# CONTRACTOR DECISION MAKING

# and

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PREFACE

SUMMARY

CONCLUSIONS

RECOMMENDATIONS

## Preface

Interest in incentive contracting has grown with the procurement of complex weapons and space systems – procurements involving a preponderance of R&D effort and uncertainty in the cost outcomes. Because of these cost uncertainties, contracts for R&D procurements are typically negotiated with Cost Plus Fixed Fee provisions. Two objections are generally imputed to the CPFF contract: it encourages an understatement of the target cost; it is not conducive to efficient contractor performance. The incentive contract fee arrangements (the Cost Plus Incentive Fee and the Fixed Price Incentive), on the other hand, incorporate contractor participation in cost overruns or underruns through a variable fee schedule and, in theory, generate conscious cost control efforts on the part of the contractor through an appeal to the profit motive.

The efficacy of the incentive contract has been questioned and defended at some length; to date, its proponents seem to outnumber its detractors. Incentive contracts have generally received industry commendation; industry spokesmen specifically acknowledge that these contracts contribute to efficiency. NASA has placed considerable emphasis on substituting incentive for CPFF contracts whenever possible. Because of the important status of this contract form and a specific expression of NASA's interest in a study of the "objectivity of NASA's fee policy", our investigation to date is devoted to the incentive contract fee objectives – its capability for accomplishing the ostensible contracting objectives, the validity of its rationale, the basis for the contractor interest in this type of contract, etc. [1]

We have departed from the customary form of presentation in that a somewhat lengthy summary, conclusions and recommendations are presented prior to the discussion proper. This arrangement is intended to accommodate those desiring a general discussion of our approach, but who are disinclined to pursue a lengthy discussion incorporating mathematical notation.

## Summary

This paper is an inquiry into the rationale of the Cost Plus Incentive Fee and the Fixed Price Incentive contract forms. Experience with the incentive contract is well documented and there is little new or revealing to be expected from further surveys or data collection in this field. Despite this, we are unaware of any effort to systematically analyze this contract form in the presence of these facts. (The recent papers of Frederick Scherer's are an exception). In short, we believe that the known experience with incentive contracts should be sufficient to determine the influence of the contract fee arrangements on the contractor's efficiency, the contractor's evaluation of this type of contract, and the desirability of continuing with this contract form in pursuit of the ostensible objectives offered for its use.

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[1] The direction and objectives of our research were reoriented at the beginning of this year at the request of the Office of Policy Planning (NASA) in conformity to that office's objective of identifying and analyzing NASA's present and emerging policy problems.

The principal thesis of this paper is that the incentive contract form must be examined in terms of elementary decision theory (analysis incorporating the contractor's utility function and the question of choice under uncertainty) We are convinced that this approach clarifies the contractor's decision problem and develops the necessary insight as to the value of this contract for stimulating contractor efficiency. The development in the earlier part of the paper is the necessary background for this thesis.[2]

There are certain assumptions (both explicit and implicit) associated with incentive contracting which are useful to keep in mind.

(1) A contractor acts to maximize his short-run profits.

(2) The contractor can control the cost outcome of the contract and participate in the benefits (rewards) arising from the additional profit associated with improvements in efficiency; i.e., the contractor will reap rewards of efficiency and pay the penalty of inefficiency.

Experience with incentive contracts, however, clearly indicates that the cost sharing by the contractor is not sufficient to stimulate efficiency, and there is constant admonishment on the part of the procurement officials to negotiate sharing fractions which are considerably greater than the present experience. (See the quotations from the speech of Thomas Morris, page 5.)

This leads us to a fundamental inconsistency in incentive contracting. The contractor's efficiency is not a factor to be associated with risk; efficiency is directly and completely controllable by the contractor. (The association of efficiency and risk can be noted in the quotation on page 5.) Therefore the contractor's choice between inefficient procedures or profits (the choice that the fee schedule attempts to influence) is a choice which can only be made under conditions of cost certainty. But the cost uncertainties associated with incentive contracting conditions are patent – uncertainties which, in most instances, exceed by a considerable magnitude the range of cost under the influence of the contractor. The final cost outcome has but very limited relationship to the contractor's efforts or capabilities at cost control. Therefore, at the time of negotiation, the contractor is more concerned with establishing a favorable position with respect to the cost uncertainties involved in the contract than the possibility of rewards for future efficiency. It is not logical to design an appeal to a contractor's profit motive through a cost outcome over which he has little control. Maximization of profit is a logical objective under conditions of cost certainty, since the maximum utility will always occur at the point of maximum profit. However, the contractor will not necessarily act to maximize the expected profit, since maximizing expected profit maximizes the contractor's utility only in exceptional circumstances. This point receives considerable elaboration in the paper.

Furthermore, the procurement manuals, contracting guides, etc. convey another implicit assumption – that the contractor's risk is primarily a function of the sharing fraction. The contractor's participation in cost sharing is certainly an element in the risk assumed, but risk, by any measure, is a sensitive function of the distribution of

[2] We will not elaborate here on the background material and support for our arguments which is presented in the first part of the discussion material. We have limited the summary to our primary points and these are summarized in order of their occurrence in the discussion.

the possible cost outcomes and, even more particularly, of the relation of the target cost to the distribution of possible cost outcomes. Incentive contract negotiations attempt to maximize the risk to the contractor through the negotiation of tight target costs while, at the same time, urging greater contractor participation in this risk through greater sharing fractions – objectives which are mutually incompatible. The topic of risk and the negotiation of these contracts under these conditions of uncertainty is a primary concern of this paper

One must view the incentive contract negotiations as the arrangement of a satisfactory gamble for the contractor. Any decision problem under uncertainty is properly considered a gamble; there is a unique aspect, however, to the arrangement of this lottery under the terms of an incentive contract which makes it particularly interesting from a decision theory point of view. To explain, first consider a contractor faced with bidding on a sealed-bid contract and with a certain confidence that he must bid-in at $x in order to get the contract. He has but one option –either not to bid (not to gamble), in which case he gains or loses nothing, or to bid the contract, with a certain expectation of gaining a certain profit and another expectation of losing a certain profit. The expected winnings (expected profits) is the probability (degree of belief) that he will receive a positive profit times the amount of that profit. The expected loss will be the probability of a loss times the amount of the loss if a loss occurs. The sum of the two represents his expected monetary gain. Obviously any change in (a) the amount to be gained (or lost) or (b) the probabilities associated with a given gain (or loss) will influence his interest in either accepting or rejecting the gamble. However, under the conditions described, the contractor has no capability of changing either the probabilities associated with a cost outcome or the amount of the expectation; his choice is either accept or reject the lottery.

On the other hand, the incentive contract provides a plethora of opportunities to vary the expected gain or loss, via (a), through the multiplicity of fee arrangements – the upper and lower fee swings, the upper and lower sharing fractions, the target fee and the target cost. [We are assuming that the probabilities (b) attached to the uncertain cost outcomes essentially remain unchanged.] If the contract negotiations involve multiple incentives, the opportunities to vary the expected fee increase by the multiple of the incentive goals. (See page 18) The contractor's options may be best described as a choice from a spectrum of alternatives, with a CPFF contract fee (which is considered by some as a riskless contract) at one end and the FFP fee arrangement at the other.

Our primary point of interest centers on the negotiation of an expected fee outcome of maximum advantage to the contractor and how this is determined. There are certain related questions:

(a) Does the incentive contract provide increased benefits to the contractor (as opposed to a CPFF contract), and do these benefits arise through the exercise of efficiency?

(b) Does the maximum advantage accrue to the contractor with the maximum or minimum of cost-sharing participation, or does it occur with partial fee sharing arrangements?

The explanation of our approach begins with an illustration of a probability distribution of cost outcomes (called a subjective density function) together with the fee function. (See page 11 and Figure 1). Figures 2 and 3 illustrate the sensitivity of the probable or expected loss (for particular contracting conditions) to the measure of cost uncertainty (the standard deviation of the probability distribution of cost outcomes) and describes the "chances" that a contractor has in achieving the theoretical limits of the fee outcome given certain contracting conditions.

This example is offered only to demonstrate what is often overlooked – that the expected fee is considerably less than the maximum fee swing, unless the target cost is negotiated untenably high.

Before answering the above questions it is important to examine the assumption that a contractor operates to maximize his short-run profits. The subject of contractor motivation has been extensively investigated by managerial economists and there is significant agreement that the contractor does not act to maximize short-run profits. This background is reviewed, page 13 to 16. The empirical data on contractor behavior with incentive contracts apparently supports this argument. If the contractor intended to maximize the expected profits he should always elect a maximum sharing fraction – if the target cost is negotiated greater than the expected cost outcome (as illustrated in Figure 4 and in the discussion on page 12). Since the contractor never accepts maximum sharing, one can only conclude two things; either the target cost is typically negotiated less than the expected cost outcome (extremely doubtful) or the contractor is not maximizing the expected profits.

A decision is always a choice between the utilities of the alternatives present in the decision situation. Utility is a concept long used by economists to indicate a measure or index of the satisfaction afforded by an economic good. Since a cardinal measure of utility (an "absolute" measure, like gallons, inches, etc.) is impossible to devise, the concept of utility has generally been left to theoretical discussions. Recent developments in this field have resurrected a measure of utility called the "Bayesian" or "Bernoullian" Utility index which is capable of being determined for an individual. While this index is not strictly a cardinal measure of utility, it is sufficient to allow the determination of an individual's choices under situations of uncertainty – if he follows certain assumptions defining rational behavior. Under these assumptions, the contractor will attempt to maximize his expected utility (the sum of the probabilities associated with each outcome times the utility measure of each outcome) if he behaves rationally.

The fee function is a Bernoullian Utility function only if there is a linear or proportional relationship between the amount of the fee and the "satisfaction" which it generates. Studies of contractor motivation indicate that this is never the case, (unless the fee outcome is comparatively inconsequential to the decision maker, which per se eliminates it from our interest). Rather, the contractor has a decreasing marginal utility for fee (i.e. fees increase at a decreasing rate with an increase in the fee); the change in utility for a given change in fee is represented by a curve which is concave, as illustrated in Figure 5.

One can conclude without further elaboration that the contractor's maximum benefit with incentive contracts does not lie at the FFP end of the fee spectrum. Conversely, then, is the contractor's maximum satisfaction attained with a CPFF contract or with partial sharing fractions? And why? This is an important question to answer in any analysis of incentive contracts, and the general model of contractor behavior, (beginning, page 17) provides the basis for this answer. This model assumes only a general class of utility functions, those which are concave, and symmetrical cost densities. It examines the changes in the contractor's expected utility with changes in the target cost and in the sharing fraction.

This examination yields the following interesting fact. If the target cost is negotiated greater than the contractor's expected cost outcome on the contract there is always a range of sharing fractions which will provide the contractor with a greater expected utility from the incentive contract than he would receive from the CPFF contract, even if they are both negotiated with the same target fee. This is illustrated in Figure 6. Furthermore, there is always one value of this sharing fraction which will provide the maximum expected utility. (This is illustrated by the dotted line, Figure 6). In other words, for every target cost greater than the expected cost, the sharing fraction can take a large number of values which will make the gamble on the profit outcome of the incentive contract a preferable choice to the CPFF contract.

The model of contractor behavior was developed incorporating only one of the several negotiated contract terms – the sharing fraction. It can readily be seen that the negotiation of the fee swings and the target fee, as well as the target cost, will present additional opportunities for increasing the contractor's expected utility; i.e., these additional contract arrangements all present opportunities for effectively accomplishing the same end as would be achieved by increasing the target cost itself. The success of the incentive contract form for achieving greater post facto utility (increased fee) will obviously depend upon the contractor's competence in cost estimation and negotiation. But given a schedule of probable cost outcomes (subjective cost density), the contractor has several paths by which to negotiate a fee arrangement more favorable than that offered by a CPFF contract – or a greater expected utility. We have attempted to illustrate the sensitivity of the expected utility to the various cost and contract parameters in Figures 7 and 8. The utility function assumed for this illustration is presented on page 18.

We believe that this model of contractor behavior is both normative and descriptive; it is not only a model for optimum contractor behavior, it describes present contractor behavior and the basis for the contractor's present satisfaction with this contract form. We believe it is a formal description of the contractor's approach to contract negotiations; it describes how the contractor can improve his situation with an incentive contract – not through cost efficiency efforts but through negotiations.

In a sense, the section discussing risk in incentive contracts (beginning page 19) is somewhat redundant after the discussion of contractor behavior in maximizing expected utility in incentive contracts; a contractor who operates to maximize his expected utility has properly considered risk. However, there may be some who feel that utility measures of contractor behavior remain somewhat esoteric for the pragmatic issues of government procurement and who prefer to consider risk as either the chance of an outright loss or as a measure of the expected amount of that loss, if a loss occurs. Using these latter two definitions of risk together with measures of uncertainty interpreted from the contracting manuals, we have attempted to demonstrate through simple algebra that it would be quite easy to negotiate a CPIF contract with more risk than a FFP contract, even with fairly small sharing fractions. (This development has certain elements of obviosity except for the fact that there is a very strong inclination to forget the measure of cost uncertainty when considering risk in fee arrangements and measuring risk strictly in terms of the sharing fraction). If the target costs are customarily negotiated close to the expected cost outcome, it is unlikely, on this basis alone, that incentive contracts will be negotiated with greater sharing fractions than are presently experienced.

The final section of the paper (beginning on page 23) examines a "fee" function which is an expression of the marginal gain or increment that a given cost-plus contract will add to the firm's gross profit. [This function is hereafter called the marginal fee expression and in the Text portion it is labelled F*(x).] The net increment o[illegible]ed by a cost-plus contract is equal to the fee only if the reimbursement for costs is actually equal to the cost which was incurred. If all costs were direct costs, then the net gain to the contractor would only be that normally associated with the fee. The manner of determining overhead costs, however, is such that for many contracting situations the net gain derivable from the overhead payment is considerably greater than that normally associated with the fee proper. This is because the reimbursement paid to a contractor for overhead is determined by multiplying the direct costs on the contract by a burden factor – the ratio of all overhead to all direct costs. Unless (1) all of the sales are cost-plus, (2) the actual overhead on the contract is of the same proportion as that existing on the other sales in the firm and (3) the overhead varies proportionally with the direct costs, this method of allocating overhead provides essentially payment for costs which were not incurred. For those who believe that the contractor is motivated by the prospects of short-run profit, it is important to note that in many contract cost situations and contracting fee sharing arrangements now being negotiated, the contractor will usually find it to his advantage to incur overruns, rather than underruns.

This net or marginal fee function is the logical one to use for decision making and evaluation purposes by both contracting parties. Although this function was necessarily developed under specific assumptions, it should still be possible to apply it to a broader range of contracting situations, albeit approximate answers will result. The empirical evidence does not support an ascertion that contractors control their cost outcomes to maximize the marginal fee; however, our evidence in[illegible]s that they are generally aware that a considerable portion of their net "fee" arises from the overhead cost reimbursement. Furthermore, it is difficult to explicitly state the proportion of contracting situations in which the marginal fee expression presents a si[illegible]ntly different fee outcome than the more traditional fee expression. Our examples, although approx[illegible] to actual contracting situations, indicate that a firm with 50% or possibly more, cost plus sales would [illegible] significant difference in the two fee expressions.

**Conclusions**

1. The contractor is not greatly motivated by the prospect of a fee greater than that which he considers "fair"; i.e., he has a rapidly decreasing marginal utility for fees. Rather, contractors receive greater motivation from the prospects of future sales.
2. The incentive contract philosophy incorporates the explicit assumption that contractors can and will significantly control their costs to participate in the incentive "rewards" for efficiency. The possibilities for controlling the cost outcomes are insignificant in comparison to the cost uncertainties associated with incentive contracts. Therefore, the contractor seeks the most desirable position in terms of the uncertainties of the cost outcome. This position does not coincide with the negotiation of high sharing fractions and wide fee

swings – terms which would be a significant inducement to the contractor to hold costs.

3. The contractor has an exceptional opportunity with incentive contracts to negotiate fee arrangements providing for a greater expected utility than that derivable from a CPFF contract; i.e., he can derive an acceptably higher expected fee from the incentive contract than from a CPFF contract. However, this increase in utility does not arise from an increase in the contractor's efficiency, but through the opportunities associated with the negotiation of a multiplicity of contract arrangements. Therefore the contractor probably has ample reason, on the average, to prefer incentive contracts.
4. Tight target costs are completely incompatible with the negotiation of strong incentive provisions (high sharing fractions, etc.). Assuming that the target costs are reasonably "tight" , on the average (a factor difficult to measure), there is a definite indication that sharing fractions larger than those now being negotiated would result in incentive contracts with greater risk than FFP contracts – unless the target fee is considerably greater than the legal limits.
5. Furthermore, there appears to be little inducement for the government contract negotiators to negotiate sharing arrangements which would provide a significant inducement for the contractors; in fact, it is usually to their (contract officer's) advantage to arrange moderate to small sharing arrangements.
6. Incentive contracts have an advantage of flexibility in the fee arrangements, assuming that this attribute is a rational and desirable objective. Despite certain past criticisms, the incentive contract has the quasi-theoretical facade of promoting the efficiency through a basic appeal to the profit motive. Since it is unlikely that incentive contracts will ever be developed with significant sharing arrangements, there is only limited support for arguments that a high fee outcome is the product of efficiency, and the profit outcomes are likely to be vulnerable to greater criticism in the future. Another possible advantage to incentive contracts may be generated in the inherent emphasis on meeting the target cost; the psychological effects of such emphasis may prevent or mitigate large overruns. (This particular and important aspect was not investigated in this paper). This possible advantage, however, is independent of the incentive fee arrangements.
7. The method of pricing cost-plus contracts can result in a greater fee for overruns than for underruns. This is a significant factor to consider if one is designing a fee schedule to appeal to a short-run profit motivation. However, if contractors are oriented towards the short-run profit rewards one would expect a greater number of cost overruns on incentive contracts than are presently indicated by empirical data. Therefore, there is reason to believe that the contractors are generally unconcerned about the marginal aspects of the fee situation, or that they are more concerned with the positive benefits to their long-run sales associated with a reputation for achieving target costs.
8. Our conclusions point to the general absence of a potential for negotiating fee arrangements which will stimulate contractor efficiency. We believe, as does Scherer, that the contractor's performance can be better controlled through some organized method of contractor performance evaluation or, generally, through inducements directed at long-run rather than short-run interests of the contractor.

### Recommendations

1. If contractor efficiency and control of the contract costs are the primary basis for using incentive contracts, NASA should seek other fee arrangements than the CPIF or the FPI.
2. Motivation of contractors should be based primarily on appeals to their long-run rather than their short-run interests. The Contractor Performance Evaluation Program was intended to measure contractor performance over the long-run, and if a comprehensive program of this type can be developed by NASA, contractor performance evaluation together with a CPFF fee arrangement should be considered as a substitute for the CPIF/FPI contracts.
3. On the other hand, if those responsible for policy select contractor motivation as a primary objective of the fee arrangement, then other contract forms should be emphasized. In this regard, our initial appraisal indicates that the Award Fee contract should receive more emphasis and be considered for wider application. Also the British preference for negotiating a CPFF contract with a switch to FFP as rapidly as the costs are determinate is a more logical contracting form for applying maximum incentive from the fee arrangement than the present CPIF or FPI fee arrangements.

TEXT

**Introduction**

The pricing of government contracts is an anomaly to economic theory. Not only is the traditional market place absent, but the buyer, to a large extent, determines the pricing mechanism. Yet the government-consumer is typically uneasy over the price outcome and the justification of the fee. The contract forms and pricing procedures associated with this singular market have developed from temporizing between the urgency of procurement, the criticisms of wastage and inordinate profits, and the counter claim of unsatisfactory profits. The procurement agencies persistently seek improved pricing and procurement procedures, and the last five years have seen an increasing emphasis on the incentive contracts, weighted guideline pricing techniques, as well as procurement innovations indirectly related to pricing, e.g., the Contractor Performance Evaluation Program.

Incentive contracts are the most prominent (from the standpoint of interest) of the present contracting arrangements, and it appears to be an acceptable contracting arrangement to those immediately concerned. Contractors, to all appearances, prefer this contracting form to the Cost Plus Fixed Fee. The government (particularly DOD) claims remarkable cost savings through the use of incentive contracts; all parties (with the possible exception of the Renegotiation Board and the GAO) attribute the preponderance of cost underruns experienced with incentive contracts to the efficiency induced by the incentive features of the contract arrangements. At the same time, the majority of the contracts are negotiated with terms which significantly limit contractor participation in the profits resulting from cost savings, and the claims of significant savings resulting from incentive contracts are even more interesting in view of such limitations to the contractor's inducement. The government procurement officials recognize the inconsistency of the usual contract arrangements and incentives, and they constantly urge the negotiation of contract terms incorporating greater contractor participation in possible profits or losses (higher sharing fractions) applicable over a greater range of cost experience (wider fee swings) with target costs negotiated closer to the expected cost outcome. In spite of this, the pattern of negotiation continues to favor low profit sharing fractions for the contractor. This phenomena, among others, appears to offer an interesting subject for investigation.

Our study of this contracting form was undertaken in response to a list of topics from NASA expressing interest in a study which would improve the "objectivity" of its pricing procedures. Incentive contracting is an obvious beginning point. It is unlikely that another study of incentive contract pricing would greatly change the understanding of this contracting form which has been the subject of scrutiny for some years. However, examination of this subject from the standpoint of decision theory and Bernoullian Utility concepts provides useful additional insight as to contractor reactions and the effectiveness of this contracting arrangement for promoting efficiency. The incentive or inducement associated with the contract fee should also be considered in the context of its marginal content and contribution to the firm's profits, particularly in view of the manner of contract pricing. Contractor's response to marginal fee situations have previously been observed, but the critical features of this subject have never been presented. A study ot this topic forms a second portion of the paper.

**Contracting Forms**

The primary types of contracting forms in use today are the Cost Plus Fixed Fee (CPFF), the Cost Plus Incentive Fee (CPIF), the Fixed Price Incentive (FPI) and the Firm Fixed Price (FFP). Under the CPFF contract, the contractor and the government negotiator arrive at an estimated target cost (a best estimate of what the contract performance will cost). A fee is then negotiated on this cost estimate, usually around 6%. The government then reimburses the contractor for any costs incurred. Under the CPIF contract, the government is also responsible for all costs incurred. However the target fee is subject to an increase or a decrease depending on the extent to which costs underrun or overrun the target cost. The rate of increase or decrease in the fee as a function of cost is called the "sharing factor" or "sharing fraction"; the range between the upper and lower limits of the fee is called the "fee swing". In other words, the negotiated fee will vary up to an agreed upon limitation in cost variation about the target cost. The FPI incentive contract is the same as the CPIF contract except that a ceiling price is negotiated: i.e., there is a negotiated price above which the government will not reimburse the contractor for costs incurred on the contract. The FFP contract is a fairly self explanatory title – the contract is negotiated for a firm price all overruns or underruns are absorbed by the contractor.[1] The differences and similarities of the contracting forms can be determined more explicitly by examining them in notation. This notation will be used throughout the paper. Let

$k$ = contractor's sharing fraction $[0 \leq k \leq 1.0]$

$x_0$ = target cost

$x_1$ = cost associated with maximum value of fee swing

$x_2$ = cost associated with minimum value of fee swing

$p$ = upper price limit (FPI only)

$f$ = target fee = $\lambda x_0$ [where $0 \leq \lambda \leq 1.0$]

$x$ = final cost outcome

CPFF contract:

$$F(x) = f + k(x_0 - x) = \lambda x_0 \qquad 0 \leq x \leq \infty, \ k = 0$$

CPIF contract:

$$\begin{aligned} F(x) &= f + k(x_0 - x_1) && \text{for } 0 \leq x < x_1, \quad 0 \leq k \leq 1.0 \\ &= f + k(x_0 - x) && \text{for } x_1 \leq x \leq x_2, \quad 0 \leq k \leq 1.0 \\ &= f + k(x_0 - x_2) && \text{for } x_2 < x \leq \infty, \quad 0 \leq k \leq 1.0 \end{aligned}$$

FPI contract:

$$\begin{aligned} F(x) &= f + k(x_0 - x) && \text{for } 0 \leq x < \frac{p - f - kx_0}{1 - k}, \quad 0 \leq k \leq 1.0 \\ &= f + (x_0 - x) && \text{for } \frac{p - f - kx_0}{1 - k} \leq x \leq \infty \end{aligned}$$

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[1] The above distinctions in the contracting forms are gross ones. There are other differences: e.g. under CPIF contracts costs are established by NPR, while under FPI they are negotiated at contract termination. These other differences are not considered pertinent to this discussion.

FFP Contract:

$$F(x) = f + (x_0 - x) \qquad \text{for } 0 \leq x \leq \infty$$

**The Rationale for Other Than CPFF Contracts**

It is the basic tenet of incentive contracting that the contractor is motivated principally (if not entirely) in terms of the profit outcome; i.e., if the contractor is given the opportunity of increasing his expected fee outcome, his actions will be consistent with maximizing this outcome. A profit maximizing objective will necessarily involve the elimination of "unnecessary" costs. Since the government pays all incurred costs it benefits through this saving and can therefore "afford" to pay additional fees to encourage this performance.

There is an additional conviction that the CPFF contract, per se, is a basic factor leading to excesses and wastage in contract performance. This is an obvious corollary to the motivational assumption just stated.

> *Horror cases of lagging program schedules, performance failures, massive overruns in cost, and so forth were widely publicized . . .* [11, p. 2]
>
> *. . . fixed fee arrangements tend too strongly to create an environment in which cost and time are irrationally subordinated to insignificance . . . the contractor has little or no measurable stake in the outcome, since in the CPFF contract, his fee remains static.* [11, p. 7]
>
> *Widespread use of the CPFF contract is therefore considered a major cause of the unsatisfactory results of many large development projects.* [11, p. 7]

One can find many similar statements in the speeches of various responsible government officials.

The contract form, however, is by no means the sole explanation for cost overruns, lagging schedules, etc. Such contracting innovations as the Program Definition Phase of DOD, etc. implicitly recognize this fact. In places the contracting manuals recognize this fact also:

> *But in another sense, recourse to this contract form is itself regarded as a* symptom *of a deeper problem. Behind the tendency to "go CPFF" lie the more basic defects of insufficient planning and identical treatment of all research and development work. The end objective, then, of a revitalized approach to research and development procurement is not just the elimination of a contract type, but the correction of these fundamental deficiencies.*
>
> *It is obvious that the same balance between cost, time and performance that characterizes DOD's program decisions must also be a motivating factor in the behavior of Government contractors. This motivation will be supplied by tying profits to contract results.* [11, p. 7]

Unfortunately, this symptomatic view of the CPFF contract is not consistently carried through in discussion of the role of this contract. Since the CPFF contract form is often considered a primary causal factor of poor contract performance, the following generalization is also encountered frequently:

> *. . . if the Defense Department achieves its goal of $3.2 billion reduction in CPFF contracts in fiscal 1963, as compared with fiscal 1961 . . . it would avoid about $320 million in overruns.* [2]

[2] Testimony of Thomas E. M[illegible]s (ASDIL) before House Military Operations Subcommittee. *Aviation Week*, 3 June 1963, p. 85.

*. . . We are convinced that this (the CPFF contracting form) is a major contributing factor to the high costs of weapons today.* [21, p. 17]

The bases for such conclusions have been difficult to determine, but they apparently arise from gross comparisons of the cost outcomes of incentive and CPFF contracts. There is a definite underrun bias in the cost outcomes of incentive contracts while the opposite is true with CPFF contracts – ergo a cost savings from the former contract type.[3] Since the CPFF contracts are let for uncertain research endeavors with, often, grossly specified objectives (quite contrary to the conditions specified for incentive contracts) it is difficult to accept that the CPFF contract form is the causal factor of overruns or that the implied savings will result from some other contract type.

**Experience with Incentive Contracting Provisions**

If the contractor acts to maximize the expected fee outcome, and the negotiated cost is equal to or greater than the contractor's expected cost outcome, there is no apparent rationale for the selection of a partial sharing arrangement, or limited fee swings.[4] Present and past experience with CPIF contracts indicates that the contractor is reluctant to accept large sharing fractions or fee swings.[5] This experience is a prima facie refutation of the hypothesis that entrepreneurs have the singular motivation of maximizing expected profits. There is an obvious observation that the contractor may prefer to take a small gamble rather than a large one, but this is inconsistent with the hypothesis of fee (or expected fee) maximization. The contracting manuals explicitly urge the negotiation of contract provisions with greater contractor participation in the profit outcome. Again this counsel is repeated in speeches by government officials.

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[3] There are numerous and rather fragmentary collections of data comparing the two types of contracts on cost outcomes. A better one is given by Dr. Fred Moore [19, p. 42 and 48] who, it must be added, does not draw these conclusions (p.49). By his data it is calculated that CPFF contracts have overruns about 55% of the time while CPIF contracts have cost underruns about 75% of the time. Case history data examined by Harvard Weapons Acquisition Research Project found a similar frequency of underruns. [22, p. 224] also [23]. Data submitted to the McClellan Committee hearings also substantiate these findings. [26, p. 674 and 818]. The Air Force presentation (p. 818) indicates that even when the total dollar value of incentive contracts (totaling more than $4 billion) is evaluated there is an underrun of 3.4%. Their comparison of total cost experience on CPFF contracts with that of incentive contracts finds that the former run 10.6% greater than the latter. This apparently matches the estimate used by DOD in calculating "savings" attributable to switching from CPFF to incentive contracts.

As of Dec. 31, 1964, only six incentive contracts had been completed for NASA for a total of $2.8 million. (This does not include award fee contracts.) This experience represents about a 4% overrun, but it is insufficient for determining separate NASA experience on CPIF contracts.

The Contractor Performance Evaluation Program appears to be the only potential source of accurate information as to the causal factors for the cost experience. This program, practically speaking, is just underway; it is a primary endeavor to measure contractor performance. Such an evaluation demands some insight as to the factors underlying contract performance and cost outcome. Despite the difficulties and ambiguities of such evaluations, an examination of the data generated by this program may establish the responsibility of the contract form to the cost outcome.

[4] See p. 12.

[5] Moore [19] shows data on 130 CPIF contracts which indicate that 59% of them had sharing arrangements of 20% or less, while 96% had 25% sharing or less. Personal investigation of this experience with DOD contracting personnel indicates this experience is still fairly general. "In fixed price incentives we seem to be wedded to an 80/20 or 75/25 sharing." See speech by T.D. Morris (ASDIL) [21, p. 55]. Our examination of contracting arrangements on the CPIF contracts negotiated by NASA indicates similar outcomes. (Also see p. 12.)

*I am not satisfied with these (sharing) percentages for two reasons. First, they do not give the contractor a big enough incentive to go all out to reduce costs – I am not sure that an extra $1 or $2 million on a $100 million contract is sufficient to induce sustained superior performance. My second objective is that they do not impose sufficient penalties for poor performance. In his endeavors in non-governmental business a contractor must be prepared to accept out-of-pocket losses for substandard performance – but he also has full opportunity to achieve higher rewards for superior performance. . . . we should be willing to accept, and impose, far greater risks than we now do . . . such spreads in the ranges of allowable fees and profits impose greater responsibilities and greater risks, but they also offer the promise of greater rewards for superior performance and suitable penalties for substandard performance.* [21, p. 56]

This quotation illustrates the general attitude that the cost sharing fractions typically negotiated on incentive contracts do not provide sufficient incentives. It also illustrates the general belief that these sharing fractions can be considerably increased and that the contractor should be willing to accept greater sharing fractions since he must be prepared for this experience in non-governmental work. The basic error in this logic, as will be discussed later, is the implicit assumption that risk is solely determined by the size of the sharing fraction. This quote further describes a rather curious inconsistency in view of the known uncertainties of the cost situation – that the contractor has a significant control over the fee outcome and, second, that the fee outcome is associated with risk.

*. . . any substantial progress we make in this area will require risks – risks (that the contractor must accept) that the contract could result in little or no fee if his performance is poor.* [21, p. 54]

A contractor's efficiency is not a matter of uncertainty or a phenomenon associated with risk; its level is at the option of the contractor. Risks, on the other hand, are associated with the cost uncertainties which any contractor, efficient or otherwise, cannot accurately predict. The fee outcome, however, is a product of both the contractor's actions and the cost uncertainties; the latter, under R&D and advanced systems procurement, probably exceeds the range of cost outcomes subject to the control of the contractor by a considerable multiple. In view of the extreme cost uncertainties associated with incentive contracts it is patently obvious that the contractor will not likely welcome large participations in cost outcomes over which he has little control – assuming that the target cost is tightly negotiated.

**The General Problem of Establishing Target Costs**

The cost outcome of a contract, therefore, is a function of both deterministic and random variables. The contractor has a limited amount of control over the cost outcome through his own actions, while at the same time the vagaries associated with any contracting operation will affect cost outcome in an unknown manner. Both elements are recognized in the contracting manuals.

*Unfortunately, no amount of preplanning or conscientious negotiation can determine exactly what the cost outcome of a project will be . . . The target cost is, in fact, an estimate, a prediction of future events. As such it will inevitably contain some degree of uncertainty either large or small – some probability that at contract completion costs will be, for example, 10, 15, or 30 percent above or below target. Each procurement, then, is concerned not with a single cost, but* *with a band of possible cost outcomes. The narrower this band, the more risk of performance being shifted to the contractor by means of firm fixed price or incentive contracts. (The negotiator) must formulate an estimate of the amount by which final costs might deviate from target.* [20, p. 203]

*. . . when the upper limit (of the expected cost outcome) is less than +10% the government should concentrate on negotiation of a firm fixed price arrangement. For an upper limit between 10% and 25%, the fixed price incentive type will usually be appropriate. When confidence decreases to a level of more than 25%, a CPIF. Thereafter, the CPIF arrangement is appropriate, at least theoretically, regardless of the deterioration of the confidence level.* [20, p. 201]

On the other hand, the philosophy of incentive contracting supposes that the contractor is in control of the cost outcomes.

*Profit and fee is thus tuned to the contractor's control of a variable on which his management skills can have a noticeable effect.* [20, p. 103]

*At what level should an incentive target cost be set? . . . the target cost* should *represent the best mutually determined estimate of what costs* will actually *be when the work is complete. The target cost should be set that the contractor, if he performs with more than usual efficiency, has a real chance to do the work for less than target . . . these are the same criteria as used for the CPFF or FFP contracts. In other words, differences in risk inherent in various contract types should be reflected, not in the cost estimation process but in the establishment of profit (or fee) rates.* [20, p. 202]

For obvious reasons contractor capabilities for predicting cost outcomes are difficult to evaluate objectively. Some general perspective of this matter is important to the evaluation of the contractor's decision process since the average errors in cost estimates could dominate any margin of control that the contractor may have over costs. Since this capability for prediction would vary so greatly by contractor and contract the matter can only be examined in generalities.

Marshall and Meckling [18] have presented data for a number of missile and aircraft models comparing the increases in the cumulative costs of production with the cost estimates. Their investigation involved the preparation of adjusted ratios of the cumulative average cost of production of the mentioned items for comparison to the earliest of such cost estimates as were available. As might be anticipated, the data leave much to be desired as to accuracy and lack of ambiguity, [18, footnote 9, p. 468] and two sets of ratios were determined for each program because of the equivocality of the price level adjustments. Using one set of factors (ratios subject to a given set of adjustments), two of four cargo and tanker aircraft programs experienced cost outcomes equal to the estimated costs, while the other two had 40% and 50% overruns. Using the second set of adjustments for the same programs, the results were measured as an 80% and 90% underrun and a 50% and 60% overrun. The other groups (fighters, bombers, missiles) indicated much greater variations. It was the authors' conclusion that "The factors (the adjusted ratios) for (4 different) cargo and tanker aircraft (programs) probably represent an upper bound on the currently attainable level of accuracy in cost estimating." [18, p. 471] (The conclusion would obviously pertain to this type of procurement).

Alchian [3] recently reported a study of the prognostic reliability of progress (or learning) curves based upon data recently declassified; he was interested in the accuracy to which direct labor costs could be predicted from knowledge of past experience with the production of similar aircraft types. Progress or learning curves were developed from certain post war inquiries of RAND personnel who found a "log-log" relationship between the direct labor per pound of airframe and the nth airframe. Since direct labor is the important component in the building of airframe, a prediction of the labor hours involved would be an important element to estimating the contract cost. It might be noted, that the aircraft examined in this particular study were composed of traditional or propeller driven types. It is unlikely that, even for the period under consideration, the development of these aircraft pushed the state-of-art – at least in the same sense as many of the aerospace systems under development today. While the theory of learning curves is not profound or involved, it remains one of the important methods for predicting costs. Therefore the reliability of this procedure provides some understanding of the importance of the known versus the unknown factors in cost outcomes – again for systems not generally suitable for FFP contracts.

> *Nevertheless, for practical purposes it may be appropriate to use an average of individual progress functions. One such practical purpose would be the prediction of total direct labor requirements for the first 1000 airplanes of a particular model. The average error of prediction is shown to be about 25 percent. For the entire output of any particular airframe model produced in one facility the error of prediction is also 25 percent.* [3, p. 679]

> *If specific curves are fitted to the past performance of a particular manufacturing facility in order to predict its future requirements, the margins of error or prediction average about 20 percent. All these margins of error, while averaging about 20 to 25 percent, represent specific errors which in .9 of the cases range between -40 and +70 percent.* [3, p. 679]

These studies of cost estimates on unsophisticated airframes (by today's standards) suggest that cost prediction for aerospace systems would appeal to the more adventurous. It also suggests that the uncertainties of the cost outcomes should be the most prominent factors in the contractor's decision process.

#### The Subject of Unnecessary Costs

CPIF contracts are negotiated with the explicit viewpoint that savings are made by the elimination of "unnecessary" expenses – the discharge of unnecessary personnel, the elmination of overbuilding (goldplating), etc. "Wastage", "unnecessary costs" are ambiguous terms, and the contractor and contracting agency have different bases for such evaluations. Past experience with contractor operations, especially during the early fifties, might support the contention there is always considerable wastage in large systems contracts. But it does not necessarily follow that the elimination of cost items, prima facie excessive or unnecessary to a contract, are accomplished at no cost to the contractor. For example, an excess of personnel on a contract may illustrate the contractor's efforts to maintain the continuity of the organization. This is an objective which occupies a place of primacy among managerial problems, and it is not often realized by the firing of scarce technical personnel. An excess of personnel may also denote the development of a new capability for the contractor. Furthermore, the rigid enforcement of cost controls necessary and expected in an organization oriented to civilian goods production are difficult to apply to firms commited to sales involving a preponderance of R & D. These observations are not made in defense of costs considered items of waste from the contracting officer's point of view. They are to observe what contracting manuals ignore – cost reduction demands effort in any circumstance,[6] and perhaps more so in R & D types of contracts. A dollar reduction in cost does not produce an additional profit equal to the sharing factor times the reduction in the accounting cost; in fact it might require a rather sizeable sharing fraction to "break even".

As noted later (with the discussion of contractor motivation) the contractor is seldom in a position to select an alternative which will maximize profit outcome. A firm will tend to operate at a level at which, by some devised criteria, the managers are (or are not) satisfied. (This is usually referred to in the literature as "satisfycing".) There is considerable evidence that certain CPIF contracting situations have resulted in shock situations to the contractor, and an incipient failure to obtain a certain fee (one of the firm's goals) has resulted in cost reduction actions. On occasions the threats of a potential loss have resulted in the exercise of particular care not to exceed the established cost goals which may or may not be efficient or austere ones. However, unless the contractors are in a potential or actual loss situation it appears that little will goad them into an efficiency drive, or any change in the status quo of cost control activities. But it is important to note that the contractor is highly unlikely to enter into the contract or conduct his negotiations with the attitude that he will later improve his efficiency. Few firms upon introspection would admit to material inefficiency, particularly following a period of financial adversity. Cost reduction here could only be achieved at high opportunity costs. The contractor will, therefore, base his judgments at contract negotiations upon his evaluation of the cost outcome under his present state of operations.[7]

[6] This point is explicitly made by Scherer, [22], [23]. Scherer's model considers only the contractor's decision making as a decision between the discounted future profits which would result from any given cost reduction programs and the incentive fee which would result from that cost reduction. The contractor would then conduct his actions so as to operate at a level of "efficiency" to maximize the difference between the fee associated with a negotiated incentive contract minus the user costs (or discounted future profits). Scherer's first presentation of this model [23] did not consider the cost outcome as a random variable. His second presentation [22] differs from the first only in that for the cost outcome (X) the expression E(X) was substituted in all equations; i.e., the contractor acted to "optimize" in his decision model on the expected cost outcome. In this latter instance Scherer recognizes that the cost outcome is uncertain – but that these uncertainties do not change the decision process of the manager. A decision model of this kind could only find empirical application in circumstances in which the "limits" of the contractor's subjective density of cost outcomes were small in comparison to the range over which the contractor could control the output; i.e., cost uncertainties are relatively unimportant.

[7] Our interviews and discussions with contracting personnel often verred to the subject of wastage in contracts; contractor's cost control efforts, etc. Individuals who were experienced with contracting, cost outcomes, etc. had widely different opinions as to the contractor's capability to control the cost outcomes.

**Incentive Contracting as Gambling**

A gamble, in its barest essentials, consists of choosing between two alternatives – opting a given sum of money with certainty (meaning, usually, not playing) or an uncertain outcome consisting of either a larger sum or a loss, each associated with a given probability. More elaborate gambling situations involve more than a dichotomy of outcomes each associated with a given probability, but the essentials remain the same.

There is a reason for this patent definition of a gamble. Many decision problems made under uncertainty are described by such a lottery; a decision to accept an incentive contract with a variable fee can be viewed as accepting either the chance of a fee larger or smaller than that connected with a CPFF contract, or the CPFF fee for certain.[8] It is true that the contractor does not have the same freedom of choosing between the CPFF or the incentive contract form as might be associated with a choice in a casino; however, the selection of the contract form is a matter of negotiation as well as being dictated by the cost uncertainties. What is distinctive with an incentive contract is the multiplicity of fee arrangements which, in effect, further determine the contract type; i e., the contractor has the potential through the sharing fractions, fee swings, etc. to approach a CPFF fee arrangement as one limiting form and the FFP fee arrangement, which offers the maximum chance for gain or loss, as the other. As will be discussed later, a choice between gambling or not gambling can be determined by changing either; (a) the probabilities associated with the uncertain outcomes, (b) the values of these uncertain outcomes or both. The incentive contract provides the potential for the latter. The negotiation of the incentive contract should therefore be viewed as the negotiation of an acceptable lottery to the contractor.

A proper analysis of CPIF contracting should incorporate principles applicable to decision making under uncertainty. There have been various attempts to apply game theoretical analysis to the problem of contracting,[9] but it appears more reasonable to consider the problem as one of decision theory instead of game theory. (The distinction here follows Baumol in that decision theory is game theory where the opponent is not a rational individual. This is sometimes described as "games against nature", i.e. the factors which condition the outcome are, in general, inexplicable.) Furthermore, in a typical contracting situation it does not seem reasonable to consider the contractor and the contracting agency as opponents seeking antithetical goals, although casual impressions of negotiation proceedings may appear to the contrary. The long-run goals of the two parties are the same – that of producing a quality product and maintaining the financial health of the firm. Both parties in most instances are

[8] It is often said that a CPFF contract is a "riskless" contract. This is not a sound generalization. Large overruns on contracts are often associated with difficult problems which the contractor encountered in the exercise of the contract. Although his fee is not altered under the terms of the CPFF contract, overruns usually mean that he is employing expensive technical personnel for no reward, unless the contractor has no alternative use for these personnel at the time. This uncompensated overtime constitutes a real opportunity cost. Although the fee is constant, the margin may not be. However, losses of this kind do not generate the same levels of concern, since opportunity costs are seldom measured under accepted accounting principles. (This point is well stated by Scherer [23, p. 185].) On the other hand the methods of pricing contracts and allocating overhead costs may produce situations in which there is a considerable net gain associated with overruns on contracts. (This is covered later in the paper.) "Riskless" CPFF contracts depend upon the contractor's cost circumstances and the ratio of CPFF to FFP sales as well as the possible alternative uses of the firm's resources.

[9] Our information is limited to two such efforts – unpublished and proprietary information.

genuinely concerned with the negotiation of a reasonable fee, despite certain differences on its proper measure. The contractor and agency are likewise constrained to a fee outcome which is defensive under congressional committee criticism, etc.

On the other hand, there are two different points of view as to how, or by what route, the contractor will achieve the given fee outcome. The contracting agency attempts to bargain the contractor to a target cost from which these additional profits would be achieved only through increased efforts at cost control or where losses would accrue if there is indifference to efficiency. The contractor considers the uncertainties of the cost outcome as the primary basis for this decision making (i.e., decision making at the time of negotiation). Rewards from undershooting the target cost are desirable, but they are to be secured in the probable outcome of the contract under status quo levels of efficiency; the contractor does not mentally revise his expected methods of operation with each increment of possible fee.[10] (A particularly good statement of this philosophy was made by General Davis to the McClellan Committee. [26, p. 861])

It is only after the contract has developed and the cost outcome becomes definite that the contractor will react to possible incentives for holding costs. At this time the contractor's problem is one of evaluating the alternatives under a condition of certainty, whereas at the time of the contract negotiation his problem is one of evaluation of alternatives under conditions of uncertainty. These are widely different decision problems, involving different criteria for evaluation on the part of the contractor. (The criteria upon which contractor decisions are based are discussed throughout the subsequent sections). As a result, the contract arrangements of most interest to the contractor under certainty are of the least interest to him under conditions of uncertainty.

**The Target Cost and the Expected Cost Outcome**

The negotiation of the target cost is the first step in the contract price negotiations. The responsible contractor must obviously develop some best estimate of the cost outcome with his bid proposal, although this estimate is not necessarily that introduced into the negotiations. He will also have an estimate or judgment of the upper and lower cost limits associated with the contract. And it would seem quite reasonable to expect that the final cost outcome experienced at the end of the contract is "more likely" to occur in the vicinity of the best estimate than at some extreme. It is reasonable, therefore, to represent the uncertain cost situation with a subjective density function given an assumption as to the form of that function and knowledge of a few of the contractor's cost estimates together with their associated gross uncertainties. The existence of subjective probabili-

[10]A target cost negotiated at the best estimate of the cost outcome implies a cost based upon efficient operation. Therefore in an ideally negotiated contract the slack upon which the contractor can draw for increased profits should be non-existent. This would indicate a major inconsistency between the stated objectives of CPIF contracting (rewarding the contractor for efficiency) and the goals of the negotiations (an accurate target cost). On the other hand there is considerable evidence that the government is not as interested in attempting to vary the contractor's levels of efficiency as it is to mitigate the embellishment of the engineering product beyond that necessary for the fulfillment of the contract requirements.

ties associated with the estimates of the target costs are explicitly recognized by the NASA contracting manual [20, p. 201] (quoted in this paper, page 6). The discussion hereafter will be developed as if these subjective densities are formally determined by the decision makers. All evidence indicates that the subjective density of cost outcomes is an important element in the decision process of both the contractor and the contracting agency, and the methodological incorporation of this function is a necessary element in the analysis of contract negotiations – especially incentive contracting. Hereafter the word "probability" denotes the contractor's degree of belief in the cost outcome; i.e., subjective probability.

Since the cost outcome is uncertain, an assumption of profit maximization as a factor in contractor motivation implies maximization of the *expected* fee outcome, E[F(x)], where

$$E[F(x)] = \int_{-\infty}^{x_1} n(x)\,g(x_1)\,dx + \int_{x_1}^{x_2} n(x)\,g(x)\,dx + \int_{x_2}^{\infty} n(x)\,g(x_2)\,dx \qquad (1)$$

Figure 1 is a diagram of a subjective cost density [n(x)] superimposed on a CPIF fee function [g(x)]. This diagram is drawn with the target cost negotiated at the contractor's expected cost outcome (here normalized at 100). For convenience of illustration the density function was assumed to be normal [$n(\mu, \sigma) = n(100, 20)$], the sharing factor on the CPIF contract was set at .20, and the fee swing was set at the target cost $\pm$ \$25 (i.e., the fee would vary over a cost range from 75 to 125). Therefore the contractor's fee could range from a maximum of 13% to a minimum of 3%, if the target fee corresponding to these sharing arrangements is taken as (typically) equal to 8%.

Figure 2 shows the expected fee outcome as a function of the level at which the target cost was negotiated for various arbitrarily selected parameters. Obviously, if the target cost is negotiated at the same value as the expected cost outcome, the contractor cannot expect (average) more than the target cost. The expected fee would theoretically never reach the contractural upper limit of 13% and the target cost would have to be considerably above the expected cost outcome for this expected fee to even reach 12%. Similarly, the larger the value of $\sigma$, the greater the spread between $x_0$ and $\mu$ associated with a given expected fee outcome.[11] An increase in the sharing factor (k) and the fee swing would increase the expected fee outcome for all values of $x_0 > \mu$. A contractor optimizing the expected fee outcome would obviously elect the maximum sharing factors and fee swings, as well as attempting to negotiate the target as far above the expected cost outcome as possible.

The function plotted in Figure 2 prima facie suggests that the uncertainty ($\sigma$) associated with the cost outcome of the contract would be of little importance to the individual contractor. The marginal difference in E[F(x)] for the two assumed values of $\sigma$ was, at best 2% under the assumed conditions. By way of providing perspective, Figure 3 indicates the probability of obtaining a fee less than 6% (the typical target fee for a CPFF contract) for various values of $x_0$. For a sharing factor of .1, a fee swing of $\pm$ \$25 and $\sigma = 25$, the contractor

[11] One may wish to change the assumption of a fee swing over a constant cost range ($\pm$ \$25 in the above example) to one where the fee swing ranges over a constant percentage of the target cost. In this case the E[F(x)] would increase monotonically to exceed the contractural limit of 13% and would be higher for all values of $x_0 > \mu$ than the previous function.

would anticipate better than a 20% chance of getting less than 6% if the target cost is negotiated at his expected cost outcome. When $\sigma = .1$, this probability is considerably lower. An increase in the sharing factor from .1 to .2 would increase this probability (of a fee less than 6%) to approximately 35% (if $\sigma = 25$). This is still significantly high even when the target cost is negotiated 20% above the expected cost outcome. By the same token, a large uncertainty attached to the subjective density of outcomes should increase the probability (the contractor's belief in his receipt) of a large fee outcome (say one larger than 10%). These observations suggest that a measure of risk (uncertainty of cost outcome) is an important factor in CPIF contracting.[12]

It is important to observe that the contractor's behavior, if he is maximizing $E[F(x)]$, is entirely dependent upon the sign of $(x_0 - \mu)$ and is independent of $\sigma$. For a fee function $g(x)$ symmetrical about the target cost, and $x_0 = \mu$, the expected fee is equal to the target fee (which for purposes of this explanation is assumed fixed). Or

$$E[F(x)] = f \qquad\qquad \text{for } x_0 = \mu$$

This expected cost outcome would be unrelated to the sharing fraction negotiated; i.e., $E[F(x)] = f$, for $0 \leq k \leq 1.0$. A target cost negotiated at $(x_0 - \mu) > 0$ would result in $E[F(x)] > f$ for $0 < k \leq 1.0$ and would be maximized at $k = 1.0$. If, on the other hand, $(x_0 - \mu) < 0$ then $E[F(x)] < f$, for $k > 0$.[13] Therefore, if the contractor acted to maximize the expected value of the profit, one would expect the contracting arrangements to polarize at either a FFP or a CPFF contract. (For a contractor maximizing the expected fee outcome, the optimum value of k as a function of the negotiated target cost is given in Fig. 4). This behavior is patently absent from empirical data of contracting negotiations. Customarily the contractor negotiates for low sharing fractions; i.e., contracts are seldom negotiated for sharing fractions exceeding .30. These fractions are accompanied by fee swings which also limit the expected cost outcome. Since experience indicates that underruns are more frequent than overruns there is clear indication that the target cost is negotiated above the expected cost outcome. Therefore, if profit maximization is the contractor's objective, the evidence is substantial that contractors are not attempting to maximize fee outcome, and a model of contractor behavior should seek other explanations.

**Utility in Gambling**

The association of utility with gambling behavior has been formally recognized since the 17th century. The recent resurgence of Bernoullian utility theory has developed from a desire to explain preference between uncertain choices. Choices made under conditions of uncertainty are made in response to the utility (or subjective

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[12]This point is discussed here at this length because of the lack of reference in the contracting manuals to cost uncertainty when imputing fee to the risk function (See NPR, Section 3.80_. For a further discussion of risk measure as a probable loss of fee see pp. 19-22)

[13]This behavior is independent of the fee swings if these are negotiated symmetrical about $x_0$. However, $E[F(x)]$ would be maximized by unlimited fee swings at $(x_0 - \mu) > 0$ and by zero fee swings for $(x_0 - \mu) < 0$. Since the sharing fraction and the fee swing are two methods for achieving the same ends, the latter parameter is omitted from the analysis for simplicity.

evaluation, of the presented alternatives; under the Bernoullian utility axioms however, decisions between uncertain alternatives are made in response to the expected utility of the outcomes. The proliferation of papers developing the concept of the Bayes principle is sufficient argument for its use here, and the rationale of this decision rule will not be argued. Much of the literature on applied decision theory develops the subject in terms of the less ambiguous measure of the expected monetary outcome of the alternatives, while recognizing that utility is the sole criteria governing the choice of alternatives. Schlaiffer presents a "test for the validity of expected monetary values as a guide to action":

> *To sum up: business men tend to treat acts which must have one or the other of just two possible consequences as being "really worth" their expected monetary value as long as the worst of the two consequences is not too bad and the best of the consequences is not too good.* [24, p. 28]
>
> *. . . we will realize that whether or not they (businessmen) formally compute monetary value they act in accordance with expected monetary value when the amounts at stake are not too large.* [24, p. 29]

Generalizing on Schlaiffer's test, the expected monetary outcome is a guide whenever the monetary outcome is not very important to the decision maker, in which case the utility of the outcomes will also be a linear function of the fee. If this situation is generally true, the individual fee outcomes do not warrant particular attention or study, and neither could they be considered as a source for contractor motivation. The first point of interest is the nature of this utility function for the contractor; the second is to generalize from this function to certain conclusions concerning incentive contracting.

**Managerial Motivation and Profits**

A principle assumption of traditional micro-economic texts has been the goal of profit maximization. This assumption has traditional origins in earlier economic writings which were primarily concerned with the rational behavior of the owner-entrepreneur. In addition, there are pedogogical advantages to this assumption, since it is necessary for a precise analysis of normative behavior. There are several instances in which economists have supported this assumption for no other reason than this convenience.[14] The prominence of the profit maximization

[14] Stigler takes the contrary position to that of this paper and states that . . ."profit maximization is the strongest, the most universal, and the most persistent of the forces governing entrepreneurial behavior" [25, p. 149] . . . "The profit maximizing assumption confers great definiteness on economics. The variables in a firm's policy that affect profits are usually more or less quantifiable, and the rule of maximum profits is simple; equate marginal cost to price." [25, p. 150]

On the other hand, it should not be implied that theoretical economists have been exclusively preoccupied with the principle of profit maximization. Chamberlin has distinguished between "Imperfect Competition" (Mrs. Robinson) and his theories of "Monopolistic Competition". The former theory, exclusively preoccupied with marginal analysis, is closely identified with the profit maximizing principle. Chamberlin points out, somewhat impatiently, that his theories are completely compatible with full cost pricing and other forms of entrepreneurial behavior not compatible with the principle of profit maximization. [9] But for that matter, marginal analysis is not an exclusive technique with profit maximizers; e.g., even the Russian planners use it. [4, p. 129] assumption in the teaching of economic principles has undoubtedly led to its incorporation as a principle of incentive contracting.

> *The incentive principle holds, in brief, that a contractor should be motivated, in calculable monetary terms . . .* [11, p. 101]

The unqualified use of this assumption in college texts may be a disservice to the general student, for there is little evidence that profit maximization is at all descriptive of behavior in other than very limited situations. A vast amount of recent literature in economics, sociology, and psychology is devoted to the topic of the large corporation as the primary entity in the economic-social organization, its organizational objectives and the goals of its professional managers. This literature expresses a common belief that the professional manager or executive is differently motivated than those individuals assuming the entrepreneurial functions – the stockholders or the entrepreneurial manager. It is not surprising to find that the professional managers pursue diverse and frequently conflicting objectives rather than the unique aim of maximizing profits. One authoritative early study of managerial motivations does not even mention profits as an activating influence.[15]

Another thesis common to students of managerial behavior is labelled "satisfycing" [10], [6]. This term describes decision making in circumstances precluding the determination of optimal objectives. The decision maker must select a lesser goal by some predetermined criteria – one which is satisfactory or "good enough". For example, profit maximization is attained at the point at which marginal cost equals marginal revenue. As yet there is no evidence of a business man who has satisfactorily determined this point. Instead, it is universal business behavior to establish a price and output by some other criteria, and the final relationship to the maximum profit point is unknown and, at times, incidental.

These other criteria are usually aggregative indices – turnover, aggregative profits and, particularly, total sales. One thesis holds that the accounting budget is a primary determinant of organizational objectives. [10] The budget, by its very nature, becomes a prediction and schedule of performance, as well as a criterian for adequate and acceptable performance. The budget specifies relationships between sales and costs which become guideposts to the achievement of a satisfactory, but not a maximum level of profits. But more important the budget prescribes the year-to-year standards of behavior.

> *It defines the decisions of one year and thereby establishes a prima facie case for continuing existing expenditures. Only in quite exceptional cases do firms in fact reexamine the rationale of existing functions, for example, or alter, radically the expenditures for them. This tends to be particularly true of overhead functions . . .* [10, p. 51]

The evidence and accumulation of experience has led other students to assert that profits are not the prime objective of the large modern business enterprise. To Baumol, [8] [6] business managers have a dominant desire to maximize the firm's sales and will constantly pursue this end subject to profits as a minimum constraint; i.e., (although it is somewhat of an over-generalization), profits are a necessary nuisance.[16] A recent treatise by

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15 "The most important spurs to action by the business man are . . . the urge for power, the desire for prestige, group loyalty, security . . ." [14, p. 137]

16 This is particularly true in short-run situations. See [8, p. 1085]

Marris (based somewhat more strongly upon the psychological studies of managerial behavior) is consonant with Baumol's conclusions and asserts that the manager will attempt to maximize the growth rate of the firm, subject to the constraint of security (security from corporate raiding). Furthermore, the manager's rate of advance is determined exclusively by his peers and superiors (not by stockholders).

> *". . . it is more likely to be governed by criteria derived from the collective situation of the managerial class which . . . means favouring expansion . . . a man is unlikely to be judged by his ability as a profit maximizer."* [17, p. 102]

We may conclude from this that contractors do not approach the contract negotiation with particular attention to fee outcome, particularly to maximizing it. They come with notions of an acceptable fee for the contract (a level they wish greatly to attain but not necessarily exceed) and a preoccupation for increasing or holding the sales level. But more important, this evidence strongly suggests that the average contractor has a rapidly decreasing marginal utility function for fees. Instead of maximizing fees this person operates towards acceptable or constrained minimal profit objectives. He will find little additional utility in exceeding this profit objective – it will not be a matter leading to professional eclat nor to an increase in salary, perhaps the contrary. On the other hand, the manager will find considerable disutility in any profit outcome which falls short of some minimal or acceptable goal.

If the professional executive or manager is not motivated by the anticipation of high profits it would be even more difficult to project such motivation to the typical project manager. This individual is usually highly trained and competent in a particular skill, as well as having managerial ability – an individual whose professional approbation is more likely to arise from an association with a competent engineering accomplishment than through a measurement of success related to profit return. On the other hand, a monetary loss associated with a project is likely to be an indictment or symptom of technical failure. It is doubtful if the motivations of project managers – individuals most likely to be responsible for performance outcome on the contract (cost outcomes included) – would include profit maximization.

To our knowledge, Scherer was the first to clearly point out that defense firms have a decreasing marginal utility for profits. [23, p. 243]. His case histories indicated that contractors strive to improve efficiency when confronted with a loss but are indifferent to a reward for such efforts. (This phenomenon was unequivocally verified in our discussions with procurement personnel.) This empirical evidence, plus the failure of his model to explain observed phenomena, led Scherer to suggest the need of a "pressure theory" for contracting as opposed to a "reward theory", and a more general theory of contracting which includes "risk aversion" as a contractor goal [22, p. 276]. Perhaps there are two factors to explain Scherer's proposal for additional developments in a theory of contractor motivation. First, he incorporates utility into his analyses through the expedient of indifference curves, but indifference curve analysis is not useful to the explanation of behavior under conditions of uncertainty. Second, Scherer's indifference curves relate expected profits to the probability of outright loss. That is, his utility curves ignore the expected loss or the size of the loss, if a loss occurs. This is a fundamental omission of decision-bearing information from the problem.

As previously demonstrated, the strict profit maximization model, (i.e., contractor decisions based on expected fee outcome) does not yield a realistic representation of contractor behavior (this discussion was formally presented on pages 11 and 12 of this text). We feel that a decision model based on a Bernouillian utility function for profits or fees would give a more accurate representation of contractor behavior. The observed actions of managers with regard to the fee outcome of a contract (or profits in general) can be completely supported by the theory that these individuals have a decreasing marginal utility for profits and will act to maximize their expected utility. Most of the previously mentioned contractor goals e.g., risk aversion, satisfycing, attainment of "acceptable" levels of profits, etc., are easily contained in the notion that contractors have decreasing marginal utility for profits and act to maximize their expected utility. The problem now is one of examining incentive contracting in a utility context – specifically within the framework of Bernoullian utility theory.

**Utility Theory as Decision Theory**

Utility theory can best be incorporated into the explanation of contractor behavior through the concept of Bernoullian Utility Theory, rather than indifference curve analysis, or concepts of ordinal utility. A Bernoullian Utility Function (following Adams, [1, p. 169]) can be defined as follows: If x is a possible positive monetary outcome associated with an uncertain even and s a possible negative outcome, then these alternatives can be represented by an ordered-pair vector $< px, (1 - p)y >$, where p is the probability associated with outcome x. There are two assumptions defining a Bernoullian Utility Function

> *. . . there exists a function u with domain K (where K is the set of all alternatives) such that for all x and y in K, x is preferred or indifferent to y if and only if*
>
> $$u(x) \geq u(y)$$
>
> *and for all* $0 \leq p \leq 1$,
>
> $$u(< px, (1 - p)\, y >) = pu(x) + (1 - p)\, u(y)$$

This is to say that the utility associated with two outcomes of an uncertain event (where outcome x is preferred to outcome y) is equal to the expected utility associated with each outcome. The decision theory utilizing this utility function would incorporate the Bayes solution.[17] The outcome x, in our problem, can be a positive fee outcome, or it could be a fee greater than 6%, or some other selected value. The outcome y could be any other fee outcome so long as $y<x$. The possible fee outcomes on a contract should be considered as a continuous function over a given range rather than a dichotomy of outcomes, and the probabilities associated with this would be a continuous density function.

A utility function therefore can be substituted for the fee function g(x) in (1). The fee function is obviously well determined for a given contract situation; a measure of the utility function associated with this fee function is another matter. It is well recognized that utility cannot be measured in a cardinal manner such that the utility

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[17] "The intuitive idea of the Bayes principle is that in forming his preferences among the acts, the individual must act as though he forms estimates of the probabilities of the states in S and that his preferences are in accordance with the expected utilities derived from these probabilities." [1, p.233]

values can be added. On the other hand, it is possible under the Bernoullian utility definition to measure utility up to a linear transformation. A contractor acting in a manner consistent with the axioms associated with Bernoullian utility will act to maximize his expected utility, i.e., this utility concept can be used to explain contrac- behavior when confronted with choices under uncertainty. A utility measure is a psychological index and its measurement is difficult and often inconsistent for this reason. Being psychological the utility measure is individualistic, and the measure of a "firm's" utility for fees may not be uniquely determined by a simple experiment. But an awareness of the capability of measuring utility uniquely up to a linear transformation is valuable if only to avoid analyses based upon a vacuous concept. In the usual event the responsible individual in the firm is no more aware of its utility function for the fee outcome than he is for his own utility function for money outcome – but he will act in accordance with some unconscious recourse to such a function.

**A General Contractor Decision Model**

The fee function is again written

$$F(x;k) = f + k\,(x_0 - x) \qquad \text{for: } 0 \leq x \leq \infty,\quad 0 \leq k \leq 1.0$$

With little loss in generality assume that $f = 0$ and define a Bernoullian Utility function,

$$U[F(x;k)] = U[k(x_0 - x)] \qquad \text{for: } a \leq x \leq b$$

(The fee swing is again considered unlimited for purposes of simplicity). Zero on the Bernoullian Utility Scale is

$$U[F(x_0;k)] = F(x_0;k) = 0$$

Assume that $U[k(x_0 - x)]$ is strictly concave; i.e., the function is continuous and

$$\frac{d^2U[F(x;k)]}{dx^2} < 0$$

Assume, also, a rectangular subjective density of cost outcomes

$$h(x) = 1/(b-a) \qquad a \leq x \leq b$$

Figure 5 shows this density, $h(x)$, together with the fee and utility functions for $k = 1$; i.e., $F(x;1)$ and $U[F(x;1)]$. Where $x_0 = b$, the fee and the utility will be maximized for $k = 1.0$ since the partial expectation of a loss equals zero. If $x_0$ approaches $\mu$, the partial expectation of a negative fee increases and when $x_0 = \mu$, the expected fee will equal zero. However, the expected utility, because of the nature of the defined function, will become zero at $\mu < x_0 = c < b$.

Decreasing the value of k from 1.0 to 0 for any value of $x_0$ will rotate the fee function 45° to an expected outcome of zero. The utility function will rotate and change shape with decreasing values of k; when $k = 0$, utility is zero for all values of x. By definition, at $x_0 = c$ the expected utility will be zero for $k = 1.0$ and also for $k = 0$. The utility function reaches a maximum for some value of $0 < k < 1.0$ at $x_0 = c$.[18] Therefore for every $x_0 > \mu$ there

[18] A mathematical proof is given in the Appendix.

is some value of $k > 0$ which will maximize the expected utility at some positive value. These maximums for specific values of $x_o$ are illustrated by the dotted curve in Figure 6 [19]

An increase in f (the target fee) has the effect of shifting the fee function (and the utility function) upward and, in turn, $U[F(x; 1)] = 0$ for $x < c$. Thus increasing f has the obvious effect of increasing the expected utility. It is also apparent that the fee swing can be used to control the partial expectations of a loss and a gain. Thus the negotiation of incentive contract terms presents a multiplicity of arrangements for effectively increasing the contractor's expected utility. There is no guarantee that the bargaining will necessarily result in this increase. But if the bargaining is not heavily weighted in favor of the government, it is difficult to believe that bargaining over five different contract fee terms ($x_o$, k, $x_1$, $x_2$ and f),[20] all with the potential of increasing the expected fee and the expected utility, would not usually result in an increased utility over that presented by a CPFF contract.

Contracts with multiple incentives obviously increase the complexity of the contract negotiations. Consider a contract with incentive provisions for performance and for cost. At the time of negotiation both outcomes are unknown and uncertain and are described by a joint density $f(x_a, x_b)$. If $U[F(x_a, x_b)]$ is the utility function, then

$$E[u] = \int_R \int_R U[F(x_a, x_b)]\, f(x_a, x_b)\, dx_a\, dx_b \qquad (2)$$

The contractor must now negotiate the target costs and target performance, the multiple sharing fractions, etc. such that $E[u] > 0$, where zero is the utility of the CPFF (fee). The facility and confidence with which the contractor can arrange terms which insure this subjective outcome is a moot point, especially as the complexity of the arrangements multiply. Nevertheless the increasing multiplicity of fee arrangements increases the opportunity of negotiating an expected utility greater than zero.

It might be useful to illustrate this general model with an example. For purposes of convenience assume a normal subjective density of cost outcomes, and a utility function for profits which is concave. No brief is made that the following selected utility function is representative or "average" for all contractors; however, it is a strictly concave function as assumed in the model.

| fee | utiles |
|---|---|
| 18 | 1.3 |
| 12 | 1.0 |
| 6 | 0.0 |
| 0 | -3.0 |
| -6 | -9.3 |

The continuous utility function based on these weights or utiles can be described by a third degree function

$$U(F) = .0010F^3 - .0458F^2 + .7389F - 3 \qquad \text{for } -6 \le F \le 18$$

since $F = g(x) = f + k(x_o - x)$ the expected utility is

$$E\left\{U[g(x)]\right\} = \int_{-\infty}^{\infty} U[g(x)]\, f(x)\, dx \qquad \text{for } \frac{f + kx_o - 18}{k} \le x \le \frac{f + kx_o + 6}{k}$$

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[19] We wish to repeat for sake of emphasis that Figure 4 illustrates contractor behavior when maximizing expected fee, while Figure 6 pertains to contractor behavior while maximizing expected utility.

[20] This would increase to six terms, if different values of k are negotiated for overruns and for underruns.

Figure 7 shows the expected utility outcome as a function of the negotiated target cost ($x_o$), for selected values of k. In this example $\mu = 100$, $\sigma = 20$, f = 8 and $x_1 = x_o - 25$ and $x_2 = x_o + 25$. For those situations in which the target cost is negotiated less than 125% of the expected cost outcome (over the values considered) there is a noticeable decrease in utility with an increase in k. As the target cost is negotiated at higher values, (say upward of 125% of expected cost) there is no material increase in the utility with increases in k. On the other hand, only contracting situations with low values of k have positive utility (produce a utility greater than that associated with a CPFF contract) when the target cost is negotiated in the vicinity of the expected cost outcome, and a large value of k produces an extremely high disutility in such situations.

There seems little to be gained through the analysis of a wide spectrum of possible contract arrangements. The example just examined is one which demonstrates the high disutility of a large sharing arrangement if the target cost is negotiated close to the expected cost outcome. Since high sharing fractions and tight target costs are the primary negotiation objectives of the government, an additional example is provided in Figure 8 – one providing an extreme or upper boundary to our assumed set of parameters. In this case the sharing fraction was selected as 1.0; and in keeping with this selection the target fee was assumed as 12, instead of 8. The utility function was again determined as a function of $x_o$ for two values of uncertainty ($\sigma = 20$ and $\sigma = 40$). The lower value of the standard deviation probably represents a level of uncertainty, according to the contracting manuals, at which one would negotiate a different type of contract (FPI), and it is probably one in which the government contracting negotiators would argue for the largest sharing fraction on the part of the contractor (commensurate with the large target fee). The contractor's reaction to these large sharing arrangements when the uncertainty reaches levels for which CPIF contracts are supposedly written ($\sigma = 40$) would be readily apparent from the examination of the appropriate function in Figure 3.

The assumption of a normal density also deserves some comment. A more logical density function for these analyses would be one which is skewed right, since an argument is easily made that large overruns are more likely to occur than large underruns. The introduction of other densities would only complicate the interpretations with additional parameter assumptions, as well as the problem of distinguishing the correct measure of central tendency to associate with the contractor's "expected cost outcome" – a problem we have not encountered with symmetrical distributions. And a normal function is a very reasonable approximation for many skewed densities. Extreme right skewness however, would only increase both the probability for a given overrun and the contractor's propensity for electing small sharing fractions.

**Risk, the Sharing Fraction, and the Target Fee**

It is acknowledged that the FFP contract is the most "risky" of all contract forms and consequently should have the greatest monetary reward in the way of target fee. The government's efforts to negotiate larger sharing fractions (to negotiate the incentive contract terms closer to those of the fixed price contract) have been noted. It may be informative to compare the FFP contract with a FPI contract, as they are typically negotiated today, on some measure of risk.

One measure of risk is the probability of the contractor obtaining a negative fee outcome. A second measure of risk would be the expected dollar loss associated with the contract. Thus if n(x) is the density associated with the cost outcome, g(x) is the fee function and $x_1$ is the particular cost outcome at which the fee is zero, then risk by the first measure is

$$R_1 = \int_{x_1}^{\infty} n(x)\,dx$$

While risk measured by criteria two is

$$R_2 = \int_{x_1}^{\infty} n(x)\,g(x)\,dx$$

A third measure of risk might be stated in terms of a utility measure, and the utility function u(x) could replace g(x) in $R_2$ with proper adjustment of the limits. However, a discussion in terms of the utility of the expected loss would be more complicated and the point to be illustrated does not depend upon this improvement. Furthermore, it was previously stated that the (total) expected utility of the fee outcome is the basis upon which the contractor makes his decisions, not the partial expectations. This assertion is not being contradicted here. Since the imputation of fees to risk is prevalent in contract negotiations a discussion in terms of the partial expectations of the cost outcome seems relevant.

The FPI contract form (rather than the CPIF) was selected for this comparison to the FFP since the two have identical fee functions beyond the maximum price level of the FPI contract. This selection and the following assumptions are for convenience of exposition; we do not feel they provide a "special case" for the general conclusions to be drawn. Assume: (1) that the "uncertainty" associated with a cost density function is measured by the standard deviation, (2) the expected cost outcomes for the two contracts are equal, (3) the subjective cost densities are both normal (4) the target costs for both contracts are negotiated at the expected cost outcomes, (5) the FPI contract has a negotiated maximum price at a cost equivalent to a fee of zero. Under these assumptions, the two contracts would have the same risk if their respective fee equations equalled zero at the same t-value.

If

$$t = \frac{x - \mu}{\sigma}$$

then

$$F = 0 = f_1 + k_1(x_o - \sigma_1 t - \mu) = f_2 + k_2(x_o - \sigma_2 t - \mu) \tag{3}$$

where the subscripts 1 and 2 pertain to the FFP and FPI contracts respectively. If these two contracts are negotiated, as assumed, at $x_o = 100 = \mu$, then

$$\frac{k_2}{k_1} = \frac{\sigma_1}{\sigma_2} \tag{4}$$

One can also examine (3) to determine the relationships of the $f_i$ given $k_1 = k_2 = k$. Thus

$$f_2 = f_1 - kt(\sigma_1 - \sigma_2) \tag{5}$$

It should be also noted that negotiating the target cost of the contracts at a value greater than the expected cost outcome will tend to decrease the value of the sharing fraction necessary to equate the risk of the FPI and the FFP contracts. Assume the two contracts being compared were negotiated at some target cost, both greater than their respective expected cost outcomes. If

$$f_1 = f_2 = f$$

$$t_1 = t_2 = t$$

$$x_o - \mu = d$$

from equation (3)

$$\frac{k_2}{k_1} = \frac{d - \sigma_1 t}{d - \sigma_2 t} \tag{6}$$

Since $\sigma_1 t < \sigma_2 t$, then the addition of a constant to both the numerator and the denominator will reduce the ratio.

Assume initially that the contracts are both FFP and $f_1 = f_2 = 14$, $\sigma_1 = 10$ and $\sigma_2 = 25$. (These values of sigma correspond to the upper levels of uncertainty suggested in the contracting manuals quoted on page 5). In other words, the two contracts have equal expected costs but there is a greater uncertainty attached to the second contract. In order for the two contracts to have the same risk under definition (3), the second contract would have to be let with a target fee of 35.

Since a fee of this size is clearly beyond regulations, it is reasonable to attempt to reduce the risk by using a sharing fraction less than 1.0. The value of k which would produce the same risk on the contract would be .40 (equation 4). This is to say, a FPI contract and a FFP contract with specified uncertainty levels commensurate with the contracting guide, both with target costs negotiated at expected cost outcome and identical target fees, could not have a sharing fraction negotiated greater than 60/40 without the FPI contract having a greater risk than the FFP contract. Because the FPI contract is assumed to be less risky than the FFP contract, the target fee is usually reduced in accordance with this expectation. Therefore, if a lesser target fee is associated with the FPI contract, the value of the contractor's sharing factor would have to be less than .40 in order for the two contracts to have the same measure of risk.

Consider again the two contract forms having density functions with the same parameters as specified above. However, in this case specify that the target cost for both contract situations was negotiated at 110; i.e., the target cost was 10% higher in both cases than the expected cost outcome. In the case of the FFP contract, with an assumed standard deviation of 10, the probability of a negative fee becomes quite small. However, this 10% increase in the target cost over the expected cost outcome does not operate to proportionally reduce the risk on the FPI contract. (See equation 6). In order for the FPI contract to have the same risk as the FFP contract under these conditions, the sharing fraction would have to be reduced to $k = .27$. And this assumes that the target fee for the FPI contract was at the premium risk level of the FFP contract – unlikely with a sharing fraction this low. If the target fee is correspondingly reduced with the decrease in the sharing fraction, the FPI contract with a sharing fraction of .27 implies a greater risk than the FFP contract. In other words, although both contracts become less risky with $(x_0 - \mu) > 0$, the risk factor for the fixed price contract decreases more rapidly than for a FPI contract unless the sharing fraction is negotiated quite low.

The discussion so far has been limited to $R_1$ as a measure of risk. Under the assumptions, the same values for k and f which equate the risk on the two contracts under assumption $R_1$ will do so for $R_2$. One can also generalize somewhat more by assuming unlimited fee swings. Because of the symmetry of the normal distribution, if the target cost is negotiated at the expected cost outcome, the adjustments in the sharing fraction (or in the target fee) which will equate the risk of a negative fee between the two types of contract forms will also equate the risk both as measured by the expected monetary loss or in terms of the expected utility. Whenever $x_0 > \mu$, the adjustment in the sharing fraction to equate the two utilities is greater than the ratio of the two standard deviations but becomes a more complex function than equation (4). However, for a given value of $x_0$, the k-value equating risk between the contracts would be lower if the risk is measured by $R_2$ rather than $R_1$.

The contracting manuals measure risk as some uncertain function of the sharing fraction only; as demonstrated here the parameters of the subjective cost densities and the target costs are also fundamentally involved. The previous examples indicate that it is doubtful if the sharing arrangements could be negotiated much above their present level without making the incentive contracts more risky than the FFP.

**Motivation of the Contracting Officer**

The discussion in this paper has considered only the contractor's behavior model. To a limited extent this includes an implicit assumption that the government's position is antithetical to the contractor's which, as asserted previously, is not the case; i.e., the contracting officer and the contractor have similar goals.

As discussed in the preceding section, large contractor participation in the cost outcome would have to be associated with a comparatively high target cost. This would be accompanied by a high probability of underruns with the resulting criticism of the profit outcome from groups unsympathetic to the view that large underruns are the result of efficiency. [See 26, Part 2]. Contracting officers, as noted, characteristically demand small sharing fractions when aware of such situations. But assume that the negotiations have produced a tight target cost. Even in this circumstance there is no particular motivation for the contracting officer to negotiate large sharing fractions and fee swings. If he is concerned about the defensibility of the profit outcome, there still remains a significant probability of underruns and large sharing fractions are not to his advantage. On the other hand, the contracting officer (and the government) has nothing to gain from sharing-terms which have a high probability of putting the contractor in a tight cost situation. While there is ample evidence that the contractor becomes more cost conscious in such situations, the cost control efforts are likely to produce completely inconsequential results when confronted with cost uncertainties of the magnitude associated with incentive contracts. The net result is to precipitate profit situations in which the contractor is, at best, uninterested in further contract participation, except to the extent that he considers his future sales and reputation involved. Also, the recent pronouncements of NASA officials indicate a primary concern over final costs which considerably exceed their target costs (the opposite of the criticism of the McClellan Committee in the Boeing case). This factor, per se, should induce the negotiating team to shade the target cost to the high side with the corresponding protection of low, or certainly moderate, sharing fractions and fee swings.

**The Question of Overhead and Fee Determination**

There is a limited awareness that cost-plus pricing and the methods of allocating overhead cost can produce a fee outcome in a manner contrary to the objectives of incentive contracting. An article in *Aviation Week* [21] describes a situation involving a contractor with a CPIF contract and other sales of a firm-fixed-price category. By deliberately incurring large overruns this contractor could increase his profits although this action resulted in a minimum fee allowable under the CPIF contract arrangements. This situation is a consequence of the method of costing contracts. Under cost-plus contracts the contractor is reimbursed for the incurred variable costs together with an overhead loading. This overhead rate is determined prior to the signing of the contract. If the contractor succeeds in increasing his direct costs on the contract at a more rapid rate than the fixed charges, he will, in actuality, receive a payment for overhead charges which were not incurred. The costing procedure, in effect, shifts a portion of the company's overhead to the CPIF contract. This "unearned" overhead payment, as in the example, could be greater than the reduction in profit which would occur from the overruns and the penalty arrangements negotiated with the contract. It is true that the costs (variable and overhead) on a cost-plus contract are determined at the termination of the contract, and the extent to which the firm accrues a net incremental benefit through the overhead payment depends partially upon the ratio of cost-plus to firm-fixed sales.

This situation is not unknown to NASA. The *Incentive Contracting Guide* contains an example quite similar to that presented in the article in *Aviation Week*. In NASA's example the firm had negotiated a CPIF contract with a 95/5 sharing arrangement and experienced a net gain in profits from deliberate overruns. The example also demonstrates that the negotiation of an 80/20 sharing arrangement would have decreased the profits to be gained from an overrun and the maximum profit would have been obtained from an underrun.

> *What is critically important for NASA negotiators is not precision in estimating these fixed expenses, but strong efforts to incorporate the steepest possible share lines in those situations where the contribution to fixed overhead is likely to be an important factor; namely, when the contractor is operating at less than full capacity (or will be at full capacity but have no backlog) and a significant part of his business is performed under firm fixed price contracts.* [20, p. 134]

These examples are the product of a classical problem that has faced economists and business decision makers since the beginning of the industrial revolution. Overhead costs are difficult to allocate to services performed, and the problem becomes most onerous in industries in which overhead costs are a dominant factor, railroads being the most widely cited example. Small increments of variable cost (such as adding another passenger) are seldom associated with corresponding increases in the overhead or fixed costs, and allocations of such overhead costs are basically arbitrary.

21Bruce Backe. "How Fees May Undermine Incentive Goal", *Aviation Week*, January 11, 1965, p. 69.

The full implications of reimbursing overhead costs to incentive contracts are generally overlooked. The opportunity to increase the net revenue through overhead reimbursement is not a situation associated with contractor duplicity or circumstances involving excess capacity. This situation is present in every cost-plus contract negotiation, although both parties to the contract may be unaware of its existence. Except for those circumstances in which the contract overhead varies proportionally with the variable costs (a contradition, by definition) and the ratio of overhead to variable costs is identical to that on the fixed price sales, the burden factor represents a variable fee schedule which is a positive function of the variable cost. Incentive contracts, therefore incorporate two variable fee arrangements, both functions of the cost outcome but with opposite sign. The fee increments which arise from this costing procedure can operate as "reverse incentive" to the extent that a contractor is motivated by profit considerations.

**Marginal Costing and the Fee Formula**

The limited recognition of this phenomena is ascribable to the costing methods; the overhead cost allocation is essentially an average cost. Incentive contracting, on the other hand, was designed to appeal to an overt profit interest in a marginal situation – the marginal gain or loss in fee associated with an incremental change in the cost outcome. Therefore the proper cost values to measure this gain or loss are associated with marginal, not average, costing procedures. Optimal decision making implies marginal analysis.

> *This is the heart of marginal decision making – the statement that an action merits performance if and only if, as a result, the actor can expect to be better off than he was before.* [7, p. 20]
>
> *. . . the best interest of a firm, a consumer, or any other economic unit requires that any decision take into account the magnitude of the marginal yield which it promises.* [7, p. 21]

As Baumol points out, this may appear to be an obvious principle, but it is frequently ignored. It is not always obvious what the marginal yield of a given action will be, and the data available to the decision maker are often inadequate for this determination.

> *In business operations one often encounters rule of thumb calculations . . . when these business calculations are explicit, they are frequently made in terms of average rather than marginal quantities . . . yet it is tempting to reason on the basis of unit (average) costs of revenues or profits, largely because of the difficulty of marginal data collection.* [7, p. 32]

The significance of the following examination as a descriptive model of contractor behavior is dependent upon the assumption that contractors base their decisions on marginal cost analyses. We have no information as to the extent to which contractors consider the marginal cost in their contract negotiations; this would be the subject of a very interesting study, although a difficult one from the standpoint of obtaining access to accurate data. However, there should be no fundamental quarrel that incorporation of marginal analyses would provide a considerably more accurate basis for normative behavior. Furthermore, as will be demonstrated later, the magnitude of the "fee" return from the overhead reimbursement is considerably greater, in many cost situations, than that from the fee proper. Therefore the contractor's decision process is much less dependent upon accurate marginal cost determinations, and rather gross estimates of the marginal costs are adequate for the decision process. This factor mitigates a prime difficulty in using marginal costing in the determination of optimum actions in incentive contracting.

Our objectives here are:

(1) to define a more accurate gross profit function to incorporate in a model of normative contractor behavior and,

(2) to describe the fee outcome in terms of the various cost parameters and the sharing fraction. The latter parameter should receive particular emphasis in view of the interest in reversing the incentive to overrun through the negotiation of larger sharing fractions (See [20, p. 134] quoted in this paper on page 23).

The basic fee formula remains

$$F(X) = f + k(X_o - X)$$

where f is the target fee; k, the sharing factor; $X_o$, the target cost and X, the final cost outcome. In addition, define

$x_o$ = target variable cost = 100 (i.e. variable cost is normalized at 100)

$x$ = final variable cost outcome

$c_2$ = overhead associated with firm-fixed price sales

$c_1$ = direct cost associated with firm-fixed price sales

$\beta_o$ = the overhead loading factor for target cost

$\beta_1$ = overhead loading factor as determined post facto or at end of contract

$\Delta c_2$ = additional increment in overhead necessary for the fulfillment of the contract

$\gamma$ = target fee rate ($0 \leq \gamma \leq 1.0$)

$k$ = contractor's incentive sharing fraction ($0 \leq k \leq 1.0$)

$x_1$ = direct cost associated with maximum value of the fee swing

$x_2$ = direct cost associated with minimum value of the fee swing

$$\beta_o = \frac{c_2 + \Delta c_2}{c_1 + x_o}$$

$$\beta_1 = \frac{c_2 + \Delta c_2}{c_1 + x}$$

$$X_o = x_o + \beta_o x_o$$

$$X = x + \beta_1 x$$

The target cost is determined by first estimating the variable costs to be incurred in the fulfillment of the contract. The loading factor (or burden) is the ratio of the total estimated overhead to the direct labor costs of the firm (or the accounting unit) for the period of the contract. The burden is multiplied by the direct labor costs to obtain the compensation for the overhead.[22] This method of allocating overhead assumes that the overhead cost is a positive linear function of the variable cost and that the average dollar overhead cost per dollar variable cost is a proper measure of this factor of proportionality for all contracts.

The following analysis assumes: (1) there is no ambiguity in the determination of the variable costs and that unallowable or unsubmitted costs are excluded, although these may be quite important in practice. (2) The firm's other sales are of a firm-fixed price category and the other business operations of the company do not change to materially affect the existing ratio of fixed to variable costs, $(c_2/c_1)$. (3) Prior to the contract the increment of overhead associated with that contract is properly estimated and that any overrun associated with the contract will be made primarily in the variable cost portion. This defines a situation similar to that described in the previously cited references (see footnote 21, and [20]). Also, to provide a better perspective, a second function will incorporate the antithesis of assumption (3) and assume that the overhead increases proportionally to changes in the variable cost – the implicit assumption of incentive contracting. The true situation will be somewhere between these two extremes, but in most instances, probably much closer to the first.

The "fee" equation (incorporating assumption (3)) is now written to represent the net gain to be made on the contract. The following discussion is limited to the CPIF contract, but the fee equations apply to all cost-plus contracts with suitable selection of the parameters.

Net Gain = $F^*(X)$ = Sales - Cost of Goods Sold

= fee + reimbursement for cost - cost of goods sold

$$F^*(x) = \left\{\gamma(x_o + \beta_o x_o) + k[x_o(1 + \beta_o) - x(1 + \beta_1)]\right\} + [(x + \beta_1 x) - (x + \Delta c_2)] \qquad \text{for } x_1 \le x \le x_2 \qquad (7a)$$

$$= \gamma(x_o + \beta_o x_o) + k[x_o(1 + \beta_o) - x_1(1 + \beta_{1.1})] + [(x + \beta_1 x) - (x + \Delta c_2)] \qquad \text{for } 0 \le x \le x_1 \qquad (7b)$$

$$= \gamma(x_o + \beta_o x_o) + k[x_o(1 + \beta_o) - x_2(1 + \beta_{1.2})] + [(x + \beta_1 x) - (x + \Delta c_2)] \qquad \text{for } x_2 \le x \le \infty \qquad (7c)$$

where

$$\beta_{1.1} = \frac{c_2 + \Delta c_2}{c_1 + x_1}$$

[22]There are many methods for estimating loading or burden; e.g., direct labor hours may be used instead of direct labor costs. Furthermore, there are a variety of overhead pools to be distributed, not one. So far as the details of an actual negotiation are concerned, the above is a simplified representation. However the consideration of overhead as one term representing the summation of perhaps numerous overhead pools distributed in a like manner does not invalidate the model for the purposes of this discussion.

As a practical matter, the overhead associated with the contract is likely to consist of variable, semi-variable and fixed burden or overhead. The classification of many cost items as "overhead" or "variable" is a product of the individual accounting system. There are certain cost items (e.g., electric power) which are generally considered variable costs, but which are convenient to cost as overhead items. Some cost accounting systems, on the other hand, require extensive time records of personnel normally considered "overhead" (even top managers) and allocate their time to contracts as actually expended. An accurate determination of marginal increments of cost implies a sufficiently detailed costing system, but there will never be one which will completely resolve the ambiguities of overhead costing.

$$\beta_{1.2} = \frac{c_2 + \Delta c_2}{c_1 + x_2}$$

A CPIF contract becomes a CPFF contract whenever the cost outcome exceeds the fee swing; therefore equations (7b) and (7c) are expressions for a CPFF fee over the defined range. For a FPI contract the fee expression is the same as for a CPIF contract except the upper limit of the range of definition would be finite. From (7b) and (7c) it is apparent that any increase in the variable cost will increase the fee over the range of definition. Therefore there is this "incentive" to increase costs in CPIF contracts whenever the cost exceeds the fee swings in either direction. Between the fee swings, however, the CPIF contract has the variable second term (the variable fee term) as a basis for offsetting this gain. This fee function will obviously vary with the various cost parameters – the sharing arrangements and the fee swings, as well as the cost outcome. The peculiarities of this fee function will be examined in the following examples.

For purposes of illustration, the following combinations of cost situations are examined; (a) the CPIF direct target costs equal to the direct costs on the fixed price sales, (b) the CPIF direct target costs equal to .1 of the fixed price costs, (c) overhead or burden factors of 2:1 and 1:1, plus (d) various selected values of sharing arrangements and fee swings. The numerous parameters complicate the analysis and the following examination will attempt to generalize from these selected situations.

Consider first a fee arrangement with unlimited fee swings. The inclusion of fee swings results in a piecewise function which complicates but does not greatly modify the subsequent evaluation. Since there is considerable insistence in the contracting manuals that the fee swings be made as large as possible, this will provide commentary on the situation in which this objective is attained. With this assumption we need only to consider equation (7a).(Examples are considered later in which the complete fee expression is used). The first derivative of equation (7a) is

$$\frac{dF^*}{dx} = -k + \frac{c_1 (1-k)(c_2 + \Delta c_2)}{(c_1 + x)^2} \tag{8}$$

The second derivative is

$$\frac{d^2F^*}{dx^2} = \frac{-2c_1 (1-k)(c_2 + \Delta c_2)}{(c_1 + x)^3} \tag{9}$$

For all positive values of x, the fee will be maximized where the first order condition is zero.

Setting (8) equal to zero and solving for k determines the sharing fraction which will maximize the fee function at a given cost outcome, x.

$$k_o = \frac{c_2 + \Delta c_2}{c_1 + c_2 + \Delta c_2 + \frac{x^2}{c_1} + 2x} \tag{10}$$

$F^*(x)$ is concave and there is a value of k, $k = k_o^+$, which will maximize $F^*(x)$ at $x = x_o$. As k varies from k = 1.0 to k = 0, $F^*(x)$ rotates counter-clockwise about $x_o$ and changes shape; also, $F^{*\prime}(x_o; k) > 0$ for $k < k_o^+$ and $F^{*\prime}(x_o; k) < 0$ for $k > k_o^+$. The effects of changes in k can be noted in Figures 10 to 15. As noted, with the addition of the fee swings $F^*(x)$ becomes a piece-wise function; once the fee swing is exceeded, the net return is always increased by increasing the variable cost. Given $k < k_0^+$, the fee can be maximized by overruns, and for $k > k_0^+$, by underruns. This is in contrast to the fee function, $F(x)$, in which the maximum fee for a given k is always attained at the lowest cost. However, given that an overrun occurred, the maximum fee occurs when $k = 0$ and vice versa; i.e., for fee maximization the contractor would select a sharing fraction of either one or zero.

Function (10) is plotted in Figure 9 for various selected cost parameters. Figure 9a pertains to a cost situation in which the negotiated CPIF contract adds variable cost equal to those of the firm-fixed price sales. Figure 9b pertains to a cost situation in which the CPIF contracts increase the firm's variable expense by 10%. Various levels of additional overhead ($\Delta c_2$) were assumed as pertaining to the CPIF contract; but under the assumptions, these values do not vary with variations in the variable cost.

Referring to Figure 9a: If the CPIF contract is accepted with an increase in the overhead of $\Delta c_2 = 100 = x_0$ and $c_2/c_1 = 2$, a sharing factor of 43% would maximize the fee function for a variable cost outcome equal to the target cost (function a). If the CPIF contract was accepted with the same addition to overhead ($\Delta c_2 = 100$) and $c_1/c_2 = 1$, then a sharing factor of .33 would maximize the fee function at the target cost (function b). An initial overhead to variable cost of 2:1 combined with no additional fixed cost associated with the CPIF contract ($\Delta c_2 = 0$) is also illustrated by function (b). Finally, a CPIF contract with $c_2/c_1 = 1$ and no incremental overhead would find the function $[k_0(x)]$ illustrated by function c. The same parameter [illegible] are illustrated in Figure 9b, except that the absolute levels of $c_2$ and $c_1$ are increased by an order of magnitude.

Some judgments might be made by comparing the information from these figures with the sharing arrangements being negotiated or likely to be negotiated under present contracting conditions. If the CPIF contract involves costs on the order of 10% of the firm's fixed price costs, it would appear that the range of sharing fractions which would maximize the fee function at underruns considerably exceed most all of the sharing arrangements now being negotiated; a contractor with a sharing arrangement less than .40 and cost parameters given in Figure 9b would maximize fee at practically unlimited overruns. For the parameter descriptions of Figure 9a only a contractor with a cost situation given by c would find the present average level of sharing arrangements maximizing the fee at underruns.

It is important to examine this fee function in its entirety, i.e., including the fee swings. Figure 10 through 15 illustrate fee functions 7a, 7b, and 7c (solid lines) for selected parameters. The dotted line illustrates function 7a when $x_1 = 0$ and $x_2 = \infty$. All figures assume the target percentage ($\gamma$) is equal to .08. Except for Figure 13, each figure is given with two cost ratios, $c_2/c_1 = 2$ and $c_2/c_1 = 1$. Because of the number of parameters involved in the function, we have "bracketed" the more probable cost situations by assuming certain reasonable limits to these parameter values. For example, data from the files of NASA indicate that the burden ratio seldom is less than 1:1 or more than 2:1. On the other hand, we have no information as to the distribution of the ratio of variable costs on accepted CPIF contracts to the variable costs of fixed price contracts – nor is it obvious how such information would be interpreted if available. However, the assumptions of tw ontract situations, one doubling the firm's variable costs and one increasing it 10%, might be reasonable boundary values for this parameter.

At first glance the size of the "fee" may be somewhat startling; this, of course, is explained by the third term of equation 7a. Lower values of the ratio $c_2/c_1$ serve to reduce the overall level of this function, $F^*(x)$; but in the cases illustrated, the net gain on the contract will be comparatively large in terms of the fee arrangement, i.e., the fee is comparatively unimportant. The fee swings convert the fee function from a CPIF contract to a CPFF contract for values of the cost outcome exceeding those corresponding to the upper and lower fee swings. So long as an increase in the direct cost on the contract is not accompanied by a proportional increase in the overhead cost, and unless all sales are cost-plus there is always a net gain from increasing the costs on a CPFF contract. It is apparent that the existence of a fee swing would give the contractor faced with an overrun condition an option of increasing his fee either by increasing the overrun or by cutting costs.

Figures 10 and 11 also demonstrate that selection $k \doteq k_0^+$ would produce a comparatively "flat" fee function and a probable indifference (on the contractor's part) as to the cost outcomes between the fee swings. This is more noticeable when the cost-plus contracts form a smaller percentage of the firm's total sales. (See Figures 14 and 15). The sharing arrangements must be quite high compared with those negotiated today for the fee function to have an important increase associated with a decrease in cost between the defined limits of the fee swing. (See Figure 12). Once the fee swings are exceeded in either direction, however, the fee is more sensitive to the cost outcome, except for extreme values of k. On the premise that a given overrun is easier to attain than an underrun, it is reasonable to anticipate that if the contractor elects to increase his fee by control of the cost outcome (which he is in a position to do later in the contract) he will most likely do so by overruns.

Changing the burden from a ratio of 2:1 to 1:1 changes the overall level of the fee, but it does not materially affect the shape of the curves and the previous observations. On the other hand reducing the burden ratio decreases the values of $k_0$ for any given value of cost outcome x. This is especially noticeable when the initial burden is large compared to that for the CPIF contract. Also increasing $c_2$ will serve to increase the overall level of the fee function, ceteris paribus.

## Contractor Decision Making with $F^*(x)$

A contractor aware of his marginal costs could logically consider the fee function $F^*(x)$ in place of the traditional function, $F(x)$, for decision making under uncertainty. Fundamentally the contractor should consider the transformation of this fee function into his utility function in the manner discussed earlier. It does not seem particularly useful to assume another utility function as a basis for additional examples; rather, it would seem sufficient to examine the fee function under certain parameter assumptions recognizing the importance of the utility transformation (or subjectively weighting the probable loss versus the probable gain).

Maximization of the function $E[F^*(x,k)] = \int_R F^*(x,k)h(x)dx$ would occur where $dE[F^*(x,k)]/dk = 0$.[23] Since

[23] The optimum choice of k with the function $E[F^*(x,k)]$ will change at a value of $x_0 > \mu$, as in contrast with the function $E[F(x)]$, where the optimum choice changes at $x_0 = \mu$.

k is independent of x, the function under the integral can be differentiated. The derivative is a linear function of k, so the function will either maximize for $k = 0$ or $k = 1.0$. As before, the contractor will base his decisions on the utility function of fees, which is again assumed to be concave. To review, a utility function based upon the function F(x) will rotate to a value of U(x) = constant as k approaches zero. Since the fee function $F^*(x)$ is concave and pivots about $x_0$ as k changes, the utility function will do likewise; utility as a function of $F^*(x)$ will swing from a strictly concave function of x monotonically increasing (when $k = 0$) to a strictly concave monotonically decreasing function of x when $k = 1$. And, as before, the maximum utility can occur with partial sharing fractions. A contractor negotiating a contract on the basis of $F^*(x)$ might, therefore, be expected to have some indifference to $x_0$ so long as he is in a reasonable position to negotiate the sharing fraction since, as noted, the fee function is comparatively flat for values of k approximately equal to those maximizing the fee at $x_0$. It is possible that contracts negotiated upon the function $F^*(x)$ would have a tendency to stabilize about the expected cost. Certainly, if the k were negotiated at the value $k_0$ there would be every incentive for the contractor to attain his target cost since the utility is maximized at that cost value. However, under the cost parameters assumed in our examples, $k_0^+$ considerably exceeds the values of sharing fractions now being negotiated.

The previous comments were confined to $F^*(x)$ as defined by 7a where $x_1 = 0$ and $x_2 = \infty$. The opposite extreme would be a function with narrow fee swings. Narrow fee swings tend to produce a function which, except for a narrow range is essentially an increasing function of x. Therefore, narrow fee swings would tend to make it of less interest to the contractor to negotiate high target costs and high sharing fractions. Since the values of $x_1$ and $x_2$ define $F^*(x)$, one cannot determine an optimum combination of fee swings, sharing fractions and target costs. Each of these parameters are subject to separate negotiations and it is difficult to generalize as to how the contractor should negotiate in this instance. However, the contractor in most situations should find wide fee swings acceptable to the government negotiators. One "optimizing" procedure might be to negotiate in terms of unlimited fee swings and attempt to improve the contracting position with the fee swings last, as suggested by the other terms negotiated.

The advantage of using the function $F^*(x)$ in the decision process is the unequivocal advantage of a more precise statement of the fee outcome, unless the proportion of cost-plus to total sales is high.[24] If, as in some cases, the other cost-plus contracts are sufficiently near completion that the costs can be fairly determined, the expected net fee and the selection of an optimum k can be calculated with slight modifications of $F^*(x)$ and a moderate increase in computational complexity. If, on the other hand, the firm's other sales involve several incentive contracts with unknown costs, then the selection of an optimum sharing fraction becomes the evaluation of a joint density and a fee function involving multiple cost outcomes. However, in this case there would be a specific k for the contract which would maximize the expected net utility on all contracts.

[24] Access to a limited amount of proprietary data indicates that of the top 100 defense contractors in 1962-63, 48% of them had better than 50% of their sales in firm-fixed price category. These firms would certainly be more than casually concerned about the effects of marginal pricing of their CPIF contracts. On the other hand, it must be noted that the large companies among this group had less than 50% FFP sales, and these are the firms which have a larger percentage of government contracts.

The previous comments have been based upon the assumption of a symmetrical cost density function. A distribution highly skewed to the right would, in effect, serve to restrict the range of possible underruns, especially if the target cost is in the vicinity of the modal, rather than the expected cost outcome. A contractor who bases his negotiation decisions on the expected cost outcome would undoubtedly attempt to compensate for the skewed distribution by the negotiation of either a low upper limit to the fee swing or a value of k lower than that maximizing (7a) at the target cost. Other conditions may be assumed, but it does not appear that important increases in our understanding of the problem would be obtained from an examination of skewed distributions.

**Marginal Fee Function with Variable Overhead Assumptions, $F^{**}(x)$**

As stated earlier it is desirable to examine this model under the assumption that $\Delta c_2$ is considered to vary proportionally with changes in x. This is equivalent to an assumption that all overhead is variable overhead, and is but one of the infinity of overhead situations which could be assumed. This assumption should offer a "reasonable other bound" to our model.

In this case the fee function (following equations 7a, 7b, 7c) is

$$F^{**} = \gamma x_o(1+\beta_o) + k\left[x_o(1+\beta_o) - x - \frac{x\left(c_2 + \frac{x\Delta c_2}{x_o}\right)}{c_1 + x}\right] + \frac{x\left(c_2 + \frac{x\Delta c_2}{x_o}\right)}{c_1 + x} - \frac{x\Delta c_2}{x_o}. \qquad \text{for } x_1 \le x \le x_2 \tag{11a}$$

$$= \gamma x_o(1+\beta_o) + k\left[x_o(1+\beta_o) - x_1 - \frac{x_1\left(c_2 + \frac{x_1\Delta c_2}{x_o}\right)}{c_1 + x_1}\right] + \frac{x\left(c_2 + \frac{x\Delta c_2}{x_o}\right)}{c_1 + x} - \frac{x\Delta c_2}{x_o} \qquad \text{for } 0 \le x \le x_1 \tag{11b}$$

$$= \gamma x_o(1+\beta_o) + k\left[x_o(1+\beta_o) - x_2 - \frac{x_2\left(c_2 + \frac{x_2\Delta c_2}{x_o}\right)}{c_1 + x_2}\right] + \frac{x\left(c_2 + \frac{x\Delta c_2}{x_o}\right)}{c_1 + x} - \frac{x\Delta c_2}{x_o} \qquad \text{for } x_2 \le x \le \infty \tag{11c}$$

Again assuming an unlimited fee swing [equation 11a where $0 \le x \le \infty$]

$$\frac{dF^{**}}{dx} = -k + \frac{(1-k)\left(c_1 c_2 + \frac{2c_1\Delta c_2 x}{x_o} + \frac{\Delta c_2 x^2}{x_o}\right)}{(c_1 + x)^2} - \frac{\Delta c_2}{x_o}$$

Setting the first derivative equal to zero and solving for k

$$k = c_1\left[\frac{c_2 - \frac{c_1\Delta c_2}{x_o}}{\left(1 + \frac{\Delta c_2}{x_o}\right)\left(\frac{x^2}{c_1}\right) + 2\left(1 + \frac{\Delta c_2}{x_o}\right)x + c_1 + c_2}\right]$$

Figure 16 is comparable to Figure 9, although it will be noted that the curves are "lower" on the scale. Under these assumptions the values of k which will maximize the fee with underruns are lower than under the assumption of non-varying overhead. For those situations in which the proportion of incremental overhead to variable cost is the same on the contract as for the original burden in the firm (that burden existing prior to contract negotiation) the fee function will be identical to F(x). Therefore contract situations with cost parameters meeting these conditions are not shown on these figures. But unless the ratios of overhead to variable cost meet these conditions, or, to put it conversely, if $\Delta c_2/x_0 < c_2/c_1$, there is additional fee associated with the overhead payments.

Figure 17 should be examined in comparison to Figure 13. Both consider the same cost parameters and sharing arrangements. The assumption of completely variable overhead changes the slope of the fee function (or the value of the cost outcome at which the fee function is optimized). However there remains more than one cost outcome at which the fee function will be optimized and the previous comments concerning the contractor's options hold for this case. Figure 18 illustrates a situation in which $\Delta c_2/x_1 \doteq c_2/c_1$. One will note the similarity between this fee function and those considered in previous sections of this paper. Also, as the proportion of cost-plus sales to total sales increases, $F^{**}(x)$ approaches F(x), the usual fee expression.

**Note on FPI Contracts**

The previous discussion has been confined largely to the CPIF contract form. The FPI contract has an upper price (cost) limit beyond which it converts to a fixed price contract. This difference in the two contract forms does not significantly change the previous observations about optimal behavior of contractors. The contractor would obviously be more concerned about the effect of cost uncertainty with a FPI contract than with a CPIF contract and would be induced to negotiate target costs higher than the expected costs ($x_0$ would be negotiated much higher than $\mu$ relative to the variance of the subjective cost density) and low sharing fractions for a given target fee. If the contractor establishes his behavior on marginal cost outcome, the opportunity for reward from exceeding the target cost is limited, but there would still be situations in which the fee is maximized by overruns.

FIGURE 1

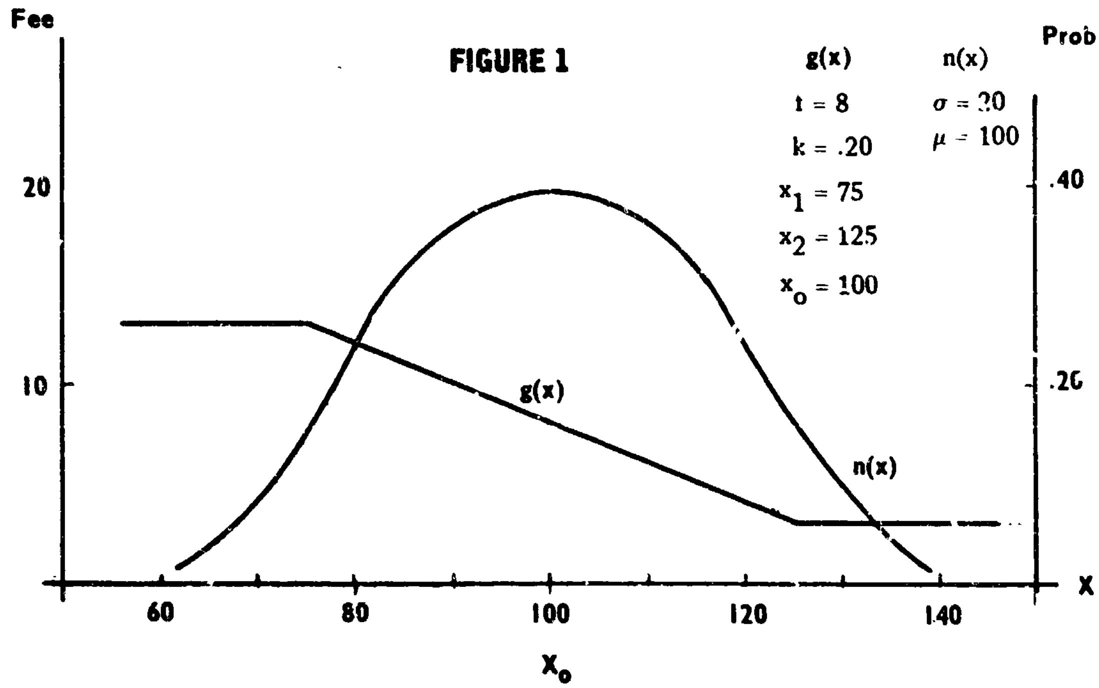
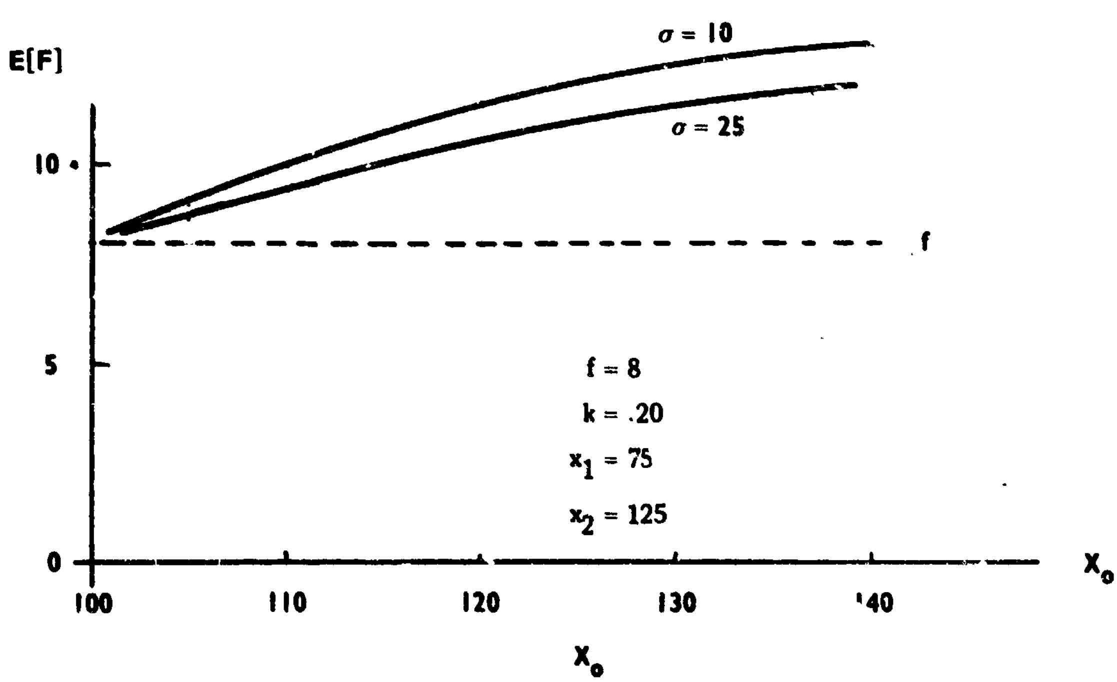



FIGURE 2



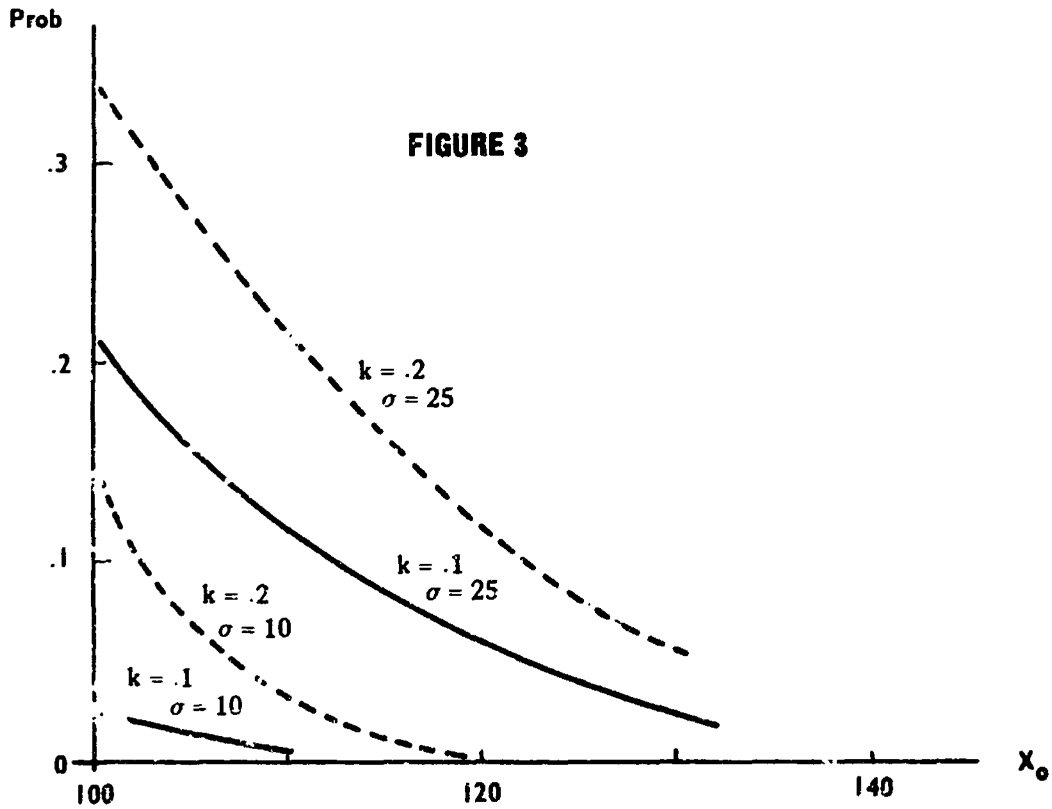



FIGURE 3

**FIGURE 4**

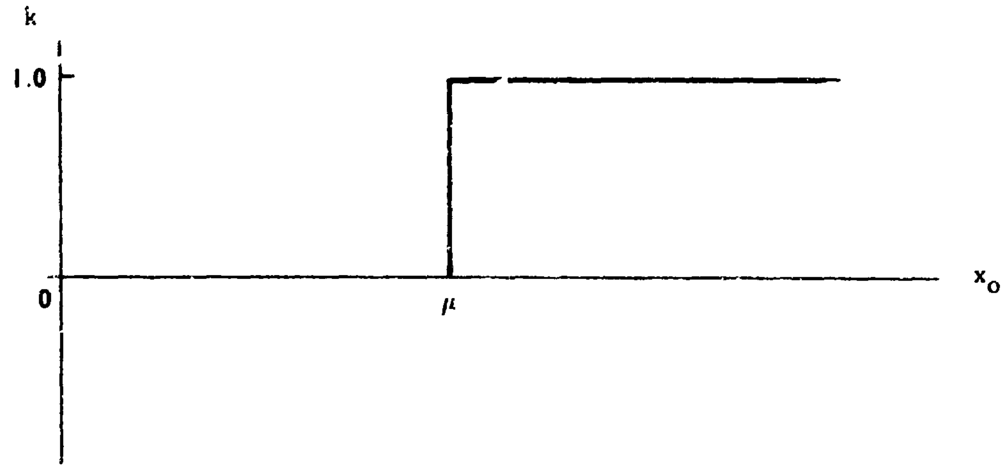



**FIGURE 5**

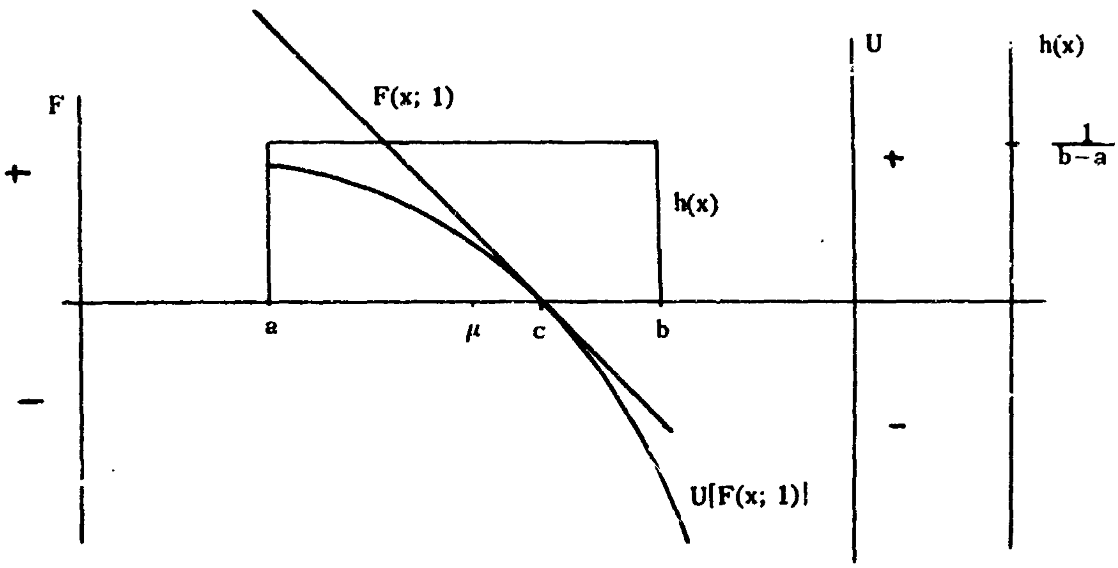



**FIGURE 6**

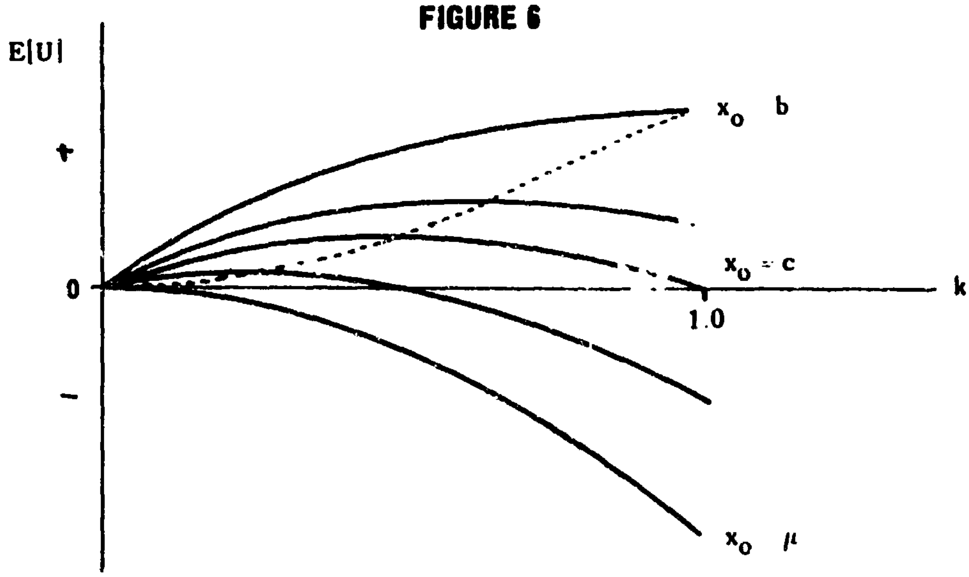


**FIGURE 7**

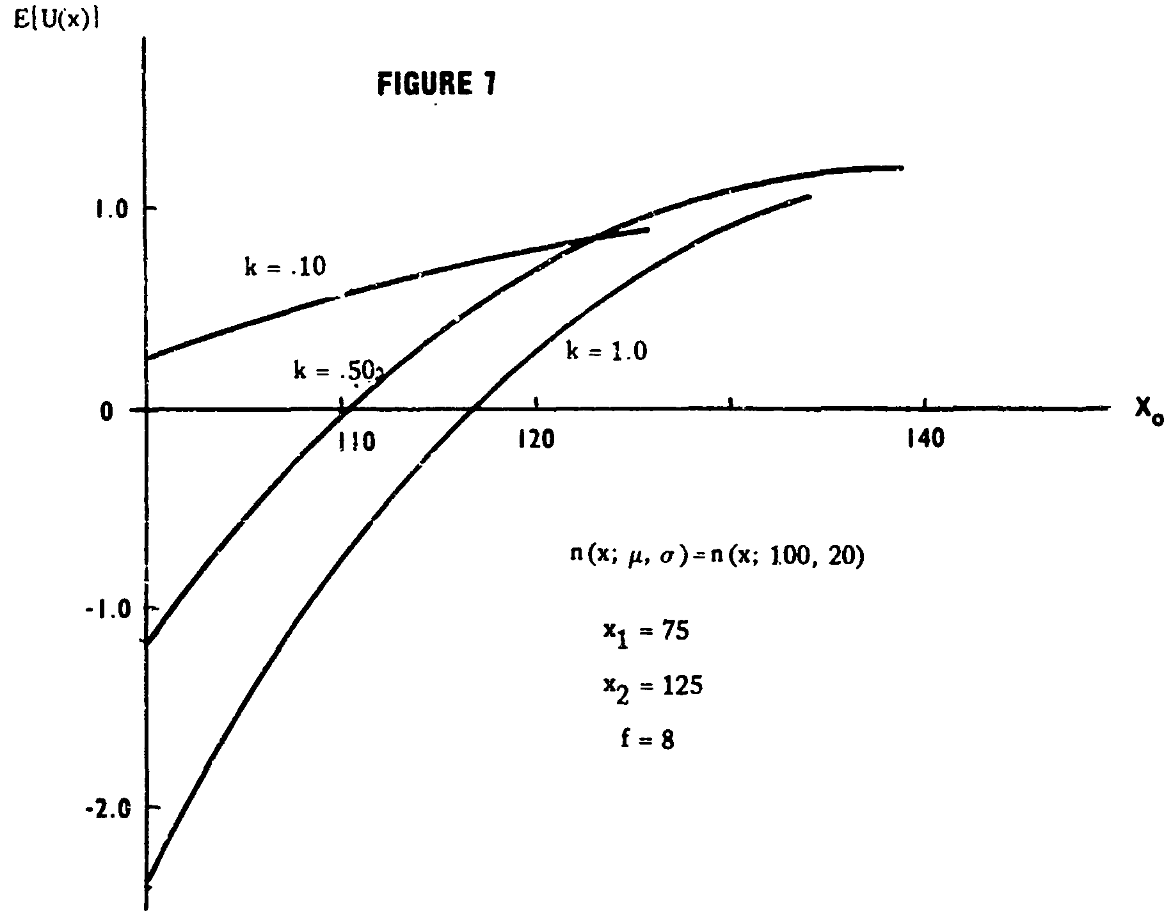



**FIGURE 8**

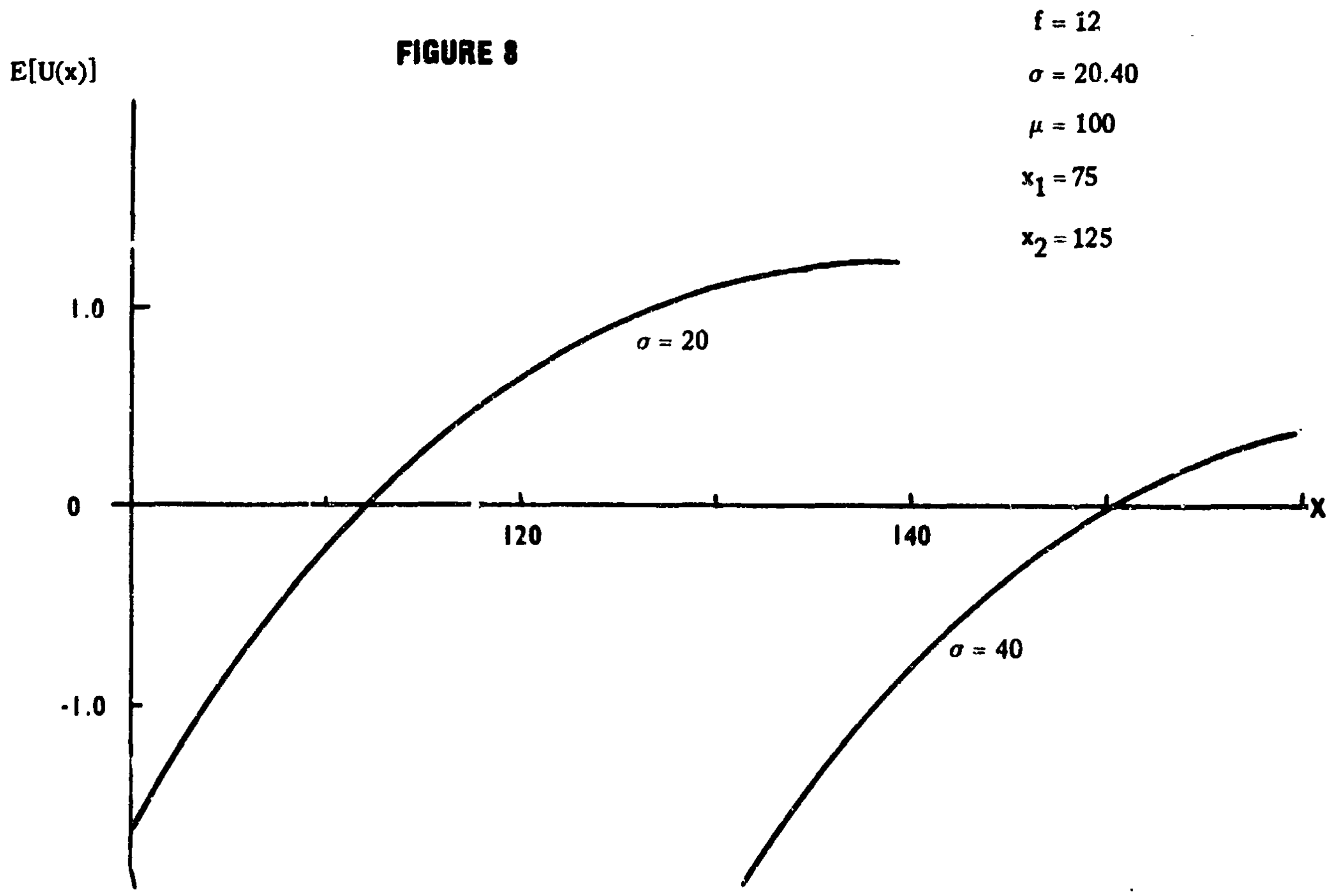
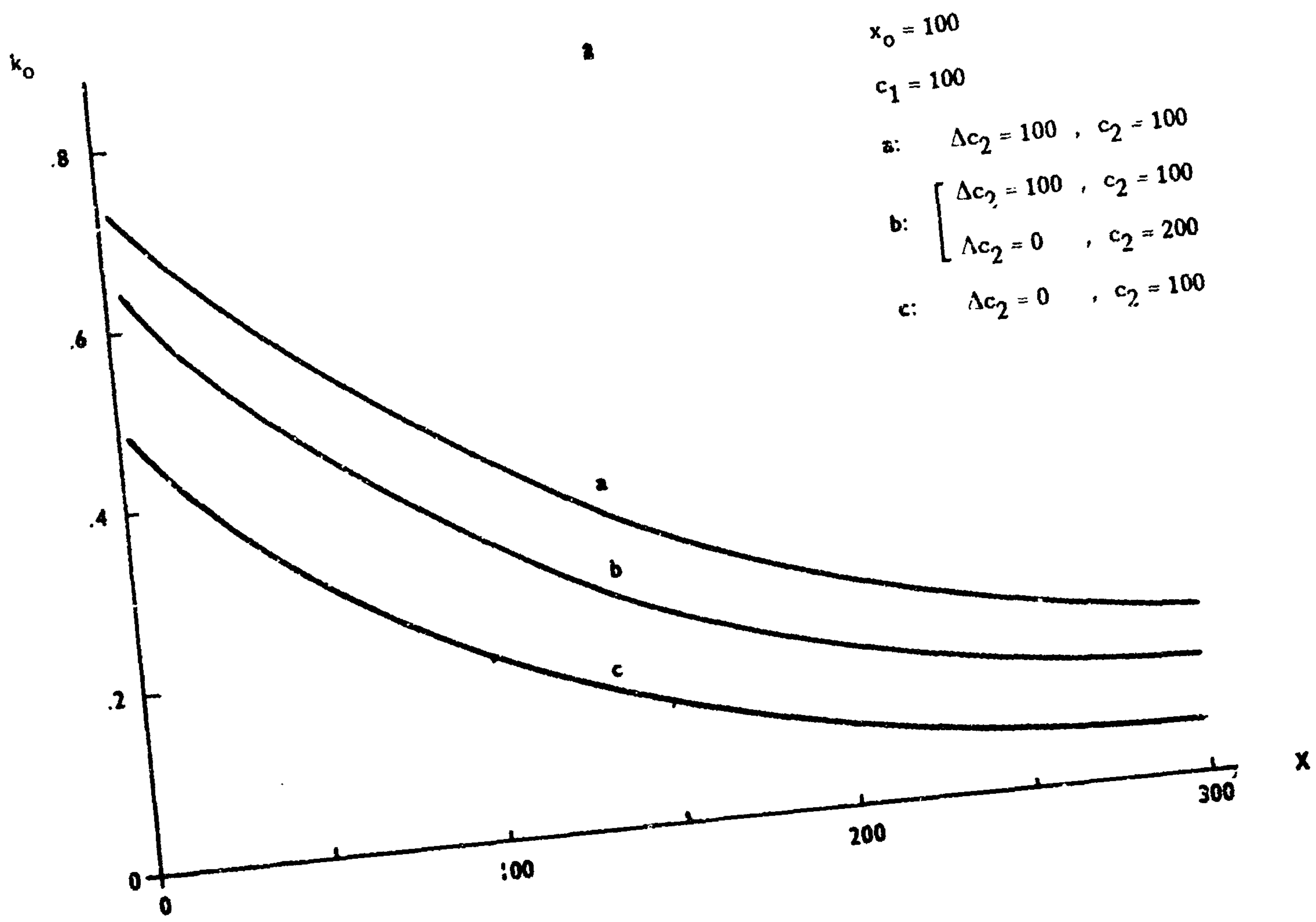
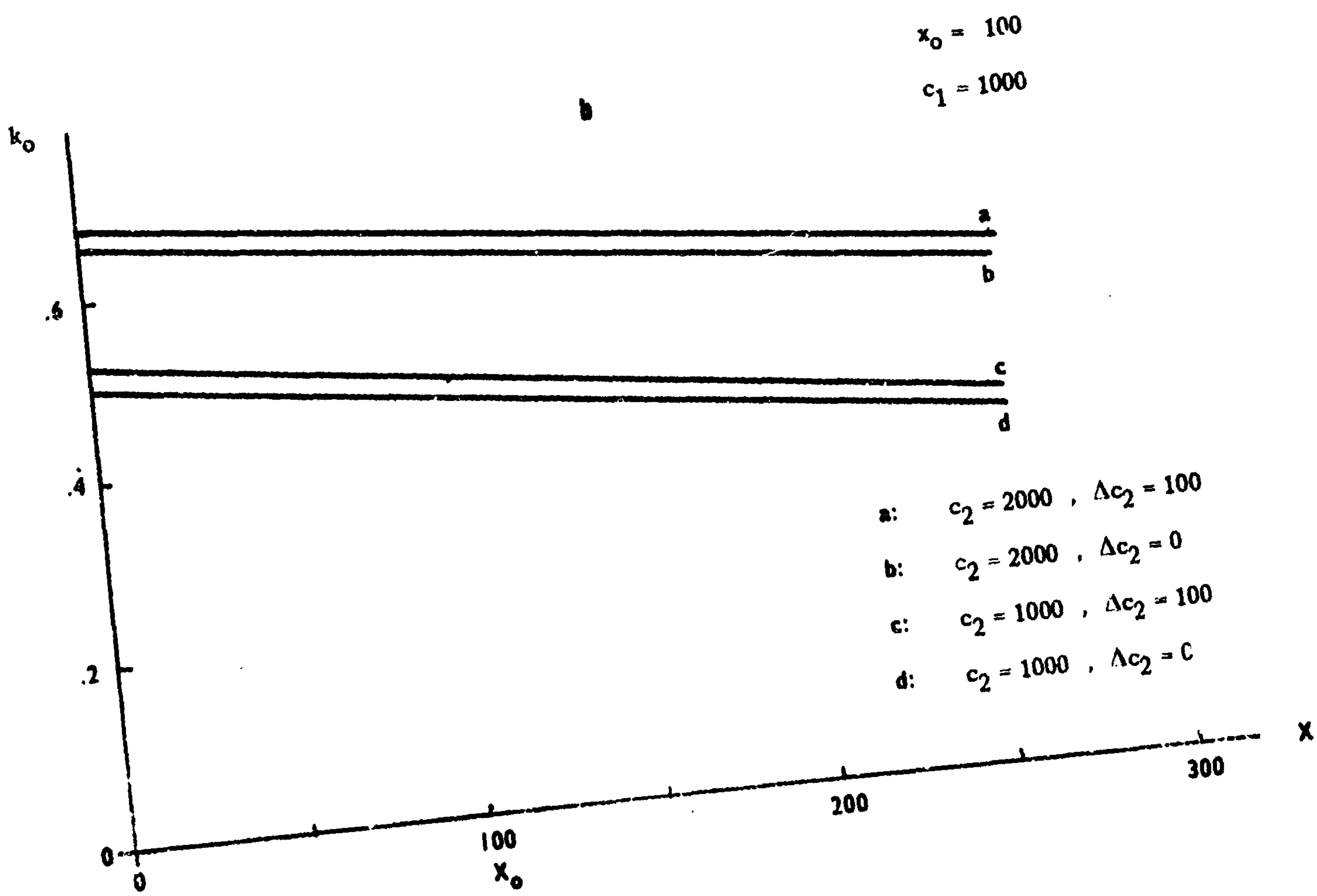


FIGURE 9



a
$x_o = 100$
$c_1 = 100$
a: $\Delta c_2 = 100$ , $c_2 = 100$
b: $\Delta c_2 = 100$ , $c_2 = 100$
$\Delta c_2 = 0$ , $c_2 = 200$
c: $\Delta c_2 = 0$ , $c_2 = 100$
$k_o$
.8
.6
.4
.2
0
a
b
c
0
100
200
300
X

$x_o = 100$
$c_1 = 1000$
b
$k_o$
a
b
c
d
.6
.4
.2
0
a: $c_2 = 2000$ , $\Delta c_2 = 100$
b: $c_2 = 2000$ , $\Delta c_2 = 0$
c: $c_2 = 1000$ , $\Delta c_2 = 100$
d: $c_2 = 1000$ , $\Delta c_2 = C$
0
100
200
300
X
$X_o$

FIGURE 10

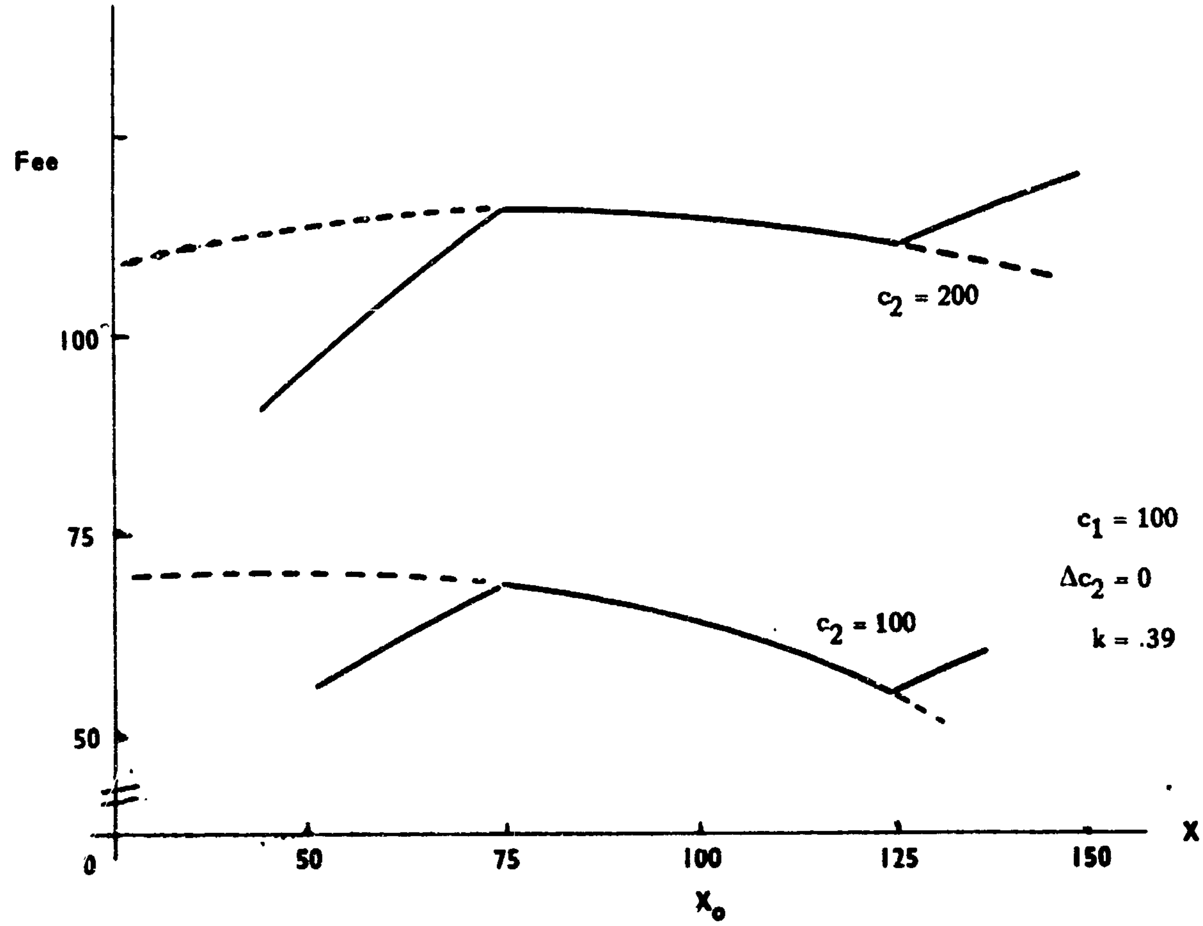
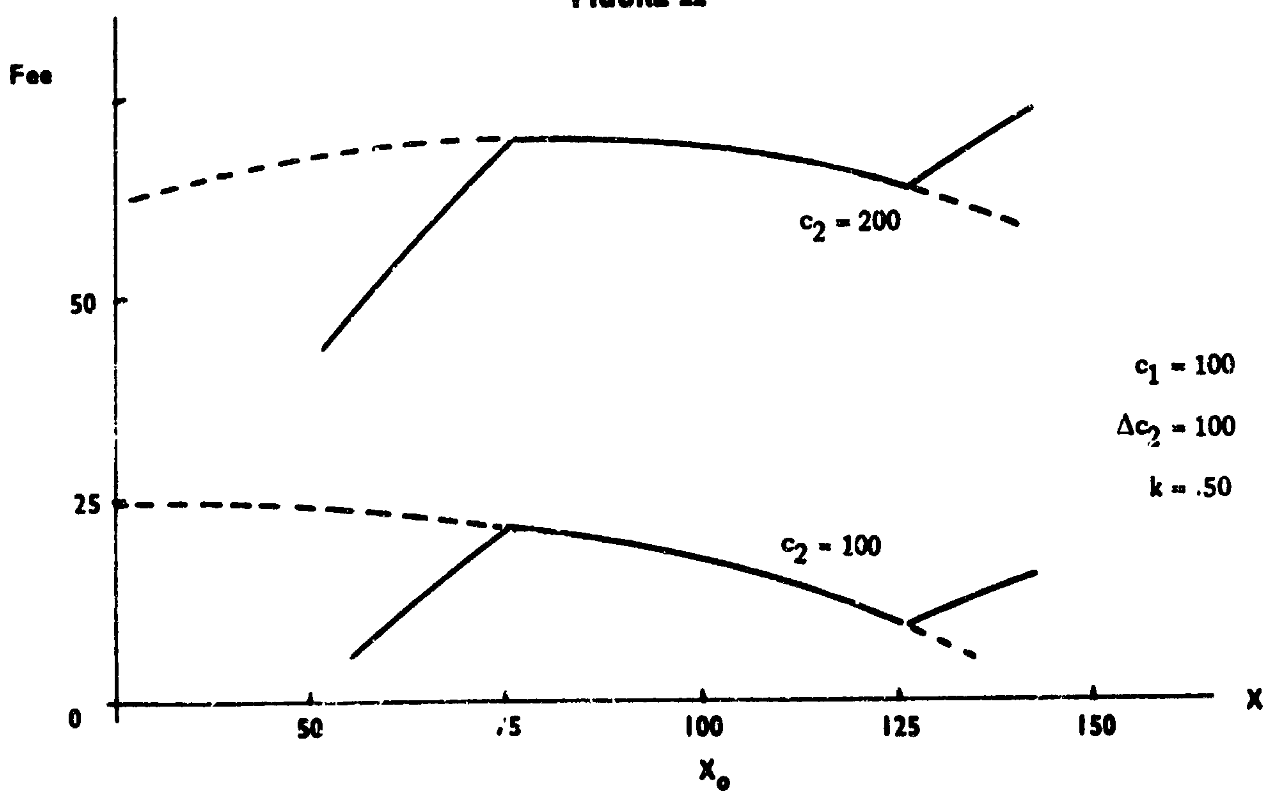
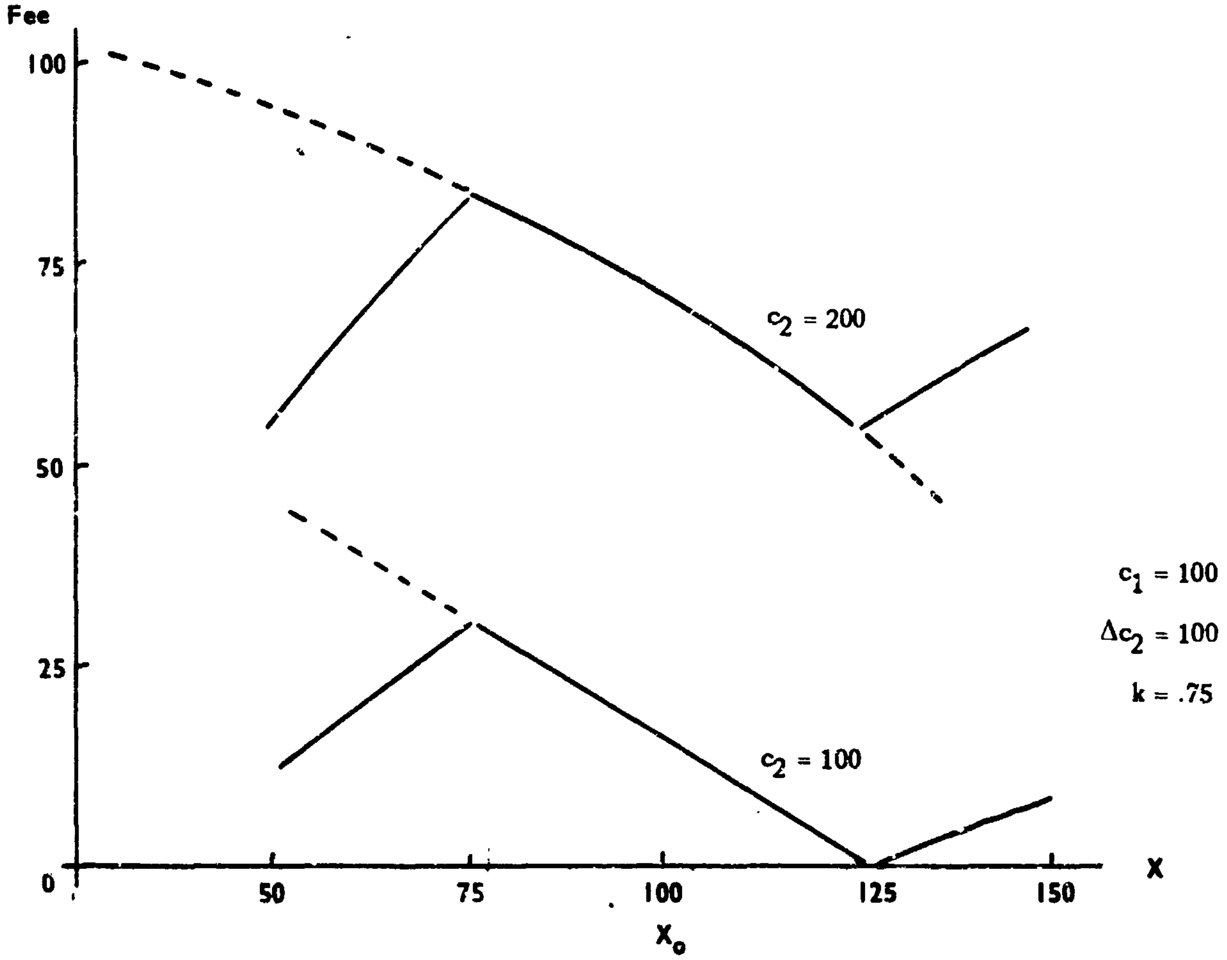



FIGURE 11



## FIGURE 12




## FIGURE 13

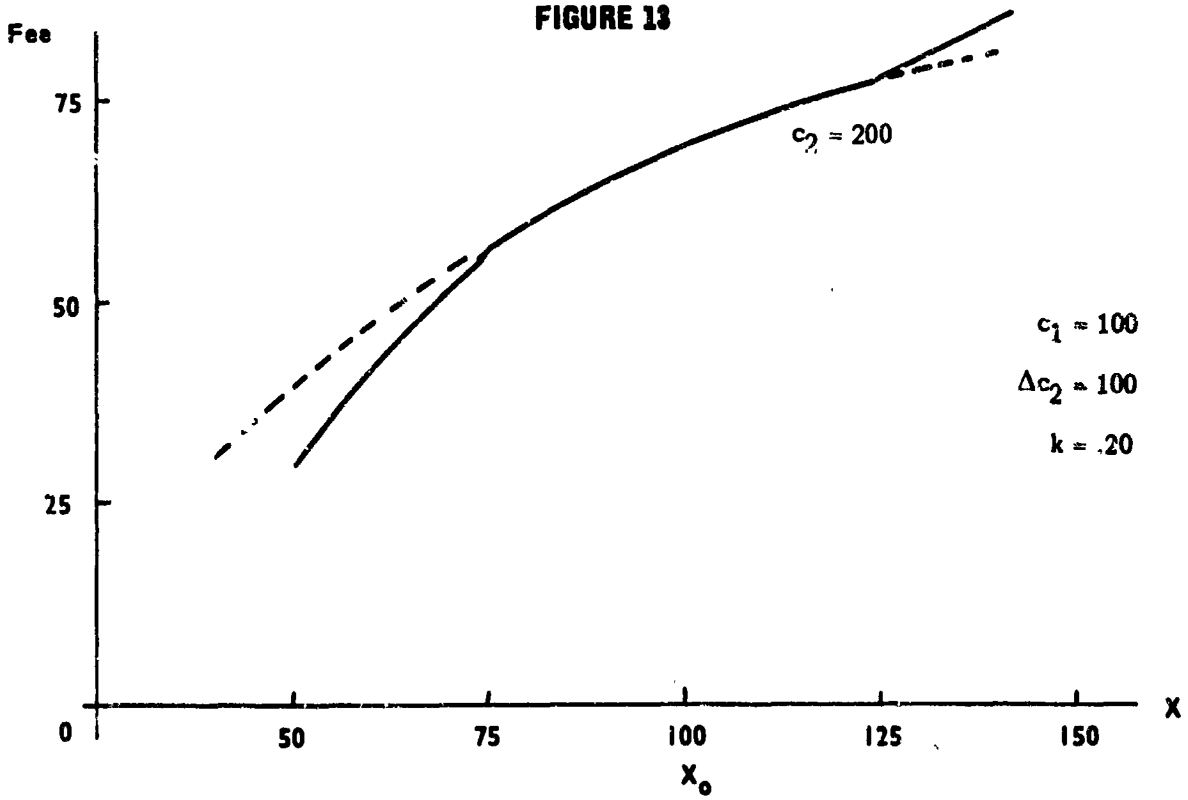


FIGURE 14

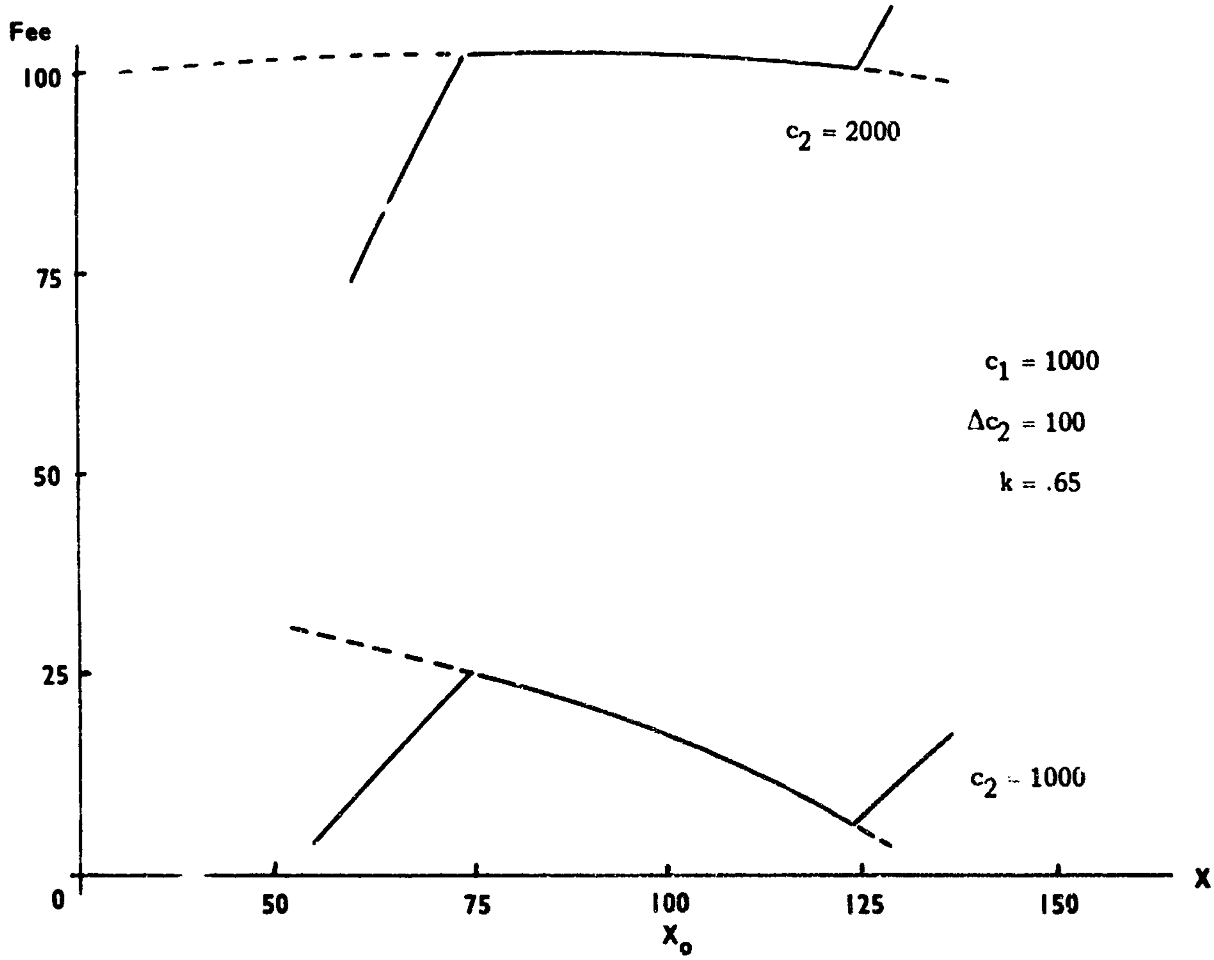



FIGURE 15

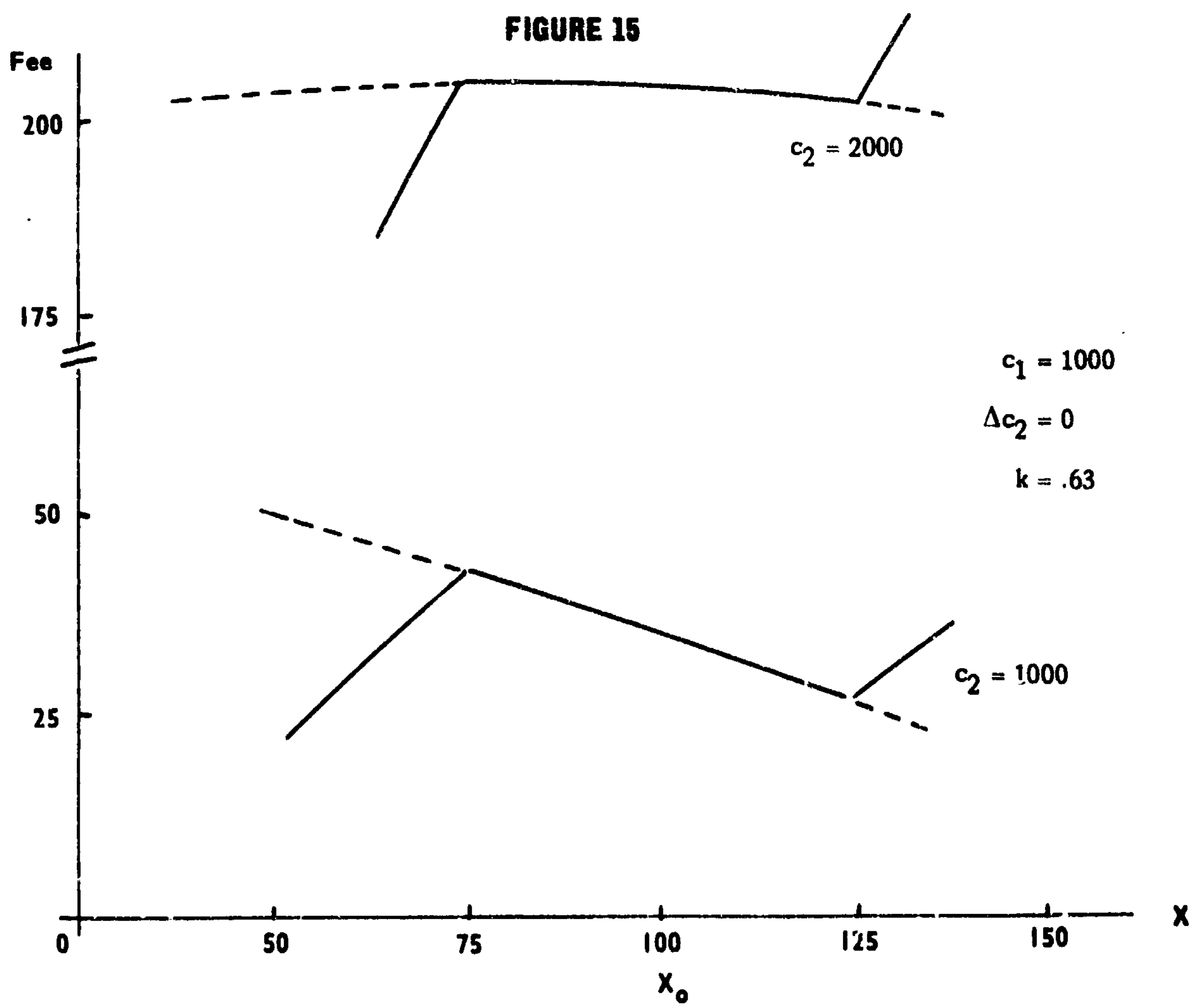
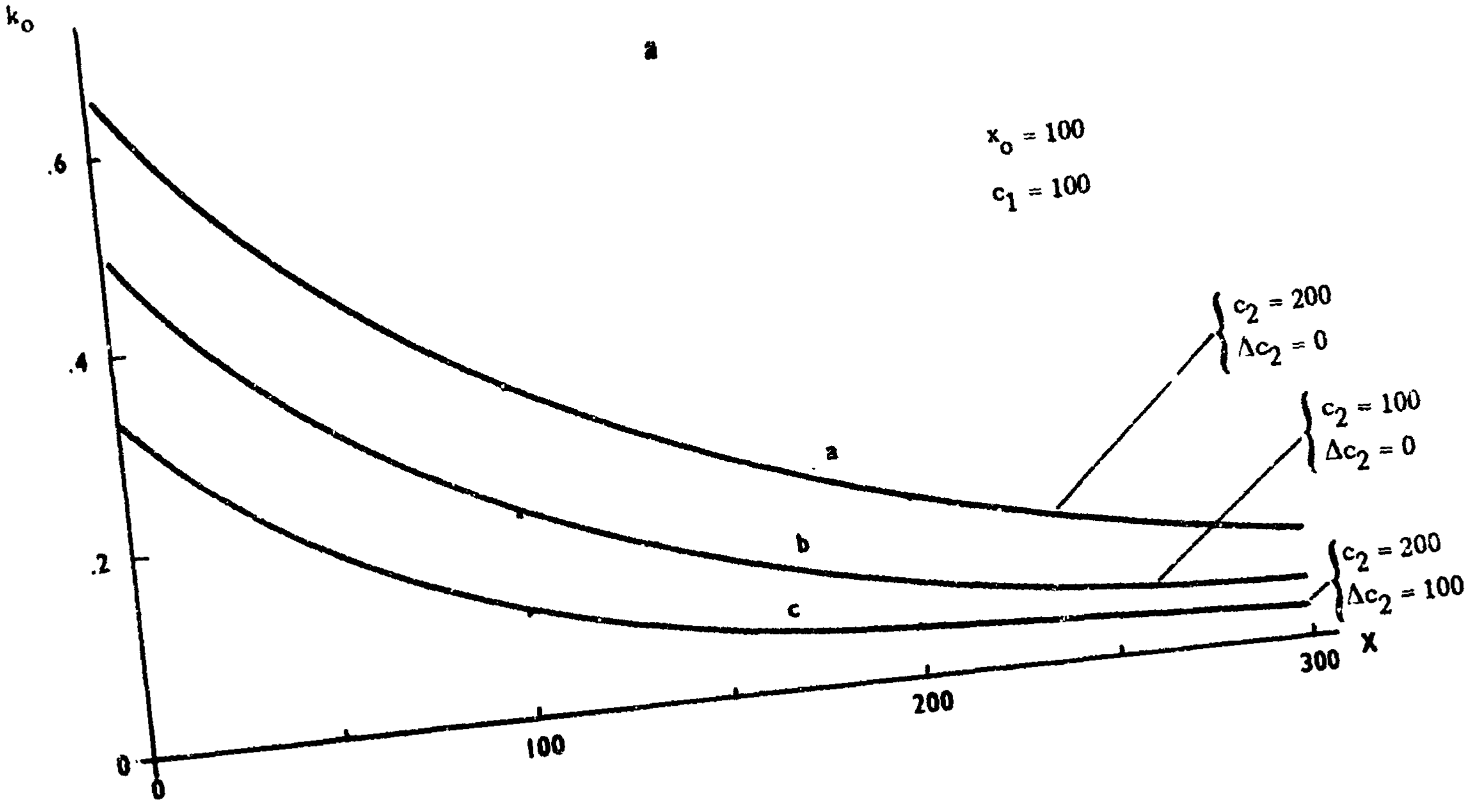
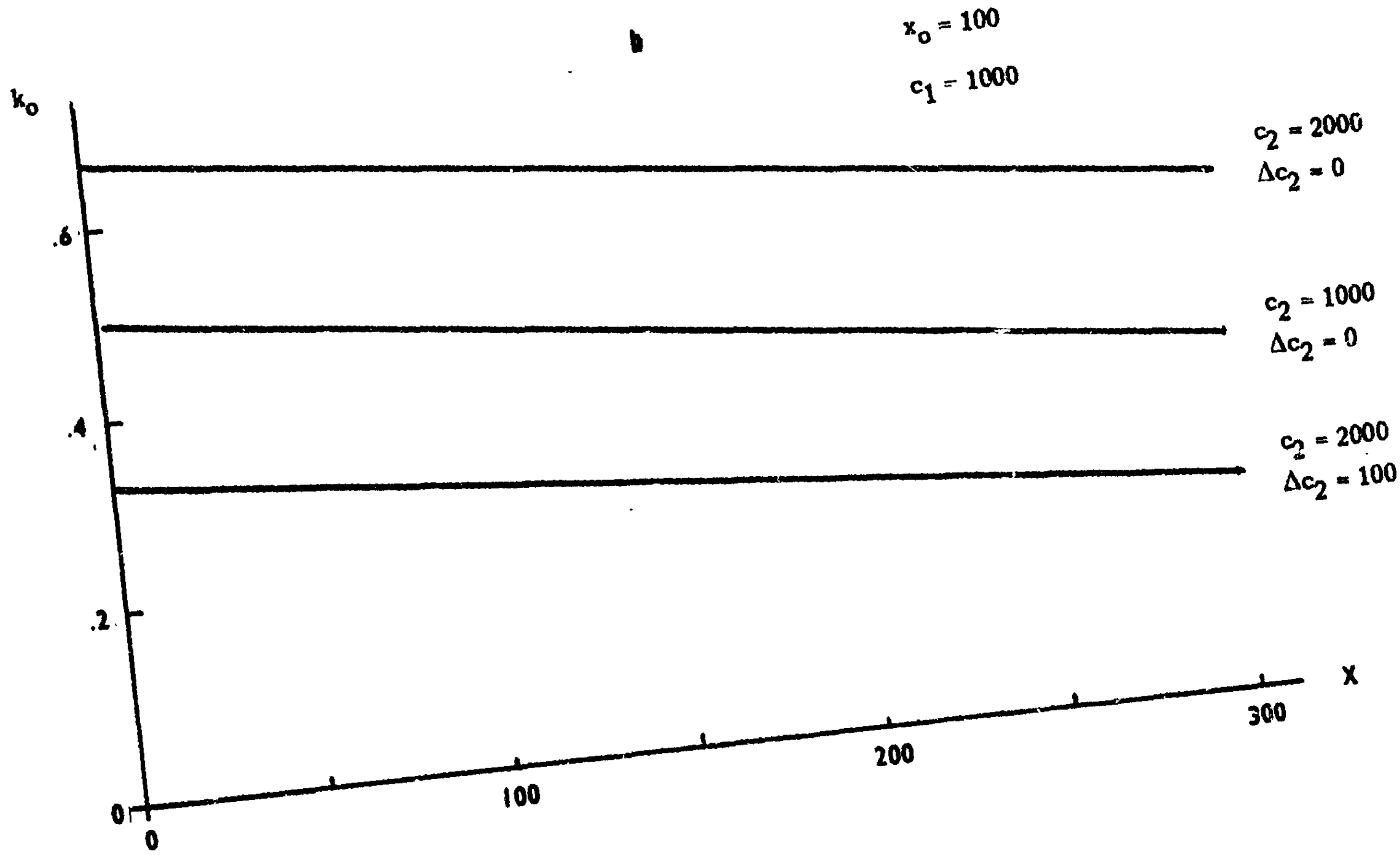


FIGURE 16



a
$k_o$
.6
.4
.2
0
0
100
200
300
X
$x_o = 100$
$c_1 = 100$
$c_2 = 200$
$\Delta c_2 = 0$
$c_2 = 100$
$\Delta c_2 = 0$
$c_2 = 200$
$\Delta c_2 = 100$
a
b
c

b
$k_o$
.6
.4
.2
0
0
100
200
300
X
$x_o = 100$
$c_1 = 1000$
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$\Delta c_2 = 0$
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$\Delta c_2 = 0$
$c_2 = 2000$
$\Delta c_2 = 100$

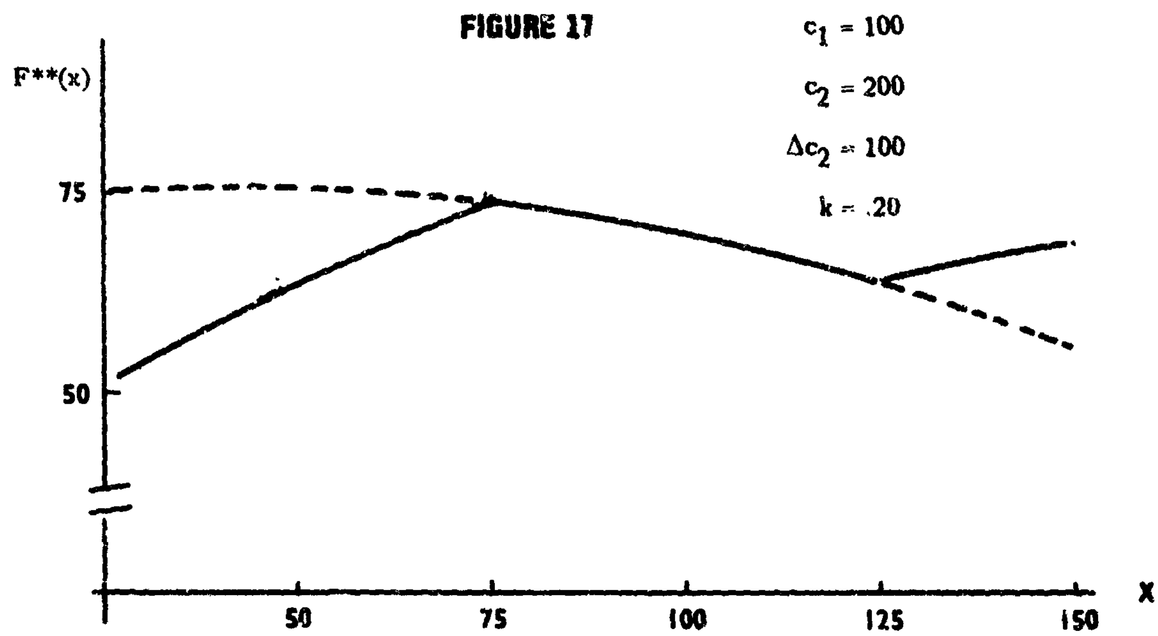

FIGURE 17

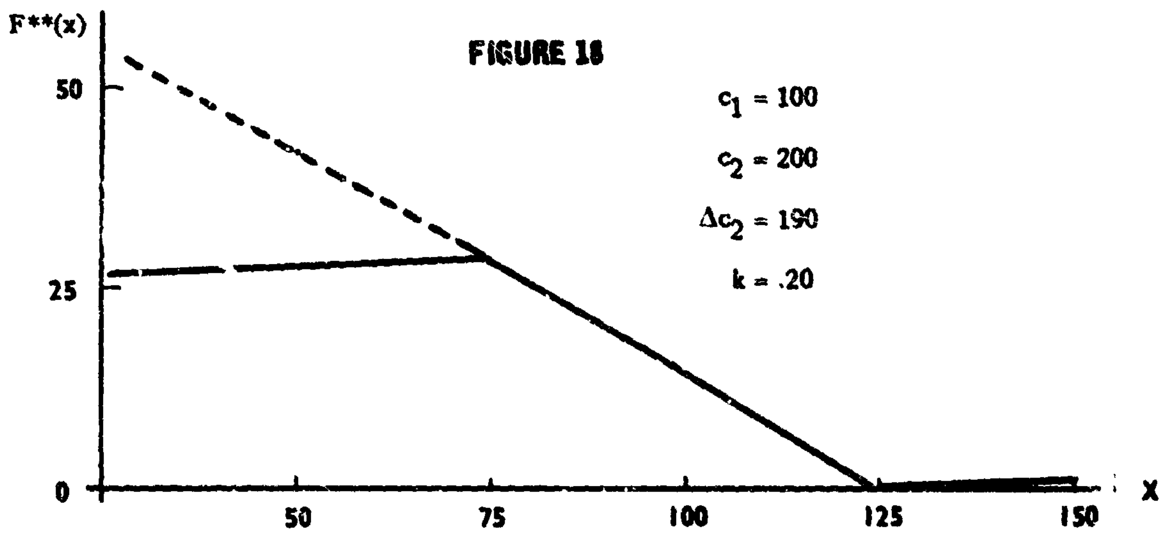

FIGURE 18

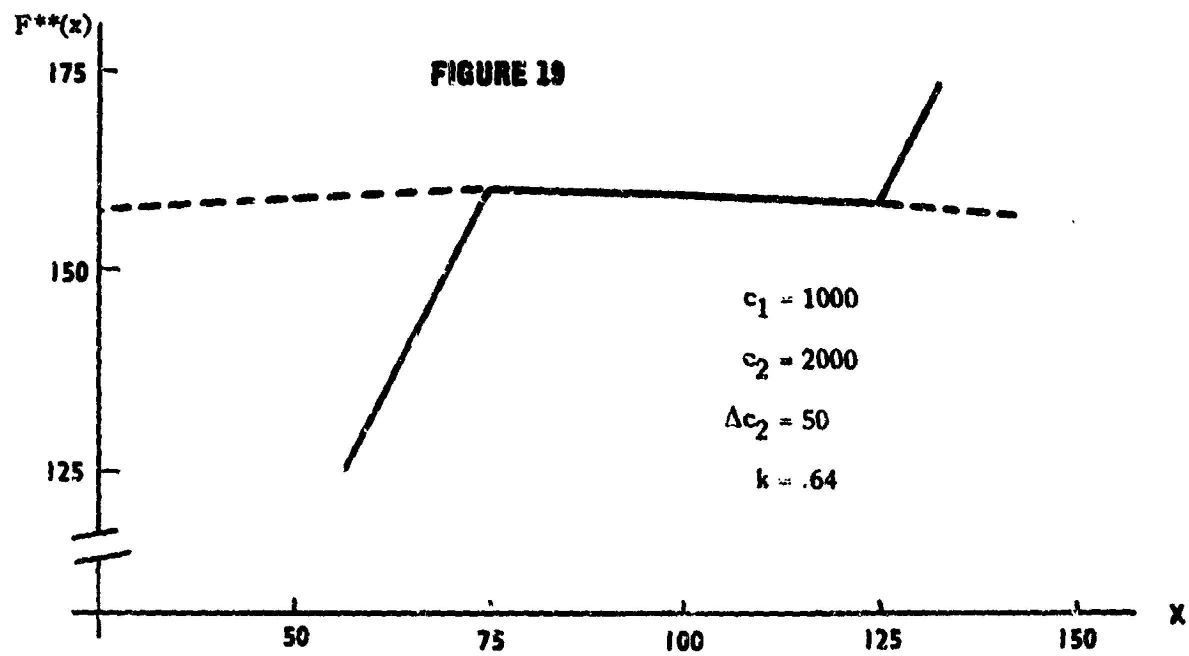

FIGURE 19

## Appendix *

Given:

$$\int_a^b U(x_o - x)\, f(x)\, dx = 0$$

$$U(0) = 0$$

$$U''\ (x_o - x) < 0$$

Show that

$$\int_a^b U[k(x_o - x)]\, f(x)\, dx > 0 \qquad \text{for } 0 < k < 1.0$$

Proof: Since $U''\ (x) < 0$, $U(x)$ is a strictly concave function.

For a strictly concave function we have the following relationship**

$$U(q_1x_1 + q_2x_2) > q_1U(x_1) + q_2U(x_2) \qquad \text{for } x_1 \neq x_2$$

where

$$q_1 + q_2 = 1$$

$$q_1 \geq 0$$

$$q_2 \geq 0$$

Letting

$$0 < k < 1.0$$

$$q_1 = k,\ x_1 = x_o - x$$

$$q_2 = 1 - k,\ x_2 = 0$$

and recalling that $U(0) = 0$

$$U[k(x_o - x)] > kU(x_o - x)$$

Then

$$\int_a^b \{U[k(x_o - x)] - kU(x_o - x)\}\, f(x)\, dx > 0$$

$$\therefore \int_a^b U[k(x_o - x)]\, f(x)\, dx - k\int_a^b [U(x_o - x)]\, f(x)\, dx > 0$$

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* This proof was provided by Professor H.W. Lilliefors, the George Washington University.

** See Hardy, Littlewood, Polya, *Inequalities*, Cambridge University Press, 1934; pp. 73-77.

Since the second term in the above expression was given as equal to zero

$$\therefore \quad H(k) = \int_a^b U[k(x_o - x)]\, f(x)\, dx > 0 \qquad \text{for } 0 < k < 1.0$$

if H(0) = H(1.0) = 0

and H(k) is strictly concave

then H(k) attains a maximum for $0 < k < 1.0$

Proof:

$$\frac{d^2 H(k)}{dk^2} = \int_a^b (x_o - x)^2\, U''\, [k(x_o - x)]\, f(x)\, dx < 0$$

since

$$(x_o - x)^2 > 0$$

$$f(x) > 0$$

$$\text{and } U''\, [k(x_o - x)] < 0$$

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