X-721-66-85

NASA TM X-55463

THE EFFECT OF A FIN TRAILING-EDGE WEDGE ON THE ROLL HISTORY OF A NIKE APACHE

BY

HOWARD L. GALLOWAY, JR.

GPO PRICE $ ______

CFSTI PRICE(S) $ ______

Hard copy (HC) 3.00

Microfiche (MF) .75

ff 653 July 65

FEBRUARY 1966

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-721-66-85

THE EFFECT OF A FIN TRAILING-EDGE WEDGE ON THE ROLL HISTORY OF A NIKE APACHE

Howard L. Galloway Jr.
Sounding Rocket Branch
Spacecraft Integration and Sounding Rocket Division

THE EFFECT OF A FIN TRAILING-EDGE WEDGE ON THE ROLL HISTORY OF A NIKE APACHE

by
Howard L. Galloway Jr.

SUMMARY

The Nike Apache 14.28 GT was launched on February 12, 1964, from Wallops Island, Virginia. The rocket performed a "textbook" flight, following the theoretical trajectory to within 0.6 percent altitude at apogee and 0.3 percent range at impact. The roll rate/time history proved to be typical for Apaches having deflection wedges on the trailing edge of the fins. The plots of roll rate/time and pitching frequency/time cross rapidly early in flight, where restoring moments are high, allowing a stable flight to occur.

CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

THE EFFECT OF A FIN TRAILING-EDGE WEDGE

ON THE ROLL HISTORY OF A NIKE APACHE

by
Howard L. Galloway Jr.

ABSTRACT

When roll-time histories of missiles carrying fin wedges revealed an unexpected relationship of roll rate as a function of velocity, a heavyweight fin with trailing-edge wedge was tested for greater aeroload resistance. The fins were thermally protected. The rocket carrying these fins achieved a flight close to the theoretical trajectory. The flight was stable and the resulting roll-rate history was what is now known to be the prescribed roll-rate history. The report includes the method of calculating Apache natural pitching frequency, as well as methods for avoiding pitch-roll lock-in.

ABBREVIATIONS AND SYMBOLS

LaRC	Langley Research Center	
GSFC	Goddard Space Flight Center	
θ	Displacement Angle of Free Oscillating Balance Wheel	radians
ϕ_{input}	Displacement Angle of Shaft of Forced Oscillating Balance Wheel	radians
ϕ_{output}	Displacement Angle of Wheel of Forced Oscillating Balance Wheel	radians
p	Input Frequency to Oscillating System Roll Rate of Rocket	CPS CPS
ω	Natural Frequency of Oscillating System	CPS
n	Damping Constant	CPS
MF	Magnification Factor = $\text{Amplitude}_{output}$ / Amplitude_{input}	
CP	Location of Center-of-Pressure Forward of Nozzle Exit Plane	feet
CG	Location of Center-of-Gravity Forward of Nozzle Exit Plane	feet
F_N	Normal Force Acting at CP	pounds
α	Angle of Attack	radians
β	Angle of Sideslip	radians
$\bar{\alpha}$	Complex Angle of Attack = $\beta + i\alpha$	radians
$\bar{\alpha}_{trim}$	Trim Complex Angle of Attack $\beta_{trim} + i\alpha_{trim}$	radians
m	Mass	slugs
M	Restoring Moment Acting at CP about CG	foot-pounds
M_α	Slope of Moment/Angle of Attack Curve at $\alpha = 0$	ft-lbs/radians
$(I_P)_X$	Pitching Moment-of-Inertia About Point X	slug-feet^2
$(I_P)_{CG}$	Pitching Moment-of-Inertia About Center-of-Gravity	slug-feet^2
SM	Static Margin = (CG - CP)/d	

d	Apache Diameter = 0.542 ft	feet
A	Apache Cross Section Area $= \pi d^2/4 = 0.23\ ft^2$	$feet^2$
V	Velocity of Apache	feet/sec
ρ	Atmospheric Density	$slugs/feet^3$
CPS	Cycles Per Second	
GE MASS program	General Electric Missile and Satellite Systems program	
q	Dynamic Pressure $= \rho V^2/2$	lb/ft^2
W_S	Weight of Sustainer	pounds
W_P	Weight of Propellent	pounds
$W_{P/L}$	Weight of Payload	pounds
W_T	Weight of Total Vehicle	pounds
NEP	Nozzle Exit Plane	

THE EFFECT OF A FIN TRAILING-EDGE WEDGE ON THE ROLL HISTORY OF A NIKE APACHE

Howard L. Galloway Jr.

INTRODUCTION

When the roll rate of a missile reaches a value near its natural pitching frequency, any pitching motion is magnified. The resulting coning motion, if occurring in the atmosphere, will at least degrade performance and may even cause break-up of the vehicle.

The natural pitching frequency of a sounding rocket is easily calculated as a function of time. The roll-rate/time history of the rocket must be selected so that (1) early in flight, while restoring moments are large, the roll-rate/time curve crosses the natural pitching-frequency/time curve as rapidly as possible (minimum time at same frequency), or (2) the roll-rate/time curve stays <u>below</u> the natural pitching-frequency curve until after burnout. The latter choice will result in pitch-roll coupling and, consequently, poor altitude of the Apache.

Roll-rate/time histories of Apaches using deflection wedges on the trailing edge of the fins revealed a completely unexpected result: The roll rate/time curve was contrary to the expected curve, that obtained for a canted fin. Nike Apache 14.28 GT was flown to better understand this phenomenon and to validate the theories put forth to explain this roll behavior.

HISTORY

Until recently, the roll rate of a missile bearing fin wedges was believed to be, like that of the canted-fin missile, directly proportional to the velocity of the missile.

Increasing experience with roll-time histories has shown that such rockets were not flying the expected roll history, but were performing in almost the opposite manner.

The Nike Apache sounding rocket used in this study has the following flight history: The Nike fires for 3.5 sec. At this time, the Nike Apache is about 1 statute mile high and travelling at about a Mach number of 3. The relative high drag on the Nike causes it to separate from the Apache. Apache ignition occurs 20 seconds after launch, at an altitude of about 8 statute miles and a Mach number of 1.5.

After firing for 6.4 seconds, the Apache reaches an altitude between 56,000—70,000 ft with a Mach number of between 5.1 and 6.5. Depending on the launch angle, drag configuration, and payload weight, the Apache then coasts to an apogee of 368,000—710,000 ft.

Starting at launch, the roll rate increased rapidly, peaking at about Nike burnout (Figure 1). This was expected. Then, as the forward velocity of the Apache decreased, the roll rate increased again, peaking at Apache ignition. This was not expected. As the Apache velocity increased during burning, the roll rate decreased until, just before Apache burnout (when the roll rate was about two-fifths its maximum value), it once again started to climb, reaching a plateau at about half the peak value.

Because this was unexplainable, and because it was really believed that the fin with trailing-edge wedge should behave like the well understood canted fin, theories were immediately put forth as to the cause of the failure to obtain expected roll histories.

Most experts thought that the problem arose from a physical change in the fins (warping under load, twisting under aerothermal stress, aeroelastic effects, etc.*).

Thus, it was decided to fly a test rocket, Nike Apache 14.28 GT, with fins sufficiently sturdy to resist any potential aeroloads, and protected against high temperature by a coating of Thermo-Lag** to keep their strength high.

The Test Rocket

Nike Apache 14.28 GT was flown from Wallops Island on February 12, 1964, with the cooperation of Langley Research Center. LaRC supplied the Apache motor and Magline fins; GSFC supplied the Nike, the payload, and associated hardware.

*See Sounding Rocket Branch request for Project Initiation 14.28-14.29 GT, Appendix A.
**Proprietary product of Emerson Electric of St. Louis (Mo.)

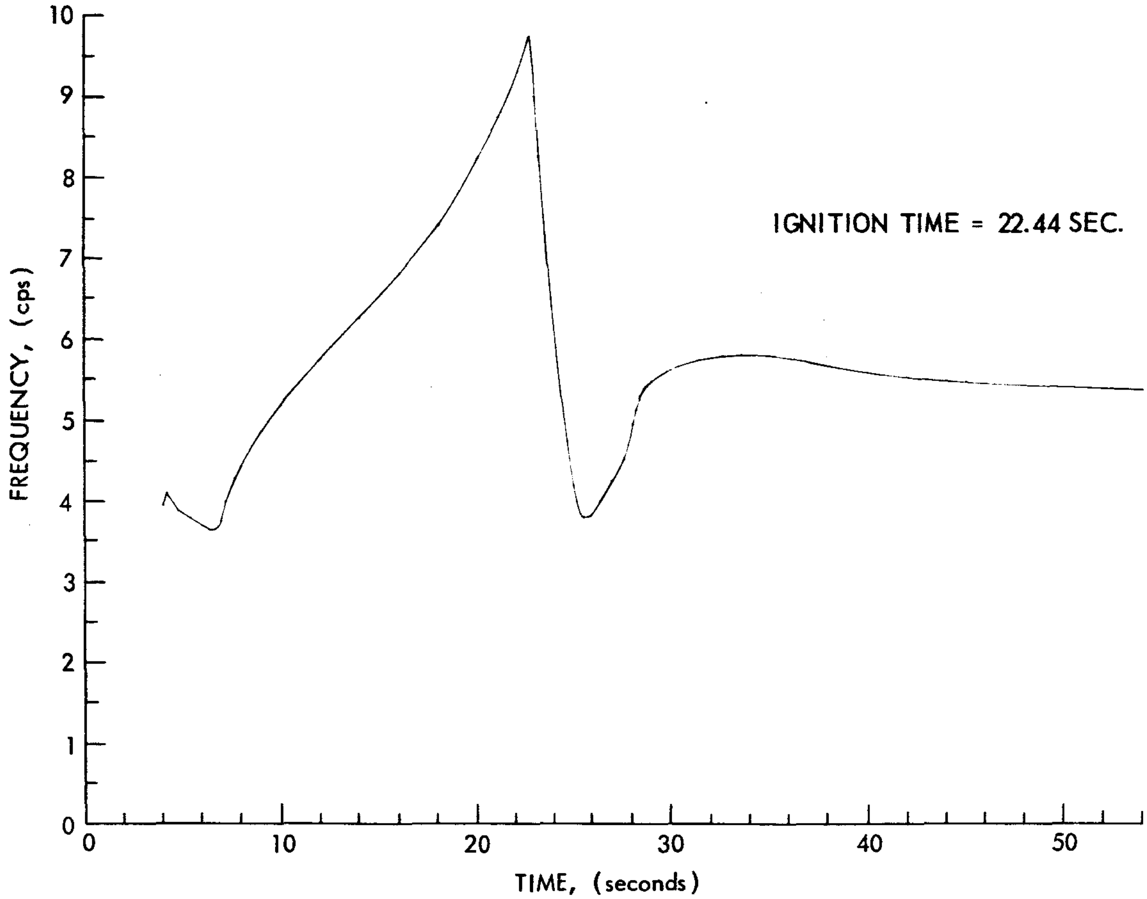


Figure 1 – Roll-Rate/Time History, Nike Apache

The first stage of the 14.28 GT (Figure 2) was a standard Nike M5-E1. The second stage (Figure 3) consisted of:

- A standard Apache motor
- A Magline hinged-fin assembly
- Two body shrouds 180 degrees apart in line with fins
- Four 45-degree turnstile antennas in line with fins
- A standard 11-degree nosecone and cylinder
- A pitch yaw ogive tip

Figure 4 shows details of each of the four turnstiles.

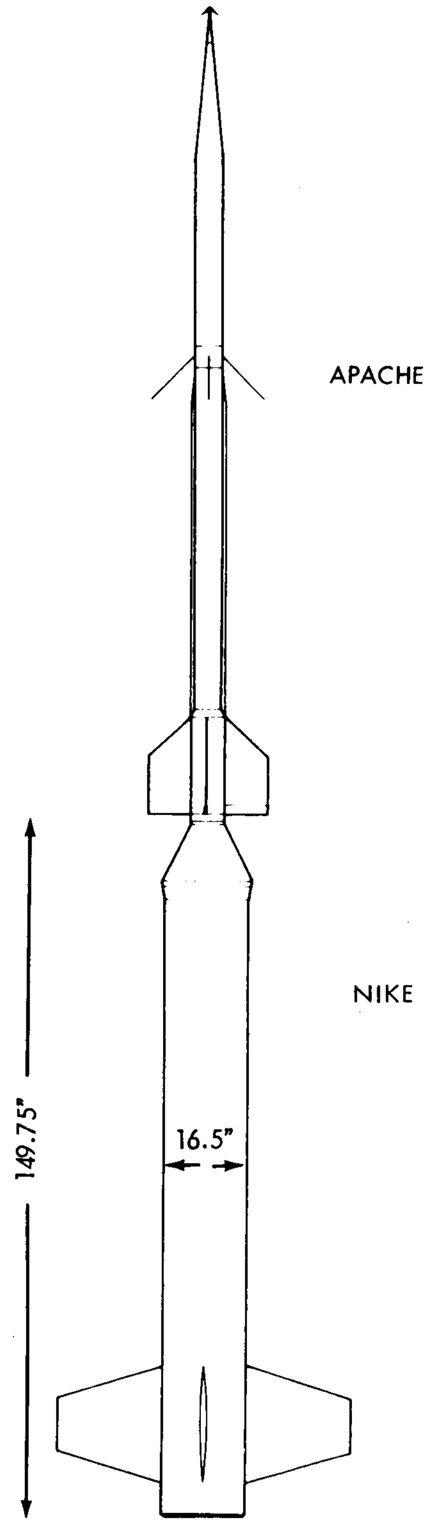


Figure 2 – Nike Apache 14.28 GT

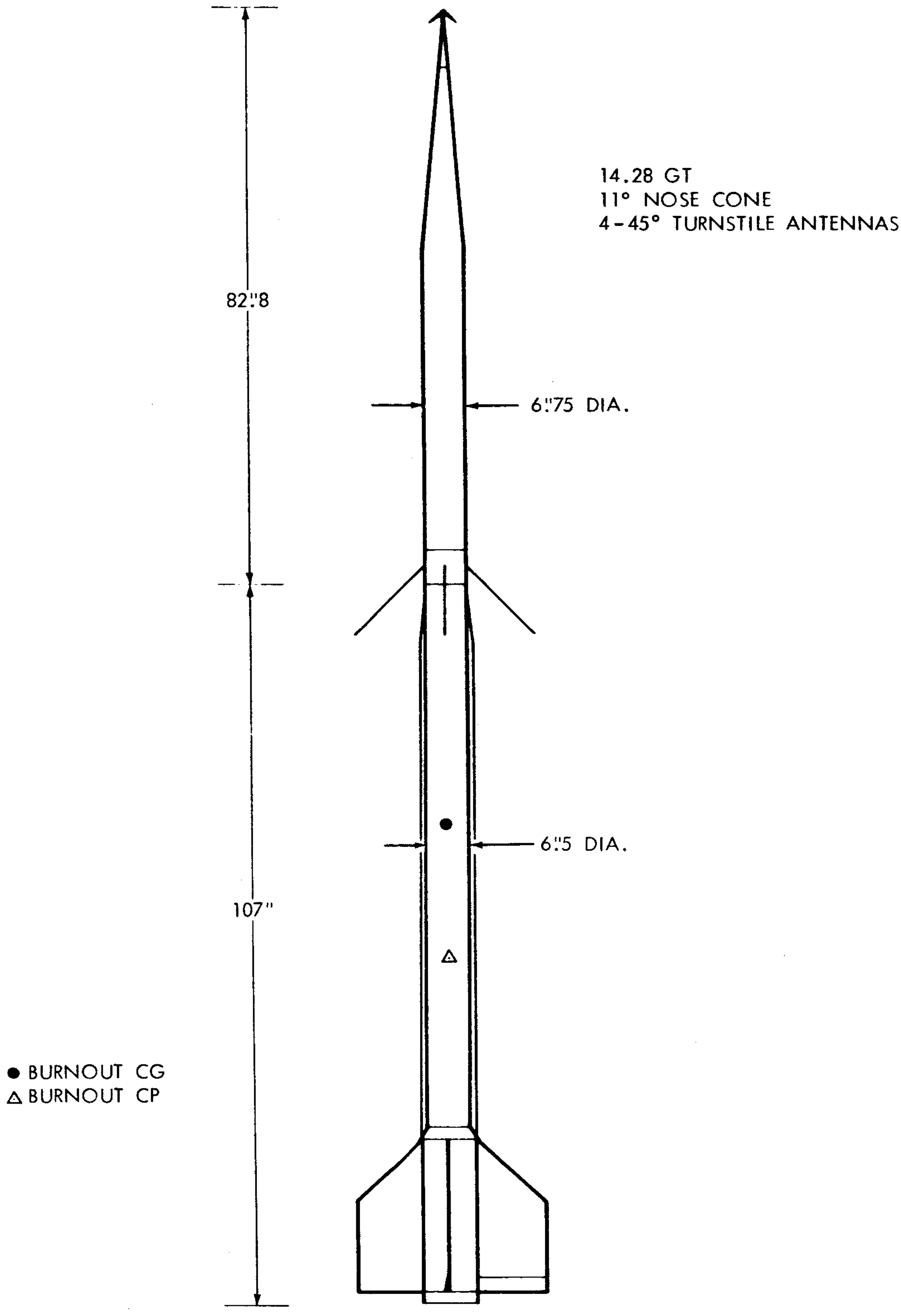


Figure 3 – Apache Second Stage, 14.28 GT

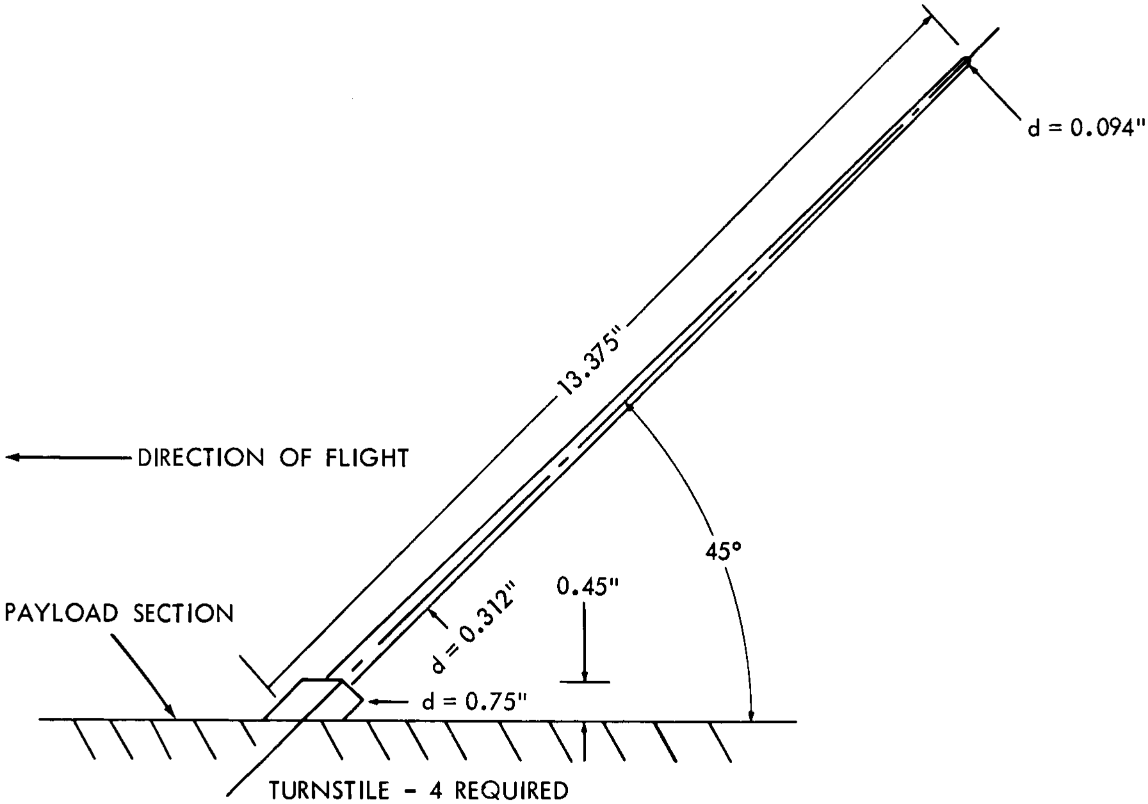


Figure 4 – Detail of Turnstile Antennas, Nike Apache 14.28 GT

Figure 5 shows the completed Nike Apache, mounted on the launch rail and ready for firing.

Effective payload weight (total second-stage weight, less the weight of a GSFC standard Apache) was 68 lb; its length was 80 inches plus pitch yaw tip.

The 14.28 GT differed from a standard Apache in the following respects:

- Heavy Magline hinged fin assembly replaced lighter Atlantic Research Corporation fin assembly normally used.
- Thermo-Lag was coated on the fins.
- Two body shrouds were run the length of the Apache motor to protect the temperature leads.

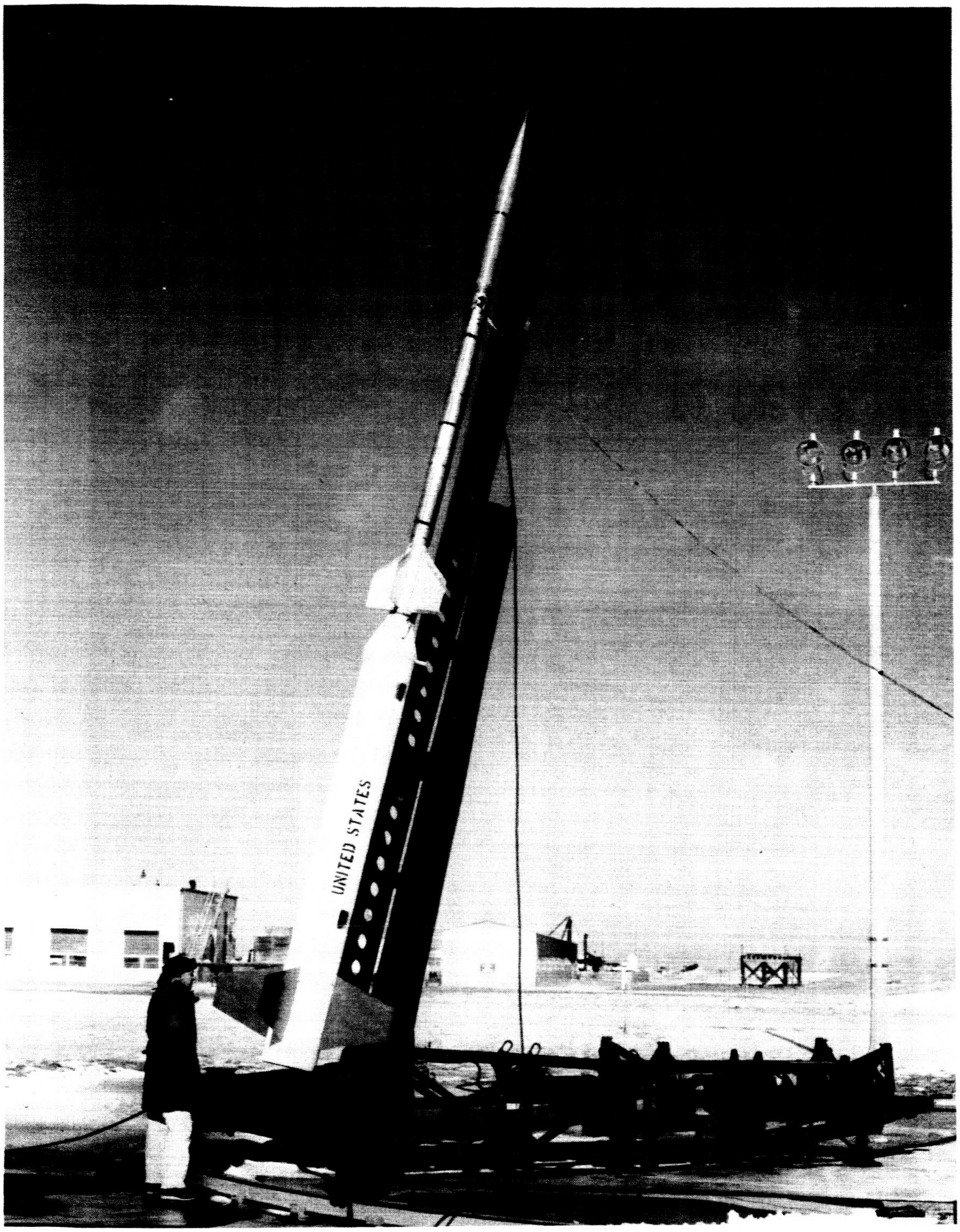


Figure 5 – Nike Apache on Launcher

Results of Flight

The 14.28 GT followed the theoretical trajectory to within 0.6 percent of the theoretical altitude at apogee and to within 0.3 percent of range at impact. The flight was stable, as evidenced by the complex angle-of-attack history (Figure 6).

The roll-rate/time history of this Apache with the sturdy fins was the same as the roll history of a standard Apache (Figure 1). In fact, the roll-rate history was what is now known to be the prescribed roll rate.*

Figure 7 shows that the roll history of 14.28 GT approximated that of curve C described in the following discussion of resonance. The roll acceleration was large, and the roll-time curve rapidly crossed the natural pitching-frequency curve.

The crossing occurred at 7 seconds, and the complex angle-of-attack/time curve shown in Figure 6 reveals that a slight oscillation did occur there. This motion, however, damped out.

DISCUSSION

Resonance

If a rocket, flying at some complex angle-of-attack (Figure 8), experiences a roll rate at or near its natural pitching frequency, this complex angle-of-attack will increase with time. This phenomenon is known as resonance.

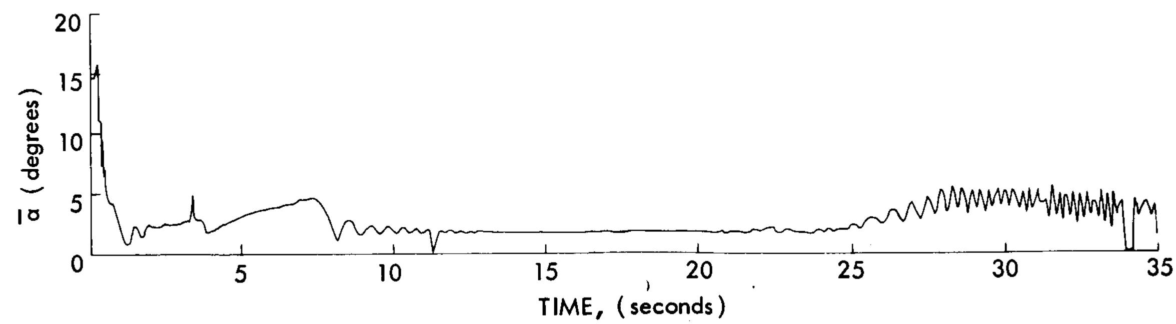


Figure 6 – Angle-of-Attack History, 14.28 GT

*See Roger J. Hawks, "A Theoretical Investigation of the Rolling Motion of the Nike Cajun and Nike Apache Rocket Vehicles," GSFC X-721-65-335.

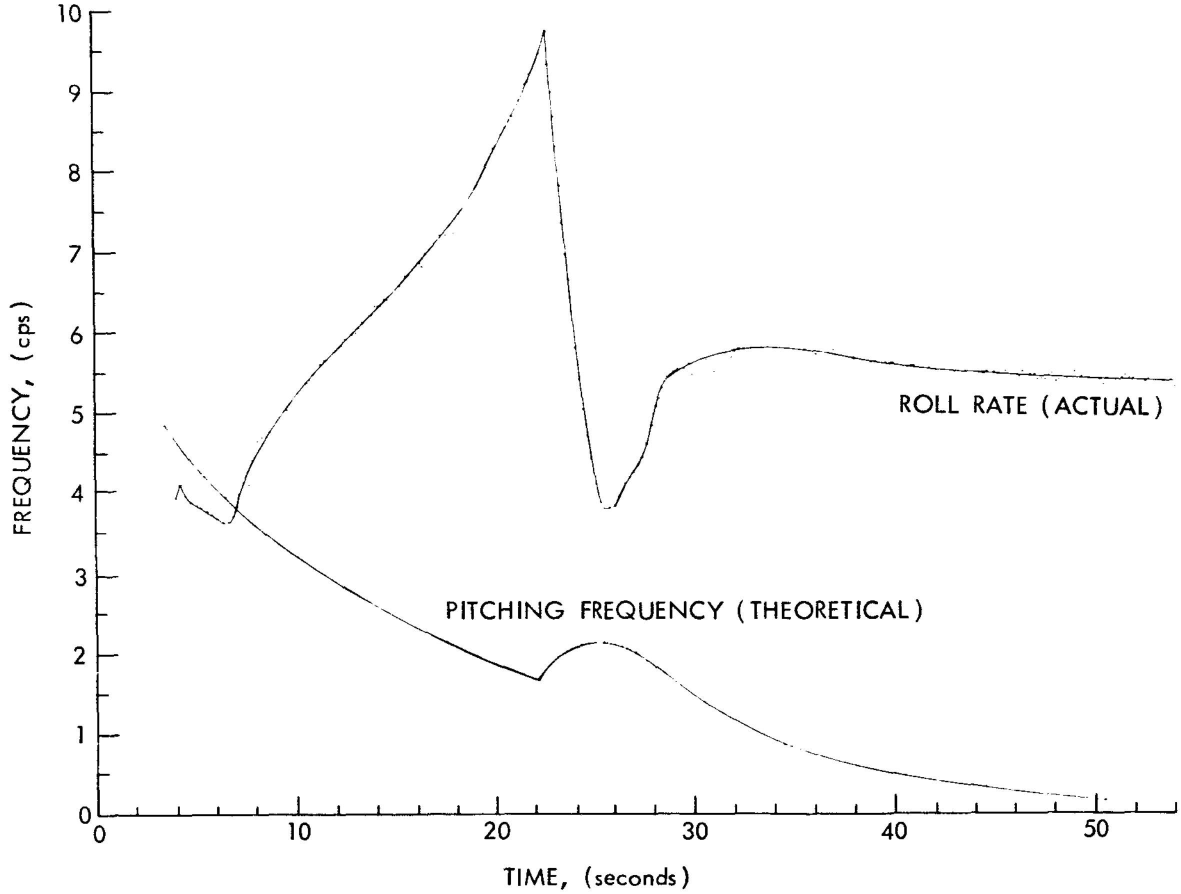
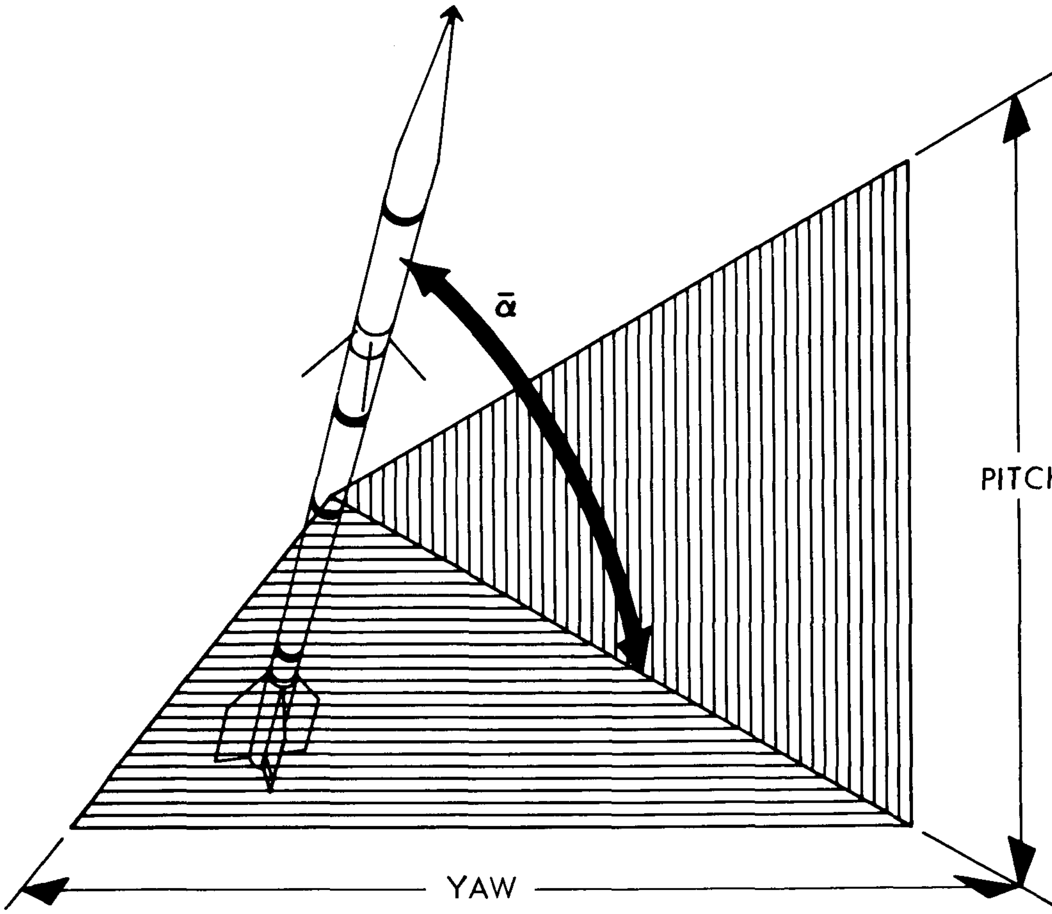


Figure 7 – Roll History, 14.28 GT

A simple example shows the importance of resonance. Consider the motion of a wheel of mass m, connected to a shaft by a spirally wound spring (Figure 9a). If the wheel is rotated through a displacement angle, θ_0, with respect to the shaft, energy is stored in the spring and a restoring moment is felt by the wheel. If the wheel is released, it starts back to its neutral position, the restoring moment decreasing with decreasing displacement angle until, when $\theta = 0$, the restoring moment also equals zero. However, inertia carries the wheel along, winding the spring in the opposite direction. A restoring moment now appears in the direction opposite to the first one. This stops the wheel at a displacement angle $-\theta_1$, where $|-\theta_1| \leq |\theta_0|$. If there is zero damping, $|-\theta_1| = |\theta_0|$. If damping is present, of course $|-\theta_1| < |\theta_0|$. The wheel now will return through $\theta = 0$ to θ_2, where $|\theta_2| \leq |-\theta_1|$. This oscillatory motion has a natural frequency $= \omega$ and is a function of the inertia of the wheel and the restoring moment of the spring.

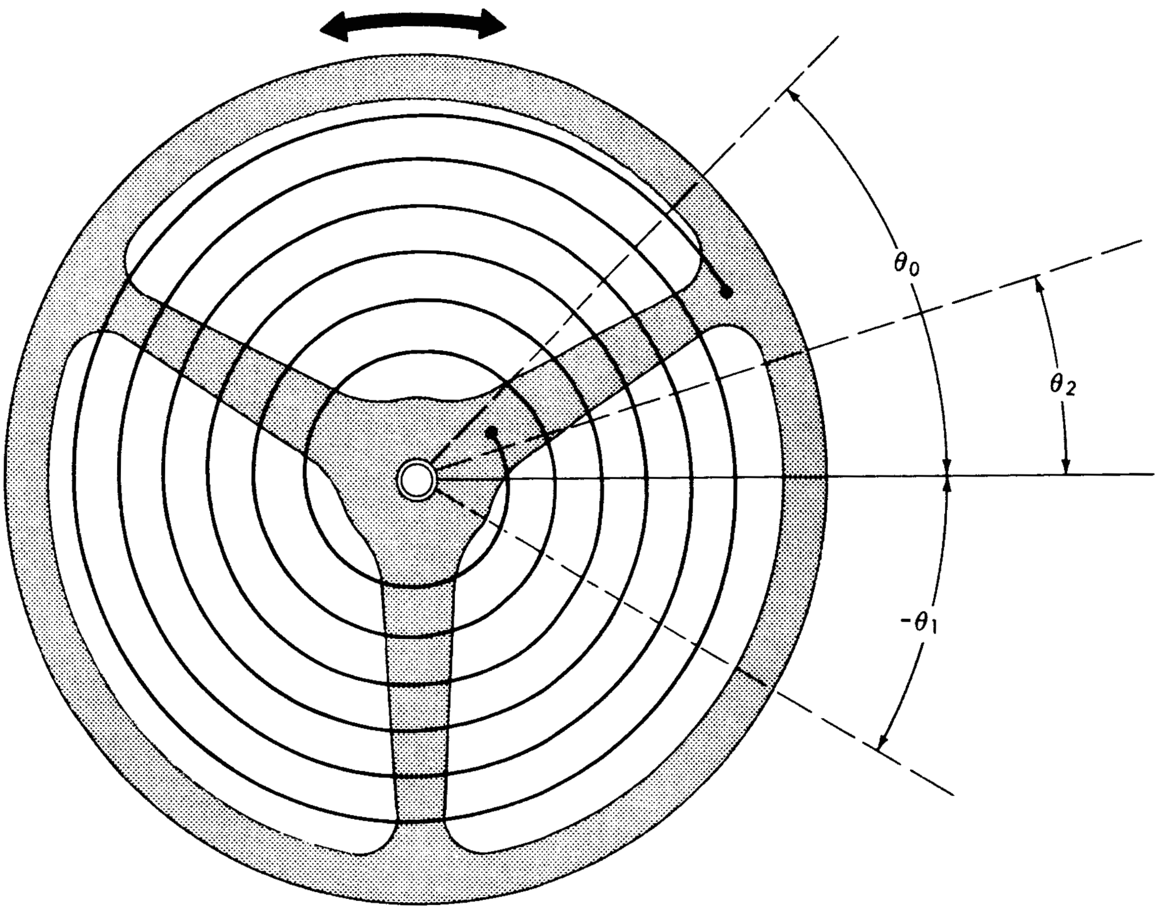


$\bar{\alpha}$ = COMPLEX ANGLE OF ATTACK
= $\beta + i\alpha$

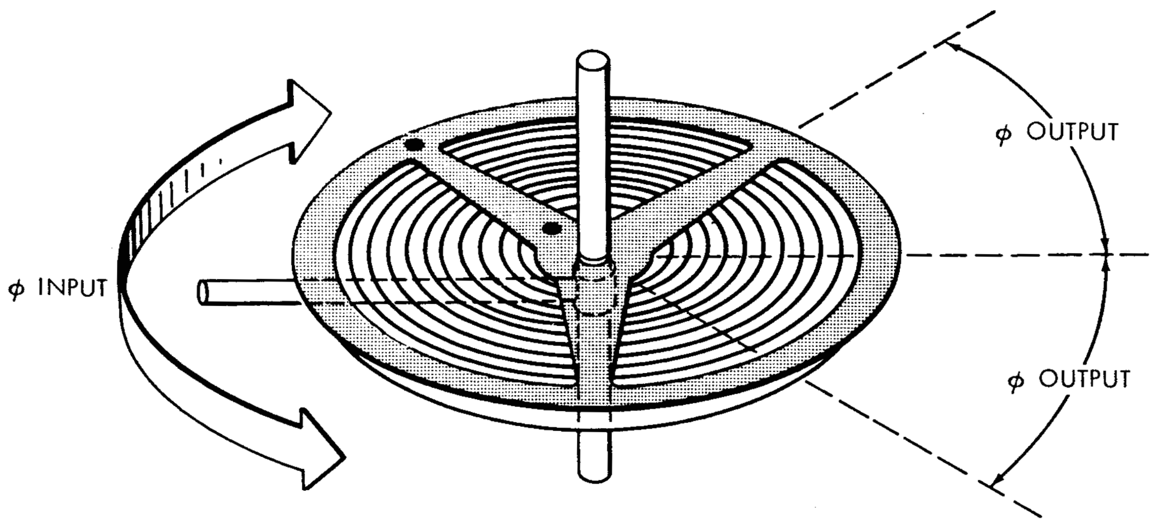
Figure 8 – Rocket at Complex Angle-of-Attack, $\bar{\alpha}$

The motion just described is free oscillatory motion. Consider now forced oscillatory motion. If the shaft is caused to oscillate through some angle $\pm\phi_{input}$, the wheel will oscillate through some angle $\pm\phi_{output}$ (Figure 9b). The ratio of ϕ_{output} to ϕ_{input} is defined as the magnification factor, MF, where

$$MF = \frac{\phi_{output}}{\phi_{input}} = \left| \frac{1}{\sqrt{\left[1 - \left(\frac{p}{\omega}\right)^2\right]^2 + \left(\frac{2n}{\omega}\right)^2 \left(\frac{p}{\omega}\right)^2}} \right| \tag{1}$$



(a) Free Oscillatory Motion



(b) Forced Oscillatory Motion

Figure 9 – Resonance Model: Wheel and Spring

where

p = input circular frequency to shaft

ω = natural circular frequency of system

n = damping constant

Figure 10 is a plot of this equation.

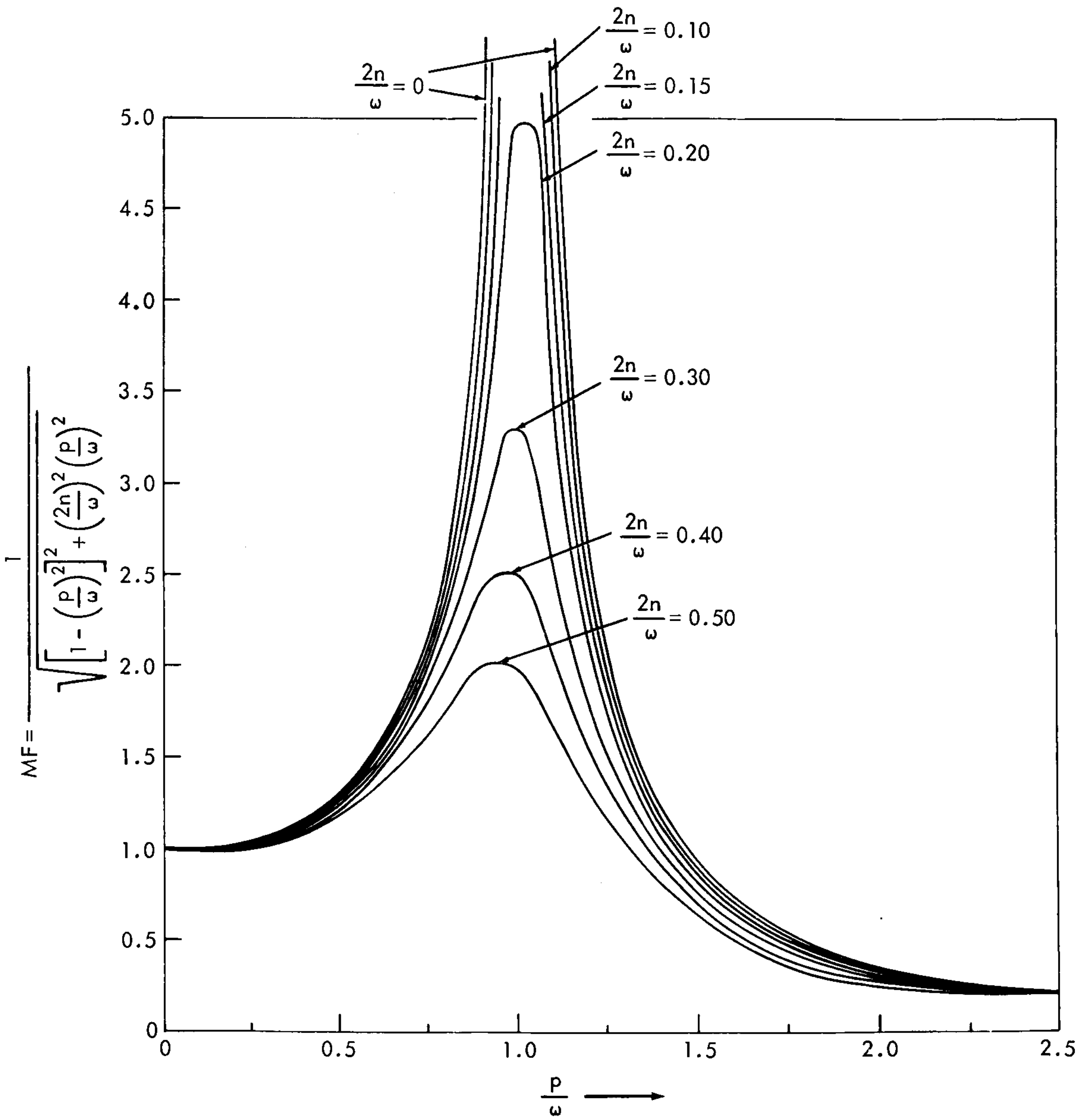


Figure 10 – Plot of Equation, Magnification Factor

The magnification factor becomes a maximum when $p^2/\omega^2 = 1-2n^2/\omega^2$. Since n is usually small compared to ω, the maximum magnification factor usually occurs when p/ω is about unity.

For the case of small damping, i.e. $(2n/\omega)^2 << 1$, Equation (1) reduces to

$$MF = \frac{\Phi_{output}}{\Phi_{input}} = \left| \frac{1}{1 - \left(\frac{p}{\omega}\right)^2} \right| \tag{2}$$

The characteristics of the spring-wheel system may be determined by examining Equation 2 or Figure 10. When the input frequency p is very small compared to the natural frequency of the system (i.e., $p/\omega << 1$), then $\Phi_{output} \approx \Phi_{input}$ and the wheel rotates as though the spring were rigid. However, as p increases, the magnification factor grows, until, when the input frequency equals the natural frequency, the theoretical maximum output angle tends to approach infinity. In a real case, where damping is present, $\Phi_{output}/\Phi_{input}$ has a finite value, which, however, may be very large.

As the input frequency is increased beyond the natural frequency, MF decreases. When $p/\omega = \sqrt{2}$, MF = 1, and once again the spring acts as though it were rigid. As p increases beyond $(\sqrt{2})(\omega)$, the MF decreases rapidly and (since it is less than 1) shows that attenuation occurs above $p/\omega = \sqrt{2}$. In fact, as $p \to \infty$, the wheel cannot follow and remains motionless.

Resonance of a Rocket

Consider a rocket moving through the atmosphere. (In the model already described, replace the wheel by the rocket, as shown in Figure 11). An oscillatory motion applied to the shaft will cause a pitching motion in the rocket. When a rocket moving through the atmosphere is at some angle-of-attack, a normal restoring force, F_N, will appear at the rocket's center of pressure, CP (Figure 12). The rocket oscillates about its center-of-gravity, CG, and this restoring force, acting on the lever arm (CG-CP), produces a restoring moment, $M = F_N$ (CG-CP). This restoring moment may now be considered to be acting instead of the spring (Figure 13).

A missile with some assymetries (thrust, mass, configurational, etc.) will fly at some trim complex angle-of-attack, $\bar{\alpha}_{trim} = \beta_{trim} + i(\alpha)_{trim}$. If this missile is now caused to roll, this complex angle-of-attack will rotate about the velocity vector; that is, the body axis will rotate about the velocity vector. This motion is called coning (Figure 14) and is an example of lunar motion where

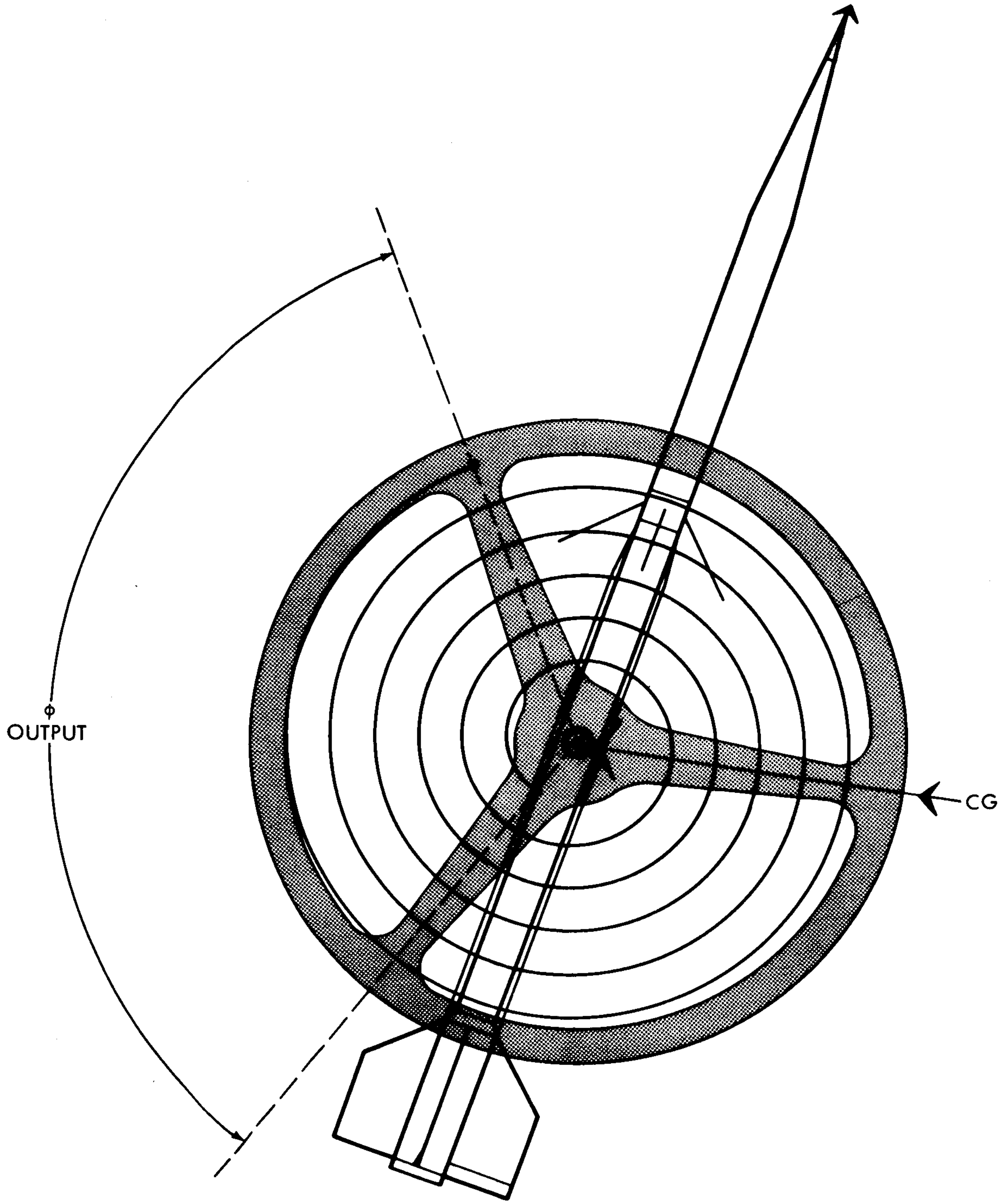


Figure 11 – Resonance Model: Rocket in Atmosphere

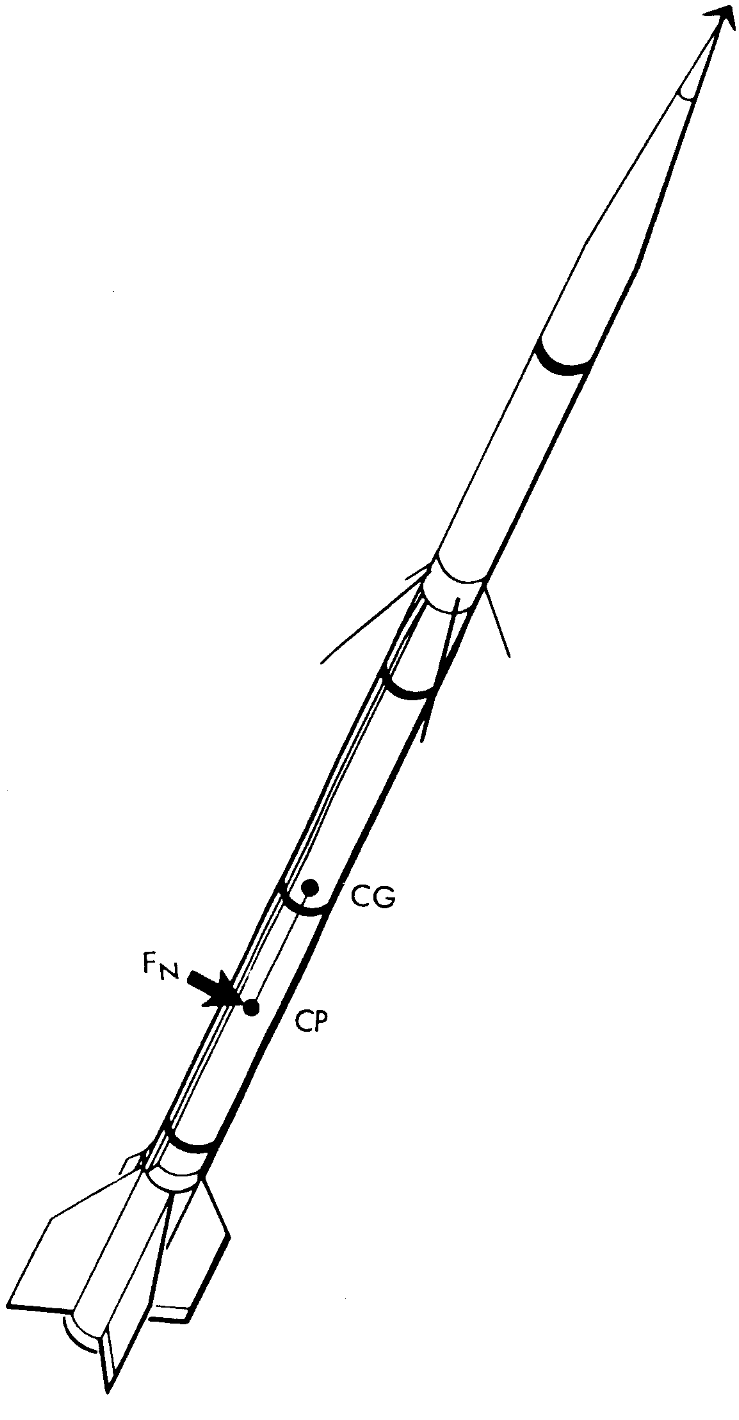
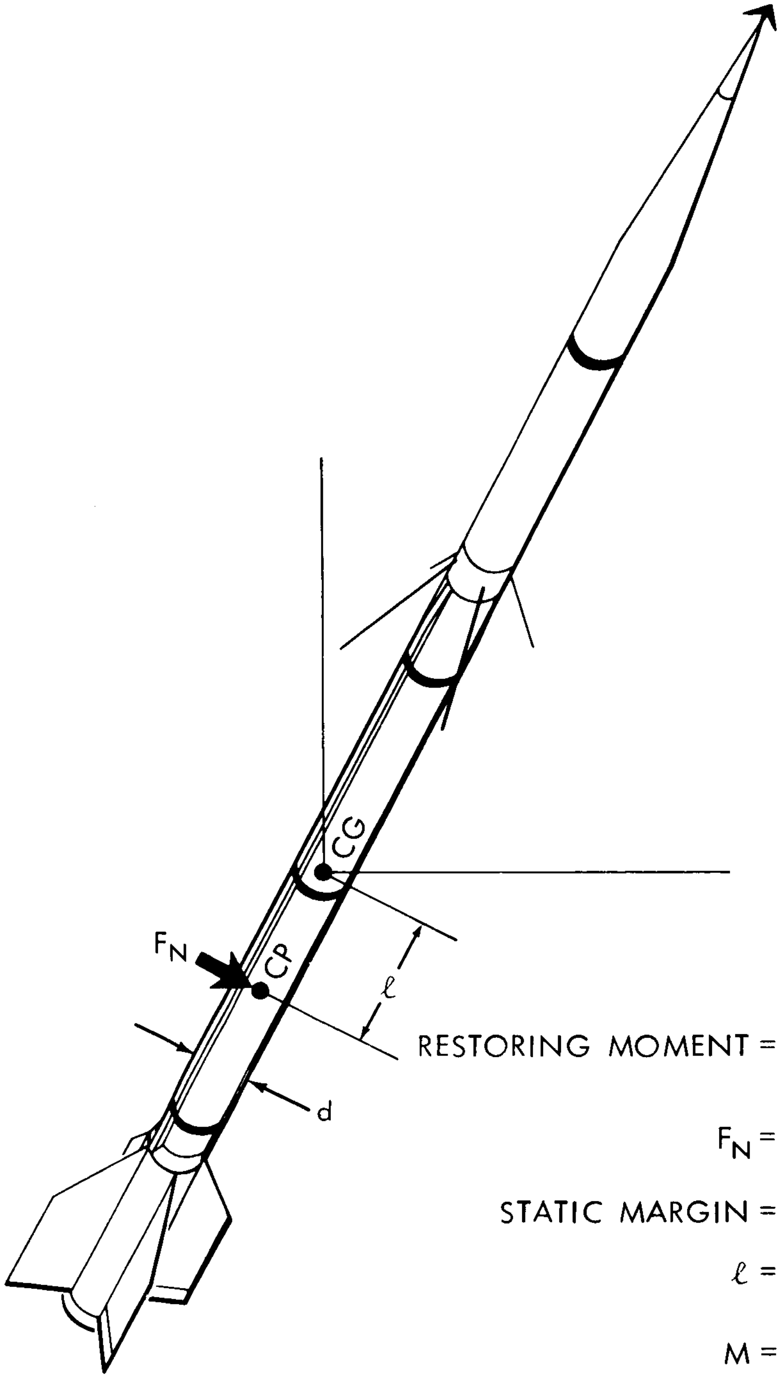


Figure 12 – Restoring Force at Rocket Center of Pressure



$$\text{RESTORING MOMENT} = M = \left[F_N\right](\ell)$$

$$F_N = C_{N_\alpha}\,\alpha\,\frac{\rho V^2}{2}\,A$$

$$\text{STATIC MARGIN} = SM = \frac{CG - CP}{d}$$

$$\ell = (SM)\,d$$

$$M = \left[C_{N_\alpha}\,\frac{\alpha \rho V^2}{2}\,A\right](SMd)$$

$$\text{WHERE } A = \frac{\pi d^2}{4}$$

Figure 13 – Resonance Model: Rocket with Restoring Moment

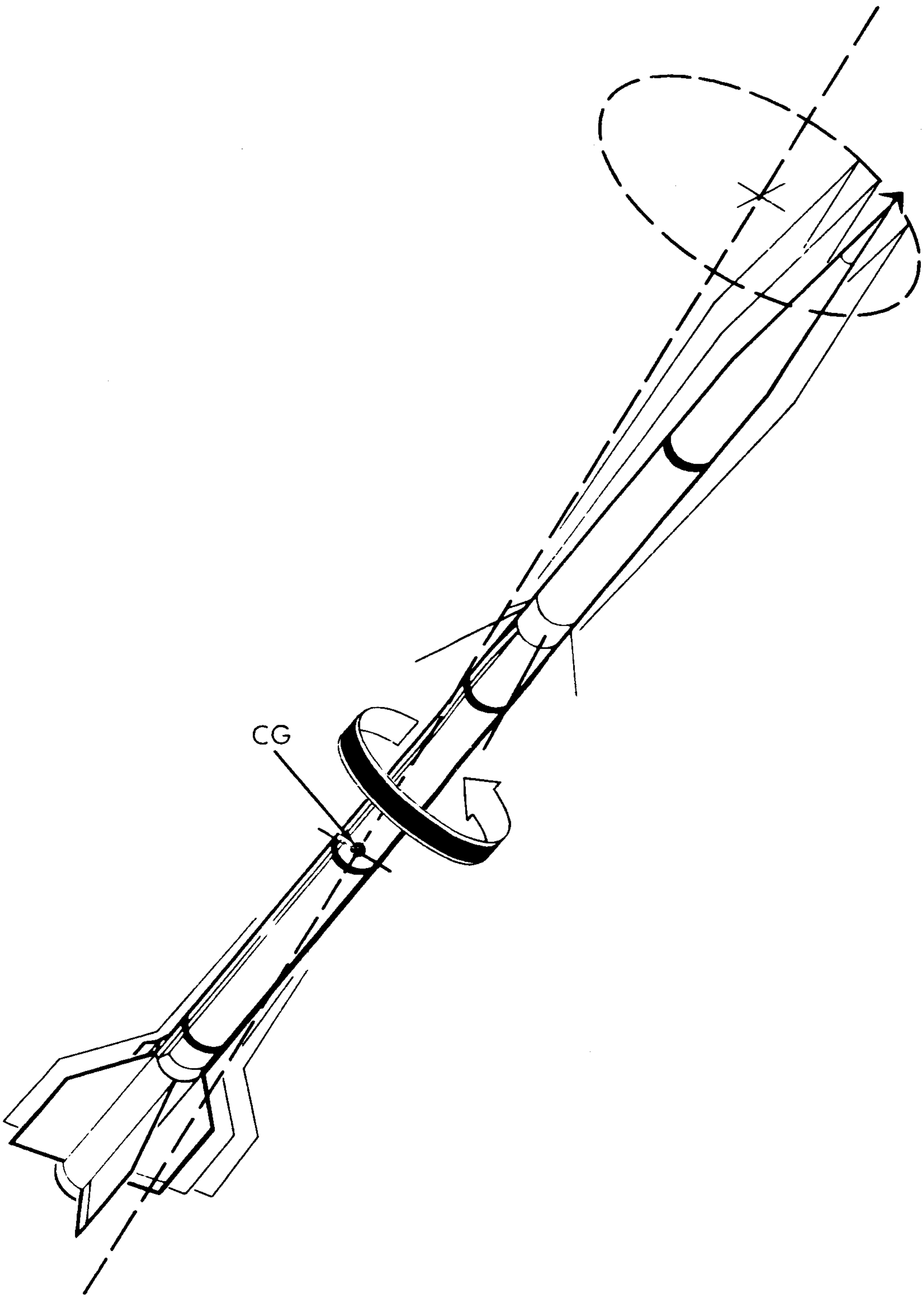


Figure 14 – Coning of Rocket in Flight

one side of the rocket always faces the velocity vector. This oscillatory pitching motion is the forcing function which replaces the shaft in Figure 11.

Thus, a free rocket moving through the atmosphere is analogous to the spring-wheel system where, in each case, a forcing function works on a system that contains inertia, a restoring moment, and damping.

Since the trim angle rotates at the roll rate, the input frequency p = roll rate. The rocket system possesses a natural frequency = ω. Consequently, the magnification factor/frequency ratio curve and its analysis apply to the rocket system, where MF = $\left(\bar{\alpha}_{trim}\right)_{roll} / \left(\bar{\alpha}_{trim}\right)_{no\ roll}$.

To restrict the resulting pitching motion to small amplitudes, where pitching magnitude

$$\left(\bar{\alpha}_{trim}\right)_{roll} = MF\left(\bar{\alpha}_{trim}\right)_{no\ roll} ,$$

MF must be kept small. That is to say, p should at all times be different from ω. Or, never roll the missile at the natural pitching frequency.

In sounding rockets the MF is usually of the order of 15 or greater. Thus, a rocket with a no-roll trim angle-of-attack of 2 degrees would, at resonance, have a trim angle of 30 degrees or more.

The natural pitching frequency of a sounding rocket is a function of (1) the slope of the restoring-moment/angle-of-attack curve, and (2) its moment-of-inertia:

$$\omega = \sqrt{\frac{M_\alpha}{\left(I_p\right)_{CG}}}$$

where

$$M_\alpha = C_{N_\alpha} SM \frac{\rho V^2}{2} A d$$

where

C_{N_α} = Normal Force Coefficient/Angle of Attack Slope at $\alpha = 0$

SM = Static Margin = (CG - CP)/d

ρ = Ambient Atmospheric Density

V = |Velocity| of Rocket

A = Reference Area of Rocket

d = Reference Diameter of Rocket

$(I_p)_{CG}$ = Pitching Moment-of-Inertia of Rocket about Center-of-Gravity

These factors are constantly changing in the near-vertical flight of a sounding rocket. Hence, the density ρ decreases above sea level; the velocity and the Mach number decrease after burnout; and the C_{N_α} increases with decreasing Mach number. The natural pitching frequency continually changes with time. Figure 15 is a typical pitching-frequency time curve.

Curves a, b, and c of Figure 16 represent three of many possible roll histories.

Curve a: Here the missile is decelerating in roll. The roll rate and the natural frequency coincide so that the ratio $p/\omega = 1$ for a time. During this time, the MF takes effect, overcoming inertia of the missile and allowing the coning motion to grow. Sometimes the rocket "locks in"; that is, the rolling frequency equals the natural pitching frequency for a long time. During this time, the coning angle grows, staying at $(\bar{\alpha}_{trim})_{roll} = MF(\bar{\alpha}_{trim})_{no\ roll}$. As the

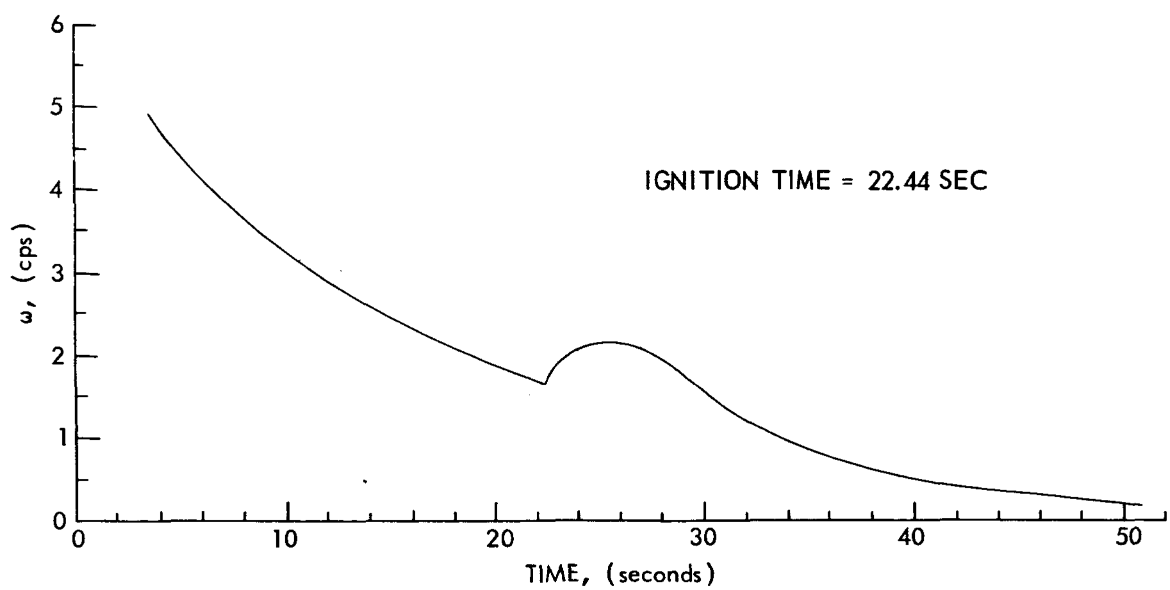


Figure 15 – Typical Natural Pitching-Frequency/Time Curve

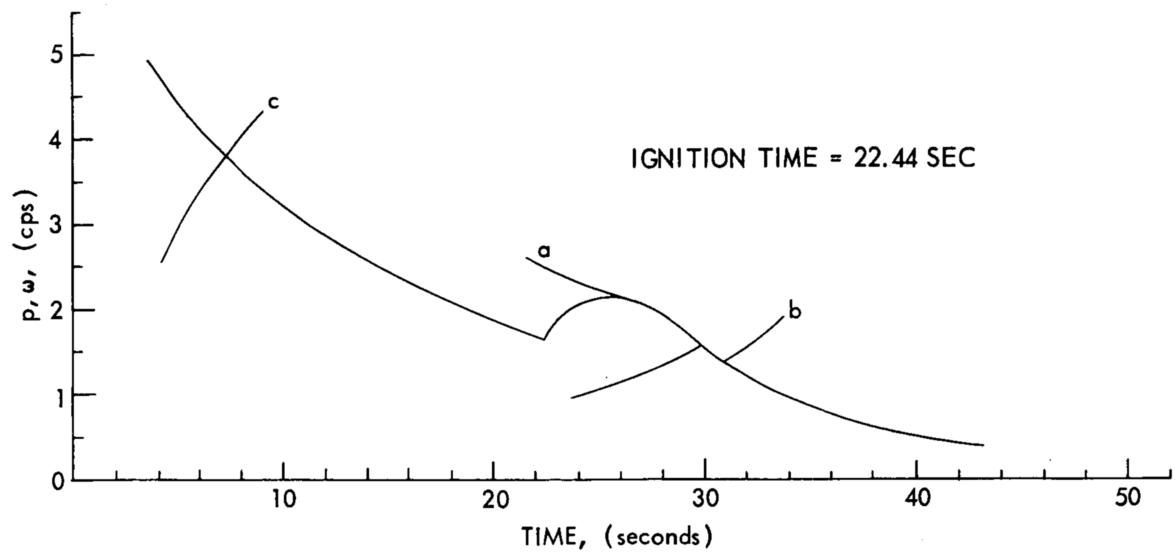


Figure 16 – Possible Roll Histories Shown with Typical Natural Pitching-Frequency/Time Curve

rocket goes higher, the damping grows smaller with decreasing density of the atmosphere. Figure 10 shows that, for decreased damping, $(MF)_{max}$ increases. Thus, for a locked-in rocket leaving the atmosphere, the motion will grow into large-amplitude coning.

Curve b: Here the missile is accelerating in roll in a region where the natural pitching frequency is decreasing. Although the motion starts to lock in, the moments inducing roll are great enough to allow the missile to break out of lock-in by increasing p. Then the ratio p/ω suddenly increases beyond 1, and the MF decreases suddenly. If the missile is dynamically stable, the increased coning motion it gains while being locked in will soon damp out.

Curve c: Here, the rocket's roll acceleration is so high that it passes quickly through the resonance point.

To ensure safe passage through the resonance point, the ratio p/ω should grow rapidly.

Coning motion occurring early enough in flight will cause, at least, a decrease in performance because of increase in drag; at worst, it may cause vehicle breakup because of high aerodynamic loading. This type of motion is usually undesirable and roll history of types a and b usually should be avoided. Ideally, the roll history of Apache-type sounding rockets should cross the natural pitching-frequency curve in the manner of curve c (above) and remain above the pitching curve.

To review the desirability of such a roll history:

- The ratio p/ω increases rapidly; i.e., the roll frequency moves rapidly away from the natural pitching frequency, thus reducing the magnification factor (Figure 10).
- Early-in-flight restoring moments are large, increasing static stability.
- Early-in-flight damping forces are large, increasing dynamic stability.

To prevent lock-in, the burnout roll rate of the Apache should be above 5 cps.

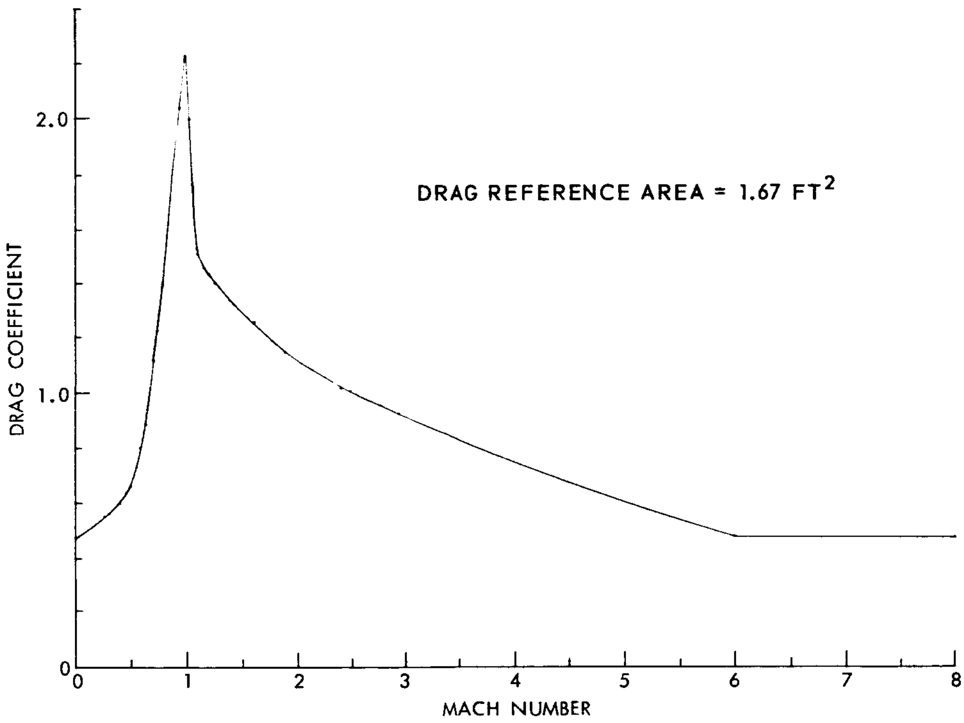


Figure 17 – Drag Coefficient as a Function of Mach Number (Nike Apache)

Particle Trajectories

Trajectories were run on the GSFC version of the GE MASS program. Standard inputs used for the preflight calculations were:

Drag coefficient vs Mach number (Nike Apache) (Figure 17)

Drag coefficient vs Mach number (Apache) (Figure 18)

Thrust vs time (Nike) (Figure 19)

Thrust vs time (Apache) (Figure 20)

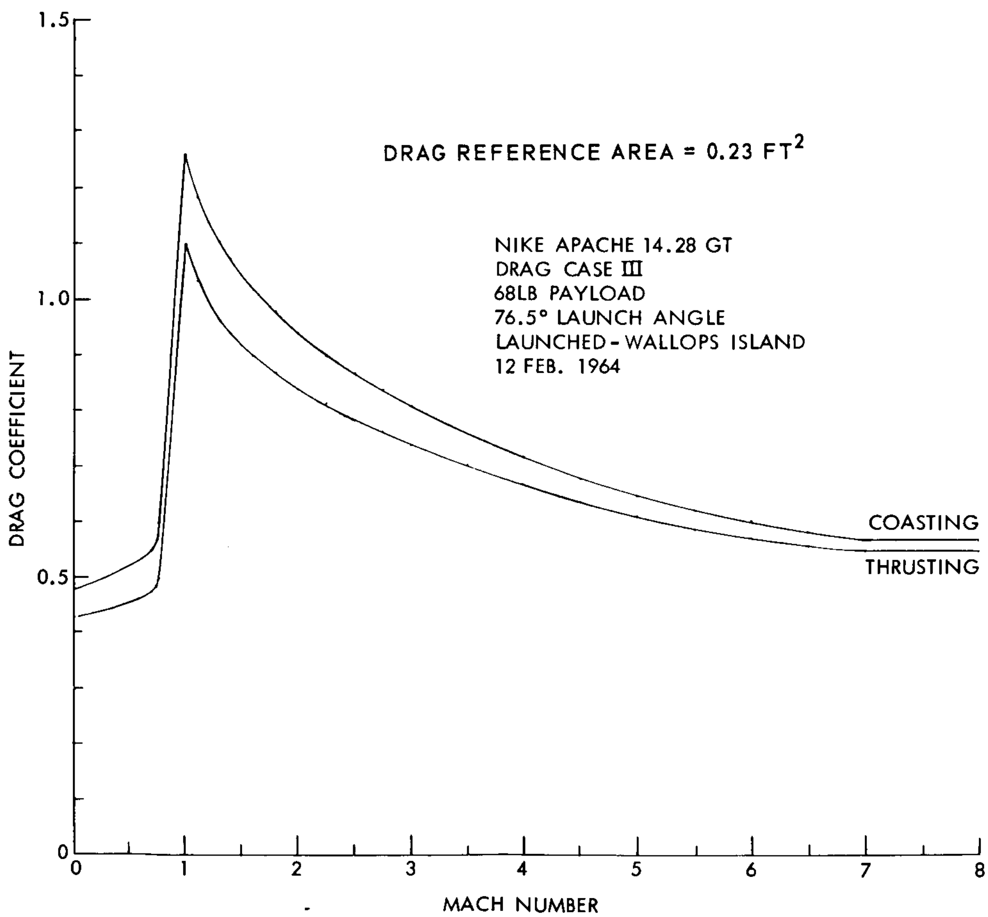


Figure 18 – Drag Coefficient as a Function of Mach Number (Apache)

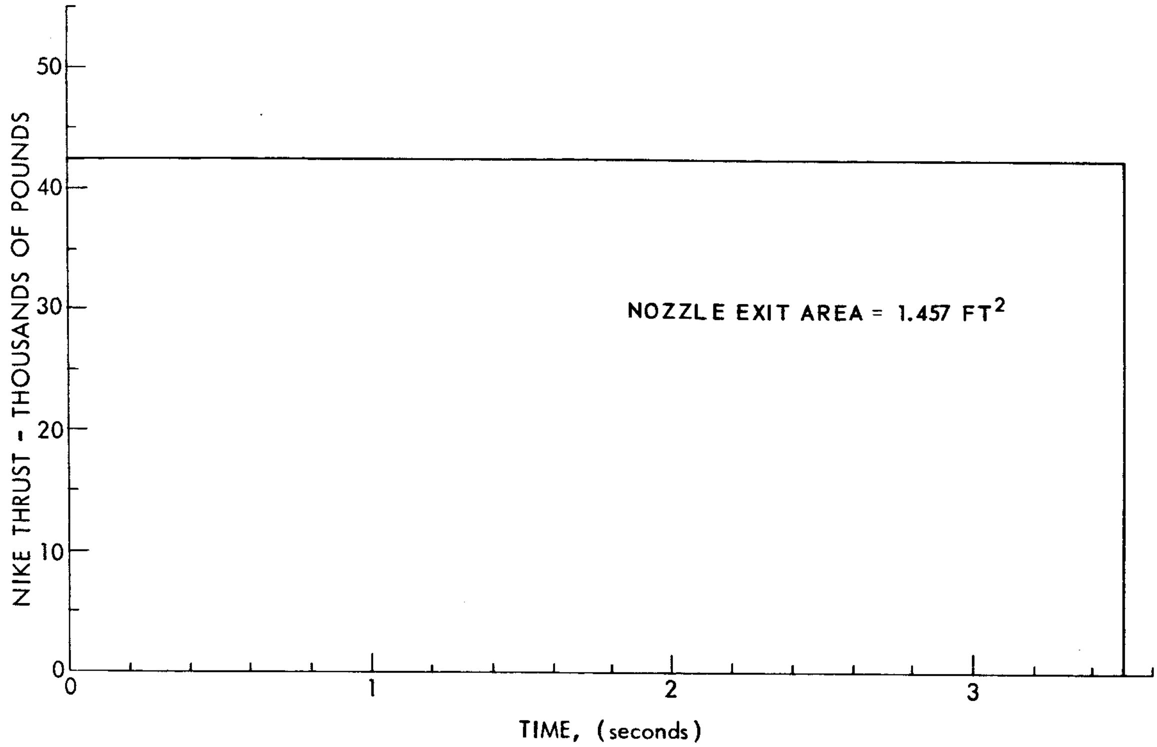


Figure 19 – Thrust as a Function of Time (Nike)

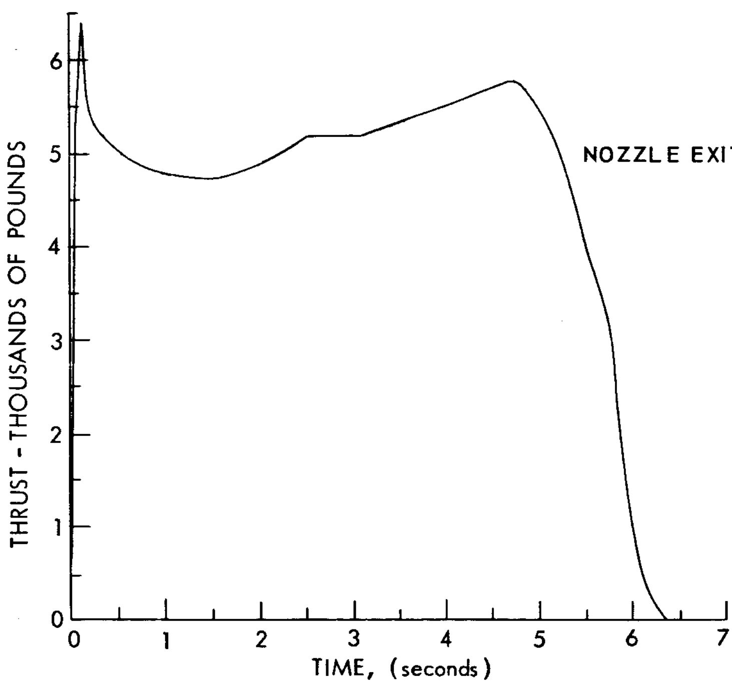


Figure 20 – Thrust as a Function of Time (Apache)

Burn time -

Nike 0-3.5 sec.

Apache 20-26.4 sec.

Special inputs were:

Payload weight = 60 lb

Launch angle = 77 degrees

Appendix B, the flight plan, contains results of these trajectory calculations.

Actual flight values used for calculating postflight trajectories were:

	Obtained from:
Payload weight = 68 lb	Small-scale records
Launch angle = 76.5 degrees	Radar data (flight-path angle) (Figure 26)
Second-stage ignition = 22.44 sec	Radar data (velocity)

On the carpet plot (Figure 21), the launch angle appears to be less than 76.5 degrees. This is because the 14.28 GT had a delayed ignition (22.44 sec), although the carpet plot is drawn for the normal ignition (20.0 sec).

Results of these calculations agreed with the actual flight trajectory (Figures 22-27) as shown in the following summary:

Time	Event	Parameter	Postflight Calculation	Actual
3.5	Nike burnout	Velocity	3220 ft/sec	---
		Altitude	5285 ft.	---
22.44	Apache ignition	Velocity	1400 ft/sec	1408 ft/sec
		Altitude	42054 ft.	40626 ft.
28.80	Apache burnout	Velocity	5840 ft/sec	5891 ft/sec
		Altitude	63074 ft.	62027 ft.
Peak	Apogee	Time	192.03 sec.	193.92 sec.
		Altitude	470867 ft.	473835 ft.
		Horizontal range	366564 ft.	371061 ft.
Impact	Splashdown	Time	375.08 sec.	---
		Horizontal range	737493 ft.	740000* ft.

*Extrapolated from radar

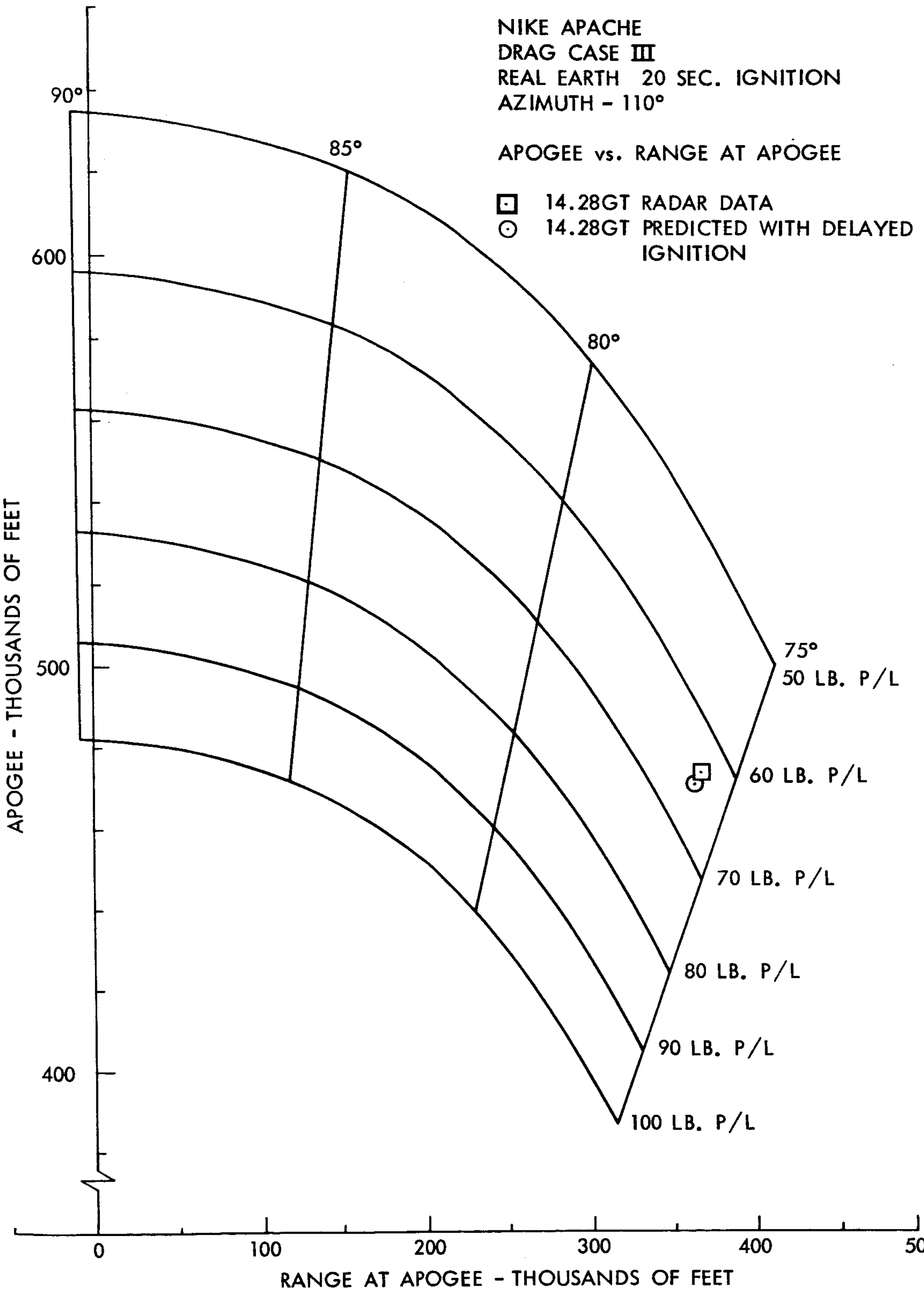


Figure 21 – Carpet Plot, Nike Apache Drag Case III

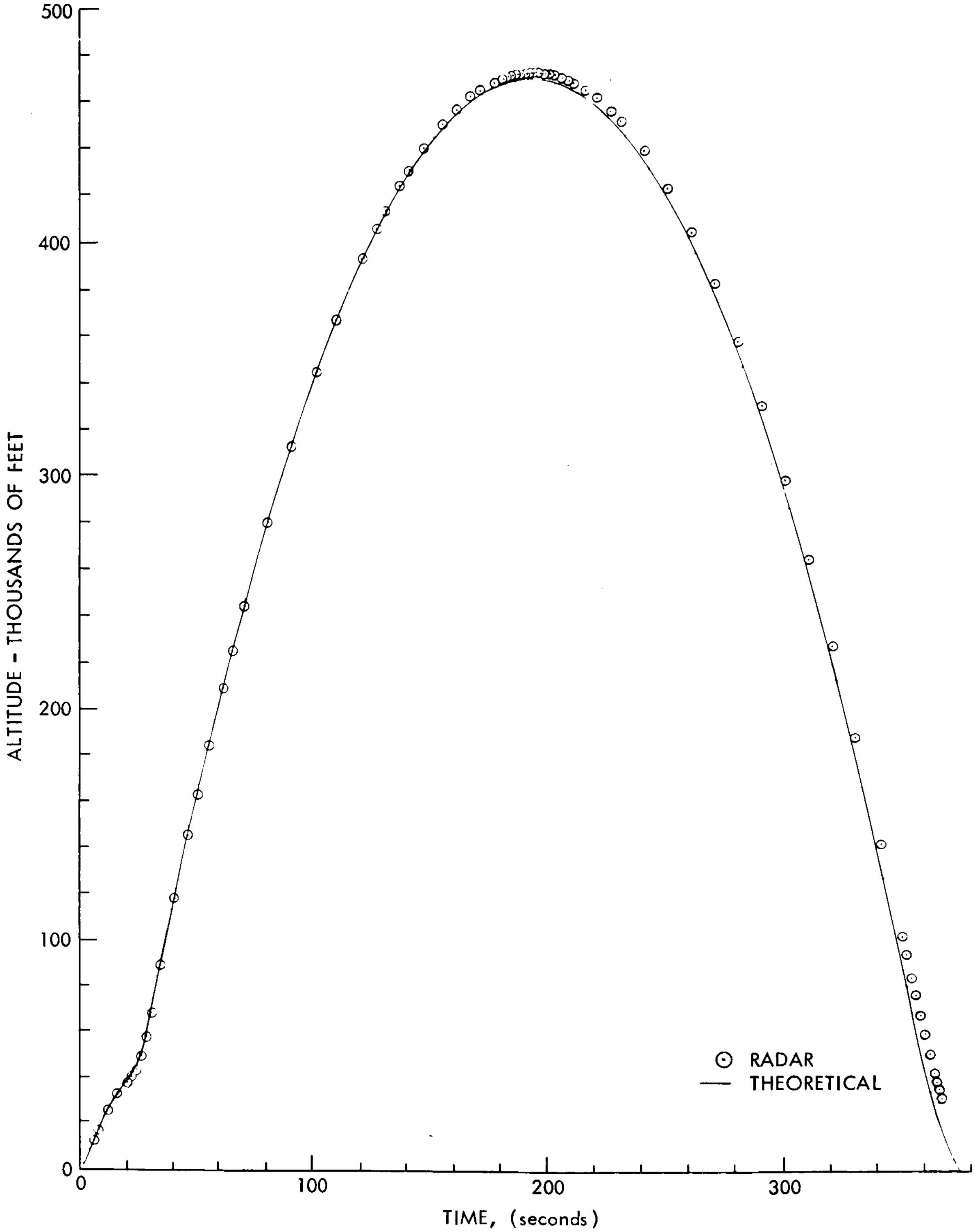


Figure 22 – Altitude as a Function of Time, 14.28 GT

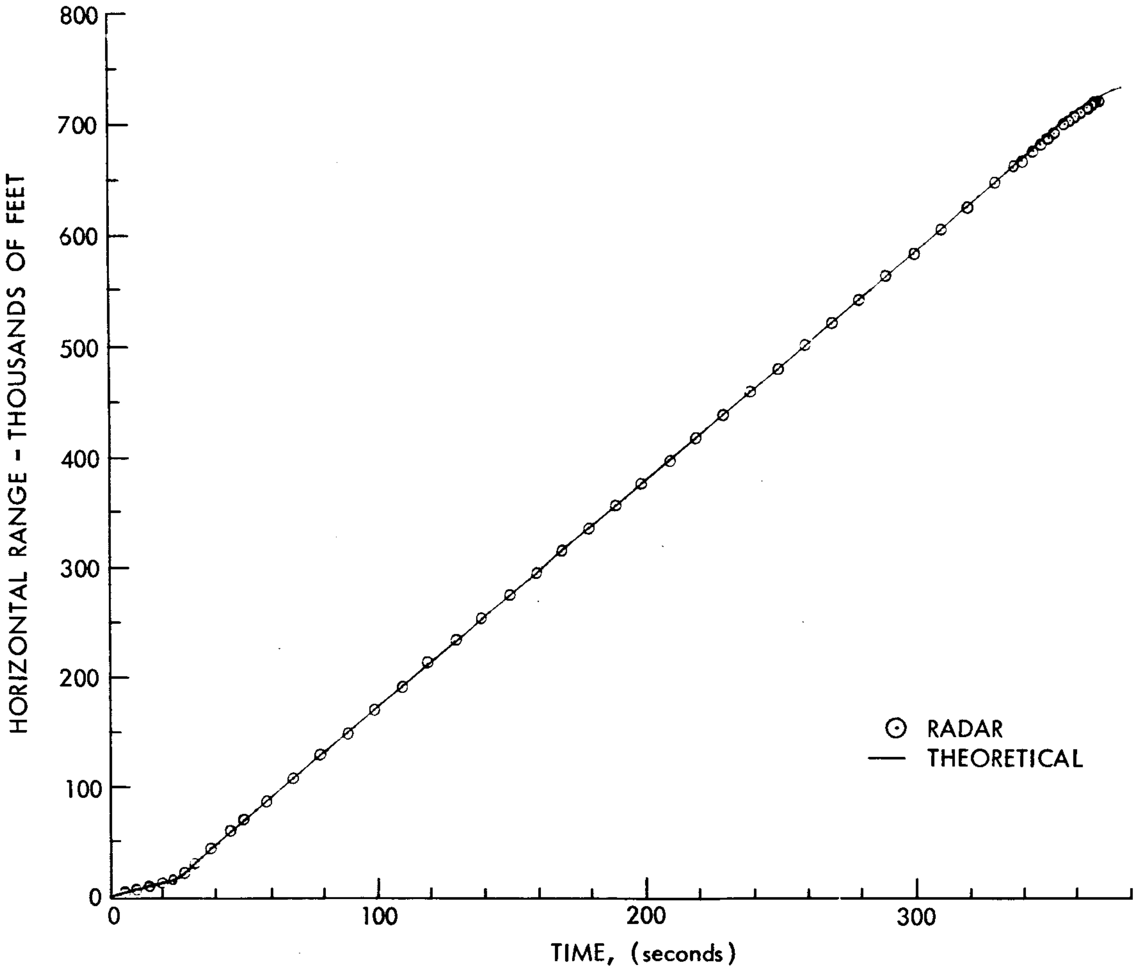


Figure 23 – Horizontal Range as a Function of Time, 14.28 GT

Natural Pitching Frequency

The natural pitching frequency may be determined by the following expression

$$\omega = \sqrt{\frac{M_\alpha}{I_{P_{CG}}}}$$

where the pitching moment slope is given by

$$M_\alpha = C_{N_\alpha} (SM) \frac{\rho V^2}{2} Ad$$

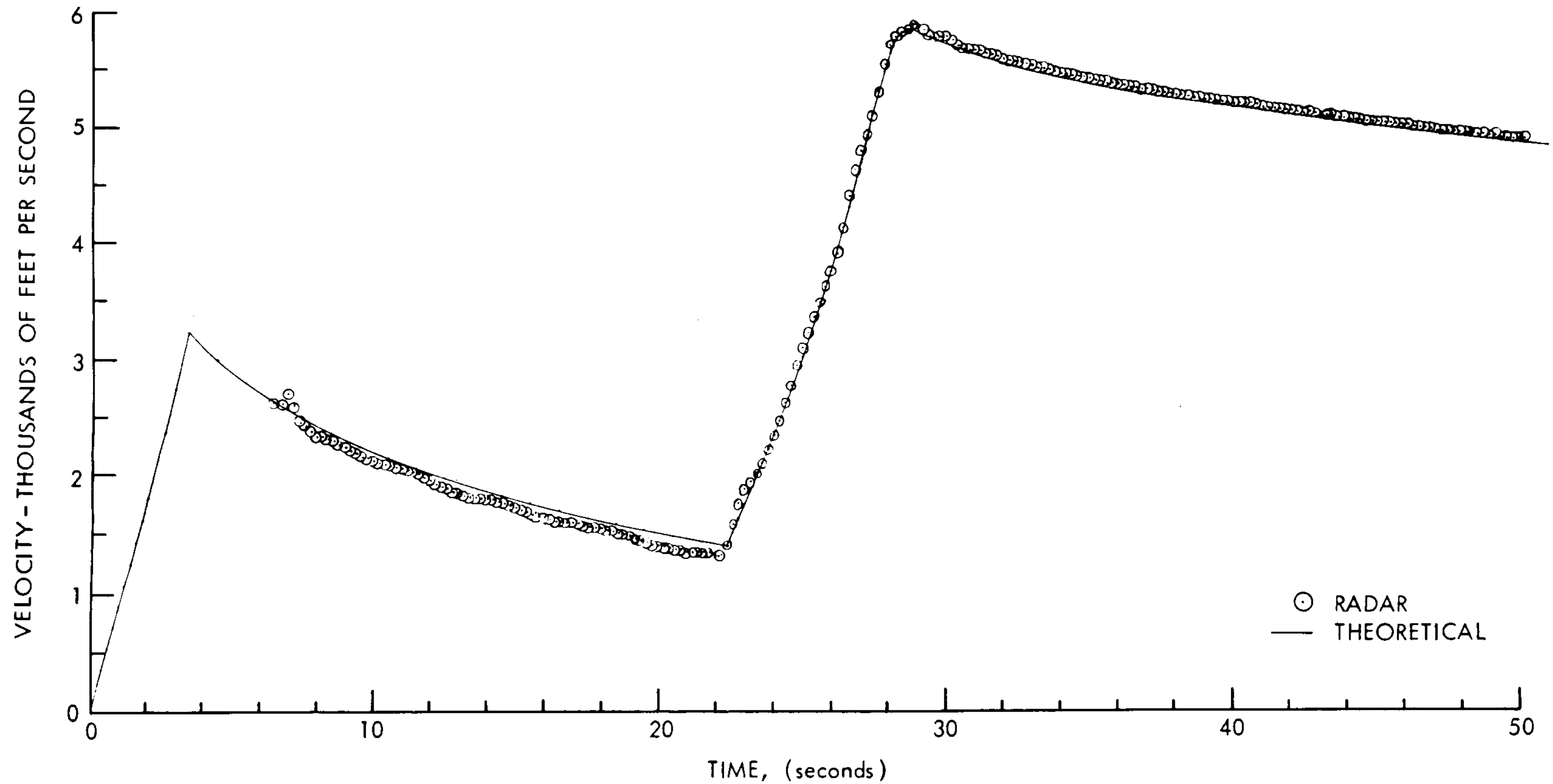


Figure 24 – Velocity as a Function of Time (0 to 50 Sec), 14.28 GT

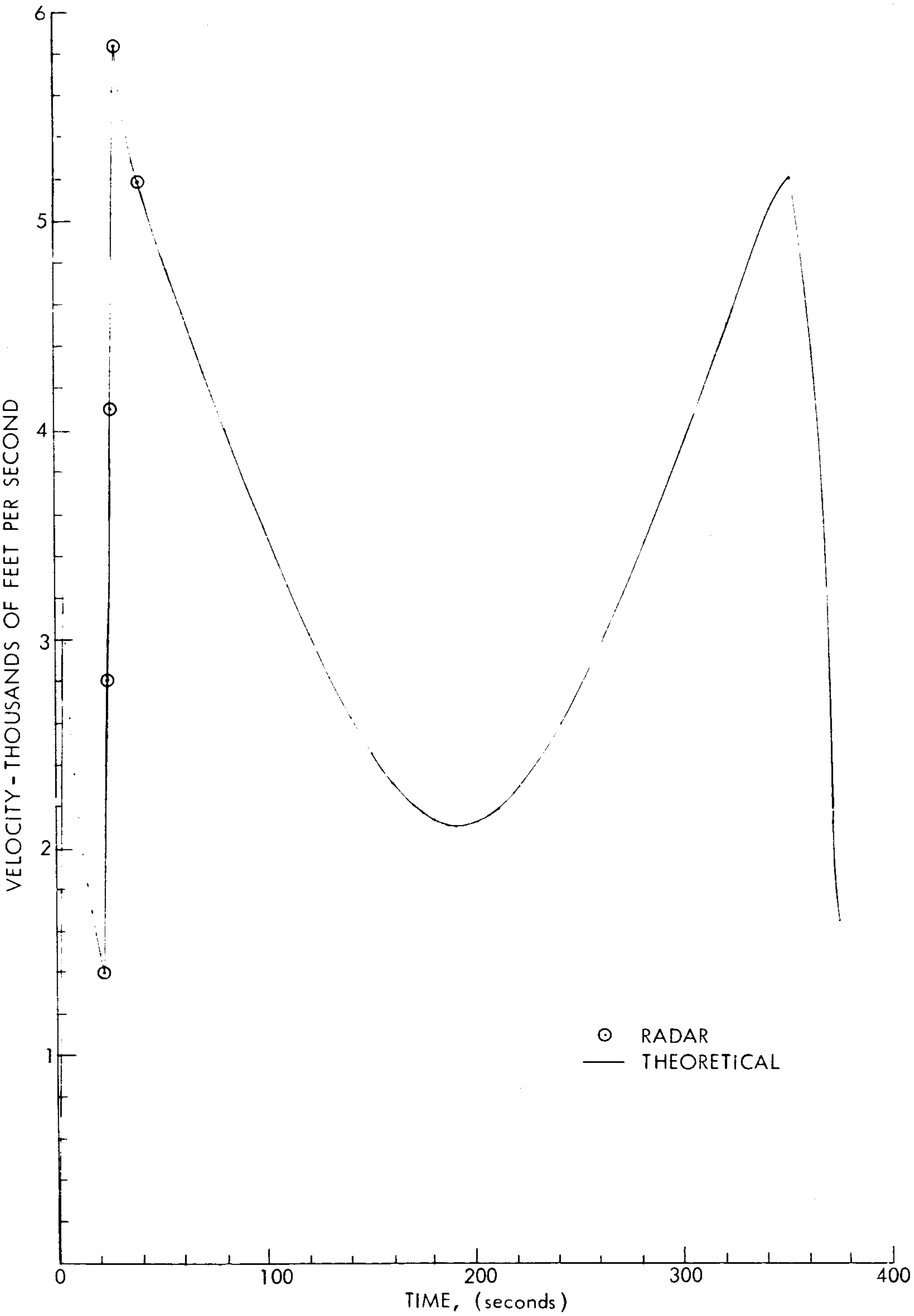


Figure 25 – Velocity as a Function of Time (Entire Flight), 14.28 GT

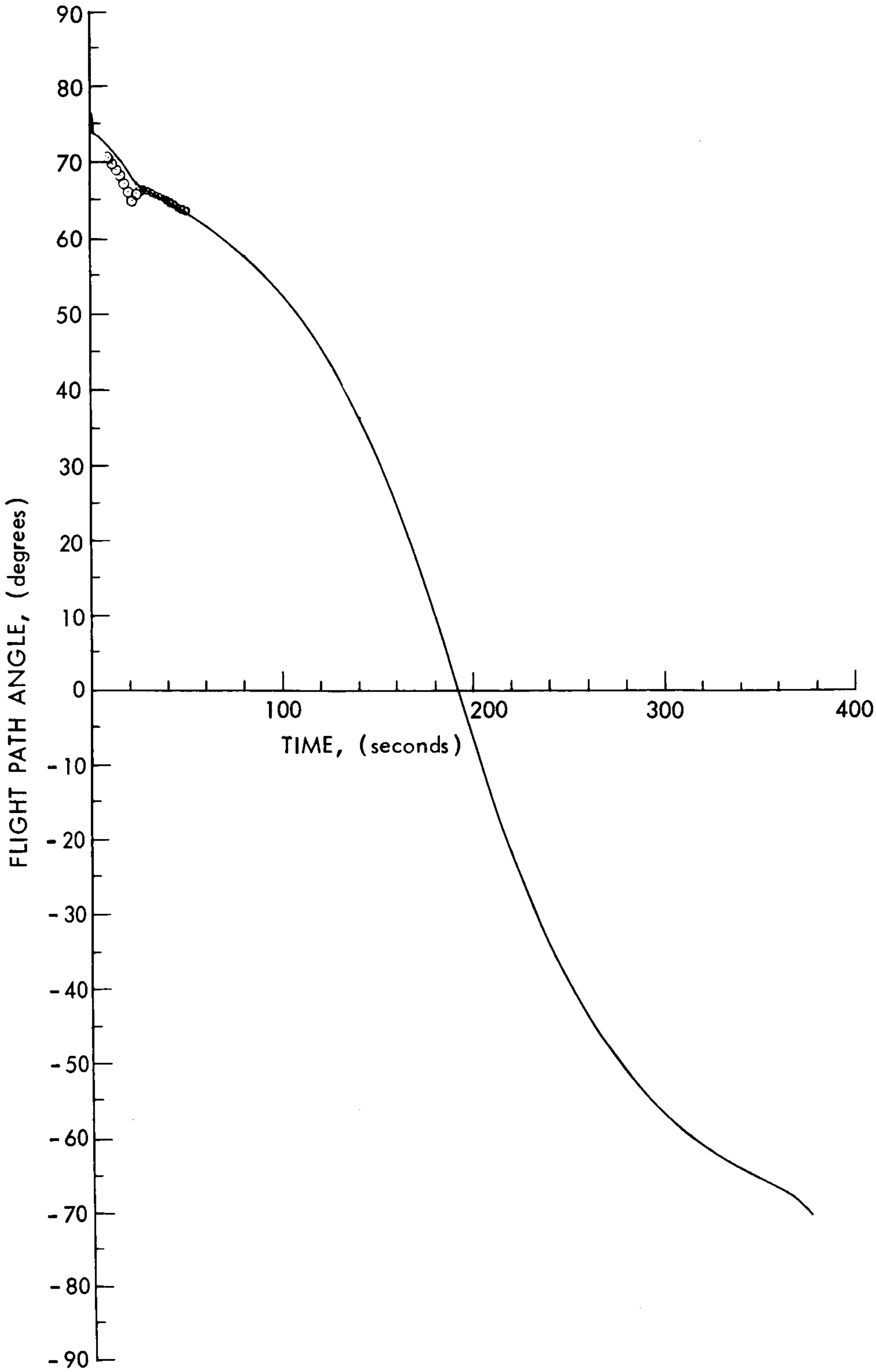


Figure 26 – Flight Path Angle as a Function of Time, 14.28 GT

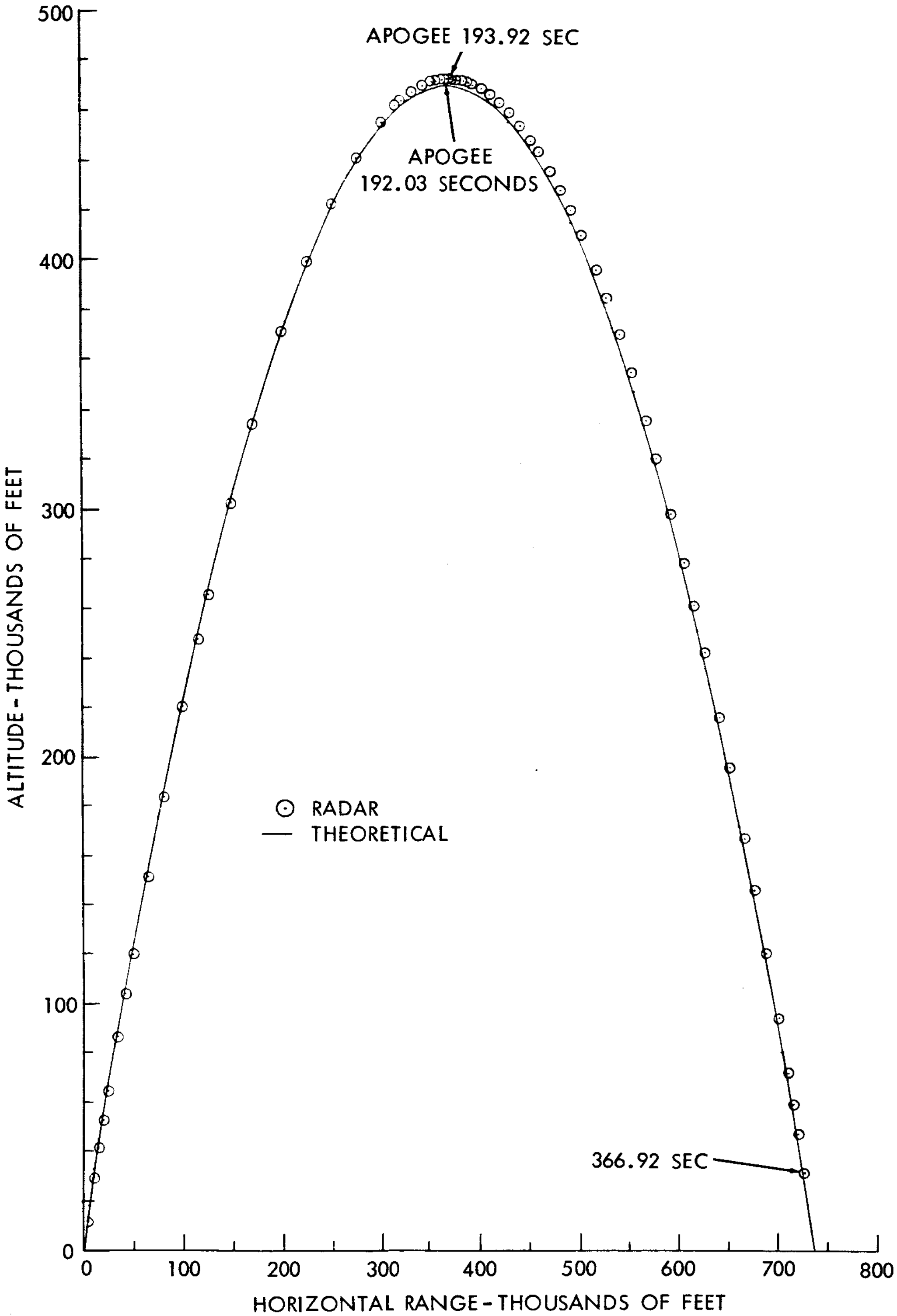


Figure 27 – Altitude as a Function of Horizontal Range, 14.28 GT

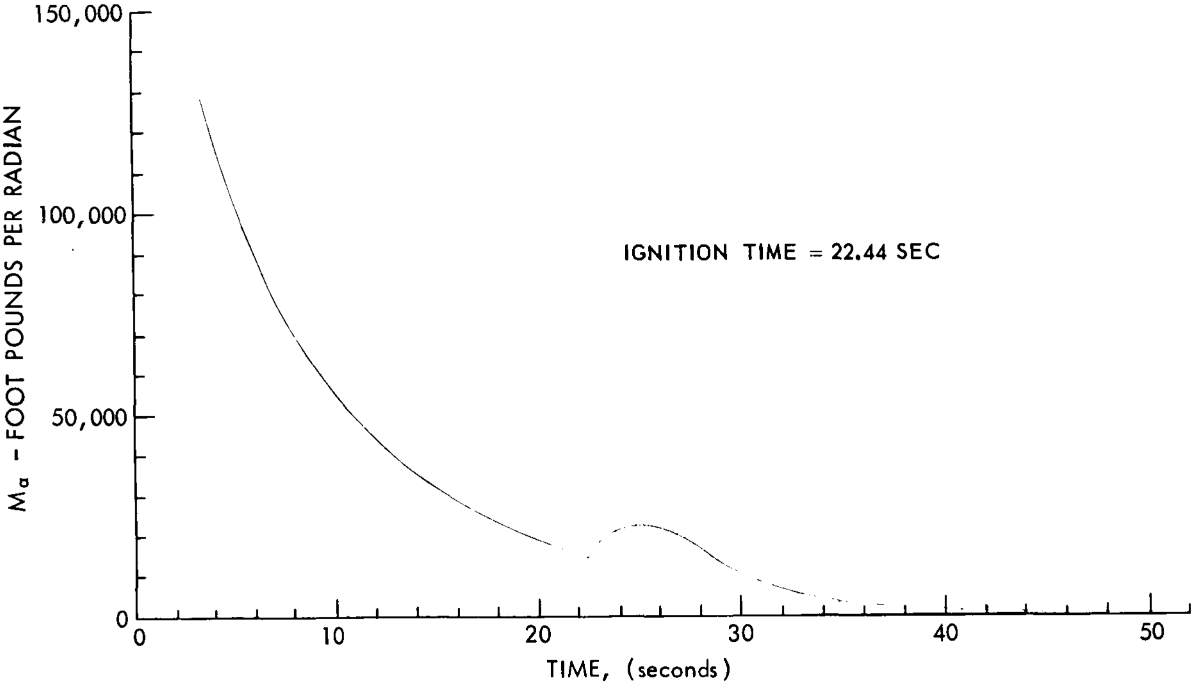


Figure 28 – Restoring Moment/Angle-of-Attack Slope @ α = 0 as a Function of Time

Figure 28 is the slope of the pitching moment/angle-of-attack curve at α = 0 plotted as a function of time.

The pitching moment-of-inertia about the rocket's CG is given by

$$(Ip_{CG})_{TOTAL} = (Ip_{CG})_{P/L} + m_{P/L}\left[CG_{P/L} - CG_{TOTAL}\right]^2$$

$$+ (Ip_{CG})_{sustainer} + m_{sustainer}\left[CG_{sustainer} - CG_{TOTAL}\right]^2$$

where the first two terms concern the payload section and the last two terms concern the Apache sustainer.

The value of Ip_{CG} for the payload and for a "normal" Apache sustainer was calculated and compared to the actual measured value. This method yielded values accurate to within 1.5 percent of the measured values. The Ip_{CG} was then determined for the 14.28 GT motor: that is, allowance was made for heavy fins, Thermo-Lag, and shrouds.

The moment-of-inertia of the motor was considered to vary linearly with time for Apache burning, going from 133.9 slug-ft^2 at ignition to 105.8 slug-ft^2 at burnout (Figure 29).

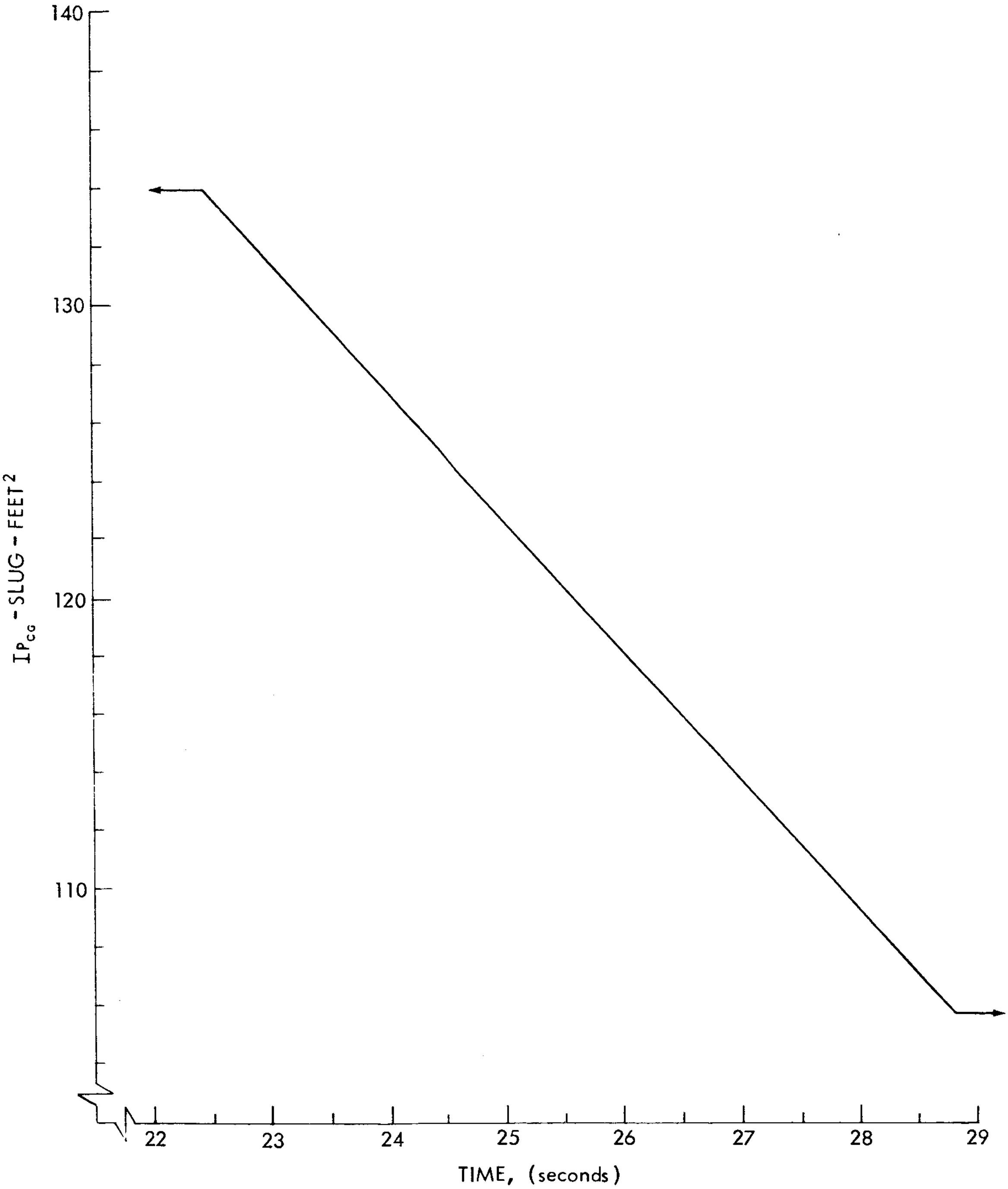


Figure 29 – Moment-of-Inertia as a Function of Time

Appendix C contains sample calculations. Table 4 shows results of these calculations.

RECOMMENDATION

It is recommended that Nike Apaches have their second stages rolled so that the roll/time curve either (1) crosses the pitch-frequency curve early in flight, where restoring moments are greatest, or (2) remains at all times less than 0.1 rps (note that coupling occurs out of the atmosphere, and that this often results in poor attitude of the Apache). This crossing should be rapid so that the roll rate and pitch frequency are the same for only a short duration.

To prevent roll lock-in, the burnout roll rate of the Apache should be above 5 rps.

Appendix A

Sounding Rocket Branch
Request for Project Initiation

SOUNDING ROCKET BRANCH

REQUEST FOR PROJECT INITIATION

14.2

April 14 '68

A. GENERAL

1. Cognizance:
   (a) NASA Scientist E. F. Sorgnit
   (b) Experimenter (s) H. L. Galloway
   Name (s)
   Affiliation GSFC
   Location

2. Project Initiated at: NASA Hqs. ☐ GSFC ☒

3. Project Responsibility
   (a) NASA Hqs.: Name ____ Code ____
   Contract let to Experimenter: ☐ Yes ☒ No
   (b) GSFC: Name H. L. Galloway Code 671.2

B. PROJECT

1. Scientific Discipline Rocket Test

2. Brief of objectives, experiment technique and instrumentation
   The objective of these tests is to provide evaluation data for an interim replacement fin for the second stage of Nike Cajun and Nike Apache. Recent flight results with the standard Capache fins with spin tabs indicate a marginal condition exists which is tentatively identified as an aeroelastic effect. Langley Research Center has some flight data on a different fin (same planform) with wedges. However, additional data are needed for evaluation. LRC will furnish two sets of fins and two Apache Motors for the tests with GSFC providing remaining hardware and instrumented payload.

3. Rocket type Nike Apache No. of flights 2

4. Proposed Launch Date (s)
   Date (s) 11-12 Feb '64 Time ____ Site Wallops Island

5. Trajectory Requirements (Vehicles may be launched on sa[illegible] day.)
   (a) Maximum expected altitude
   (b) Minimum acceptable attitude
   (c) Special requirements (if any)

6. Radioactive sources will ☐ will not ☒ be required at rocket range (site).

C. APPROVALS

(a) Originating Branch ____ Date ____

(b) Originating Division [signature] Date 1/27/64

(c) Assistant Director John W. Townsend Jr Date 1/24/64

(d) Headquarters (when req.) ____ Date ____

GSFC 19-2B (7/63)

SOUNDING ROCKET BRANCH
REQUEST FOR PROJECT INITIATION

14.28
14.29 GT

D. SUPPORT REQUIRED FROM SI&SR DIVISION

Instrumentation Support Required: Yes [X] No [ ]

If yes, which of the following are required:

Telemetry Transmitter X DOVAP ______ Batteries ______
Wiring ______ Other ______

Structural and Mechanical Support Required: Yes [ ] No [ ]

Consultation ______ Design of Payload ______

Special Devices ______ Other ______

Analytical Support Required:

Preliminary Data ______ Performance Calculations X

Wind Weighting ______ Aero Analysis ______

Ground Support Required: Yes [ ] No [ ]

Telemetry ______ DOVAP ______ Ionosonde ______

Other ______

Special Systems Required: Yes [ ] No [X]

ACS ______ FACS ______ Spin Control ______ Recovery ______

Recovery: Land ______ Sea ______

Other ______

E. SUPPORT REQUIRED FROM T&E DIVISION

Support Required: Yes [ ] No [X]

If yes, which of the following are required:

Vibration ______ Temperature ______

Thermal Vacuum ______ Shock ______

Spin ______ Balance ______

Other ______

GSFC 10-281 (7/63)

Appendix B

Flight Plan
Nike Apache Rockets NASA 14.28 and 14.29 GT

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND 20771

February 5, 1964

Mr. Robert L. Krieger
NASA Wallops Station
Wallops Island, Virginia

Dear Mr. Krieger:

Enclosed are copies of the Flight Plan for NASA Nike Apaches 14.28 and 14.29 GT for your information.

The Goddard Space Flight Center Project Scientist will be Mr. E. F. Sorgnit.

Sincerely,

John W. Townsend Jr

John W. Townsend, Jr.
Assistant Director

Copies to:

Distribution
List Attached

DISTRIBUTION LIST

Codes		Copies
NASA Headquarters:		
S	Dr. H. E. Newell	1
	Dr. J. F. Clark	1
SG	Dr. J. E. Naugle	1
	Dr. N. C. Roman	1
	Dr. R. F. Fellows	1
	Mr. M. Dubin	1
	Mr. J. R. Holtz	1
SP	Miss G. Stansell	1
SV	Dr. R. B. Morrison	1
	Cdr. Wm. Schubert	1
T	Mr. E. C. Buckley	1
AFPD	Mr. Jack Keany	2
NASA Wallops Station:		
	Mr. Robert L. Krieger, Director	70
Naval Research Laboratory:		
	Dr. Herbert Friedman Code 7100 Washington 25, D. C.	1
Air Force Cambridge Research Laboratory:		
	Mr. P. E. Gustafson (CRZER) L. G. Hanscom Field Boston, Massachusetts	1
New Mexico State University:		
	Physical Science Laboratory P. O. Box 548 University Park, New Mexico Mr. Keith Hennigh Mr. C. I. Ricketts Mr. Herbert Haas Mr. R. A. Bumgarner	4
New Mexico State University Field Representative		
	Mr. Fred Lemon P. O. Box 35 Pocomoke City, Maryland	3
Langley Research Center:		
	R. Rhinehart	10

Distribution List (Continued)

Codes		Copies
Internal Distribution:		
100	Dr. H. J. Goett	2
	Mr. E. W. Wasielewski	1
600	Dr. J. W. Townsend, Jr.	1
	Mr. H. E. LaGow	1
	Mr. C. R. Hamilton	1
202	Mr. A. Rosenthal	1
	Mr. E. Mason	1
	Mr. H. W. Harloff	1
230	Miss S. Deremer	2
250	Mr. D. C. Hutchison	1
326	Mr. K. R. Mercy	1
	Mr. M. J. Kramer	1
500	Mr. J. T. Mengel	1
610	Dr. L. H. Meredith	1
610.1	Dr. J. C. Lindsay	1
610.5	Mr. J. C. Seddon	1
620	Dr. D. G. Mazur	2
630	Mr. N. W. Matthews	2
640	Dr. W. N. Hess	2
650	Mr. W. G. Stroud	2
651	Mr. N. W. Spencer	1
611	Dr. F. B. McDonald	1
613	Dr. J. E. Kupperian, Jr.	1
615	Mr. R. E. Bourdeau	1
670	Mr. R. C. Baumann	1
671	Mr. K. R. Medrow	2
671.1	Mr. G. E. MacVeigh	3
671.2	Mr. E. F. Sorgnit	10
	Mr. H. L. Galloway, Jr.	2
671.3	Mr. E. E. Bissell	10
671.4	Miss E. C. Pressly	4
672	Mr. A. L. Franta	1
673	Mr. F. T. Martin	1
545	Mr. D. W. Dembrow	1
	Mr. L. Lohr Wallops Island, Virginia	2

TABLE OF CONTENTS

NIKE APACHE ROCKETS NASA 14.28 AND 14.29 GT

FLIGHT PLAN

NIKE APACHE ROCKETS NASA 14.28 AND 14.29 GT

SECTION I - GENERAL OPERATIONS

A. Firing Plan

Date:	11 February 1964 - 14.28 GT 12 February 1964 - 14.29 GT
Alternate Date:	As soon as conditions warrant (until fired)
Time:	1600 Z
Time Span:	1600 - 2130 Z

B. Flight Objectives

The primary objective of these flights is to test a hinged fin assembly on the Apache vehicle. The flights will be made as a cooperative effort with Langley Research Center. The fins will be equipped with a modified fin wedge to provide approximately 5 rps roll rate at Apache burnout. Another objective is to investigate the influence of thermal protection on the fin assembly during flight. The first rocket in the series will be flown without thermal protection. Test results will be compared with Langley Research Center data obtained on previous flights. The second vehicle will be flown with or without thermal protection as the above comparison will indicate.

C. Flight Information

With a 60 pound payload, 11° nose cone with a pitch yaw ogive tip, and four rigid turnstile antennae, the rocket will theoretically be capable of attaining an altitude of 94.7 statute miles if fired at a 77° elevation angle from Wallops Island, Virginia.

SECTION II - ROCKET INFORMATION

A. Rocket

Langley Research Center will furnish the two Nike rockets with fins and the adapter section, as well as the two sets of fins and wedges for the Apache. Both the Nike and Apache fins will be Magline hinged fin assemblies. (The planform of all these fins is identical to that of the presently used Nike Apache Atlantic Research Corporation - SVD fins). The Apache motors and igniters will be Goddard furnished. The Apache vehicle is to have a 20 second delay head inserted igniter. The Apache motor will have two shrouds (180° apart and in line with fins) to protect wires leading to the payload section from temperature gages on the Apache fins.

B. Performance

Performance for this vehicle, based on a 60 pound payload and a 77° launch angle, is estimated to be as follows:

Event	Time (sec.)	Altitude (ft.)	Range (ft.)	Velocity (ft./sec.)
Nike Burnout	3.5	5,335	1,480	3,237
Apache Ignition	20	38,810	12,000	1,499
Apache Burnout	26.4	61,100	20,600	6,038
Apogee	196.5	500,250 (94.7 st. mi.)	356,000	-
Impact	385.2	-	713,000 (135.0 st. mi.)	

C. Recovery is not required

D. Weights and Dimensions

1. Weights

Nike Apache launch weight total	1,594.5 lb.
Payload weight	60 lb.
Apache burnout weight	146.5 lb.

2. Rocket Dimensions

Payload length	80 in.
Apache length	107 in.
Nike stage length	149 in.
Total length	336 in.
Apache body diameter	6.5 in.

E. Modifications

1. Four standard GSFC Turnstile Antenna Whips will be used for telemetry transmission from launch to impact.

2. A pitch yaw ogive nose tip will be flown to determine angle of attack data.

3. The Apache fins are to be fitted with full span wedges with a chord length of 1.5 inches and an 8^{o} wedge angle. The wedges will be supplied and installed by Langley Research Center. Langley Research Center will align the fins on the Apaches at Wallops Island. The wedges are to provide a 5 rps roll rate at Apache burnout. The roll rates are to be clockwise, as viewed from the rear of the vehicle.

F. Installations

1. Pitch yaw ogive

2. Accelerometers

3. Turnstile Antennae

4. FM/FM Telemetry systems

5. Magnetometers

6 Five (5) Fin temperature gages

G. Pyrotechnic Installations

Umbilical release will be by explosive bolts, type GCA 3061A4002. R = 0.2Ω, max. no fire current = 1 amp, 100% fire current = 5 amp.

SECTION III - EXPERIMENT AND INSTRUMENTATION

The rocket will carry instrumentation to measure mechanical and temperature stresses on the rocket and fins. There will also be instrumentation aboard for determining vehicle attitude.

SECTION IV - FIRING RANGE

A. Radio Frequencies

FM/FM 240.2 Mc/s - ¼ watts.

B. Range Safety

Normal for Nike Apache firings.

C. Ground Station Support

1. Operation of NASA Wallops Island telemetry ground station and GSFC telemetry ground station located in the Aerobee Blockhouse.

2. Complete recording of the receiver video on ½ inch direct recorded magnetic tape with servo control on 17 kc/s and 100 kc/s tape speed compensation will be required for this flight. A copy of all tapes is to be provided to the Telemetry Engineer, Code 671.3, GSFC.

3. It is requested that complete recording by direct recording oscillograph be made of discriminated subcarrier data on the FM/FM telemetry system. These records are to be made at a speed of 10 inches per second until apogee, and at one inch per second from apogee to splash.

D. Ionosonde

No requirement.

E. Radar Beacon

None.

F. Meteorological Support

Standard meteorological support as required for launch of this vehicle will be conducted and evaluated by Wallops Island personnel. Three copies each of the last three wind weighting calculations are to be forwarded to Code 671.4, GSFC.

G. Ballistic Data

Operation of all available tracking equipment for trajectory data is required. FPS-16, Spandar and MIT radar tracking are particularly desirable. Six copies each of tabulated and reduced trajectory data are to be forwarded to Code 671.4, GSFC. Distribution to Project Scientist will be made from that office.

H. In-Flight Data

The following information should be provided to the Project Scientist immediately after the flight:

1. Absolute time of take-off.

2. Preliminary radar trajectory plot.

3. Second stage ignition time, altitude, and velocity.

4. Second stage burnout time, altitude, and velocity.

5. Peak time.

6. Peak altitude.

7. Impact time, range, and azimuth.

I. Communications

No special requirements.

SECTION V - PHYSICAL RECOVERY

None

SECTION VI - FACILITIES AND SERVICES

A. The following services and facilities will be needed during rocket preparation and firing operation:

1. Range instrumentation as described in SECTION IV.

2. Meteorological support as described in SECTION IV is required.

3. Rocket preparation, weight and C.G., igniter installation and firing, individual stage weight, C.G., location, and length are to be measured. It is requested that weight, C.G. location, and length be determined from these measurements for the total vehicle before launch and for ignition and burnout of the second stage.

4. Laboratory Space - Use of 200 sq. ft. of assembly and laboratory space in one of the bays adjacent to the Small Scale Shop is required.

5. Photographs - The normal requirement for movies and stills for flight information and documentation will be required. At least three copies each of all stills are required by the Project Scientist.

6. Messing Facilities

No requirement.

7. Nike and Apache motor numbers are to be supplied to the Vehicle System Manager.

8. Facilities to charge flight batteries are required.

SECTION VII - PERSONNEL

The following personnel will be present:

Name	Organization	Function
E. F. Sorgnit	GSFC	Project Scientist
H. L. Galloway, Jr.	GSFC	Project Engineer
J. H. Lane	GSFC	Engineer
J. S. O'Brien	GSFC	Mechanical Engineer
W. B. McAlister	GSFC	Technician
D. Tackett	GSFC	Instrumentation Engineer
L. Bauter	GSFC	Instrumentation Technician
G. Lippencott	GSFC	Instrumentation Technician
C. M. Hendricks	GSFC	Vehicle System Manager
R. Kramer	GSFC	Vehicle System Engineer
R. Rhinehart	Langley	Engineer
J. Timmons	Langley	Engineer
C. A. Sandahl	Langley	Engineer

Admission of all of the above persons to all Wallops Island areas involved in these launchings is requested. Orange badges will be needed by all persons.

SECTION VIII - SCHEDULE OF OPERATIONS

10 February 1964	Equipment and personnel arrive at Wallops Island
10 February 1964	Instrumentation check

11 February 1964	Horizontal check
11 February 1964	Fire rocket 14.28 GT

A similar schedule will be used for 14.29 GT.

SECTION IX - TELEMETERING ALLOCATION

The telemeter channel allocation is as follows:

FM/FM #1 (240.2 Mc/s)

Additional information will be furnished by the GSFC Instrumentation Engineer to the cognizant range people as required.

SECTION X - WIRING DIAGRAMS

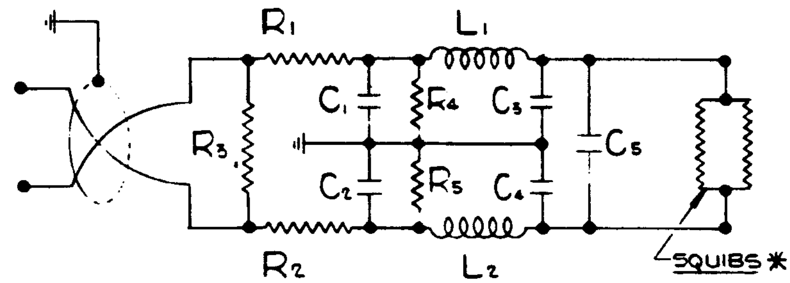
The following drawings are attached:

Enclosure 1 - Nike Apache Ignition Circuit.

E11067

SYM	NO.	DESCRIPTION	BY	DATE	CHKD

NEXT ASSY	USED ON
E11066	TE-356
E11066	TE-354



R_1, R_2, - 24 Ω 2 WATT CARBON LIMITING RESISTORS (OPTIONAL) 10% OR LESS.

R_3, - 51 Ω 2 WATT CARBON LIMITING RESISTOR.

R_4, R_5, - 0.1 M Ω 1/2 WATT CARBON.

E11067

C_1, C_2, C_3, C_4, - 0.001 MFD, CRL. DD102 ± 10% OR EQUIV.

C_5, - 0.01 MFD, DISC CERAMIC CRL, DD1032 ± 20% OR EQUIV.

L_1, L_2, - 19 TURNS, AWG #20 ENAMEL ON 3/16 DIA x 1 1/2 LONG ACRYLIC ROD.

* SQUIBS TO BE FURNISHED ACCORDING TO CUSTOMERS REQUIREMENTS: X-287-DELAY (0 TO 22 SECONDS) OR EQUIV. (REF)

ITEM NO.	PART NO.	NO. REQD	DESCRIPTION	MATERIAL	SPECIFICATION
1		1	WIRING DIAGRAM	———	———

UNLESS OTHERWISE SPECIFIED:
BREAK ALL EDGES .003 — .015"
ALL SMALL FINISHED FILLETS .020 — .040 R
ALL DIMENSIONS ARE IN INCHES
TOLERANCES
0.000 ± .010; 0.00 ± .030;
ANGLES ± 1/2°; FRACTIONS ± 1/16
FINISHED SURFACES PER MIL—STD—10
TOLERANCES ARE PER MIL—STD—8
THREADS ARE PER FEDERAL HANDBOOK H-28 AND SUPPLEMENT.
WELDING SYMBOLS ARE PER JAN—STD—19.

	BY	DATE
DRWN	EWH	10 MAR 61
CHKD	AN	15 MAR 61
ENGR	[illegible]	15 MAR
ENGR		
USER		
STRESS	[illegible]	[illegible] 61
SAFETY		
APPD	[illegible]	[illegible] 61

WIRING DIAGRAM, PYROGEN

Thiokol Chemical Corporation
Elkton Division
Elkton Maryland

WEIGHT CALC ACTUAL
SCALE AND AS NOTED NONE
NEXT ASSY SEE "USED ON" BLOCK
TE-SEE "USED ON" BLOCK
B
E11067

Enclosure 1

Appendix C

The Method of Calculating Pitching Frequency of an Apache

The natural pitching frequency of an Apache is given by

$$\omega = \sqrt{\frac{M_\alpha}{I_{P_{CG}}}} \tag{1}$$

where

$$M_\alpha = \left(C_{N_\alpha}\right)(SM)(q)(A)(d) \tag{2}$$

where

d = Apache body diameter (Fixed)

A = Apache body cross section area (Fixed)

q = dynamic pressure

SM = static margin = (CG–CP)/d

CG = center-of-gravity forward of nozzle-exit plane (NEP)

CP = center of pressure forward of nozzle-exit plane (NEP)

C_{N_α} = slope of normal force coefficient/angle-of-attack curve at $\alpha = 0$ (function of Mach number)

$I_{P_{CG}}$ = pitching moment-of-inertia of the Apache about its CG

The terms remaining to be determined depend on the results of the trajectory calculation.

CP = center of pressure (depends on Mach number)

q = dynamic pressure (depends on density of air and velocity of missile)

C_{N_α} = slope of normal force coefficient/angle-of-attack curve (depends on Mach number)

It is now, therefore, necessary to use the results of the GE MASS trajectory calculation.

Starting with the end of equation (2) we will now determine each term.

d = body diameter of the Apache motor = 6.50 in = 0.542 ft (fixed)

A = body cross section area of the Apache motor $= \pi d^2/4$ = 33.2 in² = 0.230 ft² (fixed)

q = dynamic pressure (obtained from trajectory calculations) lb/ft²

$= \rho V^2/2$

where

ρ = density of air (function of altitude)

V = velocity of Apache

q is one of the outputs of the GE MASS trajectory program. To determine CG, see subsequent discussion and Table 2.

Figure 30 shows CG as a function of time.

CP = center of pressure

Figure 30 shows CP as a function of time.

Figure 31 shows CP as a function of Mach number.

SM = static margin

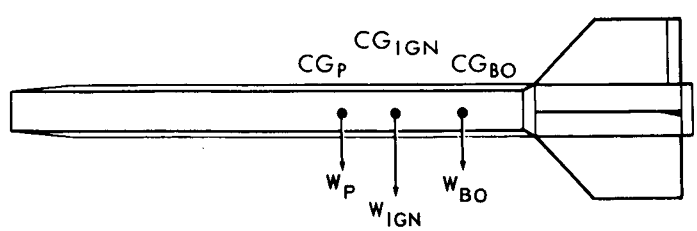
$$= \frac{CG-CP}{d}$$

Figure 32 shows SM as a function of time.

C_{N_α} = slope of the normal force/angle-of-attack curve at α = 0.

Figure 33 shows C_{N_α} as a function of Mach number.

To determine CG shift vs time for Apache burning:



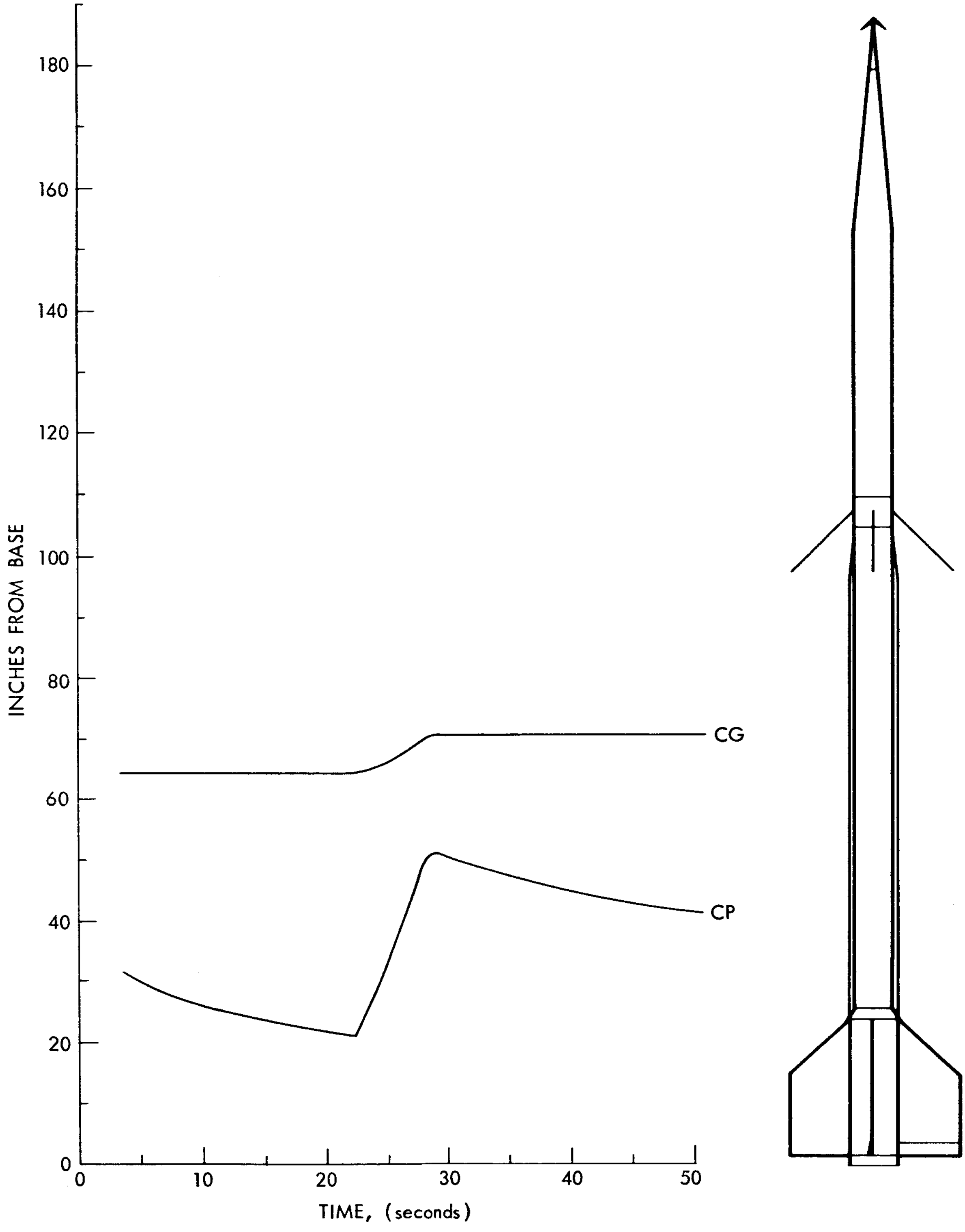


Figure 30 – Variations in CG and CP with Time

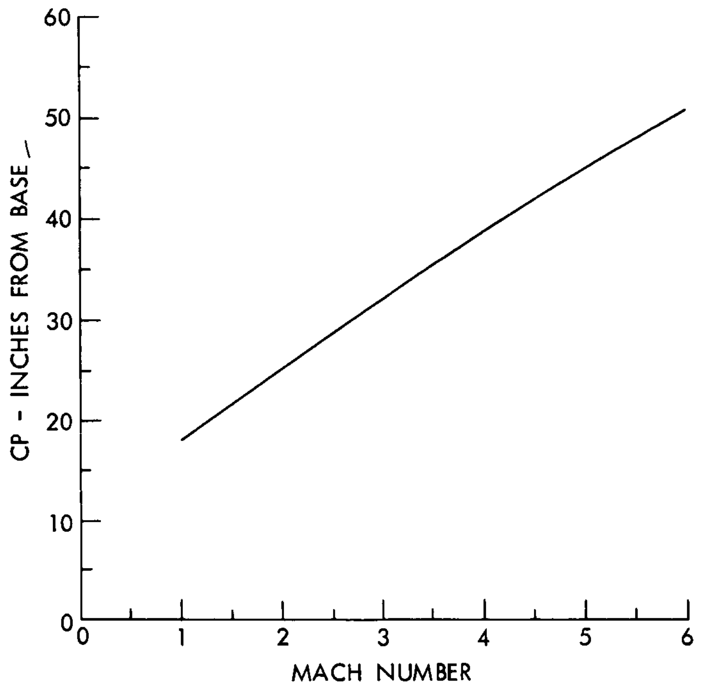


Figure 31 – Variations in Center-of-Pressure with Mach Number

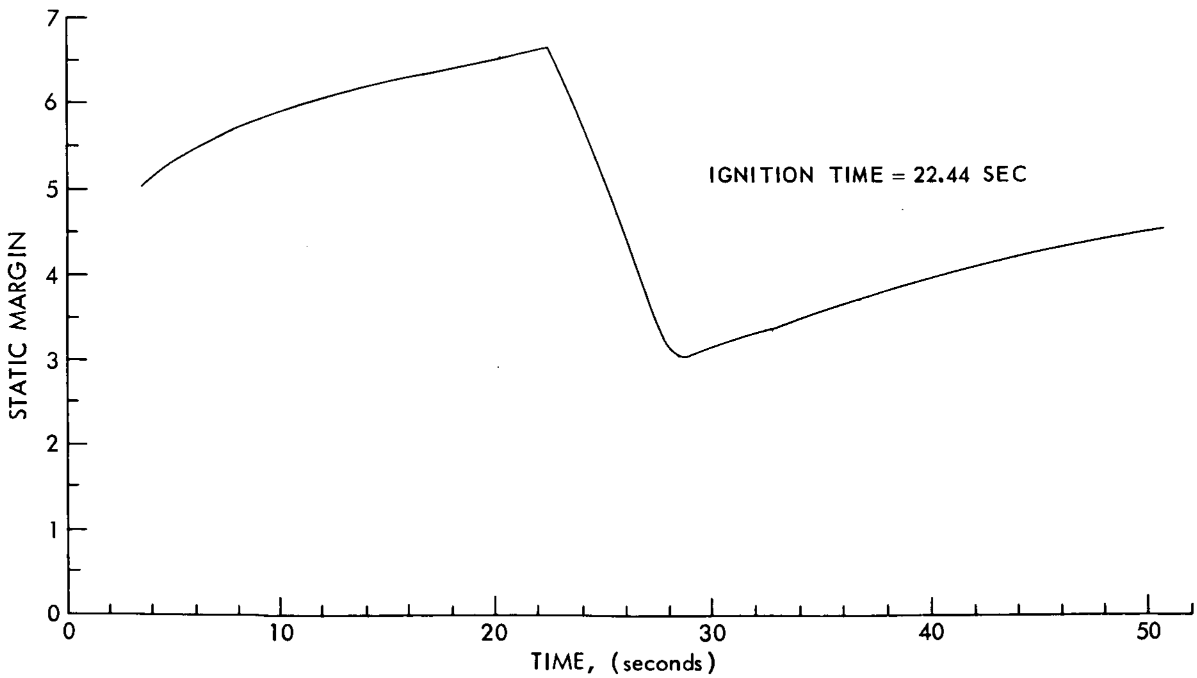


Figure 32 – Variations of Static Margin (SM) with Time

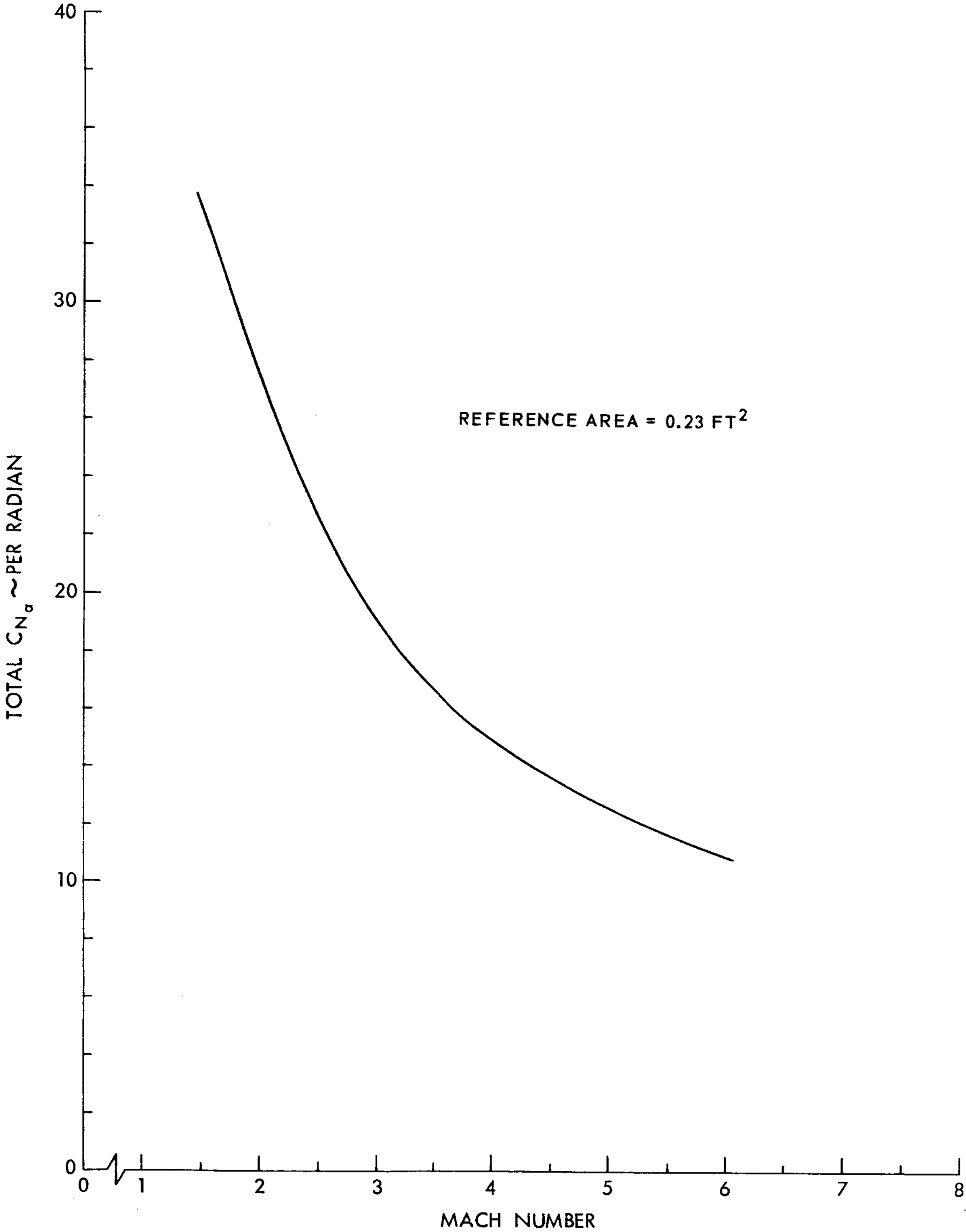


Figure 33 – Normal Force Coefficient/Angle-of-Attack Slope as a Function of Mach Number

Given

$W_{S_{ign}}$ (weight of Apache sustainer at ignition) = 230.4 lb

$W_{S_{bo}}$ (weight of Apache sustainer at burnout) = 99.4 lb

W_p (weight of Apache propellant) = 131.0 lb

$CG_{S_{ign}}$ (center-of-gravity of Apache 14.28 GT sustainer at ignition) = 48.3 in (NEP)

$CG_{S_{bo}}$ (center-of-gravity of Apache 14.28 GT sustainer at burnout) = 37.2 in (NEP)

CG_p (center-of-gravity of Apache propellant) = to be calculated.

Note that

$$\left(W_{S_{ign}}\right)\left(CG_{S_{ign}}\right) = \left(W_{S_{bo}}\right)\left(CG_{S_{bo}}\right) + \left(W_p\right)\left(CG_p\right) \quad . \tag{3}$$

Thus,

$$CG_p = \frac{\left(W_{S_{ign}}\right)\left(CG_{S_{ign}}\right) - \left(W_{S_{bo}}\right)\left(CG_{S_{bo}}\right)}{\left(W_p\right)} \tag{4}$$

$$CG_p = \frac{(230.4)(48.3) - (99.4)(37.2)}{(131.0)} \tag{5}$$

$$CG_p = \frac{11128.0 - 3698.0}{131.0} \tag{6}$$

$$CG_p = \frac{7430.0}{131.0} = 56.72 \text{ in (NEP)} \tag{7}$$

Assume that this CG of the propellant is constant during burning (this is a reasonable assumption, because the propellant burns radially).

To calculate the center-of-gravity of the Apache 14.28 GT sustainer, (CG_S), at any time, t, use

$$\left\{\left(W_S\right)\left(CG_S\right)\right\}_t = W_{S_{bo}}\, CG_{S_{bo}} + \left(W_p\right)_t \left(CG_p\right)_t \tag{8}$$

$(W_S)_t = \left(W_{S_{bo}}\right) + (W_p)_t$ = total weight of Apache 14.28 GT sustainer at any time, t

$(CG_s)_t$ = center-of-gravity of Apache 14.28 GT sustainer @ any time, t

$(CG_p)_t$ = constant = (CG_p) = 56.72 in (NEP)

Subscript t = Value at time, t

Thus equation (8) becomes:

$$\left[\left(W_{S_{bo}} + W_p\right)\left(CG_S\right)\right]_t = \left(W_{S_{bo}}\right)\left(CG_{S_{bo}}\right) + \left(W_p\right)_t\left(CG_p\right) \tag{9}$$

$$\left(99.4 + W_{P_t}\right)\left(CG_S\right)_t = (99.4)(37.2) + \left(W_P\right)_t (56.72) \tag{10}$$

$$\left(CG_S\right)_t = \frac{(99.4)(37.2) + \left(W_P\right)_t (56.72)}{\left(99.4 + W_{P_t}\right)} \tag{11}$$

$$\left(CG_S\right)_t = \frac{(3698) + \left(W_{P_t}\right)(56.72)}{\left(99.4 + W_{P_t}\right)} \tag{12}$$

Check end points:

At t = ign, , W_p = 131.0 lb

$$CG_{S_{ign}} = \frac{(3698) + (131.0)(56.72)}{(99.4 + 131.0)}$$

$$\frac{(3698) + 7430}{230.4} = \frac{11128}{230.4} \tag{13}$$

$$CG_{S_{ign}} = 48.3 \text{ in (NEP)} \tag{14}$$

At t = bo, W_p = 0

$$CG_{S_{bo}} = \frac{(3698) + (0)(56.72)}{(99.4 + 0)}$$

$$\frac{3698}{99.4} \tag{15}$$

$$CG_{S_{bo}} = 37.2 \text{ in. (NEP)} \tag{16}$$

Figure 34 shows W_P as a function of time.

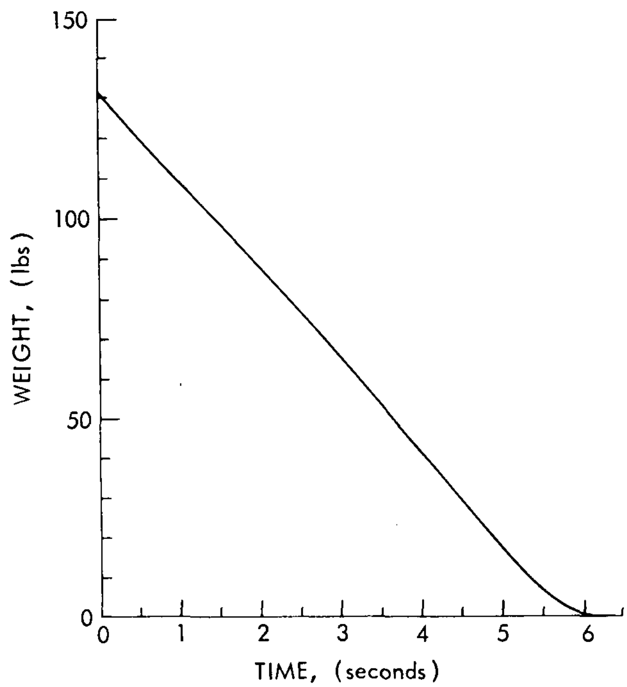


Figure 34 – Propellant Weight as a Function of Time

Using equation (12), prepare Table 1:

Table 1

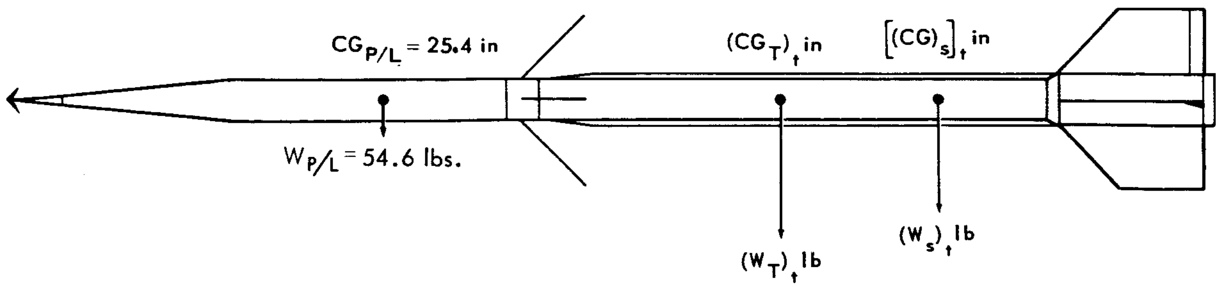
Center-of-Gravity of Sustainer as a Function of Time

(1) Time Seconds	(2) W_P pounds	(3) (W_P) (56.72) inch pounds	(4) (3698) + (W_P) (56.72) inch pounds	(5) $W_s = (99.4 + W_P)$ pounds	(6) $(CG_s)_t$ = (4) divided by (5) inches
3.5	131.0	7430	11128	230.4	48.3
5.5	131.0	7430	11128	230.4	48.3
6.5	131.0	7430	11128	230.4	48.3
12.5	131.0	7430	11128	230.4	48.3
17.5	131.0	7430	11128	230.4	48.3
20.50	131.0	7430	11128	230.4	48.3
22.44	131.0	7430	11128	230.4	48.3
23.04	112.3	6370	10068	211.7	47.6
23.54	106.9	6063	9761	206.3	47.3
23.84	100.5	5700	9398	199.9	47.0
24.04	96.3	5462	9160	195.7	46.8
24.24	92.0	5218	8916	191.4	46.6
24.64	83.5	4736	8434	182.9	46.1

Table 1 (Continued)

(1) Time Seconds	(2) W_P pounds	(3) $(W_P)(56.72)$ inch pounds	(4) $(36.98) + (W_P)(56.72)$ inch pounds	(5) $(99.4 + W_P)$ pounds	(6) $(CG_s)_t$ = (4) divided by (5) inches
24.84	78.9	4475	8173	178.3	45.8
24.94	76.7	4350	8048	176.1	45.7
25.04	74.5	4226	7924	173.9	45.6
25.14	72.1	4090	7788	171.5	45.4
25.24	69.9	3965	7663	169.3	45.3
25.44	65.3	3704	7402	164.7	44.9
25.54	63.0	3573	7271	162.4	44.8
25.64	60.7	3443	7141	160.1	44.6
25.84	56.0	3176	6874	155.4	44.2
26.04	51.2	2904	6602	150.6	43.8
26.44	41.8	2371	6069	141.2	43.0
26.64	36.8	2087	5785	136.2	42.5
26.84	31.9	1809	5507	131.3	41.9
27.04	27.0	1534	5232	126.4	41.4
27.24	21.9	1242	4940	121.3	40.7
27.64	12.3	698	4396	111.7	39.4
28.04	4.8	272	3970	104.2	38.1
28.24	2.1	119	3817	101.5	37.6
28.54	0.3	17	3715	99.7	37.3
28.80	0.0	0	3698	99.4	37.2
30.8	0.0	0	3698	99.4	37.2
32.8	0.0	0	3698	99.4	37.2
36.8	0.0	0	3698	99.4	37.2

Next, calculate the center-of-gravity shift of the entire Apache stage as a function of time.



$$\left[(W_T)\ (CG_T)\right]_t = (W_{p/1})\ (CG_{p/1}) + \left[(W_S)\ (CG_S)\right]_t$$

$W_T = W_{p/1} + W_S$ = total weight of Apache stage

GG_T = center-of-gravity of Apache stage

$W_{p/1}$ = weight of payload = 54.6 lb

$CG_{p/1}$ = center-of-gravity of payload = (25.4 + 107.0) = 132.4 in. (NEP)

$$\left[(W_{p/1} + W_S)\ (CG_T)\right]_t = (54.6)\,(132.4) + \left[(W_S)\ (CG_S)\right]_t \tag{17}$$

$$(CG_T)_t = \frac{(7229) + \left[(W_S)\ (CG_S)\right]_t}{54.6 + (W_S)_t} \tag{18}$$

Check:

At t = ign, W_S = 230.4 lb., CG_S = 48.3 in.

$$(CG_T)_{ign} = \frac{(7229.0) + \overset{11128}{(230.4)\,(48.3)}}{54.6 + 230.4} \tag{19}$$

$$(CG_T)_{ign} = \frac{18357}{285.0 \text{ in}} = 64.41 \text{ in (NEP)}. \tag{20}$$

Note that the measured value for $(CG_T)_{ign}$ = 65.75 in. (NEP).

At t = bo, W_S = 99.4 lb, CG_S = 37.2 in.

$$(CG_T)_{bo} = \frac{(7229) + \overset{3698}{(99.4)\,(37.20)}}{54.6 + 99.4} \tag{21}$$

$$(CG_T)_{bo} = \frac{10927}{154.0} = 70.95 \text{ in (NEP)}. \tag{22}$$

Using equation (18), prepare Table 2.

Table 2

Center-of-Gravity of Total Apache 14.28 GT as a Function of Time

(1) Time seconds	(2) W_S pounds	(3) CG_S inches	(4) W_S CG_S inch pounds	(5) (7229) + W_S CG_S inch pounds	(6) 54.6 + $(W_S)_t$ pounds	(7) $(CG_T)_t$ = (5)/(6) inches
3.5	230.4	48.3	11128	18357	285.0	64.41
5.5	230.4	48.3	11128	18357	285.0	64.41
6.5	230.4	48.3	11128	18357	285.0	64.41
12.5	230.4	48.3	11128	18357	285.0	64.41
17.5	230.4	48.3	11128	18357	285.0	64.41
20.5	230.4	48.3	11128	18357	285.0	64.41
22.44	230.4	48.3	11128	18357	285.0	64.41
23.04	211.7	47.6	10077	17306	266.3	64.99
23.54	206.3	47.3	9758	16987	260.9	65.11
23.84	199.9	47.0	9395	16624	254.5	65.32
24.04	195.7	46.8	9159	16388	250.3	65.47
24.24	191.4	46.6	8919	16148	246.0	65.64
24.64	182.9	46.1	8432	15661	237.5	65.94
24.84	178.3	45.8	8166	15395	232.9	66.10
24.94	176.1	45.7	8048	15277	230.7	66.22
25.04	173.9	45.6	7930	15159	228.5	66.34
25.14	171.5	45.4	7786	15015	226.1	66.41
25.24	169.3	45.3	7669	14898	223.9	66.54
25.44	164.7	44.9	7395	14624	219.3	66.68
25.54	162.4	44.8	7276	14505	217.0	66.84
25.64	160.1	44.6	7140	14369	214.7	66.92
25.84	155.4	44.2	6869	14098	210.0	67.13
26.04	150.6	43.8	6596	13825	205.2	67.37
26.44	141.2	43.0	6072	13301	195.8	67.93
26.64	136.2	42.5	5788	13017	190.8	68.22
26.84	131.3	41.9	5501	12730	185.9	68.48
27.04	126.4	41.4	5233	12462	181.0	68.85
27.24	121.3	40.7	4937	12166	175.9	69.16
27.64	111.7	39.4	4401	11630	166.3	69.93
28.04	104.2	38.1	3970	11199	158.8	70.52
28.24	101.5	37.6	3816	11045	156.1	70.76
28.54	99.7	37.3	3719	10948	154.3	70.95
28.80	99.4	37.2	3698	10927	154.0	70.95
30.8	99.4	37.2	3698	10927	154.0	70.95
32.8	99.4	37.2	3698	10927	154.0	70.95
36.8	99.4	37.2	3698	10927	154.0	70.95

To calculate M_α:

$$M_\alpha = \text{slope of the pitching moment/angle of attack curve at } \alpha = 0$$

$$= \left(C_{N_\alpha}\right)(SM)\,(q)\,(A)\,(d) \qquad (23)$$

where

$$Ad = (0.230 \text{ ft}^2)\,(0.542 \text{ ft}) = 0.125 \text{ ft}^3$$

Using Equation (23), prepare Table 3. Figure 28 shows M_α as a function of time.

To calculate $(I_p)_{CG}$:

$(I_p)_{CG}$ = the pitching moment-of-inertia of the Apache about its CG, and is determined in a step-wise fashion.

Given for a standard Apache,*

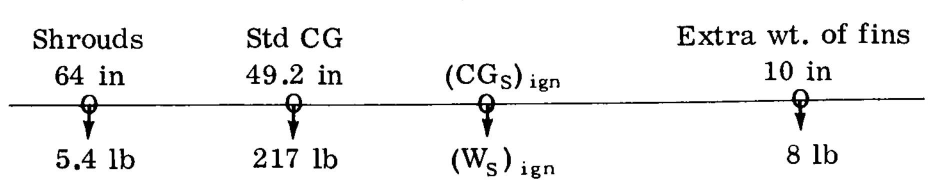
Time	Pitching moment-of-inertia about CG (slug-ft^2)	Weight (lbs)	Distance CG Forward of NEP (inches)
ign	58	217	49.2
bo	29.6	86	38.1

The 14.28 GT was a standard Apache whose fins were replaced with Magline fins having a CG 10 inches forward of the nozzle exit plane (NEP) and weighing 8 lb more than the standard fins.

A set of 2 shrouds weighing 5.4 lb, with a CG 64 inches forward of the NEP, was mounted along the casing in line with one pair of fins.

This changed the CG and the pitch moment-of-inertia as follows:

At ignition,



*See Memo of 10 April 1964, Hershfield to Baumann, Subj: Principal Axis Tilt (PAT) of Apache and Payload.

Table 3

M_α as a Function of Time

Time seconds	Mach* Number	C_{N_α}	CP inches (NEP)	CG inches (NEP)	CG-CP inches	$\underset{\sim}{SM}$	q* pounds/foot^2	(a) = $(C_{N_\alpha})(\underset{\sim}{SM})$	(b) = (a) q pounds/foot^2	M_α = (b) Ad foot pounds
3.5	2.94	19.4	31.8	64.41	32.6	5.02	10527	97.4	1,025,000	128000
5.5	2.60	21.6	29.3	64.41	35.1	5.40	6632	116.6	773300	96700
6.5	2.47	22.7	28.4	64.41	36.0	5.54	5403	125.8	679700	85000
12.5	1.96	28.0	24.8	64.41	39.6	6.09	1968	170.5	335500	41900
17.5	1.69	31.2	22.9	64.41	41.5	6.38	996	199.0	198200	24800
20.5	1.54	32.9	21.8	64.41	42.6	6.55	668	215.5	144000	18000
22.44	1.45	33.7	21.25	64.41	43.2	6.65	522	224.1	117000	14600
23.04	1.82	29.7	23.8	64.99	41.2	6.34	790	188.3	148800	18600
23.54	2.12	26.1	25.9	65.11	39.2	6.04	1034	157.6	163000	20400
23.84	2.31	24.1	27.3	65.32	38.0	5.85	1188	141.0	167500	20900
24.04	2.43	23.0	28.2	65.47	37.3	5.74	1294	132.0	170800	21400
24.24	2.56	22.0	29.0	65.64	36.6	5.64	1404	124.1	174200	21800
24.64	2.83	20.1	31.0	65.94	34.9	5.37	1637	107.9	176600	22100
24.84	2.97	19.2	31.9	66.10	34.2	5.27	1761	101.2	178200	22300
24.94	3.05	18.8	32.5	66.22	33.7	5.19	1825	97.57	178100	22300
25.04	3.12	18.4	33.0	66.34	33.3	5.13	1890	94.39	178400	22300
25.14	3.20	18.0	33.6	66.41	32.8	5.05	1956	90.90	177800	22200
25.24	3.27	17.7	34.0	66.54	32.5	5.00	2022	88.50	178900	22400
25.44	3.42	17.0	35.1	66.68	31.6	4.87	2154	82.79	178300	22300
25.54	3.50	16.7	35.5	66.84	31.3	4.82	2220	80.49	178700	22300
25.64	3.58	16.4	36.2	66.92	30.7	4.73	2287	77.57	177400	22200
25.84	3.74	15.8	37.25	67.13	29.9	4.60	2422	72.68	176000	22000
26.04	3.91	15.2	38.3	67.37	29.1	4.48	2559	68.10	174300	21800
26.44	4.26	14.2	40.7	67.93	27.2	4.19	2838	59.50	168900	21100
26.64	4.45	13.7	41.9	68.22	26.3	4.05	2980	55.48	165300	20700
26.84	4.65	13.3	43.1	68.48	25.4	3.91	3122	52.00	162300	20300
27.04	4.85	12.8	44.3	68.85	24.6	3.79	3265	48.51	158400	19800
27.24	5.06	12.4	45.6	69.16	23.6	3.63	3407	45.01	153300	19200
27.64	5.47	11.7	48.0	69.93	21.9	3.37	3643	39.43	143600	18000
28.04	5.81	11.2	49.9	70.52	20.6	3.17	3738	35.50	132700	16600
28.24	5.94	11.0	50.6	70.76	20.2	3.11	3720	34.21	127300	15900
28.54	6.03	10.9	51.2	70.95	19.8	3.05	3542	33.24	117700	14700
28.80	6.03	10.9	51.2	70.95	19.8	3.05	3322	33.24	110400	13800
30.8	5.82	11.2	50.0	70.95	21.0	3.23	1874	36.18	67801	8475
32.8	5.66	11.4	49.2	70.95	21.8	3.35	1090	38.19	41630	5204
34.8	5.43	11.8	47.8	70.95	23.2	3.57	635	42.13	26752	3344
36.8	5.24	12.1	46.7	70.95	24.2	3.72	384	45.01	17280	2160
40.8	4.92	12.7	44.7	70.95	26.2	4.03	152	51.18	7779	972
44.8	4.65	13.3	43.1	70.95	27.8	4.28	65.9	56.92	3751	469
50.8	4.38	13.9	41.4	70.95	29.6	4.55	22.0	63.24	1391	174

*Values read from Run 237D

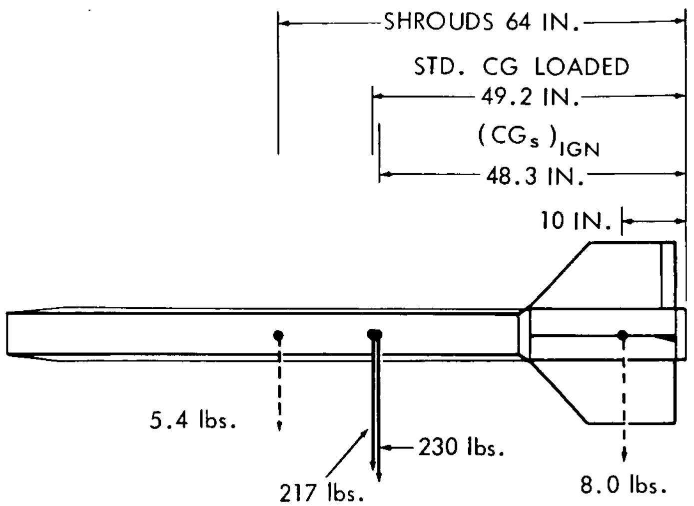
Taking moments

$$(5.4 + 217.0 + 8.0)\ (CG_S)_{ign} = (5.40)\ (64.0) + (217.0)\ (49.2) + (8.0)\ (10.0)$$

$$(230)\ (CG_S)_{ign} = 346 + 10676 + 80$$

$$(230)\ (CG_S)_{ign} = \frac{11102}{230}$$

$$(CG_S)_{ign} = 48.3 \text{ in (NEP)}$$



At burnout

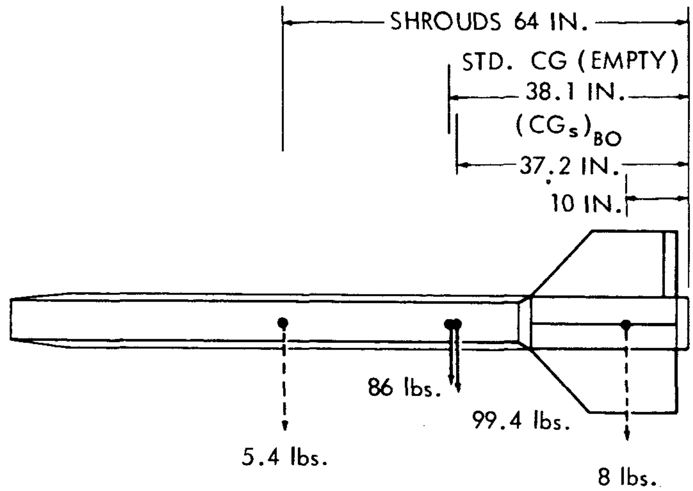
Shrouds	Std CG (empty)	$(CG_S)_{bo}$	Fins
64 in.	38.1 in.		10 in.
5.4 lb	86 lb	$(W_s)_{bo}$	8 lb

$$(5.4 + 86.0 + 8.0)\ (CG_S)_{bo} = (5.4)\ (64.0) + (86.0)\ (38.1) + (8.0)\ (10.0)$$

$$(99.4)\ (CG_S)_{bo} = 346 + 3277 + 80$$

$$(CG_S)_{bo} = \frac{3703}{99.4}$$

$$(CG_S)_{bo} = 37.2 \text{ in. (NEP)}$$



Thus the center-of-gravity of the 14.28 GT Apache sustainer as flown had a slightly different location from that of the standard Apache sustainer. The new moment-of-inertia is caused by three factors:

1) The standard Apache sustainer

2) The extra weight of the Magline fins

3) The shrouds

NOTE: The following expression for transfer of moment-of-inertia from about the center of gravity to about a new point will be used:

$$I_{P_X} = I_{P_{CG}} + m(CG - X)^2 \qquad \text{(A)}$$

$I_{P_{CG}}$ = value of pitching moment-of-inertia about CG

I_{P_X} = value of pitching moment-of-inertia about a given point X

m = mass under consideration

CG = location of center-of-gravity of mass, m

X = location of point about which I_{P_X} was calculated

Equation A shows that the pitching moment-of-inertia of a body is a minimum when taken about the body's CG.

Transfer moments-of-inertia to the center-of-gravity of the 14.28 GT motor. Here X = 48.3 in at ignition, and 37.2 in at burnout.

1) Transfer of the standard Apache pitching moment-of-inertia to the new CG yields:

At ignition,

$$\left(I_{P_{48.3}}\right)_{\substack{STD\\ \text{sustainer}}} = \left(I_{P_{CG}}\right)_{S.S} + m\left(CG - CG_{48.3}\right)^2$$

$$= 58.0 + \frac{217.0}{32.2}\left(\frac{49.2 - 48.3}{12}\right)^2$$

$$= 58.0 + (6.74)\left(\frac{.09}{16}\right)$$

$$= 58.0 + (.04)$$

$$= 58.0 \text{ slug-ft}^2 \text{ (change too small to detect)}$$

At burnout,

$$\left(I_{P_{37.2}}\right)_{\substack{STD\\ \text{sustainer}}} = 29.6 + \frac{86.0}{32.2}\left(\frac{38.1 - 37.2}{12}\right)^2$$

$$= 29.6 + (2.67)\left(\frac{.09}{16}\right)$$

$$= 29.6 + (.02)$$

$$= 29.6 \text{ slug-ft}^2 \text{ (change too small to detect)}$$

2) Transfer of moment-of-inertia caused by extra weight of the fins:

Consider extra weight to be at a point, the CG of the fins.

Then $\left(I_{P_{CG}}\right)_{Fins} = 0.$

Thus:

At ignition,

$$\left(I_{P_{48.3}}\right)_{Fins} = 0 + m(48.3 - CG)^2$$

$$= 0 + 8/32.2 \left(\frac{48.3 - 10}{12}\right)^2$$

$$= 0 + (.25)\ (10.0)$$

$$= 2.5 \text{ slug-ft}^2$$

At burnout,

$$\left(I_{P_{37.2}}\right)_{Fins} = 0 + m(37.2 - CG)^2$$

$$= 0 + 8/32.2 \left(\frac{37.2 - 10}{12}\right)^2$$

$$= 0 + (.25)\ (5.14)$$

$$= 1.3 \text{ slug-ft}^2$$

3) Moment-of-inertia due to shrouds:

a) Moment-of-inertia of shrouds about their own center-of-gravity is

$$\left(I_{P_{CG}}\right)_{Shrouds} = m\left(\frac{l^2}{12}\right)$$

where l = length of shrouds and m = mass of shrouds.

$$\left(I_{P_{CG}}\right)_{Shrouds} = \frac{5.4}{32.2}\left(\frac{86}{12}\right)^2 \frac{1}{12}$$

$$= (.17)\ (52)\ (.083)$$

$$= .73 \text{ slug ft}^2$$

Transfer this to the center-of-gravity of the Apache sustainer:

At ignition,

$$\left(I_{P_{48.3}}\right)_{\text{Shrouds}} = \left(I_{P_{CG}}\right)_{\text{Shrouds}} + m(CG - 48.3)^2$$

$$= 0.73 + \frac{5.4}{32.2}\left(\frac{64 - 48.3}{12}\right)^2$$

$$= 0.73 + (.17)\,(1.71)$$

$$= 0.73 + .29$$

$$= 1.02 \text{ slug-ft}^2$$

At burnout,

$$\left(I_{P_{37.2}}\right)_{\text{Shrouds}} = \left(I_{P_{CG}}\right) + m(CG - 37.2)^2$$

$$= 0.73 + \frac{5.4}{32.2}\left(\frac{64 - 37.2}{12}\right)^2$$

$$= 0.73 + (.17)\,(4.99)$$

$$= 0.73 + .85$$

$$= 1.58 \text{ slug-ft}^2$$

Hence, the moment-of-inertia of all three components about the new center-of-gravity is:

$$\left(I_{P_{CG}}\right)_S = \left(I_{P_X}\right)_{\substack{\text{STD}\\ \text{Sustainer}}} + \left(I_{P_X}\right)_{\text{fins}} + \left(I_{P_X}\right)_{\text{shroud}}$$

At ignition,

$$\left(I_{P_{48.3}}\right)_S = 58.0 + 2.5 + 1.02$$

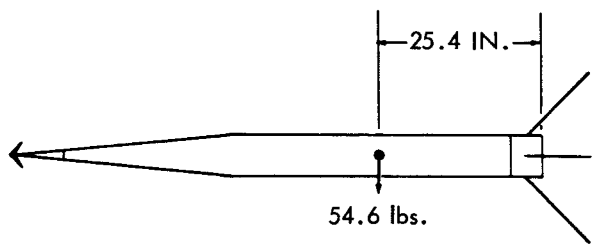
$$= 61.5 \text{ slug-ft}^2$$

At burnout,

$$\left(I_{P_{37.2}}\right)_S = 29.6 + 1.3 + 1.58$$

$$= 32.5 \text{ slug-ft}^2$$

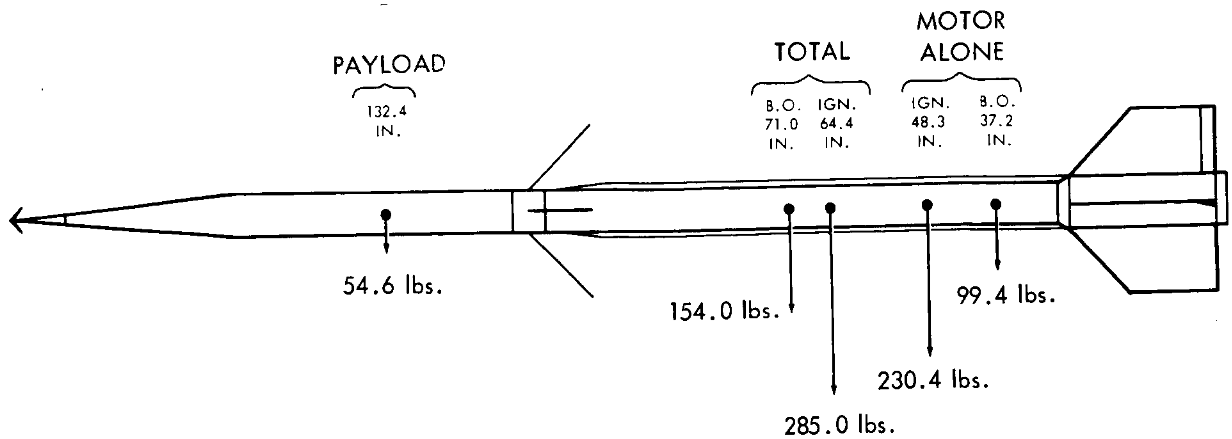
Consider the payload:



The following quantities derive from measurements made by the Test & Evaluation group at GSFC.

Element	Pitching Moment-of-Inertia about CG (slug-ft^2)	Weight (lb)	CG from Aft end (inches)
Payload	4.25	54.6	25.4

The center-of-gravity of the entire second stage, CG_T, may now be calculated as was shown in Table 2.



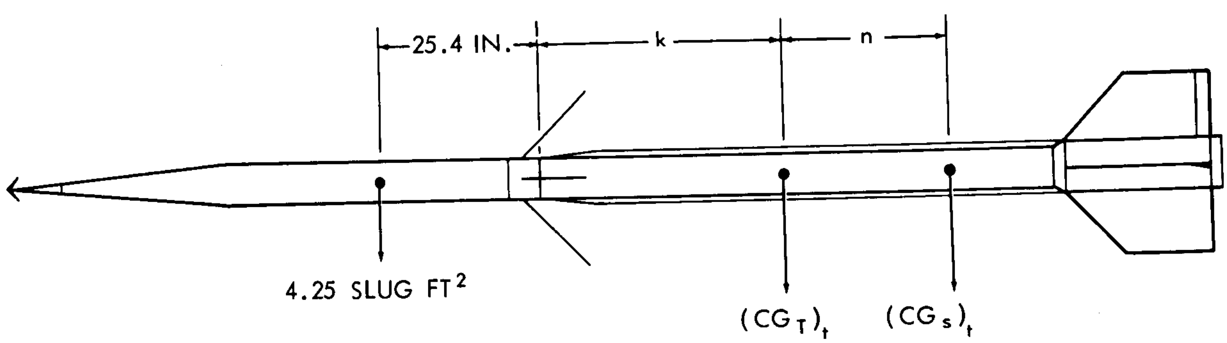
At ignition,

$$CG_T = 64.4 \text{ in (NEP)}.$$

At burnout

$$CG_T = 71.0 \text{ in (NEP)}.$$

Finally, transfer the moment-of-inertia to the center-of-gravity of the total Apache:



$$\left(I_{P_{CG}}\right)_{T_t} = \left(I_{P_{CG}}\right)_{p/1} + m_{p/1} \left(\frac{25.4 + k_t}{12}\right)^2 + \left(I_{P_{CG}}\right)_{S_t} + \left[m_s \left(\frac{n}{12}\right)^2\right]_t$$

where

$\left(I_{P_{CG}}\right)_{T_t}$ = pitching moment-of-inertia about CG of complete 14.28 GT rocket as a function of time

$\left(I_{P_{CG}}\right)_{p/1}$ = pitching-moment-of-inertia about CG of payload

$(m)_{p/1}$ = mass of payload = 54.6/32.2 slug ft²

$\left(I_{P_{CG}}\right)_{S_t}$ = pitching moment-of-inertia about CG of 14.28 GT sustainer as a function of time

m_{S_t} = mass of 14.28 GT sustainer as a function of time

$(k)_t$ = distance from CG complete 14.28 GT to head cap

$(n)_t$ = distance from CG of complete 14.28 GT to CG of 14.28 GT sustainer

At ignition,

$$\left(I_{P_{CG}}\right)_{Tign} = (4.25) + \left(\frac{54.6}{32.2}\right)\left[\frac{25.4 + (107.0 - 64.4)}{12}\right]^2$$

$$+ (61.5) + \left(\frac{2304}{32.2}\right)\left(\frac{64.4 - 48.3}{12}\right)^2$$

$$= (4.25) + (1.70)(32.5) + (61.5) + (7.16)(1.80)$$

$$= (4.25) + (55.25) + (61.5) + (12.9)$$

$$= 133.9 \text{ slug-ft}^2$$

At burnout,

$$\left(I_{P_{CG}}\right)_{Tbo} = (4.25) + \left(\frac{54.6}{32.2}\right)\left[\frac{25.4 + (107 - 71)}{12}\right]^2$$

$$+ (32.5) + \left(\frac{99.4}{32.2}\right)\left(\frac{71.0 - 37.2}{12}\right)^2$$

$$= (4.25 + (1.70)(26.2) + (32.5) + (3.09)(7.95)$$

$$= (4.25) + (44.5) + (32.5) + (24.6)$$

$$= 105.8 \text{ slug-ft}^2$$

Assume linear change in moment-of-inertia as a function of time during Apache burning.

Figure 29 shows this moment-of-inertia/time history. According to equation (1), the natural pitching frequency is given by

$$\omega = \sqrt{\frac{M_\alpha}{I_{P_{CG}}}}$$

Figure 28 shows M_α vs time.

Figure 29 shows $I_{P_{CG}}$ vs time.

Form Table 4.

Figure 15 shows the natural pitching frequency, ω, plotted as a function of time.

Table 4

ω as a Function of Time

t(sec)	ft. lbs/rad m_α	slug-ft^2 $I_{P_{CG}}$	rad^2/sec^2 $M_\alpha/I_{P_{CG}}$	rad/sec $\omega = \sqrt{M_\alpha/I_{P_{CG}}}$	cyc/sec $\omega = \sqrt{M_\alpha/I_{P_{CG}}}/2\pi$
3.5	128000	133.9	955.9	30.92	4.92
5.5	96700	133.9	722.2	26.87	4.27
6.5	85000	133.9	634.8	25.20	4.01
12.5	41900	133.9	312.9	17.69	2.81
17.5	24800	133.9	185.2	13.61	2.16
20.5	18000	133.9	134.4	11.59	1.84
22.44	14600	133.9	109.0	10.44	1.66
23.04	18600	131.2	141.8	11.91	1.89
23.54	20400	129.0	158.1	12.57	2.00
23.84	20900	127.7	163.7	12.79	2.03
24.04	21400	126.8	168.8	12.99	2.06
24.24	21800	125.9	173.2	13.16	2.09
24.64	22100	124.1	178.1	13.34	2.12
24.84	22300	123.3	180.8	13.45	2.14
24.94	22300	122.9	181.4	13.46	2.14
25.04	22300	122.4	182.2	13.50	2.15
25.14	22200	122.0	182.0	13.49	2.14
25.24	22400	121.5	184.4	13.58	2.16
25.44	22300	120.6	184.9	13.60	2.16
25.54	22300	120.2	185.5	13.62	2.16
25.64	22200	119.8	185.3	13.61	2.16
25.84	22000	118.9	185.0	13.60	2.16
26.04	21800	118.0	184.7	13.59	2.16
26.44	21100	116.2	181.6	13.48	2.14
26.64	20700	115.3	179.5	13.40	2.13
26.84	20300	114.5	177.3	13.32	2.12
27.04	19800	113.6	174.3	13.20	2.10
27.24	19200	112.7	170.4	13.05	2.07
27.64	18000	110.9	162.3	12.74	2.02
28.04	16600	109.1	152.2	12.34	1.96
28.24	15900	108.3	146.8	12.12	1.93
28.54	14700	106.9	137.5	11.73	1.87
28.80	13800	105.8	130.4	11.42	1.82
30.80	8475	105.8	80.10	8.950	1.42
32.80	5200	105.8	49.19	7.014	1.12
36.80	2160	105.8	20.41	4.518	0.719
40.80	972	105.8	9.19	3.032	0.483
45.80	469	105.8	4.43	2.105	0.335
50.80	174	105.8	1.64	1.281	0.204
34.8	3344	105.8	31.61	5.622	0.895