X-547-66-94

NASA TM X-55465

A LUNAR EPHEMERIS, WITH THE EARTH'S EQUATORIAL PLANE AS REFERENCE

BY

THEODORE L. FELSENTREGER

MARCH 1966

NASA

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-547-66-94

A LUNAR EPHEMERIS, WITH THE EARTH'S EQUATORIAL PLANE AS REFERENCE

Theodore L. Felsentreger

March 1966

Goddard Space Flight Center
Greenbelt, Maryland

A LUNAR EPHEMERIS, WITH THE EARTH'S EQUATORIAL PLANE AS REFERENCE

Theodore L. Felsentreger

SUMMARY

24401

Values for the angular orbital elements of the moon and their daily motions, with the earth's equatorial plane as reference, are given at 10-day intervals for a period of five years. In addition, the formulas used in the computations are derived. Results from harmonic analyses of the sine and cosine of the longitude of the node and inclination are also displayed, in addition to graphs which reveal the libratory natures of these two elements.

Author

CONTENTS

A LUNAR EPHEMERIS, WITH THE EARTH'S EQUATORIAL PLANE AS REFERENCE

INTRODUCTION

Most astronomers and navigators find it expedient to use the ecliptic as the basic reference plane in defining the angular orbital elements of the moon. For this reason, most ephemerides and nautical almanacs present formulas for and tables of the elements with reference to the ecliptic (see References 1 and 2).

However, with the advent of artificial satellites and a need for adequate knowledge of the forces perturbing their motions, many scientists investigating the effects of luni-solar forces on such satellites have found it more convenient to use, as the fundamental plane, the earth's equatorial plane (see References 3, 4, and 5). Consequently, it was felt that an ephemeris of the angular orbital elements of the moon referred to the earth's equatorial plane would be worthwhile.

This paper presents formulas for the computation of the following angular elements, along with their daily motions:

$\lambda_{\mathfrak{C}}$: mean longitude of the moon, measured in the earth's equatorial plane from the mean equinox of date to the mean ascending node of the moon's orbit, and then along the orbit.

$\omega_{\mathfrak{C}}$: argument of perigee of the moon's orbit, measured from the mean ascending node on the earth's equator.

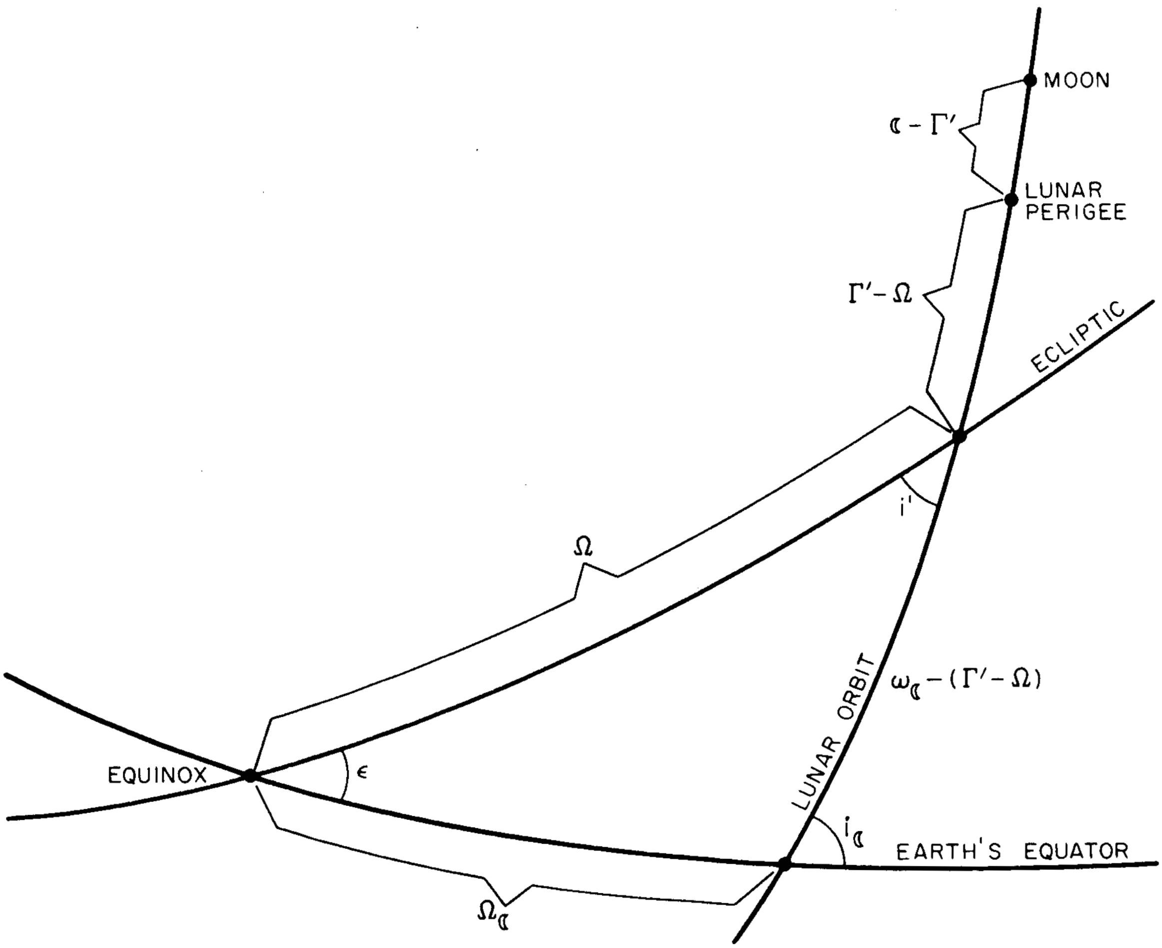
$\Omega_{\mathfrak{C}}$: longitude of the mean ascending node of the moon's orbit on the earth's equator, measured from the mean equinox of date.

$i_{\mathfrak{C}}$: inclination of the moon's orbital plane to the earth's equatorial plane.

In addition, the elements and their daily motions are given in tabular form at 10-day intervals for a period of 5 years, beginning January 1.0, 1964. Graphs are exhibited which show the oscillatory natures of $\Omega_{\mathfrak{C}}$ and $i_{\mathfrak{C}}$, the period of oscillation in each case being about 6800 days. Plots of $\sin \Omega_{\mathfrak{C}}$, $\cos \Omega_{\mathfrak{C}}$, $\sin i_{\mathfrak{C}}$, and cos $i_☾$ as functions of time are also presented, along with results from harmonic analyses of these functions. The harmonic analyses were undertaken in an effort to illustrate a method of handling terms in the lunar disturbing function whose periods are commensurate with those of $\Omega_☾$ and $i_☾$ (see also Reference 6).

FORMULAS

The following diagram illustrates the relations between the various angular elements:



where (see References 1 and 2)

ϵ = obliquity of the ecliptic
$= 23°452294 - 0°013013\ T - 0°000002\ T^2$

$☾$ = mean longitude of the moon, measured in the ecliptic from the mean equinox of date to the mean ascending node of the lunar orbit, and then along the orbit

= $270\overset{\circ}{.}43416 + 481267\overset{\circ}{.}88314\ T - 0\overset{\circ}{.}00113\ T^2$

Γ' = mean longitude of the lunar perigee, measured in the ecliptic from the mean equinox of date to the mean ascending node of the lunar orbit, and then along the orbit

= $334\overset{\circ}{.}32956 + 4069\overset{\circ}{.}03403\ T - 0\overset{\circ}{.}01033\ T^2 - 0\overset{\circ}{.}00001\ T^3$

Ω = longitude of the mean ascending node of the lunar orbit on the ecliptic, measured from the mean equinox of date

= $259\overset{\circ}{.}18328 - 1934\overset{\circ}{.}14201\ T + 0\overset{\circ}{.}00208\ T^2$

i' = inclination of the moon's orbital plane to the ecliptic

= $5\overset{\circ}{.}1453964$

T = number of Julian centuries since January 0.5, 1900.

Then,

$$\cos i_☾ = \cos i' \cos \epsilon - \sin i' \sin \epsilon \cos \Omega$$

$$\sin \Omega_☾ = \frac{\sin i' \sin \Omega}{\sin i_☾}$$

$$\cos \Omega_☾ = \frac{\cos i' - \cos \epsilon \cos i_☾}{\sin \epsilon \sin i_☾}$$

$$\sin [\omega_☾ - (\Gamma' - \Omega)] = \frac{\sin \epsilon \sin \Omega}{\sin i_☾}$$

$$\cos [\omega_☾ - (\Gamma' - \Omega)] = \frac{\cos \epsilon - \cos i' \cos i_☾}{\sin i' \sin i_☾}$$

$$\lambda_☾ = \Omega_☾ + \omega_☾ + (☾ - \Gamma').$$

In addition, the following expressions for the mean motions may be obtained by differentiating the preceding equations with respect to time:

$$\frac{di_{☾}}{dt} = \frac{\dot{\epsilon}(\cos i' \sin \epsilon + \sin i' \cos \epsilon \cos \Omega) - \dot{\Omega} \sin i' \sin \epsilon \sin \Omega}{\sin i_{☾}}$$

$$\dot{\Omega}_{☾} = \frac{\sin i'}{\sin i_{☾} \cos \Omega_{☾}} \left[\dot{\Omega} \cos \Omega - \frac{\cos i_{☾} \sin \Omega}{\sin i_{☾}} \frac{di_{☾}}{dt} \right]$$

$$\dot{\omega}_{☾} = \dot{\Gamma}' - \dot{\Omega} + \frac{1}{\sin i_{☾} \cos [\omega_{☾} - (\Gamma' - \Omega)]} \left[\dot{\epsilon} \cos \epsilon \sin \Omega + \dot{\Omega} \sin \epsilon \cos \Omega \right.$$

$$\left. - \frac{\sin \epsilon \cos i_{☾} \sin \Omega}{\sin i_{☾}} \frac{di_{☾}}{dt} \right]$$

$$\dot{\lambda}_{☾} = \dot{\Omega}_{☾} + \dot{\omega}_{☾} + (\dot{☾} - \dot{\Gamma}').$$

The table of values for $\lambda_{☾}$, $\omega_{☾}$, $\Omega_{☾}$, $i_{☾}$, and their daily motions may be found in the Appendix. Figures 1 and 2 (also in the Appendix) reveal the libratory natures of $\Omega_{☾}$ and $i_{☾}$, with a basic period of about 6800 days (the period of Ω).

HARMONIC ANALYSES

Figures 3, 4, 5, and 6 are plots of $\sin \Omega_{☾}$, $\cos \Omega_{☾}$, $\sin i_{☾}$, and $\cos i_{☾}$, respectively, as functions of time. Representations of each of these functions as a trigonometric series of the form

$$A_0 + \sum_{j=1}^{n} B_j \sin jx + \sum_{j=1}^{n} C_j \cos jx \quad (\text{period of } x \simeq 6800 \text{ days})$$

were obtained by least squares fits, with the following results (for $n = 10$):

$$\begin{aligned}\sin \Omega_{☾} = &- .00000062 + .04420680 \sin x + .22004150 \cos x - .02131752 \sin 2x \\ &+ .00892511 \cos 2x - .00213527 \sin 3x - .00315217 \cos 3x + .00048020 \sin 4x \\ &- .00048970 \cos 4x + .00010752 \sin 5x + .00007122 \cos 5x - .00001058 \sin 6x \\ &+ .00002353 \cos 6x - .00000594 \sin 7x - .00000095 \cos 7x - .00000087 \sin 8x \\ &- .00000103 \cos 8x - .00000052 \sin 9x - .00000005 \cos 9x + .00000004 \cos 10x\end{aligned}$$

$$\begin{aligned}\cos \Omega_{☾} = &\ .98714554 + .00262086 \sin x - .00052636 \cos x - .00475573 \sin 2x \\ &- .01135891 \cos 2x + .00212412 \sin 3x - .00143672 \cos 3x + .00036945 \sin 4x \\ &+ .00036334 \cos 4x - .00005682 \sin 5x + .00008739 \cos 5x - .00001938 \sin 6x \\ &- .00000786 \cos 6x + .00000101 \sin 7x - .00000424 \cos 7x + .00000109 \sin 8x \\ &- .00000001 \cos 8x + .00000022 \sin 9x + .00000018 \cos 9x + .00000002 \cos 10x\end{aligned}$$

$$\begin{aligned}\sin i_{☾} = &\ .40131012 + .08016325 \sin x - .01610321 \cos x + .00194116 \sin 2x \\ &+ .00462109 \cos 2x - .00042448 \sin 3x + .00029001 \cos 3x - .00004642 \sin 4x \\ &- .00004868 \cos 4x + .00000815 \sin 5x - .00000876 \cos 5x + .00000357 \sin 6x \\ &+ .00000064 \cos 6x + .00000164 \sin 7x + .00000029 \cos 7x + .00000142 \sin 8x \\ &+ .00000129 \sin 9x - .00000001 \cos 9x\end{aligned}$$

$$\begin{aligned}\cos i_{☾} = &\ .91376036 - .03498530 \sin x + .00702753 \cos x - .00000259 \sin 2x \\ &+ .00000038 \cos 2x - .00000172 \sin 3x + .00000015 \cos 3x - .00000129 \sin 4x \\ &+ .00000008 \cos 4x - .00000102 \sin 5x + .00000005 \cos 5x - .00000085 \sin 6x \\ &+ .00000004 \cos 6x - .00000072 \sin 7x + .00000003 \cos 7x - .00000062 \sin 8x \\ &+ .00000003 \cos 8x - .00000055 \sin 9x + .00000002 \cos 9x + .00000002 \cos 10x.\end{aligned}$$

Approximately 5-place accuracy was obtained with these representations. Obviously, more accuracy requires retention of more terms in the series. In each case, the residuals show a period of about 600-700 days, which is the period of $10x$.

REFERENCES

1. The American Ephemeris and Nautical Almanac, U. S. Government Printing Office, Washington, D.C.

2. Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac, Her Majesty's Stationery Office, 1961.

3. Kozai, Y.,"On the Effects of the Sun and the Moon Upon the Motion of a Close Earth Satellite," Smithsonian Institution Astrophysical Observatory Special Report No. 22, March 20, 1959.

4. Musen, P., Bailie, A., and Upton, E., "Development of the Lunar and Solar Perturbations in the Motion of an Artificial Satellite," NASA TN-D-494, January, 1961.

5. Murphy, J. P., and Felsentreger, T. L., "An Analysis of Lunar and Solar Effects on the Motion of Close Earth Satellites," GSFC X-547-65-251, June, 1965.

6. Felsentreger, T. L., "Long Period Lunar and Solar Effects on the Motion of Relay 2," GSFC X Report in publication.

APPENDIX

Table 1

Date 1964	$\lambda_{☾}$	$\dot{\lambda}_{☾}$	$\omega_{☾}$	$\dot{\omega}_{☾}$	$\Omega_{☾}$	$\dot{\Omega}_{☾} \times 10^2$	$i_{☾}$	$\frac{di_{☾}}{dt} \times 10^2$
Jan. 1	116.33439	13.176602	46.47006	.11174141	13.026741	-.01318805	22.959541	.47484565
11	248.10035	13.176592	47.58796	.11184214	13.024866	-.02430912	23.007023	.47477841
21	19.86620	13.176583	48.70687	.11194367	13.021882	-.03536334	23.054495	.47467142
31	151.63199	13.176573	49.82681	.11204405	13.017797	-.04635028	23.101954	.47452500
Feb. 10	283.39767	13.176564	50.94775	.11214420	13.012614	-.05726961	23.149398	.47433929
20	55.16325	13.176555	52.06967	.11224376	13.006344	-.06812112	23.196822	.47411457
Mar. 1	186.92875	13.176546	53.19260	.11234254	12.998992	-.07890445	23.244220	.47385109
11	318.69416	13.176537	54.31651	.11244065	12.990563	-.08961937	23.291592	.47354910
21	90.45949	13.176528	55.44141	.11253802	12.981070	-.10026569	23.338929	.47320885
31	222.22472	13.176519	56.56727	.11263477	12.970514	-.11084313	23.386231	.47283055
Apr. 10	353.98980	13.176510	57.69408	.11273085	12.958904	-.12135152	23.433494	.47241443
20	125.75491	13.176501	58.82186	.11282617	12.946247	-.13179673	23.480713	.47196080
30	257.51988	13.176492	59.95061	.11292072	12.932547	-.14216044	23.527884	.47146983
May 10	29.28475	13.176483	61.08028	.11301461	12.917815	-.15246058	23.575006	.47094180
20	161.04953	13.176474	62.21089	.11310788	12.902056	-.16269103	23.622072	.47037692
30	292.81422	13.176464	63.34243	.11320037	12.885281	-.17285170	23.669079	.46977552
June 9	64.57883	13.176455	64.47490	.11329214	12.867491	-.18294235	23.716025	.46913775
19	196.34334	13.176446	65.60828	.11338324	12.848695	-.19296297	23.762906	.46846388
29	328.10776	13.176437	66.74256	.11347365	12.828899	-.20291344	23.809717	.46775416
July 9	99.87207	13.176429	67.87774	.11356336	12.808114	-.21279374	23.856455	.46700883
19	231.63631	13.176419	69.01382	.11365233	12.786342	-.22260386	23.903119	.46622813
29	3.40046	13.176410	70.15078	.11374058	12.763596	-.23234370	23.949700	.46541233
Aug. 8	135.16450	13.176401	71.28861	.11382820	12.739877	-.24201315	23.996200	.46456163
18	266.92847	13.176391	72.42735	.11391499	12.715196	-.25161241	24.042611	.46367632
28	38.69233	13.176382	73.56691	.11400119	12.689556	-.26114121	24.088933	.46275660
Sept. 7	170.45611	13.176373	74.70736	.11408663	12.662969	-.27059966	24.135161	.46180274
17	302.21979	13.176364	75.84865	.11417141	12.635439	-.27998788	24.181294	.46081492
27	73.98340	13.176355	76.99079	.11425541	12.606975	-.28930583	24.227323	.45979350
Oct. 7	205.74691	13.176346	78.13376	.11433874	12.577582	-.29855347	24.273250	.45873863
17	337.51032	13.176337	79.27756	.11442140	12.547266	-.30773096	24.319071	.45765055
27	109.27363	13.176328	80.42217	.11450338	12.516037	-.31683830	24.364779	.45652954
Nov. 6	241.03687	13.176318	81.56762	.11458459	12.483901	-.32587553	24.410374	.45537584
16	12.80001	13.176309	82.71387	.11466514	12.450862	-.33484278	24.455854	.45418965
26	144.56305	13.176300	83.86091	.11474503	12.416934	-.34374008	24.501211	.45297125
Dec. 6	276.32601	13.176291	85.00876	.11482421	12.382119	-.35256746	24.546447	.45172086
16	48.08888	13.176282	86.15740	.11490268	12.346422	-.36132519	24.591555	.45043870
26	179.85165	13.176273	87.30680	.11498047	12.309856	-.37001330	24.636533	.44912505

Angles are in degrees, mean motions are in deg/day
All dates are at 0 hrs. U.T.

Table 2

Date 1965		$\lambda_{☾}$	$\dot{\lambda}_{☾}$	$\omega_{☾}$	$\dot{\omega}_{☾}$	$\Omega_{☾}$	$\dot{\Omega}_{☾} \times 10^2$	$i_{☾}$	$\frac{di_{☾}}{dt} \times 10^2$
Jan.	5	311.61435	13.176264	88.45701	.11505754	12.272423	-.37863184	24.681378	.44778015
	15	83.37694	13.176255	89.60796	.11513398	12.234132	-.38718105	24.726088	.44640417
	25	215.13944	13.176246	90.75968	.11520971	12.194989	-.39566087	24.770658	.44499739
Feb.	4	346.90185	13.176237	91.91214	.11528477	12.155002	-.40407156	24.815087	.44356006
	14	118.66418	13.176227	93.06537	.11535912	12.114177	-.41241329	24.859370	.44209240
	24	250.42640	13.176218	94.21933	.11543284	12.072521	-.42068607	24.903505	.44059463
Mar.	6	22.18855	13.176209	95.37402	.11550586	12.030042	-.42889015	24.947488	.43906705
	16	153.95062	13.176201	96.52945	.11557819	11.986746	-.43702574	24.991316	.43750983
	26	285.71257	13.176192	97.68559	.11564988	11.942639	-.44509289	25.034988	.43592319
Apr.	5	57.47444	13.176183	98.84245	.11572088	11.897729	-.45309176	25.078499	.43430740
	15	189.23622	13.176174	100.00001	.11579124	11.852023	-.46102257	25.121849	.43266267
	25	320.99791	13.176165	101.15826	.11586094	11.805526	-.46888541	25.165033	.43098924
May	5	92.75953	13.176156	102.31722	.11592995	11.758248	-.47668057	25.208046	.42928735
	15	224.52103	13.176147	103.47685	.11599834	11.710193	-.48440824	25.250889	.42755722
	25	356.28246	13.176138	104.63718	.11606605	11.661369	-.49206885	25.293556	.42579912
June	4	128.04379	13.176129	105.79818	.11613312	11.611783	-.49966156	25.336047	.42401323
	14	259.80504	13.176120	106.95983	.11619955	11.561438	-.50718767	25.378358	.42219973
	24	31.56621	13.176112	108.12217	.11626530	11.510346	-.51464698	25.420486	.42035897
July	4	163.32728	13.176103	109.28515	.11633044	11.458512	-.52203971	25.462428	.41849110
	14	295.08826	13.176094	110.44878	.11639492	11.405940	-.52936604	25.504184	.41659634
	24	66.84916	13.176085	111.61304	.11645877	11.352640	-.53662623	25.545746	.41467497
Aug.	3	198.60997	13.176077	112.77795	.11652198	11.298618	-.54382035	25.587117	.41272717
	13	330.37069	13.176068	113.94348	.11658457	11.243878	-.55094875	25.628292	.41075315
	23	102.13132	13.176059	115.10963	.11664652	11.188430	-.55801152	25.669268	.40875320
Sept.	2	233.89187	13.176051	116.27641	.11670785	11.132280	-.56500897	25.710042	.40672749
	12	5.65232	13.176042	117.44378	.11676856	11.075430	-.57194133	25.750613	.40467625
	22	137.41270	13.176033	118.61177	.11682864	11.017892	-.57880870	25.790977	.40259971
Oct.	2	269.17300	13.176025	119.78035	.11688809	10.959672	-.58561145	25.831131	.40049811
	12	40.93320	13.176017	120.94953	.11694694	10.900771	-.59234966	25.871075	.39837163
	22	172.69332	13.176008	122.11929	.11700518	10.841203	-.59902354	25.910805	.39622051
Nov.	1	304.45334	13.175999	123.28963	.11706281	10.780969	-.60563346	25.950319	.39404500
	11	76.21330	13.175991	124.46053	.11711983	10.720078	-.61217953	25.989614	.39184527
	21	207.97317	13.175982	125.63202	.11717626	10.658537	-.61866191	26.028687	.38962156
Dec.	1	339.73294	13.175974	126.80406	.11723207	10.596348	-.62508099	26.067536	.38737410
	11	111.49264	13.175966	127.97666	.11728730	10.533521	-.63143688	26.106161	.38510305
	21	243.25226	13.175958	129.14982	.11734191	10.470063	-.63772985	26.144557	.38280872
	31	15.01179	13.175949	130.32350	.11739594	10.405979	-.64396012	26.182722	.38049126

Table 3

Date 1966		$\lambda_{\mathbb{C}}$	$\dot{\lambda}_{\mathbb{C}}$	$\omega_{\mathbb{C}}$	$\dot{\omega}_{\mathbb{C}}$	$\Omega_{\mathbb{C}}$	$\dot{\Omega}_{\mathbb{C}} \times 10^2$	$i_{\mathbb{C}}$	$\frac{di_{\mathbb{C}}}{dt} \times 10^2$
Jan.	10	146.77124	13.175941	131.49773	.11744940	10.341272	-.65012788	26.220654	.37815089
	20	278.53060	13.175932	132.67248	.11750226	10.275953	-.65623334	26.258351	.37578783
	30	50.28990	13.175925	133.84777	.11755454	10.210028	-.66227686	26.295811	.37340233
Feb.	9	182.04910	13.175917	135.02358	.11760624	10.143501	-.66825851	26.333031	.37099454
	19	313.80822	13.175908	136.19990	.11765736	10.076378	-.67417852	26.370009	.36856475
Mar.	1	85.56727	13.175900	137.37672	.11770792	10.008667	-.68003720	26.406743	.36611310
	11	217.32623	13.175892	138.55404	.11775790	9.940374	-.68583477	26.443231	.36363982
	21	349.08512	13.175885	139.73188	.11780731	9.871503	-.69157141	26.479470	.36114515
	31	120.84392	13.175876	140.91019	.11785616	9.802062	-.69724740	26.515459	.35862930
Apr.	10	252.60264	13.175869	142.08900	.11790445	9.732055	-.70286285	26.551195	.35609242
	20	24.36129	13.175861	143.26828	.11795218	9.661491	-.70841809	26.586676	.35353480
	30	156.11985	13.175853	144.44803	.11799936	9.590375	-.71391335	26.621900	.35095660
May	10	287.87835	13.175845	145.62826	.11804599	9.518710	-.71934871	26.656868	.34835803
	20	59.63676	13.175838	146.80895	.11809207	9.446506	-.72472449	26.691573	.34573933
	30	191.39511	13.175830	147.99011	.11813758	9.373768	-.73004093	26.726015	.34310067
June	9	323.15338	13.175822	149.17170	.11818257	9.300500	-.73529828	26.760192	.34044228
	19	94.91155	13.175815	150.35375	.11822701	9.226709	-.74049665	26.794103	.33776436
	29	226.66967	13.175807	151.53624	.11827091	9.152403	-.74563637	26.827744	.33506712
July	9	358.42770	13.175800	152.71917	.11831428	9.077585	-.75071757	26.861115	.33235075
	19	130.18566	13.175792	153.90252	.11835711	9.002261	-.75574040	26.894214	.32961547
	29	261.94355	13.175786	155.08631	.11839942	8.926438	-.76070521	26.927038	.32686149
Aug.	8	33.70137	13.175778	156.27051	.11844119	8.850122	-.76561228	26.959585	.32408900
	18	165.45910	13.175771	157.45513	.11848246	8.773318	-.77046153	26.991855	.32129823
	28	297.21677	13.175763	158.64016	.11852319	8.696032	-.77525341	27.023844	.31848934
Sept.	7	68.97437	13.175756	159.82559	.11856340	8.618269	-.77998809	27.055551	.31566255
	17	200.73190	13.175749	161.01142	.11860310	8.540036	-.78466569	27.086976	.31281807
	27	332.48936	13.175742	162.19766	.11864229	8.461338	-.78928649	27.118115	.30995610
Oct.	7	104.24675	13.175736	163.38427	.11868095	8.382180	-.79385066	27.148967	.30707682
	17	236.00405	13.175728	164.57126	.11871912	8.302569	-.79835845	27.179530	.30418049
	27	7.76131	13.175721	165.75965	.11875678	8.222510	-.80281000	27.209803	.30126726
Nov.	6	139.51850	13.175715	166.94641	.11879394	8.142010	-.80720557	27.239782	.29833734
	16	271.27560	13.175708	168.13452	.11883061	8.061072	-.81154525	27.269469	.29539093
	26	43.03265	13.175701	169.32301	.11886676	7.979703	-.81582939	27.298860	.29242824
Dec.	6	274.78964	13.175695	170.51186	.11890243	7.897908	-.82005799	27.327954	.28944942
	16	306.54655	13.175688	171.70107	.11893760	7.815693	-.82423139	27.356749	.28645474
	26	78.30340	13.175681	172.89062	.11897229	7.733063	-.82834972	27.385245	.28344436

Table 4

Date 1967		$\lambda_{☾}$	$\dot{\lambda}_{☾}$	$\omega_{☾}$	$\dot{\omega}_{☾}$	$\Omega_{☾}$	$\dot{\Omega}_{☾} \times 10^2$	$i_{☾}$	$\frac{di_{☾}}{dt} \times 10^2$
Jan.	5	210.06017	13.175675	174.08051	.11900648	7.650024	-.83241326	27.413438	.28041847
	15	341.81689	13.175669	175.27074	.11904019	7.566582	-.83642203	27.441328	.27737729
	25	113.57356	13.175663	176.46131	.11907342	7.482743	-.84037632	27.468912	.27432104
Feb.	4	245.33015	13.175656	177.65221	.11910617	7.398509	-.84427633	27.496190	.27124984
	14	17.08668	13.175650	178.84344	.11913843	7.313889	-.84812219	27.523161	.26816396
	24	148.84315	13.175644	180.03498	.11917023	7.228886	-.85191399	27.549824	.26506354
Mar.	6	280.59956	13.175638	181.22683	.11920155	7.143508	-.85565216	27.576174	.26194879
	16	52.35590	13.175632	182.41900	.11923240	7.057758	-.85933662	27.602214	.25881995
	26	184.11219	13.175626	183.61148	.11926278	6.971642	-.86296765	27.627938	.25567716
Apr.	5	315.86842	13.175620	184.80426	.11929271	6.885166	-.86654550	27.653348	.25252064
	15	87.62459	13.175614	185.99733	.11932216	6.798334	-.87007015	27.678441	.24935059
	25	219.38071	13.175608	187.19070	.11935114	6.711154	-.87354184	27.703218	.24616720
May	5	351.13677	13.175603	188.38436	.11937967	6.623628	-.87696083	27.727675	.24297063
	15	122.89276	13.175597	189.57829	.11940774	6.535763	-.88032723	27.751811	.23976112
	25	254.64871	13.175592	190.77250	.11943536	6.447565	-.88364118	27.775626	.23653885
June	4	26.40460	13.175587	191.96700	.11946251	6.359037	-.88690279	27.799118	.23330398
	14	158.16043	13.175581	193.16176	.11948923	6.270186	-.89011220	27.822287	.23005673
	24	289.91621	13.175575	194.35678	.11951549	6.181016	-.89326969	27.845129	.22679731
July	4	61.67194	13.175570	195.55207	.11954130	6.091533	-.89637533	27.867646	.22352587
	14	193.42763	13.175565	196.74761	.11956667	6.001743	-.89942935	27.889834	.22024265
	24	325.18325	13.175560	197.94340	.11959160	5.911650	-.90243180	27.911693	.21694782
Aug.	3	96.93884	13.175555	199.13944	.11961609	5.821258	-.90538278	27.933223	.21364154
	13	228.69435	13.175550	200.33573	.11964013	5.730575	-.90828251	27.954421	.21032403
	23	0.44982	13.175546	201.53223	.11966375	5.639603	-.91113109	27.975288	.20699549
Sept.	2	132.20526	13.175540	202.72899	.11968691	5.548349	-.91392876	27.995820	.20365607
	12	263.96063	13.175536	203.92597	.11970966	5.456819	-.91667555	28.016019	.20030601
	22	35.71597	13.175531	205.12319	.11973196	5.365017	-.91937163	28.035882	.19694547
Oct.	2	167.47125	13.175526	206.32061	.11975382	5.272946	-.92201713	28.055408	.19357464
	12	299.22649	13.175522	207.51826	.11977527	5.180614	-.92461222	28.074596	.19019373
	22	70.98170	13.175517	208.71612	.11979629	5.088026	-.92715684	28.093447	.18680293
Nov.	1	202.73685	13.175514	209.91418	.11981688	4.995185	-.92965136	28.111957	.18340238
	11	334.49196	13.175509	211.11245	.11983705	4.902097	-.93209583	28.130126	.17999233
	21	106.24702	13.175505	212.31092	.11985679	4.808768	-.93449026	28.147954	.17657293
Dec.	1	238.00205	13.175501	213.50958	.11987612	4.715200	-.93683480	28.165442	.17314435
	11	9.75703	13.175496	214.70844	.11989502	4.621402	-.93912978	28.182583	.16970685
	21	141.51198	13.175493	215.90748	.11991351	4.527376	-.94137493	28.199382	.16626056
	31	273.26689	13.175489	217.10671	.11993158	4.433129	-.94357076	28.215834	.16280568

Table 5

Date 1968		$\lambda_{☾}$	$\dot{\lambda}_{☾}$	$\omega_{☾}$	$\dot{\omega}_{☾}$	$\Omega_{☾}$	$\dot{\Omega}_{☾} \times 10^2$	$i_{☾}$	$\frac{di_{☾}}{dt} \times 10^2$
Jan.	10	45.02175	13.175485	218.30612	.11994924	4.338664	-.94571700	28.231943	.15934240
	20	176.77657	13.175482	219.50569	.11996648	4.243988	-.94781400	28.247703	.15587094
	30	308.53138	13.175478	220.70544	.11998331	4.149103	-.94986172	28.263117	.15239143
Feb.	9	80.28614	13.175474	221.90536	.11999973	4.054016	-.95186044	28.278182	.14890406
	19	212.04086	13.175470	223.10544	.12001573	3.958733	-.95381017	28.292897	.14540906
	29	343.79554	13.175467	224.30567	.12003133	3.863256	-.95571095	28.307263	.14190661
Mar.	10	115.55019	13.175464	225.50606	.12004653	3.767592	-.95756297	28.321278	.13839684
	20	247.30482	13.175461	226.70660	.12006132	3.671745	-.95936621	28.334942	.13488002
	30	19.05940	13.175458	227.90728	.12007570	3.575720	-.96112076	28.348255	.13135628
Apr.	9	150.81397	13.175454	229.10812	.12008968	3.479523	-.96282686	28.361213	.12782583
	19	282.56850	13.175451	230.30908	.12010325	3.383157	-.96448443	28.373820	.12428884
	29	54.32298	13.175448	231.51018	.12011643	3.286628	-.96609374	28.386070	.12074552
May	9	186.07746	13.175446	232.71141	.12012921	3.189940	-.96765469	28.397907	.11719601
	19	317.83190	13.175443	233.91277	.12014159	3.093098	-.96916731	28.409509	.11364054
	29	89.58631	13.175440	235.11423	.12015357	2.996108	-.97063185	28.420696	.11007927
June	8	221.34071	13.175438	236.31583	.12016515	2.898973	-.97204828	28.431526	.10651239
	18	353.09506	13.175435	237.51754	.12017634	2.801699	-.97341678	28.441998	.10294011
	28	124.84940	13.175432	238.71936	.12018713	2.704292	-.97473726	28.452113	.09936263
July	8	256.60371	13.175430	239.92129	.12019753	2.606754	-.97600994	28.461870	.09578004
	18	28.35800	13.175428	241.12330	.12020753	2.509091	-.97723473	28.471269	.09219262
	28	160.11228	13.175426	242.32543	.12021714	2.411309	-.97841179	28.480309	.08860051
Aug.	7	291.86652	13.175423	243.52765	.12022636	2.313410	-.97954113	28.488988	.08500389
	17	63.62075	13.175422	244.72996	.12023519	2.215402	-.98062283	28.497310	.08140298
	27	195.37495	13.175420	245.93235	.12024362	2.117288	-.98165704	28.505269	.07779795
Sept.	6	327.12914	13.175418	247.13483	.12025167	2.019072	-.98264366	28.512868	.07418896
	16	98.88331	13.175416	248.33738	.12025934	1.920760	-.98358276	28.520107	.07057623
	26	230.63748	13.175415	249.54002	.12026660	1.822358	-.98447444	28.526983	.06695996
Oct.	6	2.39161	13.175413	250.74271	.12027349	1.723867	-.98531868	28.533500	.06334025
	16	134.14574	13.175412	251.94549	.12027998	1.625295	-.98611564	28.539652	.05971736
	26	265.89985	13.175410	253.14832	.12028608	1.526646	-.98686529	28.545441	.05609146
Nov.	5	37.65394	13.175409	254.35121	.12029181	1.427923	-.98756759	28.550870	.05246269
	15	169.40802	13.175408	255.55415	.12029715	1.329133	-.98822268	28.555935	.04883131
	25	301.16209	13.175407	256.75714	.12030210	1.230281	-.98883063	28.560635	.04519745
Dec.	5	72.91616	13.175406	257.96018	.12030666	1.131369	-.98939135	28.564974	.04156129
	15	204.67021	13.175405	259.16327	.12031086	1.032404	-.98990492	28.568948	.03792306
	25	336.42425	13.175404	260.36641	.12031465	0.933390	-.99037135	28.572558	.03428294
	35	108.17828	13.175403	261.56957	.12031806	0.834331	-.99079063	28.575805	.03064104

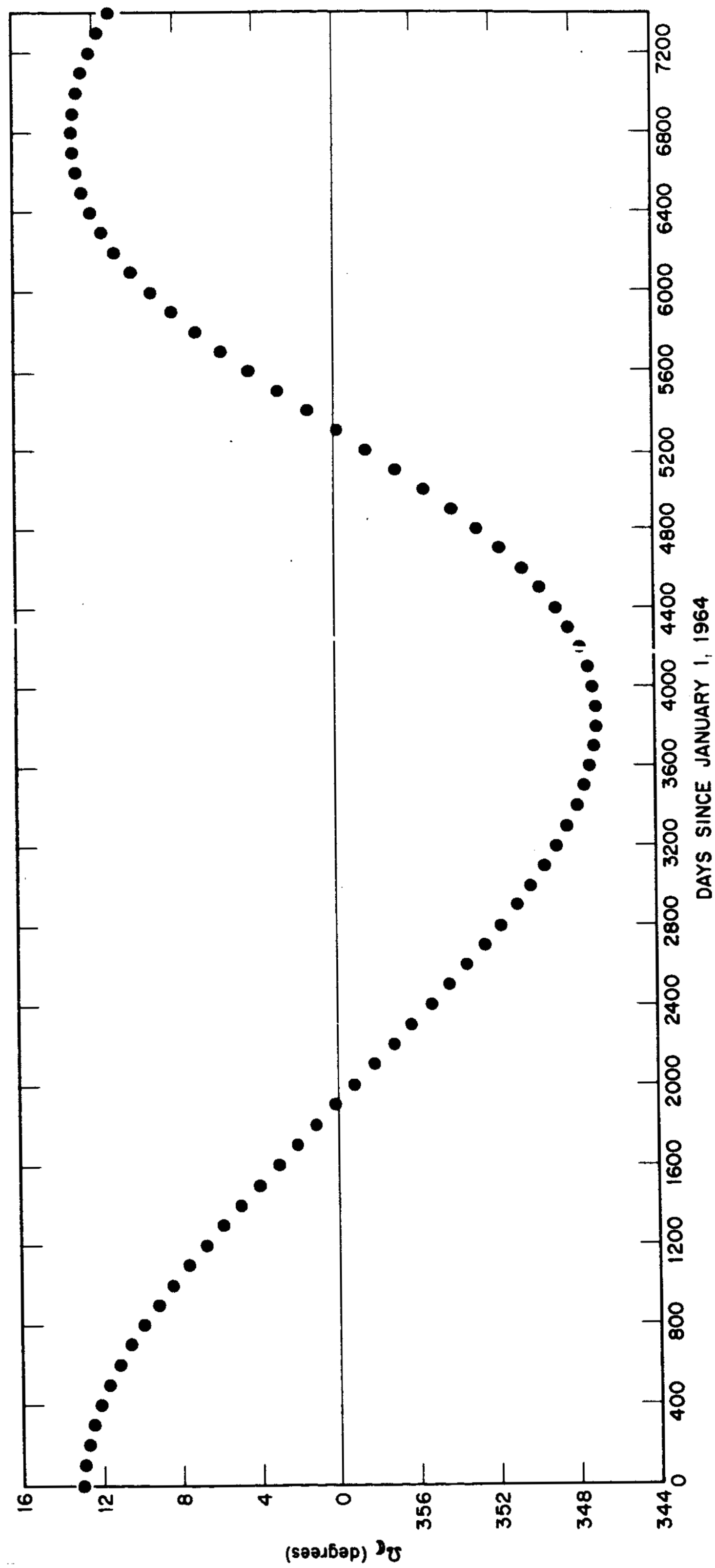


Figure 1—$\Omega_{☾}$ vs. time.

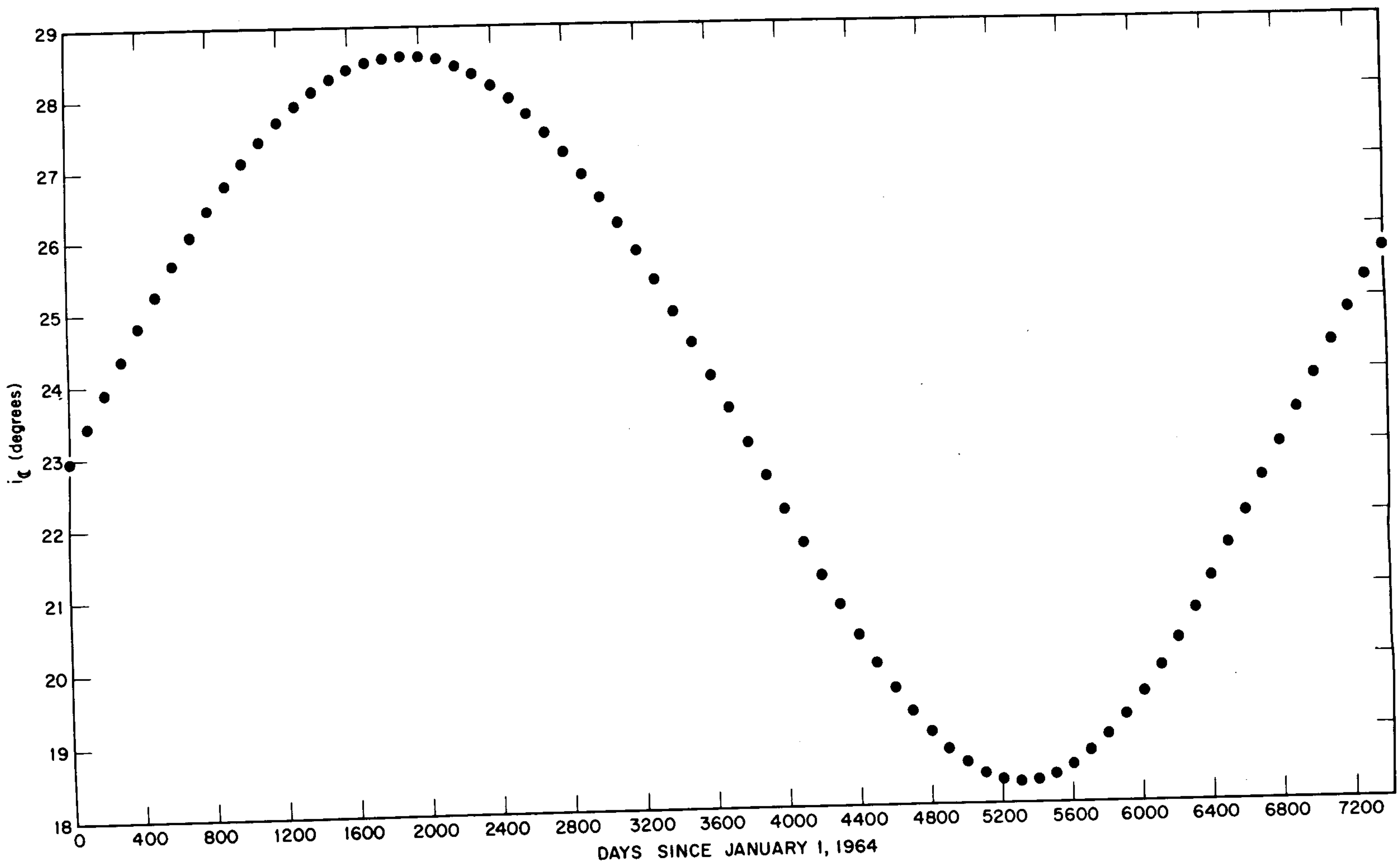


Figure 2–$i_{☾}$ vs. time.

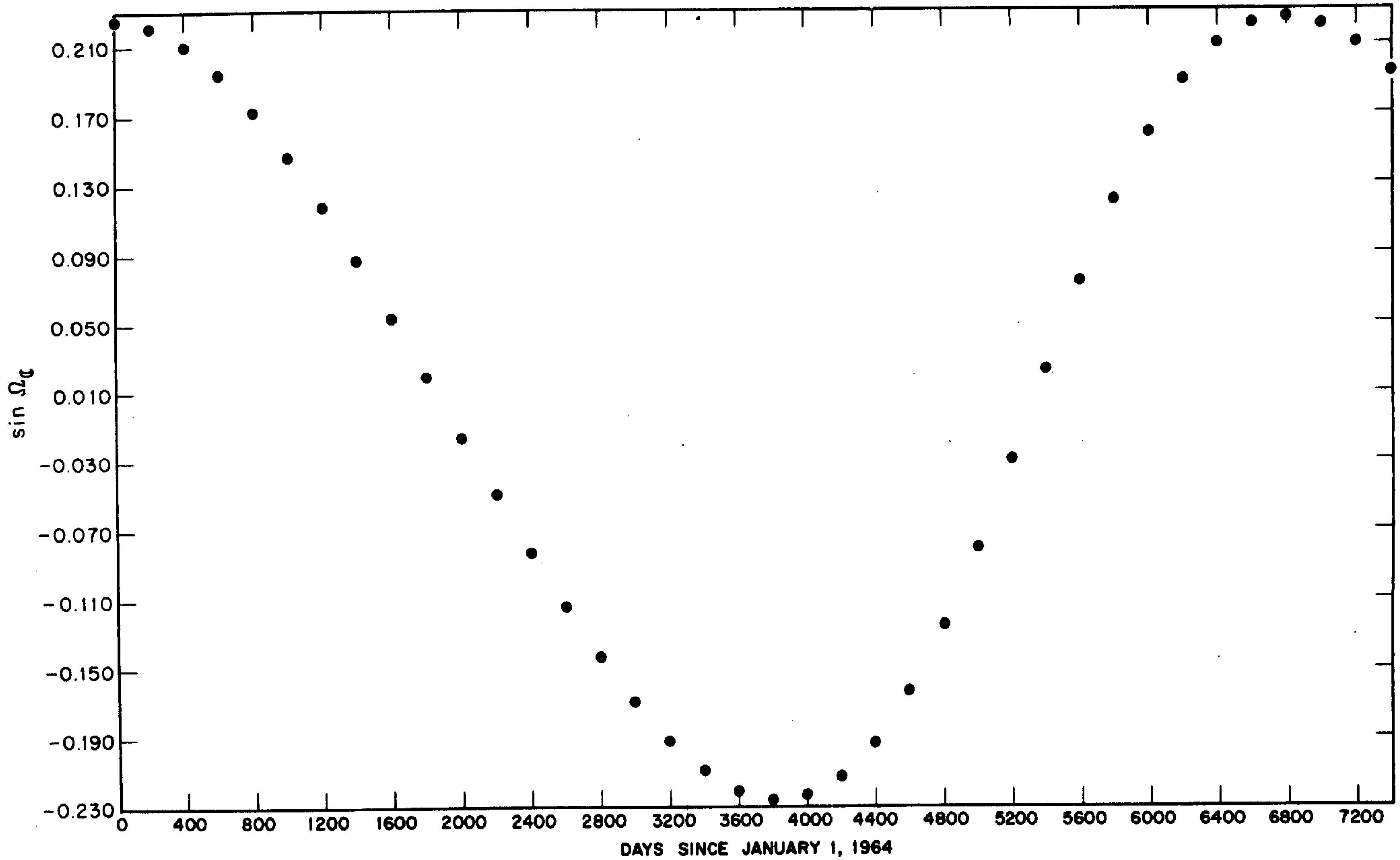


Figure 3–$\sin \Omega_{\mathbb{C}}$ vs. time.

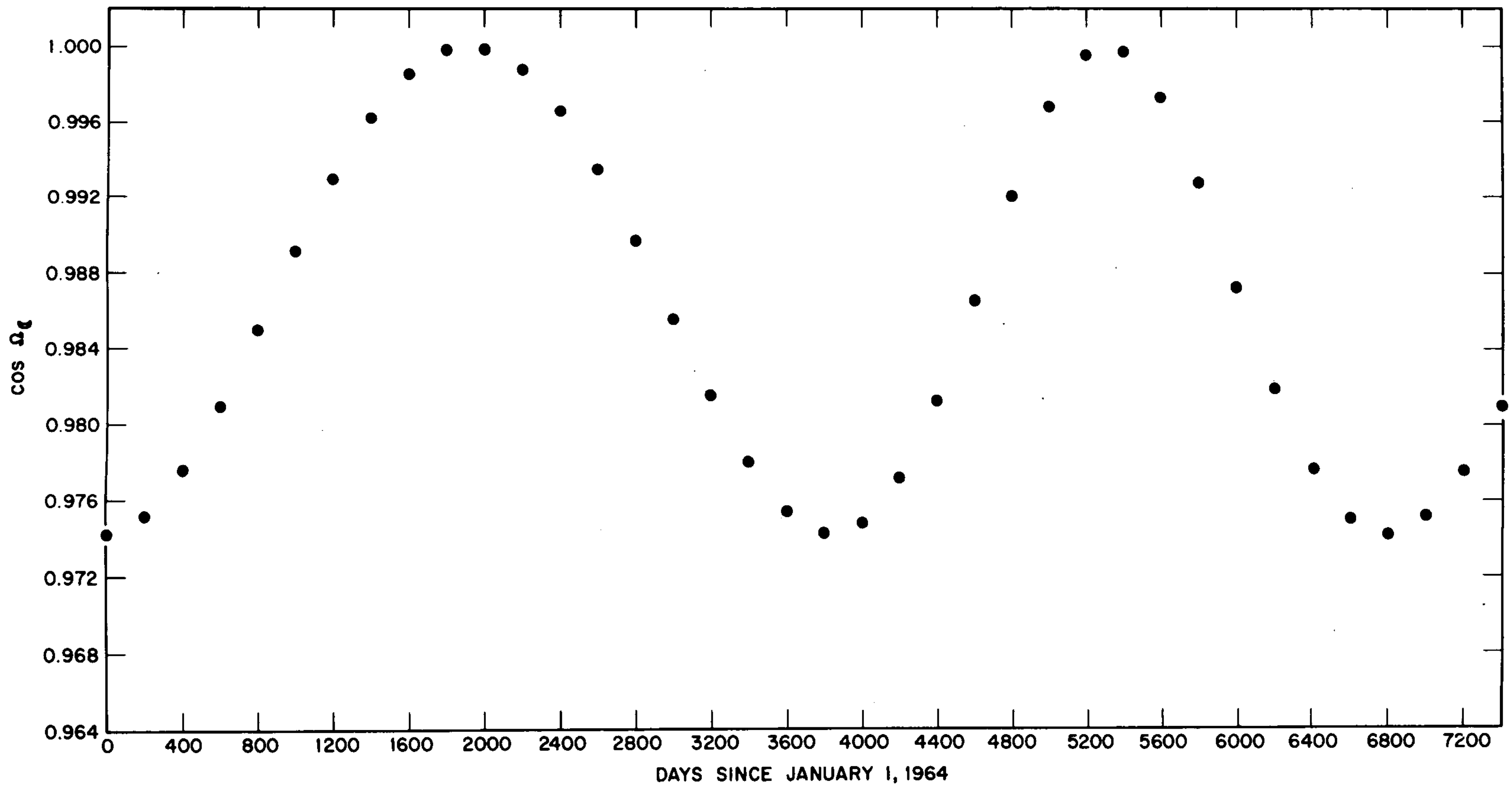


Figure 4—cos $\Omega_{\mathbb{C}}$ vs. time.

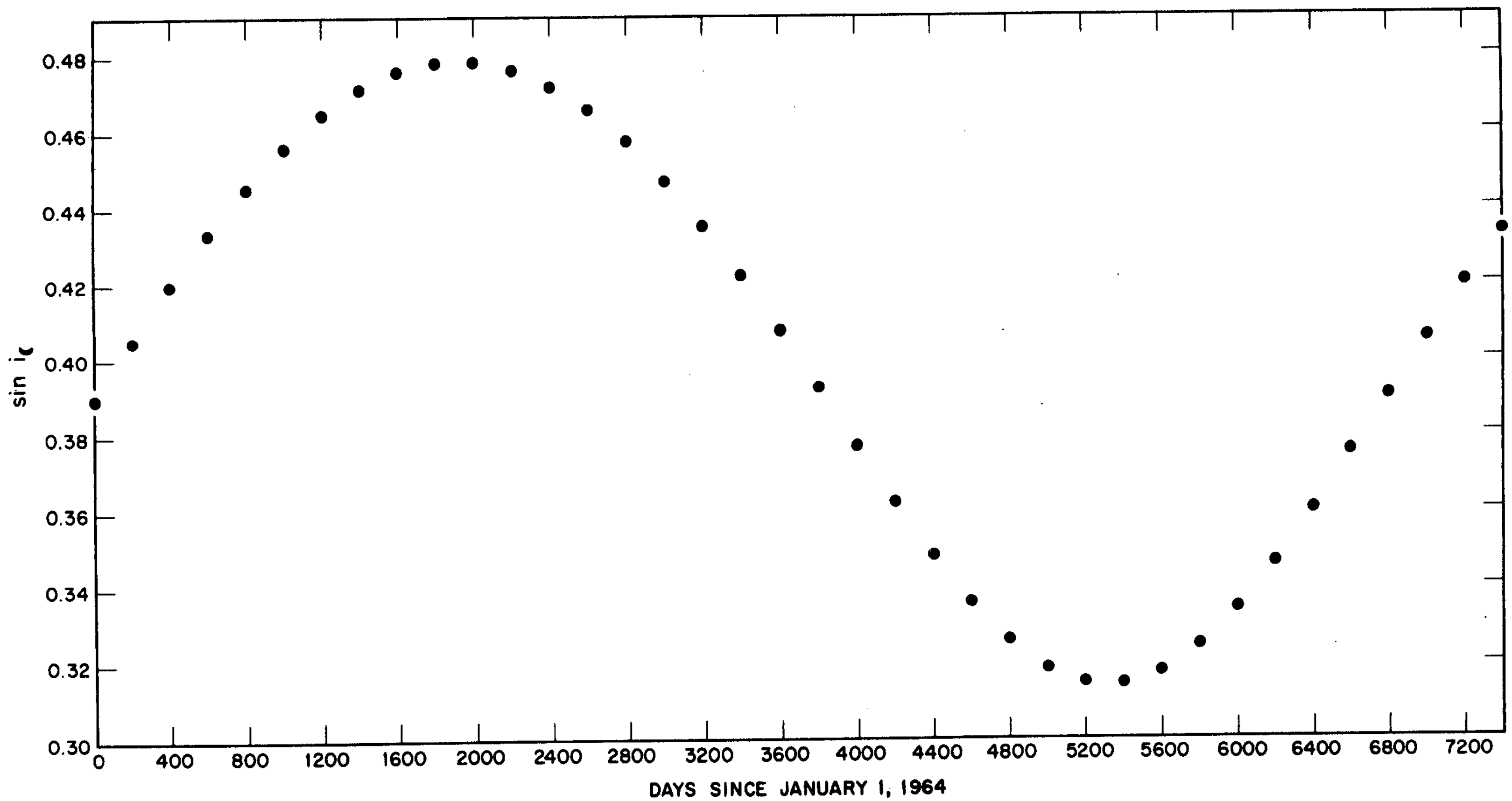


Figure 5–sin $i_{☾}$ vs. time.

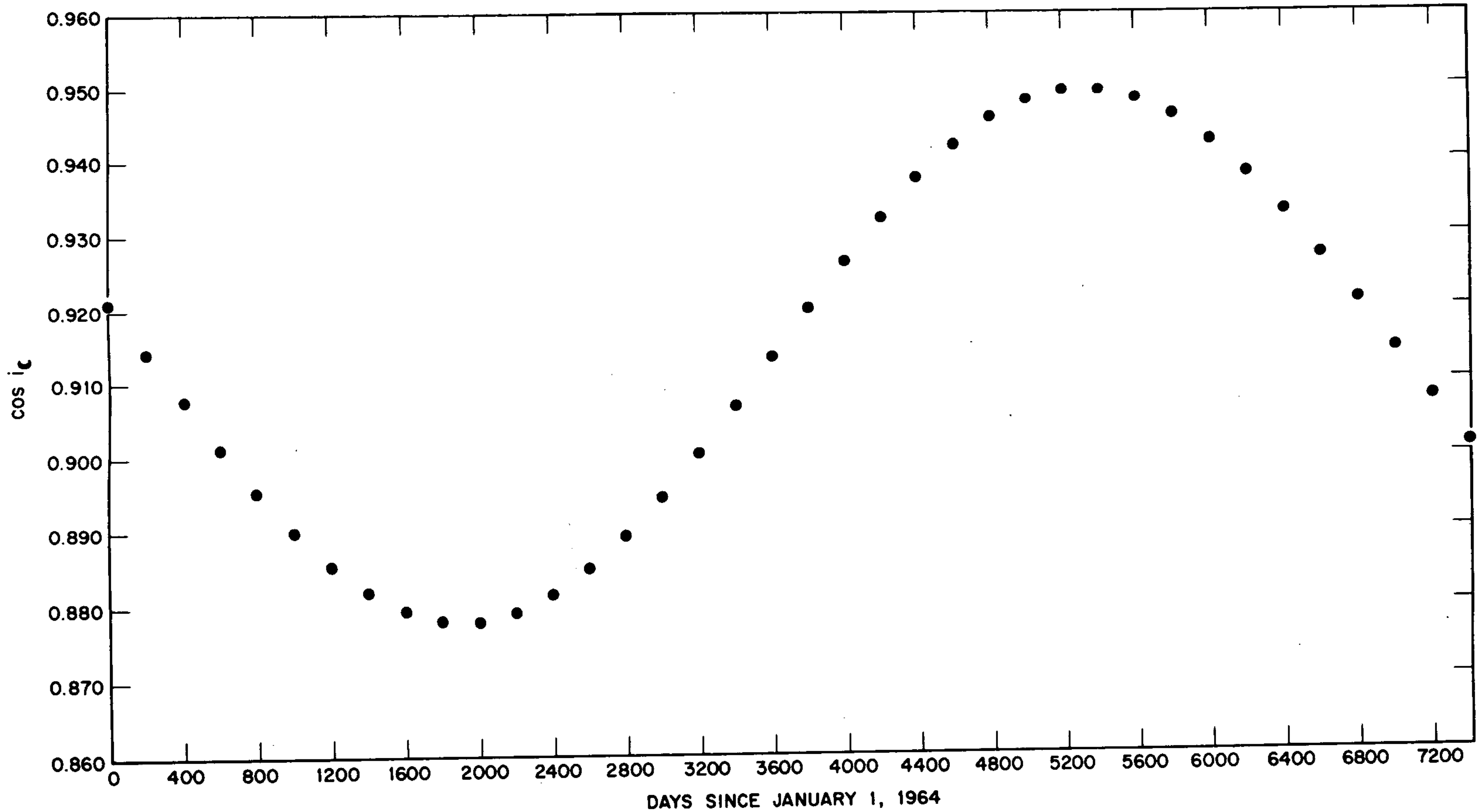


Figure 6—cos $i_{☾}$ vs. time.