obtained in a similar manner. Most of the time, only one leg or the other is in contact simultaneously. Usually, the first leg will leave the surface before the second leg comes in contact, and the vehicle will be in free flight. When the normal force from Equation (33) becomes negative, the forces are set to zero and the leg is off the surface. The value of δ_1 at the moment the leg leaves the surface is designated as δ_{1ff}. As long as subsequent values of δ_1 remain less than δ_{1ff}, the leg is off the surface and the forces on the leg remain set to zero.

As the second leg contacts the surface for the first time ($\delta_2 \geqq 0$), forces in this leg will begin to develop. The stroke parallel to the vehicle's longitudinal axis for the second leg is

$$\delta_2 = L_2 - Y \sec(\varphi - \theta) - L_3 \tan(\varphi - \theta), \tag{38}$$

and the stroking rate is

$$\dot{\delta}_2 = - \bar{C} - \dot{d} \tan(\varphi - \theta), \tag{39}$$

where

$$\bar{C} = \sec^2(\varphi - \theta) \left\{ \dot{Y} \cos(\varphi - \theta) + \left[Y \sin(\varphi - \theta) + L_3 \right] \dot{\varphi} \right\}. \tag{40}$$

Before the stroking rate can be determined from Equation (39), the rate at which the dimension, d, is varying must be obtained. This depends upon the rate that the second foot is sliding. The position of the second foot along the X-axis may be expressed by

$$S_2 = X + X_3, \tag{41}$$

where

$$X_3 = (L_2 - \delta_2) \sin(\varphi - \theta) + L_3 \cos(\varphi - \theta). \tag{42}$$

Therefore,

$$\dot{S}_2 = \dot{X} + (L_2 - \delta_2)\dot{\varphi} \cos(\varphi - \theta) - \dot{\delta}_2 \sin(\varphi - \theta) - L_3\dot{\varphi} \sin(\varphi - \theta)$$
$$+ \dot{d} \cos(\varphi - \theta). \tag{43}$$

Since

$$Y = (L_2 - \delta_2) \cos(\varphi - \theta) - L_3 \sin(\varphi - \theta), \tag{44}$$

then,

$$\dot{S}_2 = \dot{X} + Y\dot{\varphi} - \dot{\delta}_2 \sin(\varphi - \theta) + \dot{d} \cos(\varphi - \theta). \tag{45}$$

Substitution of Equation (39) into Equation (45) and solving for $\dot{d}$ yields:

$$\dot{d} = -\bar{C} \sin(\varphi - \theta) \cos(\varphi - \theta) - (\dot{X} + Y\dot{\varphi} - \dot{S}_2) \cos(\varphi - \theta). \tag{46}$$

As before, it is first assumed that the sliding rate, $\dot{S}_2$, is zero and $\dot{d}$ and $\dot{\delta}_2$ is obtained. The deflection rates in the main member and support member are then obtained:

$$\dot{\delta}_{m2} = (e\dot{\delta}_2 - d\dot{d})/L_{m2} \tag{47}$$

$$\dot{\delta}_{s2} = (f\dot{\delta}_2 - d\dot{d})/L_{s2} \tag{48}$$

where

$$e = b_o - \delta_2 \tag{49}$$

$$f = c_o - \delta_2 \tag{50}$$

$$L_{m2} = (d^2 + e^2)^{1/2} \tag{51}$$

$$L_{s2} = (d^2 + f^2)^{1/2} . \tag{52}$$

The deflection in the members is obtained from

$$\delta_{m2} = L_{m2_o} - L_{m2} \tag{53}$$

$$\delta_{s2} = L_{s2_o} - L_{s2}. \tag{54}$$

The force in the main member is then obtained from one of the following equations:

$$F_{c2} = C_{m2}\delta_{m2} + R_{m2}\dot{\delta}_{m2} \tag{55}$$

$$\delta_{m2} \geqq 0$$

$$\delta_{m2} - \delta_{m20} < 0$$

$$\delta_{m2pp} - \delta_{m20} < 0$$

$$F_{c2} = C_{m20} \tag{56}$$

$$\delta_{m2} \geqq 0$$

$$\delta_{m2} - \delta_{m20} \geqq 0$$

$$\delta_{m2} - \delta_{m2pp} > 0$$

$$F_{c2} = C_{m2}(\delta_{m2} - \delta_{m2pp} + \delta_{m20}) + R_{m2}\dot{\delta}_{2} \tag{57}$$

$$\delta_{m2} \geqq 0$$

$$\delta_{m2} - \delta_{m2pp} \leqq 0$$

$$\delta_{m2} - \delta_{m2pp} + \delta_{m20} > 0$$

$$F_{c2} = 0 \tag{58}$$

$$\delta_{m2} \geqq 0$$

$$\delta_{m2} - \delta_{m2pp} + \delta_{m20} \leqq 0$$

$$F_{c2} = C_{m2}\delta_{m2} + R_{m2}\dot{\delta}_{m2} \qquad \delta_{m2} < 0. \tag{59}$$

The force in the support member of the second leg is obtained from one of the following equations:

$$F_{s2} = C_{s2}\delta_{s2} + R_{s2}\dot{\delta}_{s2} \tag{60}$$

$$\delta_{s2} \geqq 0$$

$$\delta_{s2} - \delta_{s20p} < 0$$

$$\delta_{s2pp} - \delta_{s20p} < 0$$

$$F_{s2} = C_{s20p} \tag{61}$$

$$\delta_{s2} \geqq 0$$

$$\delta_{s2} - \delta_{s20p} \geqq 0$$

$$\delta_{s2} - \delta_{s2pp} > 0$$

$$F_{s2} = C_{s2}(\delta_{s2} - \delta_{s2pp} + \delta_{s20p}) + R_{s2}\dot{\delta}_{s2} \tag{62}$$

$$\delta_{s2} \geqq 0$$

$$\delta_{s2} - \delta_{s2pp} \leqq 0$$

$$\delta_{s2} - \delta_{s2pp} + \delta_{s20p} > 0$$

$$F_{s2} = 0 \qquad (63$$

$$\delta_{s2} \geqq 0$$

$$\delta_{s2} - \delta_{s2pp} + \delta_{s20p} \leqq 0$$

$$F_{s2} = C_{s2}\delta_{s2} + R_{s2}\dot{\delta}_{s2} \tag{64}$$

$$\delta_{s2} < 0$$

$$\delta_{s2} - \delta_{s20n} > 0$$

$$\delta_{s2pn} - \delta_{s20n} > 0$$

$$F_{s2} = C_{s20n} \tag{65}$$

$$\delta_{s2} < 0$$

$$\delta_{s2} - \delta_{s20n} \leqq 0$$

$$\delta_{s2} - \delta_{s2pn} < 0$$

$$F_{s2} = C_{s2}(\delta_{s2} - \delta_{s2pn} + \delta_{s20n}) + R_{s2}\dot{\delta}_{s2} \tag{66}$$

$$\delta_{s2} < 0$$

$$\delta_{s2} - \delta_{s2pn} \geqq 0$$

$$\delta_{s2} - \delta_{s2pn} + \delta_{s20n} < 0$$

$$F_{s2} = 0 \tag{67}$$

$$\delta_{s2} < 0$$

$$\delta_{s2} - \delta_{s2pn} + \delta_{s20n} \geqq 0.$$

Once the forces F_{c2} and F_{s2} are obtained (having assumed $\dot{S}_2 = 0$), the normal and tangential forces acting on the foot are obtained:

$$F_{n2} = F_{c2} \cos(\eta_2 + \varphi - \theta) + F_{s2} \cos(\xi_2 + \varphi - \theta) \tag{68}$$

$$F_{t2} = -F_{c2} \sin(\eta_2 + \varphi - \theta) - F_{s2} \sin(\xi_2 + \varphi - \theta) \tag{69}$$

where

$$\eta_2 = \tan^{-1}(d/e) \tag{70}$$

$$\xi_2 = \tan^{-1}(d/f). \tag{71}$$

Then,

$$\tan \mu_2 = F_{t2}/F_{n2}. \tag{72}$$

If $|\tan \mu_2| \leqq \tan \mu_{o2}$, then the assumption that $\dot{S}_2 = 0$ is correct and the foot is at rest. If $|\tan \mu_2| > \tan \mu_{o2}$, the foot cannot be at rest but is sliding. A value for $\dot{S}_2$ is then chosen with the sign opposite to that obtained for $\tan \mu_2$ and new values for $\dot{d}$ and $\dot{\delta}_2$ obtained from Equations (46) and (39). The proper $\dot{S}_2$ is obtained by iteration such that $|\tan \mu_2| = \tan \mu_{o2}$.

After the forces have been determined, the accelerations are obtained from the equations of motion. From Newton's second law,

$$m\ddot{Y} = F_{n1} + F_{n2} - W \cos \theta - (T_1 + T_2) \cos (\varphi - \theta) \tag{73}$$

$$m\ddot{X} = F_{t1} + F_{t2} - W \sin \theta + (T_1 + T_2) \sin (\varphi - \theta) \tag{74}$$

$$mk^2\ddot{\varphi} = (F_{t1} + F_{t2})Y + F_{n2}X_3 - F_{n1}X_1 + T_1L_1' - T_2L_3' \tag{75}$$

where T_1 and T_2 are downward directed stabilization rockets.

The stabilization rockets are represented by

$$T_1 = K_1 \qquad K_1 \neq 0,\ t < t_{1b} \tag{76}$$

$$T_1 = K_{1o} \qquad K_{1o} \neq 0,\ t_{\delta 2 \geqq 0} < t < t_{\delta 2 \geqq 0} + t_{1b} \tag{77}$$

$$T_1 = 0 \qquad K_1 = 0,\ K_{1o} = 0 \tag{78}$$

$$T_2 = K_2 \qquad K_2 \neq 0,\ t < t_{2b} \tag{79}$$

$$T_2 = K_{2o} \qquad K_{2o} \neq 0,\ t_{\delta 2 \geqq 0} < t < t_{\delta 2 \geqq 0} + t_{2b} \tag{80}$$

$$T_2 = 0 \qquad K_2 = 0,\ K_{2o} = 0. \tag{81}$$

This representation provides sufficient flexibility to account for a number of rocket stabilization schemes. For example, one scheme might be a single rocket motor which ignites upon initial contact and directed downward through the vehicle's center of gravity. For this case,

$$K_1 \neq 0, \quad t_{1b} \neq 0$$

$$K_{10} = K_2 = K_{20} = t_{2b} = L_1' = L_3' = 0.$$

If the rocket does not ignite upon contact of the first leg but does upon contact with the second leg, then

$$K_{10} \neq 0, \quad t_{1b} \neq 0.$$

If rocket motors are attached along the side of a three-legged vehicle, then,

$$L_1' = L_5, \quad L_3' = L_7 = \tfrac{1}{2}L_5, \quad K_{20} = 2K_{10}.$$

The accelerations obtained from the equations of motion are integrated numerically for new values of velocity and position.

$$\dot{Y} = \dot{Y}_o + \int \ddot{Y}\, dt \tag{82}$$

$$\dot{X} = \dot{X}_o + \int \ddot{X}\, dt \tag{83}$$

$$\dot{\varphi} = \dot{\varphi}_o + \int \ddot{\varphi}\, dt \tag{84}$$

$$Y = Y_o + \int \dot{Y}\, dt \tag{85}$$

$$X = \int \dot{X}\, dt \tag{86}$$

$$\varphi = \varphi_o + \int \dot{\varphi}\, dt \tag{87}$$

$$a = a_o + \int \dot{a}\, dt \tag{88}$$

$$d = d_o + \int \dot{d}\, dt \tag{89}$$

$$L_1 = L_5 + a \tag{90}$$

$$L_3 = L_7 + d. \tag{91}$$

The entire process is repeated starting with Equation (4) for a new time step. While the first leg is in contact, very small computing intervals are used in order to retain accuracy and also to keep the iteration procedure on $\dot{S}_1$ under control. After the first leg leaves the surface and the vehicle is in free flight, larger computing intervals can be used. When the second leg impacts, the smaller computing interval is again necessary. After the second leg leaves the surface and the vehicle is again in free flight ($\delta_1 < \delta_{1ff}$ and $\delta_2 < \delta_{2ff}$), the larger computing interval is again used.

For the downhill landing (positive V_h and negative θ), the vehicle impacts on the first (uphill) leg, goes into free flight with a rotational motion toward tumbling ($\dot{\varphi} < 0$), impacts on the second leg, and then again goes into free flight. As the second leg leaves the surface ($\delta_2 < \delta_{2ff}$), the rotational rate, $\dot{\varphi}$, may be positive. If so, the run is stopped since the vehicle will not tumble, but merely continue to bounce on first one leg and then the other as it continues to move downhill with less and less amplitude of the bouncing motion. If $\dot{\varphi}$ is negative, the motion is continued until the second leg impacts again and $\dot{\varphi}$ becomes positive (for a stable landing), or until the vehicle's center of gravity passes over the downhill foot ($\alpha_2 < 0$) for a tumble. The angle, α_2, is given by

$$\alpha_2 = \tan^{-1}\left(\frac{L_3}{L_2 - \delta_{2ff}}\right) + \varphi \qquad \delta_2 < \delta_{2ff} \tag{92}$$

$$\alpha_2 = \tan^{-1}\left(\frac{L_3}{L_2 - \delta_2}\right) + \varphi \qquad \delta_2 \geqq \delta_{2ff}. \tag{93}$$

Two equations are necessary since the vehicle can go unstable in either free flight or while the second leg is in contact.

For an uphill landing (negative V_h and negative θ), the vehicle impacts on the first (uphill) leg and subsequently either tumbles uphill or begins rotating toward the second (downhill) leg. The run is stopped if the center of gravity passes over the uphill foot ($\alpha_1 < 0$). The angle, α_1, is given by

$$\alpha_1 = \tan^{-1}\left(\frac{L_1}{L_2 - \delta_{1ff}}\right) - \varphi \qquad \delta_1 < \delta_{1ff} \tag{94}$$

$$\alpha_1 = \tan^{-1}\left(\frac{L_1}{L_2 - \delta_1}\right) - \varphi \qquad \delta_1 \geqq \delta_{1ff} \tag{95}$$

Determination of when to print was rather tedious. The print-out operation is a time consuming process so that the printing is held to just enough print-out at special times to obtain a clear picture of the motion. As the vehicle impacts on the first leg, it is desirable to know when the main and support members start crushing and when they stop crushing. It is desirable to know when the foot starts or stops sliding, when the vehicle's rotational motion changes direction, when the vehicle goes into free flight, the maximum distance from the surface the vehicle attains in free flight, when the second leg impacts, and so forth. After some consideration of this problem, the program has been coded to print whenever the events occur that are listed in Table I. The data that is printed is listed in Table II, and the input data needed to begin a run are listed in Table III.

In some cases, it is desirable to continue the solution beyond the time when the stability tests would normally stop the run. This is true, for example, when maximum crushing strokes are the data desired or when the total travel distance downhill from the instant of impact is desired. For such cases, the stability tests for stopping the run is bypassed and the run is stopped manually.

SECTION III. CONCLUSIONS

1. While the equations described in this report will more accurately represent the touchdown dynamics motion during lunar landing than those used previously, more exact duplication of the landing strut loads and deflections in three dimensions may be required for refined results.

2. These equations should be adequate, however, for general investigations of the touchdown dynamics problem. Since the STL program assumes the crushing force is constant normal to the lunar surface rather than along the strut member and also does not account for the changes in the leg spread as the leg deflects, these equations will yield somewhat different results than would be obtained with the STL program.

TABLE I
PRINT-OUT EVENTS

A print-out is made whenever any of the following events occur:

$\dot{S}_1$ or $\dot{S}_2$ changes from $\neq 0$ to 0 or from 0 to $\neq 0$

F_{n1} or F_{n2} changes from $\neq 0$ to 0 or from 0 to $\neq 0$

$\dot{\delta}_{m1}$, $\dot{\delta}_{m2}$, $\dot{\delta}_{s1}$, or $\dot{\delta}_{s2}$ changes sign

$\dot{Y}$, $\dot{X}$, or $\dot{\phi}$ changes sign

α_1 or α_2 becomes negative

$\delta_{m1} \geqq \delta_{m10}$ or $\delta_{m2} \geqq \delta_{m20}$ for the first time

$\delta_{s1} \geqq \delta_{s10p}$ or $\delta_{s2} \geqq \delta_{s20p}$ for the first time

$\delta_{s1} \leqq \delta_{s10n}$ or $\delta_{s2} \leqq \delta_{s20n}$ for the first time

$F_{c1} > C_{m10}$ or $F_{c2} > C_{m20}$ from Equations (20) or (55)

$F_{c1} < 0$ or $F_{c1} > C_{m10}$ from Equation (22)

$F_{c2} < 0$ or $F_{c2} > C_{m20}$ from Equation (57)

$F_{s1} > C_{s10p}$ or $F_{s2} > C_{s20p}$ from Equations (25) or (60)

$F_{s1} < 0$ or $F_{s1} > C_{s10p}$ from Equation (27)

$F_{s2} < 0$ or $F_{s2} > C_{s20p}$ from Equation (62)

TABLE I (Cont'd)

$F_{s1} < C_{s10n}$ or $F_{s2} < C_{s20n}$ from Equations (29) or (64)

$F_{s1} > 0$ or $F_{s1} < C_{s10n}$ from Equation (31)

$F_{s2} > 0$ or $F_{s2} < C_{s20n}$ from Equation (66)

TABLE II

PRINT-OUT DATA

t sec	$\ddot{X}$ in/sec^2	$\ddot{Y}$ in/sec^2	$\ddot{\varphi}$ rad/sec^2	$\dot{X}$ in/sec
$\dot{Y}$ in/sec	$\dot{\varphi}$ deg/sec	X in	Y in	φ deg
δ_1 in	δ_2 in	F_{n1} lb	F_{n2} lb	F_{t1} lb
F_{t2} lb	α_1 deg	α_2 deg	a in	d in
$\dot{S}_1$ in/sec	δ_{m1} in	δ_{s1} in	δ_{m2} in	δ_{s2} in
$\dot{S}_2$ in/sec	F_{c1} lb	F_{s1} lb	F_{c2} lb	F_{s2} lb

TABLE III

INITIAL DATA

V_v	m/sec	δ_{m10}	in
V_h	m/sec	δ_{m20}	in
θ	deg	δ_{s10p}	in
φ_o	deg	δ_{s20p}	in
$\dot{\varphi}_o$	deg/sec	δ_{s10n}	in
tan μ_{o1}	-	δ_{s20n}	in
tan μ_{o2}	-	δ_{1to}	in
k^2	in^2		
g	in/sec^2	a_o	in
W_e	lb	b_o	in
		c_o	in
C_{m1}	lb/in	d_o	in
C_{m2}	lb/in	L_2	in
C_{s1}	lb/in	L_5	in
C_{s2}	lb/in	L_7	in
C_{m10}	lb	K_1	lb
C_{m20}	lb	K_{10}	lb
C_{s10p}	lb	K_2	lb
C_{s20p}	lb	K_{20}	lb
C_{s10n}	lb	t_{1b}	sec
C_{s20n}	lb	t_{2b}	sec
		L_1'	in
R_{m1}	lb-sec/in	L_3'	in
R_{m2}	lb-sec/in		
R_{s1}	lb-sec/in		
R_{s2}	lb-sec/in		

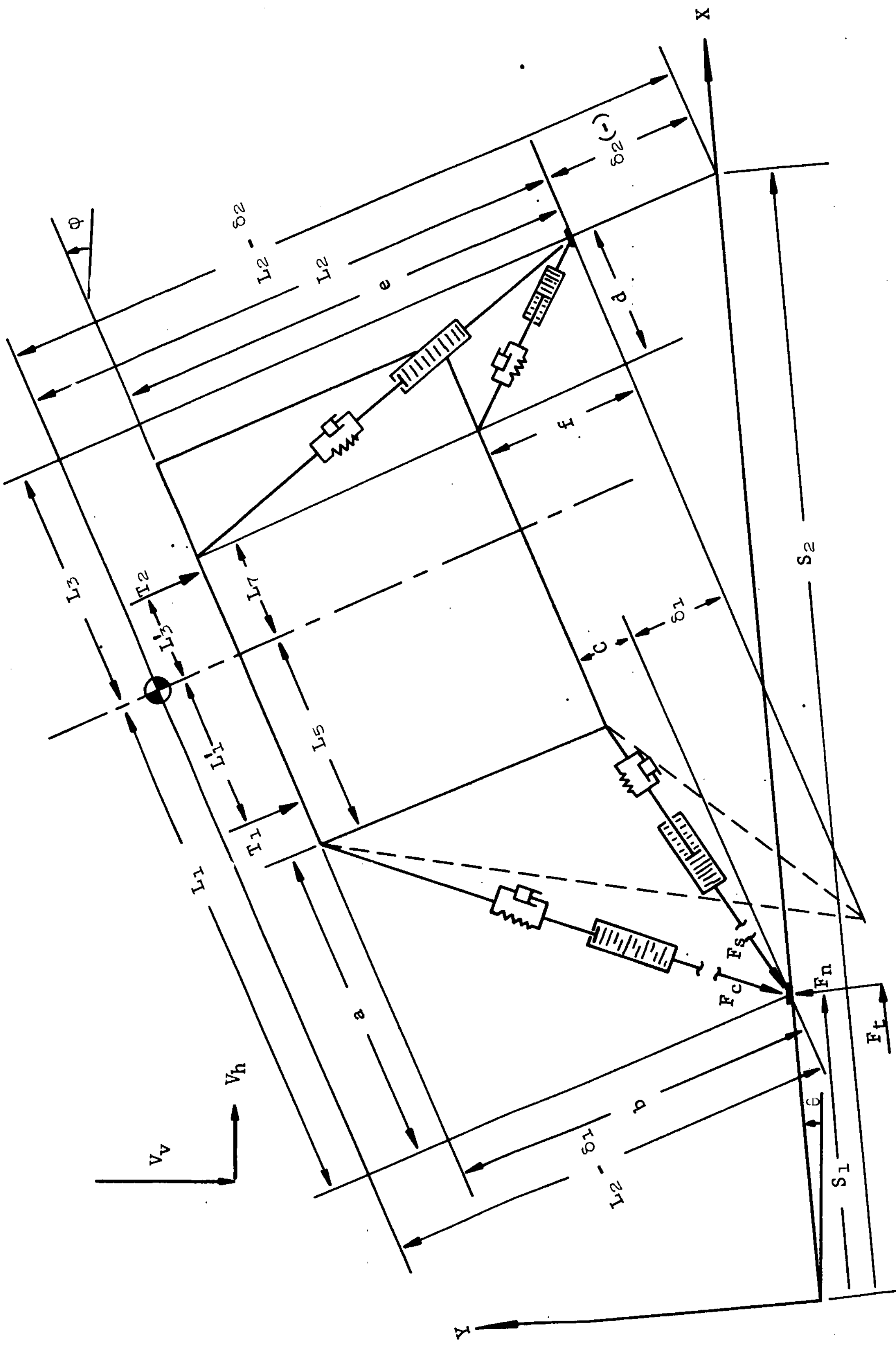

FIG. 1. TOUCHDOWN DYNAMICS MODEL

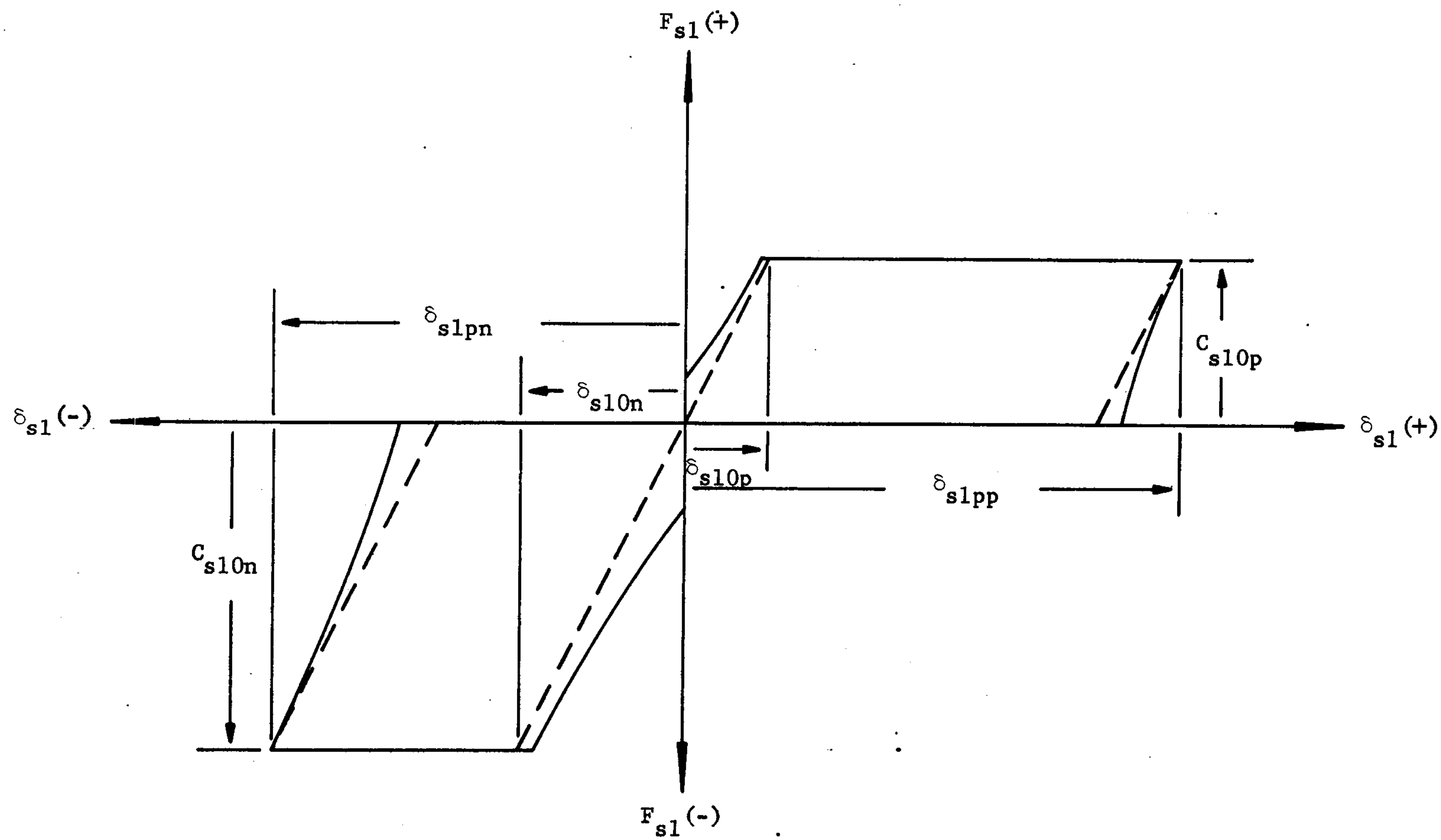

FIG. 2. SUPPORT MEMBER LOAD-DEFLECTION CURVE

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EQUATIONS FOR TWO-DIMENSIONAL ANALYSIS OF TOUCHDOWN DYNAMICS OF SPACECRAFT WITH HINGED LEGS INCLUDING ELASTIC, DAMPING, AND CRUSHING EFFECTS

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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