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USE OF THE CURTIS-GODSON APPROXIMATION IN CALCULATIONS
OF RADIANT HEATING BY INHOMOGENEOUS HOT GASES

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ABSTRACT

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*Messrs. Krakow, Babrov, Maclay, and Shabott are associated with
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THERMAL ENVIRONMENT BRANCH
AERODYNAMICS DIVISION
AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

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SUMMARY

Calculations of radiant heating by inhomogeneous hot gases require knowledge of spectral transmittances of inhomogeneous optical paths. Determination of these transmittances is a difficult problem that can be attacked by means of the Curtis-Godson approximation.

We have tested the accuracy of the Curtis-Godson approximation experimentally. Infrared spectral transmittances of inhomogeneous hot samples of H_2O and CO_2 were measured. Each inhomogeneous hot gas specimen consisted of two or three homogeneous zones in series. The transmittance of each zone was measured, as was the transmittance of the entire multi-zone assembly. The measured transmittance of each inhomogeneous path was compared with a transmittance calculated from the homogeneous zonal transmittances, using the Curtis-Godson approximation in conjunction with a random band model. The measured and calculated inhomogeneous transmittances agreed within about .01 to .02. The error appeared to be due more to the band model theory than to the Curtis-Godson approximation.

I. INTRODUCTION

The irradiance of any target by a hot gas is the integral of the contributions from all spatial regions and spectral intervals. Spatial variations are usually slow enough so that quadrature may be accomplished using reasonably large intervals. However, in a straightforward numerical integration, to obtain a reasonably accurate answer, the size of the wave number interval in the spectral integration has to be made so small ($\sim .01 \text{ cm}^{-1}$) as to make the calculation impractical. The reason the wave number interval must be made small is that for a numerical integration the integrand must be essentially constant (slowly varying) over each increment of the independent variable. For the frequency integration, this means that radiance must not change rapidly over the mesh size, $\Delta\nu$. But a typical spectral line has a half-width of the order of $.01 \text{ cm}^{-1}$ at a pressure of 0.1 atmosphere, so that the size of $\Delta\nu$ should be less than $.01 \text{ cm}^{-1}$ for a straightforward calculation at this pressure. Band models were invented to get around this difficulty¹. The band model relies on the substitution in the spectral interval of interest of an equivalent and mathematically tractable spectrum for the true spectrum.

Tourin and Krakow² have shown that spectral irradiance of the target along any line of sight depends only on the local Planck blackbody functions and the transmittances of the optical paths. The Planck functions can be considered constant over reasonably large spectral intervals. Determination of transmittance is, therefore, the only barrier to the use of large spectral intervals in radiance calculations.

Infrared spectral transmittances of gases are usually measured and reported for homogeneous samples. Band models have been used quite successfully on such homogeneous samples. In most practical problems, however, hot specimens have gradients of temperature, pressure, and concentration. The transmittances of these inhomogeneous gases are needed for the calculation of their radiance.

A number of methods have been proposed for determining transmittances of inhomogeneous gases. There is, of course, the Beer-Lambert law that works so well for liquids but is not good for gases. Babrov and Maclay³ obtained fairly good results for inhomogeneous hot gases by using Sakai's approximate overlap correction⁴. In this paper we will examine another method, namely, the Curtis-Godson approximation^{1,5}. Meteorologists have found this method useful for calculating transmittances of inhomogeneous atmospheric paths. In this approximation, the transmittance through the inhomogeneous path is obtained by substituting a certain hypothetical homogeneous path with the same transmittance. The homogeneous path can then be treated by established band model methods. Studies have been made of the errors incurred when using the Curtis-Godson approximation^{1,5}, and the errors have been found to be negligible for most cases of meteorological interest. The objective of this research was to test the Curtis-Godson approximation for gases obtained by combustion of common fuels.

We have tested the accuracy of the Curtis-Godson approximation and other methods of calculating transmittance of an inhomogeneous path (e.g., that of Ref. 3) experimentally by measuring spectral absorptances of hot water vapor and carbon dioxide. To simplify the problem of predicting the transmittance through an inhomogeneous gas and to permit easy comparison of theoretical and experimental transmittances, we used inhomogeneous paths that were composed of series of smaller homogeneous zones.

II. THEORY

A. Radiance Equations

Rewriting equation (6) of Ref. 2 in terms of the wave number, ν , instead of the wavelength, λ , we obtain

$$H(\nu_j, \Delta\nu_j) = \sum_{i=1}^n N_b(\nu_j, T_i) \left[\bar{\tau}_{i-1}(\nu_j, \Delta\nu_j) - \bar{\tau}_i(\nu_j, \Delta\nu_j) \right]. \quad (1)$$

The terms appearing in equation (1) will be defined below. The radiant flux at an element of the irradiated area is

$$\frac{q^+}{A} = \int_{\omega} \sum_{j=1}^m H(\nu_j, \Delta\nu_j) \Delta\nu_j \cos \theta d\omega \quad (2)$$

where q^+/A is the radiant flux in watts/cm². To get BTU/ft²-sec from watts/cm², we multiply by 0.8811. The wave number, ν_j , is at the center of the wave number interval, $\Delta\nu_j$ (units of ν and $\Delta\nu$ are cm⁻¹). There are m intervals in the wave number range of interest for heat flux calculations. The range of interest depends on the temperature and composition of the gas; for combustion gases, a safe range (not necessarily the minimum range) is 600 cm⁻¹ to 13,000 cm⁻¹. $H(\nu_j, \Delta\nu_j)$ is the normal spectral irradiance in the line of sight (units watts/cm²-cm⁻¹-steradian). We have written H as a function of ν_j and $\Delta\nu_j$ to emphasize that the numerical value of H depends on the size of the spectral interval, $\Delta\nu_j$, as well as on the frequency of the midpoint of the interval, ν_j . The subscript i labels a zone (approximately isothermal) in a line of sight between the element of area, A , of the target and the farthest edge of the hot gas (or point of occultation if the line of sight is occulted), θ is the angle between the normal to the element of area and the line of sight, $d\omega$ is a differential solid angle, and the limits of $d\omega$ are indicated by ω . $N_b(\nu_j, T_i)$ is the spectral radiance of a blackbody (units watts/cm²-cm⁻¹-steradian) at the temperature T_i (degrees Kelvin), and wave number, ν_j . $N_b(\nu, T)$ is given by the formula

$$N_b = \frac{2c^2 h \nu^2}{[\exp(hc\nu/kT) - 1]} = \frac{1.1909 \times 10^{-12} \nu^2}{[\exp(1.439\nu/T) - 1]}, \quad (3)$$

where e is the base of the natural logarithms, c is the velocity of light (2.998×10^{10} cm/sec), h is Planck's constant (6.625×10^{-27} erg sec),

and k is Boltzmann's constant (1.380×10^{-16} ergs/°K). $\tau_{i-1}(\nu_j, \Delta\nu_j)$ is the transmittance from the beginning of the first zone to the end of zone $i-1$; similarly, τ_i is the transmittance from the beginning of the first zone to the end of zone i .

B. Combination of Zonal Transmittances

As stated earlier, the integral over solid angle in equation (2) can usually be carried out by a straightforward numerical integration using reasonably large intervals. The Planck function in equation (1) can be considered constant over fairly large values of $\Delta\nu_j$. Calculation of the transmittances in equation (1) is more difficult because the monochromatic transmittance oscillates rapidly within any practical $\Delta\nu_j$. We will now concentrate on the problem of determining these transmittances since they are the only unknowns remaining in the radiance equations. A band model will be used as a starting point.

In making these transmittance calculations, the band model we chose to use was the random band model with constant line strengths and widths¹. According to this band model, the transmittance of a homogeneous gas over a spectral interval containing many lines, can be expressed as follows:

$$-\ln \tau = 2\pi [\gamma/d] f(x), \quad (4)$$

in which τ is the transmittance, γ is the line half-width and d the average line spacing in the spectral interval considered.

$$x = \frac{[S/d] \ell}{2\pi [\gamma/d]} \quad (5)$$

in which S is the line strength and ℓ is the length of the optical path. The function $f(x)$ is the Ladenburg and Reiche function⁶, whose mathematical properties are well known and whose values have been tabulated⁷.

If an inhomogeneous gas has a transmittance, τ , then some hypothetical homogeneous sample has the same transmittance if it has certain values of γ/d and x (or, alternatively, γ/d and S/d). The Curtis-Godson approximation gives us equations (6) and (7)

$$\frac{S}{d} \ell = \sum_i \frac{S_i}{d} \ell_i = \sum_i x_i \frac{-\ln \tau_i}{f(x_i)} \quad (6)$$

$$\frac{S}{d} \ell \frac{\gamma}{d} = \sum_i \frac{S_i}{d} \ell_i \frac{\gamma_i}{d} = \frac{1}{2\pi} \sum_i x_i \left[\frac{-\ln \tau_i}{f(x_i)} \right]^2, \quad (7)$$

which define the band model parameters of this particular homogeneous sample. The definition is given in terms of the band model parameters of the individual zones comprising the inhomogeneous path being studied. In equations (6) and (7), parameters that pertain to any individual zone i are labelled with the subscript i . The zonal band model parameters have been expressed in terms of zonal transmittances and zonal x 's on the right side of equations (6) and (7).

The term γ/d is obtained by dividing equation (7) by equation (6); x can then be determined by dividing equation (6) by $2\pi \gamma/d$. Putting these values of γ/d and x into equation (4) yields an expression for the transmittance of the inhomogeneous path in terms of zonal transmittances and zonal x 's,

$$-\ln \tau = \frac{\sum_i x_i \left[\frac{-\ln \tau_i}{f(x_i)} \right]^2}{\sum_i x_i \frac{-\ln \tau_i}{f(x_i)}} f \frac{\left[\sum_i x_i \frac{-\ln \tau_i}{f(x_i)} \right]^2}{\sum_i x_i \left[\frac{-\ln \tau_i}{f(x_i)} \right]^2}. \quad (8)$$

Generally, transmittances calculated with equation (8) are not very sensitive to errors in x_i . Therefore, with moderately accurate values of x_i , the accuracy with which τ can be calculated should be limited by the accuracy of our transmittance measurements and the suitability of the theory. This is particularly true in the high and low x regions where equation (8) reduces to a form in which τ is independent of x_i . Thus, if all the x values are low (less than 0.2), $f(x) \approx x$, and equation (8) reduces to

$$-\ln \tau \approx \sum_i -\ln \tau_i. \quad (9)$$

If all the x_i 's are high (greater than 2.0), and $f(x) \approx [2x/\pi]^{1/2}$, equation (8) reduces to

$$-\ln \tau \approx \left[\sum_i (-\ln \tau_i)^2 \right]^{1/2}. \quad (10)$$

Equations (8), (9), and (10) are the three formulas we will use for experimental evaluation of the theory. Equation (8) is the most general and the most accurate. It can be used for any x values, but the x values must be known. Equations (9) and (10) are good in the low and high x regions, respectively. In their own regions they give results that do not differ very much from the ones obtained with equation (8). They have the advantage that they do not require explicit x values as long as x is known to be in the proper range. We will apply equations (8), (9), and (10) to two- and three-zone samples. The formulas, however, are general and could be applied to any number of zones. Equation (9) agrees with the Beer-Lambert law.

C. Calculation of Inhomogeneous Transmittances from Band Model Parameters

The transmittance $\bar{\tau}_j(\nu_j, \Delta\nu_j)$ in equation (1) may be expressed by equation (4) as follows:

$$-\ln \bar{\tau}_i(\nu_j, \Delta\nu_j) = 2\pi(\bar{\gamma}/d)_i f(\bar{x}_i). \quad (4a)$$

The terms $(\bar{\gamma}/d)_i$ and \bar{x}_i are the values of γ/d and x for the hypothetical homogeneous sample that has the transmittance $\bar{\tau}_i(\nu_j, \Delta\nu_j)$. Values obtained from equations (6) and (7) for these parameters in terms of zonal band model parameters are

$$(\bar{\gamma}/d)_i = \frac{\sum_{h=1}^i p_h^a (S^\circ/d)_h \ell_h \left[(\gamma_a^\circ/d)_h p_h^a + (\gamma_b^\circ/d)_h p_h^b \right]}{\sum_{h=1}^i (S^\circ/d)_h p_h^a \ell_h} \quad (11)$$

and

$$\bar{x}_i = \frac{\left(\sum_{h=1}^i (S^\circ/d)_h p_h^a l_h \right)^2}{2\pi \sum_{h=1}^i (S^\circ/d)_h p_h^a l_h \left[(\gamma_a^\circ/d)_h p_h^a + (\gamma_b^\circ/d)_h p_h^b \right]}, \quad (12)$$

where the quantities S_i/d and γ_i/d of equations (6) and (7) have been replaced by

$$(S^\circ/d)_h p_h^a = S_h/d$$

and

$$(\gamma_a^\circ/d)_h p_h^a + (\gamma_b^\circ/d)_h p_h^b = \gamma_h/d.$$

The formulas involved in calculating $\bar{\tau}_{i-1}$ are defined analogously to those for $\bar{\tau}_i$ (sums from $h=1$ to $i-1$) except that $\bar{\tau}_0$ is defined by $\bar{\tau}_0 = 1$.

The terms p_h^a and l_h are the pressure of absorbing gas (H_2O or CO_2) and the path length of zone h , respectively, and p_b is the pressure of nonabsorbing (broadening) gas (N_2 , etc.). The quantities $(S^\circ/d)_h$ and $(\gamma^\circ/d)_h$ are the microscopic band model parameters; they are closely related to the effective absorption coefficients of Ferriso, Ludwig, and Abeyta, as will be shown below. Both S°/d and γ°/d are functions of temperature. The subscripts a and b on γ° stand for absorber and broadener, respectively. The case where more than one broadening gas is in the gas mixture for a given zone, or where two species simultaneously absorb radiation and broaden the lines of the other species (e.g., CO_2 and H_2O in the region near 3500 cm^{-1}), is easily handled by these equations. For the case of several broadening gases the term $(\gamma_b^\circ/d)_h p_h^b$

in equation (6) is merely replaced by

$$\sum_s (\gamma_s^\circ/d)_h p_h^s,$$

where the summation over the index s indicates that one should sum the product of partial pressure and $(\gamma_b^\circ/d)_h$ for each species (note that $(\gamma_b^\circ/d)_h$ depends on the absorbing gas as well as on the broadening gas) for each zone h . For the spectral regions where two gases both absorb radiant energy, we first compute $\bar{\tau}_i(\nu_j, \Delta\nu_j)$ for each absorbing gas separately, then multiply the two $\bar{\tau}_i(\nu_j, \Delta\nu_j)$ to get the effective $\bar{\tau}_i(\nu_j, \Delta\nu_j)$ to be used in equation (4a). For example, the case of overlapping H_2O and CO_2 bands mentioned above would use the formula

$$\bar{\tau}_i(\nu_j, \Delta\nu_j) = \left[\bar{\tau}_i^{H_2O}(\nu_j, \Delta\nu_j) \right] \left[\bar{\tau}_i^{CO_2}(\nu_j, \Delta\nu_j) \right]. \quad (13)$$

In the above calculation of the effective $\bar{\tau}_i(\nu_j, \Delta\nu_j)$, for spectral regions where two gases simultaneously absorb energy, one should treat the second gas as a broadening species when the first is treated as the absorbing gas, and vice versa; e.g., in the CO_2 calculation, we would use

$$(\gamma_{H_2O-CO_2}^\circ/d) p_{H_2O}$$

as one of the terms in the sum of $(\gamma_s^\circ/d)_h p_h^s$. The special case where there is only an absorbing gas in a zone is easily handled by setting $p_h^b = 0$ in equation (11) and equation (12).

In a recent report⁸, Ferriso, Ludwig, and Abeyta calculate transmittance with a random band model having a different distribution function from the one we have employed. They use the equation

$$-\ln \tau = K_{eff}(\nu) u \left[1 + \frac{K_{eff}(\nu) u}{4 a(\nu)} \right]^{-1/2} \quad (14)$$

where

$$a(\nu) = \gamma/d, \quad (15)$$

$$K_{\text{eff}} = S^0/d = 2\pi/p_a \ell \times \gamma/d, \quad (16)$$

and

$$u = p_a \ell. \quad (17)$$

Equations (15), (16), and (17) are the transformation equations from our band model parameters x , (γ/d) , p_a , and ℓ to another set of equally useful parameters K_{eff} , u , and a . With the help of these transformation equations, the transmittance of an inhomogeneous sample can be calculated from this alternative set of band model parameters using equations (6), (7), and (14). According to Plass⁹, equations (4) or (14) agree within 10 percent. Thus, the choice of equations (4) or (14) should not be too critical. However, at the risk of being redundant, it should be emphasized that K_{eff} is not a "true" absorption coefficient and was never intended to be used as such in the "exact" integral equation of radiant energy transport.

III. EXPERIMENTAL

A. Apparatus

Measurements were made at a variety of temperatures, pressures, and path lengths. Figure 1 shows a schematic diagram of the optical system used for the longest optical paths. With this system, the chopped signal from a globar makes three passes through each of two furnaces and then enters a small grating monochromator with which spectral measurements are made. Each furnace contained an 8-inch gas cell, thereby providing two individually isothermal zones in series with a 24-inch optical path per zone. The temperatures and pressures of the two zones were independently variable. The globar signal was chopped before the sample to eliminate interference by sample emission.

To make measurements with shorter optical paths we could, by changing the mirrors of the fore optics, pass the globar signal through each furnace only once as shown in Figure 2. We used this single pass mode of operation with a $1\frac{1}{2}$ -inch cell to study short optical paths and with an 8-inch cell for intermediate paths.

The furnace-heated gas cells are useful for temperatures up to about 1000°C. For measurements at higher temperatures, flat flame burners such as those illustrated in Figure 3 were employed. In the burner assemblies, each zone consisted of a 2-inch x 2-inch burner flanked by a pair of $\frac{1}{2}$ -inch x 2-inch burners. Hot CO₂ was generated by burning carbon monoxide with oxygen on the 2-inch x 2-inch burner and adding a controlled amount of cold CO₂ to each burning mixture to regulate the flame temperature. The flame propagation rate was adjusted by adding measured traces of hydrogen. "Guard" flames on the $\frac{1}{2}$ -inch x 2-inch burners produced combustion products that had the same temperature as the CO₂, but were all transparent at the wavelengths studied. Therefore, each zone was simply an isothermal specimen of CO₂ as far as the spectroscopic observations were concerned. Measurements were made on individual zones to obtain their transmittances and infrared emission-absorption temperatures as well as on two- and three-zones in series to obtain the transmittances of the inhomogeneous assemblies. A single-pass optical system was used with the flat flames.

During every measurement, the entire optical system outside the sample was flushed with dry nitrogen to minimize absorption by atmospheric H₂O and CO₂.

B. Procedure

The experimental procedure for two-zone cell measurements was as follows: We used a sample in-sample-out technique to obtain the transmittances of the individual zones and of the two-zone assembly. The procedure consisted of five steps as follows:

- (1) A spectral scan was made with both cells evacuated to measure the global signal,
- (2) the first cell was filled with sample and the spectral scan repeated to measure the transmittance of the sample in the first cell,
- (3) the second cell was filled and the spectral scan again repeated to obtain the transmittance of the two samples in series,
- (4) the first cell was evacuated and the transmittance of the second cell was then determined, and
- (5) the second cell was evacuated and the global signal was rechecked to detect any signal drift that might have occurred during the experiment.

After making these measurements, the two-zone transmittance was calculated from the transmittances of the two individual zones using the

Curtis-Godson approximation. The calculated value was then compared with the measured two-zone transmittance.

A similar sample in-sample-out technique was used for two-zone flame studies. A zonal sample was in the beam whenever the three flames of the zone were burning. For sample-out measurements, the three flames of the zone were replaced by streams of nitrogen. The same five-step procedure described for cell measurements was followed.

For the three-zone flame measurements, one of the flames had to be turned off and relit. Otherwise, the technique was the same as for two zones.

IV. RESULTS

Hot water vapor spectral transmittances were measured at two frequencies, 3990 cm^{-1} and 3503 cm^{-1} . We believe that these are representative frequencies in the water vapor spectrum, and the observations made should be typical of what may be expected at other frequencies.

Table I compares measured and calculated transmittances at 3990 cm^{-1} for thermally inhomogeneous optical paths in the high x region. Properties of zone 1 are labelled with the subscript 1 and properties of zone 2 are labelled with the subscript 2. Zone 1 had a temperature of 1273°K and the temperature of zone 2 was half of that, 637°K . The pressures of the water vapor in the two zones are given in columns 1 and 2. Columns 3 and 4 give the measured transmittance of zones 1 and 2, respectively. Column 5 shows the measured transmittance of the combined two-zone sample. The values of x_1 and x_2 are given on the line above the table. These values of x were calculated from the path length of each cell and from the value of x/l , which was determined by a modified curve-of-growth method¹⁰ from the transmittances for both short and long isothermal paths at the appropriate temperatures. The calculations involved in column 6 were made with equation (8) which is the equation that uses the exact values of x_1 and x_2 . This calculated two-zone transmittance is always higher than the measured transmittance by approximately .01 to .02, as shown by a comparison of columns 5 and 6.

The calculations involved in column 7 were made with equation (10) which is the equation that uses the high x approximation and in which the x values do not occur explicitly. Although x_2 is not much above 2.0, which we arbitrarily chose as the lower limit of the high x range, the differences between columns 6 and 7 are only .006 to .01.

Tables II and III give similar data for measurements made with both zones at the same temperature. Here the only inhomogeneities are due to pressure differences. The discrepancies between measured and

calculated two-zone transmittances are about the same as before. Note that when the pressures as well as the temperatures in the two zones are essentially the same, this discrepancy is not generally any smaller. In this case, where the inhomogeneity is negligible, the discrepancy cannot be due to the Curtis-Godson approximation. Therefore, the Curtis-Godson approximation may not be the major source of error in any case. The discrepancy may be primarily due to the error in the band model representation of the real spectrum.

For Table II, x_2 , as well as x_1 , is above 3, so the discrepancies between columns 6 and 7 are a little lower than in Table I. For Table III, x_2 , as well as x_1 , is below 3, so the discrepancies between columns 6 and 7 are a little higher than in Table I.

Table IV gives the results of medium x measurements. This is the region we were most worried about because it is the region in which the transmittances are most sensitive to errors in x and the region where the Curtis-Godson approximation is expected to be least accurate. However, the errors appear small.

In this region, neither the high nor the low x approximation is expected to be accurate. Columns 4 and 5 show that the errors observed with the high x approximation are still much smaller than those produced by the low x approximation.

Table V is concerned with measurements in which one zone was in the medium x range and the other had a low x . Transmittance of the two zones in series is very little lower than the transmittance of the 8-inch zone alone. Since the transmittance of the 8-inch zone was part of our input in our calculations, these calculations were not likely to be very wrong, and they are not. This table demonstrates the difficulty of trying to test the Curtis-Godson approximation in the low x region with water vapor. When x is low, water vapor just does not provide enough absorption.

High absorptance with low x values can be observed in the spectrum of hot carbon dioxide where many overlapping hot bands make the average line spacing very small. Table VI shows the results of measurements on two-zone samples of hot CO_2 . The first two rows are for wavelengths where x was low at the experimental temperatures. At the third wavelength, $4.179\text{-}\mu$, x was high. For the first and second wavelengths differences between measured values and those calculated with the low x approximation were well within the experimental error. At the third wavelength, the high x approximation gives a comparably accurate value of .335 for the two-zone transmittance.

Table VII shows the results of measurements on three-zone samples of hot CO_2 . For each of the three wavelengths at which

measurements were made, x is low. Thus, it is not surprising that equation (9), the low x approximation to equation (8), gives a calculated transmittance for the combined sample which is in good agreement with the experimental value.

As mentioned in the introduction, we have calculated the transmittance of inhomogeneous gas samples by methods other than the Curtis-Godson approximation. The Sakai approximation has already been mentioned. Another interesting approach has been that suggested by Plass¹¹, who discussed the generalization to two cells in series of the random band model with an exponential distribution of line strengths. We used this method (equation (58) of Ref. 11) to calculate the combined transmittance for all the cases involving H_2O . In no case was the result in significantly better agreement with experiment than the Curtis-Godson approximation. Sakai's approximation gave results much worse than the Curtis-Godson approximation for the combination of a high x zone with a high x zone, and since the form of Sakai's approximation is similar to that of equation (58) of Ref. 11, we feel that Sakai's approximation would give no better results in the intermediate and low x cases. Both Sakai's approximation and that of Ref. 11 become progressively more complex as the number of zones is increased beyond two.

CONCLUSIONS

We have now tested the Curtis-Godson approximation through the gamut from high x calculations through medium to low x calculations. Measurements of hot water vapor at 3503 cm^{-1} were so much like the results at 3990 cm^{-1} that they lead to the same conclusions. This similarity corroborates the hypothesis that it is likely that the behavior at these two frequencies is typical of the water vapor spectrum.

This study indicates that the accuracy of the Curtis-Godson approximation is at least comparable to that of the band model theory we used. Consequently, it should be possible to calculate transmittances of inhomogeneous and homogeneous specimens with similar accuracy, whenever the band model parameters are available. The other approximations which were tested resulted in more complex calculations with no gain in accuracy over the Curtis-Godson approximation.

In calculations of radiances of rocket plumes, the errors in currently available band model parameters and flow field information are likely to introduce much larger errors than the Curtis-Godson approximation or the band model theory.

Table I

Two-Zone Water Vapor Transmittance at 3990 cm^{-1}

Optical Path: 24" per zone

$T_1 = 1273^\circ\text{K}$

$T_2 = 637^\circ\text{K}$

$x_1 = 3.49$

$x_2 = 2.53$

p_1 (mm)	p_2 (mm)	Measured Transmittance			Calculated $\bar{\tau}$	
		τ_1	τ_2	$\bar{\tau}$	From Eq. (8)	From Eq. (10)
49	53	.745	.711	.622	.630	.638
100	104	.597	.503	.398	.414	.424
151	151	.443	.358	.247	.260	.270
101	51	.573	.722	.501	.518	.525
148	52	.452	.710	.402	.415	.421

Table II

Two-Zone Water Vapor Transmittance at 3990 cm^{-1}

Optical Path: 24" per zone

$$T_1 = T_2 = 1273^\circ\text{K}$$

$$x_1 = x_2 = 3.49$$

p_1 (mm)	p_2 (mm)	Measured Transmittance			Calculated $\bar{\tau}$	
		τ_1	τ_2	$\bar{\tau}$	From Eq. (8)	From Eq. (10)
53	53	.720	.729	.621	.628	.634
102	105	.571	.566	.425	.443	.450
150	153	.438	.443	.284	.306	.314
51	104	.743	.571	.515	.524	.530
56	146	.724	.464	.419	.428	.435

Table III

Two-Zone Water Vapor Transmittance at 3990 cm^{-1} Optical Path: $24''$ per zone

$$T_1 = T_2 = 637^\circ\text{K}$$

$$x_1 = x_2 = 2.53$$

$p_1(\text{mm})$	$p_2(\text{mm})$	Measured Transmittance			Calculated $\bar{\tau}$	
		τ_1	τ_2	$\bar{\tau}$	From Eq. (8)	From Eq. (10)
51	50	.721	.728	.620	.626	.634
102	100	.510	.528	.368	.384	.395
147	148	.374	.381	.221	.241	.252
148	59	.383	.677	.339	.345	.355
105	49	.513	.726	.462	.468	.477
102	153	.507	.365	.273	.286	.297

Table IV

Two-Zone Water Vapor Transmittance at 3990 cm^{-1}

Optical Path: 8" per zone

 $T_1 = 1273^\circ\text{K}$ $T_2 = 637^\circ\text{K}$ $x_1 = 1.16$ $x_2 = 0.84$

$p_1(\text{mm})$	$p_2(\text{mm})$	Measured Transmittance			Calculated $\bar{\tau}$		
		τ_1	τ_2	$\bar{\tau}$	From Eq. (8)	From Eq. (10)	From Eq. (9)
50	50	.846	.849	.771	.772	.791	.718
100	100	.744	.728	.617	.619	.648	.542
150	150	.655	.625	.496	.496	.530	.409

Table V

Two-Zone Water Vapor Transmittance at 3990 cm^{-1}

$$T_1 = 1273^\circ\text{K} \quad T_2 = 637^\circ\text{K}$$

$$l_1 = 8'' \quad l_2 = 1.5''$$

$$x_1 = 1.16 \quad x_2 = .16$$

$p_1(\text{mm})$	$p_2(\text{mm})$	Measured Transmittance			Calculated $\bar{\tau}$	
		τ_1	τ_2	$\bar{\tau}$	From Eq. (8)	From Eq. (9)
50	50	.853	.960	.840	.836	.827
100	100	.751	.923	.723	.720	.682
150	150	.650	.882	.612	.607	.563

Table VI
Two-Zone CO₂ Transmittances
Optical Path: 2" per zone
Atmospheric Pressure
 $T_1 = 1460^\circ\text{K}$ $T_2 = 2740^\circ\text{K}$

Wavelength (microns)	Measured Transmittances			Calculated $\bar{\tau}$ From Eq. (9)
	τ_1	τ_2	$\bar{\tau}$	
4.555	.085	.099	.009	.008
4.865	.959	.589	.572	.564
4.179	.340	.838	.325	.285

Table VII

Three-Zone CO₂ Transmittances

Optical Path: 2" per zone

Atmospheric Pressure

$T_1 = 1460^\circ\text{K}$; $T_2 = 2170^\circ\text{K}$; $T_3 = 2760^\circ\text{K}$

Wavelength (microns)	Measured Transmittances				Calculated $\bar{\tau}$ From Eq. (9)
	τ_1	τ_2	τ_3	τ_{total}	
4.555	.068	.014	.098	.000	.000
4.696	.745	.252	.290	.049	.055
4.865	.952	.734	.601	.414	.415

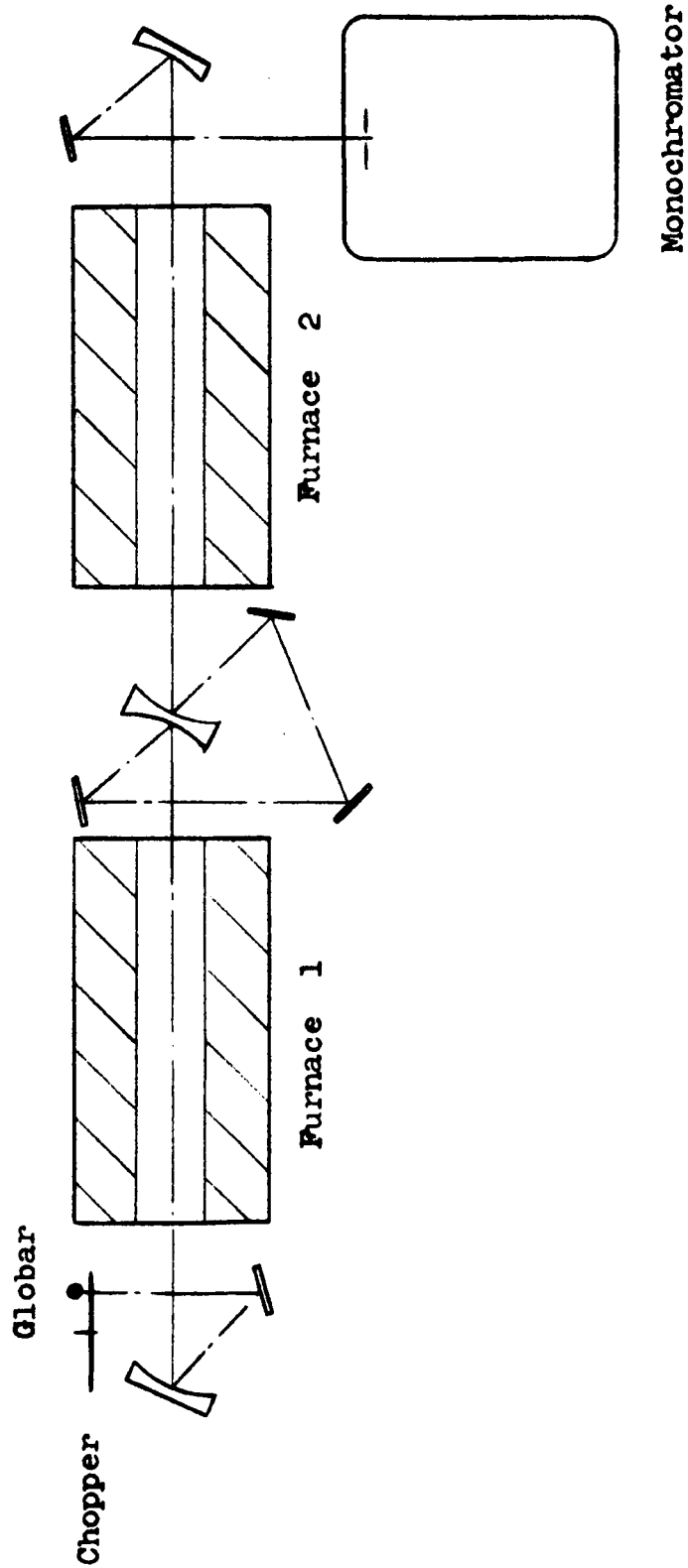


Fig. 2 Two furnace single-pass optical system.

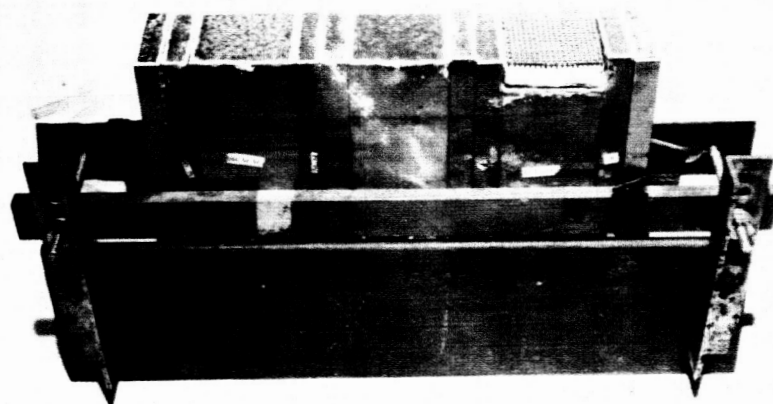


Fig. 3. Three-zone burner assembly.

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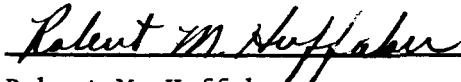
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USE OF THE CURTIS-GODSON APPROXIMATION IN CALCULATIONS
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