PROPERTIES OF MULTIELECTRODE RC DISTRIBUTED CIRCUITS*

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Introduction. RC distributed networks with shaped electrodes have been discussed before [1-8]. The purpose of this paper is to discuss some of the properties of these distributed networks.

The type of configuration to be considered is shown in Fig. 1. It is composed of a resistive layer, a dielectric and a conducting electrode. The conducting electrode is subdivided. The shape of these electrodes determines the circuit parameters. The resistive layer has uniform resistivity except for conducting tabs placed across each end.

Circuit Parameters. The schematic representation of the circuit is shown in Fig. 2. The short circuit parameters for this network have been derived [6] and are given here.

$$y_{11} = y_{22} = \frac{W \gamma \operatorname{Coth} [\gamma L]}{R} \tag{1}$$

$$-y_{21} = \frac{W\gamma}{R \operatorname{Sinh} [\gamma L]} \tag{2}$$

$$-y_{31} = \frac{W\pi}{RL} \sum_{n=1}^{\infty} \frac{nsC_{no}^{(3)}}{s + \lambda_{no}} \tag{3}$$

$$-y_{43} = \frac{CLW}{4} \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{C_{nm}^{(3)} C_{nm}^{(4)} s^2}{s + \lambda_{nm}} \tag{4}$$

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$$y_{23} = \frac{W\pi}{RL} \sum_{n=1}^{\infty} \frac{nsC_{no}^{(3)} \operatorname{Cos}(n\pi)}{s + \lambda_{no}} \tag{5}$$

$$y_{33} = sCWL \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{C_{no}^{(3)} (1-(-1)^n)}{n} - \frac{1}{4} \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{(C_{nm}^{(3)})^2 s}{s + \lambda_{nm}} \tag{6}$$

$$\text{where } \lambda_{nm} = \frac{(\frac{n\pi}{L})^2 + (\frac{m\pi}{W})^2}{RC} \tag{7}$$

$$\text{and } \gamma = \sqrt{RCs} \quad . \tag{8}$$

R is in ohms per square.
s is the complex frequency variable.
C is in farads per unit area.

The capacity of electrode number 3 to the resistance layer is a function of x and y. In the above equations this has been defined by the Fourier expansion in two variables.

$$C^{(3)} = C \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} C_{nm}^{(3)} \operatorname{Sin}(\frac{n\pi x}{L}) \operatorname{Cos}(\frac{m\pi y}{W}) \quad . \tag{9}$$

In this case $C_{nm}^{(3)}$ is a dimensionless Fourier coefficient. Parameters y_{41}, y_{42}, and y_{44} are similar to y_{31}, y_{32}, and y_{33} respectively if $C_{mn}^{(3)}$ is replaced by $C_{mn}^{(4)}$. Any number of electrodes can be accounted for in a similar manner. If only predetermined terms in the series are non-zero and these terms are under control, then the parameters are rational. This is certainly the case for short circuit transfer admittances from any electrode to terminals 1 or 2 where control of only C_{no} is required. Usually other parameters can be approximated sufficiently close for other cases when C_{nm} is important.

In Fig. 1, electrode number 3 is bounded by $(0 \leq y \leq f(x))$ and $(0 \leq x \leq L)$. In this case

$$C_{no} = \frac{2}{L} \int_0^L f(x) \, \text{Sin}\frac{n\pi x}{L} dx \quad . \tag{10}$$

The C_{no}'s are the Fourier coefficients in the expansion of $f(x)$ on the interval--$L \leq x \leq L$ where $f(x)$ is considered to be an odd function. In this report C_{00} is always zero. However, it has been shown[6] that if

$$f(x) = \frac{Wx}{L} + W\sum_{n=1}^{N} a_n \, \text{Sin}\frac{n\pi x}{L} \tag{11}$$

then

$$-y_{21} - y_{31} = \frac{W}{RL} + \frac{W\pi}{RL} \sum_{n=1}^{N} \frac{ns \, a_n}{s + \lambda_{no}} \quad , \tag{12}$$

This allows for a direct synthesis of a short circuit transfer function. If it has a zero at the origin it is possible to use either (3) or (5). If there is no zero at the origin then (12) is applicable. The only requirement is that the function $f(x)$, as defined, be within the bounds of the rectangular structure. This method is quick and easy but not perfectly general. There are some cases where $f(x)$ goes out of bounds.

An important property of this form of distributed circuit is that many moderately high degree functions can be realized with a single structure even zeros in the right half of the s plane. The poles are related by (7), but any of the residues may be zero. Flexibility can be increased by the combination of several of these networks.

<u>Example</u>. If given a desired transfer function of

$$f(s) = \frac{s^2 + s + 4}{(s+1)(s+4)} \tag{13}$$

then it may be expanded in the form of (12) as

$$-y_{21} - y_{31} = 1 - \frac{4s}{3(s+1)} + \frac{4s}{3(s+4)} \quad . \tag{14}$$

This determines the function f(x).

$$f(x) = W\left[\left(\frac{x}{L}\right) - \frac{4}{3\pi}\mathrm{Sin}\left(\frac{\pi x}{L}\right) + \frac{2}{3\pi}\mathrm{Sin}\left(\frac{2\pi x}{L}\right)\right] . \tag{15}$$

Then if $RC = (\pi/L)^2$, the desired function is realized with the multiplying constant of $W/(RL)$.

Other Rational Functions. A system with all rational parameters has been presented [3, 7]. However, this system called for a resistance in one direction to be equal to zero. The approximation of a rational driving point function can be obtained to whatever degree required with uniform resistivity. This may be accomplished by one of two methods. In (7) the pole frequency is given. If W is much less than L, then the values of $-\lambda_{no}$ may be made the dominant poles and control the function over a band of frequencies. The C_{no} values are easy to control as given by (10).

An alternate procedure is to place parallel conducting strips along the resistive layer running in the y direction. A small number of these strips is usually sufficient. An exact solution for a given number of strips can be obtained by looking at the structure as a number of structures of the type already described with interconnections as shown in Fig. 3. This is long and tedious. A simple case illustrates the problem.

Let the function f(x) which bounds electrode number 3 be

$$f(x) = .5W\mathrm{Sin}\frac{\pi x}{L} . \tag{16}$$

Then by (10) $C_{10} = .5$, all other $C_{no} = 0$. C_{nm} in general is given by

$$C_{nm} = \frac{4}{LW}\int_0^L \int_0^{f(x)} \mathrm{Cos}\frac{m\pi y}{W}\mathrm{Sin}\frac{n\pi x}{L}dydx \tag{17}$$

or

$$C_{nm} = \frac{4}{m\pi L} \int_0^L \operatorname{Sin}\left(\frac{m\pi}{W} f(x)\right) \operatorname{Sin}\frac{n\pi x}{L} dx \quad , \tag{18}$$

In the general case C_{nm} is $\frac{2}{m\pi}$ times the nth harmonic of $\operatorname{Sin}\left(\frac{m\pi}{W} f(x)\right)$.

In this example the solution of (18) yields

$$C_{nm} = \frac{4}{m\pi} J_n \left(\frac{m\pi}{2}\right) \quad . \tag{19}$$

For this case, the width modes ($m \neq 0$) correspond to higher frequency poles than the case without the strips. The total residue for a given pole is determined by the sum of the squares of the corresponding C_{nm} terms for each section; whereas the sum of the C_{nm} terms is not changed by the added strips. Hence the twofold results of the strips are to move the unwanted poles to higher frequencies and generally to reduce the values of their residues. In practice, if W is the same order of magnitude as L, only a few strips are required.

Conclusion. A single network of the type described can produce a high order short circuit transfer admittance including complex zeros in each half of the s plane. These transfer admittances can always be made rational. The allowable pole values are related. Rational driving-point admittances can be approximated if a small number of conducting strips are added to the resistive layer.

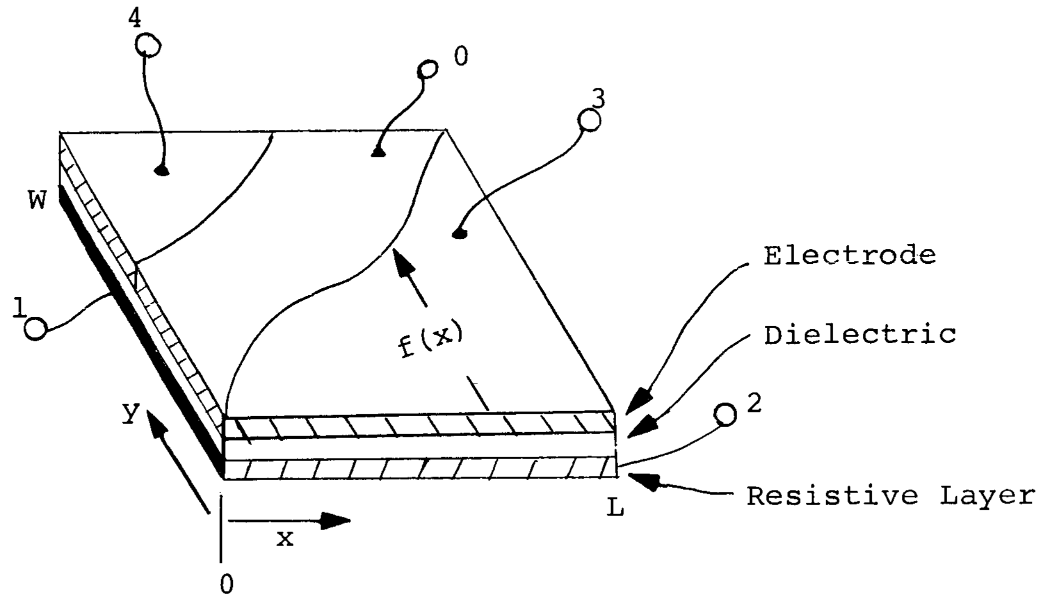

Fig. 1
Network Structure

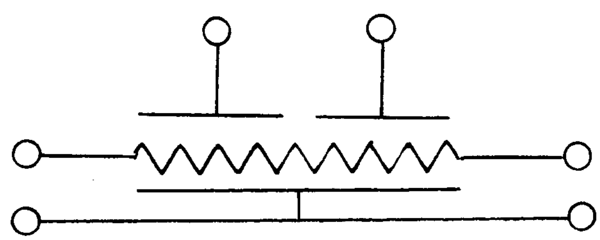

Fig. 2
Schematic Representation

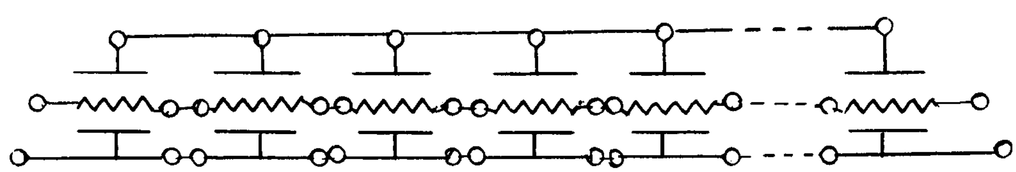

Fig. 3
Connected Components

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