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GEOMETRIC STRUCTURE OF THE EARTH'S GRAVITATIONAL FIELD AS DERIVED FROM ARTIFICIAL SATELLITES

by

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AS DERIVED FROM ARTIFICIAL SATELLITES[1]

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Abstract.--The structure of the Earth's gravitational potential and its gradient is studied at sea level and in outer space (1,000 km, 10,000 km, and 100,000 km elevation above sea level), with particular consideration given to the surfaces of constant potential and of constant gravitation. Their shape, curvature, etc., are computed and the results presented in tables and maps using the zonal part of the gravitational field as a reference frame. In a concluding section the geometry of the orthogonal trajectories of the equipotential surfaces (direction of the gradient field) is briefly touched. Among others, the curvature and torsion of the verticals are derived and the trajectories are integrated up to 10 km height for the gravity field and 100,000 km for the gravitational field. All numerical computations are based on Izsak's and Kozai's latest harmonic coefficients of the gravitational potential (Izsak, 1965; Kozai, 1964).

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The gravitational potential of outer space can be described by the function

$$U = \frac{GM}{r}\left\{1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^n (C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\, P_{nm}(\sin\varphi)\right\}, \tag{1}$$

with

a = equatorial radius,

GM = product of the gravitational constant and the mass of the Earth

r = geocentric radius, i.e., the distance from the gravity center to a point in free space,

φ = geocentric latitude, i.e., the complement of the angle that is included by r and the rotation axis,

λ = geocentric longitude, i.e., the angle between a meridian plane through r and the meridian plane through Greenwich,

C_{nm}, S_{nm} = harmonic coefficients,

$P_{nm}(\sin\varphi)$ = Legendre's associated function.

Equation (1) is valid in the case of

$$\text{div grad } U = 0 , \tag{2}$$

that is, in empty space where Laplace's equation is satisfied.

In our analysis we make two specifications:

1) we disregard the mass of the atmosphere, and

2) we extend the above function down to sea level, ignoring thereby the effect of the interfering topography. This gravitational potential is describe together with its gradient field, in four different sections of outer space:

at sea level, at 1,000 km elevation, at 10,000 km elevation, and at 100,000 km elevation. At sea level we shall also consider the case where the centrifugal potential Z is included:

$$Z = \frac{\omega^2 r^2}{2} \cos^2 \varphi \quad , \tag{3}$$

where ω = angular velocity of the Earth.

The numerical computations are performed with a set of harmonic coefficients given in the Appendix. Although these values will be subject to continuous improvement, they already give a rather close picture of the general structure of the Earth's gravitational field. In the following sections we comment only briefly on the analysis used. It is, however, not intended to give a collection of formulas, nor should the subject be treated from a theoretical point of view.

I. POTENTIAL FIELD — EQUIPOTENTIAL SURFACES

The geopotential in free space is a function of the radius vector $\vec{r}$ and the harmonic coefficients C_{nm} and S_{nm}:

$$V = U + Z = V(r, \varphi, \lambda, C_{nm}, S_{nm}; \omega) \quad . \tag{4}$$

By neglecting the influence of the nonzonal terms, we get a new potential function:

$$\begin{aligned} \overline{V} &= \overline{V}(\overline{r}, \varphi, C_{no}; \omega) \quad , \\ &= \frac{GM}{\overline{r}} \left\{ 1 + \sum_{n=2}^{\infty} \left(\frac{a}{\overline{r}}\right)^n C_{no} \, P_{no}(\sin \varphi) \right\} + \frac{\omega^2 \overline{r}^2}{2} \cos^2 \varphi \quad , \end{aligned} \tag{5}$$

which is rotational symmetric and approximates the potential V in the mean. To relate both equations (4) and (5), we put

$$V = \overline{V}$$

and determine $\overline{V}$ in such a way that at sea level

$\overline{r} = 6{,}378{,}165$ m for $\varphi = 0$, and $\omega \neq 0$ or $\omega = 0$,[3]

and similarly in outer space

$$\left.\begin{aligned} \overline{r} &= 6{,}378{,}165 \text{ m} + 1{,}000 \text{ km} \\ \overline{r} &= 6{,}378{,}165 \text{ m} + 10{,}000 \text{ km} \\ \overline{r} &= 6{,}378{,}165 \text{ m} + 100{,}000 \text{ km} \end{aligned}\right\} \text{ for } \begin{aligned} \varphi &= 0 \\ \omega &= 0 \end{aligned} \quad .$$

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[3] $\omega \neq 0$ simply means that the centrifugal potential is taken into consideration.

To get the geometric structure of an equipotential surface V = const, we first compute the relatively simple (rotational symmetric) structure of $\overline{V}$ = const and add a small correction term, leading to the geometrical structure of V = const. The advantage of this stepwise procedure is also a practical one: the geometric results of $\overline{V}$ = const can be presented in tables, while the small corrections are easily plotted as equilevel curves in maps.

1. Shape of the equipotential surfaces

To determine the geocentric radii of an equipotential surface V = const, we split r into a spheroidal part $\overline{r}$ and a correction term Δr (see Figure 1):

$$r(\varphi,\lambda) = \overline{r}(\varphi) + \Delta r(\varphi,\lambda) \quad . \tag{6}$$

While $\overline{r}$ as a function of φ only is computed from equation (5), we obtain, by introducing the latter into equation (4),

$$V = V(\overline{r} + \Delta r,\varphi,\lambda;C_{nm},S_{nm};\omega) \quad , \tag{7}$$

the "undulations" Δr in the geocentric latitude φ and longitude λ .

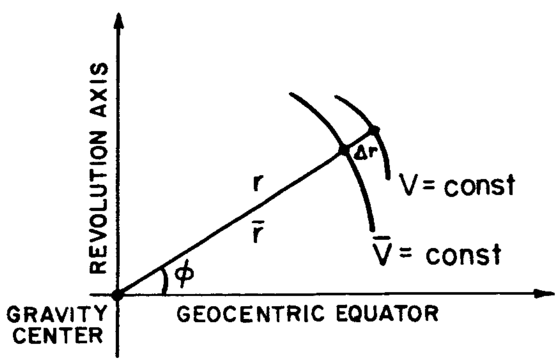


Figure 1.

Example: The geocentric radius r in the geocentric latitude $\varphi = 20^\circ$ and longitude $\lambda = 40^\circ$ is for the equipotential surface $V(\omega = 0)$ = const at 1,000 km elevation above sea level:

$$\left.\begin{aligned} r_{\substack{\varphi = 20^\circ \\ \lambda = 40^\circ}} &= \bar{r}_{\varphi = 20^\circ} + \Delta r_{\substack{\varphi = 20^\circ \\ \lambda = 40^\circ}} \\ &= 7{,}377{,}109 + 4 \\ &= 7{,}377{,}113 \text{ m} \end{aligned}\right\} \text{see 1a), b)} \; .$$

a) Spheroidal[4] (zonal) part $\bar{r}$ of the geocentric radius — shape of the equipotential surface $\bar{V}$ = const.

Table 1.

$\bar{r}$ (m) / φ°	Sea level		Elevation above sea level ($\omega = 0$)		
	$\omega \neq 0$	$\omega = 0$	1,000 km	10,000 km	100,000 km
90	6 356 792.84	6 367 812.46	7 369 216.03	16 374 133.15	106 377 544.02
80	6 357 432.57	6 368 122.11	7 369 484.57	16 374 254.56	106 377 562.74
70	6 359 277.55	6 369 015.89	7 370 258.62	16 374 604.17	106 377 616.65
60	6 362 110.55	6 370 389.36	7 371 446.05	16 375 139.93	106 377 699.24
50	6 365 590.83	6 372 075.63	7 372 903.81	16 375 797.32	106 377 800.56
40	6 369 299.88	6 373 871.18	7 374 456.29	16 376 497.19	106 377 908.38
30	6 372 791.63	6 375 560.68	7 375 916.75	16 377 155.19	106 378 009.71
20	6 375 643.11	6 376 939.98	7 377 109.46	16 377 691.97	106 378 092.33
10	6 377 511.56	6 377 846.07	7 377 890.96	16 378 042.72	106 378 146.26
0	6 378 165.00	6 378 165.00	7 378 165.00	16 378 165.00	106 378 165.00
-10	6 377 518.83	6 377 853.31	7 377 896.75	16 378 043.92	106 378 146.29
-20	6 375 657.39	6 376 954.22	7 377 119.49	16 377 693.99	106 378 092.38
-30	6 372 804.89	6 375 573.89	7 375 926.91	16 377 157.34	106 378 009.77
-40	6 369 308.04	6 373 879.32	7 374 463.20	16 376 498.68	106 377 908.42
-50	6 365 593.98	6 372 078.76	7 372 905.38	16 375 797.47	106 377 800.56
-60	6 362 102.61	6 370 381.43	7 371 439.32	16 375 138.34	106 377 699.20
-70	6 359 254.55	6 368 992.98	7 370 241.97	16 374 600.88	106 377 616.57
-80	6 357 398.44	6 368 088.12	7 369 460.19	16 374 250.01	106 377 562.64
-90	6 356 754.75	6 367 774.51	7 369 188.79	16 374 128.14	106 377 543.90

[4] The word spheroid is used in the following for "zonal part" and vice versa.

The geocentric radii at the poles are always smaller than the corresponding equatorial radius. At sea level the difference is about 21.4 km ($\omega \neq 0$) and about 10.4 km ($\omega = 0$), respectively, and decreases at 100,000 km elevation to 0.62 km ($\omega = 0$). For infinity ($\omega = 0$) the difference is obviously zero.

The phrase "sea level" has, of course, only physical meaning for $\omega \neq 0$; in the case $\omega = 0$, only the equatorial radius $\bar{r}$ ($\varphi = 0$) is supposed to be at sea level (see also page 4).

b) Corrections (undulations), Δr

See Contour Map 1, page 70. The undulations are referred to the spheroidal part of the equipotential surface (equation (5)). As shown in Table 2, we have at sea level heights up to 60 m and depths down to -70 m. In outer space these undulations are smoothed depending on the height above sea level. At 100,000 km elevation only the C_{22} and S_{22} terms show up clearly and give the gravitational field some similarity with that of a 3-axial ellipsoid. Beyond 100,000 km the potential field assumes more and more a spherical shape according to equation (4). At ground level, only the case Δr ($\omega \neq 0$) has been plotted, because Δr ($\omega = 0$) is equal to Δr ($\omega \neq 0$) within some decimeters.

Table 2.

Elevation above sea level (km)	Range of Δr (m)	
0	60	-70
1,000	47	-51
10,000	16	-14
100,000	1.8	-1.8

c) Difference of the geocentric radii of the Northern and Southern Hemispheres of the spheroid

Table 3.

$\bar{r}(\varphi)-\bar{r}(-\varphi)$ (m) / $\lvert\varphi^\circ\rvert$	Sea level		Elevation above sea level ($\omega = 0$)		
	$\omega \neq 0$	$\omega = 0$	1,000 km	10,000 km	100,000 km
90	+38.09	+37.95	+27.24	+5.01	+0.117
80	+34.13	+33.99	+24.38	+4.55	+0.106
70	+23.00	+22.91	+16.65	+3.29	+0.078
60	+ 7.94	+ 7.93	+ 6.73	+1.59	+0.038
50	- 3.15	- 3.13	- 1.57	-0.15	-0.003
40	- 8.16	- 8.14	- 6.91	-1.49	-0.035
30	-13.26	-13.21	-10.16	-2.15	-0.051
20	-14.28	-14.24	-10.03	-2.02	-0.048
10	- 7.27	- 7.24	- 5.79	-1.20	-0.029
0	0.00	0.00	0.00	0.00	0.000

The differences $\bar{r}(\varphi) - \bar{r}(-\varphi)$ shown in Table 3 point out an asymmetric spheroid whose northern part is longer and tighter while its southern part is shorter but broader.[5]

d) Difference of a spheroid and an ellipsoid with equal axes

The deviation of an equipotential spheroid from an ellipsoid with coincident axes is obtained by comparison of their geocentric radii $\bar{r}$ and E along a meridian plane (see Table 4). At the Northern Hemisphere the equipotential spheroid is completely surrounded by the corresponding ellipsoid, while for the Southern Hemisphere the ellipsoid stays within the spheroid. The largest deviation – 19.2 m – is found at sea level ($\omega \neq 0$) at about -50° latitude.

[5]Often called pear shaped; this is, however, not quite correct because these spheroids have no negative gaussian curvature (see page 30).

Table 4.

$\overline{r}$ - E (m) / $\varphi°$	Sea level		Elevation above sea level (ω = 0)		
	$\omega \neq 0$	$\omega = 0$	1,000 km	10,000 km	100,000 km
90	0.00	0.00	0.00	0.00	0.0000
80	- 1.59	- 1.77	- 0.83	-0.13	-0.0034
70	- 4.27	- 4.97	- 2.56	-0.46	-0.0125
60	- 5.18	- 6.51	- 4.17	-0.91	-0.0243
50	- 6.40	- 8.13	- 5.77	-1.34	-0.0351
40	- 8.56	-10.28	- 7.26	-1.60	-0.0410
30	-10.13	-11.45	- 7.95	-1.58	-0.0396
20	-10.69	-11.40	- 7.02	-1.25	-0.0306
10	- 5.85	- 6.03	- 3.72	-0.67	-0.0161
0	0.00	0.00	0.00	0.00	0.0000
-10	+ 2.58	+ 2.37	+ 2.90	+0.68	+0.0163
-20	+ 8.09	+ 7.30	+ 6.20	+1.35	+0.0312
-30	+12.71	+11.28	+ 9.05	+1.82	+0.0407
-40	+15.43	+13.58	+10.94	+1.96	+0.0423
-50	+19.20	+17.31	+11.81	+1.75	+0.0364
-60	+15.51	+14.06	+ 9.56	+1.26	+0.0254
-70	+ 6.41	+ 5.64	+ 4.87	+0.67	+0.0130
-80	+ 1.25	+ 1.03	+ 1.22	+0.19	+0.0036
-90	0.00	0.00	0.00	0.00	0.0000

e) <u>Difference of the spheroid ($\omega \neq 0$) at sea level and an ellipsoid with the flattening f = 1/298.25</u>

The undulations of the "satellite geoid" referred to the ellipsoid with the above flattening f = 1/298.25 are obtained by adding the values $\overline{r}$ - E of the Table 5 to the undulations at sea level (page 70).

Example: At $\varphi = 20°$ and $\lambda = 40°$ we obtain the undulation in question:

Δr	13 m
$\overline{r}$ - E	-9 m
undulation	4 m

Table 5.

φ°	$\bar{r}$ - E (m)
90	+13.13
80	+11.16
70	+ 7.35
60	+ 4.70
50	+ 1.34
40	- 3.10
30	- 6.81
20	- 9.14
10	- 5.44
0	0.00
-10	+ 1.83
-20	+ 5.15
-30	+ 6.43
-40	+ 5.06
-50	+ 4.50
-60	- 3.24
-70	-15.65
-80	-22.96
-90	-24.95

The spheroid and the ellipsoid were supposed to have the same equatorial radius — $\bar{r}$ = 6,378,165 m. However, if we refer the satellite geoid to an ellipsoid with the same potential and the same zonal harmonic coefficient of the second degree $C_{20} = -J_2$, we have to add to the above values a constant term of about 0.75 m. It should be noted that the flattening used is a rounded value of Kozai's (1964, p. 12).

f) <u>Difference of the spheroids ($\omega = 0$) and ($\omega \neq 0$) at sea level</u>

The opposite results are obtained from the geocentric radii of section a) at sea level along a meridian. The spheroid ($\omega \neq 0$) is completely surrounded by the corresponding spheroid ($\omega = 0$) so that the difference of the geocentric radii increases from 0 at the equator according to definition to 11.02 km at the poles.

Table 6.

φ°	$\bar{r}(\omega = 0) - \bar{r}(\omega \neq 0)$ (m)
90	11,019.61
80	10,689.54
70	9,738.34
60	8,278.81
50	6,484.80
40	4,571.30
30	2,769.05
20	1,296.87
10	334.51
0	0.00
-10	334.48
-20	1,296.83
-30	2,769.00
-40	4,571.28
-50	6,484.78
-60	8,278.82
-70	9,738.43
-80	10,689.68
-90	11,019.76

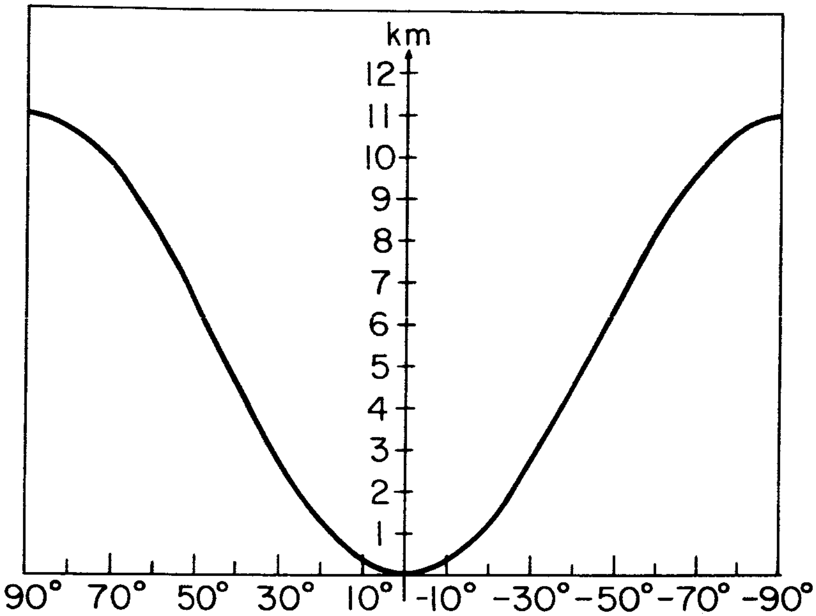


Figure 2.

g) Mean flattening of the spheroids ($\omega = 0$) as a function of elevation above sea level

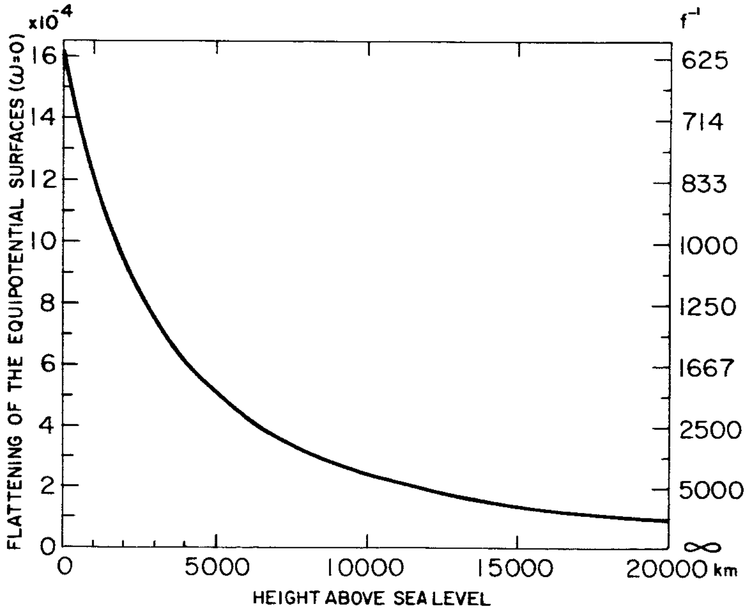


Figure 3.

Definition of the mean flattening:

$$f = 1 - \frac{\bar{r}_{north} + \bar{r}_{south}}{2\,\bar{r}_{equator}} \quad . \tag{8}$$

As we can see from Figure 3, between sea level and 3,000 km elevation the flattening of the equipotential spheroids ($\omega = 0$) decreases very strongly and tends afterward asymptotically to zero at infinite height.

h) Distance of the actual equator of the spheroids from the gravity center

Because of the odd zonal coefficients C_{no} ($n = 3,5,...$), the actual equator (surface normal is orthogonal to revolution axis) of the spheroid $\bar{V}$ = const does not coincide with the geocentric equator (see Figure 4).

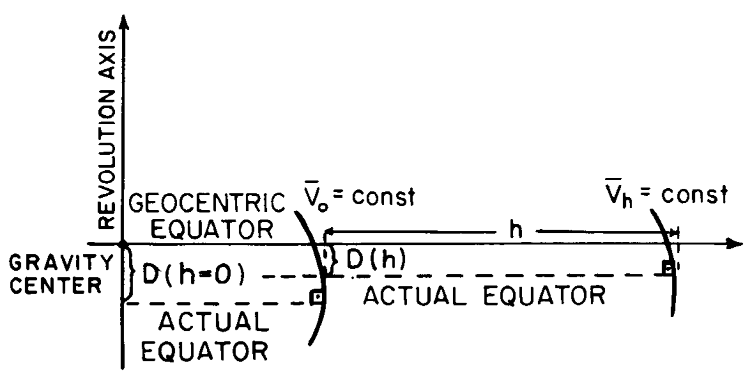


Figure 4.

The polar coordinates $\bar{r},\varphi$ of the actual equator derive from

$$\bar{r}\,\frac{\partial \bar{V}}{\partial \bar{r}} \sin\varphi + \frac{\partial \bar{V}}{\partial \varphi}\cos\varphi = 0,$$

and the distance D:

$$D \approx a\left[-\frac{3}{2}\,C_{30}\left(1+\frac{h}{a}\right)^{-2} + \frac{15}{8}\,C_{50}\left(1+\frac{h}{a}\right)^{-4} - \frac{35}{16}\,C_{70}\left(1+\frac{h}{a}\right)^{-6}\right.$$
$$\left. + \frac{315}{128}\,C_{90}\left(1+\frac{h}{a}\right)^{-8} - \frac{693}{256}\,C_{11\;0}\left(1+\frac{h}{a}\right)^{-10} + \frac{3003}{1024}\,C_{13\;0}\left(1+\frac{h}{a}\right)^{-12} - \dots\right], \tag{9}$$

where h = average elevation of the spheroid above sea level. Figure 5 shows that the actual equator moves at first slightly southward and then reapproaches asymptotically the geocentric equator as the flattening decreases.

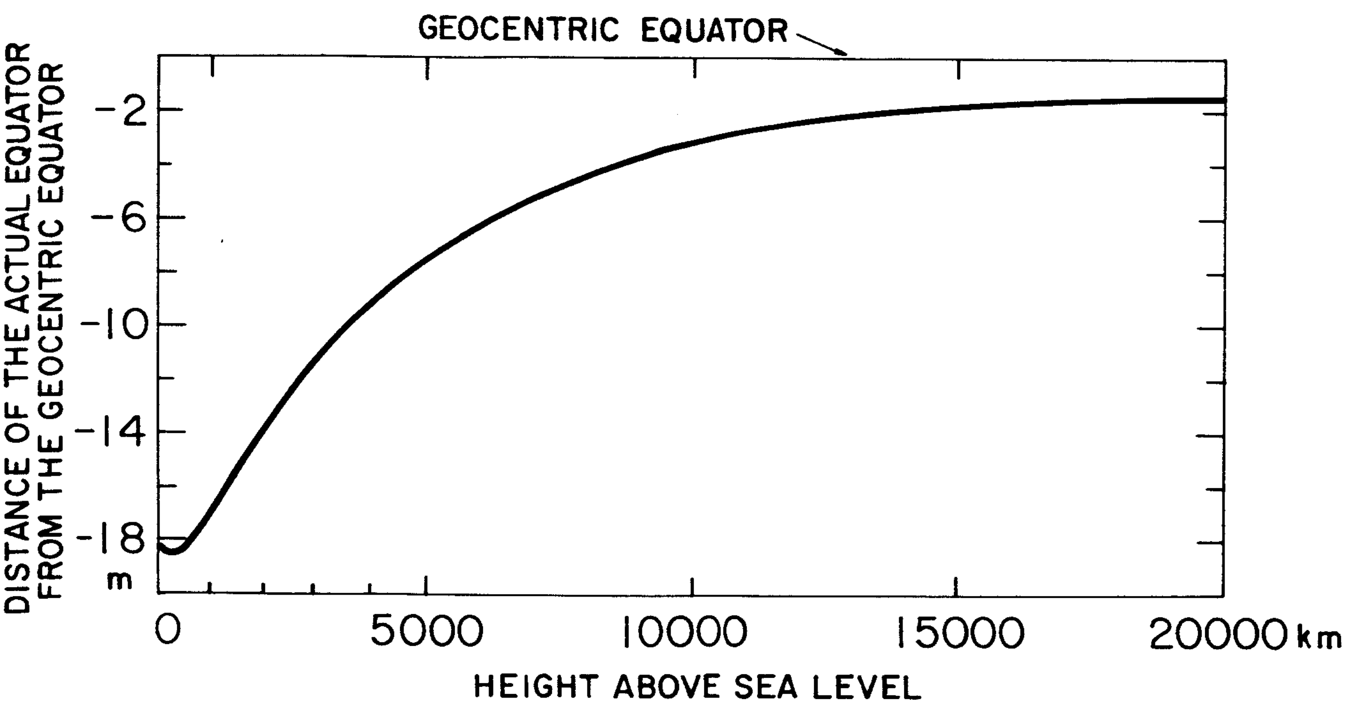


Figure 5.

2. Gravitation[6] (gravity) along an equipotential surface

By partial differentiation of equation (4) with respect to the rectangular coordinates x^i we obtain

$$\text{grad } V = \frac{\partial V}{\partial x^1} \vec{e}^1 + \frac{\partial V}{\partial x^2} \vec{e}^2 + \frac{\partial V}{\partial x^3} \vec{e}^3 \tag{10}$$

if

$$\left.\begin{aligned} x^1 &= r \cos \varphi \cos \lambda \\ x^2 &= r \cos \varphi \sin \lambda \\ x^3 &= r \sin \varphi \end{aligned}\right\} \quad \text{see page} \quad 109. \tag{11}$$

and similarly with equation (5)

$$\text{grad } \overline{V} = \frac{\partial \overline{V}}{\partial \overline{x}^1} \vec{e}^1 + \frac{\partial \overline{V}}{\partial \overline{x}^2} \vec{e}^2 + \frac{\partial \overline{V}}{\partial \overline{x}^3} \vec{e}^3 \quad . \tag{12}$$

Introducing $\overline{r}$ and $r = \overline{r} + \Delta r$ of the previous section into equations (12) and (10), we derive the variation of the gravitation along the spheroid:

$$\begin{aligned} \overline{g} &= \left|\text{grad } \overline{V}\right| = \overline{g}(\overline{r}, \varphi, C_{no}; \omega) \\ &= \left[\left(\frac{\partial \overline{V}}{\partial \overline{r}}\right)^2 + \frac{1}{\overline{r}^2}\left(\frac{\partial \overline{V}}{\partial \varphi}\right)^2\right]^{\frac{1}{2}} \quad , \end{aligned} \tag{13}$$

[6] "Gravitation" is used for $\omega = 0$ and "gravity" for $\omega \neq 0$.

and the correction $\Delta g = g - \bar{g}$:

$$\Delta g = |\text{grad } V| - |\text{grad } \bar{V}| = \Delta g(r, \bar{r}, \varphi, \lambda, C_{nm}, S_{nm}, \omega)$$

$$= \left[\left(\frac{\partial V}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial V}{\partial \varphi}\right)^2 + \frac{1}{r^2 \cos^2 \varphi}\left(\frac{\partial V}{\partial \lambda}\right)^2\right]^{\frac{1}{2}} - \left[\left(\frac{\partial \bar{V}}{\partial \bar{r}}\right)^2 + \frac{1}{\bar{r}^2}\left(\frac{\partial \bar{V}}{\partial \varphi}\right)^2\right]^{\frac{1}{2}} . \qquad (14)$$

Example: In section a) below we find, at the equipotential spheroid ($\omega = 0$) in the latitude $\varphi = 20^\circ$ for 1,000 km elevation,

$$\bar{g} = 733.0085 \text{ cm sec}^{-2} \quad .$$

If we add to this value the Δg correction (of b) below) in $\varphi = 20^\circ$ and $\lambda = 40^\circ$, then we obtain the gravitation $g_{\substack{\varphi = 20^\circ \\ \lambda = 40^\circ}}$ at the equipotential surface V = const at 1,000 km elevation:

$$\begin{array}{lcll} \bar{g} & = & 733.008 & \text{cm sec}^{-2} \\ \underline{\Delta g} & \underline{=} & \underline{+\ 0.004} & \\ g & = & 733.012 & \text{cm sec}^{-2} \end{array}$$

a) Spheroidal (zonal) part $\overline{g}$

Table 7.

φ° \ $\overline{g}$ ($cm\ sec^{-2}$)	Sea level		Elevation above sea level ($\omega = 0$)		
	$\omega \neq 0$	$\omega = 0$	1,000 km	10,000 km	100,000 km
90	983.222	979.833	732.2265	148.59713	3.522 376 640
80	983.066	979.879	732.2528	148.59823	3.522 377 260
70	982.616	980.014	732.3291	148.60139	3.522 379 044
60	981.931	980.226	732.4468	148.60625	3.522 381 778
50	981.087	980.485	732.5914	148.61221	3.522 385 132
40	980.185	980.760	732.7453	148.61855	3.522 388 702
30	979.335	981.020	732.8902	148.62452	3.522 392 057
20	978.638	981.231	733.0085	148.62939	3.522 394 793
10	978.187	981.372	733.0869	148.63258	3.522 396 579
0	978.033	981.424	733.1148	148.63370	3.522 397 200
-10	978.188	981.374	733.0879	148.63260	3.522 396 581
-20	978.644	981.236	733.0107	148.62943	3.522 394 796
-30	979.338	981.024	732.8920	148.62456	3.522 392 060
-40	980.185	980.761	732.7464	148.61858	3.522 388 704
-50	981.090	980.488	732.5921	148.61221	3.522 385 132
-60	981.930	980.225	732.4459	148.60622	3.522 381 776
-70	982.608	980.006	732.3256	148.60133	3.522 379 039
-80	983.053	979.867	732.2474	148.59814	3.522 377 252
-90	983.209	979.820	732.2204	148.59703	3.522 376 632

As we can see from Table 7, the gravity $\overline{g}$ ($\omega \neq 0$) increases at sea level along an equipotential surface $\overline{V}$ ($\omega \neq 0$) = const, with about 5.2 gal from the equator to the poles; further, the value of $\overline{g}$ at the North Pole is around 13 mgal larger than the corresponding value at the South Pole. If we exclude the centrifugal force, then the gravitation $\overline{g}$ ($\omega = 0$) along the spheroid $\overline{V}$ ($\omega = 0$) = const decreases toward the poles. The difference between pole and equator amounts at sea level to about 1.6 gal, while the value at the North Pole is again 13 mgal larger than $\overline{g}$ at the South Pole. With increasing elevation these numbers tend to zero for infinity.

Note that $\overline{g}$ is always computed along an equipotential surface $\overline{V}$ = const ($\omega \neq 0$ or $\omega = 0$) compared to section IIA where the surface $\overline{g}$ = const is considered.

b) Corrections, Δg

See Contour Map 2. At higher elevations Δg decreases percentagewise faster than the undulations Δr (see Table 8). At sea level we have values of Δg running from -26 mgal to 22 mgal, while at 100,000 km elevation Δg varies only from -0.000062 to 0.000066 mgal. Similarly to Δr the gravitational correction Δg ($\omega = 0$) at sea level is very close to Δg ($\omega \neq 0$), and hence only one case was plotted.

Table 8.

Elevation above sea level (km)	Range of Δg (mgal)	
0	22	-26
1,000	8	-10
10,000	0.18	-0.17
100,000	0.000066	-0.000062

3. Oscillation of the surface normals; latitude and longitude curves

The surface normals are collinear with the gradients of equations (10) and (12). From Figure 6 we see that the angle ν of two corresponding normal vectors (in φ and λ) on $V = \text{const}$ and $\overline{V} = V$ follows from the scalar product

$$\cos \nu = \frac{1}{\overline{g}\, g} \operatorname{grad} \overline{V} \cdot \operatorname{grad} V \quad , \tag{15}$$

and its components in and orthogonally to a meridian plane from the expressions

$$\begin{aligned} \xi &= \nu \cos \alpha \ , \\ \eta &= \nu \sin \alpha \ ; \end{aligned} \tag{16}$$

α is the azimuth of the projection of ν on $\overline{V} = \text{const}$, with its cosine

$$\cos \alpha = - \frac{(\operatorname{grad} \overline{V} \times \operatorname{grad} V)\cdot(\operatorname{grad} \overline{V} \times \vec{e}^{\,3})}{|\operatorname{grad} \overline{V} \times \operatorname{grad} V| \ |\operatorname{grad} \overline{V} \times \vec{e}^{\,3}|} \quad , \tag{17}$$

whereby $\vec{e}^{\,3}$ is the unit vector in the x^3 direction. Because ν, ξ, and η are

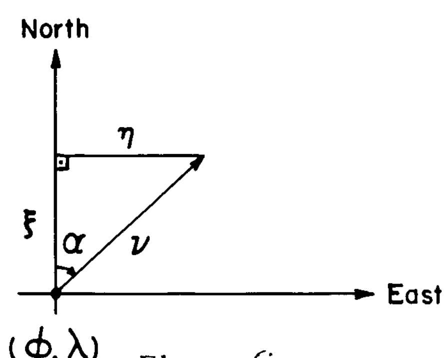


Figure 6.

small values, equations (15) and (16) can be replaced by the differential expressions

$$\nu = \left[\left(\frac{1}{\bar{r}}\frac{\partial\bar{r}}{\partial\varphi} - \frac{1}{r}\frac{\partial r}{\partial\varphi}\right)^2 + \frac{1}{r^2\cos^2\varphi}\left(\frac{\partial r}{\partial\lambda}\right)^2\right]^{\frac{1}{2}} ,$$

$$\xi = \frac{1}{\bar{r}}\frac{\partial\bar{r}}{\partial\varphi} - \frac{1}{r}\frac{\partial r}{\partial\varphi} , \tag{18}$$

$$\eta = -\frac{1}{r\cos\varphi}\frac{\partial r}{\partial\lambda} ,$$

wherein

$$\frac{\partial r}{\partial p^i} = -\frac{\partial V/\partial p^i}{\partial V/\partial r} \qquad \text{(analogously for } \bar{r}\text{)} , \tag{19}$$

with $\varphi = p^1$ and/or $\lambda = p^2$. The sign of ξ, η was defined according to geodetic usage. If we use the normal of an equipotential surface for latitude and longitude definition, as in section 3d), then ξ is positive if the latitude B of a point on V = const is larger than the latitude $\bar{B}$ of the corresponding point on $\bar{V}$ = const, and analogously for the longitude (Bomford, 1962).

a) Oscillation components ξ and η

See Contour Maps 3 and 4. The absolute oscillation $\nu = (\xi^2 + \eta^2)^{\frac{1}{2}}$ takes the largest values where the inclination of the undulations is a maximum. If we split ν up into its components, then ξ represents the inclination in the meridional direction while η accounts for the effect orthogonally to it (see Tables 9 and 10). Superposing the ξ, η graphs with the corresponding Δr graphs, the zero curves of ξ intersect the undulation lines along points where the tangents of the Δr lines coincide with the meridians; analogously, the zero curves of η intersect at points where the latitude lines touch the Δr lines.

Table 9.

Elevation above sea level (km)	Range of ξ (seconds of arc)	
0	5".7	-5".4
1,000	3".1	-2".9
10,000	0".25	-0".29
100,000	0".0039	-0".0038

Table 10.

Elevation above sea level (km)	Range of η (seconds of arc)	
0	5".7	-6".8
1,000	2".9	-3".9
10,000	0".34	-0".42
100,000	0".0070	-0".0074

b) The angle between the spheroid normal ($\omega \neq 0$) and the ellipsoid normal ($f^{-1} = 298.25$) in the same geocentric latitude; deflection of the vertical

By adding $\delta\xi$ of Table 11 to ξ of sea level, we get the oscillations of the surface normals of the satellite geoid relative to Kozai's reference ellipsoid (see Figure 7); η remains unchanged. In geodesy these values $\xi + \delta\xi$ and η are known as components of the deflection of the vertical (Bomford, 1962).

Example: At $\varphi = 20^\circ$ and $\lambda = 40^\circ$ we find (see pages 74, 76) the deflection components of the vertical:

$$\begin{array}{rcl@{\qquad}rcl} \xi & = & -1''5 & \eta & = & 2''0 \\ \delta\xi & = & +0''02 & & & - \\ \hline \xi + \delta\xi & = & -1''5 & \eta & = & 2''0 \end{array}$$

Table 11.

φ°	$\delta\xi$
90	0"00
80	-0"66
70	-0"63
60	-0"45
50	-0"81
40	-0"75
30	-0"66
20	+0"02
10	+1"18
0	+0"59
-10	+0"37
-20	+0"67
-30	-0"22
-40	-0"07
-50	-0"52
-60	-2"24
-70	-1"98
-80	-0"78
-90	0"00

$$\delta\xi = \sphericalangle\,(\vec{n}_{Ell}, \vec{n}\ (\omega \neq 0))$$

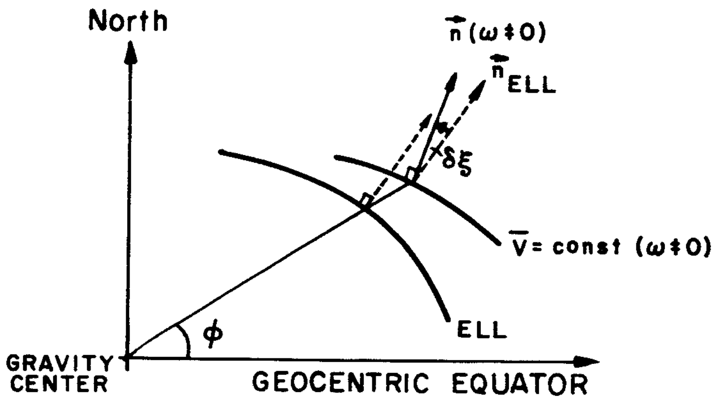


Figure 7.

c) Angle between the spheroid normal ($\omega \neq 0$) and the spheroid normal ($\omega = 0$) along $\overline{V}$ = const ($\omega \neq 0$) at sea level

Table 12 and Figure 8 show the angles with which the spheroids ($\omega = 0$) intersect the spheroid ($\omega \neq 0$) at sea level. The sign of $\Delta\xi$ follows again from the definition of ξ.

Table 12.

φ°	$\Delta\xi$
90	0' 0"
80	2' 1"
70	3' 48"
60	5' 8"
50	5' 51"
40	5' 52"
30	5' 10"
20	3' 50"
10	2' 3"
0	0' 0"
-10	-2' 3"
-20	-3' 50"
-30	-5' 10"
-40	-5' 52"
-50	-5' 51"
-60	-5' 8"
-70	-3' 48"
-80	-2' 1"
-90	0' 0"

$$\Delta\xi = \sphericalangle\,(\vec{n}\ (\omega = 0),\ \vec{n}\ (\omega \neq 0))$$

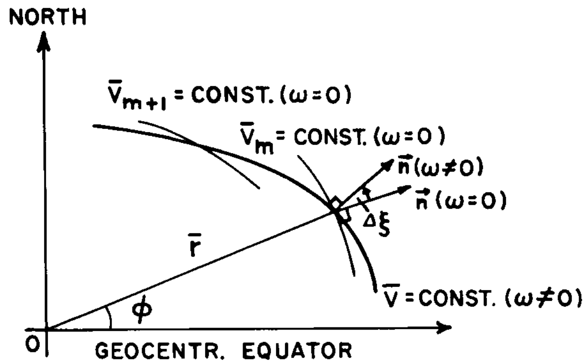


Figure 8.

d) <u>Shape of the latitude and longitude curves</u>

In this section we consider the latitude and longitude of a point defined by the surface normal rather than by the geocentric radius. Let us call latitude B the complement of the angle included by a surface normal and the revolution axis, and analogously, let us call longitude L the complement of the angle of a plane determined by a surface normal and the revolution axis with the x^2 axis. Then the latitude and longitude curves of an equipotential surface have the equations:

$$B = \text{const},$$

$$L = \text{const}.$$

To obtain their amplitudes relative to the spheroidal part we consider equations (10) to (12); the explicit expression for the latitude curves,

$$B = \text{const} = \arc \sin \frac{\sin \varphi - \frac{1}{r} \frac{\partial r}{\partial \varphi} \cos \varphi}{\left[1 + \frac{1}{r^2} \left(\frac{\partial r}{\partial \varphi}\right)^2 + \frac{1}{r^2 \cos^2 \varphi} \left(\frac{\partial r}{\partial \lambda}\right)^2\right]^{\frac{1}{2}}} \quad , \tag{20}$$

shows that B is mainly a function of φ plus a small correction term accounting for the undulation effect in V = const. Within a <u>small</u> range

$$B = f(\varphi) + \Delta \quad , \tag{21}$$

with

$$\Delta = \text{const} \; ; \tag{22}$$

by differentiation

$$dB = \frac{df}{d\varphi} d\varphi \; . \tag{23}$$

Equating ξ to dB, we get the corresponding angle $d\varphi$ by which the latitude curve B = const is shifted along the meridian. Because $\frac{df}{d\varphi}$ is close to 1, its amplitude can be expressed by

$$A_B \approx - r \cdot \xi \quad , \tag{24}$$

and similarly the amplitude of the longitude curve

$$A_L \approx - r\,\eta \quad . \tag{25}$$

In the ξ and η plottings (see Contour Maps 3 and 4) only the amount of the equi-level line numbers must be changed, according to Table 13, to obtain the corresponding graphs of A_B and A_L.

Table 13.

Sea level		1,000 km elevation		10,000 km elevation		100,000 km elevation	
ξ, η	A_B, A_L (m)	ξ, η	A_B, A_L (m)	ξ, η	A_B, A_L (m)	ξ, η	A_B, A_L (m)
				0".30	-24		
5"	-155			0".25	-20	0".006	-3.1
4"	-124			0".20	-16	0".004	-2.1
3"	- 93	3"	-107	0".15	-12	0".003	-1.6
2"	- 62	2"	- 72	0".10	- 8	0".002	-1.0
1"	- 31	1"	- 36	0".05	- 4	0".001	-0.5
0"	0	0"	0	0"	0	0"	0
-1"	31	-1"	36	-0".05	4	-0".001	0.5
-2"	62	-2"	72	-0".10	8	-0".002	1.0
-3"	93	-3"	107	-0".15	12	-0".003	1.6
-4"	124	-4"	143	-0".20	16	-0".004	2.1
-5"	155			-0".25	20	-0".006	3.1
-6"	186			-0".30	24		
				-0".35	28		
				-0".40	32		

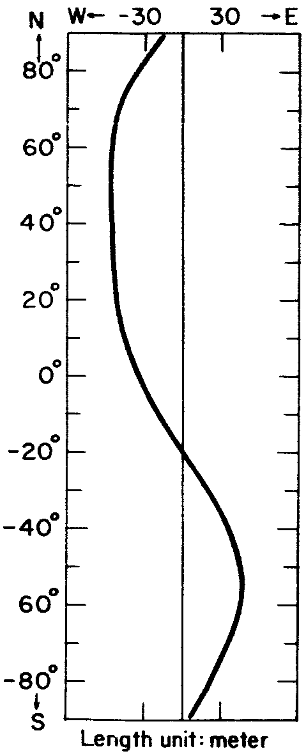
A_B + latitude curve oscillates northward
A_B - latitude curve oscillates southward

A_L + longitude curve oscillates eastward
A_L - longitude curve oscillates westward

The zero curves of A_B, A_L and the level lines of the undulations Δr intersect each other as already mentioned for ξ, η.

Examples: 1) The latitude curve B = 20°, on the satellite geoid, is at the geocentric longitude $\lambda = 40°$ about $A_B = 46$ m northward of the corresponding B = 20° latitude circle of its spheroid.

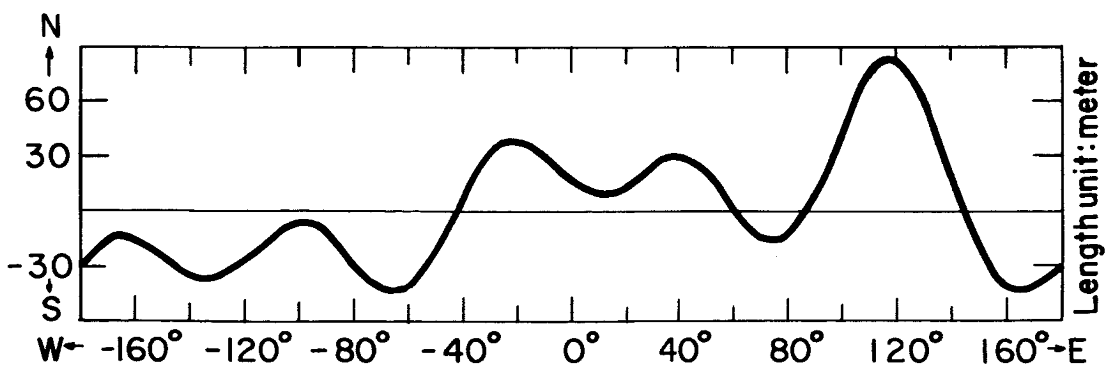
2) See Figure 9.



a)

Figure 9.--a) Shape of the longitude curve L = 0 at sea level.

b) Shape of the equator curve B = 0 at sea level.



b)

4. Gaussian and mean curvature

With the position vector

$$\vec{x} = \vec{x}\,(p^1, p^2) \qquad\qquad p^1,\ p^2 \text{ stand for } \varphi,\ \lambda \tag{26}$$

of an equipotential surface V = const and its unit normal (to the surface)

$$\vec{n} = -\frac{\text{grad } V}{g} \quad , \tag{27}$$

we get by partial differentiation of equation (26) a covariant vector

$$\vec{x}_i = \frac{\partial \vec{x}}{\partial p^i} \quad , \tag{28}$$

which leads to the metric tensor

$$a_{ij} = \vec{x}_i\,\vec{x}_j \quad . \tag{29}$$

Similarly, we proceed with the normal vector $\vec{n}$:

$$\vec{n}_i = \frac{\partial \vec{n}}{\partial p^i} \quad , \tag{30}$$

wherein

$$\frac{\partial^2 r}{\partial p^i\,\partial p^j} = -\frac{1}{\frac{\partial V}{\partial r}}\left(\frac{\partial^2 V}{\partial p^i\,\partial p^j} + \frac{\partial^2 V}{\partial r\,\partial p^i}\frac{\partial r}{\partial p^j} + \frac{\partial^2 V}{\partial r\,\partial p^j}\frac{\partial r}{\partial p^i} + \frac{\partial^2 V}{\partial r^2}\frac{\partial r}{\partial p^i}\frac{\partial r}{\partial p^j}\right) \quad . \tag{31}$$

The covariant vector $\vec{n}_i$ defines together with equation (28) a covariant tensor

$$b_{ij} = - \vec{n}_i \, \vec{x}_j \quad , \tag{32}$$

from which we obtain with equation (29) the gaussian curvature in question:

$$K = \frac{\det b_{ij}}{\det a_{ij}} \quad . \tag{33}$$

An analogous expression is found for the mean curvature

$$H = \frac{1}{2} a^{ij} \, b_{ij} \quad , \tag{34}$$

with a^{ij} the contravariant complement of equation (29).

In our numerical computations we did not consider K and H directly, but the radii of curvature instead:

$$\frac{1}{\sqrt{K}} = \frac{1}{\sqrt{K}}(r, \varphi, \lambda, C_{nm}, S_{nm}; \omega) \quad , \tag{35}$$

and

$$\frac{1}{H} = \frac{1}{H}(r, \varphi, \lambda, C_{nm}, S_{nm}; \omega) \quad , \tag{36}$$

which we split up into a zonal part $\frac{1}{\sqrt{\bar{K}}}$, $\frac{1}{\bar{H}}$ and a correction term

$$\Delta \frac{1}{\sqrt{K}} = \frac{1}{\sqrt{K}} - \frac{1}{\sqrt{\bar{K}}} \tag{37}$$

and

$$\Delta \frac{1}{H} = \frac{1}{H} - \frac{1}{\bar{H}} \quad . \tag{38}$$

Although $\frac{1}{\sqrt{\bar{K}}}$ and $\frac{1}{\bar{H}}$ are fairly different along the equipotential surface $\bar{V}$ = const except at the pole $\frac{1}{\sqrt{\bar{K}}} = \frac{1}{\bar{H}}$, the corrections $\Delta \frac{1}{\sqrt{K}}$ and $\Delta \frac{1}{H}$ agree everywhere within several meters.[7] This is easily explained by the following: calling $\bar{R}_1$ and $\bar{R}_2$ the principle radii of curvature of the spheroid, and

$$\begin{aligned} R_1 &= \bar{R}_1 + \Delta R_1 \\ R_2 &= \bar{R}_2 + \Delta R_2 \end{aligned} \tag{39}$$

the analogous radii for the equipotential surface V = const, we obtain from equations (37) and (39)

$$\begin{aligned} \Delta \frac{1}{\sqrt{K}} &= (R_1 R_2)^{\frac{1}{2}} - (\bar{R}_1 \bar{R}_2)^{\frac{1}{2}} \approx (\bar{R}_1 \bar{R}_2)^{\frac{1}{2}} \left(1 + \frac{\Delta R_1}{\bar{R}_1} + \frac{\Delta R_2}{\bar{R}_2}\right)^{\frac{1}{2}} - \\ &- (\bar{R}_1 \bar{R}_2)^{\frac{1}{2}} \approx \tfrac{1}{2}(\Delta R_1 + \Delta R_2) \quad , \end{aligned} \tag{40}$$

[7] At ground level; at higher elevations, correspondingly closer.

and similarly

$$\Delta \frac{1}{H} = \frac{2R_1 \ R_2}{R_1 + R_2} - \frac{2\bar{R}_1 \ \bar{R}_2}{\bar{R}_1 + \bar{R}_2} \approx \frac{1}{2} (\Delta R_1 + \Delta R_2) \ ; \tag{41}$$

this means

$$\Delta \frac{1}{\sqrt{K}} \approx \Delta \frac{1}{H} \quad . \tag{42}$$

In the contour maps we considered, therefore, only $\Delta \frac{1}{\sqrt{K}}$ in the different sections of outer space.

Example: With the help of sections a) and b) we obtain for the radii of the gaussian and mean curvature at $\varphi = 20^\circ$, $\lambda = 40^\circ$ at sea level ($\omega \neq 0$)

$$\frac{1}{\sqrt{\bar{K}}} = 6,361,929 \text{ m} \qquad \frac{1}{\bar{H}} = 6,361,902 \text{ m}$$

$$\Delta \frac{1}{\sqrt{K}} = - 330 \qquad \Delta \frac{1}{H} = - 330$$

$$\frac{1}{\sqrt{K}} = 6,361,599 \text{ m} \qquad \frac{1}{H} = 6,361,572 \text{ m}$$

or the curvature

$$\bar{K} = 0.247\ 071\ 3 \times 10^{-13} \text{ m}^{-2} \quad K = 0.247\ 097\ 0 \times 10^{-13} \text{ m}^{-2}$$

$$\bar{H} = 0.157\ 185\ 7 \times 10^{-6} \text{ m}^{-1} \quad H = 0.157\ 193\ 8 \times 10^{-6} \text{ m}^{-1}$$

These values may also be used in geodesy for best approximating spheres in local geodetic reference systems, etc.

a) Radius of curvature of the spheroid

Table 14.

Radius of the gaussian curvature (in meters)

φ° \ $\frac{1}{\sqrt{K}}$	Sea level		Elevation above sea level (ω = 0)		
	$\omega \neq 0$	$\omega = 0$	1,000 km	10,000 km	100,000 km
90	6 399 497	6 388 407	7 387 065.7	16 382 189.50	106 378 785.7547
80	6 398 257	6 387 831	7 386 545.1	16 381 947.14	106 378 748.3272
70	6 394 651	6 386 131	7 385 022.6	16 381 249.23	106 378 640.5556
60	6 388 942	6 383 350	7 382 643.3	16 380 179.61	106 378 475.4296
50	6 381 954	6 379 959	7 379 725.1	16 378 866.90	106 378 272.8535
40	6 374 557	6 376 400	7 376 629.5	16 377 469.00	106 378 057.2485
30	6 367 540	6 372 999	7 373 718.5	16 376 154.05	106 377 854.6113
20	6 361 929	6 370 340	7 371 355.0	16 375 080.35	106 377 689.3805
10	6 358 110	6 368 452	7 369 762.9	16 374 377.58	106 377 581.4904
0	6 356 693	6 367 706	7 369 183.1	16 374 131.19	106 377 543.9652
-10	6 358 130	6 368 472	7 369 743.9	16 374 371.64	106 377 581.3461
-20	6 361 781	6 370 192	7 371 288.5	16 375 070.40	106 377 689.1396
-30	6 367 496	6 372 954	7 373 675.6	16 376 143.43	106 377 854.3560
-40	6 374 602	6 376 444	7 376 617.7	16 377 461.50	106 378 057.0731
-50	6 381 861	6 379 868	7 379 696.4	16 378 865.87	106 378 272.8385
-60	6 388 932	6 383 340	7 382 654.1	16 380 187.25	106 378 475.6191
-70	6 394 817	6 386 294	7 385 118.8	16 381 265.84	106 378 640.9439
-80	6 398 472	6 388 043	7 386 695.1	16 381 970.55	106 378 748.8591
-90	6 399 725	6 388 634	7 387 230.4	16 382 215.42	106 378 786.3390

The gaussian and the mean curvature of the potential spheroids resulted in positive values (see Tables 14 and 15). Hence the radius of the gaussian curvature is always larger than or equal to the radius of the mean curvature over the whole surface. At the poles both radii are identical; that is, the principal radii of curvature are equal (spherical surface section) and Dupin's indicatrix is circular. For infinite elevation the radius of the gaussian curvature becomes equal to the radius of the mean curvature according to the spherical shape assumed by the equipotential spheroid.

Table 15.

Radius of the mean curvature (in meters)

φ° \ $\frac{1}{H}$	Sea level		Elevation above sea level ($\omega = 0$)		
	$\omega \neq 0$	$\omega = 0$	1,000 km	10,000 km	100,000 km
90	6 399 497	6 388 407	7 387 065.7	16 382 189.50	106 378 785.7547
80	6 398 257	6 387 831	7 386 545.1	16 381 947.14	106 378 748.3272
70	6 394 651	6 386 131	7 385 022.6	16 381 249.23	106 378 640.5556
60	6 388 940	6 383 349	7 382 643.0	16 380 179.58	106 378 475.4295
50	6 381 948	6 379 958	7 379 724.2	16 378 866.82	106 378 272.8532
40	6 374 545	6 376 397	7 376 627.7	16 377 468.83	106 378 057.2479
30	6 367 520	6 372 994	7 373 715.5	16 376 153.77	106 377 854.6103
20	6 361 902	6 370 334	7 371 350.8	16 375 079.96	106 377 689.3791
10	6 358 076	6 368 444	7 369 757.8	16 374 377.12	106 377 581.4886
0	6 356 657	6 367 697	7 369 177.6	16 374 130.69	106 377 543.9634
-10	6 358 096	6 368 464	7 369 738.7	16 374 371.17	106 377 581.3444
-20	6 361 752	6 370 185	7 371 284.3	16 375 070.02	106 377 689.1382
-30	6 367 475	6 372 949	7 373 672.5	16 376 143.15	106 377 854.3549
-40	6 374 590	6 376 441	7 376 615.9	16 377 461.33	106 378 057.0725
-50	6 381 855	6 379 866	7 379 695.5	16 378 865.78	106 378 272.8382
-60	6 388 930	6 383 340	7 382 653.7	16 380 187.22	106 378 475.6190
-70	6 394 816	6 386 294	7 385 118.8	16 381 265.83	106 378 640.9439
-80	6 398 472	6 388 043	7 386 695.1	16 381 970.55	106 378 748.8591
-90	6 399 725	6 388 634	7 387 230.4	16 382 215.42	106 378 786.3390

b) Corrections, $\Delta \frac{1}{\sqrt{K}}$ and $\Delta \frac{1}{H}$

See Contour Map 5. The corrections $\Delta \frac{1}{\sqrt{K}}$ and $\Delta \frac{1}{H}$ usually have their maxima where the undulations Δr have their minima, and vice versa. However, there are also considerable deviations possible, depending on the shape of the undulations. At the poles the radius of curvature is not identical with the spheroidal part, but the difference is practically negligible. The diminution of $\Delta \frac{1}{\sqrt{K}}$, $\Delta \frac{1}{H}$ with higher elevation is faster than that of the undulations.

Table 16.

Elevation above sea level (km)	Range of $\Delta \frac{1}{\sqrt{K}}$, $\Delta \frac{1}{H}$ (m)	
0	567	-470
1,000	331	-271
10,000	43	- 45
100,000	3.8	- 4.1

(see Table 16). The numbers drop at sea level from -470 m and/or 567 m to -4.1 m and/or 3.8 m at 100,000 km elevation. Again at sea level only the case of $\omega \neq 0$ was considered, because of the close agreement with that of $\omega = 0$.

II. GRADIENT FIELD

A. Magnitude of the Gradient Field — Equigravitational (Equigravity) Surfaces

From the equation

$$g = \left[\left(\frac{\partial V}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial V}{\partial \varphi}\right)^2 + \frac{1}{r^2 \cos^2\varphi}\left(\frac{\partial V}{\partial \lambda}\right)^2\right]^{\frac{1}{2}} , \tag{43}$$

we get the magnitude of the gradient V in each point of outer space. If we put

$$g = \text{const} , \tag{44}$$

then equation (43) describes an equigravitational (equigravity) surface whose shape will be determined in the next section. Analogous to section I we define a rotational surface

$$\bar{g} = \text{const} = \left[\left(\frac{\partial \bar{V}}{\partial \bar{r}}\right)^2 + \frac{1}{\bar{r}^2}\left(\frac{\partial \bar{V}}{\partial \varphi}\right)^2\right]^{\frac{1}{2}} , \tag{45}$$

which follows from equation (43) by simply neglecting the nonzonal terms. Further, g and $\bar{g}$ are related by the equation

$$g = \bar{g} = \text{const} , \tag{46}$$

wherein $\bar{r}$ assumes the following values: at sea level

$$\bar{r} = 6{,}378{,}165 \text{ m} , \quad \text{for } \varphi = 0;\ \omega \neq 0 \text{ or } \omega = 0 ,$$

and for the outer space

$$\left.\begin{array}{l} \bar{r} = 6{,}378{,}165 \text{ m} + 1{,}000 \text{ km} \\ \bar{r} = 6{,}378{,}165 \text{ m} + 10{,}000 \text{ km} \\ \bar{r} = 6{,}378{,}165 \text{ m} + 100{,}000 \text{ km} \end{array}\right\} \quad \begin{array}{l} \varphi = 0 \\ \omega = 0 \;. \end{array}$$

The results concerning the geometric structure of the equigravitational (equigravity) surfaces are again split up into two parts. At first the portion of $\bar{g}$ = const is computed and then a correction term is added, leading to the geometrical structure of the equigravitational (equigravity) surfaces g = const. The only difference from section I is that $\bar{g}$ = const no longer approximates g = const in the mean. However, this discrepancy is very small and can often be neglected.

1. Shape of the equigravitational (equigravity) surfaces

Similar to section I we obtain the spheroidal part $\bar{r}$ of the geocentric radius (see Figure 10) from the equation

$$\bar{g} = \bar{g}(\bar{r}, \varphi, C_{no}; \omega) = \text{const} \quad , \tag{47}$$

and the correction term from

$$g = g(\bar{r} + \Delta r, \varphi, \lambda, C_{nm}, S_{nm}; \omega) = \text{const} \quad . \tag{48}$$

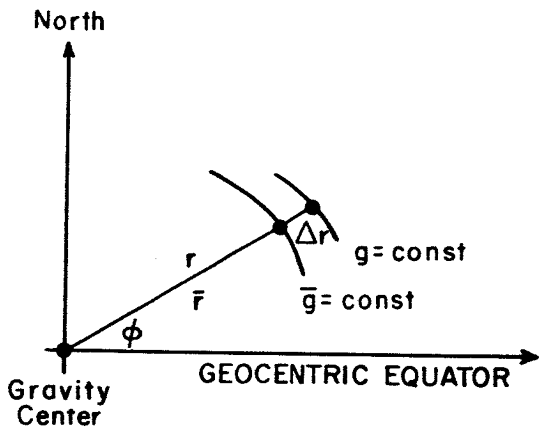


Figure. 10.

Their sum,

$$r(\varphi, \lambda) = \bar{r}(\varphi) + \Delta r(\varphi, \lambda) \quad ,$$

leads to the geocentric radius along an equigravitational (equigravity) surface whose zonal part has the above-mentioned characteristics (pages 33-34).

Example: With the help of sections a) and b) we obtain the geocentric radius r at $\varphi = 20^\circ$ and $\lambda = 40^\circ$ at 10,000 km elevation ($\omega = 0$):

$$\bar{r} = 16{,}377{,}455 \text{ m}$$
$$\Delta r = -4$$
$$r_{\substack{\varphi = 20^\circ \\ \lambda = 40^\circ}} = 16{,}377{,}451 \text{ m}$$

a) Spheroidal (zonal) part of the geocentric radius

Table 17.

φ° \ $\bar{r}$ (m)	Sea level		Elevation above sea level ($\omega = 0$)		
	$\omega \neq 0$	$\omega = 0$	1,000 km	10,000 km	100,000 km
90	6 373 689.54	6 362 631.37	7 364 739.30	16 372 117.71	106 377 233.56
80	6 373 816.80	6 363 091.80	7 365 140.74	16 372 299.76	106 377 261.64
70	6 374 191.84	6 364 426.28	7 366 300.04	16 372 824.01	106 377 342.49
60	6 374 784.84	6 366 489.70	7 368 081.84	16 373 627.39	106 377 466.37
50	6 375 512.59	6 369 021.52	7 370 269.12	16 374 613.21	106 377 618.34
40	6 376 284.31	6 371 713.45	7 372 597.30	16 375 662.79	106 377 780.07
30	6 377 013.98	6 374 247.99	7 374 786.99	16 376 649.69	106 377 932.06
20	6 377 605.10	6 376 310.69	7 376 575.00	16 377 454.93	106 378 055.98
10	6 378 011.24	6 377 677.53	7 377 750.54	16 377 981.27	106 378 136.88
0	6 378 165.00	6 378 165.00	7 378 165.00	16 378 165.00	106 378 165.00
-10	6 378 022.37	6 377 688.70	7 377 761.57	16 377 983.67	106 378 136.94
-20	6 377 638.43	6 376 344.11	7 376 596.06	16 377 458.94	106 378 056.08
-30	6 377 038.57	6 374 272.61	7 374 806.58	16 376 653.97	106 377 932.16
-40	6 376 294.59	6 371 723.70	7 372 609.49	16 375 665.78	106 377 780.14
-50	6 375 524.34	6 369 033.32	7 370 273.96	16 374 613.55	106 377 618.34
-60	6 374 773.82	6 366 478.63	7 368 070.50	16 373 624.25	106 377 466.29
-70	6 374 142.71	6 364 376.89	7 366 265.76	16 372 817.40	106 377 342.33
-80	6 373 743.74	6 363 018.43	7 365 088.90	16 372 290.59	106 377 261.42
-90	6 373 608.45	6 362 549.97	7 364 681.22	16 372 107.59	106 377 233.32

At sea level the geocentric radii at the poles are about 4.5 km ($\omega \neq 0$) and/or 15.6 km ($\omega = 0$) smaller than the corresponding equatorial radius, as shown in Table 17. At higher elevations this difference decreases and finally becomes zero for infinity. Note that the polar radii at sea level are bigger for ($\omega \neq 0$) than for ($\omega = 0$), which is opposite to the results obtained for the equipotential surfaces.

b) <u>Correction term, Δr</u>

See Contour Map 6, page 80. The size of the corrections Δr is much bigger than the undulations of section I1. They run now at sea level from -158 m to 116 m (see Table 18); this roughing process is easily explained by the differentiation involved (see equation (43)). At 100,000 km, however, the C_{22} and the S_{22} terms are again the most significant ones.

Table 18.

Elevation above sea level (km)	Range of Δr (m)	
0	116	-158
1,000	85	-104
10,000	26	- 24
100,000	2.9	- 2.7

c) <u>Difference between the geocentric radii of the Northern and Southern Hemispheres of the spheroid</u>

See page 36. From Table 19 we see that the equigravitational (equigravity) surfaces show a stronger asymmetry of the Northern compared to the Southern Hemispheres than the equipotential surfaces (see section IIc)).

Table 19.

$\overline{r}_{(\varphi)} - \overline{r}_{(-\varphi)}$ (m) / $\lvert\varphi^\circ\rvert$	Sea level		Elevation above sea level ($\omega = 0$)		
	$\omega \neq 0$	$\omega = 0$	1,000 km	10,000 km	100,000 km
90	+81.09	+81.40	+58.08	+10.12	+0.234
80	+73.06	+73.37	+51.84	+ 9.17	+0.213
70	+49.13	+49.39	+34.28	+ 6.61	+0.155
60	+11.02	+11.07	+11.34	+ 3.14	+0.076
50	-11.75	-11.80	- 4.84	- 0.34	-0.006
40	-10.28	-10.25	-12.19	- 2.99	-0.070
30	-24.59	-24.62	-19.59	- 4.28	-0.102
20	-33.33	-33.42	-21.06	- 4.01	-0.096
10	-11.13	-11.17	-11.03	- 2.40	-0.058
0	0.00	0.00	0.00	0.00	0.000

d) Difference of the spheroid and an ellipsoid with equal axes

Table 20.

$\overline{r} - E^*$ (m) / φ°	Sea level		Elevation above sea level ($\omega = 0$)		
	$\omega \neq 0$	$\omega = 0$	1,000 km	10,000 km	100,000 km
90	0.00	0.00	0.00	0.00	0.0000
80	- 7.56	- 6.31	- 2.33	- 0.20	-0.0065
70	-20.74	-16.32	- 5.98	- 0.76	-0.0243
60	-22.68	-14.43	- 7.01	- 1.52	-0.0475
50	-24.96	-14.20	- 8.46	- 2.28	-0.0687
40	-30.40	-19.64	-11.63	- 2.80	-0.0805
30	-31.27	-22.94	-14.70	- 2.86	-0.0781
20	-35.88	-31.34	-15.69	- 2.33	-0.0607
10	-18.67	-17.40	- 8.55	- 1.28	-0.0320
0	0.00	0.00	0.00	0.00	0.0000
-10	- 5.09	- 3.77	+ 4.24	+ 1.41	+0.0327
-20	+ 6.95	+11.66	+12.18	+ 2.86	+0.0630
-30	+13.62	+22.14	+19.47	+ 3.94	+0.0824
-40	+13.43	+24.38	+24.63	+ 4.36	+0.0861
-50	+34.41	+45.51	+30.53	+ 3.99	+0.0743
-60	+27.15	+35.66	+25.26	+ 2.94	+0.0518
-70	+ 1.75	+ 6.23	+11.05	+ 1.58	+0.0267
-80	- 1.96	- 0.71	+ 2.17	+ 0.44	+0.0073
-90	0.00	0.00	0.00	0.00	0.0000

*E is the geocentric ellipsoidal radius.

The results presented in Table 20 resemble those in section IId); however, by the differentiation process, the amplitudes increase or, in other words, the spheroidal surfaces become rougher (see $\varphi = -10°, -80°$ at sea level); the maximum is reached at sea level ($\omega = 0$) around the latitude $\varphi = -50°$.

e) <u>Difference of the spheroids ($\omega \neq 0$) and ($\omega = 0$) at sea level</u>

Table 21.

$\varphi°$	$\bar{r}(\omega \neq 0) - \bar{r}(\omega = 0)$ (m)
90	11,058.17
80	10,725.00
70	9,765.56
60	8,295.14
50	6,491.07
40	4,570.86
30	2,765.99
20	1,294.41
10	333.71
0	0.00
-10	333.67
-20	1,294.32
-30	2,765.96
-40	4,570.89
-50	6,491.02
-60	8,295.19
-70	9,765.82
-80	10,725.31
-90	11,058.48

The results from Table 21 are opposite to those in section IIf), i.e., the equigravity spheroid ($\omega \neq 0$) encloses the equigravitational spheroid ($\omega = 0$) at sea level.

f) Mean flattening[8] of the spheroids ($\omega = 0$) as a function of the elevation above sea level

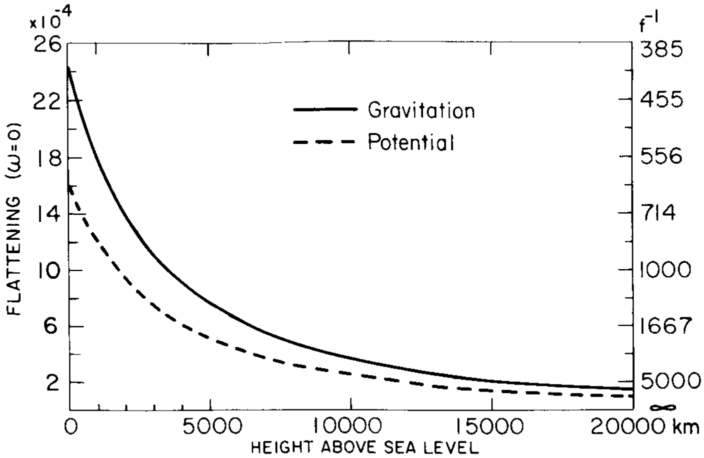


Figure 11.

The mean flattening of the equigravitational spheroids decreases asymptotically with increasing elevation above sea level. For contrast we also show in Figure 11 the mean flattening of the equipotential spheroids ($\omega = 0$).

Comparison: Flattening of the equigravitational (equigravity) spheroids and equipotential spheroids:

Table 22.

$\bar{r}(\varphi = 0)$	f^{-1} ($\bar{V}$ = const)	f^{-1} ($\bar{g}$ = const)	$\bar{r}_{90°}(\bar{V}) - \bar{r}_{90°}(\bar{g})$ (km)	Elevation
6,378,165 m ($\omega \neq 0$)	298	1,412	-16.9	sea level
6,378,165 m ($\omega = 0$)	615	409	5.2	sea level
7,378,165 m ($\omega = 0$)	823	548	4.5	1,000 km above sea leve
16,378,165 m ($\omega = 0$)	4,060	2,706	2.0	10,000 km above sea leve
106,378,165 m ($\omega = 0$)	171,290	114,193	0.3	100,000 km above sea leve

[8] For the definition of the mean flattening see page 12.

Including the centrifugal force ($\omega \neq 0$), the equipotential spheroid stays at sea level within the equigravity spheroid. Otherwise, if no centrifugal force is included ($\omega = 0$), the equigravitational spheroid is surrounded by the equipotential spheroid. At infinite elevation, both the equipotential and the equigravitational spheroid fall together (see also Figure 11). In column 4 of Table 22 we also give the polar distance between an equipotential and equigravitational spheroid assuming that the equatorial radius is identical. Note the decrease of the numbers with higher elevation.

g) Distance of the actual equator of the spheroid from the gravity center

As in section I1h), we find the distance D of the actual equator from the gravity center:

$$D \approx a\left[-3\, C_{30}\left(1+\frac{h}{a}\right)^{-2} + \frac{45}{8}\, C_{50}\left(1+\frac{h}{a}\right)^{-4} - \frac{35}{4}\, C_{70}\left(1+\frac{h}{a}\right)^{-6}\right.$$
$$\left. + \frac{1575}{128}\, C_{90}\left(1+\frac{h}{a}\right)^{-8} - \frac{2079}{128}\, C_{11\,0}\left(1+\frac{h}{a}\right)^{-10} + \frac{21021}{1024}\, C_{13\,0}\left(1+\frac{h}{a}\right)^{-12} \cdots\right] . \tag{49}$$

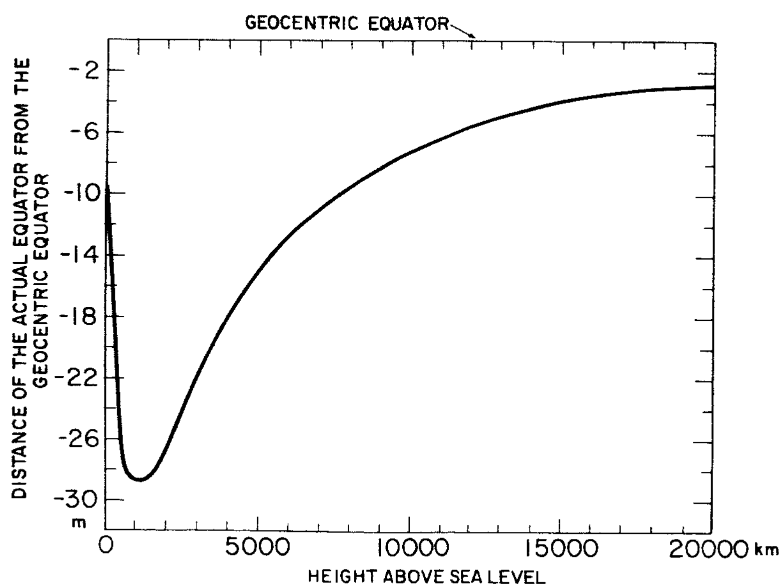


Figure 12.

Near sea level the actual equator moves at first southward and returns then asymptotically to the geocentric equator with increasing height, as is shown in Figure 12.

2. Oscillation of the surface normals

See section I3 and Contour Maps 7 and 8. The only basic difference here is the use of the function $\bar{g}$ and g instead of $\bar{V}$ and V for the determination of ξ and η. Both results in section I3 and IIA2 are needed in the next section to obtain the plane, defined by the corresponding surface normals of V = const and g = const.

Table 23.

Elevation above sea level (km)	Range of ξ (seconds of arc)	
0	15"	-15"
1,000	7".8	- 7".7
10,000	0".43	- 0".52
100,000	0".0061	- 0".0060

Table 24.

Elevation above sea level (km)	Range of η (seconds of arc)	
0	16"	-17"
1,000	8".0	- 9".1
10,000	0".54	- 0".72
100,000	0".011	- 0".011

The surface normals of the equigravitational (equigravity) surfaces oscillate in a far larger range than those of the equipotential surfaces. Tables 23 and 24 show the magnitudes of the components ξ, η at different elevations. The results at sea level were again practically the same for $g(\omega \neq 0)$ = const and $g(\omega = 0)$ = const, and hence only one contour map was plotted.

3. Intersection of the equigravitational (equigravity) surfaces with the equipotential surfaces

The angle by which the surfaces V = const and g = const intersect each other (see Figure 13) is obtained from the scalar product of their gradients

$$\cos \mu = \frac{\text{grad } V \cdot \text{grad } g}{g \left|\text{grad } g\right|} \quad ; \tag{50}$$

in the case of the spheroids, this becomes

$$\cos \bar{\mu} = \frac{\text{grad } \bar{V} \cdot \text{grad } \bar{g}}{\bar{g} \left|\text{grad } \bar{g}\right|} \quad . \tag{51}$$

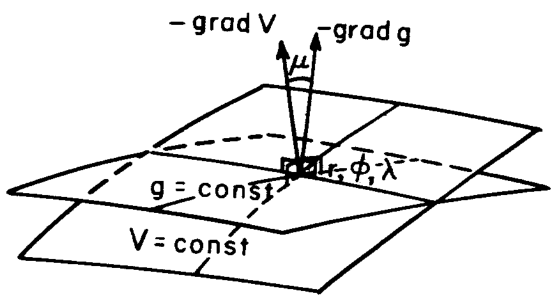


Figure 13.[9]

We determine μ stepwise again by

$$\mu = \bar{\mu} + \Delta\mu \quad , \tag{52}$$

wherein $\Delta\mu$ is found from equation (50) with the help of equations (51) and (52).

It should be pointed out that the angles μ are computed along the equipotential surfaces of section I.

[9] The negative sign is due to the opposite direction of grad V and grad g with respect to the surface normals.

Example: The angle μ included by the normal of the equipotential surface at 1,000 km elevation and the normal of the equigravitational surface at $\varphi = 20^\circ$ and $\lambda = 40^\circ$ is obtained from sections a) and b) below.

$$\begin{array}{rcr} \bar{\mu} & = & 1'20''.6 \\ \Delta\mu & = & -1''.2 \\ \hline \mu & = & 1'19''.4 \end{array}$$

Oscillation of the surface normals:

1. Equipotential surface (see Contour Maps 3 and 4)
   $\xi = -0''.9 \quad \eta = 1''.6$
2. Equigravitational surface (see Contour Maps 7 and 8)
   $\xi = -2''.1 \quad \eta = 1''.6$

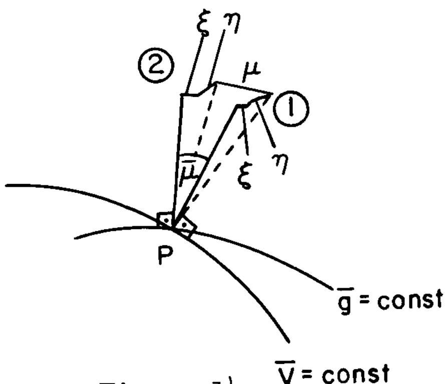


Figure 14.

For the determination of the plane in which μ has to be taken, we describe around P a unit sphere and project on its surface the relative position of the normals of the equipotential and equigravitational surface. While the spheroidal part $\bar{\mu}$ lies always in the meridian, the plane of μ is obtained by ξ, η of the equipotential surface 1 and by ξ, η of the equigravitational surface 2, as shown in Figure 14. Because the η values in this example are incidentally both the same, $\Delta\mu$ is immediately seen to be the difference of the ξ values 2 and 1.

a) Spheroidal (zonal) part

Table 25.

$\Delta\bar{\mu}$ / φ°	Sea level		Elevation above sea level (ω = 0)		
	$\omega \neq 0$	$\omega = 0$	1,000 km	10,000 km	100,000 km
90	0' 0".0	0' 0".0	0' 0".00	0".00	0".000
80	3' 6".7	0'55".5	0'42".21	8".66	0".206
70	5'48".6	1'46".7	1'20".01	16".28	0".387
60	7'48".2	2'25".6	1'48".22	21".94	0".521
50	8'54".5	2'44".0	2' 2".85	24".96	0".593
40	8'54".4	2'44".7	2' 2".87	24".98	0".593
30	7'51".1	2'24".1	1'48".00	21".99	0".521
20	5'49".7	1'47".2	1'20".65	16".35	0".387
10	3' 2".3	1' 0".9	0'43".92	8".73	0".206
0	0' 0".29	0' 0".29	0' 0".33	0".05	0".000
-10	3' 5".0	0'58".2	0'42".84	8".65	0".206
-20	5'51".0	1'45".9	1'20".20	16".31	0".387
-30	7'48".0	2'27".2	1'48".78	22".01	0".521
-40	8'54".8	2'44".2	2' 3".24	25".05	0".593
-50	8'54".8	2'43".7	2' 3".44	25".08	0".593
-60	7'44".0	2'29".8	1'50".10	22".07	0".522
-70	5'45".1	1'50".2	1'21".98	16".40	0".387
-80	3' 5".2	0'56".9	0'43".27	8".73	0".206
-90	0' 0".0	0' 0".0	0' 0".00	0".00	0".000

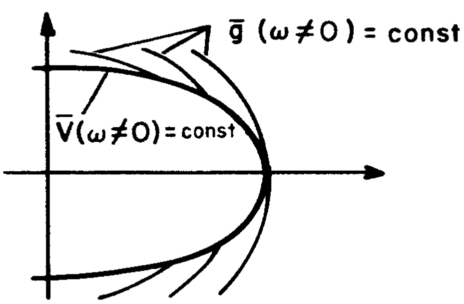


Figure 15.

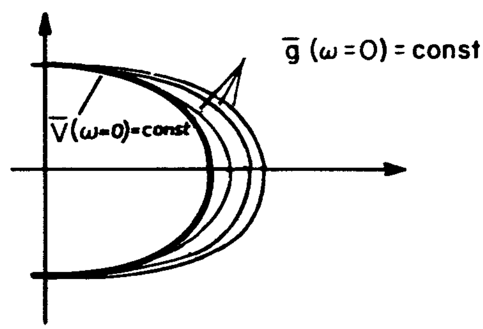


Figure 16.

At sea level the two surfaces $\bar{g}$ ($\omega \neq 0$) and $\bar{V}$ ($\omega \neq 0$) intersect each other, as shown in Figure 15. If we ignore the centrifugal force at sea level and higher elevations, Figure 16 reflects the situation for (ω = 0) (see Table 25).

b) Corrections, $\Delta\mu$

See Number Map 1, page 88 . In the immediate neighborhood of the equator and the poles, $\Delta\mu$ changes very rapidly with latitude. This is due to the facts that:

1. at the equator the $\overline{\mu}$ values are very small compared with the $\mu = \overline{\mu} + \Delta\mu$ values; and
2. at the poles we consider only a point value instead of values along a latitude curve.

In the number maps the geocentric longitude (first horizontal line) has to be multiplied by 10. The value $\lambda = -5$ actually means $\lambda = -50^{\circ}$, etc. The geocentric latitude is shown in the first vertical line.

B. Direction of the Gradient Field — Orthogonal Trajectories[10] of the Equipotential Surfaces

In the previous sections we were concerned with the geometric structure of surfaces of constant potential or constant gravitation (gravity). Here we shall investigate briefly the characteristics of the curves orthogonally to the equipotential surfaces. Because their tangent is collinear with the gradient of V = const (see Figure 17), the differential equation of the orthogonal trajectories is immediately found to be

$$\frac{d\vec{x}}{ds} = -\frac{\text{grad } V}{g} \qquad (s = \text{curve length}) \quad . \tag{53}$$ [11]

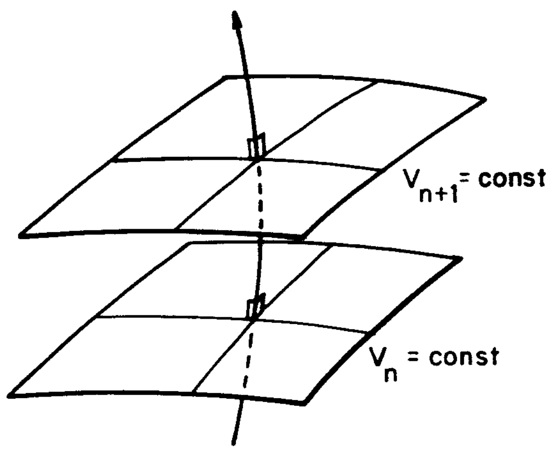


Figure 17.

[10] Also called plumb line, vertical, gradient curve, etc.

[11] The negative sign accounts for the direction of the orthogonal trajectory toward outer space.

1. <u>The gradient (direction) field in a spherical coordinate system</u>

With the help of equation (53) it is possible to determine the angle δ included by the geocentric radius vector and the tangent of the orthogonal trajectory at a certain point (see Figure 18):

$$\cos \delta = - \frac{\text{grad } V \cdot \vec{r}}{g \cdot r} \quad . \tag{54}$$

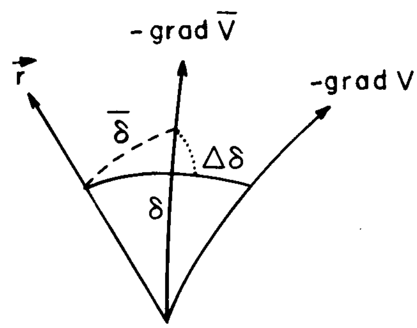


Figure 18.

This angle δ represents the structure of the gradient field and the direction of the trajectories. The spheroidal part is obtained from

$$\overline{\delta} = \text{arc sin} \frac{1}{\overline{r}\,\overline{g}} \left| \frac{\partial \overline{V}}{\partial \varphi} \right| \quad , \tag{55}$$

and the correction term $\Delta\delta = \delta - \overline{\delta}$:

$$\Delta\delta = \text{arc sin} \frac{1}{rg} \left[\left(\frac{\partial V}{\partial \varphi} \right)^2 + \frac{1}{\cos^2 \varphi} \left(\frac{\partial V}{\partial \lambda} \right)^2 \right]^{\frac{1}{2}} - \text{arc sin} \frac{1}{\overline{r}\,\overline{g}} \left| \frac{\partial \overline{V}}{\partial \varphi} \right| \quad . \tag{56}$$

On pages 50 and 91 to 92 we give only the magnitudes of $\bar{\delta}$ and $\Delta\delta$. But with the previous results from section I3 we can easily determine the direction in which δ has to be taken (see also section IIA3).

<u>Example</u>: The angle δ included by the geocentric radius and the gradient V at $\varphi = 20^{\circ}$ and $\lambda = 40^{\circ}$ in 1,000 km elevation can be found with the help of sections a) and b) below.

$$\begin{array}{rcr} \bar{\delta} & = & 2'41''.5 \\ \Delta\delta & = & -\;0''.9 \\ \hline \delta & = & 2'40''.6 \end{array}$$

To get the direction in which δ has to be taken, we proceed similarly to section IIA3. The oscillation of the gradient vector is obtained from section I3:

$$\xi = -0''.9$$

$$\eta = 1''.6 \quad ,$$

which gives, together with $\bar{\delta}$, the direction in question (see Figure 19). Although we take, throughout section III, the distance above sea level along the geocentric radius r as elevations, the ξ, η values (which actually refer to an equipotential surface) remain, for practical purposes, unchanged.

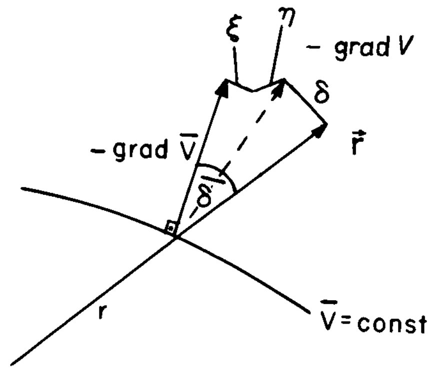


Figure 19.

a) <u>Zonal part of the gradient field</u>

Table 26.

$\bar{\delta}$ / $\varphi°$	Sea level		Elevation above sea level (ω = 0)		
	$\omega \neq 0$	$\omega = 0$	1,000 km	10,000 km	100,000 km
90	0' 0".0000	0' 0".0000	0' 0".0000	0".0000	0".00000
80	3'55".5234	1'54".2138	1'25".5555	17".3829	0".41190
70	7'23".5148	3'35".2924	2'40".9575	32".6677	0".77410
60	9'58".4738	4'50".5056	3'36".9644	44".0105	1".04292
50	11'21".0097	5'30".1226	4' 6".6982	50".0454	1".18595
40	11'21".8628	5'30".2499	4' 6".7332	50".0480	1".18594
30	10' 0".2821	4'50".4767	3'37".0957	44".0210	1".04292
20	7'26".4583	3'36".1446	2'41".4673	32".6912	0".77414
10	3'58".8622	1'56".1838	1'26".3763	17".4209	0".41200
0	0' 0".5934	0' 0".5913	0' 0".4717	0".0460	0".00017
-10	3'57".3108	1'54".6377	1'25".4978	17".3438	0".41172
-20	7'25".7662	3'35".4555	2'41".0780	32".6546	0".77400
-30	10' 1".1638	4'51".3566	3'37".4094	44".0400	1".04299
-40	11'22".6848	5'31".0699	4' 7".4158	50".1225	1".18622
-50	11'22".3416	5'31".4534	4' 7".7750	50".1608	1".18637
-60	10' 1".1708	4'53".2003	3'38".5202	44".1406	1".04339
-70	7'26".1276	3'37".9031	2'42".4809	32".7801	0".77450
-80	3'56".9709	1'55".6602	1'26".4569	17".4481	0".41212
-90	0' 0".0000	0' 0".0000	0' 0".0000	0".0000	0".00000

The values at sea level from Table 26 are referred to an equipotential surface $\overline{V}$ = const ($\omega \neq 0$) with $\overline{r}$ = 6,378,165 m for $\varphi = 0$. Adding the elevation to the geocentric radius, we obtain the angle $\overline{\delta}$ at higher elevations, and analogously, for section b) we obtain $\Delta\delta$ by considering V = const ($\omega \neq 0$). The angle $\overline{\delta}$ is, of course, zero at the poles; however, at the equator, we get nonzero results, since the actual equator of an equipotential spheroid does not coincide with the geocentric equator of our coordinate system (see section I1h)). The maximum values of $\overline{\delta}$ appear as expected in the latitude between $|40^\circ|$ and $|50^\circ|$.

b) Corrections, $\Delta\delta$

See Number Map 2, page 91 . The rapid change of $\Delta\delta$ near the equator and near the poles is due to the same fact already mentioned in section IIA3b). In Table 27 we give the range of the $\Delta\delta$ for different elevations; the polar and equatorial areas are, however, excluded because of the above reasons. Note that the range of the $\Delta\delta$ values agrees closely with that of the ξ values of the equipotential surfaces. For explanation see Figure 19.

Table 27.

Elevation above sea level (km)	Range of $\Delta\delta$ (seconds of arc)	
0	5".5	-5".8
1,000	3".0	-3".1
10,000	0".29	-0".23
100,000	0".0039	-0".0037

2. Curvature and torsion of the orthogonal trajectories

The second and third derivatives of equation (53) with respect to s lead to the radii of curvature and of torsion of the orthogonal trajectories: radius of curvature,

$$\rho = \frac{1}{\sqrt{\frac{d^2\vec{x}}{ds^2} \cdot \frac{d^2\vec{x}}{ds^2}}} , \tag{57}$$

radius of torsion (attached with a sign),

$$\tau = \frac{1}{\rho^2} \frac{1}{\left(\frac{d\vec{x}}{ds}\, \frac{d^2\vec{x}}{ds^2}\, \frac{d^3\vec{x}}{ds^3}\right)} . \tag{58}$$

If we define a local coordinate system y^i, which coincides with the moving trihedral of the trajectory, then the canonical representation of an orthogonal trajectory can be expressed (see also Kreyszig, 1959):

$$y^1 = s - \frac{1}{6\rho^2} s^3 + \frac{\rho'}{8\rho^3} s^4 + \frac{s^5}{120\ \rho^2} \left[\frac{1}{\rho^2} + \frac{1}{\tau^2} + \frac{4\rho''}{\rho} - \frac{11(\rho')^2}{\rho^2}\right] + \cdots,$$

$$y^2 = \frac{s^2}{2\rho} - \frac{\rho'}{6\rho^2} s^3 + \frac{s^4}{24\rho} \left[\frac{2(\rho')^2 - 1}{\rho^2} - \frac{1}{\tau^2} - \frac{\rho''}{\rho}\right] \tag{59}$$

$$+ \frac{s^5}{120\ \rho} \left[\frac{3\tau'}{\tau^3} + \frac{3\rho'}{\rho\tau^2} + \frac{6}{\rho^3}\rho' - \frac{6(\rho')^3}{\rho^3} + \frac{6\rho'\rho''}{\rho^2} - \frac{\rho'''}{\rho}\right] + \cdots,$$

$$y^3 = \frac{s^3}{6\rho\tau} - \frac{s^4}{24\rho\tau}\left[\frac{2\rho'}{\rho} + \frac{\tau'}{\tau}\right]$$

$$+ \frac{s^5}{120\rho\tau}\left[-\frac{1}{\rho^2} - \frac{1}{\tau^2} + \frac{6(\rho')^2}{\rho^2} + \frac{3\rho'\tau'}{\rho\tau} + \frac{2(\tau')^2}{\tau^2} - \frac{3\rho''}{\rho} - \frac{\tau''}{\tau}\right] + \cdots ,$$

wherein

$$\frac{d\rho}{ds} \equiv \rho' , \quad \frac{d^2\rho}{ds^2} \equiv \rho'' \quad \text{etc., and similar for } \tau \quad .$$

With a positive ρ the trajectory under consideration behaves for $\tau > 0$ like a right-hand screw and for $\tau < 0$ like a left-hand screw. The tangent $\vec{t}$, the normal $\vec{n}$, and the binormal $\vec{b}$ have in this case the following components in the above-defined local cartesian coordinate system:

$$\vec{t} = (1,0,0)$$

$$\vec{n} = (0,1,0)$$

$$\vec{b} = (0,0,1) \; .$$

The derivatives of $\vec{x}$ with respect to the curve length s were computed partly from analytical expressions and partly by numerical differentiation of the integrated trajectory (see the next section) in a spherical coordinate system.

a) Radius of curvature of the orthogonal trajectories

1) Zonal part, i.e., radius of curvature of the orthogonal trajectories of the equipotential spheroids $\bar{V}$ = const

Table 28.

$\bar{\rho}$ (km) / φ°	Sea level		Elevation above sea level					
	$\omega \neq 0$	$\omega = 0$	10 km ($\omega \neq 0$)	10 km ($\omega = 0$)	100 km ($\omega = 0$)	1,000 km ($\omega = 0$)	10,000 km ($\omega = 0$)	100,000 km ($\omega = 0$)
90	∞	∞	∞	∞	∞	∞	∞	∞
80	3520 E 3*	1182 E 4	3501 E 3	1188 E 4	1236 E 4	1797 E 4	1945 E 5	5328 E 7
70	1883 E 3	6145 E 3	1873 E 3	6174 E 3	6434 E 3	9476 E 3	1035 E 5	2835 E 7
60	1399 E 3	4500 E 3	1392 E 3	4522 E 3	4720 E 3	7006 E 3	7680 E 4	2104 E 7
50	1222 E 3	3996 E 3	1216 E 3	4015 E 3	4185 E 3	6172 E 3	6753 E 4	1850 E 7
40	1219 E 3	3979 E 3	1213 E 3	3998 E 3	4171 E 3	6172 E 3	6751 E 4	1850 E 7
30	1379 E 3	4548 E 3	1372 E 3	4569 E 3	4764 E 3	7022 E 3	7673 E 4	2104 E 7
20	1854 E 3	6113 E 3	1844 E 3	6141 E 3	6396 E 3	9405 E 3	1032 E 5	2834 E 7
10	3551 E 3	1076 E 4	3530 E 3	1082 E 4	1135 E 4	1727 E 4	1934 E 5	5324 E 7
0	2230 E 6	2269 E 6	2369 E 6	2413 E 6	4844 E 6	2303 E 6	3705 E 7	6467 E 10
-10	3501 E 3	1125 E 4	3481 E 3	1131 E 4	1182 E 4	1771 E 4	1951 E 5	5331 E 7
-20	1848 E 3	6188 E 3	1838 E 3	6215 E 3	6466 E 3	9458 E 3	1035 E 5	2835 E 7
-30	1389 E 3	4451 E 3	1381 E 3	4473 E 3	4674 E 3	6973 E 3	7666 E 4	2104 E 7
-40	1218 E 3	3991 E 3	1212 E 3	4010 E 3	4179 E 3	6153 E 3	6732 E 4	1849 E 7
-50	1222 E 3	4005 E 3	1215 E 3	4023 E 3	4189 E 3	6143 E 3	6722 E 4	1849 E 7
-60	1412 E 3	4371 E 3	1404 E 3	4393 E 3	4594 E 3	6885 E 3	7634 E 4	2102 E 7
-70	1902 E 3	5949 E 3	1892 E 3	5977 E 3	6233 E 3	9245 E 3	1027 E 5	2832 E 7
-80	3548 E 3	1152 E 4	3529 E 3	1157 E 4	1204 E 4	1752 E 4	1929 E 5	5322 E 7
-90	∞	∞	∞	∞	∞	∞	∞	∞

*E 3 means $\times 10^3$, etc.

The radius of curvature $\bar{\rho}$ at the equator increases from sea level ($\omega = 0$) up to 100 km ($\omega = 0$), then drops at 1000 km height ($\omega = 0$) and starts increasing again (refer to Table 28). This behavior is due to the fact that the actual equator of the equipotential spheroids moves at first southward and finally touches the geocentric equator asymptotically with higher elevation (see section IIh)). The values $\bar{\rho}$ at sea level were computed on the spheroidal part of the Earth's surface. Elevation 10 km, for example, means 10 km above sea level along the geocentric radius.

2) Zonal and tesseral (sectorial) part, i.e., radius of curvature of the orthogonal trajectories of the equipotential surfaces V = const

In Number Map 3, the radius of curvature is given as a function of the latitude (first vertical line) and longitude (first horizontal line) at a certain elevation above sea level. As easily recognized, each goecentric latitude and longitude number has to be multiplied by 10. Further, $\pm 90^{\circ}$ in latitude actually means $\pm 89^{\circ}.875$, which had to be taken because the computer program does not work at $\pm 90^{\circ}$, for mathematical reasons, since spherical coordinates were used; a reprogramming hardly seemed worthwhile. The first four digits are significant numbers of the curvature radius, and the last number is a power to the ten, which has to be added to the scaling power to get the radius in kilometers.

<u>Example</u>: The radius of curvature is at sea level for $\omega = 0$ and $\varphi = 20^{\circ}$, $\lambda = 40^{\circ}$

$$\rho = 6\ 229 \times 10^3 \text{ km} \quad .$$

b) <u>Radius of torsion of the orthogonal trajectories</u>

See Number Map 4. Similar to the radius of curvature, we give the radius of torsion for different sections of outer space as a function of the geocentric latitude and longitude. Again the latitude $\pm 90^{\circ}$ actually stands for $\pm 89^{\circ}.875$, as stated previously. The first two digits are significant numbers of the radius of torsion, and the number behind the exponent E has to be added to the scaling power in order to get the radius of torsion in kilometers.

Example: The radius of torsion at 1,000 km elevation for $\omega = 0$ and $\varphi = 20^\circ$, $\lambda = 40^\circ$

$$\tau = - 11 \times 10^4 \text{ km} \quad .$$

The sign was attached to the radius for reasons of simplicity. Actually it means that the torsion is negative and has a radius of 11×10^4 km.

The range of the radius of torsion can go up to infinity. In the computer programs, however, only a relatively small spectrum could be recorded. If we have at a certain latitude, for example, a change in the exponent of magnitude three or larger, then the numerical results of the absolute larger exponent cannot be guaranteed, because of rounding errors.

3. Integration of the differential equation; orthogonal trajectories

The orthogonal trajectories can be obtained pointwise by integrating the differential equation system (53) by a Runge-Kutta procedure. If we apply a curve fitting, we get the trajectory as a function of the arc length s starting from an initial point on the Earth's surface.

a) Gravity field ($\overline{V} + \overline{Z}$); zonal part

We consider for the moment only the zonal part of the geopotential. By using a step length of 2.5 km up to an elevation of 10 km, we represent the orthogonal trajectory as a function of the arc length:

$$\overline{r} = \overline{r}(s) \quad ,$$

$$\varphi = \varphi(s) \quad ; \tag{60}$$

or, applying an interpolation formula,

$$\overline{r} = \overline{r}_0 + \sum_{q=1} \binom{p}{q} \overline{r}_q \quad ,$$

$$\varphi = \varphi_0 + \sum_{q=1} \binom{p}{q} \overline{\varphi}_q \quad , \tag{61}$$

wherein

$\overline{r}_0, \varphi_0$ = initial points of the numerical integration (not exactly coinciding with sea level because of the way the computer programs were set up. See also p. 59 and p. 62).

$p = \frac{s}{\Delta}$, with Δ the step length of the integration process. The values s and Δ must have the same length unit.

Table 29.[12]

Initial points		Coefficients*				
φ_0°	$\bar{r}_0$ (m)	$\bar{r}_1$ (m)	$\bar{r}_2$ (m)	$\bar{\varphi}_1$	$\bar{\varphi}_2$	$\bar{\varphi}_3$
90	--	--	--	--	--	--
80	6 357 432.285 872 9	2 499.998 369 9	-0.7 E-6	0.448 984 3 E-6	-0.734 E-10	0.5 E-12
70	6 359 277.419 177 4	2 499.994 219 4	-0.26 E-5	0.845 236 1 E-6	-0.141 8 E-9	0.9 E-12
60	6 362 110.511 335 3	2 499.989 474 4	-0.47 E-5	0.114 004 26 E-5	-0.192 8 E-9	0.12 E-11
50	6 365 590.836 372 7	2 499.986 371 0	-0.62 E-5	0.129 655 931 E-5	-0.213 98 E-9	0.136 E-11
40	6 369 299.878 656 8	2 499.986 336 8	-0.62 E-5	0.129 742 848 E-5	-0.212 49 E-9	0.137 E-11
30	6 372 791.523 500 6	2 499.989 410 6	-0.49 E-5	0.114 157 545 E-5	-0.183 59 E-9	0.120 E-11
20	6 375 642.854 393 7	2 499.994 142 4	-0.27 E-5	0.848 664 87 E-6	-0.136 09 E-9	0.89 E-12
10	6 377 511.144 032 6	2 499.998 323 3	-0.7 E-6	0.453 912 94 E-6	-0.794 4 E-10	0.51 E-12
0	6 378 164.510 543 6	2 500	0	0.112 732 E-8	-0.45 E-12	0
-10	6 377 518.386 571 7	2 499.998 345 0	-0.8 E-6	-0.450 967 25 E-6	0.731 6 E-10	-0.49 E-12
-20	6 375 657.063 778 4	2 499.994 160 5	-0.27 E-5	-0.847 348 73 E-6	0.133 12 E-9	-0.88 E-12
-30	6 372 804.700 009 8	2 499.989 379 6	-0.48 E-5	-0.114 324 701 E-5	0.189 60 E-9	-0.123 E-11
-40	6 369 307.994 930 5	2 499.986 303 9	-0.63 E-5	-0.129 899 059 E-5	0.212 97 E-9	-0.136 E-11
-50	6 365 593.939 136 6	2 499.986 317 6	-0.62 E-5	-0.129 909 398 E-5	0.215 43 E-9	-0.135 E-11
-60	6 362 102.623 656 6	2 499.989 379 4	-0.46 E-5	-0.114 517 66 E-5	0.203 3 E-9	-0.12 E-11
-70	6 359 254.691 583 3	2 499.994 151 1	-0.25 E-5	-0.850 214 4 E-6	0.151 0 E-9	-0.9 E-12
-80	6 357 398.595 474 8	2 499.998 349 8	-0.7 E-6	-0.451 744 1 E-6	0.778 E-10	-0.5 E-12
-90	--	--	--	--	--	--

* $\bar{\varphi}_1$, $\bar{\varphi}_2$, $\bar{\varphi}_3$ are in radians.

The coefficients $\bar{r}_q$ and $\bar{\varphi}_q$ of Table 29 are valid from sea level (spheroidal part) up to 10 km elevation. Within this range the trajectory can be presented with a numerical accuracy of about 13-14 digits. A short analysis shows that the orthogonal trajectory is curved toward the poles within the above range L = 10 km. The opposite result is obtained if $\omega = 0$ (see page 62).

b) Gravity field (V + Z); zonal and tesseral part

See Number Map 5 pages 99-104. Including the tesseral (sectorial) harmonics

[12] The steplength Δ is 2.5 km. For a numerical example see next section.

our integration process, the orthogonal trajectory has the equations

$$r = r(s) \quad ,$$

$$\varphi = \varphi(s) \quad ,$$

$$\lambda = \lambda(s) \quad .$$

We fit the point array obtained by the Runge-Kutta procedure again with an interpolation formula:

$$r = r_0 + \sum_{q=1} \binom{p}{q} r_q \quad ,$$

$$\varphi = \varphi_0 + \sum_{q=1} \binom{p}{q} \varphi_q \quad ,$$

$$\lambda = \lambda_0 + \sum_{q=1} \binom{p}{q} \lambda_q \quad ,$$

wherein

r_0, φ_0, λ_0 are initial points of the numerical integration with p defined as in equation (61).

Hence with the tables on page 99-104 the trajectory is fully determined in its whole run from sea level up to 10 km elevation with an accuracy of 13 significant digits.

<u>Explanation of the number maps</u>: The trajectory was computed 10 x 10° in φ and λ all over the Earth's surface. At the North and South Poles, 89°.875 in φ was taken instead of 90°; the step length of the numerical integration was $\Delta = 2.5$ km.

$\underline{r_q \text{ tables}}$: The first column and the first row are $\varphi/10$ and $\lambda/10$ of the trajectory under consideration. The set of two or three numbers to be found expresses r_0, r_1, r_2, whereby in front of r_0 (larger number) the digits 63 had to be omitted because of space problems in the printout. If we like meters as length units in r, then r_0, r_1, r_2 have to be multiplied by 10^{-6}.

$\underline{\varphi_q, \lambda_q \text{ tables}}$: The three differences φ_1, φ_2, φ_3 and λ_1, λ_2, λ_3, respectively, have to be multiplied by 10^{-13} to get radians.

<u>Example</u>: The equation of the plumb line at $\varphi_0 = 20°$, $\lambda_0 = 40°$ starting from sea level ($V = \text{const}$, $\omega \neq 0$):[13]

$$r = r_0 + p\, r_1 + \frac{p(p-1)}{2} r_2 = 6{,}375{,}657.361\,403$$

$$+ 2{,}499.994\,183\, p - 0.000\,003 \frac{p(p-1)}{2} \quad ,$$

$$\varphi = \varphi_0 + p\, \varphi_1 + \frac{p(p-1)}{2} \varphi_2 + \frac{p(p-1)(p-2)}{6} \varphi_3$$

$$= 20° + \rho° \Big(0.000\,000\,845\,730\,2\, p$$

$$- 0.000\,000\,000\,130\,8 \frac{p(p-1)}{2} \quad ,$$

$$+ 0.000\,000\,000\,000\,9 \frac{p(p-1)(p-2)}{6} \Big)$$

$$\lambda = \lambda_0 + p\, \lambda_1 + \frac{p(p-1)}{2} \lambda_2$$

$$= 40° + \rho° \Big(0.000\,000\,003\,274\,4\, p$$

$$- 0.000\,000\,000\,000\,4 \frac{p(p-1)}{2} \Big)$$

---

[13] $\rho°$ is the conversion factor from radians to degrees.

At 7 km above sea level the spherical coordinates r, φ, λ of the trajectory assume the following values:
With

$$p = \frac{7}{2.5} = 2.8 \quad ,$$

$$\frac{p(p-1)}{2} = 2.52 \quad ,$$

$$\frac{p(p-1)(p-2)}{6} = 0.672 \quad ,$$

and

$$\rho^\circ = 57.295\ 779\ 51 \quad ,$$

we get

$$\begin{aligned} r_{7\ \mathrm{km}} = &\ 6{,}375{,}657.361\ 403 \\ &+ \quad 6{,}999.983\ 712 \\ &- \qquad\quad .000\ 008 \\ \hline &\ 6{,}382{,}657.345\ 107\ \mathrm{m} \end{aligned} \quad ,$$

$$\begin{aligned} \varphi_{7\ \mathrm{km}} &= 20^\circ + \rho^\circ\ (0.000\ 002\ 368\ 044\ 6 \\ &\qquad\qquad - .000\ 000\ 000\ 329\ 6 \\ &\qquad\qquad + .000\ 000\ 000\ 000\ 6) \\ &= 20^\circ\!.000\ 135\ 660\ 111\ 0 \\ &= 20^\circ 00' 00''\!.488\ 376\ 40 \quad , \end{aligned}$$

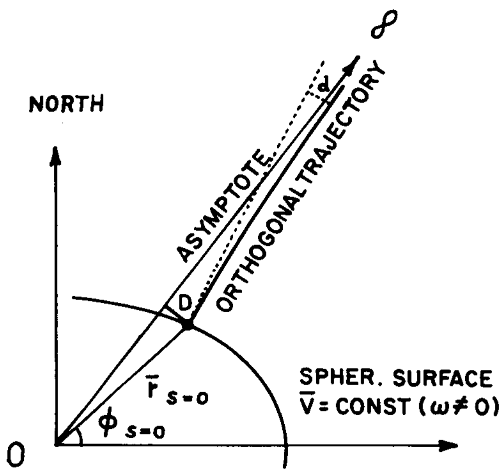
$$\begin{aligned} \lambda_{7\ \mathrm{km}} &= 40^\circ + \rho^\circ\ (0.000\ 000\ 009\ 168\ 3 \\ &\qquad\qquad - .000\ 000\ 000\ 001\ 0) \\ &= 40^\circ\!.000\ 000\ 525\ 247\ 6 \\ &= 40^\circ 00' 00''\!.001\ 890\ 9 \quad . \end{aligned}$$

c) Gravitational field ($\overline{V}$, $\overline{Z}$ = 0); zonal part

Quite similarly the zonal part of the gravitational field is treated. Starting from the Earth's spheroidal surface, the differential equations (53) are integrated up to 100,000 km elevation; the geocentric radius $\overline{r}$, the geocentric latitude φ, and the distance d of the trajectory from the tangent of the starting point are given pointwise as functions of the curve length (see Tables 31, 32, and 33). Each trajectory approaches asymptotically a certain radius vector. The distance D of the starting point from this line is shown in Table 30.

Table 30.

φ°	D (km)
90	0
80	1.76
70	3.31
60	4.47
50	5.08
40	5.08
30	4.47
20	3.33
10	1.78
0	0.00755
-10	1.76
-20	3.32
-30	4.48
-40	5.09
-50	5.10
-60	4.49
-70	3.34
-80	1.78
-90	0



Note that the orthogonal trajectory refers to $\overline{V}$ = const (ω = 0)

Figure 20.

Compared to a straight line (Figure 20), the orthogonal trajectories vary most within the range from the Earth's surface up to about 10,000 km elevation, as can be seen easily from Table 32 on page 66.

Example: At $\varphi = 40^\circ$ the geocentric latitude changes from sea level up to 10,000 km by $2'20''.2$, while for the nine-times-longer range from 10,000 km up to 100,000 km, the change is only $24''.4$, and so forth.

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Table 31.--The geocentric radius $\overline{r}$ of the trajectories, given in meters (see page 62).

s km $\times 10^3$	$\varphi = 90°$	$\varphi = 80°$	$\varphi = 70°$	$\varphi = 60°$	$\varphi = 50°$	$\varphi = 40°$	$\varphi = 30°$
0		6 357 432.285 9	6 359 277.419 2	6 362 110.511 3	6 365 590.836 4	6 369 299.878 7	6 372 791.523 5
1		7 357 432.170 2	7 359 277.009 1	7 362 109.765 6	7 365 589.872 5	7 369 298.914 2	7 372 790.777 0
4		10 357 432.034 7	10 359 276.529 8	10 362 108.894 8	10 365 588.746 0	10 369 297.786 9	10 372 789.903 8
10		16 357 431.978 0	16 359 276.329 5	16 362 108.531 0	16 365 588.275 2	16 369 297.315 6	16 372 789.538 8
20		26 357 431.963 3	26 359 276.277 5	26 362 108.436 6	26 365 588.153 1	26 369 297.193 3	26 372 789.444 1
30	$\Delta h = \Delta r$	36 357 431.960 4	36 359 276.267 4	36 362 108.418 2	36 365 588.129 3	36 369 297.169 5	36 372 789.425 7
40		46 357 431.959 5	46 359 276.264 2	46 362 108.412 4	46 365 588.121 7	46 369 297.161 9	46 372 789.419 8
50		56 357 431.959 1	56 359 276.262 9	56 362 108.410 0	56 365 588.118 6	56 369 297.158 8	56 372 789.417 4
60		66 357 431.958 9	66 359 276.262 2	66 362 108.408 8	66 365 588.117 1	66 369 297.157 3	66 372 789.416 2
70		76 357 431.958 8	76 359 276.261 9	76 362 108.408 1	76 365 588.116 2	76 369 297.156 4	76 372 789.415 6
80		86 357 431.958 8	86 359 276.261 7	86 362 108.407 8	86 365 588.115 7	86 369 297.156 0	86 372 789.415 2
90		96 357 431.958 8	96 359 276.261 5	96 362 108.407 5	96 365 588.115 4	96 369 297.155 6	96 372 789.415 0
100		106 357 431.958 7	106 359 276.261 4	106 362 108.407 4	106 365 588.115 2	106 369 297.155 4	106 372 789.414 8

s km $\times 10^3$	$\varphi = 20°$	$\varphi = 10°$	$\varphi = 0°$	$\varphi = -10°$	$\varphi = -20°$	$\varphi = -30°$	$\varphi = -40°$
0	6 375 642.854 4	6 377 511.144 0	6 378 164.510 5436	6 377 518.386 6	6 375 657.063 8	6 372 804.700 0	6 369 307.994 9
1	7 375 642.441 1	7 377 511.025 3	7 378 164.510 5401	7 377 518.270 6	7 375 656.652 8	7 372 803.950 3	7 369 307.025 3
4	10 375 641.958 4	10 377 510.887 5	10 378 164.510 537	10 377 518.135 2	10 375 656.172 1	10 372 803.075 4	10 369 305.892 1
10	16 375 641.756 9	16 377 510.830 2	16 378 164.510 536	16 377 518.078 5	16 375 655.971 2	16 372 802.710 0	16 369 305.419 0
20	26 375 641.704 7	26 377 510.815 4	26 378 164.510 536	26 377 518.063 8	26 375 655.919 0	26 372 802.615 2	26 369 305.296 4
30	36 375 641.694 5	36 377 510.812 5	36 378 164.510 536	36 377 518.060 9	36 375 655.908 9	36 372 802.596 8	36 369 305.272 6
40	46 375 641.691 3	46 377 510.811 5	46 378 164.510 536	46 377 518.060 0	46 375 655.905 6	45 372 802.590 9	46 369 305.265 0
50	56 375 641 689 9	56 377 510.811 2	56 378 164.510 536	56 377 518.059 6	56 375 655.904 3	55 372 802.588 5	56 369 305.261 8
60	66 375 641.689 3	66 377 510.811 0	66 378 164.510 536	66 377 518.059 4	66 375 655.903 6	65 372 802.587 3	66 369 305.260 3
70	76 375 641.688 9	76 377 510.810 9	76 378 164.510 535	76 377 518.059 3	76 375 655.903 3	75 372 802.586 6	76 369 305.259 5
80	86 375 641.688 7	86 377 510.810 8	86 378 164.510 535	86 377 518.059 3	86 375 655.903 1	85 372 802.586 3	86 369 305.259 0
90	96 375 641.688 6	96 377 510.810 8	96 378 164.510 535	96 377 518.059 2	96 375 655.902 9	95 372 802.586 0	96 369 305.258 7
100	106 375 641.688 5	106 377 510.810 8	106 378 164.510 535	106 377 518.059 2	106 375 655.902 9	105 372 802.585 9	106 369 305.258 5

s km $\times 10^3$	$\varphi = -50°$	$\varphi = -60°$	$\varphi = -70°$	$\varphi = -80°$	$\varphi = -90°$
0	6 365 593.939 1	6 362 102.623 7	6 359 254.691 6	6 357 398.595 5	
1	7 365 592.967 0	7 362 101.865 6	7 359 254.272 6	7 357 398.477 1	
4	10 365 591.831 3	10 362 100.984 3	10 359 253.785 9	10 357 398.339 2	
10	16 365 591.357 6	16 362 100.617 6	16 359 253.583 7	16 357 398.281 9	
20	26 365 591.235 0	26 362 100.522 8	26 359 253.531 4	26 357 398.267 1	
30	36 365 591.211 2	36 362 100.504 3	36 359 253.521 3	36 357 398.264 2	$\Delta h = \Delta r$
40	46 365 591 203 6	46 362 100.498 5	46 359 253.518 0	46 357 398.263 3	
50	56 365 591.200 4	56 362 100.496 0	56 359 253.516 7	56 357 398.263 0	
60	66 365 591.198 9	66 362 100.494 9	66 359 253.516 0	66 357 398.262 8	
70	76 365 591.198 1	76 362 100.494 2	76 359 253.515 7	76 357 398.262 7	
80	86 365 591.197 6	86 362 100.493 8	86 359 253.515 5	86 357 398.262 6	
90	96 365 591.197 3	96 362 100.493 6	96 359 253.515 3	96 357 398.262 6	
100	106 365 591.197 1	106 362 100.493 5	106 359 253.515 3	106 357 398.262 5	

Table 32.--The geocentric latitude φ of the trajectories, given in radians.

s km × 10^3	$\varphi = 90°$	$\varphi = 80°$	$\varphi = 70°$	$\varphi = 60°$	$\varphi = 50°$	$\varphi = 40°$	$\varphi = 30°$
0		1.396 263 40	1.221 730 48	1.047 197 55	0.872 664 63	0.698 131 70	0.523 598 78
1		1.396 333 66	1.221 862 73	1.047 375 84	0.872 867 22	0.698 334 24	0.523 776 88
4		1.396 436 49	1.222 056 05	1.047 636 31	0.873 163 35	0.698 630 36	0.524 037 38
10		1.396 499 33	1.222 174 16	1.047 795 44	0.873 344 33	0.698 811 38	0.524 196 64
20		1.396 525 21	1.222 222 81	1.047 861 00	0.873 418 90	0.698 885 97	0.524 262 26
30	$\frac{\pi}{2}$	1.396 532 91	1.222 237 27	1.047 880 50	0.873 441 08	0.698 908 16	0.524 281 79
40		1.396 536 19	1.222 243 44	1.047 888 81	0.873 450 53	0.698 917 62	0.524 290 11
50		1.396 537 89	1.222 246 63	1.047 893 11	0.873 455 42	0.698 922 52	0.524 294 42
60		1.396 538 88	1.222 248 49	1.047 895 62	0.873 458 28	0.698 925 37	0.524 296 93
70		1.396 539 50	1.222 249 67	1.047 897 21	0.873 460 08	0.698 927 18	0.524 298 52
80		1.396 539 92	1.222 250 46	1.047 898 27	0.873 461 30	0.698 928 39	0.524 299 59
90		1.396 540 22	1.222 251 02	1.047 899 03	0.873 462 16	0.698 929 25	0.524 300 34
100		1.396 540 44	1.222 251 43	1.047 899 58	0.873 462 78	0.698 929 88	0.524 300 89

s km × 10^3	$\varphi = 20°$	$\varphi = 10°$	$\varphi = 0°$	$\varphi = -10°$	$\varphi = -20°$	$\varphi = -30°$	$\varphi = -40°$
0	0.349 065 85	0.174 532 925	0.0	-0.174 532 925	-0.349 065 85	-0.523 598 78	-0.698 131 70
1	0.349 198 31	0.174 603 913	0.3841 E-6	-0.174 603 089	-0.349 197 94	-0.523 777 26	-0.698 334 78
4	0.349 391 93	0.174 707 315	0.8917 E-6	-0.174 705 609	-0.349 391 16	-0.524 038 01	-0.698 631 67
10	0.349 510 24	0.174 770 400	1.1089 E-6	-0.174 768 330	-0.349 509 30	-0.524 197 36	-0.698 813 03
20	0.349 558 98	0.174 796 373	1.1657 E-6	-0.174 794 207	-0.349 558 00	-0.524 263 01	-0.698 887 72
30	0.349 573 48	0.174 804 094	1.1768 E-6	-0.174 801 910	-0.349 572 49	-0.524 282 54	-0.698 909 92
40	0.349 579 66	0.174 807 387	1.1804 E-6	-0.174 805 197	-0.349 578 67	-0.524 290 86	-0.698 919 39
50	0.349 582 86	0.174 809 089	1.1819 E-6	-0.174 806 897	-0.349 581 86	-0.524 295 17	-0.698 924 29
60	0.349 584 72	0.174 810 082	1.1826 E-6	-0.174 807 888	-0.349 583 73	-0.524 297 68	-0.698 927 14
70	0.349 585 90	0.174 810 710	1.1830 E-6	-0.174 808 516	-0.349 584 91	-0.524 299 27	-0.698 928 95
80	0.349 586 70	0.174 811 134	1.1832 E-6	-0.174 808 939	-0.349 585 70	-0.524 300 34	-0.698 930 17
90	0.349 587 26	0.174 811 432	1.1834 E-6	-0.174 809 237	-0.349 586 26	-0.524 301 10	-0.698 931 03
100	0.349 587 67	0.174 811 650	1.1835 E-6	-0.174 809 455	-0.349 586 67	-0.524 301 65	-0.698 931 65

s km × 10^3	$\varphi = -50°$	$\varphi = -60°$	$\varphi = -70°$	$\varphi = -80°$	$\varphi = -90°$
0	-0.872 664 63	-1.047 197 55	-1.221 730 48	-1.396 263 40	
1	-0.872 868 08	-1.047 377 30	-1.221 864 16	-1.396 334 48	
4	-0.873 165 42	-1.047 639 32	-1.222 058 95	-1.396 438 19	
10	-0.873 346 94	-1.047 799 08	-1.222 177 61	-1.396 501 35	
20	-0.873 421 65	-1.047 864 79	-1.222 226 39	-1.396 527 31	$\frac{\pi}{2}$
30	-0.873 443 86	-1.047 884 32	-1.222 240 89	-1.396 535 02	
40	-0.873 453 33	-1.047 892 64	-1.222 247 06	-1.396 538 31	
50	-0.873 458 22	-1.047 896 95	-1.222 250 26	-1.396 540 01	
60	-0.873 461 07	-1.047 899 45	-1.222 252 12	-1.396 541 00	
70	-0.873 462 88	-1.047 901 04	-1.222 253 30	-1.396 541 63	
80	-0.873 464 10	-1.047 902 11	-1.222 254 09	-1.396 542 05	
90	-0.873 464 96	-1.047 902 87	-1.222 254 65	-1.396 542 35	
100	-0.873 465 58	-1.047 903 42	-1.222 255 06	-1.396 542 57	

Table 33.--The distance d of the trajectory from the tangent in the initial point, given in km.

s km $\times 10^3$	φ = 90°	φ = 80°	φ = 70°	φ = 60°	φ = 50°	φ = 40°	φ = 30°	φ = 20°
0		0.000 0	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0
1		0.036 8	0.070 4	0.095 8	0.108 3	0.108 5	0.095 1	0.070 9
4		0.422 2	0.802 4	1.087 2	1.232 4	1.233 7	1.083 5	0.808 3
10		1.678 1	3.179 4	4.301 3	4.881 1	4.885 1	4.294 0	3.201 9
20		4.173 8	7.897 8	10.678 3	12.122 8	12.132 3	10.667 3	7.951 3
30	0	6.813 1	12.886 2	17.418 9	19.778 3	19.793 6	17.405 1	12.971 7
40		9.503 2	17.969 8	24.288 1	27.580	27.601	24.272	18.088
50		12.216 9	23.098 1	31.217 3	35.450	35.477	31.198	23.248
60		14.943 6	28.250 8	38.179	43.358	43.391	38.158	28.434
70		17.678 3	33.418	45.162	51.288	51.327	45.138	33.634
80		20.418 0	38.596	52.157	59.233	59.278	52.131	38.843
90		23.161 4	43.779	59.161	67.189	67.239	59.132	44.060
100		25.907 2	48.968	66.172	75.151	75.208	66.141	49.281

s km $\times 10^3$	φ = 10°	φ = 0°	φ = -10°	φ = -20°	φ = -30°	φ = -40°	φ = -50°	φ = -60°
0	0.000 0	0.000 000	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0
1	0.039 6	0.000 032	0.038 1	0.070 3	0.096 6	0.108 5	0.108 4	0.098 2
4	0.443 4	0.002 21	0.431 1	0.802 9	1.094 0	1.236 0	1.236 7	1.108 3
10	1.743 5	0.010 51	1.702 4	3.183 8	4.324 9	4.897 8	4.902 8	4.372 5
20	4.316 4	0.026 59	4.223 6	7.910 4	10.733 0	12.165 8	12.179 2	10.839 6
30	7.033 8	0.043 19	6.888 4	12.907 4	17.505 8	19.848 7	19.870 7	17.671 9
40	9.802 1	0.059 92	9.603 9	18.000	24.407	27.678	27.709	24.633 0
50	12.594 3	0.076 71	12.343 1	23.137	31.369	35.576	35.615	31.654 4
60	15.399 5	0.093 51	15.095 4	28.299	38.364	43.511	43.559	38.709
70	18.212 7	0.110 31	17.855 6	33.476	45.379	51.470	51.526	45.783
80	21.031 0	0.127 13	20.620 9	38.662	52.407	59.443	59.508	52.870
90	23.852 9	0.143 95	23.389 8	43.855	59.444	67.426	67.499	59.967
100	26.677 4	0.160 78	26.161 3	49.053	66.487	75.417	75.499	67.070

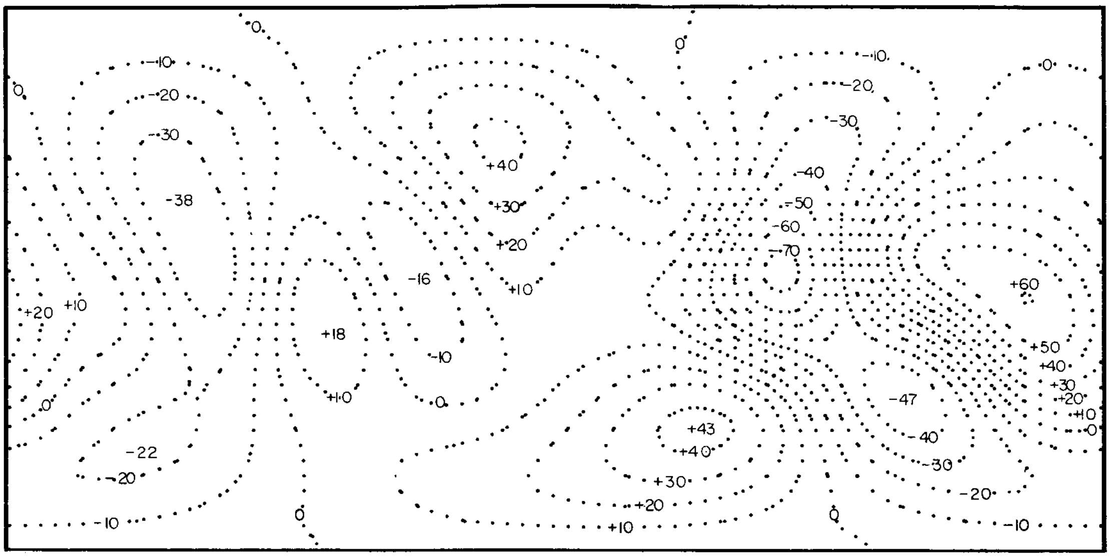
s km $\times 10^3$	φ = -70°	φ = -80°	φ = -90°
0	0.000 0	0.000 0	
1	0.072 6	0.037 8	
4	0.823 0	0.432 6	
10	3.249 5	1.715 1	
20	8.056 5	4.258 7	
30	13.134 6	6.946 6	
40	18.308 3	9.685 3	0
50	23.526 7	12.447 9	
60	28.769 6	15.223 4	
70	34.027	18.006 9	
80	39.295	20.795 5	
90	44.569	23.587 7	
100	49.848	26.382 4	

III. APPENDICES

APPENDIX A. Contour Maps

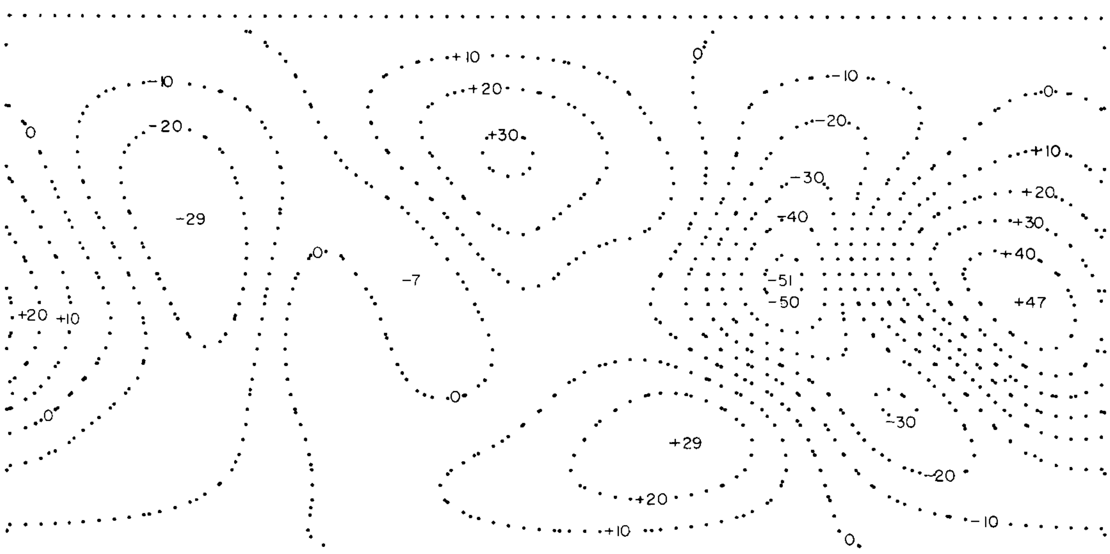
Contour Map 1.--Shape of the equipotential surfaces.

Δr at sea level ($\omega \neq 0$, $\omega = 0$)



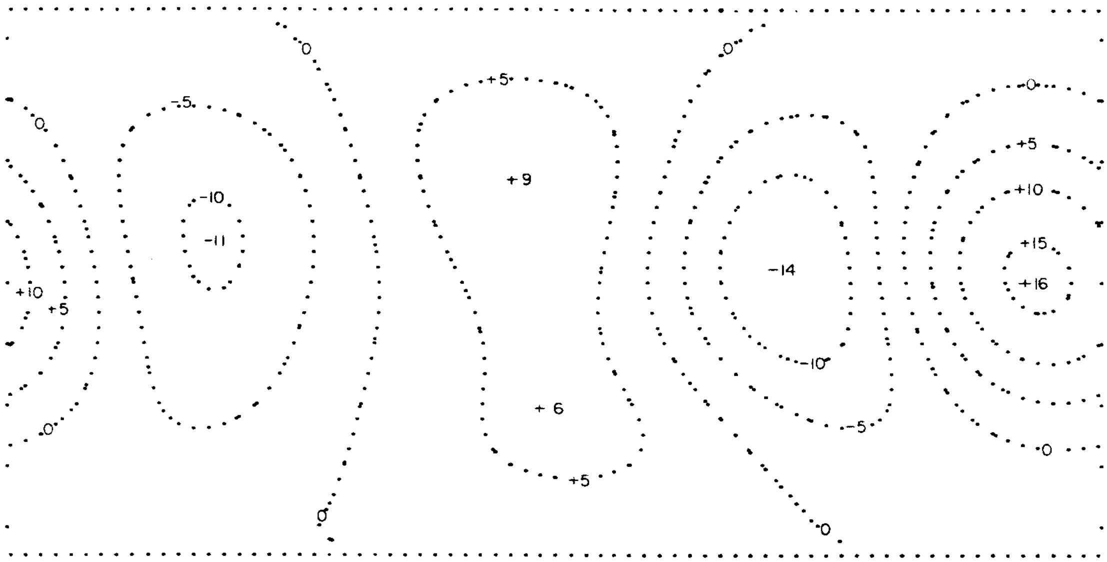
Δr in meters

Δr at 1,000 km elevation ($\omega = 0$)



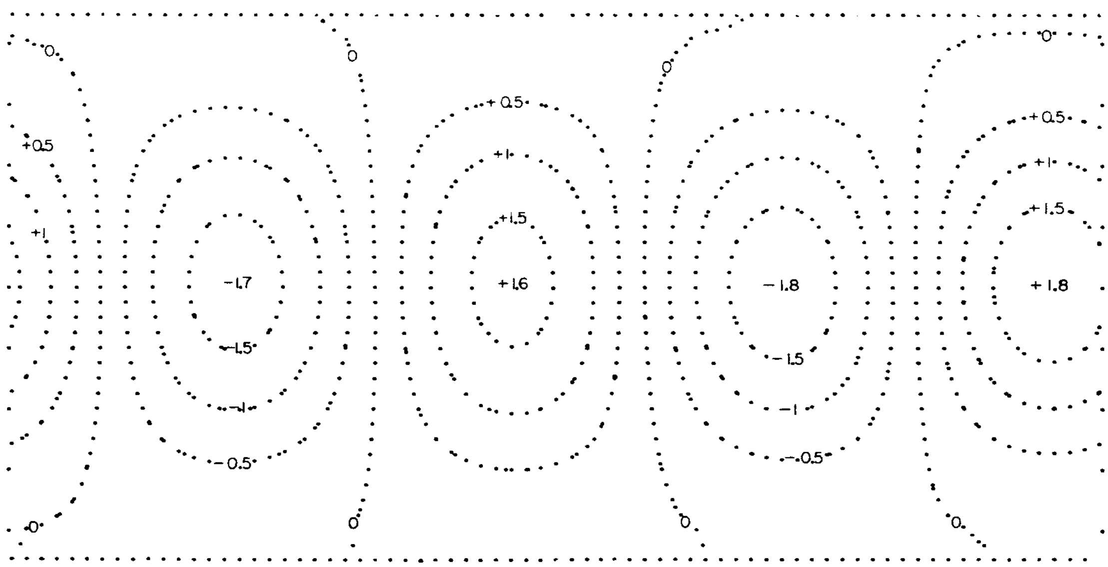
Δr in meters

Δr at 10,000 km elevation ($\omega = 0$)



Δr in meters

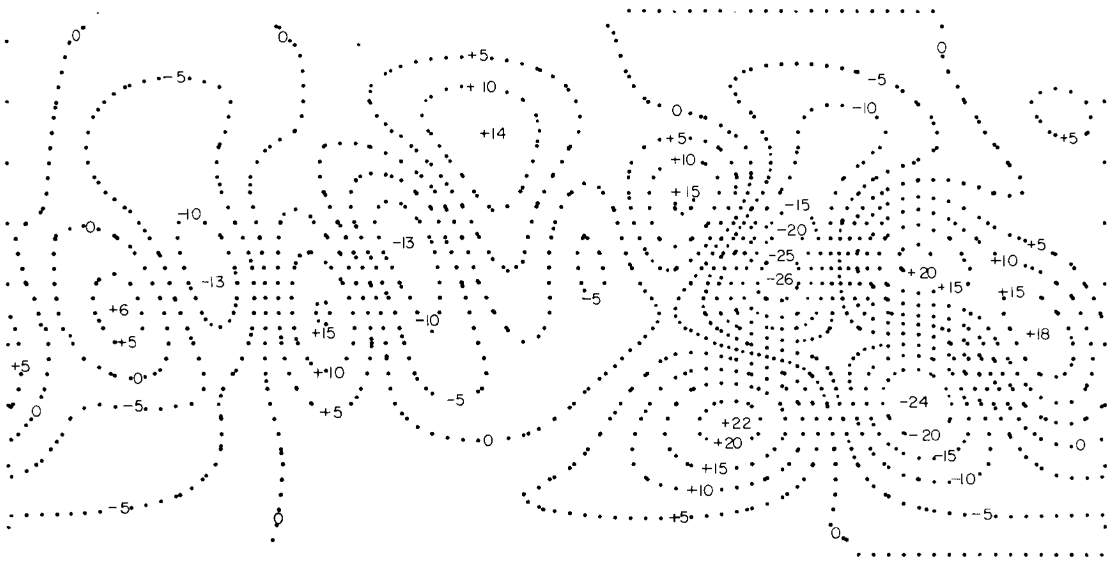
Δr at 100,000 km elevation ($\omega = 0$)



Δr in meters

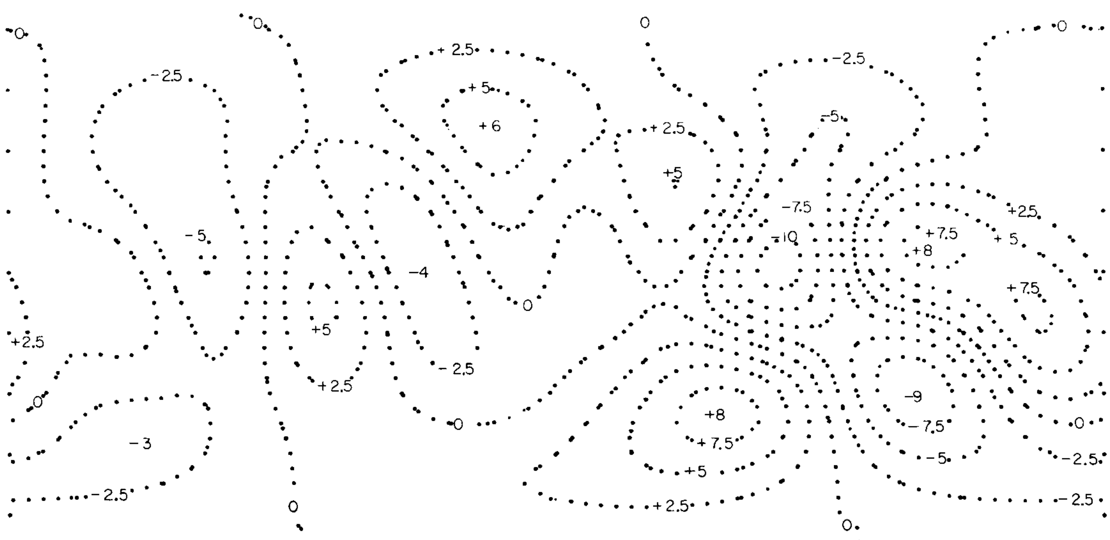
Contour Map 2.--Gravitation (gravity) along an equipotential surface.

Δg at sea level ($\omega \neq 0$, $\omega = 0$)



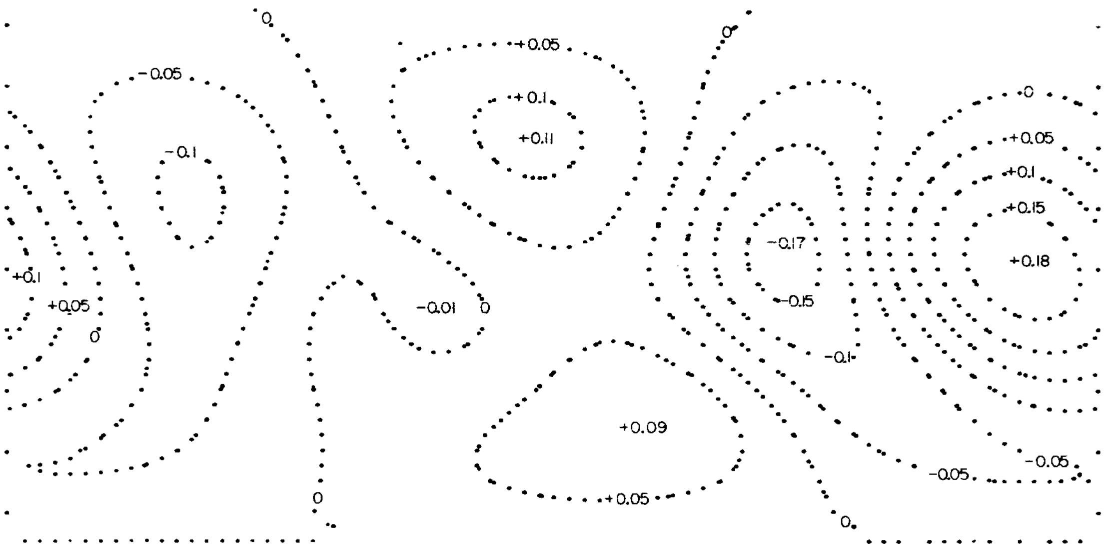
Δg in 10^{-3} cm/sec^2

Δg at 1,000 km elevation ($\omega = 0$)



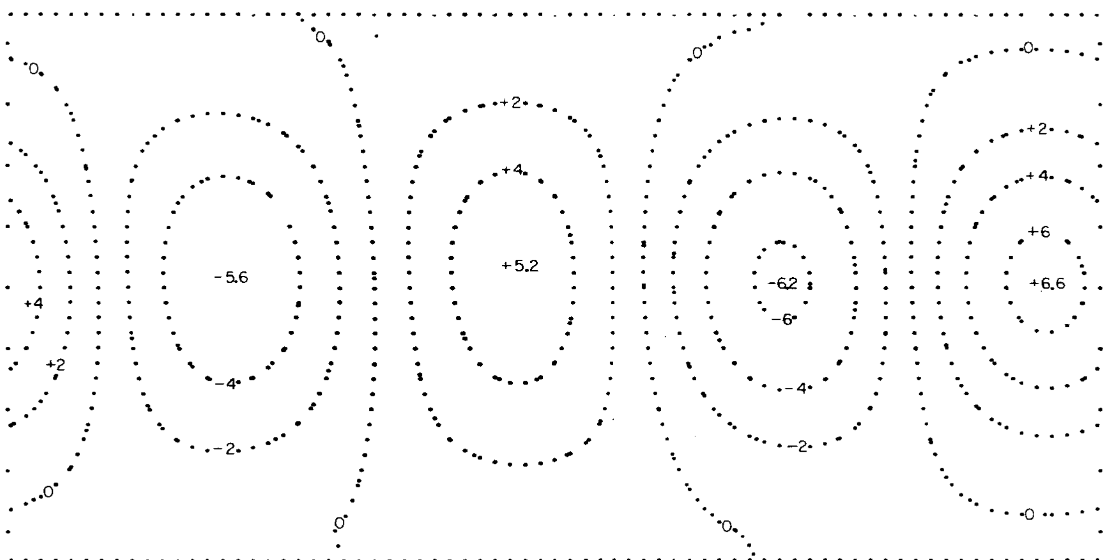
Δg in 10^{-3} cm/sec^2

Δg at 10,000 km elevation ($\omega = 0$)



Δg in 10^{-3} cm/sec^2

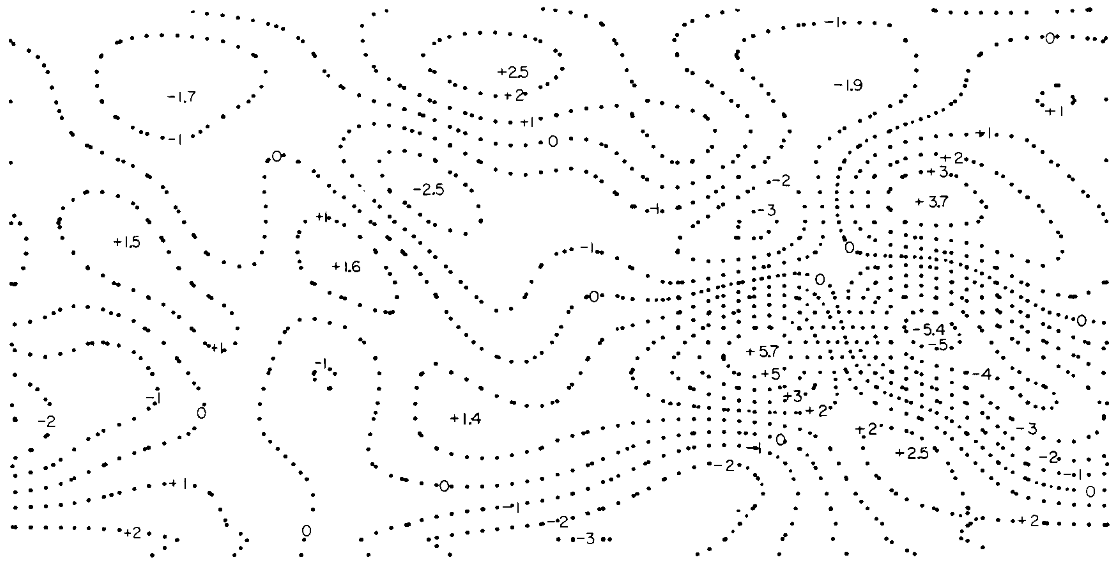
Δg at 100,000 km elevation ($\omega = 0$)



Δg in 10^{-8} cm/sec^2

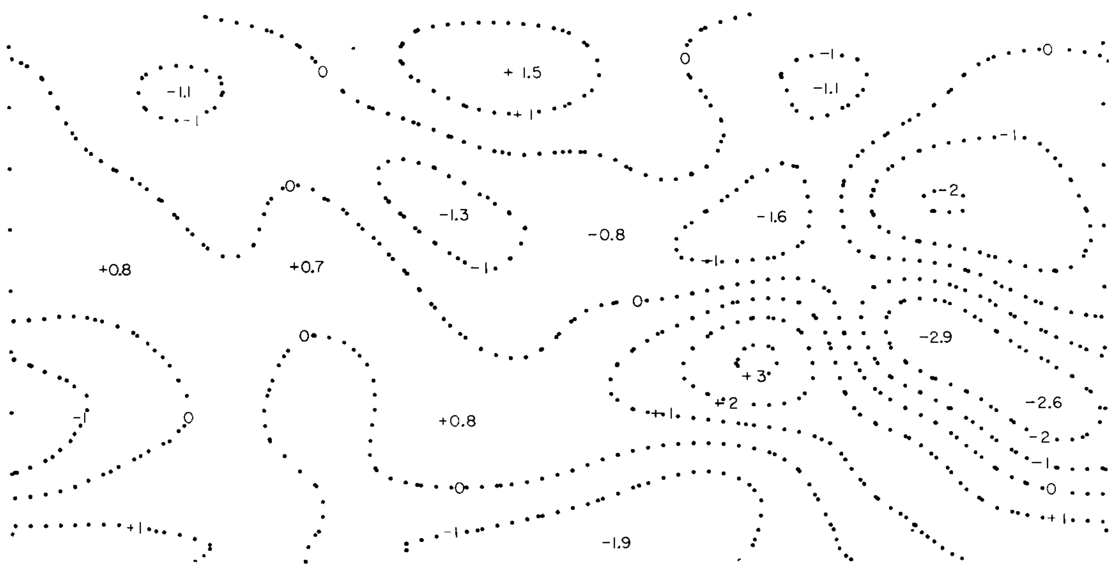
Contour Map 3.--Oscillation of the surface normals; latitude curves (equipotential surfaces).

ξ at sea level ($\omega \neq 0$, $\omega = 0$)



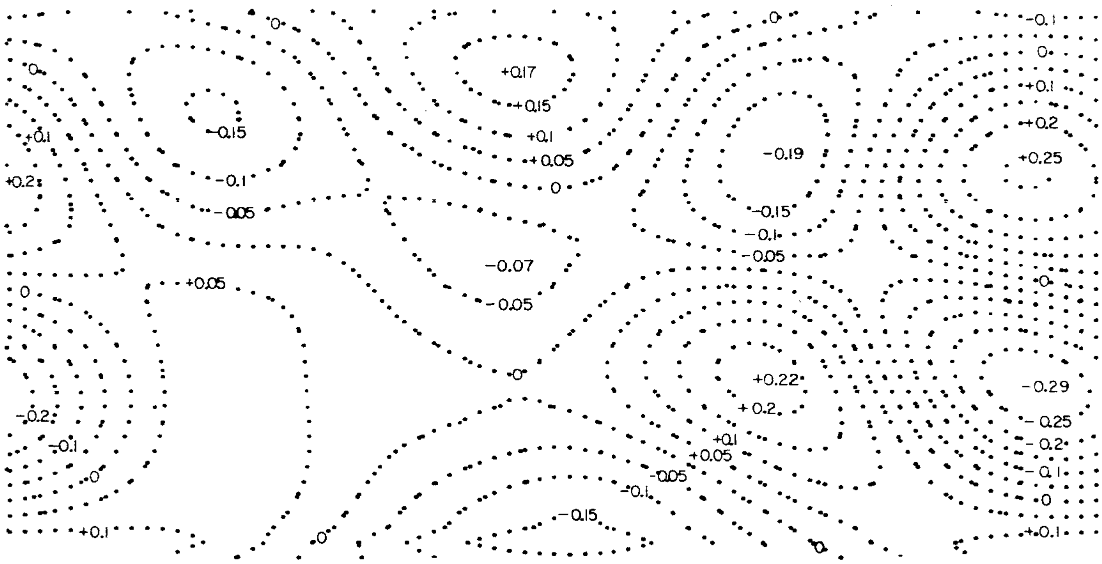
ξ in sec of arc

ξ at 1,000 km elevation ($\omega = 0$)



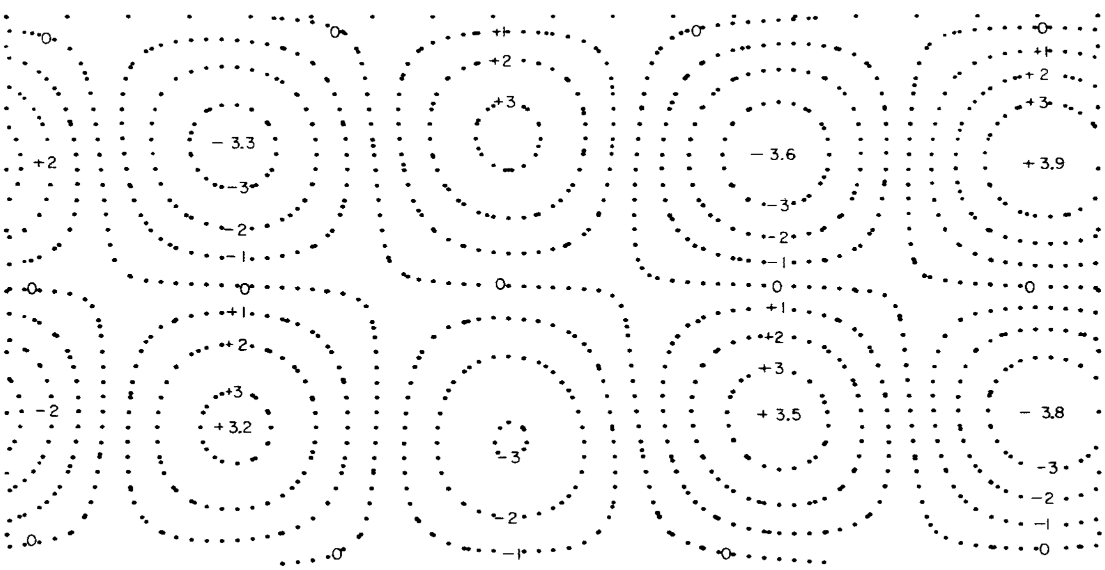
ξ in sec of arc

ξ at 10,000 km elevation (ω = 0)



ξ in sec of arc

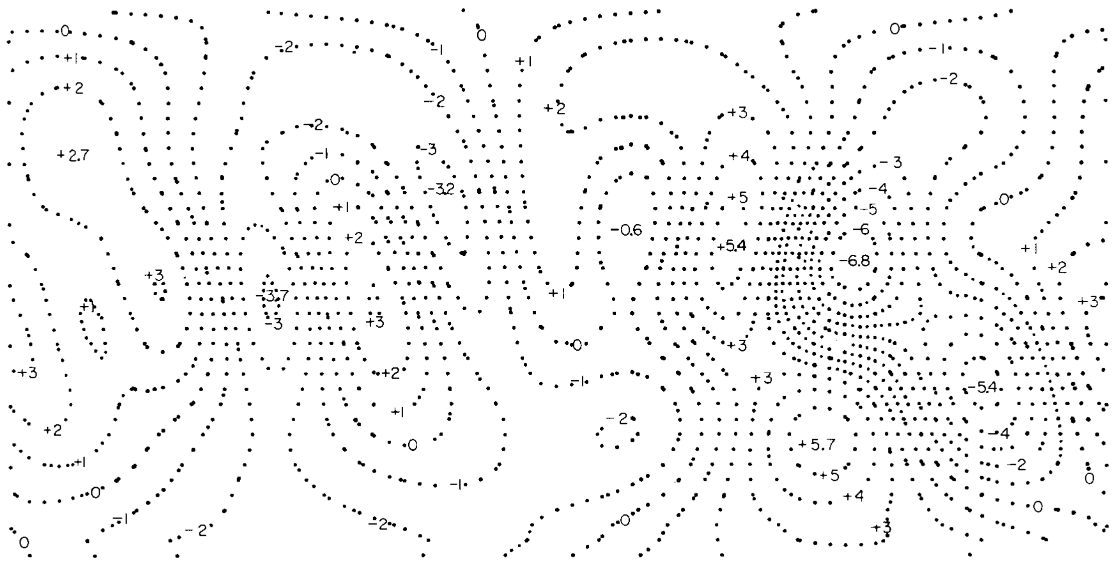
ξ at 100,000 km elevation (ω = 0)



ξ in 10^{-3} sec of arc

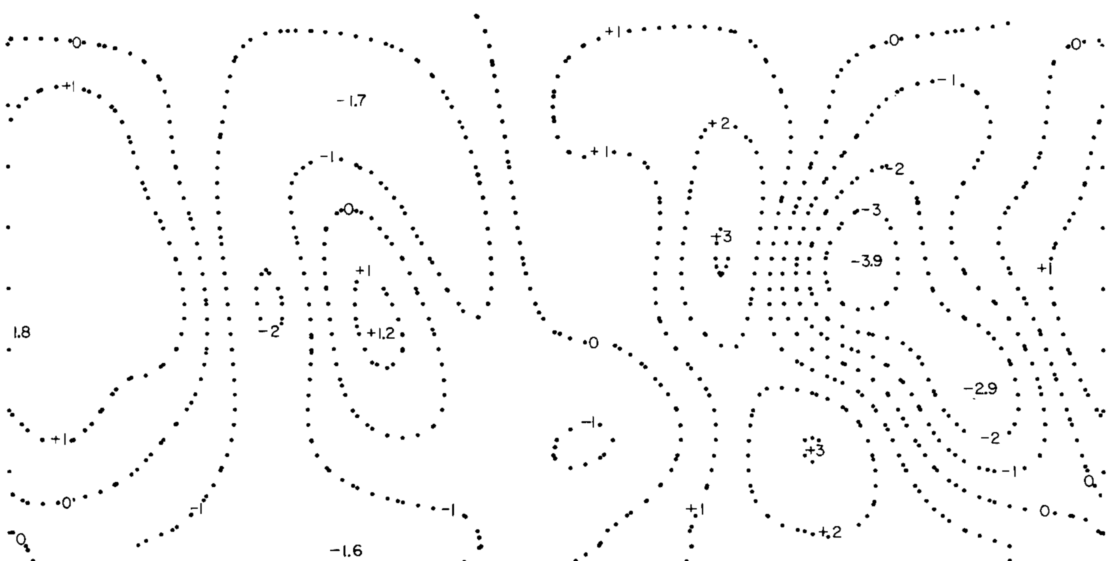
Contour Map 4.--Oscillation of the surface normals; longitude curves (equipotential surfaces).

η at sea level ($\omega \neq 0$, $\omega = 0$)



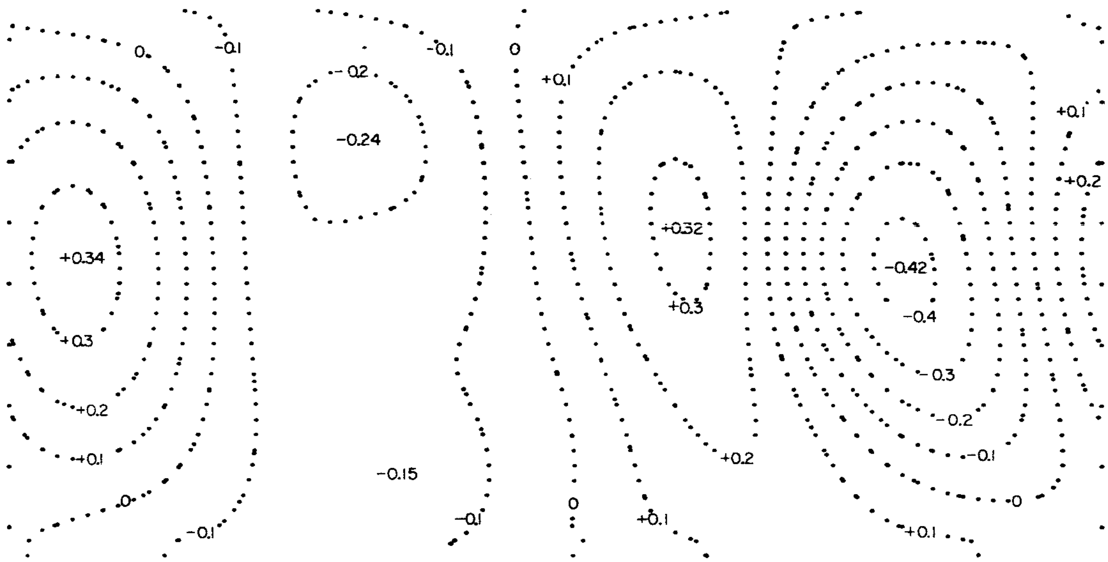
η in sec of arc

η at 1,000 km elevation ($\omega = 0$)



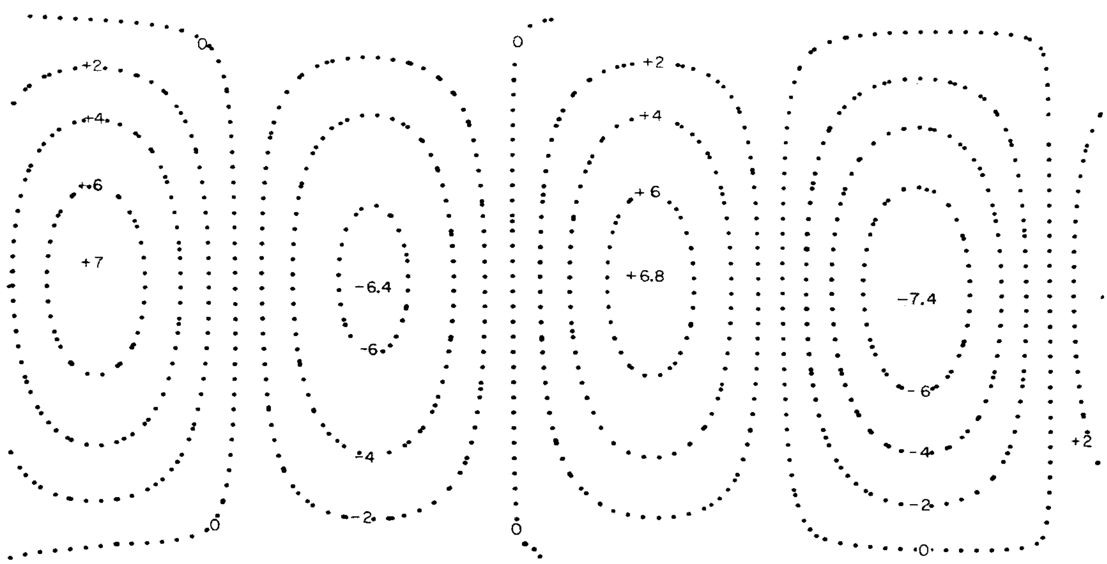
η in sec of arc

η at 10,000 km elevation (ω = 0)



η in sec of arc

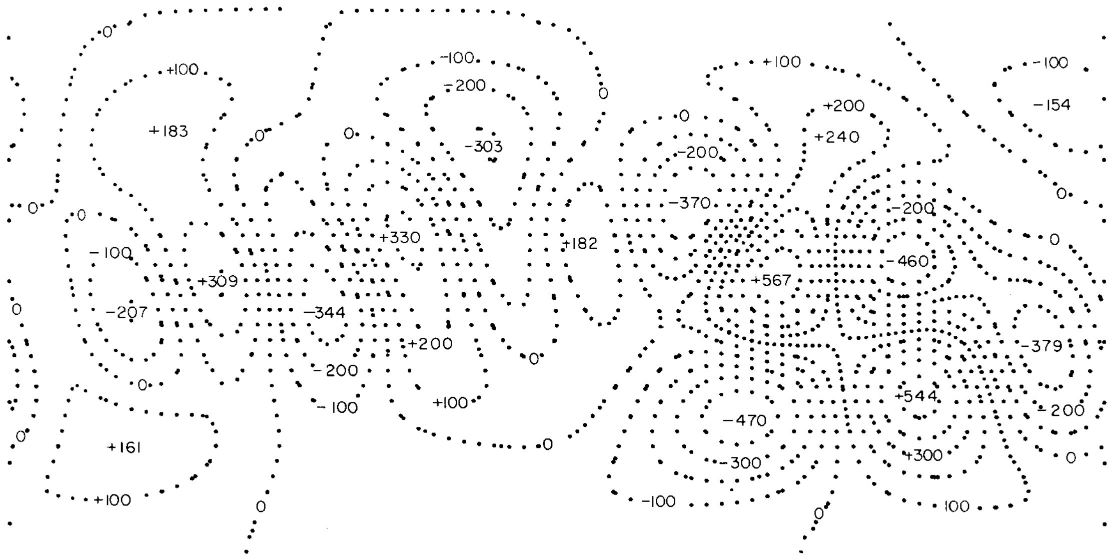
η at 100,000 km elevation (ω = 0)



η in 10^{-3} sec of arc

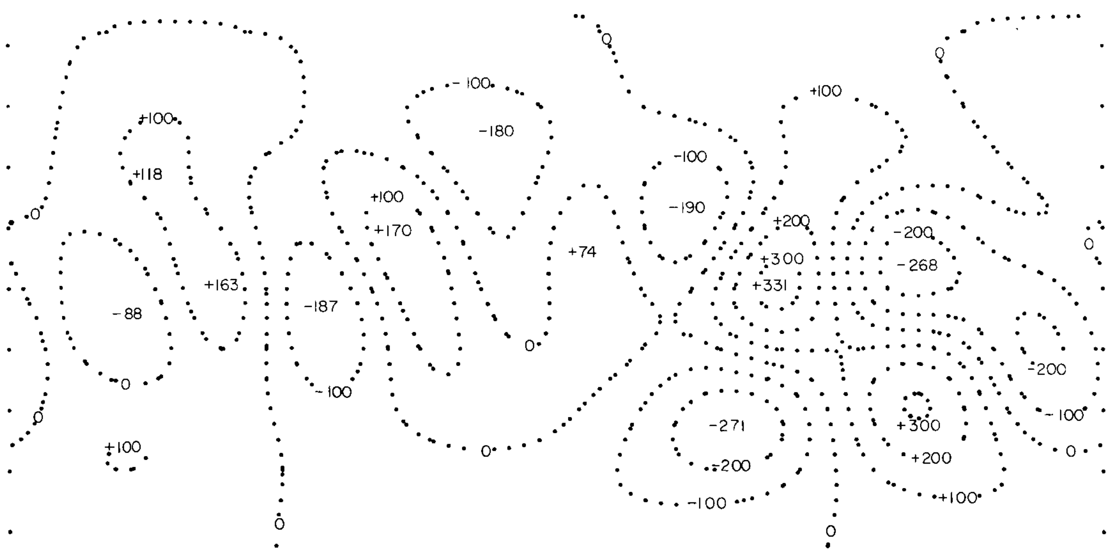
Contour Map 5.--Gaussian and mean curvature.

$\Delta \frac{1}{\sqrt{K}}, \Delta \frac{1}{H}$ at sea level ($\omega \neq 0$, $\omega = 0$)



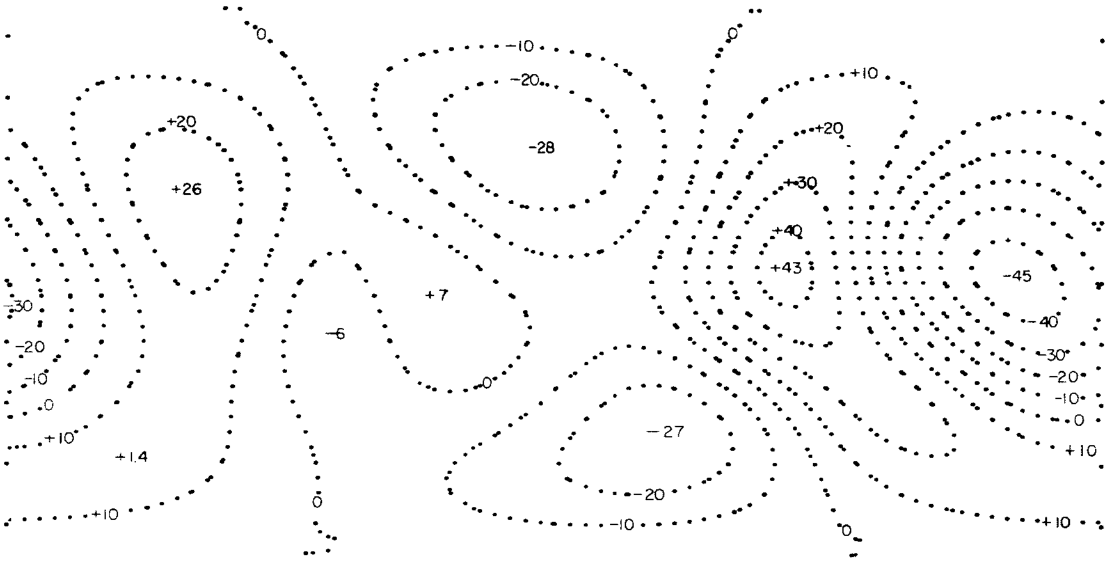
$\Delta \frac{1}{\sqrt{K}}$, $\Delta \frac{1}{H}$ in meters

$\Delta \frac{1}{\sqrt{K}}$, $\Delta \frac{1}{H}$ at 1,000 km elevation ($\omega = 0$)



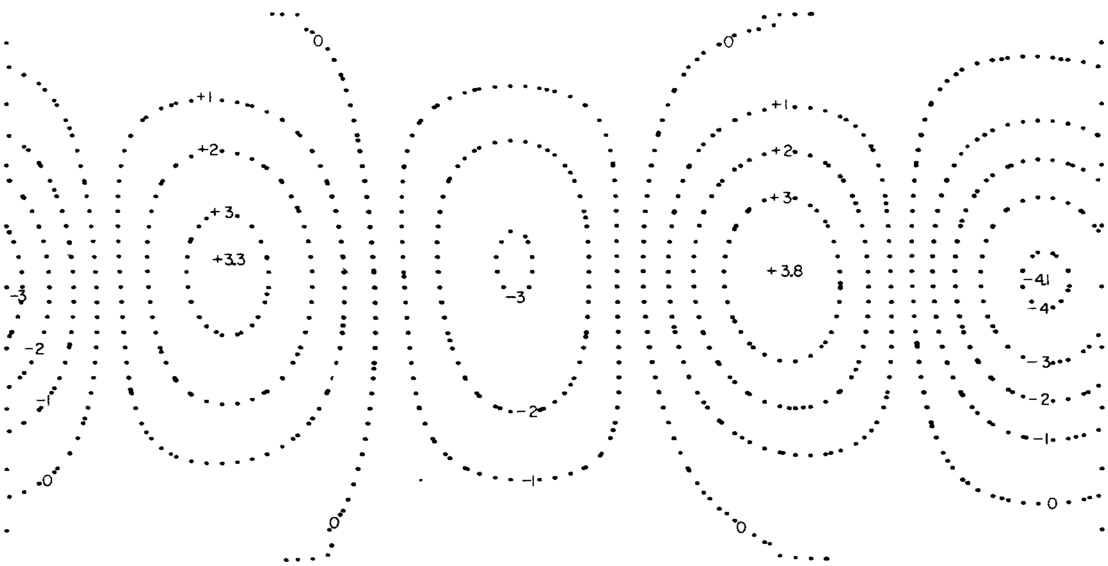
$\Delta \frac{1}{\sqrt{K}}$, $\Delta \frac{1}{H}$ in meters

$\Delta \frac{1}{\sqrt{K}}$, $\Delta \frac{1}{H}$ at 10,000 km elevation ($\omega = 0$)



$\Delta \frac{1}{\sqrt{K}}$, $\Delta \frac{1}{H}$ in meters

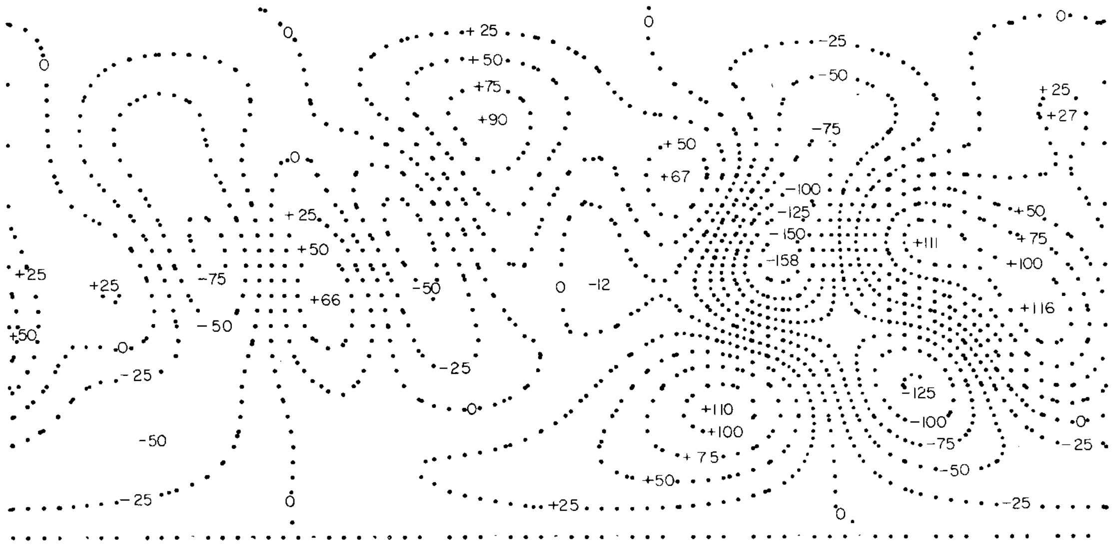
$\Delta \frac{1}{\sqrt{K}}$, $\Delta \frac{1}{H}$ at 100,000 km elevation ($\omega = 0$)



$\Delta \frac{1}{\sqrt{K}}$, $\Delta \frac{1}{H}$ in meters

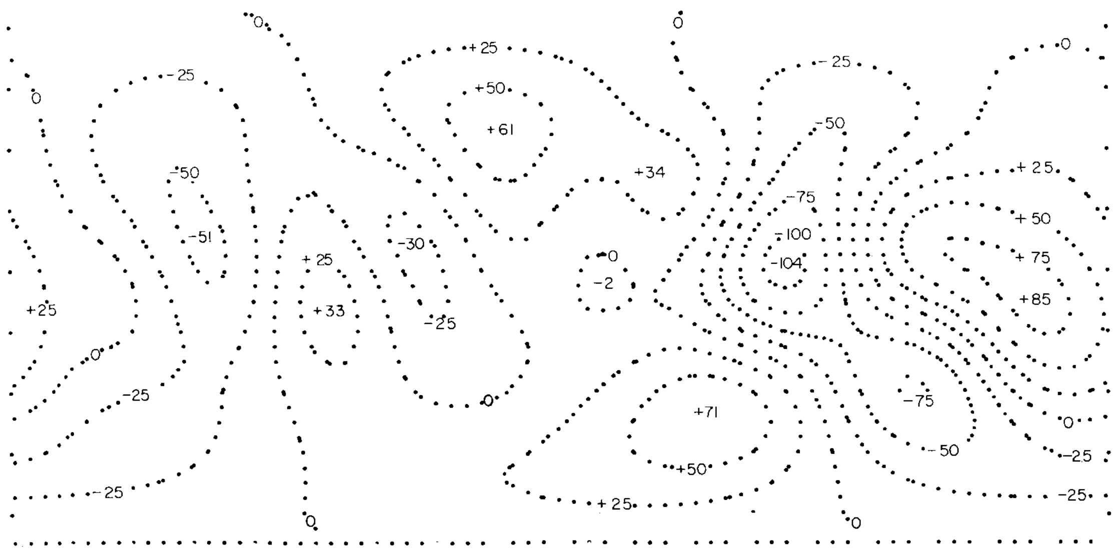
Contour Map 6.--Shape of the equigravitational (equigravity) sur

Δr at sea level ($\omega \neq 0$, $\omega = 0$)



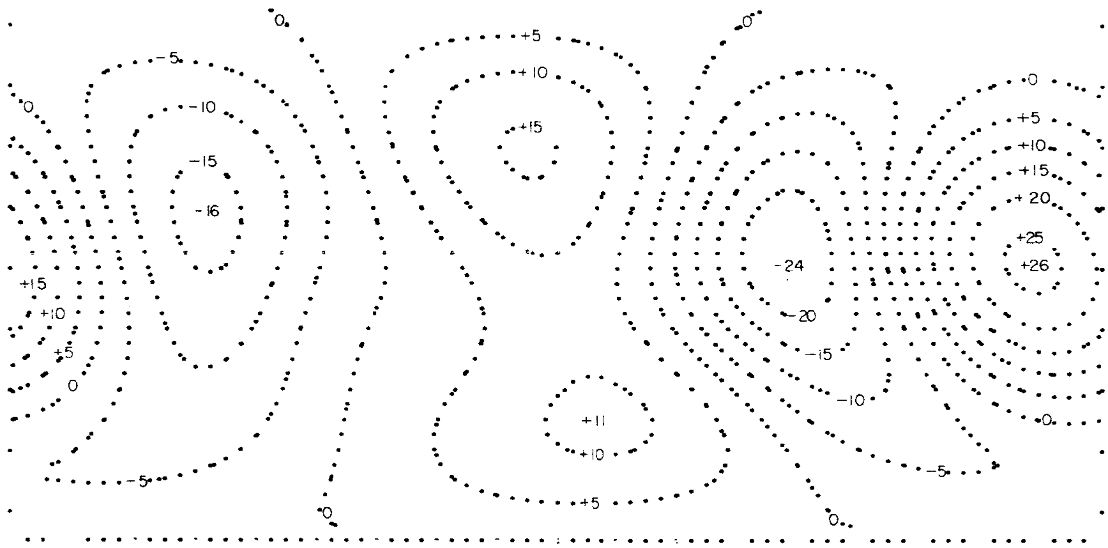
Δr in meters

Δr at 1,000 km elevation ($\omega = 0$)



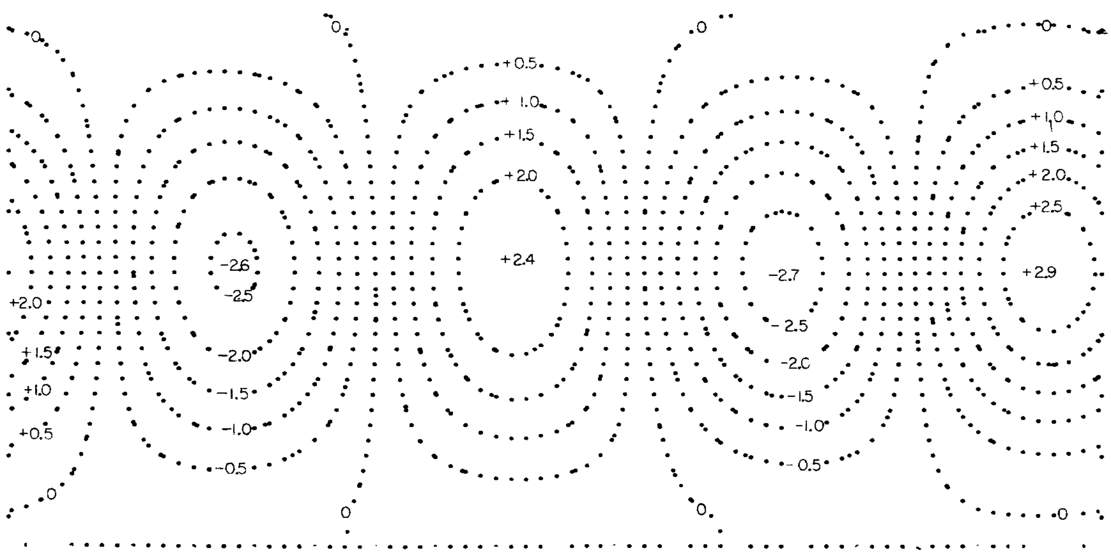
Δr in meters

Δr at 10,000 km elevation (ω = 0)



Δr in meters

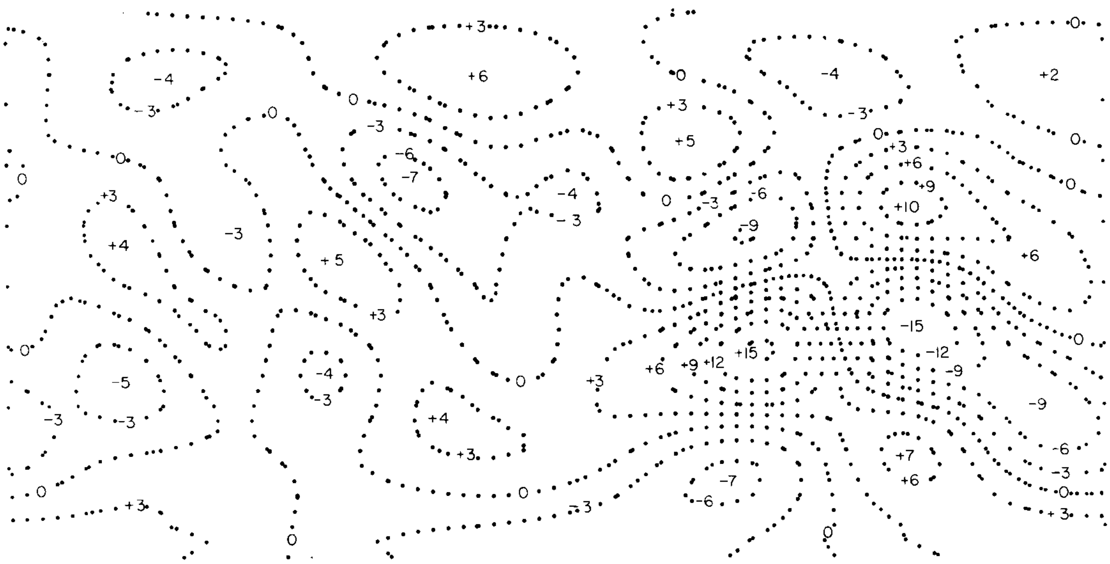
Δr at 100,000 km elevation (ω = 0)



Δr in meters

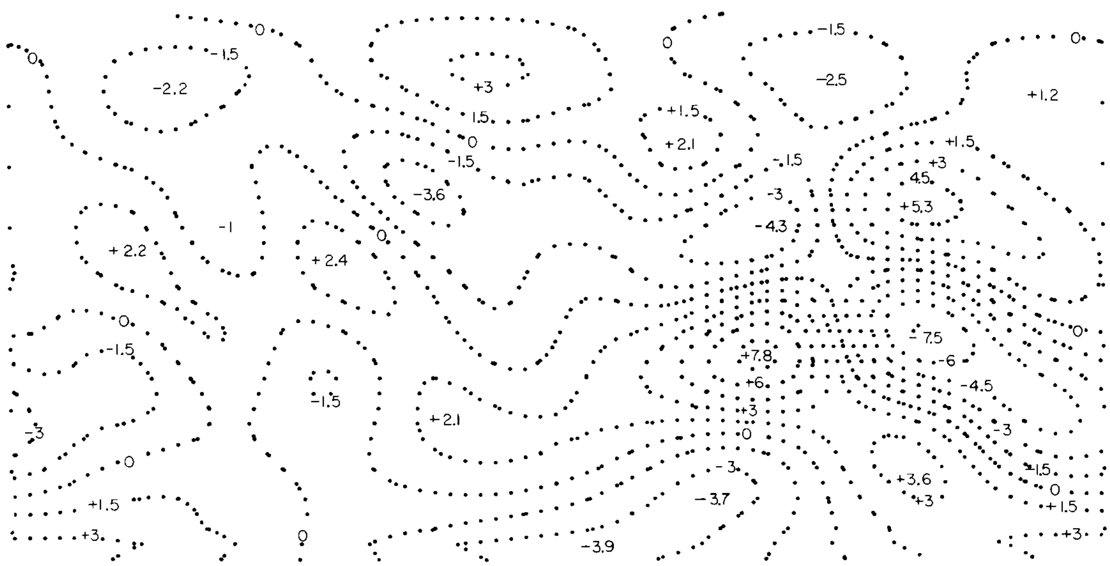
Contour Map 7.--Oscillation of the surface normals (equigravitational, equigravity surfaces).

ξ at sea level ($\omega \neq 0$, $\omega = 0$)



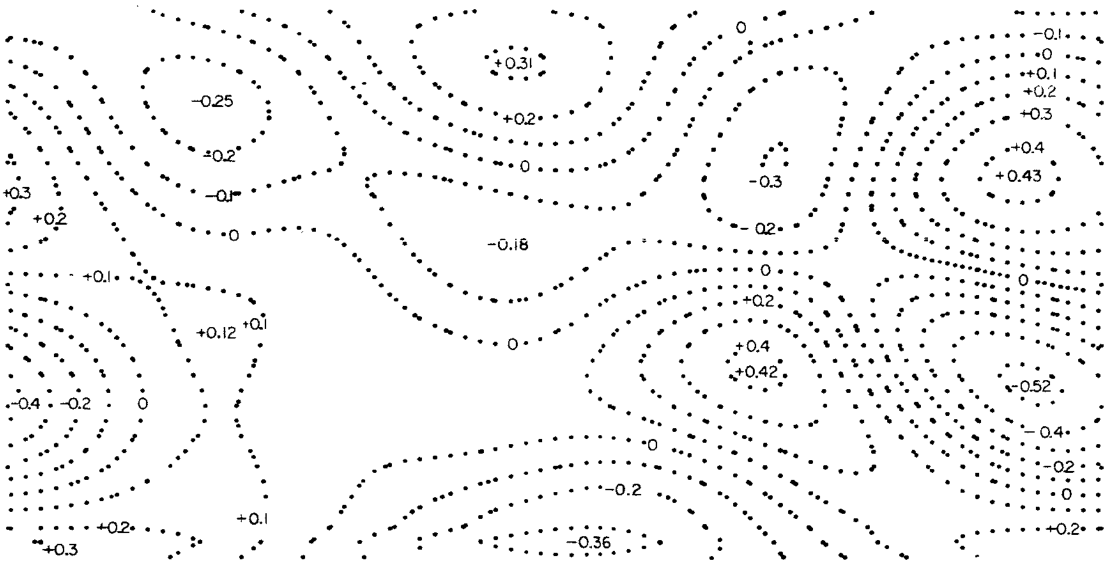
ξ in sec of arc

ξ at 1,000 km elevation ($\omega = 0$)



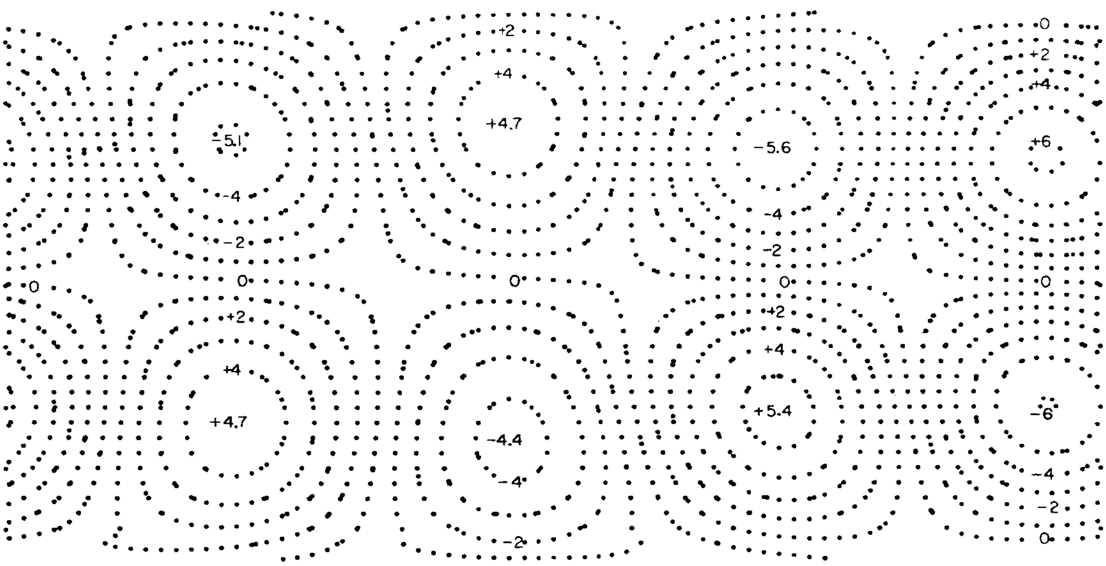
ξ in sec of arc

ξ at 10,000 km elevation ($\omega = 0$)



ξ in sec of arc

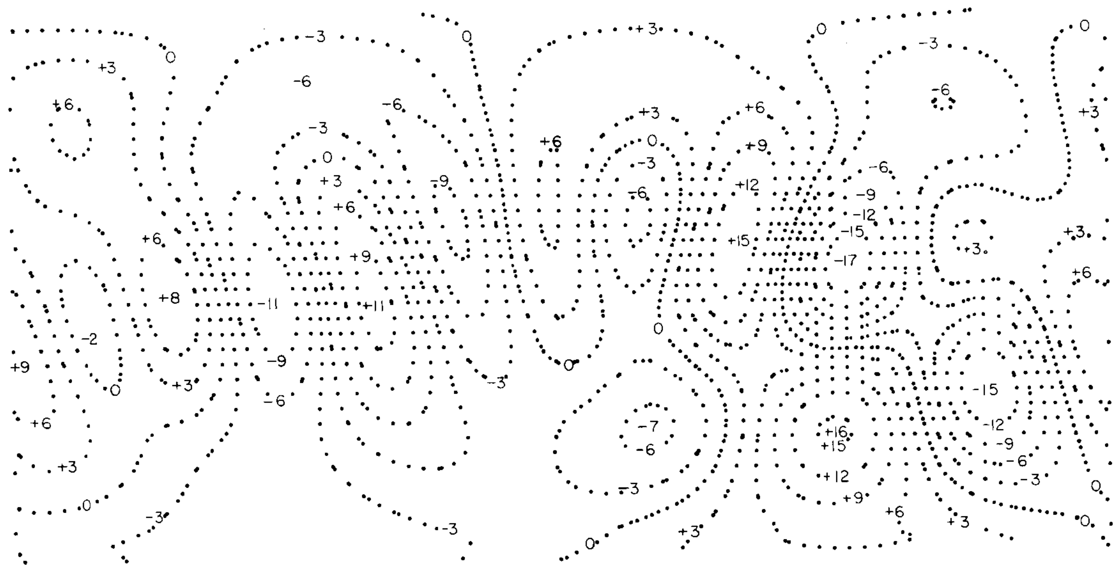
ξ at 100,000 km elevation ($\omega = 0$)



ξ in 10^{-3} sec of arc

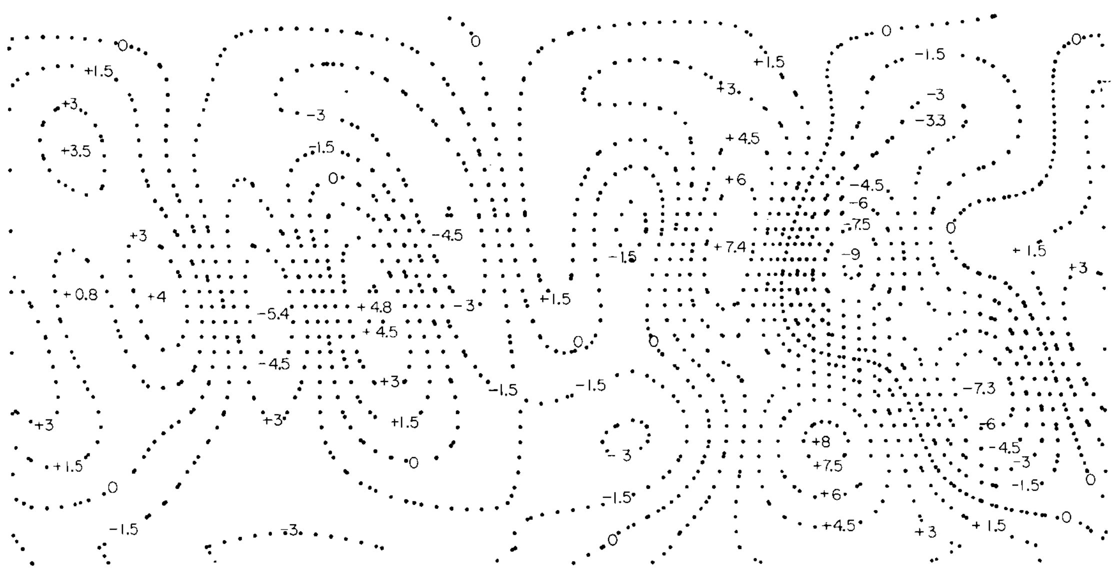
Contour Map 8.--Oscillation of the surface normals (equigravitational, equigravity surfaces).

η at sea level ($\omega \neq 0$, $\omega = 0$)



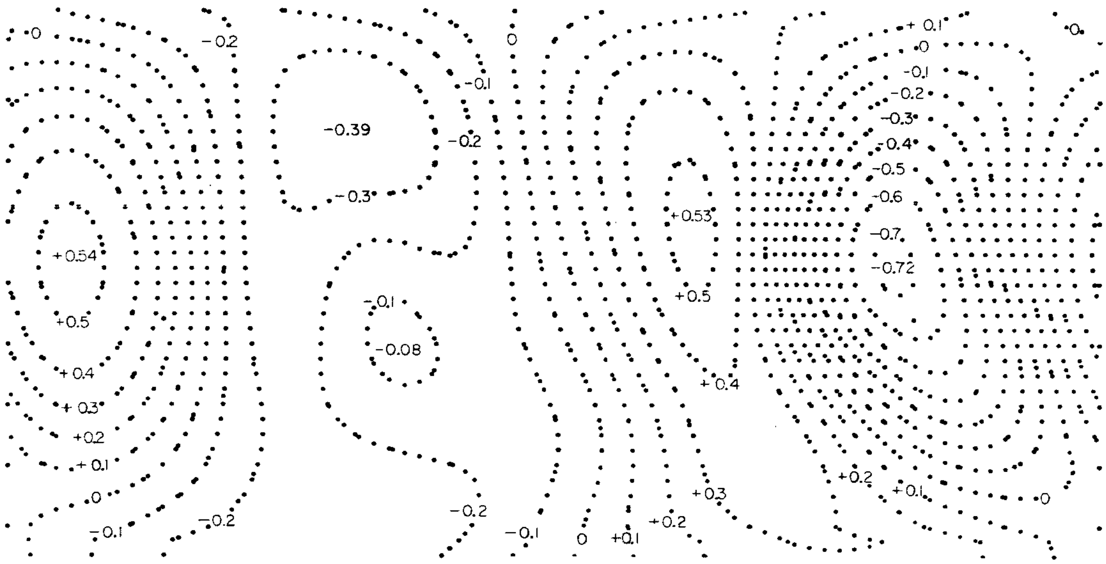
η in sec of arc

η at 1,000 km elevation ($\omega = 0$)



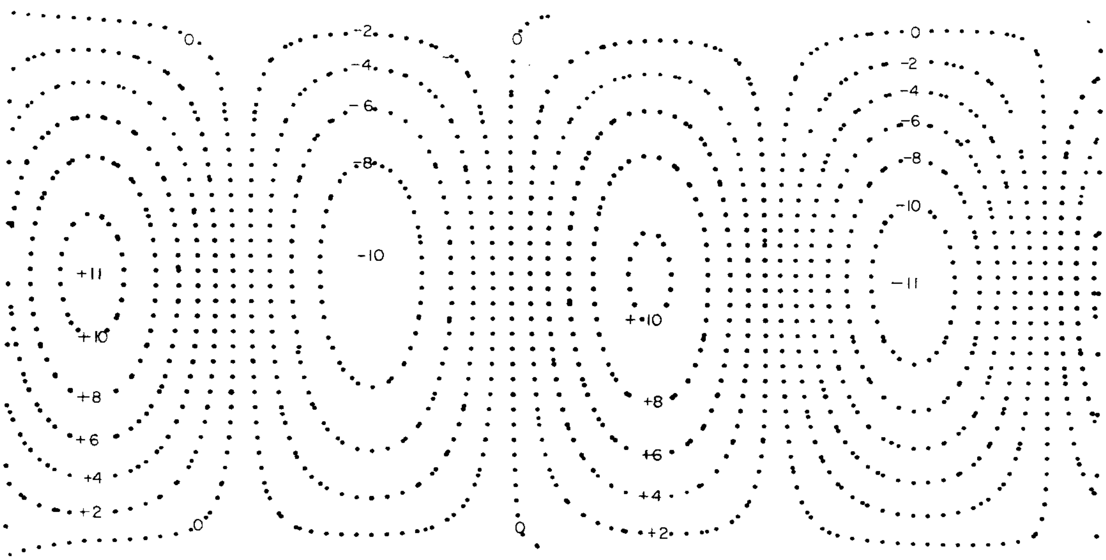
η in sec of arc

η at 10,000 km elevation (ω = 0)



η in sec of arc

η at 100,000 km elevation (ω = 0)



η in 10^{-3} sec of arc

APPENDIX B. Number Maps

Number Map 1.--Intersection of the equigravitational (equigravity) surfaces with the equipotential surfaces.

$\Delta\mu$ at sea level ($\omega \neq 0$)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12	+13	+14	+15	+16	+17	+18
90	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
85	-5	-3	-0	2	5	6	6	6	4	1	-2	-5	-8	-11	-13	-13	-13	-11	-8	-4	-0	4	7	10	12	13	13	11	9	6	3	-1	-4	-6	-7	-6	-5
80	-9	-5	1	7	11	14	15	13	10	5	-0	-7	-13	-18	-21	-23	-22	-19	-14	-8	-2	4	10	14	17	19	19	18	14	10	4	-2	-8	-11	-13	-12	-9
75	-11	-5	3	10	16	20	20	17	13	7	-0	-7	-15	-22	-27	-29	-28	-24	-18	-11	-4	3	8	13	17	21	23	23	20	15	8	0	-7	-13	-16	-15	-11
70	-11	-4	5	13	19	21	20	17	11	6	-0	-6	-13	-21	-28	-32	-31	-27	-20	-12	-6	-1	2	6	11	17	22	25	24	20	13	5	-4	-11	-15	-15	-11
65	-9	-2	6	14	19	20	17	12	7	2	-0	-3	-8	-15	-23	-29	-30	-26	-18	-11	-7	-7	-7	-5	1	9	18	24	26	23	18	10	2	-5	-10	-12	-9
60	-5	-0	7	13	17	17	12	5	0	-2	-0	2	2	-4	-14	-23	-25	-21	-13	-7	-7	-12	-18	-19	-12	-0	12	20	24	22	19	14	8	2	-3	-6	-5
55	-1	1	6	11	13	12	6	-1	-6	-6	-0	9	14	10	-1	-12	-18	-14	-6	-1	-6	-18	-29	-32	-24	-9	6	15	17	16	14	13	12	9	4	0	-1
50	3	3	4	6	8	7	2	-5	-11	-10	-0	14	25	25	14	-1	-9	-6	2	5	-3	-21	-37	-41	-31	-13	2	8	6	3	3	7	12	13	11	7	3
45	7	3	1	1	2	2	0	-6	-12	-12	-2	17	33	37	27	11	1	2	10	12	0	-22	-41	-44	-30	-10	2	2	-7	-14	-13	-4	7	14	15	11	7
40	9	4	-2	-5	-4	-1	1	-2	-10	-13	-5	15	36	45	37	21	9	9	16	18	4	-19	-37	-38	-21	-1	7	-2	-20	-33	-32	-19	-2	10	15	13	9
35	10	4	-5	-11	-10	-3	4	4	-4	-12	-9	9	32	46	43	28	15	14	20	21	8	-13	-28	-24	-5	14	15	-4	-32	-51	-50	-34	-13	3	11	13	10
30	9	3	-7	-16	-16	-5	8	12	3	-11	-14	-1	23	41	43	31	18	15	20	22	12	-4	-14	-5	16	31	25	-4	-39	-62	-63	-47	-25	-8	3	9	9
25	7	3	-9	-20	-21	-8	10	19	10	-9	-20	-13	10	31	39	32	20	15	18	20	14	5	3	17	37	47	33	-1	-40	-65	-68	-53	-34	-19	-7	2	7
20	4	3	-9	-22	-25	-11	10	23	16	-6	-25	-24	-6	18	31	29	20	13	13	15	15	14	20	36	54	57	38	2	-35	-58	-61	-51	-39	-28	-18	-6	4
15	-1	3	-8	-23	-28	-15	7	23	18	-4	-27	-33	-19	4	22	26	19	11	8	9	14	21	33	49	62	58	36	6	-23	-40	-43	-39	-37	-35	-28	-14	-1
10	-5	2	-5	-20	-27	-18	1	18	17	-2	-27	-37	-29	-8	13	22	18	9	3	3	11	25	40	53	58	47	28	8	-7	-14	-16	-20	-28	-36	-35	-22	-5
5	-10	1	-3	-16	-25	-20	-6	9	14	0	-23	-35	-32	-17	5	19	18	8	-0	-2	8	25	39	48	44	27	12	10	13	16	16	6	-15	-33	-38	-27	-10
-0	29	-0	21	14	25	55	41	10	65	68	18	55	78	39	20	53	34	16	33	11	22	33	33	71	79	28	45	97	93	61	48	36	6	19	35	41	29
-5	17	3	-3	2	12	21	20	11	3	2	6	20	30	22	5	-10	-17	-11	3	8	2	-11	-17	-5	18	34	33	9	-32	-67	-75	-56	-20	13	31	30	17
-10	17	4	-5	-7	1	14	21	18	8	-2	-4	6	19	20	8	-8	-18	-13	0	9	7	-1	0	19	47	63	53	13	-40	-82	-92	-72	-35	-1	20	25	17
-15	15	5	-8	-14	-10	5	19	21	11	-5	-13	-7	8	16	9	-6	-17	-14	-2	9	11	10	17	41	71	86	68	20	-41	-85	-98	-80	-47	-14	8	17	15
-20	11	4	-9	-20	-20	-5	13	20	11	-7	-20	-17	-0	12	11	-2	-14	-14	-4	8	15	19	31	57	85	96	75	25	-35	-77	-90	-77	-51	-26	-7	6	11
-25	4	2	-11	-25	-27	-14	6	16	10	-9	-25	-23	-6	10	14	3	-5	-12	-4	9	18	26	40	64	88	95	74	28	-24	-60	-73	-66	-50	-35	-21	-6	4
-30	-5	-2	-11	-26	-32	-21	-2	11	6	-11	-25	-24	-8	10	16	9	-3	-8	-2	9	20	28	41	60	78	82	63	28	-11	-37	-48	-47	-44	-40	-32	-18	-5
-35	-14	-6	-12	-25	-32	-25	-8	5	3	-11	-23	-22	-6	12	20	15	4	-1	2	11	20	27	36	47	58	59	47	25	2	-13	-20	-26	-34	-41	-41	-30	-14
-40	-22	-11	-12	-21	-28	-24	-11	0	0	-9	-18	-17	-3	13	22	20	12	6	7	14	19	22	25	28	32	32	27	19	13	10	6	-4	-21	-39	-46	-38	-22
-45	-28	-15	-11	-16	-21	-18	-10	-1	-1	-7	-13	-11	0	15	24	23	18	13	13	15	17	15	11	7	5	5	8	13	21	27	26	13	-9	-33	-46	-42	-28
-50	-30	-16	-9	-9	-11	-10	-5	-0	-0	-4	-8	-6	3	15	22	24	21	18	16	16	13	6	-4	-14	-19	-18	-9	6	24	37	39	26	1	-25	-41	-42	-30
-55	-28	-15	-6	-2	-2	-1	1	3	1	-2	-5	-3	4	12	19	22	21	20	17	14	8	-3	-17	-29	-36	-34	-21	0	24	41	45	33	10	-16	-33	-37	-28
-60	-22	-11	-2	4	6	7	7	6	4	0	-3	-2	2	8	14	17	18	17	15	10	1	-11	-26	-39	-45	-41	-26	-4	21	38	44	35	16	-6	-22	-27	-22
-65	-12	-5	3	9	12	13	12	9	5	1	-2	-3	-1	3	7	10	12	12	9	4	-6	-18	-31	-42	-46	-41	-26	-5	16	33	39	34	20	3	-10	-15	-12
-70	-1	3	9	14	16	16	14	11	6	1	-3	-5	-5	-3	-1	2	3	3	1	-4	-12	-22	-32	-39	-40	-35	-22	-5	12	26	32	30	22	11	2	-2	-1
-75	10	12	15	17	18	17	15	11	6	1	-4	-7	-8	-9	-8	-7	-6	-7	-9	-13	-18	-25	-30	-33	-32	-27	-17	-4	9	19	25	26	23	18	13	10	10
-80	20	20	20	20	19	17	13	9	5	-0	-5	-8	-11	-13	-14	-14	-15	-16	-18	-20	-23	-25	-26	-26	-23	-18	-11	-2	7	14	20	23	23	23	21	20	20
-85	26	25	24	22	19	16	13	8	4	-1	-5	-9	-13	-16	-18	-20	-22	-23	-24	-25	-25	-25	-23	-20	-17	-12	-6	0	6	12	17	20	23	25	26	26	26
-90	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27

$\Delta\mu$ in 10^{-1} sec of arc

$\Delta\mu$ at sea level ($\omega = 0$)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12	+13	+14	+15	+16	+17	+18
90	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
85	5	3	0	-2	-5	-6	-6	-6	-4	-1	2	5	9	11	13	13	13	11	8	4	1	-3	-7	-10	-12	-13	-13	-11	-9	-6	-2	1	4	6	7	6	5
80	9	5	-1	-6	-11	-14	-15	-13	-10	-5	1	7	13	18	21	23	22	19	14	8	2	-4	-9	-14	-17	-19	-19	-18	-14	-9	-3	3	8	11	13	12	9
75	12	5	-2	-10	-16	-19	-20	-17	-12	-6	1	8	15	22	27	29	28	25	19	11	4	-2	-8	-13	-17	-21	-23	-23	-20	-14	-7	0	8	13	16	15	12
70	11	5	-4	-13	-19	-21	-20	-16	-11	-5	1	7	14	21	28	32	31	27	20	12	6	2	-2	-6	-11	-17	-22	-25	-24	-20	-13	-4	4	11	15	15	11
65	9	3	-6	-14	-19	-20	-17	-12	-6	-2	1	4	8	15	23	29	30	26	18	11	7	7	7	5	-0	-9	-18	-24	-26	-23	-17	-10	-2	5	10	12	9
60	5	1	-6	-13	-17	-17	-12	-5	0	2	0	-2	-1	5	14	23	26	22	14	8	7	13	18	19	13	1	-12	-20	-23	-22	-18	-13	-8	-2	3	6	5
55	1	-1	-5	-10	-13	-12	-6	1	7	6	0	-8	-13	-10	1	13	18	15	7	2	6	18	29	32	24	9	-6	-15	-17	-15	-14	-13	-12	-9	-4	-0	1
50	-3	-2	-3	-6	-8	-7	-2	6	11	10	0	-14	-25	-24	-13	1	9	6	-2	-5	3	21	37	42	32	14	-2	-8	-6	-3	-3	-7	-12	-13	-11	-7	-3
45	-7	-3	-1	-0	-2	-2	0	6	12	12	2	-17	-33	-37	-26	-11	-1	-2	-10	-12	-0	22	41	44	31	12	-2	-2	7	14	13	4	-7	-14	-15	-11	-7
40	-9	-3	2	6	5	1	-1	3	10	13	5	-15	-36	-44	-36	-20	-9	-9	-16	-18	-4	19	38	38	23	3	-6	2	21	33	32	19	2	-10	-15	-14	-9
35	-10	-3	5	11	11	3	-4	-4	5	12	9	-9	-32	-46	-42	-27	-15	-13	-19	-21	-8	14	28	25	7	-12	-15	4	32	51	50	34	13	-3	-11	-13	-10
30	-9	-3	8	17	16	5	-8	-11	-2	11	15	1	-23	-41	-42	-30	-18	-15	-20	-22	-11	6	14	6	-13	-29	-24	4	40	63	63	47	25	8	-3	-9	-9
25	-7	-3	9	21	21	8	-10	-18	-9	9	20	14	-9	-31	-37	-30	-20	-15	-17	-19	-13	-3	-3	-15	-34	-45	-33	3	43	66	68	53	34	19	7	-2	-7
20	-4	-3	9	23	26	12	-10	-22	-14	7	25	26	7	-18	-29	-27	-19	-13	-12	-15	-14	-12	-20	-34	-49	-55	-38	1	39	59	61	52	39	28	18	6	-4
15	1	-3	8	23	28	16	-7	-22	-16	6	27	36	22	-4	-20	-23	-18	-10	-6	-9	-12	-19	-33	-46	-55	-55	-36	0	30	42	43	40	38	35	28	14	1
10	6	-2	6	20	28	21	-1	-17	-13	5	27	41	33	9	-11	-18	-17	-8	-1	-3	-10	-23	-40	-48	-48	-45	-26	3	18	17	16	21	29	37	35	22	6
5	11	-1	3	16	26	26	8	-9	-5	8	23	41	41	18	-3	-12	-16	-7	4	2	-6	-22	-39	-37	-27	-23	-8	12	9	-11	-16	-4	15	33	38	28	11
-0	29	-0	21	14	25	55	41	10	65	68	18	55	77	39	20	53	34	16	33	11	21	33	33	71	78	28	45	97	93	61	48	36	6	19	35	41	29
-5	-14	-3	4	-1	-11	-14	-15	-11	6	10	-5	-16	-17	-18	-5	15	19	11	-1	-8	-1	12	17	12	-8	-33	-29	7	43	69	75	55	20	-13	-30	-27	-14
-10	-15	-4	6	7	-1	-11	-19	-18	-3	8	5	-5	-13	-17	-8	10	19	13	0	-9	-7	1	-0	-16	-43	-63	-52	-9	44	82	93	73	36	1	-20	-23	-15
-15	-13	-4	8	15	10	-3	-17	-21	-8	9	14	7	-4	-14	-9	6	17	14	2	-9	-11	-10	-17	-40	-69	-85	-68	-18	41	85	98	80	48	15	-8	-16	-13
-20	-9	-4	9	21	20	6	-12	-20	-10	10	21	17	3	-11	-11	2	14	14	4	-8	-15	-19	-31	-57	-85	-96	-75	-25	35	78	91	78	54	28	7	-5	-9
-25	-2	-1	11	25	27	15	-5	-16	-9	11	25	23	8	-9	-13	-3	9	12	4	-8	-18	-25	-40	-63	-87	-95	-73	-27	24	61	73	67	54	37	21	7	-2
-30	6	2	11	26	32	22	2	-11	-6	12	25	24	9	-9	-16	-9	3	8	2	-9	-19	-28	-41	-60	-78	-81	-63	-26	12	38	48	49	48	43	33	19	6
-35	15	7	12	25	32	25	8	-5	-3	11	23	22	7	-11	-20	-15	-4	1	-2	-11	-20	-27	-35	-47	-58	-59	-45	-22	-0	14	20	27	37	44	41	30	15
-40	23	11	12	21	28	24	11	-0	-0	10	19	17	4	-13	-22	-20	-12	-6	-7	-13	-19	-22	-24	-28	-32	-31	-25	-16	-10	-9	-6	6	25	41	46	38	23
-45	28	15	11	16	21	18	10	1	1	7	13	11	-0	-15	-23	-23	-18	-13	-13	-15	-16	-14	-10	-6	-5	-4	-5	-9	-17	-26	-26	-12	12	35	46	42	28
-50	30	17	9	9	12	10	5	1	0	5	8	6	-3	-14	-22	-24	-21	-18	-16	-16	-12	-5	5	14	19	19	12	-2	-20	-36	-39	-25	1	27	42	42	30
-55	28	16	6	2	2	1	-1	-2	-1	2	5	3	-4	-12	-19	-22	-21	-19	-17	-14	-7	4	17	29	36	34	23	3	-20	-39	-45	-33	-8	17	34	37	28
-60	22	12	2	-4	-6	-7	-7	-6	-3	0	3	2	-2	-8	-14	-17	-18	-17	-15	-10	-1	12	26	39	45	42	28	7	-18	-37	-44	-35	-15	7	23	27	22
-65	12	5	-3	-9	-12	-13	-12	-9	-5	-1	2	3	1	-3	-7	-10	-12	-12	-9	-3	6	18	31	42	46	41	28	8	-14	-32	-39	-34	-19	-3	10	15	12
-70	1	-3	-9	-14	-16	-16	-14	-10	-6	-1	3	5	5	3	1	-2	-3	-3	-1	4	12	22	32	39	41	36	24	7	-10	-25	-32	-30	-22	-11	-2	2	1
-75	-10	-12	-15	-17	-18	-17	-14	-10	-5	-0	4	7	9	9	8	7	6	7	9	13	18	24	30	33	33	27	18	6	-7	-18	-25	-26	-23	-18	-13	-10	-10
-80	-20	-20	-20	-19	-18	-16	-13	-9	-4	1	5	9	11	13	14	15	15	16	18	20	23	25	27	26	24	19	12	3	-5	-13	-19	-22	-23	-23	-21	-20	-20
-85	-26	-25	-23	-21	-18	-15	-11	-7	-2	2	6	10	14	16	19	21	22	23	24	25	25	25	23	21	18	13	8	2	-4	-10	-15	-19	-22	-24	-26	-26	-26
-90	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27

$\Delta\mu$ in 10^{-1} sec of arc

Δμ at 1,000 km elevation (ω = 0)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0+	1+	2+	3+	4+	5+	6+	7+	8+	9+	10+	11+	12+	13+	14+	15+	16+	17+	18
90	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
85	0	-1	-2	-2	-3	-3	-3	-2	-1	1	3	5	6	8	9	9	9	8	7	5	3	0	-2	-4	-6	-7	-7	-7	-7	-6	-4	-3	-1	-0	0	0	0
80	3	1	-2	-4	-6	-7	-7	-6	-5	-2	1	5	8	11	13	14	13	12	10	7	4	0	-3	-6	-8	-10	-11	-10	-9	-7	-4	-2	1	3	4	4	3
75	4	1	-2	-6	-9	-10	-10	-9	-6	-3	1	4	8	12	15	16	16	15	12	9	5	1	-2	-5	-8	-11	-12	-13	-12	-9	-6	-2	2	5	6	6	4
70	5	1	-3	-7	-10	-12	-11	-9	-7	-3	-0	3	7	11	15	17	17	16	13	9	6	3	1	-2	-6	-9	-12	-14	-13	-11	-7	-3	1	5	7	7	5
65	4	1	-3	-7	-10	-12	-11	-8	-5	-3	-1	1	4	8	12	15	16	14	12	8	6	6	5	3	-1	-6	-10	-13	-13	-11	-8	-4	-0	3	5	6	4
60	3	0	-3	-7	-10	-10	-9	-6	-3	-1	-1	-1	-0	3	7	11	13	12	9	7	6	8	9	8	4	-2	-7	-11	-12	-10	-7	-4	-1	1	3	4	3
55	2	-0	-3	-6	-8	-8	-6	-3	-0	1	-1	-4	-6	-4	1	6	9	8	5	3	5	10	14	14	9	2	-5	-8	-8	-6	-4	-3	-2	-1	1	2	2
50	-0	-0	-2	-4	-5	-6	-4	-1	2	2	-1	-7	-11	-11	-7	-0	3	3	0	-0	3	11	17	17	12	4	-3	-5	-3	0	2	1	-1	-2	-1	-1	-0
45	-1	-1	-0	-1	-2	-3	-3	0	3	4	-0	-8	-15	-17	-13	-6	-2	-2	-4	-4	1	10	17	18	11	2	-3	-2	3	8	10	7	2	-1	-3	-2	-1
40	-2	-0	1	2	1	-1	-2	-0	3	5	1	-7	-17	-21	-18	-11	-6	-5	-8	-8	-2	8	15	15	7	-2	-5	-0	10	17	19	14	7	1	-2	-3	-2
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-55	16	11	6	3	1	0	-1	-1	0	2	4	3	-1	-6	-10	-12	-12	-11	-10	-7	-3	2	9	14	16	14	7	-2	-13	-20	-22	-15	-4	9	17	20	16
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-80	-13	-13	-12	-12	-11	-10	-8	-6	-3	-1	2	4	5	7	8	9	10	11	12	14	16	17	18	17	15	12	8	2	-3	-8	-11	-14	-14	-14	-14	-13	-13
-85	-16	-16	-15	-14	-12	-10	-8	-6	-3	-0	2	5	7	9	11	12	14	15	16	17	17	17	16	15	12	9	6	2	-2	-5	-9	-11	-14	-15	-16	-16	-16
-90	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17

Δμ in 10^{-1} sec of arc

Δμ at 10,000 km elevation (ω = 0)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0+	1+	2+	3+	4+	5+	6+	7+	8+	9+	10+	11+	12+	13+	14+	15+	16+	17+	18
90	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
85	-7	-7	-7	-7	-6	-5	-4	-3	-1	1	3	5	7	9	10	11	12	12	11	10	9	7	5	2	0	-2	-3	-5	-6	-6	-7	-7	-7	-7	-7	-7	-7
80	-5	-5	-6	-7	-7	-7	-6	-5	-3	-1	1	4	7	9	11	12	13	13	12	11	9	6	4	1	-2	-4	-6	-7	-7	-7	-6	-5	-5	-4	-4	-4	-5
75	-2	-3	-5	-7	-8	-8	-8	-7	-5	-3	-0	3	6	8	11	12	13	14	13	11	9	6	3	-0	-3	-6	-7	-8	-8	-7	-5	-3	-2	-1	-1	-1	-2
70	1	-2	-4	-6	-8	-9	-9	-8	-7	-5	-2	1	4	7	10	12	13	13	13	11	9	6	2	-1	-5	-7	-9	-9	-8	-6	-3	-1	1	2	3	2	1
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60	5	2	-1	-5	-8	-9	-10	-10	-8	-6	-4	-2	1	3	5	8	9	10	10	9	7	4	1	-3	-6	-9	-10	-9	-7	-3	1	4	7	9	9	8	5
55	7	4	0	-4	-7	-9	-10	-9	-8	-6	-4	-3	-1	1	3	5	6	7	7	7	5	3	-0	-4	-7	-10	-10	-9	-6	-1	3	7	10	11	11	10	7
50	9	5	1	-2	-6	-8	-9	-8	-7	-6	-4	-4	-3	-2	-0	1	3	4	4	4	3	2	-1	-4	-8	-10	-11	-9	-4	1	6	10	13	14	13	12	9
45	10	7	3	-1	-4	-6	-7	-7	-6	-5	-4	-4	-4	-4	-3	-2	-0	1	1	1	1	-0	-2	-5	-8	-11	-11	-8	-3	3	8	13	15	16	15	13	10
40	11	8	4	1	-2	-5	-6	-6	-5	-4	-3	-4	-5	-6	-6	-5	-4	-2	-2	-1	-1	-2	-4	-6	-9	-11	-11	-7	-2	4	10	14	17	17	16	14	11
35	12	9	6	3	-0	-3	-4	-4	-3	-2	-2	-4	-5	-7	-8	-7	-6	-5	-4	-4	-3	-3	-5	-7	-10	-11	-11	-7	-1	6	11	15	18	18	17	15	12
30	12	10	7	4	1	-1	-2	-2	-2	-1	-1	-3	-5	-8	-9	-9	-8	-7	-7	-6	-5	-5	-6	-8	-10	-11	-10	-6	-0	6	12	15	17	18	17	15	12
25	12	10	8	5	3	1	-1	-1	-0	1	0	-2	-4	-7	-10	-10	-10	-9	-8	-7	-6	-6	-6	-8	-10	-11	-10	-6	-0	6	11	14	16	17	16	14	12
20	12	10	8	6	4	2	1	1	1	2	2	-0	-3	-7	-9	-11	-11	-10	-9	-8	-7	-6	-6	-7	-9	-9	-8	-5	-1	4	8	11	14	14	14	13	12
15	10	9	8	7	5	4	2	2	2	3	3	1	-2	-6	-8	-10	-11	-10	-9	-8	-7	-6	-5	-6	-7	-7	-6	-4	-1	2	5	8	10	11	12	12	10
10	9	8	7	7	6	5	4	3	3	4	3	2	-1	-4	-7	-9	-10	-10	-9	-7	-6	-4	-4	-3	-4	-4	-4	-3	-2	-1	1	3	5	7	9	9	9
5	6	6	6	6	6	5	4	4	4	4	4	3	0	-3	-6	-8	-9	-8	-7	-6	-4	-2	-1	0	0	0	-1	-2	-2	-3	-3	-2	0	3	5	6	6
-0	14	16	16	15	13	9	5	7	10	9	6	3	2	-1	-2	1	2	0	-3	-3	2	8	14	17	16	8	4	13	21	25	23	17	9	1	1	8	14
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-25	15	11	8	4	1	-1	-3	-3	-2	-1	-0	-1	-2	-3	-4	-4	-4	-4	-5	-6	-8	-11	-14	-17	-20	-20	-17	-10	-2	7	14	20	22	23	21	18	15
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-40	18	15	11	7	3	0	-1	-2	-1	0	1	1	-1	-4	-6	-7	-7	-7	-7	-7	-7	-8	-9	-12	-14	-15	-15	-12	-8	-2	4	11	16	20	22	21	18
-45	17	14	10	6	3	-0	-2	-2	-1	0	1	1	-1	-3	-5	-6	-7	-6	-6	-5	-5	-5	-6	-8	-10	-12	-12	-12	-9	-5	1	7	12	17	19	19	17
-50	15	12	9	5	2	-1	-2	-2	-1	0	1	1	-1	-3	-4	-5	-6	-5	-4	-3	-2	-2	-2	-4	-6	-8	-10	-11	-10	-7	-3	3	8	13	16	16	15
-55	12	9	6	3	0	-1	-2	-2	-1	-0	0	0	-0	-2	-3	-4	-4	-3	-1	0	1	2	2	1	-2	-4	-7	-9	-10	-9	-6	-1	4	8	11	12	12
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-75	-7	-7	-7	-7	-7	-7	-6	-5	-4	-2	-1	1	2	4	5	7	8	10	11	13	13	14	13	11	9	5	2	-2	-5	-7	-9	-9	-9	-9	-8	-7	-7
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-90	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15

Δμ in 10^{-2} sec of arc

Δμ at 100,000 km elevation (ω = 0)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12	+13	+14	+15	+16	+17	+18
90	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
85	-2	-2	-3	-4	-5	-5	-5	-5	-4	-2	-1	1	3	4	6	7	7	8	7	6	5	3	2	0	-1	-2	-3	-3	-3	-3	-2	-2	-1	-1	-1	-1	-2
80	2	0	-2	-4	-6	-7	-8	-8	-7	-5	-3	-0	3	5	8	9	10	10	9	8	6	3	0	-2	-4	-5	-6	-6	-5	-3	-2	-0	1	2	3	3	2
75	5	2	-1	-4	-6	-9	-10	-10	-9	-7	-5	-1	2	6	9	11	12	12	11	9	6	2	-1	-4	-7	-9	-9	-8	-7	-4	-1	2	4	6	7	6	5
70	8	5	1	-3	-7	-10	-12	-13	-12	-10	-6	-2	2	6	10	13	14	14	13	10	6	2	-3	-7	-10	-12	-12	-11	-8	-5	-1	3	7	9	10	10	8
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55	15	11	5	-1	-7	-12	-15	-17	-16	-14	-10	-5	1	6	10	14	16	16	14	10	5	-0	-6	-12	-16	-18	-18	-15	-11	-5	1	8	13	17	19	18	15
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45	18	13	7	0	-6	-11	-15	-17	-16	-14	-10	-6	-1	4	9	12	14	14	12	9	4	-2	-8	-13	-17	-19	-19	-16	-12	-5	2	9	16	20	22	21	18
40	18	14	7	1	-5	-10	-14	-16	-16	-14	-10	-6	-1	4	8	11	13	13	11	8	4	-2	-8	-13	-17	-19	-19	-16	-11	-5	3	10	16	20	22	21	18
35	18	14	8	2	-4	-9	-13	-14	-14	-13	-10	-6	-1	3	7	9	11	11	10	7	3	-2	-7	-12	-16	-18	-18	-15	-11	-4	3	9	15	19	21	21	18
30	17	13	8	2	-3	-8	-11	-13	-13	-11	-9	-5	-2	2	5	8	9	10	8	6	2	-2	-7	-11	-15	-16	-16	-14	-10	-4	2	9	14	18	20	19	17
25	15	12	7	2	-2	-6	-9	-11	-11	-10	-8	-5	-2	1	4	6	7	8	7	5	2	-2	-6	-10	-13	-14	-14	-12	-9	-4	2	8	12	16	18	17	15
20	13	10	7	2	-1	-5	-7	-8	-9	-8	-6	-4	-2	0	3	4	5	6	5	4	1	-1	-5	-8	-10	-12	-12	-10	-7	-3	2	6	10	13	15	15	13
15	10	8	5	2	-0	-3	-5	-6	-6	-6	-5	-3	-2	-0	1	3	3	4	4	3	1	-1	-3	-5	-7	-9	-9	-8	-5	-2	1	4	8	10	11	11	10
10	7	6	4	2	0	-1	-3	-3	-4	-4	-3	-2	-1	-1	0	1	2	2	2	2	1	-0	-1	-3	-4	-5	-5	-5	-4	-2	0	3	5	6	7	7	7
5	3	3	3	2	2	1	-0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-0	0	0	1	1	1	0	-0	-1	-2	-2	-2	-2	-1	0	1	2	2	3	3	3
-0	20	29	34	34	30	23	13	3	7	16	23	27	28	26	21	14	5	2	12	22	29	32	32	27	19	7	5	17	28	36	38	35	28	16	2	8	20
-5	4	3	2	-0	-2	-3	-4	-4	-4	-3	-2	-0	1	2	3	4	4	3	3	2	0	-1	-3	-4	-5	-5	-5	-3	-2	1	3	4	6	6	6	5	4
-10	8	6	3	0	-3	-5	-6	-7	-6	-5	-4	-2	0	2	4	5	5	5	4	2	0	-2	-4	-7	-8	-9	-8	-6	-4	-0	3	6	8	10	10	9	8
-15	11	8	4	1	-3	-6	-8	-9	-8	-7	-5	-3	-0	2	4	6	6	6	5	3	1	-3	-6	-9	-11	-11	-11	-9	-6	-1	3	7	11	13	14	13	11
-20	14	10	6	1	-3	-7	-9	-11	-10	-9	-7	-4	-1	2	5	7	8	8	6	4	1	-3	-7	-10	-13	-14	-13	-11	-7	-2	3	8	13	16	17	16	14
-25	16	12	7	1	-4	-8	-11	-12	-12	-10	-8	-5	-1	2	5	8	9	9	8	5	1	-3	-7	-11	-15	-16	-16	-13	-9	-3	3	9	14	18	19	19	16
-30	18	13	8	2	-4	-9	-12	-13	-13	-12	-9	-5	-1	3	6	9	10	10	9	6	2	-3	-8	-12	-16	-17	-17	-15	-10	-4	3	9	15	19	21	20	18
-35	18	14	8	2	-4	-9	-13	-14	-14	-12	-9	-6	-1	3	7	9	11	11	10	7	3	-2	-8	-13	-16	-18	-18	-16	-11	-5	2	9	15	20	22	21	18
-40	18	14	8	2	-4	-10	-13	-15	-15	-13	-10	-6	-1	3	7	10	12	12	11	8	3	-2	-7	-12	-16	-19	-18	-16	-11	-5	2	9	15	19	21	21	18
-45	18	13	8	1	-5	-10	-13	-15	-15	-13	-10	-6	-1	3	7	11	12	13	11	8	4	-1	-7	-12	-16	-18	-18	-16	-12	-6	1	8	14	19	21	20	18
-50	17	12	7	1	-5	-10	-13	-15	-15	-13	-10	-6	-1	4	8	11	13	13	12	9	5	-0	-6	-11	-15	-17	-18	-16	-12	-6	1	7	13	17	19	19	17
-55	15	11	6	0	-5	-10	-13	-14	-14	-12	-9	-5	-1	4	8	11	13	14	12	10	6	1	-5	-9	-13	-16	-16	-15	-11	-6	-0	6	11	15	17	17	15
-60	12	9	5	-0	-5	-9	-12	-14	-13	-12	-8	-5	-0	4	8	11	13	14	12	10	6	2	-3	-8	-12	-14	-15	-13	-10	-6	-1	4	9	13	14	14	12
-65	10	7	3	-1	-5	-9	-11	-12	-12	-10	-7	-4	0	4	8	11	13	13	12	10	7	2	-2	-6	-10	-12	-13	-12	-9	-6	-1	3	7	10	11	11	10
-70	7	4	1	-2	-5	-8	-10	-11	-10	-9	-6	-3	1	4	8	10	12	12	12	10	7	3	-1	-4	-7	-9	-10	-10	-8	-5	-2	1	5	7	8	8	7
-75	4	2	-0	-3	-5	-7	-8	-9	-8	-7	-5	-2	1	4	7	9	11	11	10	9	7	4	1	-2	-5	-7	-7	-7	-6	-5	-2	0	2	4	5	5	4
-80	1	-0	-2	-3	-5	-6	-7	-7	-6	-5	-3	-1	2	4	6	8	9	9	9	8	6	4	2	-0	-2	-4	-5	-5	-5	-4	-2	-1	0	1	1	1	1
-85	-2	-3	-3	-4	-4	-5	-5	-4	-4	-2	-1	0	2	4	5	6	7	7	7	6	5	4	3	1	0	-1	-2	-2	-3	-3	-2	-2	-2	-2	-2	-2	-2
-90	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5

Δμ in 10^{-4} sec of arc

Number Map 2.--The gradient (direction) field in a spherical coordinate system.

$\Delta\delta$ at sea level ($\omega \neq 0$, $\omega = 0$)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0+	1+	2+	3+	4+	5+	6+	7+	8+	9+	10+	11+	12+	13+	14+	15+	16+	17+	18
90	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
85	-5	-5	-6	-7	-7	-7	-6	-4	-2	1	4	8	11	14	16	17	17	16	14	12	8	5	1	-2	-5	-8	-10	-10	-10	-10	-9	-7	-6	-5	-5	-4	-5
80	-0	-3	-6	-9	-11	-12	-12	-10	-7	-3	2	7	12	16	20	22	22	21	18	14	10	5	-0	-5	-9	-12	-14	-15	-14	-11	-8	-5	-2	0	1	1	-0
75	3	-1	-5	-10	-14	-16	-16	-14	-10	-6	-0	6	12	17	21	24	25	24	21	16	11	5	-0	-6	-11	-15	-17	-18	-16	-13	-9	-4	1	4	6	5	3
70	6	1	-5	-11	-15	-18	-18	-16	-12	-7	-2	4	10	16	21	24	26	24	21	17	12	7	2	-4	-9	-14	-18	-19	-18	-14	-9	-3	2	7	9	8	6
65	7	2	-5	-11	-16	-18	-18	-15	-11	-7	-3	1	6	11	17	22	24	23	19	15	12	9	5	0	-6	-12	-17	-19	-18	-14	-9	-2	3	8	10	10	7
60	7	3	-4	-10	-15	-17	-16	-13	-9	-6	-4	-3	0	5	11	16	19	19	16	13	11	10	9	5	-1	-9	-15	-18	-16	-12	-6	-1	4	8	10	10	7
55	7	3	-2	-8	-12	-14	-13	-10	-7	-5	-5	-6	-6	-3	3	9	13	13	11	8	9	11	12	9	2	-6	-13	-15	-12	-7	-2	3	6	8	10	10	7
50	7	4	-0	-5	-9	-11	-11	-8	-4	-3	-5	-9	-12	-11	-6	2	6	7	4	3	6	11	14	12	4	-5	-11	-12	-6	0	6	8	9	9	9	9	7
45	7	5	2	-2	-5	-8	-8	-6	-2	-1	-4	-11	-17	-18	-14	-6	-1	0	-2	-2	2	9	13	11	3	-7	-12	-9	0	9	15	16	14	11	10	8	7
40	7	6	5	2	-1	-5	-7	-5	-2	1	-2	-10	-19	-23	-20	-13	-7	-6	-7	-7	-7	6	11	8	-1	-11	-13	-6	7	18	24	23	19	14	11	9	7
35	8	7	7	6	3	-2	-6	-6	-2	2	1	-7	-18	-25	-24	-18	-12	-10	-12	-11	-6	1	5	1	-9	-17	-16	-5	12	26	32	30	24	18	13	9	8
30	8	8	9	10	7	1	-5	-7	-2	3	4	-3	-14	-24	-26	-21	-16	-13	-14	-14	-10	-4	-2	-7	-17	-24	-20	-5	15	31	37	35	29	22	16	11	8
25	9	9	11	13	11	4	-4	-7	-3	4	8	3	-8	-20	-24	-22	-18	-15	-15	-15	-12	-9	-9	-15	-25	-29	-23	-5	16	31	38	36	31	25	19	13	9
20	11	9	12	15	14	7	-2	-7	-4	5	11	9	-1	-14	-21	-22	-18	-15	-14	-14	-13	-13	-15	-22	-30	-31	-23	-6	13	27	33	32	30	26	22	16	11
15	11	9	11	15	16	9	0	-6	-3	6	13	14	5	-7	-17	-20	-18	-15	-13	-12	-13	-15	-19	-25	-30	-29	-21	-6	8	18	22	24	25	26	23	18	11
10	12	8	10	14	16	12	3	-3	-2	6	14	17	11	-1	-12	-17	-17	-14	-10	-10	-11	-15	-20	-24	-25	-22	-15	-5	2	5	7	11	17	22	23	18	12
5	12	7	8	12	15	13	7	1	1	6	13	17	14	4	-7	-14	-15	-12	-8	-6	-8	-13	-17	-17	-14	-10	-5	-3	-5	-9	-10	-4	6	16	20	18	12
-0	26	17	10	14	25	30	17	11	32	34	15	22	30	14	6	20	14	-1	6	2	-4	-3	18	41	43	18	19	50	58	46	30	16	4	7	18	26	26
-5	-7	-3	-2	-4	-8	-11	-11	-9	-4	-3	-6	-11	-12	-9	-1	8	12	10	4	-0	-0	2	1	-5	-15	-22	-18	-2	19	36	42	35	19	3	-7	-10	-7
-10	-4	-1	2	1	-3	-8	-12	-11	-6	-2	-2	-6	-9	-9	-3	5	10	8	2	-3	-5	-5	-9	-19	-31	-38	-30	-8	20	43	52	46	31	14	2	-4	-4
-15	1	3	5	6	2	-4	-11	-12	-7	-0	3	-0	-6	-9	-5	2	7	7	1	-5	-9	-13	-19	-32	-45	-50	-39	-14	18	44	55	52	39	24	11	4	1
-20	7	6	9	10	7	-0	-8	-11	-7	1	6	4	-2	-8	-7	-2	4	4	-1	-8	-13	-19	-27	-40	-53	-57	-45	-18	14	40	53	52	44	32	21	12	7
-25	14	11	12	14	12	4	-5	-9	-6	3	9	8	0	-7	-9	-5	0	1	-3	-10	-16	-22	-31	-44	-55	-58	-46	-21	8	31	44	48	44	38	30	21	14
-30	20	15	14	16	14	7	-2	-7	-4	4	10	9	1	-7	-11	-9	-4	-2	-5	-11	-17	-24	-32	-42	-51	-53	-43	-23	0	19	32	39	41	41	36	29	20
-35	25	18	16	16	15	9	1	-4	-2	5	10	9	1	-8	-12	-12	-8	-6	-8	-12	-17	-22	-28	-35	-41	-43	-36	-23	-7	7	18	27	35	41	40	34	25
-40	29	21	16	15	13	8	2	-2	-1	4	9	8	1	-8	-13	-14	-11	-9	-10	-13	-16	-18	-21	-24	-28	-29	-27	-21	-13	-6	3	14	27	37	41	37	29
-45	30	22	16	13	10	6	1	-2	-1	4	7	6	-0	-8	-13	-15	-13	-12	-11	-12	-12	-12	-11	-12	-13	-15	-17	-18	-18	-15	-9	3	18	31	39	37	30
-50	29	20	14	9	6	3	-1	-2	-1	2	5	4	-1	-7	-12	-14	-14	-12	-11	-10	-8	-4	-1	1	1	-2	-8	-14	-20	-22	-18	-7	8	24	33	34	29
-55	24	17	10	5	1	-1	-3	-4	-2	1	3	2	-1	-6	-10	-12	-12	-11	-9	-6	-2	3	8	12	12	8	-0	-10	-20	-25	-24	-14	0	15	25	28	24
-60	17	12	5	0	-3	-6	-6	-6	-4	-1	1	2	-0	-3	-7	-8	-9	-7	-5	-1	5	11	16	20	20	15	5	-7	-18	-25	-26	-19	-7	6	15	19	17
-65	8	4	-0	-5	-8	-9	-9	-8	-5	-2	1	2	1	-0	-2	-3	-3	-2	1	6	11	17	22	25	24	18	8	-4	-15	-23	-25	-21	-13	-3	5	9	8
-70	-2	-4	-7	-9	-11	-12	-11	-9	-6	-3	0	2	3	3	3	3	3	5	8	12	17	22	25	26	24	18	9	-1	-12	-19	-23	-21	-17	-11	-5	-2	-2
-75	-11	-12	-13	-14	-14	-14	-12	-10	-6	-3	0	3	5	7	8	9	10	12	15	18	22	25	26	26	23	17	9	0	-8	-15	-19	-21	-19	-17	-14	-12	-11
-80	-19	-19	-18	-18	-17	-15	-13	-10	-7	-3	1	4	7	10	12	15	17	19	21	23	25	26	26	24	20	15	8	1	-6	-12	-16	-19	-20	-21	-20	-19	-19
-85	-24	-24	-23	-21	-19	-17	-14	-10	-7	-3	1	5	9	13	16	19	22	24	25	26	26	26	24	21	18	13	8	2	-3	-9	-13	-17	-20	-22	-23	-24	-24
-90	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26

$\Delta\delta$ in 10^{-1} sec of arc

$\Delta\delta$ at 1,000 km elevation ($\omega = 0$)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0+	1+	2+	3+	4+	5+	6+	7+	8+	9+	10+	11+	12+	13+	14+	15+	16+	17+	18
90	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
85	-5	-5	-5	-5	-5	-5	-4	-3	-1	1	3	5	7	9	11	12	12	11	10	9	7	5	2	-0	-2	-4	-5	-6	-7	-7	-7	-6	-5	-5	-5	-5	-5
80	-2	-3	-5	-6	-7	-7	-7	-6	-4	-2	1	4	8	10	12	14	14	14	12	10	8	5	1	-2	-5	-7	-8	-9	-9	-8	-6	-4	-3	-2	-1	-1	-2
75	1	-1	-4	-6	-8	-10	-10	-9	-6	-4	-0	3	7	10	13	15	16	15	14	11	8	5	1	-2	-6	-8	-10	-11	-10	-8	-6	-3	-1	1	2	2	1
70	3	-0	-3	-7	-9	-11	-11	-10	-8	-5	-2	2	6	9	12	15	16	15	14	11	9	5	2	-2	-6	-9	-11	-12	-11	-8	-5	-2	1	4	5	4	3
65	4	1	-3	-7	-10	-11	-11	-10	-8	-6	-3	0	3	7	10	13	14	14	13	11	8	6	3	-1	-5	-8	-11	-12	-11	-8	-4	-0	3	6	7	6	4
60	5	2	-2	-6	-9	-11	-11	-10	-8	-6	-4	-2	0	3	7	10	12	12	11	9	7	6	4	1	-3	-7	-10	-11	-9	-6	-2	2	5	7	8	7	5
55	6	3	-1	-5	-8	-10	-10	-9	-7	-5	-4	-4	-3	-1	3	6	8	8	8	7	6	6	5	2	-2	-7	-10	-10	-7	-3	1	4	7	8	9	8	6
50	7	4	1	-3	-6	-8	-8	-7	-5	-4	-4	-5	-6	-5	-2	1	4	5	4	3	4	5	5	3	-2	-7	-9	-8	-4	1	5	8	9	10	10	9	7
45	7	5	2	-1	-4	-6	-7	-6	-4	-3	-4	-6	-8	-9	-7	-3	-0	1	0	0	2	4	4	2	-3	-7	-9	-7	-1	5	10	12	12	11	10	9	7
40	8	6	4	1	-2	-4	-6	-5	-3	-2	-3	-6	-10	-11	-10	-7	-4	-3	-3	-3	-1	2	3	0	-5	-9	-10	-6	2	9	14	16	15	13	11	9	8
35	8	7	5	3	1	-2	-4	-4	-2	-0	-1	-4	-9	-13	-13	-10	-7	-6	-6	-6	-4	-1	-0	-3	-8	-12	-11	-5	4	13	18	19	17	15	12	10	8
30	9	7	7	6	3	-0	-3	-4	-2	1	1	-2	-8	-12	-14	-12	-9	-8	-8	-8	-6	-4	-3	-6	-11	-14	-12	-5	6	15	20	21	19	16	13	11	9
25	9	8	7	7	5	2	-2	-3	-1	2	3	0	-5	-11	-13	-13	-11	-9	-9	-9	-7	-6	-7	-10	-14	-16	-13	-5	6	15	20	20	19	17	14	11	9
20	9	8	8	8	7	4	-1	-2	-1	3	5	3	-2	-8	-12	-13	-11	-10	-9	-9	-8	-8	-9	-12	-15	-17	-13	-5	5	12	17	18	18	17	15	12	9
15	9	8	8	9	8	5	1	-1	0	4	6	6	1	-5	-10	-12	-11	-10	-9	-8	-8	-9	-10	-13	-15	-15	-11	-4	3	8	11	13	14	15	14	12	9
10	9	7	7	8	9	6	3	0	1	4	7	7	4	-2	-7	-10	-11	-9	-8	-7	-7	-8	-10	-11	-11	-10	-8	-4	-0	2	3	6	9	12	13	11	9
5	8	6	6	7	8	7	4	2	2	4	7	8	6	1	-5	-8	-10	-8	-6	-5	-5	-6	-7	-6	-5	-4	-3	-2	-3	-4	-5	-2	2	7	10	10	8
-0	16	13	11	12	15	15	8	7	16	17	9	9	11	5	1	8	6	-2	-0	-0	-2	2	12	23	23	10	8	25	32	29	21	13	5	2	8	14	16
-5	-3	-2	-2	-3	-4	-6	-6	-5	-3	-3	-4	-6	-6	-4	0	5	7	6	3	1	0	-0	-2	-5	-9	-12	-10	-1	10	19	22	19	12	5	-1	-3	-3
-10	0	0	1	-0	-2	-5	-7	-6	-4	-2	-2	-4	-5	-4	-1	3	5	5	2	-1	-3	-4	-7	-12	-18	-20	-16	-5	10	21	27	25	19	11	5	1	0
-15	4	3	3	3	0	-3	-6	-6	-4	-1	-0	-1	-3	-4	-3	1	3	3	0	-3	-6	-8	-13	-19	-25	-27	-21	-8	8	22	29	29	24	17	11	6	4
-20	8	6	6	5	3	-1	-5	-6	-4	-0	2	1	-2	-4	-4	-2	1	1	-1	-5	-8	-12	-17	-23	-29	-31	-24	-11	6	20	28	30	27	22	16	11	8
-25	12	9	8	7	5	1	-3	-5	-3	1	3	3	-1	-4	-5	-4	-1	-1	-3	-6	-10	-14	-19	-25	-30	-31	-25	-13	2	15	24	28	27	25	21	16	12
-30	15	12	10	8	6	2	-2	-4	-2	1	4	3	-0	-4	-6	-5	-4	-3	-5	-7	-11	-14	-19	-24	-29	-29	-24	-14	-2	9	18	23	26	26	24	20	15
-35	18	14	11	9	7	3	-1	-3	-1	2	4	4	0	-4	-7	-7	-6	-5	-6	-8	-11	-13	-17	-21	-24	-25	-21	-15	-6	3	11	17	23	26	26	23	18
-40	20	15	11	9	6	3	-0	-2	-1	2	4	3	-0	-4	-7	-8	-7	-6	-7	-8	-9	-11	-13	-15	-17	-18	-17	-14	-9	-3	3	11	18	24	26	24	20
-45	20	15	11	8	5	2	-1	-2	-1	2	3	3	-0	-4	-7	-8	-8	-7	-7	-7	-7	-7	-7	-8	-10	-12	-12	-12	-11	-8	-3	4	13	20	24	23	20
-50	18	14	9	6	3	0	-1	-2	-1	1	2	2	-0	-4	-6	-8	-8	-7	-6	-5	-4	-3	-2	-1	-3	-5	-8	-11	-12	-12	-8	-2	7	15	20	21	18
-55	15	11	7	3	1	-2	-3	-3	-2	0	1	1	-0	-3	-5	-6	-6	-5	-4	-2	-0	2	4	5	4	1	-3	-8	-12	-14	-12	-6	2	9	15	17	15
-60	10	7	4	1	-2	-4	-4	-4	-2	-1	1	1	0	-1	-3	-4	-4	-3	-1	1	4	7	9	10	9	5	-0	-6	-11	-14	-14	-9	-3	4	9	11	10
-65	5	3	0	-2	-4	-5	-6	-5	-3	-1	-0	1	1	0	-0	-1	-0	1	3	5	8	11	13	13	11	8	2	-4	-10	-13	-14	-11	-7	-2	2	5	5
-70	-2	-2	-4	-5	-7	-7	-7	-6	-4	-2	-0	1	2	2	2	3	4	5	7	9	12	14	15	15	13	9	4	-2	-7	-11	-13	-12	-10	-7	-4	-2	-2
-75	-7	-7	-8	-8	-9	-9	-8	-6	-4	-2	-0	1	3	4	5	6	8	9	11	13	15	16	16	15	13	9	5	-1	-5	-9	-12	-12	-12	-10	-9	-8	-7
-80	-12	-12	-12	-11	-11	-10	-8	-7	-5	-2	-0	2	4	6	8	10	11	13	14	16	16	17	16	15	12	9	5	1	-3	-7	-10	-12	-12	-13	-13	-12	-12
-85	-15	-15	-14	-14	-12	-11	-9	-7	-5	-2	0	3	6	8	10	12	14	15	16	17	17	17	15	14	11	8	5	2	-2	-5	-8	-10	-12	-14	-15	-15	-15
-90	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17

$\Delta\delta$ in 10^{-1} sec of arc

Δδ at 10,000 km elevation ($\omega = 0$)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12	+13	+14	+15	+16	+17	+18
90	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
85	-7	-8	-8	-8	-8	-8	-7	-5	-3	-1	2	5	7	10	12	13	14	14	13	12	10	8	5	3	0	-2	-4	-5	-6	-6	-7	-7	-7	-7	-7	-7	-7
80	-4	-5	-7	-8	-9	-10	-9	-8	-6	-4	-0	3	7	10	13	15	16	16	15	13	11	7	4	0	-3	-5	-7	-8	-8	-7	-6	-5	-3	-3	-2	-3	-4
75	-0	-2	-5	-8	-10	-11	-12	-11	-9	-6	-3	2	6	10	13	16	17	17	16	14	11	7	2	-2	-6	-8	-10	-10	-9	-7	-4	-2	0	2	2	2	-0
70	4	1	-3	-7	-10	-13	-14	-13	-11	-8	-4	-0	5	9	13	16	17	18	17	14	10	6	1	-4	-8	-11	-13	-12	-10	-7	-3	1	4	6	7	6	4
65	8	4	-1	-6	-10	-13	-15	-15	-13	-10	-6	-2	3	8	12	15	17	17	16	13	9	4	-1	-6	-11	-14	-15	-14	-10	-6	-1	4	8	11	12	10	8
60	11	6	1	-5	-10	-13	-15	-16	-14	-11	-7	-3	1	6	10	13	15	15	14	12	8	3	-3	-8	-13	-16	-16	-15	-11	-5	1	7	12	15	16	14	11
55	14	9	3	-3	-9	-13	-15	-16	-14	-12	-8	-4	-0	4	7	10	12	13	12	10	6	1	-4	-10	-14	-17	-18	-15	-10	-4	4	10	16	19	19	18	14
50	17	11	5	-2	-8	-12	-14	-15	-14	-12	-9	-5	-2	2	5	8	10	10	10	8	4	-0	-6	-11	-16	-18	-18	-15	-10	-2	6	13	18	22	22	21	17
45	19	13	6	-0	-6	-10	-13	-14	-13	-11	-8	-6	-3	-1	2	4	6	7	7	5	2	-2	-7	-12	-16	-19	-19	-15	-9	-1	7	15	21	24	24	23	19
40	20	14	8	2	-4	-9	-11	-12	-12	-10	-8	-6	-4	-2	-0	1	3	4	4	3	0	-3	-8	-13	-17	-19	-18	-15	-8	0	9	16	22	25	26	24	20
35	20	15	9	3	-2	-7	-9	-10	-10	-8	-7	-6	-5	-4	-3	-1	0	1	1	0	-1	-4	-8	-13	-16	-18	-18	-14	-7	1	9	17	22	25	26	24	20
30	20	15	10	5	-0	-4	-7	-8	-8	-7	-6	-5	-5	-5	-4	-4	-3	-2	-1	-2	-3	-5	-8	-12	-15	-17	-16	-13	-6	1	9	16	21	24	24	23	20
25	19	15	10	6	1	-2	-5	-6	-5	-5	-4	-4	-5	-5	-6	-5	-5	-4	-3	-3	-4	-5	-8	-11	-14	-15	-14	-11	-6	1	8	14	19	22	22	21	19
20	17	14	10	6	3	-0	-2	-3	-3	-3	-2	-3	-4	-5	-6	-7	-6	-5	-5	-4	-4	-5	-7	-9	-11	-12	-12	-9	-5	0	6	11	15	18	19	18	17
15	14	12	9	7	4	2	-0	-1	-1	-1	-1	-2	-3	-5	-6	-7	-7	-7	-6	-5	-4	-4	-5	-6	-8	-9	-9	-7	-4	-0	4	8	11	14	15	15	14
10	10	9	8	6	5	3	2	1	1	1	1	-1	-2	-4	-6	-7	-7	-7	-6	-5	-4	-3	-3	-3	-4	-5	-5	-5	-4	-2	1	3	6	8	10	10	10
5	6	7	6	6	5	4	4	3	3	3	2	1	-1	-3	-5	-7	-7	-7	-6	-4	-2	-1	0	1	1	-0	-1	-2	-2	-2	-2	-1	1	2	4	5	6
-0	21	28	31	29	24	16	8	5	10	14	14	12	10	9	8	7	3	-2	0	8	16	23	27	27	21	10	4	17	30	37	38	33	23	9	-2	10	21
-5	4	2	-0	-2	-4	-5	-6	-6	-6	-5	-3	-2	-0	1	3	5	5	5	4	2	-1	-3	-6	-8	-9	-9	-7	-3	1	6	10	12	12	10	8	6	4
-10	9	5	2	-1	-4	-6	-7	-7	-6	-5	-4	-3	-1	0	2	3	4	4	3	0	-2	-6	-9	-12	-14	-14	-11	-7	-0	6	11	15	16	16	14	12	9
-15	13	9	5	1	-3	-6	-7	-8	-7	-6	-4	-3	-2	-0	1	2	3	3	1	-1	-4	-8	-11	-15	-17	-18	-15	-10	-2	5	12	17	20	21	19	17	13
-20	17	13	8	3	-2	-5	-8	-8	-8	-6	-4	-3	-2	-1	-0	1	2	1	0	-2	-5	-9	-13	-17	-20	-20	-18	-12	-4	4	12	19	23	24	24	21	17
-25	21	15	10	4	-1	-5	-8	-9	-8	-6	-5	-3	-2	-2	-1	-0	0	0	-1	-3	-6	-10	-14	-18	-21	-22	-20	-15	-7	3	11	19	24	27	27	25	21
-30	23	18	11	5	-0	-5	-8	-9	-8	-6	-5	-3	-2	-2	-1	-1	-0	-0	-1	-3	-6	-10	-14	-18	-22	-23	-21	-16	-9	0	10	18	24	28	29	27	23
-35	24	19	12	6	-0	-5	-8	-9	-8	-6	-5	-3	-2	-2	-1	-1	-0	-0	-1	-2	-5	-8	-13	-17	-21	-22	-21	-17	-10	-2	7	16	23	28	29	28	24
-40	24	19	12	6	-0	-5	-8	-9	-8	-6	-5	-3	-2	-2	-1	-1	-0	0	-0	-1	-3	-7	-11	-15	-19	-21	-21	-18	-12	-4	5	13	21	26	28	27	24
-45	23	18	12	5	-0	-5	-8	-9	-8	-7	-5	-3	-2	-1	-1	0	1	1	1	0	-1	-4	-8	-12	-16	-18	-19	-17	-13	-6	2	10	17	23	25	25	23
-50	20	16	10	4	-1	-5	-8	-9	-8	-7	-5	-3	-2	-1	0	1	2	3	3	3	1	-1	-5	-9	-12	-16	-17	-16	-13	-8	-1	6	13	19	22	22	20
-55	16	12	8	3	-2	-6	-8	-9	-8	-7	-5	-3	-1	0	2	3	4	5	6	5	4	2	-1	-5	-9	-12	-14	-15	-13	-9	-3	3	9	14	17	18	16
-60	11	9	5	0	-4	-7	-8	-9	-8	-7	-5	-2	-0	1	3	5	6	7	8	8	7	5	2	-1	-5	-9	-12	-13	-12	-9	-5	-0	5	9	12	13	11
-65	6	4	1	-2	-5	-7	-9	-9	-8	-6	-4	-2	0	3	5	7	8	10	11	11	10	8	5	2	-2	-6	-9	-10	-11	-9	-7	-3	1	4	6	7	6
-70	1	-0	-2	-5	-7	-8	-9	-9	-8	-6	-4	-1	1	4	6	9	11	12	13	13	12	11	8	5	1	-2	-6	-8	-9	-9	-8	-5	-3	-1	1	2	1
-75	-4	-5	-6	-7	-8	-9	-9	-9	-7	-5	-3	-0	3	5	8	10	12	14	15	15	14	12	10	7	4	0	-3	-5	-7	-8	-8	-7	-6	-5	-4	-4	-4
-80	-8	-9	-9	-10	-10	-10	-9	-8	-6	-4	-2	1	4	7	9	12	13	15	16	16	15	13	11	9	6	3	-0	-3	-5	-7	-7	-8	-8	-8	-8	-8	-8
-85	-12	-12	-12	-12	-11	-10	-9	-7	-5	-3	-0	2	5	8	10	12	14	15	15	15	15	14	12	10	7	5	2	-1	-3	-5	-7	-8	-9	-10	-11	-11	-12
-90	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14

Δδ in 10^{-2} sec of arc

Δδ at 100,000 km elevation ($\omega = 0$)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12	+13	+14	+15	+16	+17	+18
90	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
85	1	-0	-2	-4	-6	-7	-8	-8	-7	-5	-3	-0	3	5	8	10	10	11	10	8	6	3	1	-2	-4	-5	-6	-6	-5	-4	-2	-1	1	2	2	2	1
80	7	4	0	-4	-8	-11	-13	-14	-13	-10	-7	-2	3	7	12	14	16	16	14	12	8	3	-2	-6	-9	-11	-12	-11	-9	-5	-2	2	5	8	9	8	7
75	12	8	2	-4	-10	-14	-18	-19	-18	-15	-10	-4	3	9	15	19	21	21	19	14	9	2	-4	-10	-15	-17	-18	-16	-12	-7	-1	5	10	14	15	15	12
70	17	12	5	-3	-11	-17	-22	-24	-23	-19	-13	-6	2	10	17	22	25	25	22	17	10	2	-7	-14	-20	-23	-23	-20	-15	-8	-0	8	14	19	21	21	17
65	22	15	7	-3	-12	-20	-25	-28	-27	-23	-16	-8	2	11	19	25	28	28	25	19	10	1	-9	-17	-24	-27	-27	-24	-18	-9	0	10	18	24	27	26	22
60	26	19	9	-2	-13	-22	-28	-31	-30	-26	-18	-9	1	12	21	27	30	30	27	20	11	-0	-11	-20	-27	-31	-31	-27	-20	-10	1	12	22	28	31	31	26
55	30	21	10	-2	-13	-23	-30	-33	-32	-28	-20	-10	1	12	21	28	32	32	28	20	11	-1	-12	-22	-30	-34	-34	-30	-21	-11	2	14	24	32	35	34	30
50	32	23	12	-1	-13	-23	-30	-34	-33	-29	-21	-11	0	11	21	28	32	32	28	21	10	-1	-13	-24	-32	-36	-36	-31	-22	-11	2	15	26	34	38	37	32
45	33	24	13	0	-12	-23	-30	-34	-33	-29	-21	-11	-0	11	20	27	31	31	27	20	10	-2	-14	-24	-32	-37	-36	-31	-23	-11	3	16	27	35	39	38	33
40	33	25	13	1	-11	-22	-29	-33	-32	-28	-21	-11	-1	10	19	26	30	30	26	19	9	-2	-14	-24	-32	-36	-36	-31	-22	-10	3	16	27	35	39	38	33
35	32	24	13	1	-10	-20	-27	-31	-30	-27	-20	-11	-1	9	17	24	27	27	24	17	8	-2	-13	-23	-30	-34	-34	-29	-21	-10	3	15	26	34	38	37	32
30	30	23	13	2	-9	-18	-24	-28	-28	-24	-18	-10	-1	7	15	21	24	24	21	16	7	-2	-12	-21	-28	-31	-31	-27	-19	-9	3	14	24	31	35	34	30
25	27	20	12	2	-7	-15	-21	-24	-24	-21	-16	-9	-2	6	13	18	21	21	18	13	6	-2	-11	-18	-24	-27	-27	-24	-17	-8	2	12	21	28	31	30	27
20	23	17	10	2	-5	-12	-17	-19	-20	-17	-13	-8	-2	5	10	14	16	17	15	11	5	-1	-8	-15	-20	-23	-23	-20	-14	-7	2	10	18	23	26	26	23
15	18	14	8	3	-3	-9	-12	-15	-15	-13	-10	-6	-1	3	7	10	12	12	11	8	4	-1	-6	-11	-15	-17	-17	-15	-11	-5	1	8	13	17	20	20	18
10	12	10	6	3	-1	-5	-8	-9	-10	-9	-7	-4	-1	2	4	6	7	8	7	6	3	0	-3	-7	-9	-11	-11	-10	-7	-3	1	5	9	11	13	13	12
5	6	6	4	3	1	-1	-3	-4	-4	-4	-3	-2	-0	1	2	2	3	3	3	3	3	1	-0	-2	-4	-5	-5	-4	-3	-1	1	3	4	5	6	6	6
-0	38	56	67	69	63	49	29	7	13	34	50	60	63	59	48	31	10	9	30	49	62	67	64	53	35	12	11	35	55	69	73	68	54	33	8	15	38
-5	7	6	3	0	-2	-5	-7	-7	-7	-6	-3	-1	2	4	5	6	7	6	5	4	2	-1	-3	-6	-7	-8	-8	-6	-3	0	3	6	8	9	9	9	7
-10	13	10	5	0	-4	-8	-11	-13	-12	-10	-7	-3	1	5	8	10	11	11	9	6	3	-2	-6	-10	-13	-14	-14	-11	-7	-2	3	8	12	15	16	15	13
-15	19	14	8	1	-6	-12	-15	-17	-17	-15	-11	-6	0	6	10	13	15	15	13	9	4	-2	-9	-14	-18	-20	-19	-16	-11	-4	3	10	16	20	22	22	19
-20	24	17	10	1	-7	-14	-19	-22	-21	-19	-14	-7	-0	6	12	17	19	19	16	11	5	-3	-11	-17	-23	-25	-24	-21	-14	-6	3	12	20	25	28	27	24
-25	28	20	11	1	-8	-17	-22	-25	-25	-22	-16	-9	-1	7	14	19	22	22	19	14	6	-3	-12	-20	-26	-29	-29	-24	-17	-7	3	14	23	29	32	32	28
-30	31	23	13	1	-9	-18	-25	-28	-28	-25	-18	-10	-1	8	16	22	25	25	22	16	7	-3	-13	-22	-29	-32	-32	-27	-19	-9	3	15	25	32	36	35	31
-35	32	24	13	2	-10	-20	-27	-31	-30	-27	-20	-11	-1	9	17	24	27	27	24	17	8	-3	-13	-23	-31	-34	-34	-30	-21	-10	3	15	26	34	38	37	32
-40	33	25	14	1	-11	-21	-28	-32	-32	-28	-21	-11	-1	9	18	25	29	29	25	19	9	-2	-13	-24	-31	-35	-35	-31	-22	-11	2	15	26	34	38	38	33
-45	33	24	13	1	-11	-21	-29	-32	-32	-28	-21	-12	-1	10	19	26	30	30	26	20	10	-1	-13	-23	-31	-35	-35	-31	-23	-11	2	14	26	34	38	38	33
-50	31	23	12	1	-11	-21	-28	-32	-32	-28	-21	-11	-1	10	19	26	30	30	27	20	11	-0	-12	-22	-30	-34	-34	-30	-22	-12	1	13	24	32	36	36	31
-55	29	21	11	-0	-11	-20	-27	-31	-30	-26	-20	-11	-0	10	19	26	29	30	26	20	11	0	-10	-20	-28	-32	-32	-29	-21	-11	0	12	22	29	33	33	29
-60	25	18	9	-1	-11	-19	-25	-28	-28	-24	-18	-9	0	10	18	24	28	28	25	19	11	1	-9	-18	-25	-29	-29	-26	-20	-11	-0	10	19	26	29	29	25
-65	21	15	7	-1	-10	-18	-23	-25	-25	-22	-16	-8	1	9	17	23	26	26	24	18	11	2	-7	-15	-21	-25	-26	-23	-18	-10	-1	8	16	22	25	24	21
-70	16	12	5	-2	-9	-15	-20	-22	-21	-18	-13	-6	1	9	15	20	23	23	21	17	10	3	-5	-12	-17	-20	-21	-19	-15	-9	-2	6	12	17	19	19	16
-75	11	8	3	-3	-8	-13	-16	-17	-17	-14	-10	-5	2	8	13	17	19	20	18	14	9	4	-3	-8	-13	-15	-16	-15	-12	-7	-2	3	8	12	13	13	11
-80	6	3	0	-3	-7	-10	-12	-13	-12	-10	-7	-3	2	6	10	13	15	15	14	12	8	4	-0	-4	-8	-10	-11	-10	-8	-6	-2	1	4	6	7	7	6
-85	1	-1	-2	-4	-5	-7	-7	-7	-6	-5	-3	-0	2	5	7	9	10	10	10	8	6	4	2	-1	-3	-4	-5	-5	-5	-4	-2	-1	0	1	1	1	1
-90	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5

Δδ in 10^{-4} sec of arc

Number Map 3.--Radius of curvature of the orthogonal trajectories (length unit: km). (See page 55 for explanation of first column.)

ρ at sea level (ω ≠ 0)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
9	26192	26952	27782	28662	29582	30522	31452	32342	33112	33732	34112	34272	34012	33512	32742	31772	30682	29542	28422
8	35380	35290	35180	35070	34990	34930	34920	34940	35010	35100	35210	35330	35450	35550	35610	35640	35630	35560	35470
7	18890	18850	18800	18760	18730	18710	18720	18740	18770	18800	18830	18870	18900	18950	18980	19010	19000	18980	18940
6	14010	13990	13970	13950	13940	13940	13960	13980	13990	14000	13990	13990	13990	14000	14030	14060	14070	14060	14030
5	12220	12220	12220	12210	12210	12210	12220	12240	12250	12250	12230	12190	12170	12170	12190	12220	12240	12240	12220
4	12170	12190	12200	12210	12200	12200	12190	12200	12220	12220	12200	12160	12110	12090	12110	12150	12170	12170	12160
3	13770	13780	13820	13840	13840	13810	13770	13760	13790	13830	13840	13800	13730	13680	13670	13700	13740	13750	13740
2	18520	18530	18590	18660	18680	18600	18490	18430	18460	18580	18670	18670	18570	18450	18380	18390	18440	18470	18470
1	35610	35470	35610	35900	36050	35870	35490	35180	35180	35550	36030	36240	36070	35680	35270	35090	35170	35340	35460
0	20642	26353	27262	37942	23632	11412	15112	49582	97441	93461	31862	11472	81941	15622	28892	11852	17912	34452	18372
-1	34680	34930	35110	35130	34980	34750	34620	34680	34870	35040	35080	34890	34660	34640	34870	35170	35340	35250	35000
-2	18420	18450	18520	18580	18580	18500	18410	18370	18420	18510	18580	18560	18480	18410	18420	18480	18550	18550	18500
-3	13900	13890	13920	13960	13980	13950	13890	13860	13870	13920	13960	13960	13910	13860	13840	13860	13900	13910	13890
-4	12230	12210	12210	12230	12250	12240	12210	12180	12180	12200	12220	12220	12190	12150	12130	12140	12160	12170	12170
-5	12290	12250	12240	12240	12240	12240	12230	12220	12220	12230	12240	12230	12210	12180	12160	12160	12170	12180	12180
-6	14190	14160	14130	14110	14100	14100	14100	14100	14110	14120	14130	14130	14120	14100	14080	14070	14070	14070	14080
-7	19030	19010	18970	18950	18930	18930	18940	18960	18990	19020	19040	19050	19050	19040	19030	19010	19000	19000	19020
-8	35110	35110	35100	35100	35120	35160	35220	35300	35390	35490	35570	35640	35700	35730	35750	35770	35780	35800	35830
-9	12772	12832	13002	13272	13662	14182	14862	15732	16842	18272	20122	22572	25912	30542	37772	49452	71102	11633	15583

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
9	28422	27352	26392	25532	24802	24202	23722	23362	23122	22992	22982	23052	23252	23522	23892	24352	24882	25502	26192
8	35470	35350	35230	35120	35010	34930	34870	34830	34830	34860	34930	35020	35130	35250	35350	35420	35460	35440	35380
7	18940	18900	18860	18840	18820	18800	18770	18740	18710	18690	18700	18720	18760	18800	18850	18890	18910	18910	18890
6	14030	14010	14010	14030	14050	14050	14030	14000	13960	13930	13920	13930	13940	13950	13970	13990	14000	14010	14010
5	12220	12210	12230	12270	12310	12320	12300	12250	12220	12210	12210	12220	12220	12210	12200	12190	12200	12210	12220
4	12160	12150	12180	12240	12280	12280	12240	12200	12180	12200	12240	12270	12270	12240	12200	12170	12160	12160	12170
3	13740	13730	13760	13810	13830	13810	13750	13700	13720	13810	13910	13980	13980	13930	13870	13820	13780	13770	13770
2	18470	18470	18470	18470	18440	18360	18270	18250	18350	18530	18730	18850	18870	18810	18750	18690	18640	18570	18520
1	35460	35460	35300	35040	34760	34520	34440	34630	34990	35360	35640	35780	35820	35890	36060	36220	36190	35930	35610
0	18372	48432	26922	18092	18122	89241	81001	21432	13652	66051	68511	10352	12872	16992	70722	29752	17432	15012	20642
-1	35000	34840	34880	35020	35010	34670	34160	33870	34050	34760	35770	36600	36810	36400	35670	35010	34620	34540	34680
-2	18500	18430	18400	18380	18310	18190	18040	17990	18090	18350	18660	18890	18960	18890	18750	18610	18510	18440	18420
-3	13890	13860	13830	13800	13770	13710	13660	13650	13700	13810	13920	14000	14030	14030	14020	14000	13980	13940	13900
-4	12170	12150	12140	12130	12120	12120	12110	12110	12120	12140	12150	12160	12170	12190	12230	12270	12290	12270	12230
-5	12180	12180	12190	12200	12220	12250	12260	12260	12240	12200	12160	12130	12130	12160	12210	12270	12310	12310	12290
-6	14080	14090	14120	14160	14200	14240	14260	14250	14200	14130	14060	14010	13990	14020	14080	14140	14190	14210	14190
-7	19020	19050	19090	19150	19200	19240	19250	19220	19150	19050	18960	18880	18850	18860	18900	18960	19010	19040	19030
-8	35830	35880	35930	35980	36010	36000	35940	35840	35690	35520	35350	35200	35100	35040	35030	35050	35070	35090	35110
-9	15583	99582	63092	45272	35272	28992	24742	21702	19462	17752	16432	15402	14502	13982	13502	13162	12932	12802	12772

scaling: $\times 10^3$

ρ at sea level (ω = 0)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
9	86732	79972	74572	70212	66672	63792	61452	59602	58172	57152	56522	56282	55442	57352	58142	59772	62362	65112	69132
8	11631	11721	11851	11971	12071	12141	12161	12131	12051	11941	11811	11681	11551	11451	11381	11361	11371	11431	11531
7	60790	61190	61700	62210	62580	62740	62670	62430	62110	61760	61410	61050	60650	60230	59870	59660	59670	59910	60300
6	44830	44970	45190	45400	45530	45520	45370	45160	44990	44930	44990	45070	45050	44870	44570	44310	44220	44340	44580
5	40050	40020	40040	40100	40140	40120	40000	39820	39690	39710	39950	40310	40580	40570	40300	39950	39760	39820	40010
4	40020	39880	39730	39650	39680	39770	39810	39720	39550	39480	39680	40160	40680	40900	40700	40300	40010	40010	40180
3	45780	45590	45250	44960	44970	45310	45720	45840	45560	45140	45020	45430	46210	46810	46840	46450	46070	45960	46110
2	61350	61310	60650	59880	59720	60480	61720	62420	61950	60750	59750	59690	60740	62150	62840	62720	62260	61860	61830
1	10671	10801	10661	10421	10291	10411	10771	11071	10991	10671	10311	10091	10211	10611	10951	11091	11071	10911	10781
0	20602	26543	27442	38142	23562	11412	15132	49562	97521	93531	31782	11482	82021	15632	28922	11862	17922	34532	18402
-1	11551	11331	11141	11121	11281	11471	11621	11601	11321	11101	11171	11341	11501	11591	11401	11061	10911	11011	11251
-2	62410	62100	61340	60700	60760	61550	62600	63090	62460	61310	60690	60910	61720	62510	62530	61760	61060	61050	61660
-3	44330	44440	44170	43730	43570	43860	44430	44830	44690	44160	43750	43790	44250	44790	45010	44780	44410	44270	44450
-4	39370	39640	39630	39400	39240	39340	39650	39920	39920	39680	39470	39510	39830	40240	40460	40400	40200	40060	40100
-5	39320	39640	39820	39820	39760	39790	39910	40030	40030	39930	39840	39900	40120	40410	40610	40650	40570	40490	40450
-6	43070	43370	43650	43820	43890	43910	43910	43880	43810	43700	43630	43650	43780	43960	44130	44230	44260	44240	44160
-7	59420	59660	59980	60240	60390	60390	60270	60060	59790	59530	59330	59220	59220	59320	59460	59600	59680	59680	59540
-8	11941	11941	11941	11941	11911	11871	11791	11701	11601	11501	11421	11341	11291	11261	11241	11231	11221	11201	11171
-9	31982	31692	30982	29972	28752	27452	26162	24922	23792	22772	21382	21112	20452	19922	19492	19172	18942	18812	18772

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
9	69132	74362	81192	90172	10223	11853	14133	17343	21723	26563	28583	25513	20933	17023	14193	12153	10663	95352	85732
8	11531	11651	11781	11921	12041	12141	12221	12261	12271	12231	12141	12031	11901	11771	11661	11581	11551	11561	11631
7	60300	60740	61100	61370	61580	61810	62110	62470	62780	62940	62900	62630	62210	61710	61220	60830	60600	60580	60790
6	44580	44770	44780	44610	44430	44400	44600	44980	45370	45650	45750	45700	45580	45430	45250	45060	44900	44810	44830
5	40010	40090	39880	39450	39060	38960	39200	39630	40000	40160	40110	40030	40030	40140	40250	40290	40230	40130	40050
4	40180	40230	39900	39330	38910	38890	39250	39730	39940	39730	39290	38990	39020	39350	39750	40050	40160	40130	40020
3	46110	46170	45840	45300	45060	45310	45910	46420	46260	45340	44240	43580	43570	44050	44710	45250	45590	45760	45780
2	61830	61990	61910	61840	62290	63150	64030	64400	63350	61080	59010	57960	57870	58350	58990	59570	60150	60820	61350
1	10781	10811	10941	11171	11511	11681	11681	11611	11241	10711	10461	10471	10491	10411	10271	10151	10181	10391	10671
0	18402	48452	27012	18152	18152	89281	81041	21422	13662	66091	68511	10342	12882	17032	71292	29742	17422	14992	20602
-1	11251	11431	11381	11231	11251	11571	12141	12611	12341	11421	10481	98670	97190	10011	10601	11241	11661	11721	11551
-2	61660	62370	62760	63000	63770	65360	67240	68050	66600	63340	59920	57670	57010	57630	58890	60320	61520	62210	62410
-3	44450	44790	45100	45370	45780	46400	46990	47110	46490	45320	44140	43380	43100	43070	43120	43260	43550	43950	44330
-4	40100	40240	40380	40460	40520	40620	40720	40710	40540	40310	40160	40110	40040	39760	39310	38930	38820	39010	39370
-5	40450	40440	40360	40180	39940	39720	39590	39600	39760	40100	40550	40950	41030	40680	40020	39390	39030	39040	39320
-6	44160	44010	43740	43370	42950	42600	42430	42520	42890	43520	44250	44840	45040	44770	44160	43500	43050	42920	43070
-7	59540	59260	58820	58300	57790	57430	57340	57590	58200	59090	60070	60870	61270	61180	60700	60100	59610	59370	59420
-8	11171	11131	11081	11031	11001	11001	11051	11141	11281	11451	11641	11801	11931	12001	12021	12011	11981	11951	11941
-9	18772	18832	18982	19222	19572	20022	20592	21272	22072	23002	24052	25222	26482	27792	29382	30262	31212	31812	31982

scaling: $\times 10^3$

ρ at 1,000 km elevation (ω = 0)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
9	17513	14853	12963	11573	10513	96802	90302	85152	81092	77922	75522	73802	72702	72192	72262	72932	74232	76232	79012
8	17851	17941	18041	18151	18231	18291	18291	18251	18161	18051	17911	17771	17631	17521	17441	17401	17411	17461	17551
7	94190	94590	95100	95610	96000	96180	96140	95910	95570	95190	94780	94360	93900	93450	93050	92800	92770	92950	93290
6	69850	70030	70280	70520	70690	70730	70620	70430	70240	70140	70130	70150	70090	69890	69600	69340	69230	69300	69490
5	61730	61750	61820	61910	61990	62010	61920	61760	61610	61610	61790	62080	62310	62300	62060	61750	61560	61580	61710
4	61830	61740	61650	61620	61670	61780	61820	61730	61560	61480	61660	62100	62580	62810	62660	62310	62040	62010	62120
3	70320	70190	69930	69710	69750	70070	70430	70510	70240	69880	69810	70240	70990	71580	71670	71350	71000	70880	70970
2	93840	93860	93340	92740	92650	93340	94390	94940	94510	93490	92700	92820	93900	95260	96000	95960	95550	95210	95160
1	17041	17171	17071	16861	16761	16891	17221	17471	17401	17101	16771	16611	16771	17191	17571	17741	17731	17591	17471
0	37802	92972	99832	83122	41002	27282	39582	72612	23672	23402	55052	26192	19672	34312	85092	32812	47072	14613	58472
-1	18011	17821	17661	17631	17751	17941	18081	18051	17821	17641	17691	17861	18021	18071	17871	17541	17361	17441	17661
-2	94750	94570	94000	93510	93590	94310	95220	95600	95060	94120	93600	93850	94640	95340	95330	94650	94000	93970	94520
-3	69220	69390	69220	68910	68830	69140	69660	69990	69830	69350	68990	69060	69530	70080	70300	70120	69810	69690	69880
-4	60730	61010	61070	60960	60900	61040	61320	61520	61480	61250	61060	61120	61450	61860	62110	62100	61960	61860	61890
-5	60520	60830	61050	61120	61150	61220	61330	61410	61380	61250	61160	61220	61460	61760	62000	62090	62060	61990	61950
-6	68100	68380	68670	68880	69000	69060	69070	69020	68910	68780	68700	68720	68870	69080	69290	69430	69470	69430	69310
-7	92540	92720	93000	93250	93390	93400	93290	93060	92780	92500	92280	92170	92180	92280	92420	92530	92570	92480	92250
-8	18061	18051	18041	18021	17991	17931	17851	17761	17651	17541	17451	17361	17301	17251	17211	17171	17131	17091	17031
-9	62382	62242	60912	58652	55812	52742	49682	46812	44212	41912	39912	38202	36762	35572	34612	33862	33322	32962	32792

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
9	79012	82722	87562	93812	10193	11253	12653	14583	17343	21553	28533	42803	82903	20574	75213	40973	28123	21513	17513
8	17551	17671	17811	17951	18091	18221	18331	18401	18441	18431	18371	18281	18161	18031	17921	17841	17801	17801	17851
7	93290	93690	94060	94390	94690	95040	95450	95880	96240	96420	96360	96080	95640	95120	94620	94220	93990	93980	94190
6	69490	69640	69660	69550	69450	69510	69770	70170	70550	70780	70830	70720	70540	70350	70160	69980	69850	69800	69850
5	61710	61750	61560	61200	60900	60870	61130	61540	61880	61980	61870	61710	61640	61680	61760	61810	61800	61760	61730
4	62120	62120	61820	61330	60980	61000	61370	61810	61970	61720	61240	60860	60800	61030	61380	61680	61840	61870	61830
3	70970	70990	70690	70230	70020	70260	70820	71260	71070	70170	69070	68330	68200	68560	69130	69650	70020	70250	70320
2	95160	95250	95170	95100	95430	96140	96930	97190	96230	94190	92180	91010	90750	91070	91600	92120	92690	93340	93840
1	17471	17481	17591	17791	18051	18191	18211	18111	17781	17321	17051	17011	17001	16901	16731	16591	16601	16781	17041
0	58472	13713	82662	69792	56172	22192	19732	44852	34182	16122	15652	22422	34702	57072	48143	56712	34462	29942	37802
-1	17661	17841	17851	17781	17861	18201	18731	19091	18801	17901	16901	16211	16011	16281	16871	17511	17971	18111	18011
-2	94520	95220	95700	96090	96910	98360	99940	10051	99120	96100	92760	90380	89490	89890	91030	92400	93610	94420	94750
-3	69880	70220	70550	70860	71280	71870	72410	72500	71880	70710	69460	68550	68090	67950	67940	68080	68380	68820	69220
-4	61890	62020	62140	62220	62280	62390	62490	62500	62360	62120	61900	61730	61530	61180	60720	60340	60210	60390	60730
-5	61950	61900	61790	61610	61380	61200	61120	61170	61370	61690	62070	62350	62340	61950	61310	60690	60310	60290	60520
-6	69310	69100	68790	68380	67970	67660	67540	67710	68160	68830	69560	70120	70290	69990	69360	68670	68170	67990	68100
-7	92250	91870	91360	90810	90320	90020	90030	90410	91160	92170	93220	94070	94510	94450	94000	93390	92860	92570	92540
-8	17031	16961	16901	16851	16821	16841	16911	17041	17211	17421	17641	17841	17991	18091	18131	18131	18111	18081	18061
-9	32792	32802	32992	33372	33942	34722	35722	36952	38452	40222	42292	44682	47352	50302	53402	56472	59222	61312	62382

scaling: × 10^3

ρ at 10,000 km elevation (ω = 0)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
9	50643	33673	25163	20163	16903	14633	12983	11743	10783	10043	94512	90092	86602	84002	82162	81012	80512	80632	81402
8	19551	19571	19581	19601	19611	19601	19591	19561	19521	19471	19421	19361	19301	19251	19201	19171	19151	19151	19171
7	10341	10361	10371	10391	10401	10411	10411	10401	10391	10381	10361	10341	10321	10301	10291	10271	10271	10261	10271
6	76610	76720	76850	76970	77060	77130	77150	77130	77080	77010	76930	76850	76770	76690	76610	76530	76480	76460	76460
5	67290	67390	67490	67590	67680	67740	67760	67760	67720	67690	67650	67630	67610	67580	67540	67490	67450	67420	67410
4	67210	67300	67390	67490	67570	67640	67670	67660	67640	67610	67610	67620	67650	67670	67670	67650	67610	67580	67560
3	76300	76390	76490	76580	76680	76760	76800	76810	76780	76760	76760	76820	76910	76990	77040	77050	77020	76990	76960
2	10251	10261	10271	10291	10301	10311	10321	10321	10321	10311	10311	10331	10351	10371	10381	10391	10391	10391	10381
1	19151	19171	19181	19201	19221	19241	19261	19271	19271	19261	19271	19301	19361	19431	19501	19551	19561	19561	19531
0	93532	82512	80972	85192	97662	12863	17753	15273	11893	12073	15883	22183	27153	45993	68373	30733	26063	34443	89673
-1	19421	19461	19501	19541	19571	19601	19621	19621	19601	19591	19571	19571	19561	19541	19511	19481	19451	19451	19481
-2	10281	10301	10311	10331	10351	10361	10371	10371	10371	10361	10351	10351	10361	10371	10371	10361	10361	10361	10371
-3	76080	76210	76340	76470	76580	76680	76740	76750	76720	76670	76640	76660	76720	76790	76840	76860	76860	76860	76880
-4	66820	66920	67030	67140	67240	67310	67360	67360	67340	67310	67290	67300	67350	67410	67470	67500	67510	67500	67500
-5	66820	66900	66990	67090	67170	67240	67270	67270	67250	67220	67200	67210	67240	67290	67340	67370	67370	67360	67330
-6	76090	76140	76220	76300	76380	76430	76450	76440	76410	76370	76340	76330	76340	76360	76390	76390	76380	76340	76290
-7	10291	10291	10301	10301	10311	10311	10311	10301	10301	10291	10281	10271	10271	10261	10261	10251	10241	10231	10221
-8	19531	19531	19521	19511	19501	19481	19461	19431	19391	19351	19311	19261	19221	19181	19141	19101	19061	19031	19001
-9	42153	45573	36473	27073	20843	16823	14103	12193	10783	97232	89082	82722	77732	73812	70772	68442	66752	65612	64992

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
9	81402	82822	84962	87912	91802	96822	10323	11153	12213	13603	15483	18103	21963	28113	39173	63383	12064	92773	50643
8	19171	19201	19251	19301	19361	19431	19481	19531	19571	19591	19601	19601	19591	19571	19551	19541	19541	19541	19551
7	10271	10281	10291	10311	10331	10361	10381	10391	10401	10401	10401	10381	10371	10351	10341	10331	10331	10331	10341
6	76460	76500	76560	76650	76770	76900	77020	77110	77150	77120	77040	76910	76770	76640	76550	76500	76490	76530	76610
5	67410	67420	67440	67490	67560	67650	67740	67800	67820	67760	67650	67510	67370	67260	67180	67160	67170	67210	67290
4	67560	67550	67550	67560	67610	67680	67760	67820	67800	67710	67560	67390	67240	67120	67060	67050	67080	67130	67210
3	76960	76920	76900	76900	76930	77000	77080	77130	77090	76950	76740	76520	76330	76190	76120	76110	76150	76210	76300
2	10381	10371	10371	10361	10361	10371	10381	10381	10381	10361	10331	10301	10271	10251	10241	10231	10231	10241	10251
1	19531	19501	19471	19441	19421	19421	19421	19431	19431	19411	19391	19351	19321	19271	19231	19181	19151	19141	19151
0	89673	89673	26283	13833	92332	76902	83142	13023	20653	97682	65432	57872	61572	77152	12053	30623	28403	12963	93532
-1	19481	19521	19581	19631	19701	19761	19811	19811	19741	19601	19441	19291	19181	19151	19171	19231	19301	19371	19421
-2	10371	10381	10391	10411	10431	10451	10471	10471	10441	10401	10351	10291	10251	10221	10211	10221	10231	10251	10281
-3	76880	76910	76970	77050	77140	77250	77330	77340	77260	77070	76800	76520	76260	76050	75920	75860	75880	75960	76080
-4	67500	67500	67510	67530	67570	67630	67690	67720	67710	67650	67530	67370	67200	67030	66880	66780	66740	66760	66820
-5	67330	67300	67280	67270	67280	67320	67380	67440	67490	67510	67490	67410	67300	67150	67010	66880	66810	66790	66820
-6	76290	76230	76170	76140	76150	76190	76270	76380	76500	76600	76650	76660	76600	76490	76360	76230	76130	76080	76090
-7	10221	10211	10201	10201	10201	10211	10231	10251	10271	10301	10311	10331	10331	10331	10321	10311	10301	10291	10291
-8	19001	18981	18971	18981	19001	19041	19091	19151	19221	19301	19361	19421	19471	19501	19521	19531	19541	19541	19531
-9	64992	64862	65222	66082	67482	69482	72152	75622	80062	85722	92962	10233	11453	13113	15413	18753	23833	31723	42153

scaling: × 10^4

ρ at 100,000 km elevation ($\omega = 0$)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
9	50162	49902	49312	48432	47332	46082	44752	43412	42102	40862	39732	38722	37342	37102	36512	36072	35772	35622	35612
8	53240	53280	53320	53370	53420	53460	53470	53470	53450	53410	53350	53280	53210	53140	53080	53040	53010	53010	53040
7	28290	28310	28340	28370	28400	28420	28440	28440	28430	28420	28390	28360	28330	28300	28280	28260	28250	28250	28260
6	20990	21000	21030	21050	21070	21090	21100	21100	21100	21090	21080	21060	21040	21020	21000	20990	20980	20980	20990
5	18450	18470	18480	18500	18520	18540	18550	18560	18550	18550	18540	18520	18500	18490	18470	18460	18460	18460	18460
4	18450	18460	18480	18500	18520	18540	18550	18550	18550	18550	18530	18520	18510	18490	18480	18470	18460	18460	18470
3	20970	20990	21010	21030	21050	21070	21080	21090	21090	21090	21070	21060	21050	21030	21020	21010	21000	21000	21010
2	28250	28270	28290	28320	28350	28380	28390	28400	28400	28400	28390	28370	28350	28340	28320	28310	28300	28300	28300
1	53060	53090	53130	53180	53230	53270	53300	53330	53330	53330	53320	53300	53280	53250	53230	53210	53200	53190	53190
0	50182	35462	30612	30342	34172	44822	75502	24173	12623	61852	44372	37982	36622	39272	47572	69712	16723	29833	78452
-1	53110	53170	53240	53310	53380	53430	53470	53490	53480	53450	53410	53360	53310	53260	53220	53190	53180	53190	53210
-2	28250	28280	28310	28340	28380	28400	28420	28430	28430	28420	28400	28380	28360	28330	28310	28300	28290	28300	28300
-3	20970	20980	21010	21030	21050	21070	21080	21090	21090	21080	21070	21060	21040	21030	21010	21000	21000	21000	21000
-4	18440	18450	18470	18490	18510	18520	18540	18540	18540	18530	18520	18510	18500	18480	18470	18460	18460	18460	18460
-5	18440	18450	18470	18490	18510	18520	18530	18540	18540	18530	18520	18510	18490	18480	18470	18460	18450	18450	18450
-6	20970	20990	21000	21020	21040	21060	21070	21080	21080	21070	21060	21040	21020	21000	20990	20980	20970	20970	20970
-7	28270	28290	28310	28330	28360	28380	28390	28400	28390	28380	28360	28340	28310	28290	28260	28240	28230	28230	28230
-8	53200	53230	53260	53300	53340	53370	53390	53390	53370	53340	53290	53230	53170	53110	53060	53010	52990	52980	52990
-9	50782	50632	50102	49242	48112	46802	45382	43942	42532	41192	39962	38852	37902	37042	36422	35902	35542	35322	35252

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
9	35612	35742	36022	36442	36992	37682	38512	39462	40532	41702	42352	44242	45542	46302	47942	48912	49642	50072	50162
8	53040	53080	53130	53200	53270	53330	53380	53420	53430	53430	53400	53370	53320	53280	53240	53210	53200	53210	53240
7	28260	28280	28300	28340	28370	28400	28420	28430	28440	28430	28410	28380	28350	28320	28300	28280	28270	28280	28290
6	20990	21000	21020	21040	21060	21080	21100	21110	21110	21100	21080	21060	21040	21010	20990	20980	20970	20980	20990
5	18460	18470	18490	18510	18530	18540	18560	18560	18560	18550	18540	18520	18500	18480	18460	18440	18440	18440	18450
4	18470	18480	18490	18510	18530	18540	18560	18560	18560	18550	18540	18520	18490	18470	18450	18440	18430	18440	18450
3	21010	21010	21030	21050	21070	21080	21100	21110	21100	21100	21080	21060	21030	21000	20980	20970	20960	20960	20970
2	28300	28310	28330	28350	28380	28400	28420	28430	28430	28420	28390	28360	28330	28300	28270	28250	28230	28240	28250
1	53190	53200	53210	53240	53280	53320	53350	53370	53380	53370	53330	53280	53230	53170	53120	53080	53060	53050	53060
0	78452	47222	36272	32292	32622	37922	54262	12633	17183	57612	36492	29362	27582	29712	37552	61782	26243	11383	50182
-1	53210	53250	53300	53360	53430	53480	53520	53530	53520	53480	53410	53320	53240	53160	53100	53060	53050	53070	53110
-2	28300	28320	28340	28370	28400	28430	28450	28450	28450	28430	28400	28370	28330	28290	28260	28240	28230	28230	28250
-3	21000	21010	21030	21050	21070	21090	21100	21110	21110	21090	21080	21050	21020	21000	20980	20960	20950	20950	20970
-4	18460	18470	18480	18500	18520	18530	18550	18550	18550	18540	18530	18510	18490	18470	18450	18430	18430	18430	18440
-5	18450	18460	18470	18490	18510	18520	18540	18540	18540	18540	18530	18510	18490	18470	18450	18440	18430	18430	18440
-6	20970	20980	21000	21020	21040	21050	21070	21080	21080	21080	21060	21050	21030	21000	20990	20970	20960	20960	20970
-7	28230	28250	28270	28300	28320	28350	28370	28390	28390	28390	28380	28360	28330	28310	28290	28270	28260	28260	28270
-8	52990	53010	53060	53110	53170	53230	53280	53320	53340	53350	53340	53310	53280	53250	53210	53190	53180	53180	53200
-9	35252	35332	35552	35922	36432	37092	37892	38832	39902	41092	42382	43752	45152	46552	47862	49312	49932	50542	50782

scaling: $\times 10^7$

Number Map 4.--Radius of torsion of the orthogonal trajectories (length unit: km). (See page 55 for explanation of first column.)

τ at sea level ($\omega \neq 0$)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
9	73E1	88E1	12E2	18E2	51E2	-55E2	-17E2	-10E2	-73E1	-58E1	-49E1	-44E1	-42E1	-42E1	-44E1	-49E1	-56E1	-69E1	-90E1
8	-93E2	-71E2	-66E2	-71E2	-91E2	-15E3	-71E3	27E3	12E3	87E2	75E2	74E2	83E2	11E3	24E3	-81E3	-15E3	-88E2	-69E2
7	-99E2	-73E2	-69E2	-79E2	-11E3	-27E3	78E3	18E3	11E3	95E2	85E2	81E2	85E2	10E3	17E3	28E4	-17E3	-94E2	-75E2
6	-12E3	-84E2	-78E2	-95E2	-16E3	-14E4	25E3	15E3	13E3	14E3	15E3	13E3	10E3	98E2	12E3	37E3	-21E3	-94E2	-78E2
5	-19E3	-11E3	-96E2	-11E3	-24E3	70E3	16E3	13E3	16E3	39E3	25E5	98E3	18E3	97E2	89E2	16E3	-32E3	-92E2	-79E2
4	-44E3	-16E3	-12E3	-14E3	-27E3	69E3	15E3	11E3	15E3	92E4	-16E3	-15E3	63E4	10E3	65E2	92E2	-88E3	-83E2	-73E2
3	-65E4	-37E3	-24E3	-18E3	-19E3	-49E3	21E3	93E2	95E2	56E3	-98E2	-65E2	-13E3	12E3	51E2	59E2	64E3	-71E2	-61E2
2	-23E4	90E3	53E3	-42E3	-10E3	-87E2	-46E3	78E2	48E2	89E2	-95E2	-38E2	-48E2	32E3	40E2	37E2	16E3	-57E2	-45E2
1	-10E3	17E3	56E2	13E3	-47E2	-25E2	-41E2	56E2	19E2	22E2	-30E3	-18E2	-17E2	-72E2	26E2	19E2	50E2	-37E2	-24E2
0	18E1	14E0	-13E1	-84E0	31E1	39E1	25E1	-71E0	-35E1	-36E1	-13E1	42E1	62E1	39E1	-18E1	-51E1	-37E1	58E1	47E1
-1	27E2	15E3	-37E2	-35E2	15E3	23E2	23E2	54E3	-20E2	-15E2	-39E2	31E2	16E2	25E2	-16E3	-27E2	-36E2	19E3	40E2
-2	41E2	10E3	-10E3	-67E2	-61E3	61E2	51E2	29E3	-52E2	-36E2	-74E2	85E2	37E2	47E2	43E3	-78E2	-78E2	-65E3	13E3
-3	53E2	87E2	-69E3	-12E3	-31E3	16E3	11E3	57E3	-10E3	-67E2	-12E3	17E3	65E2	73E2	26E3	-19E3	-14E3	-38E3	69E3
-4	71E2	86E2	26E3	-45E3	-40E3	13E4	47E3	-94E3	-15E3	-11E3	-20E3	43E3	12E3	12E3	26E3	-91E3	-33E3	-49E3	-77E3
-5	10E3	97E2	15E3	65E3	-85E3	-53E3	-41E3	-24E3	-15E3	-14E3	-22E3	-32E4	30E3	22E3	30E3	84E3	-37E4	-90E3	-39E3
-6	17E3	12E3	16E3	35E3	-13E4	-28E3	-17E3	-13E3	-11E3	-12E3	-16E3	-31E3	-55E4	55E3	40E3	47E3	89E3	-20E4	-33E3
-7	55E3	29E3	36E3	20E4	-37E3	-16E3	-11E3	-93E2	-86E2	-90E2	-10E3	-14E3	-25E3	-70E3	28E4	88E3	11E4	-23E4	-40E3
-8	-49E3	-30E3	-19E3	-13E3	-93E2	-72E2	-60E2	-54E2	-52E2	-53E2	-58E2	-67E2	-85E2	-11E3	-17E3	-27E3	-47E3	-86E3	-17E4
-9	-13E2	-77E1	-55E1	-42E1	-34E1	-28E1	-24E1	-20E1	-17E1	-15E1	-13E1	-11E1	-89E0	-73E0	-58E0	-44E0	-32E0	-24E0	38E1

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
9	-90E1	-13E2	-25E2	-17E3	38E2	18E2	12E2	92E1	77E1	67E1	61E1	57E1	54E1	53E1	54E1	55E1	58E1	64E1	73E1
8	-69E2	-63E2	-64E2	-73E2	-93E2	-14E3	-40E3	45E3	14E3	90E2	69E2	60E2	59E2	65E2	86E2	15E3	23E4	-17E3	-93E2
7	-75E2	-73E2	-82E2	-10E3	-12E3	-16E3	-22E3	-47E3	91E3	19E3	11E3	81E2	71E2	72E2	87E2	13E3	69E3	-21E3	-99E2
6	-78E2	-92E2	-14E3	-27E3	-33E3	-21E3	-14E3	-14E3	-21E3	-40E4	21E3	11E3	91E2	84E2	92E2	12E3	30E3	-39E3	-12E3
5	-79E2	-13E3	-86E4	22E3	37E3	-26E3	-98E2	-82E2	-11E3	-35E3	37E3	16E3	13E3	12E3	12E3	13E3	21E3	10E5	-19E3
4	-73E2	-20E3	13E3	77E2	13E3	-19E3	-63E2	-58E2	-99E2	-25E4	18E3	17E3	32E3	55E3	31E3	19E3	19E3	44E3	-44E3
3	-61E2	-33E3	68E2	47E2	97E2	-99E2	-41E2	-46E2	-15E3	10E3	68E2	11E3	-63E3	-15E3	-24E3	83E3	24E3	34E3	-65E4
2	-45E2	-31E3	45E2	34E2	10E3	-47E2	-26E2	-38E2	27E3	33E2	29E2	63E2	-14E3	-57E2	-74E2	-23E3	14E4	29E4	-23E4
1	-24E2	-11E3	27E2	22E2	21E3	-19E2	-13E2	-26E2	32E2	12E2	12E2	28E2	-62E2	-27E2	-36E2	-86E2	-11E3	-75E2	-10E3
0	47E1	16E1	-26E1	-54E1	27E1	36E1	41E1	17E1	-32E1	-79E1	-68E1	-30E1	-29E1	-66E1	12E1	-10E2	19E2	38E1	18E1
-1	40E2	73E2	-97E2	-60E2	14E3	26E2	23E2	77E2	-34E2	-18E2	-23E2	-90E2	16E3	-18E3	-42E2	-48E2	14E3	28E2	27E2
-2	13E3	20E3	-25E3	-12E3	-36E3	14E3	91E2	16E3	-66E3	-33E3	33E3	13E3	85E4	-59E2	-34E2	-42E2	-48E3	51E2	41E2
-3	69E3	10E5	-18E3	-10E3	-10E3	-24E3	51E3	14E3	86E2	60E2	51E2	71E2	-12E4	-51E2	-33E2	-39E2	-14E3	83E2	53E2
-4	-77E3	-29E3	-12E3	-82E2	-78E2	-11E3	-11E4	11E3	54E2	39E2	39E2	61E2	-43E5	-58E2	-37E2	-43E2	-10E3	16E3	71E2
-5	-39E3	-17E3	-10E3	-75E2	-74E2	-11E3	-31E4	92E2	47E2	37E2	39E2	62E2	62E3	-79E2	-48E2	-52E2	-10E3	47E3	10E3
-6	-33E3	-15E3	-10E3	-82E2	-89E2	-16E3	41E3	78E2	46E2	38E2	42E2	64E2	24E3	-14E3	-73E2	-73E2	-12E3	-24E4	17E3
-7	-40E3	-20E3	-15E3	-14E3	-20E3	-98E4	14E3	70E2	50E2	44E2	48E2	66E2	13E3	11E5	-20E3	-15E3	-21E3	-71E3	55E3
-8	-17E4	97E4	74E3	28E3	14E3	88E2	62E2	49E2	42E2	41E2	44E2	51E2	67E2	10E3	17E3	41E3	36E4	-99E3	-49E3
-9	38E1	31E0	41E0	55E0	72E0	91E0	11E1	14E1	17E1	20E1	24E1	30E1	38E1	51E1	73E1	12E2	37E2	-39E2	-13E2

scaling: × 10

τ at sea level ($\omega = 0$)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
9	19E1	19E1	19E1	19E1	20E1	20E1	21E1	22E1	24E1	25E1	27E1	29E1	31E1	34E1	37E1	42E1	48E1	57E1	74E1
8	59E2	46E2	42E2	44E2	52E2	73E2	13E3	12E4	-21E3	-11E3	-92E2	-89E2	-10E3	-15E3	-47E3	36E3	13E3	87E2	72E2
7	78E2	60E2	56E2	63E2	88E2	17E3	11E5	-23E3	-14E3	-11E3	-10E3	-97E2	-10E3	-12E3	-24E3	78E3	14E3	84E2	71E2
6	11E3	78E2	73E2	90E2	16E3	24E4	-25E3	-17E3	-18E3	-24E3	-27E3	-19E3	-13E3	-11E3	-14E3	-77E3	15E3	81E2	71E2
5	19E3	10E3	92E2	12E3	32E3	-36E3	-14E3	-13E3	-22E3	21E4	27E3	49E3	-23E3	-91E2	-81E2	-17E3	19E3	69E2	62E2
4	10E4	18E3	14E3	17E3	49E3	-33E3	-13E3	-11E3	-20E3	30E3	94E2	92E2	57E3	-87E2	-55E2	-85E2	30E3	60E2	55E2
3	-42E3	13E4	46E3	31E3	27E3	91E3	-18E3	-88E2	-10E3	84E3	66E2	48E2	95E2	-98E2	-40E2	-49E2	19E4	49E2	44E2
2	-32E3	-20E3	-15E3	-64E3	12E3	80E2	25E3	-71E2	-43E2	-94E2	69E2	31E2	40E2	-21E3	-30E2	-30E2	-21E3	38E2	32E2
1	43E3	-74E2	-38E2	-60E2	90E2	29E2	36E2	-83E2	-21E2	-25E2	25E3	24E2	22E2	11E3	-25E2	-19E2	-69E2	29E2	22E2
0	18E1	13E0	-13E1	-83E0	31E1	39E1	25E1	-71E0	-34E1	-36E1	-13E1	42E1	62E1	38E1	-18E1	-51E1	-37E1	59E1	47E1
-1	-25E2	61E3	24E2	22E2	37E3	-20E2	-17E2	-84E2	21E2	15E2	35E2	-32E2	-15E2	-24E2	82E2	24E2	38E2	-96E2	-30E2
-2	-32E2	-11E3	52E2	38E2	13E3	-51E2	-34E2	-10E3	49E2	31E2	60E2	-80E2	-32E2	-41E2	48E4	54E2	64E2	-23E4	-78E2
-3	-47E2	-95E2	16E3	72E2	13E3	-18E3	-82E2	-16E3	14E3	73E2	12E3	-20E3	-70E2	-80E2	-55E3	13E3	13E3	13E4	-19E3
-4	-57E2	-77E2	-60E3	16E3	18E3	-47E4	-24E3	-54E3	23E3	12E3	21E3	-41E3	-12E3	-12E3	-34E3	46E3	34E3	26E5	-45E3
-5	-78E2	-79E2	-15E3	-33E5	35E3	42E3	65E3	39E3	20E3	17E3	24E3	-29E6	-27E3	-21E3	-28E3	-56E3	-77E3	-58E3	-94E3
-6	-13E3	-11E3	-14E3	-35E3	13E4	33E3	23E3	18E3	16E3	17E3	22E3	51E3	-16E4	-38E3	-27E3	-24E3	-23E3	-27E3	-57E3
-7	-26E3	-18E3	-21E3	-46E3	74E3	21E3	13E3	11E3	10E3	11E3	14E3	23E3	79E3	-58E3	-24E3	-17E3	-16E3	-19E3	-35E3
-8	42E3	39E3	26E3	16E3	10E3	82E2	68E2	62E2	62E2	68E2	84E2	11E3	22E3	18E4	-34E3	-18E3	-14E3	-13E3	-15E3
-9	-59E1	-66E1	-75E1	-86E1	-99E1	-11E2	-12E2	-13E2	-14E2	-14E2	-14E2	-13E2	-13E2	-12E2	-12E2	-11E2	-11E2	-10E2	-97E1

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
9	74E1	12E2	38E2	-19E2	-61E1	-31E1	-18E1	-11E1	-67E0	-48E0	-49E0	-86E0	-27E1	14E2	33E1	24E1	20E1	19E1	19E1
8	72E2	70E2	79E2	10E3	21E3	-14E4	-15E3	-83E2	-58E2	-47E2	-42E2	-41E2	-45E2	-55E2	-83E2	-20E3	34E3	96E2	59E2
7	71E2	75E2	98E2	15E3	32E3	12E4	-11E4	-36E3	-18E3	-11E3	-81E2	-66E2	-62E2	-67E2	-86E2	-15E3	75E4	14E3	78E2
6	71E2	92E2	21E3	-22E4	-47E3	-19E4	43E3	26E3	36E3	-14E4	-20E3	-11E3	-91E2	-85E2	-92E2	-12E3	-33E3	32E3	11E3
5	62E2	11E3	-43E3	-10E3	-12E3	-10E4	14E3	92E2	11E3	27E3	-66E3	-22E3	-16E3	-13E3	-11E3	-11E3	-16E3	-11E4	19E3
4	55E2	17E3	-92E2	-52E2	-73E2	29E4	75E2	58E2	89E2	50E3	-33E3	-44E3	30E4	24E4	-33E3	-14E3	-12E3	-22E3	10E4
3	44E2	24E3	-49E2	-33E2	-55E2	16E3	40E2	40E2	11E3	-12E3	-89E2	-34E3	13E3	95E2	19E3	-29E3	-12E3	-15E3	-42E3
2	32E2	21E3	-32E2	-23E2	-50E2	47E2	21E2	29E2	-25E3	-32E2	-34E2	-14E3	61E2	41E2	64E2	66E3	-17E3	-20E3	-32E3
1	22E2	99E2	-25E2	-17E2	-56E2	20E2	13E2	25E2	-32E2	-14E2	-18E2	-11E3	29E2	23E2	42E2	32E3	-58E3	25E3	43E3
0	47E1	16E1	-26E1	-54E1	26E1	36E1	41E1	17E1	-32E1	-78E1	-68E1	-30E1	-28E1	-64E1	12E1	-10E2	19E2	38E1	18E1
-1	-30E2	-49E2	95E2	47E2	-15E4	-29E2	-22E2	-10E3	23E2	18E2	40E2	-99E2	-44E2	-12E3	55E2	42E2	-15E3	-24E2	-25E2
-2	-78E2	-98E2	38E3	94E2	17E3	-17E3	-92E2	-30E3	15E3	23E3	-12E3	-64E2	-12E3	81E2	33E2	36E2	69E3	-37E2	-32E2
-3	-19E3	-23E3	36E3	10E3	87E2	12E3	32E3	-42E3	-11E3	-61E2	-47E2	-56E2	-30E3	62E2	35E2	42E2	20E3	-67E2	-47E2
-4	-45E3	-59E4	16E3	77E2	61E2	72E2	15E3	-20E3	-59E2	-38E2	-35E2	-50E2	-73E3	54E2	35E2	42E2	13E3	-10E3	-57E2
-5	-94E3	41E3	12E3	70E2	60E2	72E2	17E3	-16E3	-54E2	-37E2	-36E2	-54E2	-84E3	64E2	42E2	49E2	12E3	-20E3	-78E2
-6	-57E3	52E3	15E3	95E2	85E2	10E3	29E3	-20E3	-76E2	-54E2	-54E2	-83E2	-88E3	10E3	70E2	79E2	16E3	-52E3	-13E3
-7	-35E3	19E4	25E3	15E3	15E3	23E3	-37E4	-16E3	-88E2	-69E2	-72E2	-10E3	-41E3	23E3	12E3	12E3	21E3	67E4	-26E3
-8	-15E3	-18E3	-23E3	-27E3	-24E3	-17E3	-12E3	-92E2	-77E2	-71E2	-76E2	-96E2	-15E3	-43E3	98E3	34E3	30E3	35E3	42E3
-9	-97E1	-91E1	-87E1	-82E1	-78E1	-73E1	-69E1	-66E1	-62E1	-59E1	-55E1	-54E1	-52E1	-50E1	-49E1	-50E1	-51E1	-54E1	-59E1

scaling: × 10

τ at 1,000 km elevation (ω = 0)

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-5	-5	-4	-3	-2	-1	0
9	30E1	33E1	37E1	40E1	44E1	48E1	51E1	55E1	60E1	64E1	69E1	73E1	78E1	84E1	90E1	97E1	11E2	12E2	13E2
8	14E3	11E3	98E2	10E3	11E3	16E3	32E3	64E4	-42E3	-23E3	-19E3	-19E3	-22E3	-35E3	-15E4	54E3	23E3	16E3	13E3
7	17E3	12E3	11E3	13E3	18E3	35E3	92E4	-47E3	-27E3	-22E3	-19E3	-18E3	-19E3	-25E3	-49E3	14E4	27E3	16E3	14E3
6	24E3	15E3	14E3	17E3	29E3	18E4	-54E3	-32E3	-32E3	-38E3	-41E3	-33E3	-24E3	-21E3	-28E3	-15E4	32E3	16E3	14E3
5	43E3	21E3	18E3	23E3	51E3	-10E4	-31E3	-26E3	-38E3	-31E4	81E3	16E4	-45E3	-19E3	-17E3	-35E3	41E3	14E3	13E3
4	36E4	40E3	29E3	32E3	69E3	-89E3	-27E3	-21E3	-34E3	93E3	21E3	19E3	12E4	-18E3	-11E3	-17E3	74E3	13E3	12E3
3	-72E3	85E5	10E4	56E3	46E3	12E4	-38E3	-17E3	-18E3	10E5	14E3	10E3	18E3	-22E3	-85E2	-10E3	-49E4	10E3	97E2
2	-52E3	-33E3	-29E3	-14E4	24E3	16E3	62E3	-14E3	-88E2	-18E3	14E3	63E2	79E2	-62E3	-65E2	-63E2	-34E3	86E2	71E2
1	11E4	-12E3	-70E2	-12E3	13E3	52E2	72E2	-12E3	-37E2	-46E2	32E3	38E2	35E2	15E3	-47E2	-34E2	-10E3	59E2	42E2
0	77E1	-23E2	-87E0	-13E1	32E2	79E1	37E1	-37E1	-59E1	-60E1	-54E1	93E1	15E2	15E2	-33E1	-15E2	-11E2	-29E1	45E2
-1	-52E2	14E4	48E2	48E2	-25E4	-40E2	-36E2	-29E3	38E2	29E2	72E2	-55E2	-28E2	-44E2	20E3	48E2	71E2	-20E3	-61E2
-2	-72E2	-25E3	12E3	90E2	39E3	-10E3	-79E2	-31E3	95E2	64E2	13E3	-14E3	-63E2	-80E2	-13E4	12E3	14E3	-93E4	-17E3
-3	-96E2	-18E3	44E3	17E3	35E3	-32E3	-17E3	-52E3	21E3	13E3	24E3	-32E3	-12E3	-13E3	-54E3	32E3	27E3	16E4	-51E3
-4	-12E3	-16E3	-72E3	46E3	49E3	-35E4	-62E3	-13E5	34E3	22E3	40E3	-67E3	-21E3	-21E3	-46E3	20E4	80E3	29E4	-36E4
-5	-18E3	-17E3	-30E3	-20E4	10E4	97E3	97E3	55E3	33E3	30E3	47E3	-54E4	-45E3	-34E3	-44E3	-84E3	-17E4	-35E4	19E4
-6	-30E3	-23E3	-29E3	-63E3	38E4	62E3	39E3	30E3	27E3	28E3	39E3	10E4	-17E4	-59E3	-46E3	-47E3	-58E3	-10E4	19E4
-7	-80E3	-45E3	-51E3	-12E4	11E4	39E3	25E3	20E3	19E3	21E3	27E3	44E3	14E4	-13E4	-57E3	-45E3	-48E3	-80E3	75E4
-8	77E3	63E3	43E3	29E3	20E3	15E3	13E3	12E3	12E3	13E3	15E3	20E3	31E3	69E3	88E5	-10E4	-68E3	-67E3	-85E3
-9	-10E2	-13E2	-17E2	-23E2	-34E2	-50E2	-70E2	-88E2	-93E2	-85E2	-72E2	-60E2	-50E2	-42E2	-36E2	-31E2	-27E2	-24E2	-22E2

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
9	13E2	15E2	17E2	23E2	35E2	11E3	-61E2	-20E2	-10E2	-57E1	-32E1	-17E1	-55E0	-31E0	16E1	18E1	22E1	26E1	33E1
8	13E3	13E3	14E3	17E3	28E3	92E3	-61E3	-22E3	-14E3	-10E3	-91E2	-85E2	-88E2	-10E3	-14E3	-26E3	-14E5	27E3	14E3
7	14E3	14E3	18E3	26E3	42E3	72E3	17E4	-22E4	-51E3	-25E3	-17E3	-13E3	-12E3	-12E3	-15E3	-25E3	-12E4	37E3	17E3
6	14E3	18E3	38E3	34E4	-31E4	15E4	47E3	36E3	51E3	-25E5	-40E3	-22E3	-17E3	-15E3	-16E3	-22E3	-50E3	90E3	24E3
5	13E3	25E3	-90E3	-23E3	-29E3	37E4	23E3	16E3	20E3	51E3	-12E4	-40E3	-31E3	-26E3	-23E3	-23E3	-32E3	-15E4	43E3
4	12E3	41E3	-18E3	-10E3	-15E3	10E4	13E3	10E3	17E3	10E4	-54E3	-62E3	-48E4	85E4	-78E3	-32E3	-27E3	-45E3	36E4
3	97E2	68E3	-10E3	-69E2	-12E3	26E3	77E2	79E2	23E3	-23E3	-15E3	-43E3	31E3	18E3	31E3	-12E4	-31E3	-33E3	-72E3
2	71E2	60E3	-67E2	-49E2	-11E3	94E2	45E2	62E2	-52E3	-63E2	-64E2	-21E3	13E3	82E2	11E3	43E3	-65E3	-50E3	-52E3
1	42E2	20E3	-46E2	-33E2	-13E3	36E2	23E2	46E2	-58E2	-24E2	-28E2	-10E3	62E2	41E2	62E2	18E3	52E3	30E3	11E4
0	45E2	53E1	-58E1	-13E2	20E1	52E1	66E1	45E1	-61E1	-19E2	-16E2	-53E1	-22E1	-19E1	10E1	-32E2	-34E2	20E2	77E1
-1	-61E2	-10E3	16E3	87E2	-26E4	-54E2	-41E2	-15E3	49E2	34E2	61E2	-43E3	-97E2	-29E3	11E3	95E2	-24E3	-48E2	-52E2
-2	-17E3	-24E3	48E3	17E3	31E3	-38E3	-18E3	-39E3	52E3	64E3	-29E3	-14E3	-31E3	15E3	69E2	78E2	10E4	-86E2	-72E2
-3	-51E3	-91E3	38E3	16E3	15E3	24E3	16E4	-37E3	-16E3	-10E3	-87E2	-11E3	-84E3	11E3	66E2	77E2	30E3	-14E3	-96E2
-4	-36E4	91E3	23E3	13E3	11E3	15E3	45E3	-27E3	-10E3	-70E2	-67E2	-10E3	-14E4	10E3	69E2	81E2	21E3	-26E3	-12E3
-5	19E4	40E3	18E3	12E3	11E3	15E3	52E3	-22E3	-94E2	-68E2	-68E2	-10E3	-96E3	13E3	85E2	96E2	20E3	-64E3	-18E3
-6	19E4	38E3	20E3	15E3	14E3	21E3	12E4	-23E3	-11E3	-83E2	-86E2	-12E3	-61E3	23E3	13E3	14E3	25E3	-67E4	-30E3
-7	75E4	55E3	30E3	24E3	27E3	53E3	-91E3	-21E3	-13E3	-10E3	-11E3	-15E3	-37E3	89E3	28E3	25E3	36E3	15E4	-80E3
-8	-85E3	-12E4	-17E4	-11E4	-58E3	-31E3	-19E3	-14E3	-11E3	-10E3	-11E3	-13E3	-17E3	-31E3	-88E3	34E4	10E4	82E3	77E3
-9	-22E2	-20E2	-18E2	-16E2	-15E2	-14E2	-13E2	-12E2	-11E2	-10E2	-95E1	-89E1	-84E1	-81E1	-79E1	-79E1	-82E1	-90E1	-10E2

scaling: x 10

τ at 10,000 km elevation (ω = 0)

omitted, because some of the results were already affected by rounding errors of the computer.

scaling:

τ at 100,000 km elevation ($\omega = 0$)

omitted, because some of the results were already affected by rounding errors of the computer.

scaling:

Number Map 5.--Orthogonal trajectories, coefficients; geocentric radius. See p. 59 for explanation of first column.

r_q

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9
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	14	14	14	14	14	14	14	14	14	14
3	11406743	11444459	11474399	11486612	11482428	11470016	11456788	11445683	11437006	11431598
	-1835	-1917	-1975	-1986	-1958	-1916	-1879	-1854	-1839	-1834
	12	12	12	12	12	12	12	12	12	12
2	8474976	8510907	8537100	8548211	8547959	8542996	8536521	8527738	8516457	8506560
	-1350	-1433	-1487	-1499	-1484	-1462	-1442	-1419	-1392	-1371
	9	9	9	9	9	9	9	9	9	9
1	4527450	4540256	4547439	4552368	4559974	4571135	4580811	4582270	4573835	4561594
	-777	-800	-823	-828	-840	-862	-881	-880	-854	-820
	5	5	5	5	5	5	5	5	5	5
0	9026	-14132	-34711	-40518	-27341	-1365	24706	39522	39651	30629
	-2	50	96	107	73	9	-52	-81	-71	-40
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	783	759	739	724	715	712	713	716	718	720
	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
-9	-62122	-53303	-44620	-36339	-28714	-21976	-16330	-11945	-8953	-7439
	12	-2	-15	-29	-40	-51	-59	-65	-69	-71

Coefficients, geocentric longitude

$$\lambda_q$$

	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9
9	-4753975 [illegible] [illegible]	-5137441 7415 [illegible]	-7290110 8717 -9	-9263310 9358 -8	-8997817 9749 -7	-9469767 9964 -6	-9661746 9694 -5	-9564708 9235 -4	-9176684 8493 -3	-8505371 7482 -2
8	33797 -114	49133 -159 1	48365 -171 1	32542 -147 1	2201 -92	-38825 -13	-85151 78	-130820 168	-170189 246	-198539 303 -1
7	59097 -147	82531 -193 1	88307 -204 1	75255 -178 1	45957 -117	6138 -36	-37047 49	-77049 124	-109463 183	-132411 222
6	58930 -115	83491 -170	93456 -185 1	77852 -158	48615 -95	9882 -16	-29217 59	-60736 112	-80938 138	-91432 146
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4	41186 -52	59755 -92	67737 -110	62410 -101	43059 -64	12034 -4	-22779 60	-48506 101	-54251 94	-40411 43
3	35495 -35	46823 -56	52960 -69	53603 -76	45121 -68	22525 -31	-12052 33	-44238 94	-54907 104	-36803 46
2	35881 -35	37770 -32	38939 -31	44380 -50	48473 -77	37102 -71	3451 -8	-38707 80	-61262 126	-45804 80
1	42198 -55	34379 -23	27692	34645 -23	49715 -79	50397 -107	19777 -53	-31426 61	-68085 147	-60269 126
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-2	66662 -139	53033 -83	28340	18970 24	32078 -29	45297 -94	30910 -88	-14033 8	-60540 123	-71722 157
-3	64652 -145	51717 -111	39113 -35	23636 8	25346 -13	30329 -56	17494 -54	-17338 14	-53747 101	-63448 130
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-5	37770 -97	55990 -123	50378 -93	33289 -45	14974 -8	-2525 16	-22029 42	-43367 76	-59649 107	-61680 111
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-8	-8171 24	-36138 71	-66160 119	-99252 176	-134735 239 -1	-170358 302 -1	-203056 359 -1	-229744 402 -1	-248018 426 -1	-256646 430 -1
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-8	-256646 430 -1	-255753 415 -1	-246671 385 -1	-231487 344 -1	-212405 300 -1	-191121 256	-168365 217	-143715 181	-115718 146	-82264 107
-9	-22956055 36925 -74	-22474510 36542 -72	-22238635 35055 -67	-20947965 32511 -61	-18962215 28987 -53	-16400218 24588 -43	-13338320 19446 -32	-9868232 13714 -20	-6094406 7565 -7	-2130982 1182 6

λ_q

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-5	169143 -366 1	142539 -328 1	81022 -200 1	183 -16	-73616 158	-114086 255 -1	-108385 240 -1	-64589 134	-7089 -1	37770 -97
-6	203233 -411 1	177050 -370 1	118195 -249 1	42268 -82	-28276 76	-72765 173 -1	-81017 185 -1	-57525 123	-18349 27	17536 -53
-7	239871 -442 1	218007 -407 1	170031 -313 1	107658 -184	46229 -57	-411 35	-24617 76	-27110 69	-15664 32	-1129 -8
-8	360462 -603 1	345975 -579 1	310035 -514 1	259348 -420 1	201929 -315 1	145507 -214	95563 -129	54306 -63	20762 -13	-8171 28
-9	23053422 -37055 74	23058911 -36720 72	22359989 -35226 68	20078684 -32673 62	18958693 -29119 53	16362881 -24675 44	13271672 -19477 32	9780312 -13686 20	5995818 -7481 7	2033617 -1052 -6

SUMMARY (Appendices A and B)

Changes of the Correction Terms with Height

correction terms \ elevation above sea level	0 km		1,000 km		10,000 km		100,000 km	
Equipotential surfaces								
$\Delta r(m)$	60	-70	47	-51	16	-14	1.8	-1.8
$\Delta g(mgal)$	22	-26	8	-10	0.18	-0.17	0.000066	-0.000062
ξ''	5.7	-5.4	3.1	-2.9	0.25	-0.29	0.0039	-0.0038
η''	5.7	-6.8	2.9	-3.9	0.34	-0.42	0.0070	-0.0074
$\Delta\frac{1}{\sqrt{K}}, \Delta\frac{1}{H}(m)$	567	-470	330	-270	43	-45	3.8	-4.1
Equigravitational surfaces								
$\Delta r(m)$	116	-158	85	-104	26	-24	2.9	-2.7
ξ''	15	-15	7.8	-7.7	0.43	-0.52	0.0061	-0.0060
η''	16	-17	8.0	-9.1	0.54	-0.72	0.011	-0.011
$\Delta\mu''$ $(\omega = 0)$	9.8	-9.6	4.9	-4.7	0.23	-0.20	0.0022	-0.0019
Orthogonal trajectories								
$\Delta\delta''$	5.5	-5.8	3.0	-3.1	0.29	-0.23	0.0039	-0.0037

APPENDIX C. Constants and Coefficients Used in the Numerical Computations

Constants and coefficients used in the numerical computations

$a = 6{,}378{,}165$ m equatorial radius

$GM = 3.986\ 032 \times 10^{20}$ cm^3 sec^{-2} gravitational constant $\times$ mass of the Ear

$\omega^2 = 5.317\ 49 \times 10^{-9}$ rad sec^{-2} (gravity field)

and/or

$\omega^2 = 0$ (gravitational field)

Zonal harmonic coefficients (Kozai, 1964)

$C_{20} = -1{,}082.645 \times 10^{-6}$	$C_{30} = 2.546 \times 10^{-6}$
$C_{40} = 1.649 \times 10^{-6}$	$C_{50} = 0.210 \times 10^{-6}$
$C_{60} = -0.646 \times 10^{-6}$	$C_{70} = 0.333 \times 10^{-6}$
$C_{80} = 0.270 \times 10^{-6}$	$C_{90} = 0.053 \times 10^{-6}$
$C_{10\,0} = 0.054 \times 10^{-6}$	$C_{11\,0} = -0.302 \times 10^{-6}$
$C_{12\,0} = 0.357 \times 10^{-6}$	$C_{13\,0} = 0.114 \times 10^{-6}$
$C_{14\,0} = -0.179 \times 10^{-6}$	

with $C_{n0} = -J_n$

Conventional tesseral (sectorial) harmonic coefficients (Izsak, 1965)[8]

$C_{22} = 1.3446 \times 10^{-6}$	$S_{22} = -8.0817 \times 10^{-7}$
$C_{31} = 1.7304 \times 10^{-6}$	$S_{31} = -0.4158 \times 10^{-7}$
$C_{32} = 0.1298 \times 10^{-6}$	$S_{32} = -0.2749 \times 10^{-6}$
$C_{33} = -0.2416 \times 10^{-7}$	$S_{33} = 0.1959 \times 10^{-6}$
$C_{41} = -0.3617 \times 10^{-6}$	$S_{41} = -0.3831 \times 10^{-6}$
$C_{42} = 0.4387 \times 10^{-7}$	$S_{42} = 0.1292 \times 10^{-6}$
$C_{43} = 0.4103 \times 10^{-7}$	$S_{43} = -0.6018 \times 10^{-8}$
$C_{44} = -0.2276 \times 10^{-8}$	$S_{44} = 0.9117 \times 10^{-8}$

[8] As obtained from a computer tape; the last two digits of a coefficient are, however, guarding figures.

$C_{51} = -\ 0.1177 \times 10^{-6}$	$S_{51} = -\ 0.3355 \times 10^{-7}$
$C_{52} = 0.3826 \times 10^{-7}$	$S_{52} = -\ 0.4381 \times 10^{-7}$
$C_{53} = -\ 0.2204 \times 10^{-7}$	$S_{53} = 0.1538 \times 10^{-8}$
$C_{54} = -\ 0.1036 \times 10^{-8}$	$S_{54} = 0.1282 \times 10^{-8}$
$C_{55} = 0.1907 \times 10^{-9}$	$S_{55} = -\ 0.1022 \times 10^{-8}$
$C_{61} = -\ 0.1734 \times 10^{-7}$	$S_{61} = 0.9182 \times 10^{-7}$
$C_{62} = 0.6073 \times 10^{-8}$	$S_{62} = -\ 0.2857 \times 10^{-7}$
$C_{63} = 0.1102 \times 10^{-8}$	$S_{63} = 0.5474 \times 10^{-12}$
$C_{64} = 0.2782 \times 10^{-9}$	$S_{64} = -\ 0.1459 \times 10^{-8}$
$C_{65} = 0.3098 \times 10^{-9}$	$S_{65} = -\ 0.3098 \times 10^{-9}$
$C_{66} = -\ 0.2696 \times 10^{-10}$	$S_{66} = -\ 0.1373 \times 10^{-9}$

The C_{21} and S_{21} coefficients were assumed to be zero.

Coordinate system

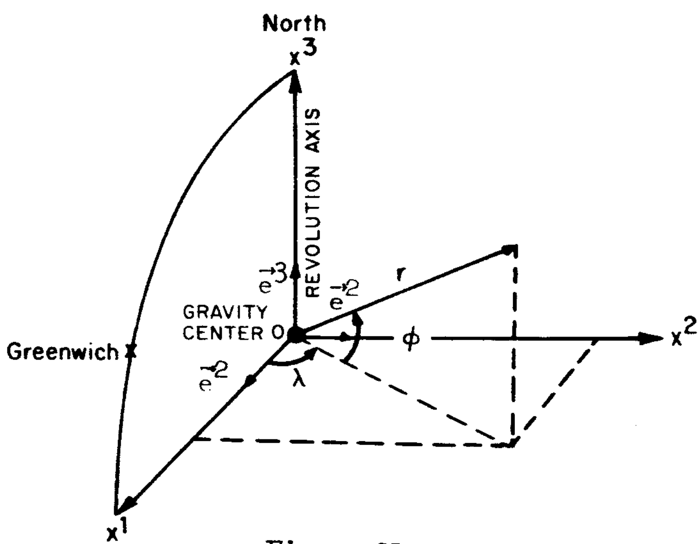

Figure 21.

Origin O is in the gravity center of the Earth ($C_{10}=C_{11}=S_{11} = 0$)

$\varphi \gtrless 0$ Northern / Southern } Hemisphere

$\lambda \gtrless 0$ eastward / westward

$\vec{e}^1$, $\vec{e}^2$, $\vec{e}^3$, unit vectors

NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

First issued to ensure the immediate dissemination of data for satellite tracking, the Reports have continued to provide a rapid distribution of catalogs of satellite observations, orbital information, and preliminary results of data analyses prior to formal publication in the appropriate journals. The Reports are also used extensively for the rapid publication of preliminary or special results in other fields of astrophysics.

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