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THE ROLE OF THERMAL
INSTABILITIES IN STAR FORMATION

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SUMMARY

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The stability of a uniform medium is discussed including thermal and magnetic effects. The new stability criteria are shown to be fundamentally different from those of Jeans (1928) and Chandrasekhar and Fermi (1953), and they reduce to Field's (1965) thermal instability criteria when self gravitation is neglected.

It is shown that Field's (1962) model for the intercloud component of the interstellar medium can lead to the formation of condensations having masses on the order of $100M_{\odot}$ in a time scale of between 10^{9} and 10^{10} years by the two step process of thermal instability at pressure equilibrium followed by gravitational collapse.

I. Introduction. - It is well known from the Virial Theorem that, assuming reasonable temperatures and densities, the smallest mass condensation that can be formed, gravitationally, out of the interstellar gas is greater than $10^3 M_\odot$. Therefore, it has been suggested that stars are formed by fragmentation processes that occur during the collapse of large interstellar gas clouds. This hypothesis is supported by the fact that young stars are frequently found in clusters.

However, groups of only two or three T Tauri stars embedded in isolated dark clouds are known, and it is possible that single T Tauri stars occur that are isolated from existing stars, and whose origin cannot be associated, kinematically, with existing associations or clusters (Herbig, 1965). Thus, it may be necessary to seek out some means of forming primary condensations out of the interstellar gas, having masses that are intermediate between those of galactic clusters and individual stars. Moreover, if a single probable example of an isolated T Tauri star can be found, it will be necessary to explain the formation of stellar mass condensations directly out of the interstellar medium, without recourse to the fragmentation hypothesis.

Strömgren (1948) showed that the observed weak component of the interstellar absorption lines could be explained by assuming that the interstellar medium has an intercloud component having a mean density of about 10^{-1} atoms/cm^3.

In 1962 Field showed that the temperature of the intercloud medium could be maintained at $\sim 10^{4\circ}$K by cosmic ray heating and that, at this temperature, the interstellar gas would be thermally unstable to wavelengths much shorter than the critical Jeans length. Field went on to show that the interstellar clouds that we observe in the galsxy could have formed by thermal instability out of a uniform intercloud medium on a time scale of about 10^9 years. The final result of such a condensation process would be a stable, cool, interstellar cloud in pressure equilibrium with the tenuous, high temperature, intercloud medium.

In a recent paper on thermal instabilities, Field (1965) generalized Weymann's (1960) thermal instability criterion to include the effects of a uniform magnetic field. However, since he did not include the self gravitation of the density perturbation in either of his analyses, he formed stable structures through thermal squeezing. In this paper, Field's calculations will be rediscussed including self gravitation, and it will be shown that 100 $M_\odot$ objects can form directly by a two step process: thermal instability followed by gravitational collapse. This mechanism was suggested by Gold and Hoyle (1958) as a means of forming galaxies in a steady state universe.

2. The stability analysis. - To make the problem tractable, we idealize the interstellar medium to be a uniform, quiescent,

radiating, thermally conducting, optically thin gas permeated by a uniform magnetic field. The new information that we are including in the present analysis that is not included in the Chandrasekhar-Fermi treatment (1953), is contained in the energy balance equation

$$\frac{Dp}{Dt} = \frac{5p}{3\rho}\frac{D\rho}{Dt} + \frac{2}{3}\left[H(p,\rho) - C(p,\rho)\right] + \frac{2\mu m_H}{3\chi}\nabla\cdot\nu\nabla\left(\frac{p}{\rho}\right), \quad (1)$$

where P is the gas pressure, ρ is the density, H (p,ρ) is heat input rate/cm^3, C (p,ρ) is the atomic cooling rate/cm^3, ν is the coefficient of thermal conduction, μ is the molecular weight, m_H is the mass of the hydrogen atom, and χ is Boltzmann's constant. Since a magnetic field is present, the coefficient of thermal conduction has a directional dependence. Accordingly, we separate ν into a component parallel to the magnetic field, ν_1, and a component perpendicular to the magnetic field, ν_2. In dimensionless variables we can write the linear equations of hydromagnetics, the conservation of thermal energy, and Poisson's equation in the form:

$$\frac{\partial y}{\partial \tau} + \frac{5}{3}BR\vec{\nabla}\cdot\vec{u} = 0, \quad (2)$$

$$\frac{\partial \vec{u}}{\partial \tau} + BR\vec{\nabla}z + R\vec{\nabla}\phi + GR[\vec{j}\times(\vec{\nabla}\times\vec{h})] = 0, \quad (3)$$

$$R^2\nabla^2\phi = -Ay, \quad (4)$$

$$\frac{\partial \vec{h}}{\partial \tau} - IR[\vec{\nabla}\times(\vec{u}\times\vec{j})] = 0, \quad (5)$$

$$\left[\frac{\partial}{\partial\tau} + F - K_1\frac{\partial^2}{\partial\xi_1^2} - K_2\left(\frac{\partial^2}{\partial\xi_2^2}+\frac{\partial^2}{\partial\xi_3^2}\right)\right]z =$$

and
$$\left[\frac{5}{3}\frac{\partial}{\partial\tau} + D - K_1\frac{\partial^2}{\partial\xi_1^2} - K_2\left(\frac{\partial^2}{\partial\xi_2^2}+\frac{\partial^2}{\partial\xi_3^2}\right)\right]y\,, \tag{6}$$

where $\vec{j}$ is the cartesian unit vector along the axis of the magnetic field in the undisturbed model. In these equations

$$\rho = \rho_0(1+y)\,,\ p = p_0(1+z),\ \vec{V} = V_s\vec{u}\,,\ \vec{\mathcal{H}} = \mathcal{H}_0\vec{n}\,,\ \vec{r} = R\vec{\xi}\,,$$
$$t = \rho_0 V_s^2\tau/C(p_0,\rho_0),\ \phi = \rho_0 V_s\Theta/[C(p_0,\rho_0)R]\,,$$
$$B = p_0 V_s/[C(p_0,\rho_0)R],\ F = \frac{10}{9}\left[\left.\frac{\partial C}{\partial p}\right)_\rho - \left.\frac{\partial H}{\partial p}\right)_\rho\right]\frac{p}{C(p_0,\rho_0)}\,,$$
$$D = \frac{10}{9}\left[\left.\frac{\partial H}{\partial \rho}\right)_p - \left.\frac{\partial C}{\partial \rho}\right)_p\right]\frac{\rho_0}{C(p_0,\rho_0)}\,,\ I = \rho_0 V_s^2/[R\,C(p_0,\rho_0)],$$
$$K_1 = 2\nu_1\mu m_H V_s^2/[3kR^2C(p_0,\rho_0)],\ K_2 = 2\nu_2\mu m_H V_s^2/[3kR^2C(p_0,\rho_0)],$$
$$G = I V_A^2/V_s^2\,,\ A = 4\pi\Gamma\rho_0^2 V_s R/C(p_0,\rho_0)\,,$$

$\vec{\mathcal{H}}_0$ is the magnetic field strength in the undisturbed medium, $\vec{V}$ is the hydrodynamic velocity, Θ is the gravitational potential, Γ is the constant of gravitation, R is a characteristic length, and V_A and V_S are, respectively, the Alfvén speed and the adiabatic sound velocity in the undistrubed model.

After eliminating the variables in the usual way, we obtain the equation

$$\left[\frac{\partial}{\partial\tau} + F - K_1\frac{\partial^2}{\partial\xi_1^2} - K_2\left(\frac{\partial^2}{\partial\xi_2^2}+\frac{\partial^2}{\partial\xi_3^2}\right)\right]\left\{\frac{\partial^2}{\partial\tau^2}\left[\frac{\partial^2}{\partial\tau^2} - \frac{5BA}{3}\right.\right.$$
$$\left.\left. - GI\left(\frac{\partial^2}{\partial\xi_1^2}+\frac{\partial^2}{\partial\xi_2^2}+\frac{\partial^2}{\partial\xi_3^2}\right)\right] + \frac{5BIGA}{3}\frac{\partial^2}{\partial\xi_1^2}\right\}y \tag{7}$$
$$- \frac{5B^2}{3}\left(\frac{\partial^2}{\partial\xi_1^2}+\frac{\partial^2}{\partial\xi_2^2}+\frac{\partial^2}{\partial\xi_3^2}\right)\left(\frac{\partial^2}{\partial\tau^2} - IG\frac{\partial^2}{\partial\xi_1^2}\right)\left[\frac{5}{3}\frac{\partial}{\partial\tau} + D\right.$$
$$\left. - K_1\frac{\partial^2}{\partial\xi_1^2} - K_2\left(\frac{\partial^2}{\partial\xi_2^2}+\frac{\partial^2}{\partial\xi_3^2}\right)\right]y = 0\,.$$

We assume a solution of the form exp $[\, i(k_1\xi_1+k_2\xi_2+k_3\xi_3)+\omega\tau\,]$ and find that ω must satisfy the characteristic equation

$$[\omega+F+k_1^2K_1+(k_2^2+k_3^2)K_2]\{\omega^2[\omega^2-5BA/3+(k_1^2+k_2^2+k_3^2)GI]-5BAGIk_1^2/3\}+(5B^2/3)(k_1^2+k_2^2+k_3^2)(\omega^2+k_1^2GI)[5\omega/3+k_1^2K_1+(k_2^2+k_3^2)K_2]=0. \quad (8)$$

For brevity, we let $J=(5/3)BA$, $\alpha_1=F+k_1^2K_1$, $\alpha_2=F+(k_2^2+k_3^2)K_2$, $\beta_1=(5Bk_1/3)^2$, $\beta_2=(k_2^2+k_3^2)(5B/3)^2$, $\psi_1=D+k_1^2K_1$, $\psi_2=D+(k_2^2+k_3^2)K_2$, $M_1=k_1^2GI$, and

$$M_2=(k_2^2+k_3^2)GI.$$

We will now examine the stability of wavelengths propagating parallel and perpendicular to the magnetic field.

Case a) Propagation parallel to the magnetic field; $k_2=k_3=0$.

The characteristic equation becomes

$$[\omega^2+M_1][\omega^3+\alpha_1\omega^2+(\beta_1-J)\omega+3\beta_1\psi_1/5-\alpha_1J]=0. \quad (9)$$

When $\omega=\pm i M_1^{1/2}$, the modes correspond to pure transverse Alfvén waves that are uncouples from the thermal and gravitational behavior of the gas. However, the modes satisfying the condition

$$\omega^3 + \alpha_1\omega^2 + (\beta_1 - J)\omega + 3\beta_1\psi_1/5 - \alpha_1 J = 0 \qquad (10)$$

represent two non-adiabatic sound waves (modified by self gravitation) and a non-propagating thermal-gravitational mode. With the exception of the J terms, which represeent the effects of self gravitation, equation (10) was derived originally by Field (1965) in different notation. When the thermal terms, α_1 and ψ_1 , are ignored, we recover Jeans' criterion (1928) for gravitational instability.

We observe that, for finite temperature, the thermal effects appear to persist even when the condensation becomes arbitrarily large. In this limit the roots of equation (10) become $\omega = \pm J^{1/2}$ and $\omega = -\alpha_1$, representing two free fall collapse modes and a pure thermal mode. However, as $R \rightarrow \infty$ the amplitude of the thermal mode approaches zero and only the pure gravitational modes remain.

The instability of the medium is assured when the constant term in equation (10) < 0. If we define an index of Jeans stability by $\epsilon_1 = \beta_1 - J$, where $\epsilon_1 > 0$, the medium is unstable provided

$$\psi_1 < 5(1 - \epsilon_1/\beta_1)\alpha_1/3 \qquad (11)$$

When ϵ_1 is small, we are considering wavelengths on the order of the critical Jeans length. However, when $\epsilon_1 \sim \beta_1$, we are considering short wavelengths where self gravitation is unimportant and pressure gradients dominate the dynamical behavior of the gas.

According to Field (1962), it is possible that cosmic rays are the most important source of heating in the intercloud medium. Thus, $H \sim \text{const} \times \rho$ (Field, 1962). If cooling occurs through atomic processes according to the general relation $C(p,\rho) = \sum_i' L_i \rho^2 Q_i(p,\rho)$, where the L_i's are constants and the Q_i's are arbitrary functions, $\psi_1 = \alpha_1 - 10/9$. With these assumptions the instability condition becomes

$$\alpha_1 \left[1 - 5\epsilon_1/(2\beta_1)\right] > -5/3 \quad . \tag{12}$$

Case b) Propagation perpendicular to the magnetic field; $k_1 = 0$.

In this case the velocity of the slow magneto-accoustic mode goes to zero, and the characteristic equation becomes

$$\omega^3 + \alpha_2\omega^2 + (\beta_2 - J - M_2)\omega + 5\beta_2\psi_2/3 + \alpha_2(M_2 - J) = 0. \tag{13}$$

When $J = 0$ we recover Field's dispersion relation for wave propagation transverse to a uniform magnetic field. In addition, if α_2 and ψ_2 are zero, we recover the Chandrasekhar-Fermi (1953) result for gravitational instability perpendicular to a uniform magnetic field.

Again, the instability condition is that the constant term in equation (13) be < 0. For the specific case of cosmic ray heating and radiative cooling, the instability criterion is

$$\alpha_2\left[1 - 5\epsilon_2/(2\beta_2) - 5M_2/(2\beta_2)\right] > -5/3 \,. \qquad (14)$$

As we expect, it is more difficult for a collapse to occur transverse to the magnetic field than along the field. It is important to note that, in both cases, the medium can be unstable, even when it is stable according to Jeans' criterion.

If the magnetic pressure is much larger than the gas pressure ($M_2/\beta_2 \gg 1$), the thermal instability criterion becomes $\alpha_2 < 0$. However, Field's (1962) cooling calculations for the interstellar medium show that $\alpha_2 > 0$ for $T > 15^\circ$K. Hence, a "strong" magnetic field ($\gg 10^{-6}$G) stabilizes the intercloud gas, and wavelengths shorter than the critical Chandrasekhar-Fermi length would be thermally stable to disturbances perpendicular to the magnetic field.

3. An estimate of the smallest mass primary condensation that can form out of the intercloud medium by gravitational collapse following a thermal instability when magnetic pressure is unimportant. - For our model of the intercloud medium ($\rho_0 = 10^{-25}$ gm/cm^3, and $T_0 = 10^{4\,\circ}$K) if the magnetic field strength is considerably less than 10^{-6}G, $M/\beta \ll 1$, and the magnetic pressure can be initially neglected. However, assuming that the magnetic field lines remain frozen into a spherical cloud during the thermal collapse, the initial magnetic field strength must be $\ll 10^{-6}\ (\rho_0/\rho)^{4/3}$ in order for the magnetic pressure to be negligible after the cloud density has been increased by thermal instability from initial value ρ_0

to final value ρ . This result follows from the facts that a) The "pure thermal instability" would develop very nearly at pressure equilibrium with the intercloud medium, and b) The magnetic field strength in the collapsing configuration would grow in proportion to the square of the redius of the cloud (Mestel and Spitzer, 1956)[1]

If the above inequality is not satisfied in the intercloud medium, then the collapse would occur preferentially along magnetic field lines. The development of such a cloud would be very complicated and will not be considered here. Instead, attention will be focused on the most favorable case for the formation of gas clouds out of the intercloud medium, in which magnetic pressure is negligible throughout the collapse. Since current estimates of the mean magnetic field strength in the interstellar medium lie between 10^{-5}G and 10^{-6}G (Spitzer, 1963), we observe that this condition will not be satisfied generally. However, since the galactic magnetic field may be quite variable, it is consistent with present knowledge to suppose that we are considering a local region in the intercloud medium where the magnetic field strength is abnormally low (i.e., a magnetic hole).

For our assumed intercloud conditions, the mass contained within a condensation having a diameter equal to one Jeans wavelength, λ_J, is about $2.85 \times 10^7\ M_\odot$. However, at a temperature of 10^4 °K, Field's calculations indicate that, providing

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1. The author is indebted to an anonymous referee for pointing out this fact

thermal conduction is small, $\alpha \sim 0.2$. This would be the case since we are considering an H I region. Consequently, the intercloud medium would be thermally unstable to short wavelength perturbations. Following a thermal instability, as the density increases and the temperature drops, the critical Jeans' length decreases rapidly . Assuming that the thermal collapse occurs under pressure equilibrium conditions (as it would for short wavelengths), the Jeans' length, $\lambda_J(T, \rho) = \lambda_J(T_o, \rho_o)T/T_o$. Consequently, the mass contained within a growing condensation, M_C, would be gravitationally unstable according to Jeans' criterion if

$$M_c > \frac{\pi}{6} \lambda_J^3(T_o, \rho_o)\, \rho_o \left(\frac{T}{T_o}\right)^{\gamma} . \tag{15}$$

Field's calculations show that, for an initial temperature of 10^{4}°K, the stable equilibrium temperature for an interstellar gas cloud collapsing under pressure equilibrium condition is about 32°K. We can estimate the smallest mass that can be formed by gravitational collapse following a thermal instability by letting $T_0 = 10^{4}$°K and $T = 32$°K. Thus, we find $M_C \sim 300 M_\odot$.

Nonetheless, Jeans' criterion again overestimates the smallest unstable mass, since the interstellar medium does not behave adiabatically at 32°K. At this temperature $\alpha \sim 5$, which implies that the shortest unstable wavelength is $(7/15)^{1/2} \lambda_J$. Consequently, the smallest unstable mass is about $(7/15)^{3/2} M_C$ or $\sim$ 100 $M_\odot$. However, in order for magnetic pressure to be negligible throughout such a collapse, the initial magnetic field strength in the magnetic hole would have to be extremely low; not in excess of a few x 10^{-9} G. According to this idealized picture, condensations less massive than 100 $M_{\odot}$ would form stable interstellar clouds. We note in passing that the classical Jeans criterion predicts that the smallest unstable mass should be about five orders of magnitude greater than our result

It is interesting to note that the minimum unstable mass derived above is in close agreement with the estimated masses of the smallest observed galactic clusters (Hogg, 1959). While it does not seem likely that individual stars can form directly by thermal gravitational collapse, the possibility cannot be excluded. If there exist hot regions in the interstellar medium in which the density is about two orders of magnitude greater than we have supposed, stellar mass condensations could form by this mechanism. However, in the absence of observational data pointing to the existence of such initial conditions, the formation of single stars by thermal gravitational collapse can only remain a possibility.

The time scale for forming interstellar clouds out of the intercloud medium is on the order of 10^9 years (Field, 1962). Since the free fall collapse time at cloud densities $\ll 10^9$ years, our simplified picture implies that the time scale for star formation should be only a few times 10^9 years; a full order of magnitude less than the estimated age of our galaxy. Consequently, if the ideas set forth in the previous sections are to be taken seriously, we must explain why all of the interstellar matter did not form into stars in less than 10^{10} years - i.e., why does the interstellar gas still exist ?

4. Disruptive effects. - The following effects would tend to disrupt the condensation process, and thereby prolong the time scale for star formation:

(1) Magnetic fields of appreciable strength in the intercloud medium,

(2) Turbulence in the intercloud medium,

(3) Angular momentum that would be present in a real cloud,

and (4) Collisions between collapsing clouds during the initial phases of star formation.

Presumably, effects 1), 2), and 3) vary greatly from place to place in our galaxy. However, Kahn (1955) has demonstrated that cloud-cloud collisions occur relatively frequently in the spiral arms of our galaxy, and this effect by itself would considerably prolong the time scale for star formation. Hence, while no quantitative conclusions about disruptive effects can be drawn at this time, it seems likely that there exists a sufficient variety of disruptive phenomena in the interstellar medium to increase the short time scale for star formation that we have derived, using an idealized model for the intercloud medium, so that it is consistent with the estimated age of the galaxy.

5. Conclusions. - It has been shown that, providing magnetic pressure is negligible, initial thermal instabilities in a uniform intercloud region of the interstellar medium, having a temperature of $10^{4\circ}$K and a density of 10^{-25} gm/cm^3, could lead to the formation of $\sim$ 100 $M_\odot$ gravitational condensations by the two step process of thermal instability at pressure equilibrium followed by gravitational collapse. Moreover, if there exist regions in the interstellar gas

in which the temperature is $\sim 10^{4\circ}$K and the density is two orders of magnitude greater than that of the intercloud medium, stellar mass condensations could form directly by the same process.

Since isolated groups of T Tauri stars provide examples of small scale condensations, it is urged that observational attention be focused on the question of determining what is the smallest mass primary condensation that exists in our galaxy.

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