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ELECTROMAGNETIC FORCES ON SPACE STRUCTURES

by W. M. Robbins, Jr.

Prepared under Contract No. NASw-652 *by*
ASTRO RESEARCH CORPORATION
Santa Barbara, Calif.

for

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CONTENTS

LIST OF FIGURES

ABSTRACT

The tensile and axial forces and the moments acting upon a conducting loop are examined, in general terms, for the cases that the loop is (a) completely isolated, (b) in the presence of another coaxial conducting loop, (c) in a uniform magnetic field, and (d) in a nonuniform magnetic field (the earth's). The results are presented so that the relative magnitudes of the effects and the numerical values can be evaluated for specific examples. Two such examples are worked out, one being the case of a loop in the earth's magnetic field and maintained circular by passing a current through the loop. The other example is that of reorienting the axis of symmetry of a very large paraboloidal antenna in the earth's magnetic field by generating loop currents in the antenna, e.g., around the rim.

The forces caused by an electrostatic charge placed on (a) a long wire, (b) a circular disk, and (c) a sphere, are examined. High tensile stresses are caused in a thin wire by a few hundred kilovolts but the same voltage causes only relatively very low

stresses in the other two cases. However, an electrostatic potential of 2×10^5 volts on an ECHO II balloon will cause an equivalent internal pressure which is two orders of magnitude greater than radiation pressure.

I. INTRODUCTION

Theory predicts and experience has shown that the earth's magnetic field produces torques upon earth satellites by the induction of eddy currents (Reference 1) and by interaction with internally generated currents. Such phenomena may, if uncontrolled, have an adverse effect upon a satellite's function. However, the question naturally arises as to whether various electromagnetic phenomena might be used to produce forces in space vehicles that could be utilized for structural deployment, geometrical control, attitude control, or propulsion.

In this report elementary electromagnetic theory will be used to derive expressions for the internal and external loads due to electromagnetic effects on a variety of simple structures. Examples will be worked out in a few cases to indicate the order of magnitude of the effects. It is hoped that the results presented in the report will be useful to structural engineers who may be interested in evaluating the mechanical effects of electric and magnetic fields on space structures.

Since it is not assumed that the reader is an expert in electromagnetic theory, some elementary propositions will be reviewed before considering applications.

The force acting upon an incremental moving electric charge, dq, as the result of electromagnetic phenomena can be described in terms of the electric field vector $\bar{E}$ and the magnetic field vector $\bar{B}$ as

$$\frac{d\bar{F}}{dq} = \bar{E} + \bar{V} \times \bar{B} \qquad (1)$$

where $\bar{V}$ is the velocity of the charge. If, as is usually the case, the force upon an electric conductor is of concern, then the above equation can be written as

$$d\bar{F} = \bar{E}\, dq + \bar{I} \times \bar{B}\, d\ell \qquad (2)$$

where $\bar{I}$ is the current and $d\ell$ is an increment of path length in which the current flows.

Our knowledge of the fields $\bar{E}$ and $\bar{B}$ which naturally exist in space is generally dependent upon prior measurements. However, if additional fields are generated by some known process, they can be predicted by means of Maxwell's equations, the conservation of charge, the geometry of the situation, and the nature of the materials involved. Maxwell's equations are:

$$\nabla \times \overline{E} + \frac{\partial \overline{B}}{\partial t} = 0 \tag{3}$$

$$\nabla \times \overline{H} - \frac{\partial \overline{D}}{\partial t} = \overline{J} \tag{4}$$

The conservation of charge can be written as

$$\nabla \cdot \overline{J} + \frac{\partial \rho}{\partial t} = 0 \tag{5}$$

Two other equations which are often given as Maxwell's equations follow directly from the above and are:

$$\nabla \cdot \overline{B} = 0 \tag{6}$$

$$\nabla \cdot \overline{D} = \rho \tag{7}$$

The electromagnetic properties of the medium are normally expressed as

$$\overline{J} = \sigma \overline{E} \tag{8}$$

$$\overline{D} = \epsilon \overline{E} \tag{9}$$

$$\overline{B} = \mu \overline{H} \tag{10}$$

where

$\overline{E}$ = electric field intensity

$\overline{D}$ = electric displacement

$\overline{J}$ = current density

$\overline{H}$ = magnetic field intensity

$\overline{B}$ = magnetic induction

σ = conductivity

ϵ = permittivity

μ = permeability

The determination of the electromagnetic forces upon macroscopic bodies (in the absence of relativistic effects) is made by means of mathematical manipulations upon the above relationships within the boundary conditions imposed by the bodies and their total environment. In practice, the problems can be quite complicated and only a few rather simple situations are considered in this report.

II. CONDUCTING LOOPS

A. GENERAL DISCUSSION

If an incremental length of conductor has no net charge upon it, the forces as the result of electromagnetic phenomena arise by virtue of the interaction of the current in the element with the total magnetic field due to all sources and is given by

$$d\overline{F} = I \, d\overline{S} \times \overline{B} \tag{11}$$

where $d\overline{S}$ is the increment of conductor length in the direction of $\overline{I}$.

The magnetic field may be naturally occurring, may result from current in other conductors, or may result solely from the current in other portions of the same conductor.

The magnetic induction $d\overline{B}_1$ at point 1 due to a current I_2 flowing in $d\overline{S}_2$ is

$$d\overline{B}_1 = \frac{\mu}{4\pi} I_2 \frac{d\overline{S}_2 \times \overline{r}_2}{r^3} \tag{12}$$

(the law of Biot-Savart)

where

$\overline{r}_2$ = radius vector from point 2 to point 1

$r = \left|\overline{r}_2\right|$

and the force, $d\overline{F}_1$, on $d\overline{S}_1$ at point 1 is obtained by combining (11) and (12)

$$d\overline{F}_1 = \frac{\mu}{4\pi} I_1 I_2 \frac{d\overline{S}_1 \times (d\overline{S}_2 \times \overline{r}_2)}{r^3} \tag{13}$$

Several examples of the forces on conducting loop are considered in the following subsections.

B. TENSION IN LOOP

1. Tension in Conducting Loop in Otherwise Field-Free Region.

A general procedure for calculating the force on a conductor was described in the preceding discussion. Another procedure, which is frequently useful, is to consider the work done where one of the dimensions of a structure is changed. In the case of a conducting loop (of any shape) the energy stored in the magnetic field is

$$W = \frac{1}{2} LI^2 \tag{14}$$

where L is the inductance of the loop and I is the current flowing in the loop. Thus if the radius, b, of the loop is changed the total radial force may be calculated from

$$dW = F_r db = \frac{\partial}{\partial b}\left(\frac{1}{2}LI^2\right) db \tag{15}$$

According to Stratton (Ref. 2) the self-inductance of a circular loop of wire of loop radius, b, and cross-sectional radius, r, is

$$L = b\left[\mu_2\left(\ln\frac{8b}{r} - 2\right) + \frac{1}{4}\mu_1\right] \tag{16}$$

in rational units (henry) where μ_1 is the permeability of the wire and μ_2 is the permeability of the external medium. Differentiating with respect to b and letting $\mu_1 = \mu_2 = \mu$ yields

$$\frac{\partial L}{\partial b} = \mu\left[\ln\frac{8b}{r} - \frac{3}{4}\right] \tag{17}$$

The total radial force on the loop is then

$$F_r = \frac{1}{2} I^2 \frac{\partial L}{\partial b} = \frac{1}{2} I^2 \cdot \mu\left[\ln\frac{8b}{r} - \frac{3}{4}\right] \tag{18}$$

and the tension in the loop is

$$T = \frac{Fr}{2\pi} = \frac{I^2}{4\pi}\,\mu\left[\ln\frac{8b}{r} - \frac{3}{4}\right] \tag{19}$$

The value of μ for free space in the rational m.k.s. system is

$$\mu = 4\pi \times 10^{-7} \text{ henry/meter}$$

which yields

$$\frac{T}{I^2} = \left[\ln\frac{8b}{r} - \frac{3}{4}\right] 10^{-7}\ \frac{\text{newton}}{\text{amp}^2} \tag{20}$$

This relationship is shown in Figure 1.

It will be noted that the tension in the loop is not strongly dependent on dimensions and also that the tension is small unless the current is large (1 Newton = 0.225 lbs).

2. Tension in Conducting Loop in the Presence of Coaxial Conducting Loop.

A pair of coaxial conducting loops with radii a and b, respectively, separated by the distance c, are shown in Figure 2. The currents I and I' flow in opposite directions.

Tension in the loop of radius b consists of a term due to its own current, eqn. (20), and a new term due to the current in the other loop.

It is readily shown from the results of Smythe (Ref. 3) that the tension in loop b due to current in loop a is

$$T_{ba} = -\frac{\mu}{2\pi} \cdot \frac{II'}{\left[\left(\frac{a}{b}+1\right)^2 + \left(\frac{c}{b}\right)^2\right]^{\frac{1}{2}}} \cdot \left[K + \frac{\left(\frac{a}{b}\right)^2 - 1 - \left(\frac{c}{b}\right)^2}{\left(\frac{a}{b}-1\right)^2 + \left(\frac{c}{b}\right)^2} \cdot E\right] \tag{21}$$

$$k^2 = 4 \frac{\left(\frac{a}{b}\right)}{\left(\frac{a}{b}+1\right)^2 + \left(\frac{c}{b}\right)^2} \tag{22}$$

where K and E are the complete elliptic integrals of modulus k.

The influence of finite wire diameter, which is very small for slender loops, has been neglected in deriving these equations.

Loop tension has been computed as a function of spacing c/b for $a/b = 0.9, 1.0,$ and 1.1. The results are plotted in Figure 2 together with valves for the tension in the loop of radius b due solely to current in that loop. The actual tension will be the total of that from the two sources. It may be observed that the portion which results from the presence of the second loop is significant only when the two loops are close together or when the current in the other loop is relatively large.

3. Tension in Conducting Loop in Earth's Magnetic Field.

a. General considerations. - The tension (or compression) in a circular conducting loop of radius b oriented normal to a uniform magnetic field of induction B is

$$T = IBb \tag{23}$$

For other loop orientations the component of B normal to the loop would be used to compute T, but a moment on the loop will then be present.

The approximate value of the magnetic field in the equatorial plane, as a function of geometric distance, is shown in Figure 3 (See Ref. 4) for units of the gamma and the gauss (since they are the most commonly encountered) and in terms of webers/m^2 (the unit in the rationalized m.k.s. system).

The tension which occurs as the result of the loop current with a normal uniform magnetic field is shown in Figure 4 for a number of values of Bb. Also shown in the figure are the components of tension due solely to the current in the loop, and secondly, to the current in a second coaxial loop (as discussed in the previous two subsections). It should be noted that the tension due to interaction with the earth's magnetic field will be several orders of magnitude larger than the tension due to self induction except when the loop is small and/or at a considerable distance from the earth.

<u>b. A magnetically stabilized conducting loop.</u> - The tension induced in a current-carrying loop by a uniform magnetic field tends to maintain the loop in a circular shape in exactly the same

manner that internal pressure in a pipe tends to keep the cross-section of the pipe circular. This mechanism may be used to prevent or eliminate deformations of circular loops. It is particularly effective for very long slender wires in an earth's orbit as will be shown by the following example.

Suppose a loop of 1000-meter radius and made of copper wire of 0.1-mm diameter is orbiting the earth with the plane of the loop normal to a uniform magnetic field of 2×10^{-5} weber/m^2 (corresponding to an equatorial orbit with an altitude of approximately 2000 miles). A current of 0.1 ampere is caused to flow in the loop by means of a battery. The following properties are assumed for copper:

δ = density = 8.9 gm/cm^3

ρ_e = electrical resistivity = 1.75×10^{-6} ohm·cm

E = tensile modulus = 17×10^6 psi = 1.17×10^{11} newton/m^2

Then it follows that:

T = tension = IBb = 2×10^{-3} newtons

V = volume of copper = 49.3 cm^3

m = mass of copper = 439 gm

R_e = electrical resistance = 14,000 ohms

V_e = voltage of battery = 1400 volts

P_e = electric power = 140 watts

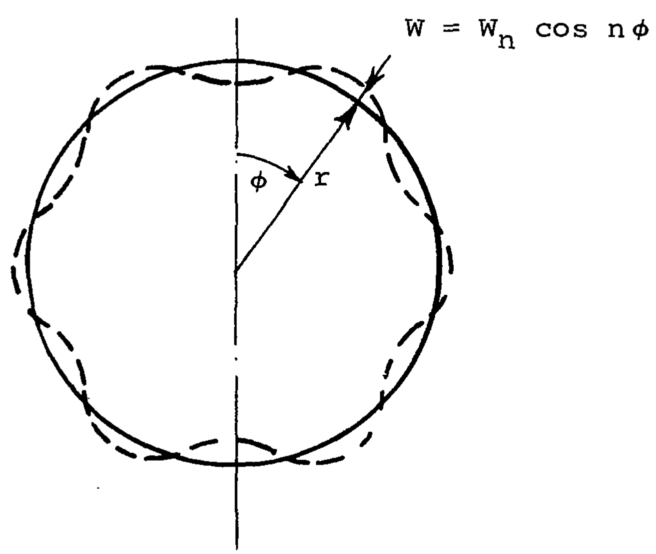
σ = tensile stress = 25.4 newton/cm^2 = 37 psi

I_x = second moment of area of wire cross section about wire diameter = $\pi \frac{d^4}{64}$ = 4.91×10^{-18} m^4

$EI_x = 5.74 \times 10^{-7}$ newton $\cdot$ m^2

We wish now to consider the resistance of the circular loop shown below to deformations caused by external forces, initial imperfections, thermal gradients, etc. Deformations are resisted by the bending stiffness of the wire and by the stiffening effect of tension. The resulting potential energy for small deformations in the plane of the loop is:

$$PE = \frac{1}{2}\int_0^{2\pi} EI\left(\frac{1}{r^2} \cdot \frac{d^2W}{d\theta^2}\right)^2 rd\,\theta + \frac{1}{2}\int_0^{2\pi} T\left(\frac{1}{r}\,\frac{dW}{d\theta}\right)^2 rd\,\theta$$



For the case of periodic deformations of sinusoidal waveform, $W = W_n \cos n\phi$ and eqn. (24) becomes:

$$PE = \frac{\pi}{2}\left[\frac{EI}{r^3} n^4 + \frac{T}{r} n^2\right] W_n{}^2 \tag{25}$$

The ratio of the first to the second term expresses the relative importance of bending stiffness and tension for resisting deformation. This ratio is

$$\frac{(PE)_{elastic}}{(PE)_{magnetic}} = \frac{EIn^2}{Tr^2} = 2.86 \times 10^{-10} n^2 \tag{26}$$

for the present example. Thus the bending stiffness is quite negligible except for extremely short wavelength.

Eqn. (25) may be used to compute the effectiveness of the magnetic field in removing kinks in the wire. Suppose that the initial deviation from a circular shape is

$$W_i = W_{in} \cos n\phi \tag{27}$$

The equilibrium shape in the presence of a magnetic field is equal to the initial deviation multiplied by the ratio of spring constants from eqn. (26)

$$\frac{W_n}{W_{in}} = \frac{EIn^2}{r^2} \tag{28}$$

The magnetic field is apparently very effective in removing the long wavelength components of initial deviations from a circular shape.

Magnetic fields can also be used to collapse loops because the sign of the stress in the loop depends on the direction of the current. The buckling criterion is obtained from eqn. (25). By reversing the sign of T and equating the potential energy to zero. For the present example, the current required to buckle the loop is very small (i.e., 1.15×10^{-10} ampere).

C. AXIAL FORCE

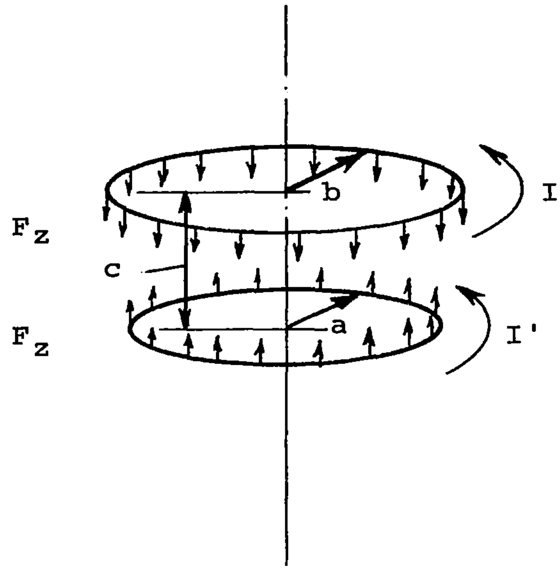
1. Axial Force Between Two Coaxial Conducting Loops.

The axial force of attraction between two coaxial conducting loops of radius a and b and of separation c is shown in Reference 3 (when conversion is made to r.m.k.s. units) to be

$$F_z = \mu I I' \cdot \frac{\frac{c}{b}}{\left[\left(\frac{a}{b}+1\right)^2+\left(\frac{c}{b}\right)^2\right]^{1/2}} \cdot \left[-K + \frac{\left(\frac{a}{b}\right)^2+1+\left(\frac{c}{b}\right)^2}{\left(\frac{a}{b}-1\right)^2+\left(\frac{c}{b}\right)^2} \cdot E\right] \qquad (29)$$

$$k^2 = 4\,\frac{\frac{a}{b}}{\left(\frac{a}{b}+1\right)^2+\left(\frac{c}{b}\right)^2} \qquad (30)$$

where K and E are the complete elliptic integrals of the modulus k.



In Figure 5, F_z/II' is shown as a function of $\frac{c}{b}$ for $\frac{a}{b}$ = 0.9, 1.0, and 1.1 .

2. Axial Force on Loop in Earth's Magnetic Field.

Since the earth's magnetic field is not uniform, but approximates that of a magnetic dipole, it is possible under some conditions to obtain an axial force on a conducting loop which is in the vicinity of the earth. This may also be seen from the discussion of the preceding section, since the magnetic field of the earth is also similar to that of a conducting loop.

Assume that the earth's magnetic field is that of a geocentric magnetic dipole and that the magnetic induction is B_o at the magnetic equator. Then

$$B_r = B_o\left(\frac{r_o}{r}\right)^3 \cos\theta \tag{31}$$

$$B_\theta = \frac{1}{2} B_o\left(\frac{r_o}{r}\right)^3 \sin\theta \tag{32}$$

where:

θ = colatitude

r = geocentric distance

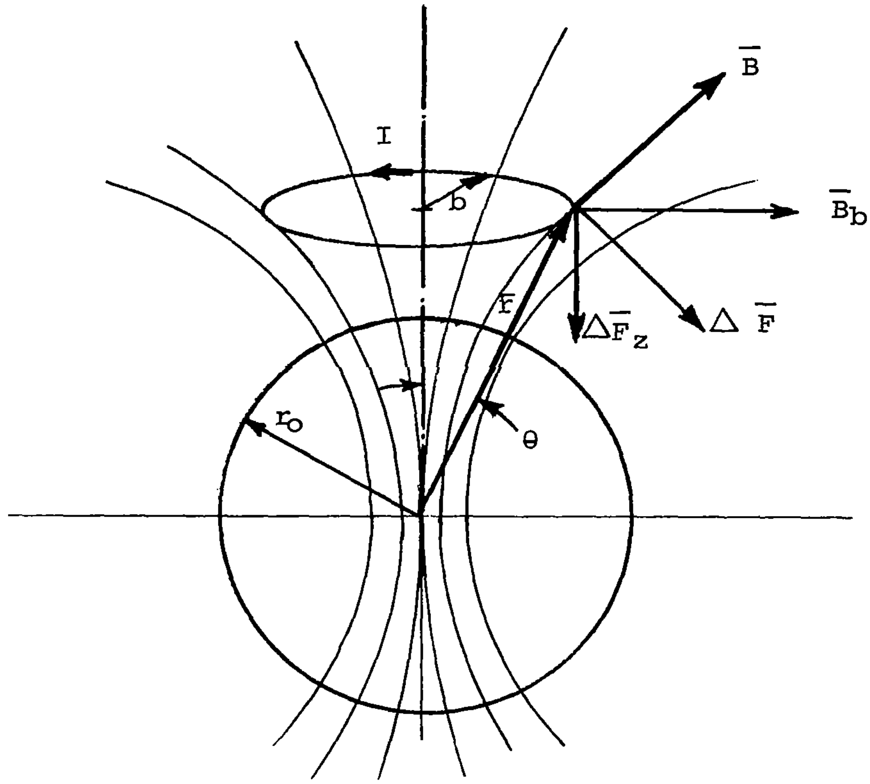
r_o = earth radius

B_r = radial component of $\bar{B}$

B_θ = meridional component of $\bar{B}$

$B_o = 4 \times 10^{-5}$ webers/m^2

In order to determine the magnitude of the possible axial force, a condition where torque on the loop is zero and the axial force is relatively large will be assumed. This condition occurs when the axis of the loop coincides with the magnetic dipole axis, as shown below.



The component of B at the loop which is in the radial (with respect to the loop) direction is

$$B_b = B_r \sin\theta + B_\theta \cos\theta = \frac{3}{2} B_o \left(\frac{r_o}{r}\right)^3 \sin\theta \cos\theta \qquad (33)$$

and since $\sin\theta = b/r$ and $b << r$

$$B_b = \frac{3}{2} B_o \left(\frac{r_o}{r}\right)^3 \frac{b}{r} \qquad (34)$$

Assuming

$$r = 8.8 \times 10^6 \text{ m (altitude = 1120 statute miles)}$$

$$r_o = 7.0 \times 10^6 \text{ m}$$

$$b = 1000 \text{ m}$$

yields $$F_z = IB_b \cdot 2\pi b = 2.15 \times 10^{-5} \text{ I newtons}$$

Reference to Figure 5 will show that this force is larger than is likely to be obtained between two coaxial conducting loops.

D. TORQUE ON CONDUCTING LOOP IN EARTH'S MAGNETIC FIELD.

The torque on a conducting loop in the earth's magnetic field can be computed with considerable precision by assuming the magnetic field to be uniform. For the case of a circular loop the torque is

$$M = \pi b^2 I \cdot \bar{B} \times \bar{n}$$

where

M = torque

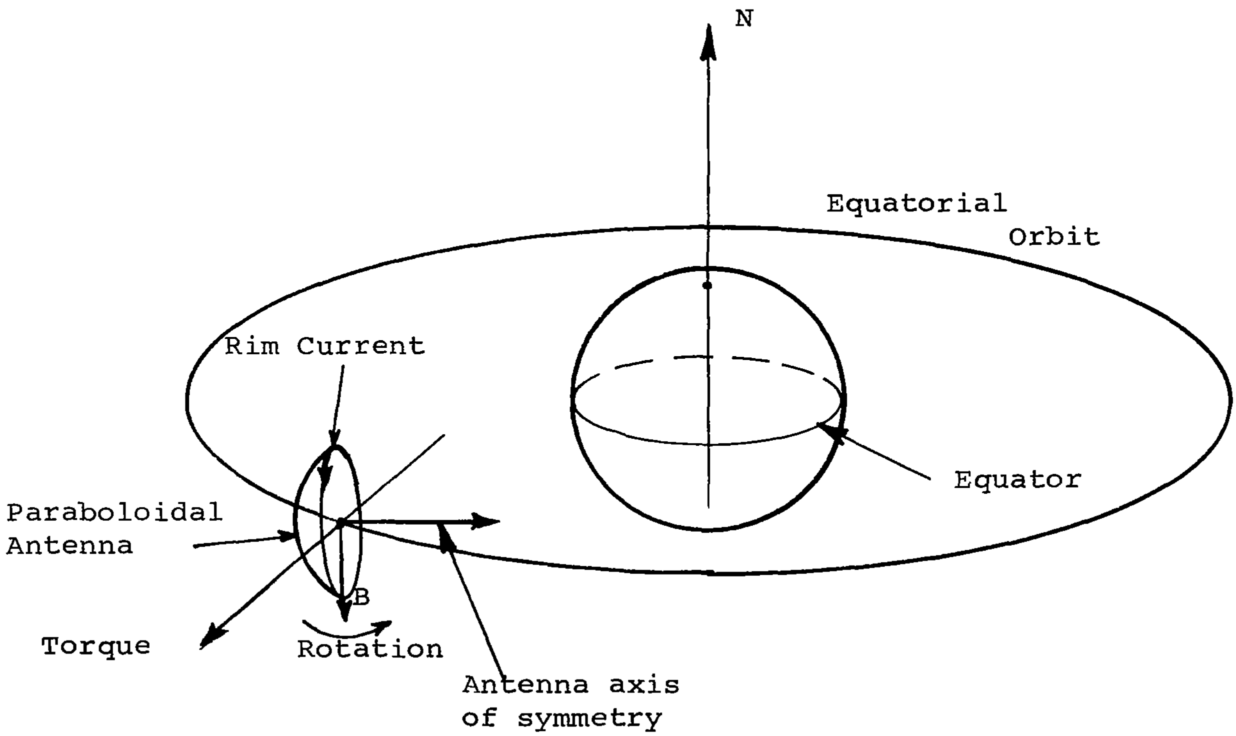
I = current

$\bar{B}$ = magnetic induction

$\bar{n}$ = unit vector normal to plane of loop

The magnitude and direction of $\bar{B}$ can be determined approximately from Figure 3 and the accompanying text.

As an example consider a paraboloidal antenna orbiting in an equatorial plane with its axis of symmetry lying approximately in the orbit plane and pointed (except during reorientation) in an inertially fixed direction as shown below.



The magnetic field is assumed to be 2×10^{-5} weber/m^2. The antenna has a conducting copper loop around its periphery and a source of electrical power causing a current to flow in the loop. The moment of inertia about a transverse axis is assumed to be the same as that of a uniform disk of the same mass and diameter. The following numerical values are assumed:

Magnetic field = $B = 2 \times 10^{-5}$ weber/m^2

Diameter = 1500 m (b = 750 m)

Total mass = m = 10,000 kg

Mass of copper loop = 10 kg

Resistivity of copper = 1.75×10^{-6} ohm·cm

Current = I = 1.0 amp

It follows that:

Transverse moment of inertia = $I_x = \frac{mb^2}{4} = 1.41 \times 10^9$ kg·m^2

Electric resistance = 346 ohms

Power = 346 watts

Torque = $M = B\pi b^2 I$ = 35.3 newton meters

Now assume that it is desired to rotate the axis of symmetry through the angle toward, or away from, the orbit plane and that the rotation is made by a constant accelleration $\ddot{\alpha}$ until $\alpha_{1/2} = \alpha/2$ is reached in the time $\tau_{1/2} = \tau/2$ and then the acceleration is reversed until α is reached in the total time τ.

Let α = 0.1 radian. Then:

$$\ddot{\alpha} = \frac{M}{I_x} = \frac{35.3}{1.41 \times 10^9} = 2.5 \times 10^{-8} \text{ rad/sec}^2$$

$$\alpha_{\frac{1}{2}} = \frac{\ddot{\alpha}}{2} \cdot \tau^2_{\frac{1}{2}}$$

$$\tau_{\frac{1}{2}} = \left(\frac{2\,\alpha_{\frac{1}{2}}}{\ddot{\alpha}}\right)^{\frac{1}{2}} = \left[\frac{(2) \quad (.05)}{2.5 \times 10^{-8}}\right]^{\frac{1}{2}} = 2000 \text{ sec}$$

$$\tau = 2\tau_{\frac{1}{2}} = 4000 \text{ sec} = 1.11 \text{ hour}$$

It is also possible to obtain other rotations of the antenna by employing additional conducting loops. Although the area of such loops is likely to be considerably less than the area of the one on the rim and the magnetic field may be somewhat less than that considered here, the feasibility of orienting a very large radio antenna by the interaction of electric currents with the earth's magnetic field appears quite feasible.

III. ELECTROSTATIC CHARGE

A. GENERAL DISCUSSION

The electrostatic energy W stored in a capacitor is

$$W = \frac{1}{2}\frac{Q^2}{C} \tag{35}$$

where:

C = capacitance

Q = electric charge

and the generalized force acting on a given geometrical coordinate θ is

$$F_\theta = -\left(\frac{\partial W}{\partial \theta}\right)_{Q\,=\,\text{constant}} = -\frac{Q^2}{2} \cdot \frac{\partial}{\partial \theta}\left(\frac{1}{C}\right) \tag{36}$$

Also, since $Q = CV$, where V is the electric potential on the surface of the capacitor (with respect to an infinitely remote point for an isolated conductor):

$$F_\theta = -\frac{C^2V^2}{2} \cdot \frac{\partial}{\partial \theta}\left(\frac{1}{C}\right) \tag{37}$$

In the following subsections the forces and stresses resulting from an electrostatic charge on several geometrical shapes are examined. The effects of a plasma sheath, bombardment by charged particles, and photoelectric emission are all very important under some conditions and can modify the capacitance and determine the power requirements for maintaining a given potential. However, they are not considered in this report.

B. ELECTROSTATICALLY CHARGED WIRE.

The capacitance of a long, straight wire in free space has been reported by a number of investigators (see, e.g., Ref. 5) and is

$$C = 2\pi\epsilon_0 \ell \left[\frac{1}{\ln\left(\frac{2\ell}{d}\right)}\right] \tag{38}$$

where:

ϵ_o = dielectric constant of free space = 8.85×10^{-12} farad/m

ℓ = length of wire

d = diameter of wire

The corresponding tension in the wire is

$$F_\ell = -\frac{C^2V^2}{2}\frac{\partial}{\partial \ell}\left(\frac{1}{C}\right) = \pi \epsilon_o V^2 \frac{\left[\ln\left(\frac{2\ell}{d}\right) - 1\right]}{\ln^2\left(\frac{2\ell}{d}\right)} \qquad (39)$$

The electric potential required versus tension obtained is shown in Figure 6 for various values of ℓ/d. It is of interest to note that these tensions are about two orders of magnitude higher than those computed by the method of Reference 6 where the same capacitance is used but it is assumed that the charge is equally divided between the ends of the wire.

As an example of the stresses obtainable, consider the following case.

$$d = 0.5 \times 10^{-3} \text{ in}$$

$$\ell/d = 10^8$$

$$\ell = 50{,}000 \text{ in} = 4{,}170 \text{ ft}$$

$$V = 100{,}000 \text{ volts}$$

then

$$T = 1.38 \times 10^{-2} \text{ newtons} = 0.0031 \text{ lbf}$$

and

$$\sigma = \frac{T}{\frac{\pi}{4} \cdot d^2} = 16{,}000 \text{ psi}$$

C. ELECTROSTATICALLY CHARGED CIRCULAR DISK

The capacitance of a circular disk (Ref. 3 or 7) is

$$C = 8\epsilon_o b \tag{40}$$

where b is the radius of the disk. The radial force is

$$F_b = 4\epsilon_o V^2 \tag{41}$$

The force per unit of circumference is

$$N_r = \frac{F_b}{2\pi b} = \frac{2\epsilon_o}{\pi b} \cdot V^2 \tag{42}$$

and the constant uniform tensile stress (in a uniform disk) is

$$\sigma = \frac{N_r}{t} = 5.63 \times 10^{-12} \frac{V^2}{bt} \frac{\text{newton}}{m^2} \tag{43}$$

when b and t are in meters, or is

$$\sigma = 1.27 \times 10^{-12} \frac{V^2}{bt} \text{ psi} \tag{44}$$

when b and t are in inches.

Two examples of the stress obtained in a disk as the result of an electrostatic charge are given below.

1. Example No. 1

$b = 750 \text{ m} = 29{,}500 \text{ in}$

$t = 0.001 \text{ in}$

$V = 2.5 \times 10^5 \text{ volts}$

The resulting tensile stress is $\sigma = 2.7 \times 10^{-3}$ psi

2. Example No. 2

$b = 9.0 \text{ in}$

$t = 0.001 \text{ in}$

$V = 20 \times 10^3 \text{ volts}$

The resulting tensile stress is

$$\sigma = 56 \times 10^{-3} \text{ psi}$$

D. ELECTROSTATICALLY CHARGED SPHERE.

The capacitance of an isolated sphere of radius R is

$$C = 4\pi\epsilon_o R \tag{45}$$

and the total radial force on the sphere is

$$F_r = 2\pi\epsilon_o V^2 \tag{46}$$

The equivalent internal pressure is

$$P_r = \frac{F_r}{4\pi R^2} = \frac{\epsilon_o}{2}\left(\frac{V}{R}\right)^2 = 4.42 \times 10^{-12}\left(\frac{V}{R}\right)^2 \text{ newton/m}^2 \tag{47}$$

The resulting tensile stress in the shell is

$$\sigma = \frac{\epsilon_0 V^2}{4Rt} = 2.21 \times 10^{-12} \frac{V^2}{Rt} \tag{48}$$

where t is the thickness of the shell.

For a sphere of the ECHO II size (i.e., with a radius of 20 meters) and a potential of 2×10^5 volts

$$P_r = 4.42 \times 10^{-4} \text{ newton/m}^2 \tag{49}$$

In comparison, solar radiation pressure at the earth's distance from the sun upon an absorbing surface is 4.5×10^{-6} newton/m^2 which is two orders of magnitude lower than the electrostatic pressure.

IV. CONCLUSIONS

The forces on a variety of simple structures due to magnetostatic and electrostatic fields have been examined in this report from the viewpoint of potential application to the deployment, attitude control, and surface contour control of large space structures. Conclusions regarding the magnitudes of achievable effects are as follows:

1. Self and mutually induced forces in conducting loops are extremely small under most conditions. They are not likely to be the predominating magnetostatic forces except for small

loops and/or very large currents.

2. Forces and moments on a conducting loop due to interaction with the earth's magnetic field will be useful for maintaining the loop deployed in the presence of various perturbing influences such as built-in imperfections and for controlling the altitude of a space vehicle. Even a very small current in the wrong direction however, can cause such a loop to buckle. Propulsion by these forces does not appear feasible.

3. Forces caused by electrostatic charge on flexible structures is applicable in some cases to contour control. Among such applications is the maintenance of a long wire in very nearly a straight line and the maintenance of a passive communication "balloon" inflated against radiation pressure.

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Santa Barbara, California,

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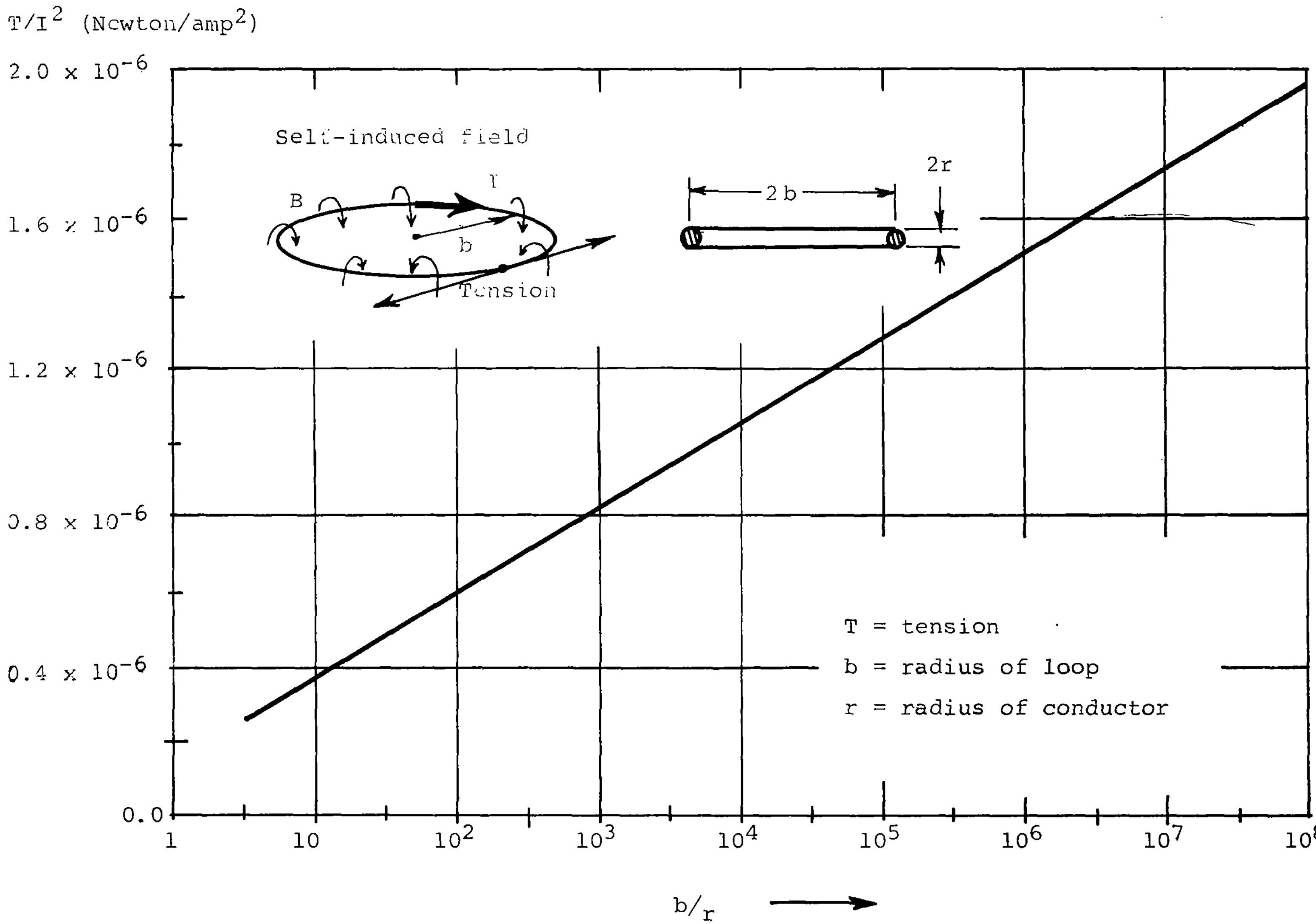


Figure 1 Tension in Current-Carrying Circulat Loop

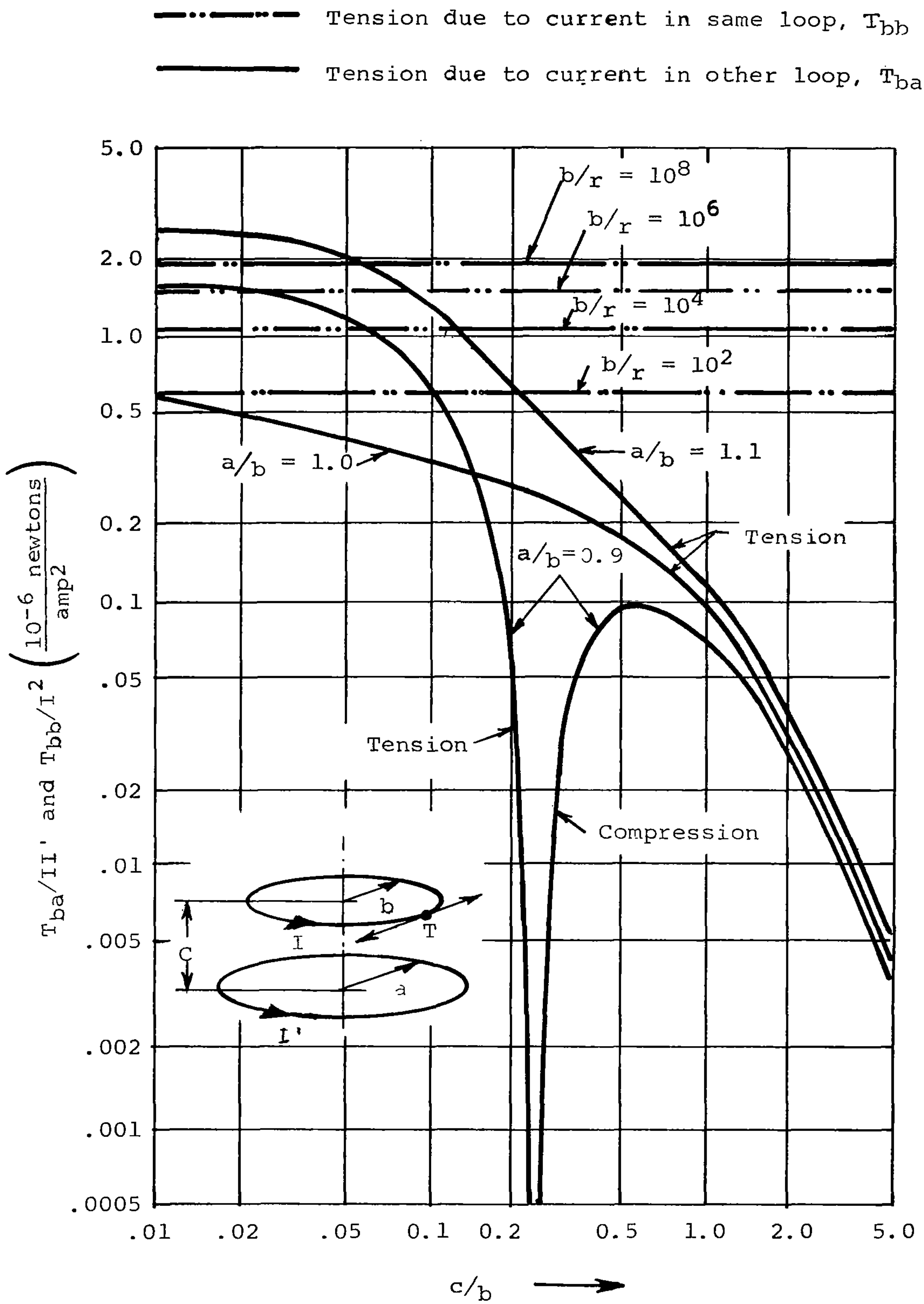


Figure 2 Tension in Conducting Loop

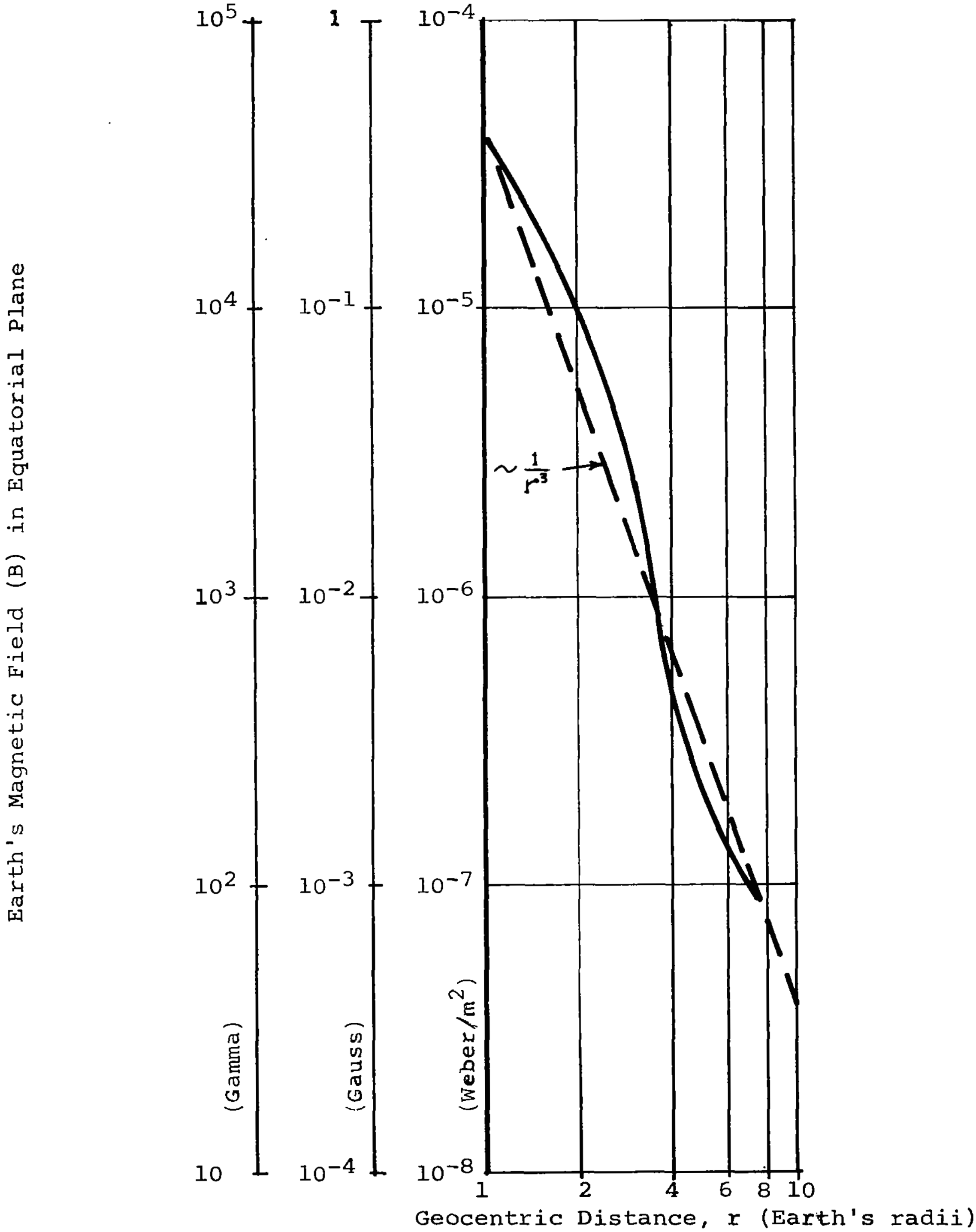
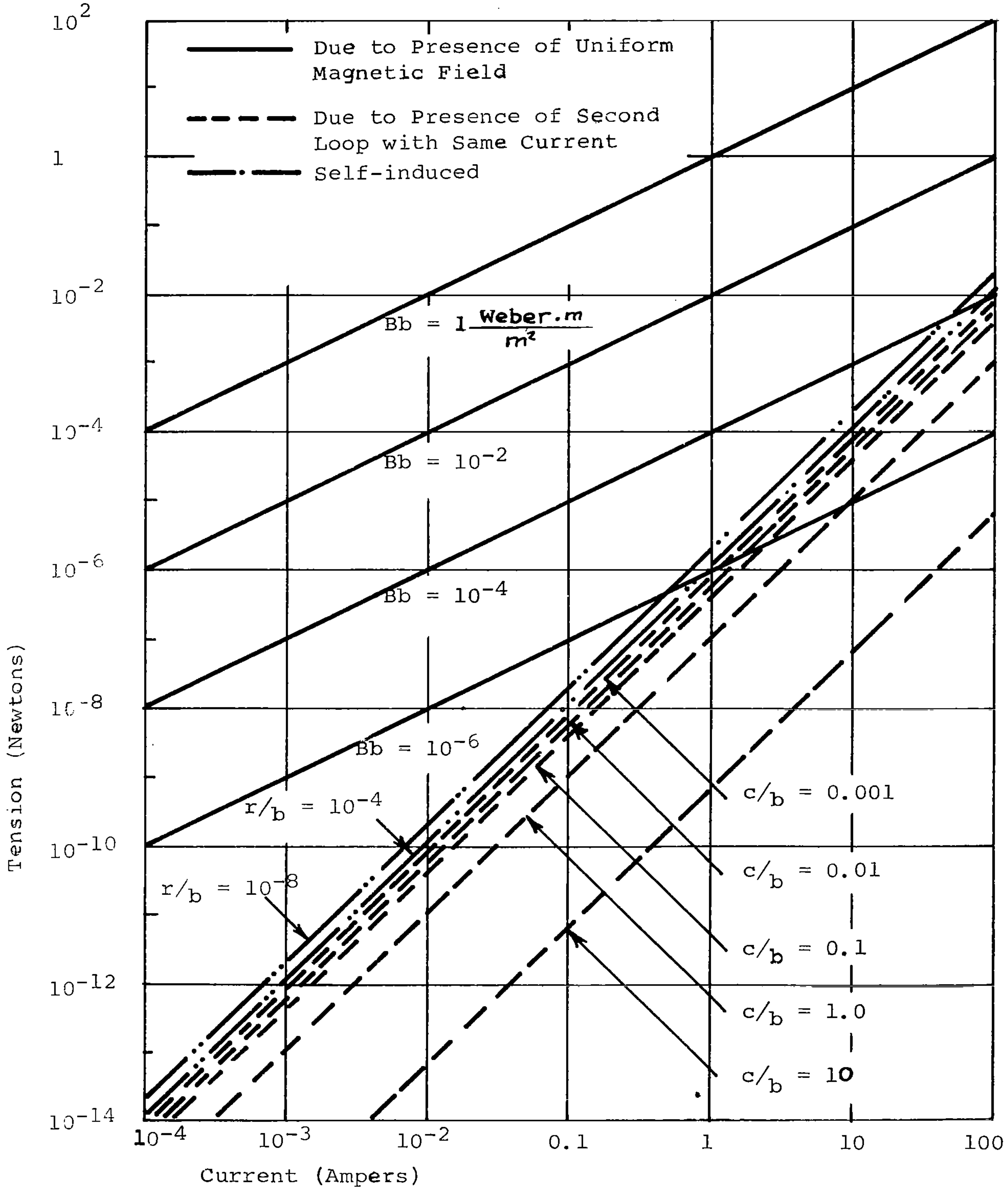


Figure 3 Approximate Magnetic Field of Earth



B = Magnetic Induction
b = radius of loop (or loops)
r = radius of conductor
c = spacing between loops

Figure 4 Tension of Conducting Loop

Note: Loops repel when currents are in opposite directions.

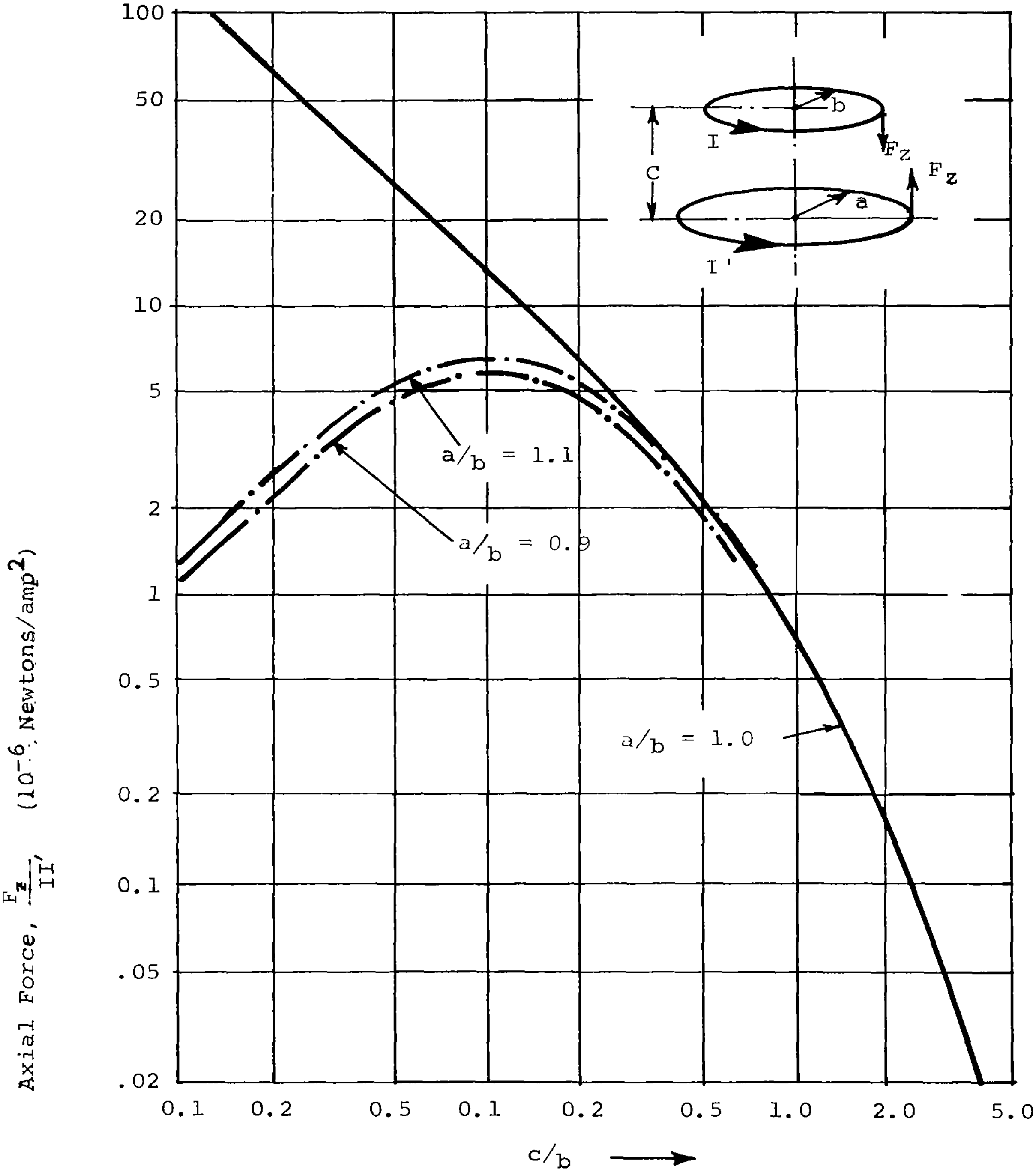


Figure 5 Axial Force Between Conducting Loops

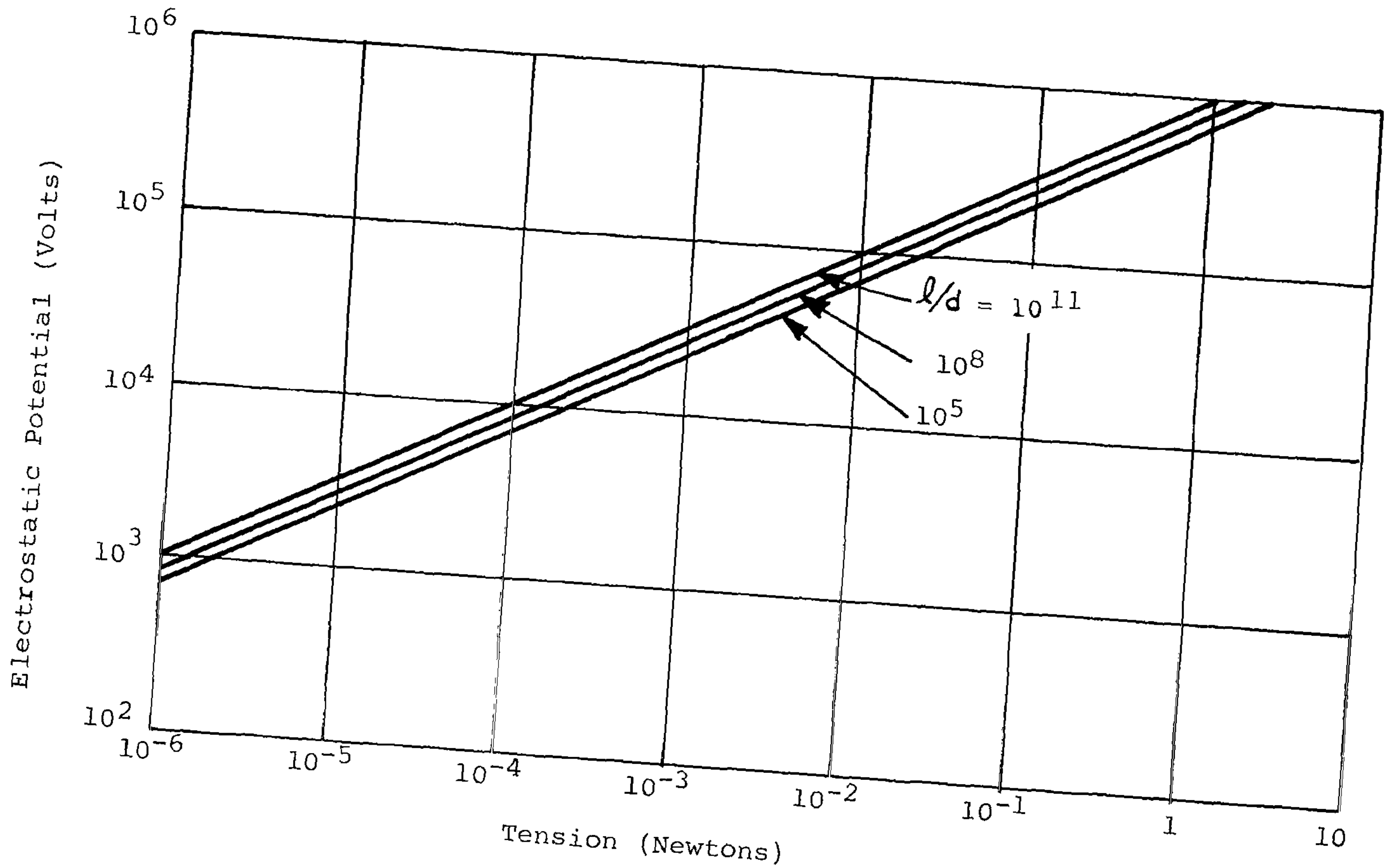


Figure 6 Tension in Wire Due to Electric Charge

NASA-Langley, 1966 CR-476