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EVALUATION OF THE USE OF POWERED FLIGHT TRACKING AND TELEMETRY DATA IN LUNAR ORBIT DIFFERENTIAL CORRECTION PROCEDURES

By D. H. Lewis

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ABSTRACT

This report describes a study of the effectiveness of including powered flight tracking and telemetry data in the differential correction of lunar satellite orbits. The powered flight maneuver used was representative of the Lunar Orbiter Photgraphic Project's main deboost required to transfer the spacecraft from the hyperbolic approach orbit to the intermediate elliptic selenocentric orbit. Range and range-rate tracking data were assumed to be taken from the Deep Space Network stations at Goldstone, Woomera, and Madrid at the rate of one set per minute. Telemetry data were assumed to consist of the body components of the thrust acceleration.

CONTENTS

SYMBOLS

a_o	initial thrust to mass ratio, meters/second^2
g_o	acceleration of gravity at sea level
I_{sp}	specific impulse, seconds
k	mass flow rate per unit initial mass, (seconds)$^{-1}$
M	observational noise covariance matrix
t_d, t_f	time of initiation and termination of the main deboost maneuver, respectively
x	6-dimensional vector of Cartesian position and velocity components, vector kilometers, kilometers/second
α,β,ϕ	angles defining orientation of vehicle pitch, yaw, and roll axes, degrees
γ	2-dimensional vector of thrust vector orientation angles, degrees
γ'	3-dimensional vector including orientation angles and accelerometer scale factor
$\delta x,\delta\gamma,\delta\lambda$	variation in x, γ, and α from reference values
$\delta x_A,\delta\gamma_A,\delta\alpha_A$	actual variation in x, γ, α
$\delta x_E,\delta\gamma_E,\delta\alpha_E$	estimated variation in x, γ, α
ε	observational noise vector
λ	2-dimensional vector of thrust vector orientation angles, degrees
$\overline{\xi},\overline{\eta},\overline{\zeta}$	unit vectors defining vehicle roll, pitch, and yaw axes, respectively
$\mathcal{L}_d,\mathcal{L}_d'$	covariance matrix including all information at t_d for complete solution vector, and state vector only, respectively
$\mathcal{L}_f,\mathcal{L}_f'$	covariance matrix including all information at t_f for complete solution vector, and state vector only, respectively
$\mathcal{L}_p$	covariance matrix at t_d, including translunar tracking from the second midcourse to initiation of the main deboost maneuver
$\mathcal{L}_t$	6 x 6 tracking covariance matrix computed from observations taken during the burn; epoch at t_d

$\mathcal{P}_{ts}$ — extended solution vector covariance matrices, computed from observations taken during the burn; 8 x 8 (including propulsion parameters)

$\mathcal{P}_{\gamma}$ — 2 x 2 a priori covariance matrix describing total thrust vector pointing errors

$\mathcal{P}'_{\gamma}$ — 3 x 3 a priori covariance matrix describing uncertainties in the thrust vector orientation and the accelerometer scale factor

$\mathcal{P}_{\lambda}$ — 2 x 2 a priori covariance matrix describing platform orientation uncertainties

ϕ_f — 6 x 6 matrix of partial derivative relating perturbations in x at t_f to those at t_d, i.e., $\partial x(t_f)/\partial x(t_{epoch})$

ϕ_s — 6 x 6 matrix of partial derivatives relating perturbations in x at t_f to those at t_d, i.e., $\partial x(t_f)/\partial x(t_d)$

ϕ_{γ} — 6 x 2 matrix of partial derivatives relating perturbations in x at t_f to those in γ, i.e. $\partial x(t_f)/\partial\gamma$

ϕ'_{γ} — 6 x 3 matrix of partial derivatives relating perturbations in x at t_f to those in γ (includes accelerometer scale factor), i.e., $\partial x(t_f)/\partial\gamma$

$\omega_r, \omega_p, \omega_q$ — vehicle angular velocities about roll, pitch, and yaw axes, respectively

Ω — 6 x 2 matrix of partial derivatives, relating perturbations in x at t_f to those in γ, i.e., $\partial x(t_f)/\partial\gamma$

<u>Subscripts</u>

A — actual

C — consider

d — deboost

E — estimate

f — final

P — a priori

s — solve; state

t — tracking

γ — propulsion parameters

EVALUATION OF THE USE OF POWERED FLIGHT TRACKING AND TELEMETRY DATA IN LUNAR ORBIT DIFFERENTIAL CORRECTION PROCEDURES

By D. H. Lewis
TRW Systems

1. SUMMARY

A study of the effectiveness of including in differential correction the tracking and telemetry data taken during powered flight maneuvers is described in this report.

The powered flight maneuver used was representative of the Lunar Orbiter Photographic Project (LOPP) main deboost required to transfer the spacecraft from the hyperbolic approach orbit to the intermediate elliptic selenocentric orbit.

Range ($\sigma = 20m$) and range-rate ($\sigma = 0.019m/s$) tracking data were assumed to be taken from the Deep Space Network (DSN) stations at Goldstone, Woomera, and Madrid at the rate of one set per minute. Telemetry data were assumed to consist of the body components of the thrust acceleration.

Uncertainties in four propulsion parameters were considered:

a) Thrust axis orientation angle α, $\sigma = 0^\circ.397$

b) Thrust axis orientation angle β, $\sigma = 0^\circ.397$

c) Accelerometer scale factor, 0.039%

d) Velocity cutoff resolution, 0.019 m/s

The state vector covariance matrix prior to the main deboost was computed using tracking from the second midcourse (70 h) to initiation of the main deboost (90 h).

Generally it was found that the use of powered flight tracking and telemetry data allows reduction of the state vector uncertainties to the level

that existed before the maneuver began. When used individually, tracking data is more useful than telemetry data; the state vector uncertainties using tracking data only are about 1/5 those which exist when no powered flight data is used, while use of telemetry data alone reduces the uncertainties to approximately one half their former values.

2. INTRODUCTION

The LOPP mission plan calls for a deboost maneuver from a hyperbolic lunar approach orbit to a selenocentric elliptic orbit. The use of a 100-lb thrust engine on a spacecraft weighting approximately 850 lb calls for long burning times. The consequent possibility of large errors in position and velocity at burn termination (introduced by propulsion system inaccuracies) raises the question of whether or not using tracking and telemetry data during the burn would be useful in reducing the uncertainties.

There are four possible tracking and telemetry burn data usages to be considered:

a) No tracking during the burn; no telemetry

b) Tracking during the burn; no telemetry

c) No tracking during the burn; with telemetry

d) Tracking during the burn; with telemetry

Consideration of all four cases allows evaluation of both the combined and individual effectiveness of tracking and telemetry data in reducing the state vector uncertainties.

The covariance matrix for the state vector immediately following termination of the main deboost, $\mathcal{Z}'_f$, will be computed for each of these cases. Suitable figures of merit (e.g., the diagonal elements) can then be used to compare the four combinations of data types. The computation procedures for these cases are discussed below (section 4).

The state vector (6 x 6) covariance matrix prior to the main deboost was computed using information from the translunar tracking from the second midcourse (70 h) to initiation of the main deboost (90 h). Tracking is assumed to take place on the free-flight arc until 1 hour prior to the main deboost maneuver. This 1-hour period is assumed to be that required to process the tracking data and compute the required maneuver parameters. It is this matrix (diagonal elements 106, 100, and 299 m; $0.57(10^{-3})$, $0.15(10^{-3})$, and $0.49(10^{-2})$m/s) which was used as a starting point in evaluating the use of telemetry data and powered flight tracking. It is important to note that the effects of systematic errors on the translunar tracking prior to the maneuver have not been included; this accounts for the very small uncertainties listed above for the a priori covariance matrix, and means that any state vector uncertainties existing after the maneuver may be attributed to propulsion parameter uncertainties.

3. MATHEMATICAL FORMULATION OF THE POWERED FLIGHT MODEL

3.1 Deboost Maneuver

The LOPP main deboost maneuver is accomplished using a fixed inertial orientation and a constant thrust level. The propulsion system is shut down when the velocity sensed by the spacecraft accelerometers reaches a preset value.

For the purposes of this study, the main deboost maneuver for the S110 reference trajectory was used (reference 1); the characteristics of this burn are listed in table I. Langley Research Center has specified the following propulsion error sources (1σ values indicated) for the main deboost:

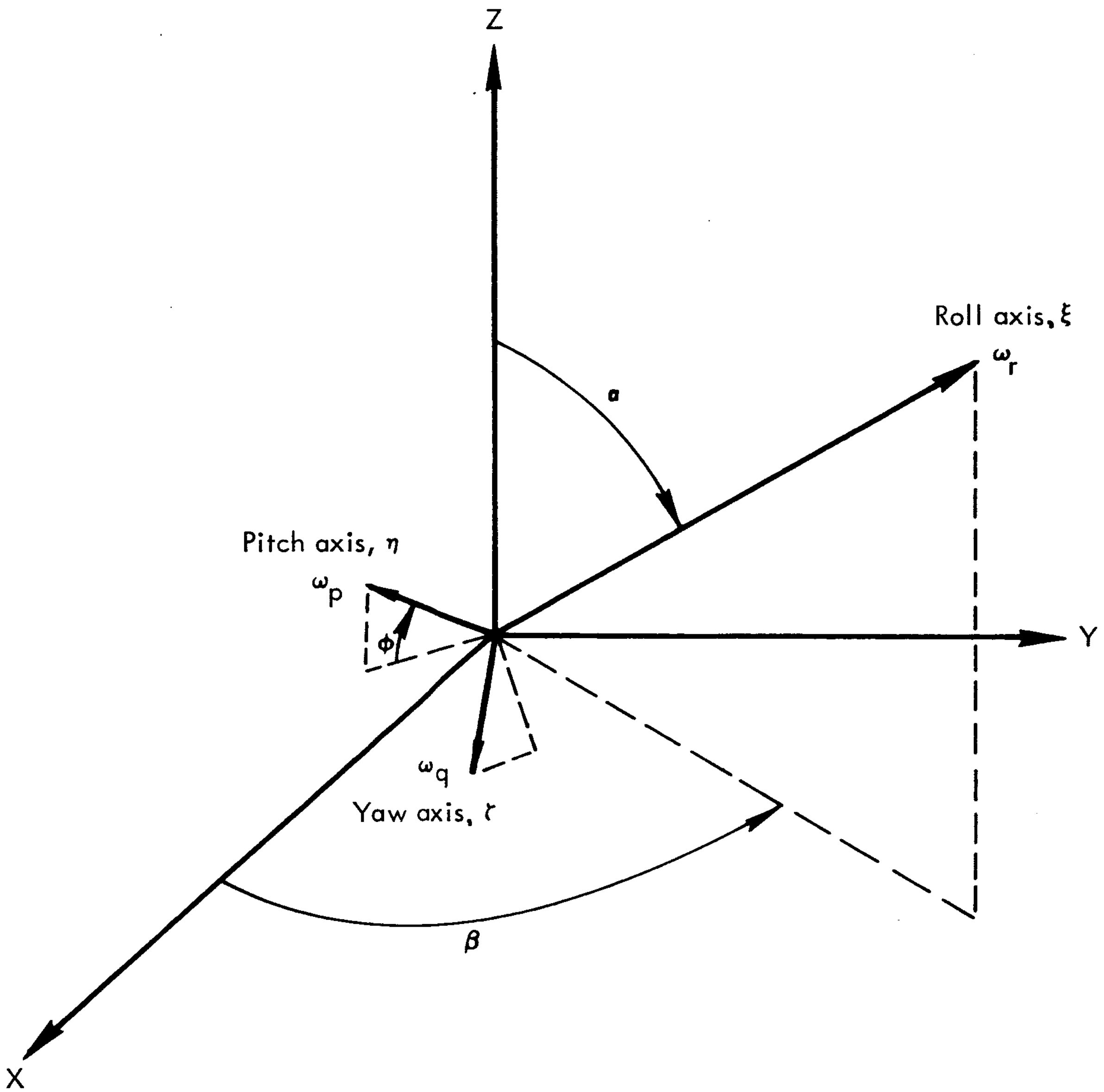
a) Thrust vector orientation uncertainties (in the angles α and β; see figure 1) 0.397°

b) Accelerometer scale factor uncertainty, 0.039%

c) Velocity cutoff resolution uncertainty, 0.019 m/s

TABLE I. - MAIN DEBOOST MANEUVER

Characteristics	Time, 1966, 27 June	
	$4^{hr}\ 0^{min}\ 27\overset{s}{.}46$	$4^{hr}\ 9^{min}\ 51\overset{s}{.}05$
	Start burn	End burn
State vector (true of date selenocentric equatorial system; X-axis, vernal equinox)		
x (km)	181.262	-887.373
y (km)	1917.015	1822.665
z (km)	1156.883	1315.704
$\dot{x}$ (km/s)	-2.2300	-1.4964
$\dot{y}$ (km/s)	-.0887	-.2130
$\dot{z}$ (km/s)	.5224	.0370
Thrust (lb)	100.24	100.24
Weight (lb)	850.0	648.232
Flow rate (lb/s)	-.358	-.358
Sensed velocity (km/s)	0.	.746
Eccentricity	1.407	.297
Inclination (deg)	34.192	33.997
Longitude of the ascending node (deg)	22.431	10.178
Argument of perigee (deg)	65.837	21.777
Period (min)	--	217.742
Semimajor axis (km)	-5518.956	2767.517



NOTES

Thrust vector aligned along roll axis
X, Y parallel to the equatorial plane
X in direction of vernal equinox

Figure 1.— Definition of Thrust Attitude Orientation Angles and Angular Rates

Since the maneuver involves a large ΔV (746 m/s), the resolution error contribution may be ignored, leaving three error sources to be considered. The assumption is made that the orientation uncertainties are due to two independent errors; that due to imperfect alignment of the platform axes (Sun-Canopus orientation error, 0.183° one sigma), and that due to imperfect orientation of the spacecraft within the platform axes (pointing error, 0.35° one sigma).

3.2 Program Modifications

To handle tracking data taken during the burn, certain modifications to the TRW Systems AT85 Orbit Determination Program were required. The powered flight option developed includes multiple stages, with the burn maneuver start and stop times, t_d and t_f, respectively, as inputs for each stage.

Included in the option is the ability to solve for the following deboost maneuver parameters:

a) Propulsion parameters

   a_o, initial thrust-to-mass ratio

   k, mass flow rate divided by initial mass

b) Orientation parameters (figure 1)

   α

   β

   ϕ

c) Angular rates (figure 1)

   ω_p

   ω_q

   ω_r

Use of telemetry (T/M) data was simulated by assuming the only contribution to orientation uncertainties was that from the Sun-Canopus sensor error, and consequently required no additional program changes (i.e., only a change in the magnitude of the uncertainties on α and β was required).

For

$$t_d \le t \le t_f$$

the equations of motion are (reference 2),

$$\ddot{\bar{r}} = \frac{a_o}{1-k(t-t_o)} \bar{\xi}(t) + \bar{G}(t) = \bar{F}(t) + \bar{G}(t) \tag{1}$$

where

$a_o = \frac{T_o}{m_o}$, the initial thrust to mass ratio (units of m/s^2, when T_o is in newtons and m_o is in kg)

$k = \frac{-\dot{m}}{m_o}$, the mass flow rate per unit initial mass (units of $(sec)^{-1}$)

$\bar{\xi}(t)$ = the unit vector in the direction of thrust

$\bar{G}$ = resultant of all gravitational forces normally included in free-flight trajectory computation.

The initial thrust-to-mass ratio might be more conveniently computed using

$$a_o = I_{sp}\, g_o\, k$$

where

I_{sp} = specific impulse (units of sec)

g_o = acceleration of gravity at sea level

For the LOPP maneuver, the thrust attitude, $\bar{\xi}$, is constant, i.e.,

$$\bar{\xi} = \begin{bmatrix} \cos\beta_o \sin\alpha_o \\ \sin\beta_o \sin\alpha_o \\ \cos\alpha_o \end{bmatrix}$$

The vehicle attitude is defined by the first-order differential equation

$$\dot{R} = RH, \; R(0) = R_o \tag{2}$$

where the colums of R are the Cartesian components of the roll, pitch, and yaw axes, and H is the skew symmetric form of the angular velocity vector, i.e.,

$$R = \begin{bmatrix} \xi_x & \eta_x & \zeta_x \\ \xi_y & \eta_y & \zeta_y \\ \xi_z & \eta_z & \zeta_z \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & -\omega_q & \omega_p \\ \omega_q & 0 & -\omega_r \\ -\omega_p & \omega_r & 0 \end{bmatrix}$$

For constant yaw, pitch, and roll rates, H is constant, and an analytic solution exists for equation (2). This solution has the form

$$R = R_o \left\{ I + \left[1 - \cos|\omega|(t - t_o)\right] \frac{H^2}{|\omega|^2} + \left[\sin|\omega|(t - t_o)\right] \frac{H}{|\omega|} \right\} \tag{3}$$

where

$$|\omega| = \sqrt{\omega_r^2 + \omega_p^2 + \omega_q^2} \; .$$

For the fixed inertial thrust attitude used for LOPP,

$$\omega_r = \omega_p = \omega_q = 0$$

and the analytic solution may be used. The values for these angular rates will be assumed nonzero in the derivation of the variational equations, to allow solution for drift rates about the vehicle body axes.

Since the magnitude and direction of the thrust vector are not a function of the local state vector, the variational equations for the initial conditions (i.e., epoch position, and velocity) are integrated through the powered flight phase with no changes. The variational equations for the powered flight parameters which have been introduced into the solution vector will have three different forms, depending upon whether the observational partials (to be included in the normal matrix) are before, during, or after the maneuver.

Before the maneuver, all partial derivatives with respect to powered flight parameters are zero, since variations in these quantities could not yet have influenced the orbit.

During the burn, variations in the powered flight parameters will affect the trajectory, and a set of variational equations (in addition to the initial condition set) must be integrated. For the i^{th} differential equation parameter,

$$\frac{d^2}{dt^2}\left(\frac{\partial \bar{r}}{\partial \gamma_i}\right) = \frac{\partial \bar{G}}{\partial \bar{r}} \frac{\partial \bar{r}}{\partial \gamma_i} + \frac{\partial \bar{F}}{\partial \gamma_i} \tag{4}$$

$$\left.\frac{\partial \bar{r}}{\partial \gamma_i}\right|_{t=t_d} = 0$$

where the first term on the right-hand side is that which describes the normal free-flight variational equations (the only difference being the null matrix initial condition at $t = t_d$), and the second term is the variation in thrust acceleration due to powered flight parameter variation. For the eight parameters of interest here, the second term takes the following forms:

$\underline{a_o (i = 1)}$

$$\frac{\partial \bar{F}}{\partial a_o} = \frac{\bar{F}}{a_o}$$

$\underline{k (i = 2)}$

$$\frac{\partial \bar{F}}{\partial k} = \frac{\bar{F}(t-t_o)}{\left[1-k(t-t_o)\right]}$$

$\underline{\alpha (i = 3)}$

$$\frac{\partial \bar{F}}{\partial \alpha} = \frac{a_o}{1-k(t-t_o)} \begin{bmatrix} 0 & 0 & \cos\beta \\ 0 & 0 & \sin\beta \\ -\cos\beta & -\sin\beta & 0 \end{bmatrix} R_o \, r$$

where

$$r = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \left[1 - \cos|\omega|(t - t_o)\right] \begin{bmatrix} \frac{\omega_r^2}{|\omega|^2} - 1 \\ \frac{\omega_r \omega_p}{|\omega|^2} \\ \frac{\omega_r \omega_q}{|\omega|^2} \end{bmatrix} + \sin|\omega|(t - t_o) \begin{bmatrix} 0 \\ \frac{\omega_q}{|\omega|} \\ -\frac{\omega_p}{|\omega|} \end{bmatrix} \right\}$$

$\underline{\beta(i = 4)}$

$$\frac{\partial \overline{F}}{\partial \beta} = \frac{a_o}{1-k(t-t_o)} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} R_o \, r$$

$\underline{\phi(i = 5)}$

$$\frac{\partial \overline{F}}{\partial \phi} = \frac{a_o \, R_o}{1-k(t-t_o)} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} r$$

$\underline{\omega_p (i = 6)}$

$$
\begin{aligned}
\frac{\partial \bar{F}}{\partial \omega_p} = \frac{a_o R_o}{1-k(t-t_o)} \Bigg\{ & \left[1 - \cos |\omega| (t - t_o)\right] \begin{pmatrix} -\dfrac{2\omega_r^2 \omega_p}{|\omega|^4} \\ \dfrac{\omega_r}{|\omega|^2} - \dfrac{2\omega_r \omega_p^2}{|\omega|^4} \\ -\dfrac{2\omega_r \omega_p \omega_q}{|\omega|^4} \end{pmatrix} \\
& + \left[\sin |\omega| (t - t_o)\right] \left[\begin{pmatrix} \dfrac{\omega_r^2}{|\omega|^2} - 1 \\ \dfrac{\omega_r \omega_p}{|\omega|^2} \\ \dfrac{\omega_r \omega_q}{|\omega|^2} \end{pmatrix} \left(\frac{\omega_p}{|\omega|} (t - t_o)\right) + \begin{pmatrix} 0 \\ -\dfrac{\omega_q \omega_p}{|\omega|^3} \\ -\dfrac{1}{|\omega|} + \dfrac{\omega_p^2}{|\omega|^3} \end{pmatrix} \right] \\
& + \left[\cos |\omega| (t - t_o)\right] \begin{pmatrix} 0 \\ \dfrac{\omega_q}{|\omega|} \\ -\dfrac{\omega_p}{|\omega|} \end{pmatrix} \left(\frac{\omega_p}{|\omega|} (t - t_o)\right) \Bigg\}
\end{aligned}
$$

$\underline{\omega_q (i = 7)}$

$$\frac{\partial \bar{F}}{\partial \omega_q} = \frac{a_o R_o}{1-k(t-t_o)} \left\{ \left[1 - \cos |\omega|(t - t_o)\right] \begin{pmatrix} -\dfrac{2\omega_r^2 \omega_q}{|\omega|^4} \\ -\dfrac{2\omega_r \omega_q \omega_p}{|\omega|^4} \\ \dfrac{\omega_r}{|\omega|^2} - \dfrac{2\omega_r \omega_q^2}{|\omega|^4} \end{pmatrix} \right.$$

$$+ \left[\sin |\omega|(t - t_o)\right] \left[\begin{pmatrix} \dfrac{\omega_r^2}{|\omega|^2} - 1 \\ \dfrac{\omega_r \omega_p}{|\omega|^2} \\ \dfrac{\omega_r \omega_q}{|\omega|^2} \end{pmatrix} \left(\frac{\omega_q}{|\omega|} (t - t_o) \right) \right.$$

$$\left. + \begin{pmatrix} 0 \\ \dfrac{1}{|\omega|} - \dfrac{\omega_q^2}{|\omega|^3} \\ \dfrac{\omega_p \omega_q}{|\omega|^3} \end{pmatrix} \right]$$

$$\left. + \left[\cos |\omega|(t - t_o)\right] \begin{pmatrix} 0 \\ \dfrac{\omega_q}{|\omega|} \\ -\dfrac{\omega_p}{|\omega|} \end{pmatrix} \left(\frac{\omega_q}{|\omega|} (t - t_o) \right) \right\}$$

$\underline{\omega_r (i = 8)}$

$$\frac{\partial \bar{F}}{\partial \omega_r} = \frac{a_o R_o}{1-k(t-t_o)} \left\{ \left[1 - \cos |\omega|(t - t_o)\right] \begin{pmatrix} \frac{2\omega_r}{|\omega|^2} - \frac{2\omega_r^3}{|\omega|^4} \\ \frac{\omega_p}{|\omega|^2} - \frac{2\omega_r^2 \omega_p}{|\omega|^4} \\ \frac{\omega_q}{|\omega|^2} - \frac{2\omega_r^2 \omega_q}{|\omega|^4} \end{pmatrix} \right.$$

$$+ \left[\sin |\omega|(t - t_o)\right] \left[\begin{pmatrix} \frac{\omega_r^2}{|\omega|^2} - 1 \\ \frac{\omega_r \omega_p}{|\omega|^2} \\ \frac{\omega_r \omega_q}{|\omega|^2} \end{pmatrix} \left(\frac{\omega_r}{|\omega|} (t - t_o) \right) \right.$$

$$\left. + \begin{pmatrix} 0 \\ -\frac{\omega_q \omega_r}{|\omega|^3} \\ \frac{\omega_p \omega_r}{|\omega|^3} \end{pmatrix} \right]$$

$$\left. + \left[\cos |\omega|(t - t_o)\right] \begin{pmatrix} 0 \\ \frac{\omega_q}{|\omega|} \\ -\frac{\omega_p}{|\omega|} \end{pmatrix} \left(\frac{\omega_r}{|\omega|} (t - t_o) \right) \right\}$$

The matrix R_o may be written in terms of α, β, and ϕ (abbreviating sine and cosine by S and C),

$$R_o = \begin{bmatrix} C_\beta S_\alpha & C_\phi S_\beta - C_\alpha C_\beta S_\phi & S_\phi S_\beta + C_\phi C_\alpha C_\beta \\ S_\beta S_\alpha & -C_\phi C_\beta - S_\phi S_\beta C_\alpha & -C_\beta S_\phi + C_\phi S_\beta C_\alpha \\ C_\alpha & S_\phi S_\alpha & -S_\alpha C_\phi \end{bmatrix}$$

At the time of termination of the burn maneuver, t_f, the matrices

$$\phi_f = \frac{\partial x(t_f)}{\partial x(t_{epoch})}$$

$$\Omega = \frac{\partial x(t_f)}{\partial \gamma}$$

are saved, where x is the free-flight state vector and γ is the vector of powered flight parameters. The free-flight variational equations are then reinitialized, allowing the partial derivatives for the postburn portion of the trajectory to be formed by the following chain rule products:

$$\frac{\partial x(t)}{\partial x(t_{epoch})} = \frac{\partial x(t)}{\partial x(t_f)} \frac{\partial x(t_f)}{\partial x(t_{epoch})}$$

$$\frac{\partial x(t)}{\partial \gamma} = \frac{\partial x(t)}{\partial x(t_f)} \frac{\partial x(t_f)}{\partial \gamma}$$

where, in both equations,

$$\frac{\partial x(t)}{\partial x(t_f)} = \phi(t)$$

is the solution to the reinitialized variational equations.

4. EVALUATION OF POWERED FLIGHT TRACKING AND TELEMETRY DATA

4.1 Error Propagation without Tracking or Telemetry

This is the problem of updating the a priori tracking covariance matrix $\mathcal{L}_p$, with consideration given to the effect of the unestimated propulsion parameters. The estimate δx_E is updated across the burn,

$$\delta x_{Ef} = \frac{\partial x_f}{\partial x_d} \delta x_{Ed} \tag{5}$$

where the subscripts d and f refer to the initiation and termination of the deboost maneuver, respectively, and $\partial x_f / \partial x_d$ is the 6 x 6 matrix, ϕ_s, relating perturbations in the state vector at time t_f to those at time t_d. The actual deviation is updated to include the effects of the (unestimated) propulsion parameter errors.

Thus

$$\partial x_{Af} = \frac{\partial x_f}{\partial x_d} \delta x_{Ad} + \frac{\partial x_f}{\partial \gamma} \delta \gamma_A \tag{6}$$

where $\delta \gamma_A$ is the actual perturbation in the propulsion parameter vector, and $\partial x_f / \partial \gamma$ is the matrix, ϕ_γ relating this perturbation to those in the state vector at time t_f. To develop the updating equation, it is covenient to expand the state vector to include $\delta \gamma$ and to realize that neglecting $\delta \gamma_A$ in the fit is equivalent to setting $\delta \gamma_E = 0$. In terms of the expanded state vector,

$$\left[\begin{array}{c} \delta x_{Ef} \\ \hline \delta \gamma_{Ef} \end{array}\right] = \left[\begin{array}{c|c} \phi_s & \phi_\gamma \\ \hline 0 & I \end{array}\right] \left[\begin{array}{c} \delta x_{Ed} \\ \hline \delta \gamma_{Ed} \end{array}\right] \tag{7}$$

and

$$\begin{bmatrix} \delta x_{Af} \\ \hline \delta \gamma_{Af} \end{bmatrix} = \left[\begin{array}{c|c} \phi_s & \phi_\gamma \\ \hline 0 & I \end{array}\right] \begin{bmatrix} \delta x_{Ad} \\ \hline \delta \gamma_{Ad} \end{bmatrix} \tag{8}$$

The error at the termination of the main deboost is then (letting $\delta\gamma_{Ed} = 0$),

$$\begin{bmatrix} \delta x_{Ef} - \delta x_{Af} \\ \hline \delta \gamma_{Ef} - \delta \gamma_{Af} \end{bmatrix} = \left[\begin{array}{c|c} \phi_s & \phi_\gamma \\ \hline 0 & I \end{array}\right] \begin{bmatrix} \delta x_{Ed} - \delta x_{Ad} \\ \hline - \delta \gamma_{Ad} \end{bmatrix} \tag{9}$$

and the covariance matrix,

$$E \begin{bmatrix} \delta x_{Ef} - \delta x_{Af} \\ \hline \delta \gamma_{Ef} - \delta \gamma_{Af} \end{bmatrix} \begin{bmatrix} \delta x_{Ef} - \delta x_{Af} \\ \hline \delta \gamma_{Ef} - \delta \gamma_{Af} \end{bmatrix}^T = \left[\begin{array}{c|c} \phi_s & \phi_\gamma \\ \hline 0 & I \end{array}\right] \left[\begin{array}{c|c} \mathcal{L}'_d & \mathcal{L}_{s\gamma} \\ \hline \mathcal{L}^T_{s\gamma} & \mathcal{L}_\gamma \end{array}\right] \left[\begin{array}{c|c} \phi^T_s & 0 \\ \hline \phi^T_\gamma & I \end{array}\right] \tag{10}$$

where

$$\mathcal{L}'_d = E(\delta x_{Ed} - \delta x_{Ad})(\delta x_{Ed} - \delta x_{Ad})^T$$

$$\mathcal{L}_{s\gamma} = E(\delta x_{Ed} - \delta x_{Ad})(- \delta \gamma_{Ad})^T$$

$$\mathcal{L}_\gamma = E(\delta \gamma_A)(\delta \gamma_A)^T .$$

Since there has not yet been any data taken during the burn,

$$\mathcal{L}_{\gamma s} = \mathcal{L}_{s\gamma} = 0$$

and

$$\mathcal{L}_f = \left[\begin{array}{c|c} \phi_s \mathcal{L}'_d \phi_s^T + \phi_\gamma \mathcal{L}_\gamma \phi_\gamma^T & \phi_\gamma \mathcal{L}_\gamma \\ \hline \mathcal{L}_\gamma \phi_\gamma^T & \mathcal{L}_\gamma \end{array}\right] \tag{11}$$

The upper left-hand 6 x 6 partition, Σ_f', of this covariance matrix describes the uncertainties in the state vector at t_f.

Preliminary perturbation runs indicated that the effects of the accelerometer scale factor error are about one order of magnitude smaller than the effects of orientation uncertainties. On the basis of these runs, the study was conducted assuming that the only propulsion error sources are the two orientation parameters, α and β. However, to verify that the effects of the accelerometer scale factor may be ignored, Σ_f (equation 11) was computed using both a 2 x 2 and 3 x 3 propulsion parameter covariance matrix, with the latter including the scale factor error.

Thus, taking the orientation parameters in the order α, β (in degrees),

$$\Sigma_\gamma = \begin{bmatrix} (0.397)^2 & 0 \\ 0 & (0.397)^2 \end{bmatrix}$$

and, adding the scale factor uncertainty (in terms of sensed velocity error in m/s),

$$\Sigma_\gamma' = \begin{bmatrix} (0.397)^2 & 0 & 0 \\ 0 & (0.397)^2 & 0 \\ 0 & 0 & (0.286)^2 \end{bmatrix}$$

Note that an additional column must be added to ϕ_γ (6 x 2 matrix) to update contributions due to the scale factor error; the resulting (6 x 3) matrix will be denoted by ϕ_γ'. The importance of the scale factor uncertainty can be evaluated by comparing the covariance matrices, Σ_f', which result from the updates including two and three propulsion error sources as described by Σ_γ and Σ_γ', respectively.

4.2 Use of Tracking Data Only

We now admit observations taken during the execution of the deboost maneuver. More information is now available about the state vector; however, the propulsion parameter uncertainties will also have affected the covariance matrix for those observations taken during the burn.

We can either consider the effect of these propulsion parameter uncertainties on the solution (without including them in the solution vector) or solve for them.

In the case where the effects of propulsion parameters are considered, the residuals can be written in the following form,

$$\delta R = A\delta x_A + B\delta\gamma_A + \varepsilon \tag{12}$$

where δR is the vector of residuals for observations taken during the burn, $\delta\gamma_A$ is the vector of propulsion parameter errors, B is the partial derivative of R with respect to γ, and ε is the vector of random noise on the observations, R.

Since $\delta\gamma_A$ is neglected in the fit,

$$\delta x_E = (A^TWA)^{-1} A^TW\delta\, R = \delta x_A + (A^TWA)^{-1} A^TW(B\delta\gamma_A + \varepsilon) \tag{13}$$

and the error in the estimate is

$$\delta x_E - \delta x_A = (A^TWA)^{-1} A^TW(B\delta\gamma_A + \varepsilon) \tag{14}$$

the covariance matrix of the error is,

$$\begin{aligned} \Sigma_t &= E(\delta x_E - \delta x_A)(\delta x_E - \delta x_A)^T \\ &= (A^TWA)^{-1} A^TW(B\Sigma_\gamma B^T + M)WA(A^TWA)^{-1} \end{aligned} \tag{15}$$

where we have assumed

$$E(\varepsilon)(B\delta\gamma_A)^T = E(B\delta\gamma_A)(\varepsilon)^T = 0$$

and

$$M = E(\varepsilon\varepsilon^T) \ .$$

If we set

$$W = M^{-1}$$

equation (15) becomes,

$$\Lambda_t = (A^TWA)^{-1} + (A^TWA)^{-1} A^TWB \Lambda_\gamma B^TWA(A^TWA)^{-1} \tag{16}$$

If the propulsion parameters, γ, are included in the solution vector, the residual equation becomes,

$$\delta R = A \begin{bmatrix} \delta x_A \\ \hdashline \delta\gamma_A \end{bmatrix} + \varepsilon$$

and

$$\begin{bmatrix} \delta x_E \\ \hdashline \delta\gamma_E \end{bmatrix} = (A^TWA)^{-1} A^TW\delta R \ . \tag{17}$$

where A is now the augmented normal matrix; i.e., it includes partial derivatives of the observations with respect to x and γ.

The covariance matrix, (dimensions are 8 x 8 since two components of γ are now included),

$$\Lambda_{ts} = (A^TWA)^{-1}$$

In both of the above cases, the powered flight tracking covariance matrices, $\mathcal{Z}_{tb}$ and $\mathcal{Z}_{tbs}$, must be combined with the a priori tracking matrix, $\mathcal{Z}_p$, and propagated across the burn to give $\mathcal{Z}_f$.

For the "consider" case, the propagation matrices will be those derived in the last section for updating including the effects of unestimated propulsion parameters. That is,

$$\mathcal{Z}f = \left[\begin{array}{c|c} \phi_s & \phi_\gamma \\ \hline 0 & I \end{array}\right] \left\{ \left[\begin{array}{c|c} \mathcal{Z}_p^{-1} & 0 \\ \hline 0 & 0 \end{array}\right] + \left[\begin{array}{c|c} \mathcal{Z}_t & \mathcal{Z}_{\gamma s} \\ \hline \mathcal{Z}_{\gamma s}^T & \mathcal{Z}_\gamma \end{array}\right]^{-1} \right\} \left[\begin{array}{c|c} \phi_s^T & 0 \\ \hline \phi_\gamma^T & I \end{array}\right] \tag{18}$$

where

$$\begin{aligned} \mathcal{Z}_{\gamma s} &= E(\delta x_{Ed} - \delta x_{Ad})(-\delta\gamma_A)^T \\ &= E\left[(A^TWA)^{-1}(A^TWB\delta\gamma_A + A^TW\varepsilon)\right]\left[-\delta\gamma_A\right]^T \\ &= -(A^TWA)^{-1}A^TWB\Sigma_\gamma \,. \end{aligned}$$

Here $\mathcal{Z}_{\gamma s} \neq 0$, because tracking data has been taken during the burn and propulsion parameter uncertainties have influenced the solution.

As in equation (11), the upper, left-hand 6 x 6 partition, $\mathcal{Z}_f'$, of this covariance matrix describes the uncertainties in the state vector at t_f.

In any expression calling for the combination of two or more covariance matrices, such as $(\mathcal{Z}_p^{-1} + \mathcal{Z}_t^{-1})^{-1}$ above, the appropriate normal matrix was used when available (rather than the inverse of the covariance matrix), to avoid multiple inversions.

When the propulsion parameters are included in the solution vector, the updating equation becomes,

$$\Lambda_f = \left[\begin{array}{c|c} \phi_s & \phi_\gamma \\ \hline 0 & I \end{array}\right] \left\{ \left[\Lambda_{ts} \right]^{-1} + \left[\begin{array}{c|c} \Lambda_p^{-1} & 0 \\ \hline 0 & \Lambda_\gamma^{-1} \end{array}\right] \right\}^{-1} \left[\begin{array}{c|c} \phi_s^T & 0 \\ \hline \phi_\gamma^T & I \end{array}\right] \tag{19}$$

The a priori information has been added to the upper 6 x 6 state vector partition of Λ_{tbs}, and the lower 2 x 2 (propulsion parameters) partition, with zeros elsewhere. Note that there is no requirement that the full (8 x 8) a priori information matrix be nonsingular. The state vector covariance matrix sought, Λ_f', is the upper left 6 x 6 partition of the result of this matrix product.

4.3 Use of Telemetry Data Only

We make the assumption that the errors in telemetered quantities are due to platform alignment errors only. This is equivalent to assuming that the magnitude and orientation of the thrust vector within the platform coordinate system is perfectly known; only the errors in the orientation of the platform itself contribute to errors in the thrust vector components. These thrust vector components are used in the integration of the equations of motion in place of the expression for $\overline{F}$ in equations (1). Then, since no additional tracking information is added,

$$\Lambda_f' = \phi_s \Lambda_d \phi_s^T + \phi_\gamma \Lambda_\lambda \phi_\gamma^T \tag{20}$$

where Λ_λ is the 2 x 2 covariance matrix describing platform alignment uncertainties.

For the purposes of this study, we will assume that the Sun-Canopus orientation error of reference 1 may be measured in terms of the angles (α and β) which have been used to describe the thrust vector orientation in the AT85 program. Under this assumption, the covariance matrix (α and β in degrees) is,

$$\Sigma_\lambda = \begin{bmatrix} (0.183)^2 & 0 \\ 0 & (0.183)^2 \end{bmatrix}$$

4.4 Use of Tracking and Telemetry

With both tracking and telemetry data available, we have the option of either considering or solving for the uncertainties in the propulsion parameters. As above (section 4.3), we make the assumption that errors in the telemetered quantities are due to platform alignment errors only.

For the case where propulsion parameters are to be solved for or considered, the update matrix is of the same form as equation (19). The solve and consider cases differ only in the covariance matrices to be updated.

The covariance matrix for the solve case will be the 8 x 8 tracking covariance matrix, Σ_{tbs}, combined with the a priori information matrices for the state vector and orientation angles; i.e.,

$$\Sigma_d = \left\{ \left[\begin{array}{c|c} \Sigma_p^{-1} & \\ \hline 0 & \Sigma_\lambda^{-1} \end{array}\right] + \left[\Sigma_{ts} \right]^{-1} \right\}^{-1} \tag{21}$$

Following the development in section 4.2, the covariance matrix for the consider case,

$$\Sigma_d = E \left[\begin{array}{c|c} (\delta x_{Ed} - \delta x_{Ad})(\delta x_{Ed} - \delta x_{Ad})^T & (\delta x_{Ed} - \delta x_{Ad})(-\delta\lambda_A)^T \\ \hline - \delta\lambda_A(\delta x_{Ed} - \delta x_{Ad})^T & (\delta\lambda_A)(\delta\lambda_A)^T \end{array}\right]$$

$$= \left[\left[\begin{array}{c|c} \Sigma_p^{-1} & 0 \\ \hline 0 & 0 \end{array}\right] + \left[\begin{array}{c|c} \Sigma_t & \Sigma_{\lambda s} \\ \hline \Sigma_{\lambda s}^T & \Sigma_\lambda \end{array}\right]^{-1} \right]^{-1}$$

where

$$\mathcal{L}_t = (A^TWA)^{-1} + (A^TWA)^{-1} A^TWB \, \mathcal{L}_\lambda \, B^TWA(A^TWA)^{-1}$$

with

$$B = \partial R/\partial \alpha$$

and

$$\begin{aligned} \mathcal{L}_{\alpha s} &= E(\delta x_{Ed} - \delta x_{Ad})(-\delta\lambda_A)^T \\ &= -(A^TWA)^{-1} A^TWB \, \mathcal{L}_\gamma \end{aligned} \tag{22}$$

This is the same form as equation (18); only $\mathcal{L}_\lambda$ has replaced $\mathcal{L}_\gamma$ in the computation of $\mathcal{L}_t$, and in the lower, right-hand corner of $\mathcal{L}_d$.

The updated covariance matrices will now be the result of the following matrix products:

Consider:

$$\mathcal{L}_f = \left[\begin{array}{c|c} \phi_s & \phi_\gamma \\ \hline 0 & I \end{array}\right] \left\{ \left[\begin{array}{c|c} \mathcal{L}_p^{-1} & 0 \\ \hline 0 & 0 \end{array}\right] + \left[\begin{array}{c|c} \mathcal{L}_t & \mathcal{L}_{\lambda s} \\ \hline \mathcal{L}_{\lambda s}^T & \mathcal{L}_\lambda \end{array}\right]^{-1} \right\}^{-1} \left[\begin{array}{c|c} \phi_s^T & 0 \\ \hline \phi_\gamma^T & I \end{array}\right]$$

Solve:

$$\mathcal{L}_f = \left[\begin{array}{c|c} \phi_s & \phi_\gamma \\ \hline 0 & I \end{array}\right] \left\{ \left[\begin{array}{c|c} \mathcal{L}_p^{-1} & 0 \\ \hline 0 & \mathcal{L}_\lambda^{-1} \end{array}\right] + \left[\mathcal{L}_{ts} \right]^{-1} \right\}^{-1} \left[\begin{array}{c|c} \phi_s^T & 0 \\ \hline \phi_\gamma^T & I \end{array}\right] \tag{23}$$

4.5 Summary of Results

The results of the study will be presented in the form of state vector covariance matrices at main deboost termination for each of the four proposed data usages. For convenience, the equations used in computing these covariance matrices are summarized below.

No Tracking; No Telemetry

Including uncertainties in α and β:

$$\Sigma'_f = \phi_s \Sigma'_d \phi_s^T + \phi_\gamma \Sigma_\gamma \phi_\gamma^T$$

Including uncertainties in α, β and accelerometer scale factor:

$$\Sigma'_f = \phi_s \Sigma'_d \phi_s^T + \phi_\gamma \Sigma'_\gamma \phi_\gamma^T$$

Telemetry Only

$$\Sigma'_f = \phi_s \Sigma_d \phi_s^T + \phi_\gamma \Sigma_\lambda \phi_\gamma^T$$

Tracking Only

Consider:

$$\Sigma_f = \left[\begin{array}{c|c} \phi_s & \phi_\gamma \\ \hline 0 & I \end{array}\right] \left\{ \left[\begin{array}{c|c} \Sigma_p^{-1} & 0 \\ \hline 0 & 0 \end{array}\right] + \left[\begin{array}{c|c} \Sigma_t & \Sigma_{\gamma s} \\ \hline \Sigma_{s\gamma}^T & \Sigma_\gamma \end{array}\right]^{-1} \right\}^{-1} \left[\begin{array}{c|c} \phi_s^T & 0 \\ \hline \phi_\gamma^T & I \end{array}\right]$$

Solve:

$$\Sigma_f = \left[\begin{array}{c|c} \phi_s & \phi_\gamma \\ \hline 0 & I \end{array}\right] \left\{ \left[\Sigma_{ts} \right]^{-1} + \left[\begin{array}{c|c} \Sigma_p^{-1} & 0 \\ \hline 0 & \Sigma_\gamma^{-1} \end{array}\right] \right\}^{-1} \left[\begin{array}{c|c} \phi_s^T & 0 \\ \hline \phi_\gamma^T & I \end{array}\right]$$

Tracking and Telemetry

Consider:

$$\Sigma_f = \left[\begin{array}{c|c}\phi_s & \phi_\gamma \\ \hline 0 & I\end{array}\right] \left\{ \left[\begin{array}{c|c}\Sigma_p^{-1} & 0 \\ \hline 0 & 0\end{array}\right] + \left[\begin{array}{c|c}\Sigma_t & \Sigma_{\lambda s} \\ \hline \Sigma_{\lambda s}^T & \Sigma_\lambda\end{array}\right]^{-1} \right\}^{-1} \left[\begin{array}{c|c}\phi_s^T & 0 \\ \hline \phi_\gamma^T & I\end{array}\right]$$

Solve:

$$\Sigma_f = \left[\begin{array}{c|c}\phi_s & \phi_\gamma \\ \hline 0 & I\end{array}\right] \left\{ \left[\begin{array}{c|c}\Sigma_p^{-1} & 0 \\ \hline 0 & \Sigma_\lambda^{-1}\end{array}\right] + \left[\begin{array}{c}\Sigma_{ts}\end{array}\right]^{-1} \right\}^{-1} \left[\begin{array}{c|c}\phi_s^T & 0 \\ \hline \phi_\gamma^T & I\end{array}\right]$$

The diagonal elements of these covariance matrices or, in the case of the consider and solve matrices, the diagonal elements of the upper left 6 x 6 partition, are presented in table II (ρ, $\dot{\rho}$ tracking from two stations) and table III (ρ, $\dot{\rho}$ plus angles from two stations and $\rho, \dot{\rho}$ from one station). Complete covariance matrices for the nominal tracking pattern (as summarized in table II) are presented in the appendix.

5. CONCLUSIONS

Looking first at the column in table II labeled 'No track; No T/M,' the effect of the accelerometer scale factor error is seen to be small (less than 1%). Consequently, all subsequent cases were run using the two error source model.

When used individually, tracking data is more useful than telemetry data; the state vector uncertainties using tracking data only are about 1/5 those that exist when no powered flight data is used, while the use of telemetry data alone reduces the uncertainties to approximately one-half their former values.

TABLE II. - STATE VECTOR UNCERTAINTIES
FOR VARIOUS POWERED FLIGHT DATA USAGES

(Uncertainties in the Cartesian components of position and velocity in km and m/s, respectively, at the termination of the main deboost)

State vector component	No tracking; no T/M		T/M(1) only	Tracking(2) only		Tracking(1) and T/M(2)	
	Two error sources	Three error sources		Consider	Solve	Consider	Solve
x	.6637	.7214	.3183	.1029	.0828	.0974	.0799
y	1.2330	1.2336	.5736	.2167	.1770	.2057	.1711
z	1.3691	1.3692	.6780	.1320	.1318	.1320	.1318
$\dot{x}$	2.439	2.459	1.125	.386	.311	.366	.299
$\dot{y}$	4.622	4.622	2.131	.803	.646	.761	.623
$\dot{z}$	4.979	4.982	2.296	.039	.038	.039	.038

(1) T/M (telemetry) assumed to consist of components of thrust acceleration.

(2) Tracking assumed to consist of range and range-rate measurements from two stations at the rate of one set per minute; $\sigma_\rho = 20$ m, $\sigma_{\dot{\rho}} = 0.02$ m.

(3) All consider and solve cases (with the exception of the third column) included two error sources; i.e., the angles α and β defining the thrust axis orientation. The third column included these orientation uncertainties plus an accelerometer scale factor error.

TABLE III. - EFFECT OF ALTERNATE OBSERVATIONAL PATTERNS ON STATE VECTOR UNCERTAINTIES

(Uncertainties in the Cartesian components of position and velocity in km and m/s, respectively, at the termination of the main deboost)

ANGULAR DATA					ONE STATION TRACKING			
Tracking(1) only		Tracking(1) and T/M(3)		State vector component	Tracking(2) only		Tracking(2) and T/M(3)	
Consider	Solve	Consider	Solve		Consider	Solve	Consider	Solve
.1029	.0828	.0974	.0799	x	.4723	.3645	.2531	.2315
.2167	.1770	.2057	.1711	y	.9933	.7685	.5337	.4893
.1320	.1318	.1320	.1318	z	.1479	.1390	.1471	.1388
.386	.311	.366	.300	$\dot{x}$	1.774	1.368	.950	.869
.803	.646	.761	.623	$\dot{y}$	3.742	2.887	2.005	1.834
.039	.038	.039	.038	$\dot{z}$	.076	.062	.055	.049

(1) Range, range-rate plus hour angle and declination from two stations at the rate of one set per minute; $\sigma_\rho = 20$ m, $\sigma_{\dot{\rho}} = .02$ m/s, $\sigma_{HA,\ DEC} = 0.06^\circ$.

(2) Range, range-rate from one station only at the rate of one set per minute; $\sigma_\rho = 20$ m, $\sigma_{\dot{\rho}} = 0.02$ m/s.

(3) T/M (telemetry) assumed to consist of components of thrust acceleration.

(4) All consider and solve cases included two error sources; i.e., uncertainties in the angles α and β defining the thrust axis orientation.

If tracking data are available (and are used), it doesn't make a great deal of difference whether the uncertainties in α and β are considered or solved for. In the operational sense, handling the tracking data in the consider mode would consist of simply modifying the equations of motion to include the thrust acceleration; no other changes would be necessary. The partial derivatives of the state vector with respect to the propulsion parameters, which were needed to compute the covariance matrix for the consider case would not be needed in real-time; i.e., the consider analysis has been completed prior to the flight and presumably the conclusion has been reached that the uncertainties when not solving for α, and β are acceptable.

The addition of telemetry data to tracking data makes only a small improvement (approximately 10%) in the knowledge of the state vector.

In table III, it can be seen that the addition of angles (0.06° one sigma) makes no visible improvement; comparison with cases without angles (table II) shows no difference to the number of figures carried. If only one station is available, telemetry-only is slightly better than tracking-only; i.e., the effectiveness of the tracking data has been sharply reduced. Hence, the addition of telemetry to one station tracking is considerably more effective than when tracking from two stations is available, with nearly a 50% reduction in the size of the uncertainties.

As with two-station tracking, there is no appreciable difference between solving for α, β and simply using the additional data taken during the burn to solve for epoch position and velocity.

6. NEW TECHNOLOGY

This section is included to comply with the requirements of the "New Technology" clause of the Master Agreement under which this report was prepared. Described herein are the results of a study to determine the utility of tracking and telemetry data taken during powered flight periods, in reducing uncertainties in determining the spacecraft orbit. The most significant new technology is the development of a procedure for updating the effects of unestimated systematic errors.

APPENDIX

Covariance Matrices at the End of the Deboost Maneuver Including Powered Flight Data

Listed below are the covariance matrices which resulted from the various powered flight data usages.

In all cases, the upper, left-hand 6 x 6 portion describes uncertainties in the Cartesian components of position and velocity in km and km/s. The lower, right-hand 2 x 2 portion of the 8 x 8 consider and solve covariance matrices describes uncertainties in the thrust vector orientation parameters α and β in deg.

1. No Tracking; No Telememetry

1.1 Including uncertainties in α and β

	1	2	3	4	5	6
1	.44051950 00	-.65538450 00	.43743201 00	.15979386-02	-.24141897-02	.15019383-02
2	-.65538450 00	.15203252 01	.21904542 00	-.24352559-02	.56787985-02	.97826183-03
3	.43743201 00	.21904542 00	.18745651 01	.14859769-02	.98649191-03	.66635827-02
4	.15979386-02	-.24352559-02	.14859769-02	.59498521-05	-.90977169-05	.54195440-05
5	-.24141897-02	.56787985-02	.98649191-03	-.90977169-05	.21358382-04	.38732068-05
6	.15019383-02	.97826183-03	.66635828-02	.54195440-05	.38732068-05	.24794779-04

1.2 Including uncertainties in α, β and accelerometer scale factor

	1	2	3	4	5	6
1	.52044161 00	-.64401071 00	.43546551 00	.15105624-02	-.24126298-02	.15485602-02
2	-.64401071 00	.15219437 01	.21876557 00	-.24476905-02	.56790205-02	.98489665-03
3	.43546551 00	.21876556 00	.18746135 01	.14881269-02	.98645353-03	.66624356-02
4	.15105624-02	-.24476905-02	.14881269-02	.60453777-05	-.90994223-05	.53685738-05
5	-.24126298-02	.56790205-02	.98645354-03	-.90994223-05	.21358412-04	.38741168-05
6	.15485602-02	.98489665-03	.66624356-02	.53685738-05	.38741168-05	.24821976-04

	1	2	3	4	5	6
1	.10128502 00	-.14491143 00	.11114267 00	.33742001-03	-.50957373-03	.31665068-03
2	-.14491143 00	.32899579 00	.27437360-01	-.51572904-03	.12028924-02	.21035788-03
3	.11114267 00	.27437360-01	.45967088 00	.31021459-03	.22164640-03	.14078814-02
4	.33742001-03	-.51572904-03	.31021459-03	.12648298-05	-.19341455-05	.11522963-05
5	-.50957373-03	.12028924-02	.22164640-03	-.19341455-05	.45406352-05	.82142273-06
6	.31665068-03	.21035787-03	.14078814-02	.11522963-05	.82142273-06	.52694845-05

3. Tracking Only

3.1 Considering the effects of uncertainties in α and β

	1	2	3	4	5	6
1	.10581494-01	-.21872096-01	.17079382-03	.39667573-04	-.82432578-04	.66548423-06
2	-.21872095-01	.46953118-01	-.57224235-02	-.81579432-04	.16916185-03	.75472680-07
3	.17079283-03	-.57224214-02	.17421691-01	-.11414096-05	.36342622-05	-.44784259-05
4	.39667573-04	-.81579433-04	-.11414059-05	.14916874-06	-.31007708-06	.29745371-08
5	-.82432580-04	.16916186-03	.36342544-05	-.31007708-06	.64475002-06	-.63321651-08
6	.66548463-06	.75471843-07	-.44784259-05	.29745385-08	-.63321680-08	.15154990-08
7	-.23102544-03	.41366951-03	.18831228-03	-.88779620-06	.18453424-05	-.88045245-07
8	-.71839452-02	.14838941-01	.25700292-05	-.26991530-04	.56100182-04	-.47289288-06

	7	8
1	-.23102541-03	-.71839452-02
2	.41366946-03	.14838941-01
3	.18831230-03	.25707017-05
4	-.88779613-06	-.26991530-04
5	.18453422-05	.56100182-04
6	-.88045253-07	-.47289314-06
7	.90000206-05	.15751114-03
8	.15751112-03	.48869885-02

3.2 Solving for α and β

	1	2	3	4	5	6
1	.68575786-02	-.14241588-01	.35434246-03	.25656917-04	-.53312209-04	.34984167-06
2	-.14241588-01	.31312160-01	-.60818488-02	-.52872580-04	.10949627-03	.71578867-06
3	.35434247-03	-.60818488-02	.17363235-01	-.44563856-06	.21881108-05	-.44437488-05
4	.25656917-04	-.52872580-04	-.44563861-06	.96455269-07	-.20051505-06	.17849709-08
5	-.53312208-04	.10949627-03	.21881109-05	-.20051505-06	.41703143-06	-.38597063-08
6	.34984167-06	.71578866-06	-.44437488-05	.17849709-08	-.38597062-08	.14813464-08
7	-.14556796-03	.23891103-03	.18306733-03	-.56616710-06	.11768535-05	-.80402127-07
8	-.46503535-02	.96475037-02	-.12240110-03	-.17459273-04	.36287915-04	-.25810746-06

	7	8
1	-.14556796-03	-.46503535-02
2	.23891103-03	.96475037-02
3	.18306733-03	-.12240111-03
4	-.56616710-06	-.17459273-04
5	.11768535-05	.36287915-04
6	-.80402127-07	-.25810746-06
7	.70173287-05	.99367616-04
8	.93240932-04	.28730321-02

4. Tracking and Telemetry

4.1 Considering the effects of uncertainties in α and β

	1	2	3	4	5	6
1	.94919548-02	-.19621885-01	.17210038-03	.35573858-04	-.73924050-04	.59340913-06
2	-.19621885-01	.42305696-01	-.57248411-02	-.73124769-04	.15158937-03	.22421969-06
3	.17209949-03	-.57248393-02	.17420857-01	-.11364126-05	.36238758-05	-.44780174-05
4	.35573858-04	-.73124770-04	-.11364093-05	.13378746-06	-.27810805-06	.27036960-08
5	-.73924050-04	.15158937-03	.36238689-05	-.27810805-06	.57830442-06	-.57692390-08
6	.59340946-06	.22421896-06	-.44780173-05	.27036974-08	-.57692416-08	.15106064-08
7	-.20711765-03	.36429897-03	.13826621-03	-.79796589-06	.16586357-05	-.86456967-07
8	-.64427677-02	.13308199-01	.16796636-05	-.24206711-04	.50312112-04	-.42386196-06

	7	8
1	-.20711763-03	-.64427677-02
2	.36429893-03	.13308198-01
3	.18826623-03	.16802675-05
4	-.79796582-06	-.24206711-04
5	.16586356-05	.50312112-04
6	-.86456974-07	-.42386220-06
7	.84750478-05	.14124743-03
8	.14124742-03	.43827900-02

4.2 Solving for α and β

	1	2	3	4	5	6
1	.63838103-02	-.13258922-01	.34246772-03	.23878140-04	-.49615143-04	.32331589-06
2	-.13258922-01	.29273870-01	-.60569418-02	-.49183161-04	.10182806-03	.77069995-06
3	.34246773-03	-.60569418-02	.17362117-01	-.49013636-06	.22805960-05	-.44440961-05
4	.23878140-04	-.49183161-04	-.49013638-06	.89776793-07	-.18663429-06	.16853457-08
5	-.49615142-04	.10182806-03	.22805960-05	-.18663429-06	.38818121-06	-.36526417-08
6	.32331589-06	.77069994-06	-.44440961-05	.16853457-08	-.36526418-08	.14797383-08
7	-.13543215-03	.21789363-03	.18330423-03	-.52811011-06	.10977546-05	-.79827997-07
8	-.43280881-02	.89790772-02	-.11432523-03	-.16249318-04	.33773106-04	-.24006356-06

	7	8
1	-.13543215-03	-.43280881-02
2	.21789363-03	.89790772-02
3	.18330423-03	-.11432524-03
4	-.52811011-06	-.16249318-04
5	.10977546-05	.33773106-04
6	-.79827997-07	-.24006356-06
7	.68001245-05	.92473030-04
8	.92473030-04	.29440310-02

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